Asynchronous Sampled-Data State Estimation for a Class of Nonlinear Complex Networks: A Matrix-Exponential-Gain-Based Approach

Luyang Yu, Zidong Wang, Yurong Liu, and Wenbing Zhang

***Abstract*—This paper is concerned with the asynchronous sampled-data state estimation problems for a class of continuous-time nonlinear complex networks. A novel asynchronous sampled-data estimator is constructed with matrix exponential gains to estimate the states of network nodes, which allows each node to independently sample and transmit the measured signals at its own designated time instants. It is demonstrated that the utilization of matrix exponential gains is capable of enlarging the maximum-allowable bound of the sampling intervals. Moreover, a modified Halanay-type inequality is derived to facilitate the analysis of estimation errors. Accordingly, by leveraging the Lyapunov stability theory, some sufficient conditions are obtained to guarantee the global exponential stability of the estimation error dynamics. In addition, the maximum-allowable bound of the sampling intervals is explicitly characterized by resorting to an algebraic inequality, and a convex optimization method is adopted with the aim of maximizing such an allowable bound. Finally, some numerical simulations are conducted to validate the feasibility and usefulness of the established theoretical results.**

***Index Terms*—State estimation, nonlinear complex network, asynchronous sampled-data estimator, matrix exponential gains, convex optimization.**

I. Introduction

In an era marked by the rapid scientific and technological progress, complex networks (CNs) are ubiquitous across a wide range of natural and social systems including, but not restricted to, the World-Wide Web, social networks, ecosystems, power grids, and transportation networks [1]–[6]. For example, the 6G networks and power grids are usually modeled as large-scale interconnected CNs [7], [8]. Typically speaking, a CN refers to a large-scale system composed of a substantial quantity of nodes and edges, where nodes stand for the agents within the network and edges signify the connections or interactions among these nodes. Owing to the characteristics of strong nonlinearity and highly organized structure, the CNs are capable of exhibiting diverse dynamical behaviors, such as consensus, synchronization and flocking. Exploring CNs in depth can not only enhance the understanding of complicated natural and social phenomena, but also provide new perspectives and methods for tackling various practical challenges. Consequently, considerable research attention has been centered on the stability, robustness, and pinning control problems of miscellaneous CNs, leading to a large body of elegant results available in the literature [9]–[15].

It has been widely recognized that the knowledge of the true system states plays a pivotal role in practical applications since it can reveal both the internal characteristics and the current operating conditions. Nevertheless, when it comes to the CNs, it is usually a formidable (if not impossible) task to access the accurate and complete state information due mainly to the inherent complexities including large scale, high dimensionality, and intricate interconnection topologies. As such, a technically feasible and economically efficient approach is to deploy some suitable sensing units and infer the network state information based on the obtained measurement outputs. In this context, the state estimation problems for CNs have garnered extensive research attention and gradually become a critical issue in the fields of signal processing and control. A wealth of significant results regarding this topic can be found in the literature, see, e.g., [16]–[20] and the references therein.

With the widespread implementation of digital hardware and the rapid advancement in communication networks, the past several decades have witnessed the growing popularity of sampled-data controllers or estimators, which aim to realize various control or state estimation objectives in a cost-effective manner [21]–[23]. In essence, the sampled-data control method is a time-driven control strategy, where the data packets are first sampled at specific time instants and then transmitted among the spatially dispersed system components over a communication network. Obviously, when designing the sampled-data controllers, a larger sampling interval is usually preferred since it is able to lower the frequency of data transmission/processing and alleviate the operational challenges such as computational burden, load limitations, and communication capacity constraints.

It should be pointed out that a persistent issue within the domain of sampled-data control is how to maximize the maximum-allowable bound (MAB) of sampling intervals while guaranteeing certain desired control objectives [24]–[26]. Recently, it has been demonstrated in [25] that the introduction of time-varying gains in a particular kind of observers with measurement delay can improve the exponential

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convergence properties. Since then, employing sampled-data controllers with time-varying gains to stabilize the controlled systems has received some initial attention, see, e.g., [27], [28]. For example, in [27], the exponential time-varying gain was introduced into the sampled-data state observer and controller, and stability criteria were derived via the input-delay approach. Nonetheless, the advantage of adopting such exponential time-varying gains in enlarging the allowable sampling intervals was only verified through numerical examples, without any rigorous theoretical proof. In the context of sampled-data state estimator design with time-varying gains for nonlinear CNs, the existing results remain rather limited. Moreover, there remains an open question regarding how to rigorously prove that introducing time-varying gains for the sampled-data state estimator of nonlinear CNs can maximize the MAB of sampling intervals. This constitutes the first motivation of the present exploration.

On the other hand, it is worth noting that nearly all existing research on sampled-data state estimation for CNs is based on the assumption that the nodes share synchronized clocks and handle the data in a periodic and synchronous manner. Nevertheless, in a practical networked environment, it might be quite challenging to ensure precise clock synchronization and accurate timing alignment across all the nodes due to the limited communication bandwidth and the unavoidable external disturbances [29]–[31]. Furthermore, different nodes are usually equipped with distinct sensors, each possessing unique physical characteristics. In this regard, it would be difficult or even impossible to unify the sampling intervals for each sensor [32]–[34]. Consequently, a preferred scheme is to set different sampling periods for each node based on the importance of the signals and the physical features of the sensors.

Recently, for the purpose of dealing with the real-world engineering challenges, researchers have shown increasing interest in the asynchronous/multi-rate sampled-data control/estimation problem for a diversity of nonlinear systems, see, e.g., [35]–[40]. In particular, the input delay approach has been widely adopted to address the challenges posed by asynchronous sampled-data (ASD) settings, due to its flexibility in accommodating diverse sampling scenarios [36], [38], [41]. However, this approach requires solving a set of linear matrix inequalities (LMIs) that involve a large number of decision variables to reduce conservatism. This often leads to a significant computational burden, especially for large-scale CNs. Consequently, developing novel methods for ASD state estimation of nonlinear CNs is essential, which forms a key motivation for this study.

Inspired by the above observations, in this paper, we endeavor to delve deeper into the sampled-data state estimation problem for a class of continuous-time nonlinear CNs. The main contributions of this paper can be outlined as follows.

1) A novel matrix-exponential-gain-based ASD state estimator is designed for a class of nonlinear CNs, where each node has a different sampling period. It is revealed that the introduction of the matrix exponential gains in designing sampled-data estimator is capable of guaranteeing a larger MAB of the sampling intervals.
2) A modified Halanay-like inequality is established to facilitate the analysis of estimation errors. Moreover, by resorting to the Lyapunov theory and the matrix inequality techniques, some easy-to-check conditions are derived to ensure that the estimation error dynamics is exponentially stable.
3) An optimization problem is formulated to design the estimator gain, which is able to maximize the MAB of the sampling intervals.

Notations: The notation used throughout this paper is fairly standard. $\mathbb{R}^p$ refers to the p-dimensional Euclidean space, and $\mathbb{R}^{p\times q}$ denotes the set of all $p\times q$ real matrices. Ξ^T and Ξ^{-1} represent, respectively, the transpose and the inverse of the matrix Ξ. For a symmetric matrix Z (i.e., $Z = Z^T$), its smallest and largest eigenvalues are, respectively, denoted by $\lambda_m(Z)$ and $\lambda_M(Z)$. For a vector $y\in\mathbb{R}^p$, its norm, indicated by $\|y\|$, is defined as the square root of y^Ty. Similarly, for a matrix Z, its norm, expressed as $\|Z\|$, is the square root of the largest eigenvalue of Z^TZ. The notation $Z_1 > Z_2$ (respectively, $Z_1 \geq Z_2$) signifies that $\lambda_m(Z_1-Z_2) > 0$ (respectively, $\lambda_m(Z_1-Z_2)\geq 0$). $\mathbb{I}_p^q \triangleq \{p, p+1, p+2, \ldots, q\}$, where p and q are integers satisfying $q > p \geq 0$.

II. Problem Formulation

Consider a nonlinear CN composed of N coupled nonidentical nodes with the following dynamics:

$$\begin{cases} \dot{s}_n(z) = C_n s_n(z) + h(s_n(z)) \\ \qquad\qquad + \sum_{m=1}^{N} a_{nm}\Upsilon(s_m(z)-s_n(z)), \\ y_n(z) = F_n s_n(z), \end{cases} \tag{1}$$

where $s_n(z)\in\mathbb{R}^p$ and $y_n(z)\in\mathbb{R}^q$ represent, respectively, the state vector and the measurement output of the n-th network node, $h:\mathbb{R}^p\to\mathbb{R}^p$ denotes a known nonlinear vector-valued function, $C_n\in\mathbb{R}^{p\times p}$ and $F_n\in\mathbb{R}^{q\times p}$ are known constant matrices, $A=(a_{nm})_{N\times N}\in\mathbb{R}^{N\times N}$ is the weighted adjacency matrix satisfying $a_{nm} > 0$ if the n-th node can receive information from the m-th node and $a_{nm}=0$ otherwise, and Υ is an inner coupling matrix of the network.

To estimate the state of CN (1), an ASD state estimator with matrix exponential gains is constructed as follows:

$$\begin{aligned} \dot{\hat{s}}_n(z) = {} & C_n\hat{s}_n(z) + h(\hat{s}_n(z)) + \sum_{m=1}^{N} a_{nm}\Upsilon(\hat{s}_m(z)-\hat{s}_n(z)) \\ & + e^{L_nF_n(z-z_r^n)}L_n(y_n(z_r^n)-F_n\hat{s}_n(z_r^n)), \\ & \qquad z\in[z_r^n, z_{r+1}^n), \end{aligned} \tag{2}$$

where $\hat{s}_n(z)\in\mathbb{R}^p$ is the estimate of $s_n(z)$, $L_n\in\mathbb{R}^{p\times q}$ denotes the estimator gain matrix to be designed later. $\{z_r^n\}_{r=0}^{+\infty}$ indicates a set of sampling instants for the n-th node, where $z_0^n = 0$, $z_{r+1}^n - z_r^n = Z_n$, and Z_n is a positive constant scalar representing the sampling period of the n-th node. Let $Z_M \triangleq \max_n\{Z_n\}$ be the MAB of the sampling intervals. Throughout this paper, suppose that there exist positive integers T_n and a positive number d such that $Z_n = T_n d$, $n\in\mathbb{I}_1^N$.

Remark 1: It should be pointed out that the matrix exponential gains are, for the first time, incorporated into the design of state estimator for the nonlinear CN. It will be demonstrated that the introduction of matrix exponential gains can effectively enlarge the MAB of the sampling intervals (see Theorem 1 and Corollary 1 for more details), which indicates that such an approach is highly beneficial to the sampled-data control systems in terms of reducing both communication frequency and computational burden.

Let $\xi_n(z) = s_n(z) - \hat{s}_n(z)$ be the state estimation error and $B = (b_{nm})_{N\times N}$ be the Laplacian matrix of the nonlinear CN (1) with $b_{nm} = -a_{nm}$ (if $n \neq m$) and $b_{nn} = \sum_{m\neq n} a_{nm}$. Then, according to (1) and (2), one has

$$\begin{aligned}\dot{\xi}_n(z) = C_n\xi_n(z) + \bar{h}(\xi_n(z)) - \sum_{m=1}^{N} b_{nm}\Upsilon\xi_m(z)\\ + e^{L_nF_n(z-z_r^n)}L_nF_n\xi_n(z_r^n),\ z\in[z_r^n, z_{r+1}^n),\end{aligned} \tag{3}$$

where $\bar{h}(\xi_n(z)) = h(s_n(z)) - h(\hat{s}_n(z))$. For the considered nonlinear CN (1) and the state estimator (2), let us denote $\xi(z) = (\xi_1^T(z), \xi_2^T(z), \ldots, \xi_N^T(z))^T$.

Definition 1: The ASD state estimator (2) with matrix exponential gains is termed as an exponential state estimator for the nonlinear CN (1) if there exist two positive real numbers $\mathfrak{M}$ and $\mathfrak{N}$ such that

$$\|\xi(z)\| \leq \mathfrak{M}e^{-\mathfrak{N}z},\ z\geq 0. \tag{4}$$

The main purpose of this paper is to design a novel ASD state estimator (2) with matrix exponential gains for the nonlinear CN (1) and derive some easy-to-check conditions to ensure that the estimation error dynamics (3) is exponentially stable.

III. Main Results

In this section, we shall deal with the state estimation problem for the nonlinear CN (1) by designing an ASD estimator (2) with matrix exponential gains. Before proceeding further, we introduce the following lemmas and an assumption.

Lemma 1: Suppose that $x : [a, b) \to [0, +\infty)$ is a continuously differentiable function. If there exist constants $\alpha > \beta_i > 0$ such that

$$\dot{x}(z) \leq -\alpha x(z) + \beta_i \max_{\varrho\in[a,z)} x(\varrho),\ z\in[a_{i-1}, a_i), \tag{5}$$

then, one has

$$x(z) \leq e^{-\lambda(z-a)}x(a),\ z\in[a, b), \tag{6}$$

where $a = a_0 < a_1 < a_2 < \cdots < a_M = b$, and λ is the real positive root to the equation $\lambda + \bar{\beta}e^{\lambda(b-a)} = \alpha$ with $\bar{\beta} = \max_i\{\beta_i\}$.

Proof: To prove (6), it suffices to show that, for any $\mathfrak{h} > 0$, the following inequality holds

$$x(z) < (x(a) + \mathfrak{h})e^{-\lambda(z-a)}, z\in[a, b). \tag{7}$$

In fact, if the inequality (7) is not true, then there must exist some $z \in (a, b)$ such that $x(z) \geq (x(a)+\mathfrak{h})e^{-\lambda(z-a)}$. Denote

$$z^* = \inf\left\{z\in(a, b) \mid x(z) = (x(a) + \mathfrak{h})e^{-\lambda(z-a)}\right\}.$$

Then, we have

$$x(z) < (x(a) + \mathfrak{h})e^{-\lambda(z-a)},\ z\in[a_0, z^*), \tag{8}$$

$$x(z^*) = (x(a) + \mathfrak{h})e^{-\lambda(z^*-a)}, \tag{9}$$

$$\dot{x}(z^*) \geq -\lambda(x(a) + \mathfrak{h})e^{-\lambda(z^*-a)}. \tag{10}$$

Suppose that $z^* \in [a_{i-1}, a_i)$. Based on (5), one has

$$\begin{aligned}\dot{x}(z^*) \leq&\ -\alpha x(z^*) + \beta_i \max_{\varrho\in[a_0,z^*)} x(\varrho)\\ <&\ -\alpha(x(a) + \mathfrak{h})e^{-\lambda(z^*-a)} + \beta_i(x(a) + \mathfrak{h})\\ \leq&\ (-\alpha + \bar{\beta}e^{\lambda(b-a)})(x(a) + \mathfrak{h})e^{-\lambda(z^*-a)}\\ =&\ -\lambda(x(a) + \mathfrak{h})e^{-\lambda(z^*-a)},\end{aligned} \tag{11}$$

which indicates a contradiction with (10), thereby affirming the validity of (7). Then, letting $\mathfrak{h} \to 0^+$, it follows from (7) that

$$x(z) \leq e^{-\lambda(z-a)}x(a),\ z\in[a, b). \tag{12}$$

The proof is now complete. ■

Lemma 2 ([42]): For any given symmetric matrix $\Upsilon \in \mathbb{R}^{p\times p}$ and positive definite matrix $\Omega \in \mathbb{R}^{p\times p}$, one has

$$x^T\Upsilon x \leq \lambda_M(\Omega^{-1}\Upsilon)x^T\Omega x,\ x\in\mathbb{R}^p. \tag{13}$$

Assumption 1: The nonlinear vector-valued function $h(\cdot)$ is continuous and satisfies the following condition

$$\begin{aligned}(h(\nu_1) - h(\nu_2) - P_1(\nu_1 - \nu_2))^T\\ \times (h(\nu_1) - h(\nu_2) - P_2(\nu_1 - \nu_2)) \leq 0,\end{aligned} \tag{14}$$

for arbitrary ν_1 and $\nu_2 \in \mathbb{R}^p$, where P_1 and P_2 are known constant matrices.

For notational convenience, let us denote

$$\begin{aligned}C &= \mathrm{diag}\{C_1, C_2, \ldots, C_N\},\\ F &= \mathrm{diag}\{F_1, F_2, \ldots, F_N\},\\ L &= \mathrm{diag}\{L_1, L_2, \ldots, L_N\}.\end{aligned}$$

The following theorem provides a sufficient condition to guarantee that the proposed ASD estimator (2) with matrix exponential gains is an exponential state estimator for the nonlinear CN (1).

Theorem 1: Let the matrix L and the positive constant α be given. Under Assumption 1, the ASD state estimator (2) with matrix exponential gains is an exponential state estimator for the nonlinear CN (1) if there exists a block diagonal positive definite matrix $U = \mathrm{diag}\{U_1, U_2, \ldots, U_N\}$, a diagonal positive definite matrix $\Phi = \mathrm{diag}\{\phi_1, \phi_2, \ldots, \phi_N\}$, and a positive constant δ such that the following inequality holds

$$\mathscr{D} = \begin{bmatrix}\mathscr{D}_0 & U + \Phi\otimes\check{P}_2 & ULF\\ * & -\Phi\otimes I & 0\\ * & * & -\delta I\end{bmatrix} < 0, \tag{15}$$

and the MAB of the sampling interval with respect to each node satisfies

$$Z_M < \frac{1}{\mu Q}\sqrt{\frac{\alpha}{\delta N\lambda_M(U^{-1})}}, \tag{16}$$

where

$$\mathscr{D}_0 = \alpha U - \Phi\otimes\check{P}_1 + U(C + LF - B\otimes\Upsilon)$$

$$
\begin{aligned}
&\quad + (C + LF - B \otimes \Upsilon)^T U, \\
&\check{P}_1 = (P_1^T P_2 + P_2^T P_1)/2, \check{P}_2 = (P_1^T + P_2^T)/2, \\
&\kappa = (\|P_1 + P_2\| + \|P_1 - P_2\|)/2, \\
&Q = X/\bar{T},\ \bar{T} = \max_n \{T_n\}, \\
&\mu = \max_n \big\{ \max\{\max_{m\neq n}\{\|b_{nm}\Upsilon\|\}, \|C_n - b_{nn}\Upsilon\|\}\big\} + \kappa,
\end{aligned}
$$

and X is the least common multiple of $T_1, T_2, \ldots, T_N$.

The proof of Theorem 1 is presented in the Appendix.

Remark 2: In Theorem 1, we have made great efforts to investigate the ASD state estimation problems for the nonlinear CNs. With the assistance of the modified Halanay-like inequality in Lemma 1, a set of sufficient conditions has been derived to guarantee the existence of the desired ASD state estimator (2) with matrix exponential gains. Different from the commonly used input delay method for handling ASD control systems, the newly proposed approach based on the modified Halanay-like inequality will introduce fewer decision variables into the sufficient conditions, thereby significantly lowering the computational burden and making it easier to verify the established conditions.

In what follows, we are going to reveal that the utilization of matrix exponential gains is able to enlarge the MAB of the sampling intervals. To this end, we construct the following ASD state estimator without matrix exponential gains

$$
\begin{aligned}
\dot{\hat{s}}_n(z) = C_n \hat{s}_n(z) + h(\hat{s}_n(z)) + \sum_{m=1}^{N} a_{nm}\Upsilon(\hat{s}_m(z) - \hat{s}_n(z)) \\
+ L_n(y_n(z_r^n) - F_n \hat{s}_n(z_r^n)), z \in [z_r^n, z_{r+1}^n). \quad (17)
\end{aligned}
$$

Then, the estimation error $\xi_n(z)$ satisfies the following dynamics:

$$
\begin{aligned}
\dot{\xi}_n(z) = C_n \xi_n(z) + \bar{h}(\xi_n(z)) - \sum_{m=1}^{N} b_{nm}\Upsilon \xi_m(z) \\
+ L_n F_n \xi_n(z_r^n),\ z \in [z_r^n, z_{r+1}^n). \quad (18)
\end{aligned}
$$

Corollary 1: Let the matrix L and the positive constant α be given. Under Assumption 1, the ASD state estimator (17) without matrix exponential gains is an exponential state estimator for the nonlinear CN (1) if there exists a block diagonal positive definite matrix $U = \mathrm{diag}\{U_1, U_2, \ldots, U_N\}$, a diagonal positive definite matrix $\Phi = \mathrm{diag}\{\phi_1, \phi_2, \ldots, \phi_N\}$, and a positive constant δ such that the inequality (15) holds, and the MAB of the sampling interval with respect to each node satisfies

$$
\bar{Z}_M < \frac{1}{\tilde{\mu} Q}\sqrt{\frac{\alpha}{\delta N \lambda_M(U^{-1})}}, \quad (19)
$$

where $\tilde{\mu} = \max_n \big\{ \max\{\max_{m\neq n}\{\|b_{nm}\Upsilon\|\}, \|C_n - b_{nn}\Upsilon\|\} + \|L_n F_n\|\big\} + \kappa$, and the remaining parameters are defined as before.

Proof: To begin with, select the same Lyapunov function as in (41). Let

$$
\bar{\varepsilon}_{n,j_n}(z) = \xi_n(z) - \xi_n(iXd + j_n Z_n),
$$

where $z \in [iXd + j_n Z_n, iXd + (j_n+1)Z_n)$, $j_n \in \mathbb{I}_0^{k_n T_n - 1}$, and $n \in \mathbb{I}_1^N$.

When $z \in [iXd, iXd + Z_1)$, it follows from (18) that

$$
\begin{aligned}
\dot{\xi}_n(z) = (C_n + L_n F_n)\xi_n(z) - \sum_{m=1}^{N} b_{nm}\Upsilon\xi_m(z) \\
+ \bar{h}(\xi_n(z)) - L_n F_n \bar{\varepsilon}_{n,0}(z),\ n \in \mathbb{I}_1^N. \quad (20)
\end{aligned}
$$

Now, we are going to provide an estimate for $\bar{\varepsilon}_{n,0}(z)$. Noting that

$$
\begin{aligned}
\dot{\bar{\varepsilon}}_{n,0}(z) &= \dot{\xi}_n(z) \\
&= C_n\xi_n(z) + \bar{h}(\xi_n(z)) - \sum_{m=1}^{N} b_{nm}\Upsilon\xi_m(z) \\
&\quad + L_n F_n \xi_n(iXd), \quad (21)
\end{aligned}
$$

and $\bar{\varepsilon}_{n,0}(iXd) = 0$, one has

$$
\begin{aligned}
\bar{\varepsilon}_{n,0}(z) = \int_{iXd}^{z} \Big(C_n\xi_n(\varrho) + L_n F_n \xi_n(iXd) \\
+ \bar{h}(\xi_n(\varrho)) - \sum_{m=1}^{N} b_{nm}\Upsilon\xi_m(\varrho)\Big) d\varrho, \quad (22)
\end{aligned}
$$

and

$$
\begin{aligned}
\|\bar{\varepsilon}_{n,0}(z)\| &\le \int_{iXd}^{z} \Big\| \Big(C_n\xi_n(\varrho) + L_n F_n \xi_n(iXd) \\
&\quad + \bar{h}(\xi_n(\varrho)) - \sum_{m=1}^{N} b_{nm}\Upsilon\xi_m(\varrho)\Big)\Big\| d\varrho \\
&\le \int_{iXd}^{z} \Big(\Big\| C_n\xi_n(\varrho) - \sum_{m=1}^{N} b_{nm}\Upsilon\xi_m(\varrho)\Big\| \\
&\quad + \big\|\bar{h}(\xi_n(\varrho))\big\| + \big\|L_n F_n\big\|\|\xi_n(iXd)\|\Big) d\varrho \\
&\le Z_1 \tilde{x}_n \max_{\varrho\in[iXd,z)} \|\xi(\varrho)\| \\
&\le Z_1 \tilde{\mu} \max_{\varrho\in[iXd,z)} \|\xi(\varrho)\|, \quad (23)
\end{aligned}
$$

where $\tilde{x}_n = \max\{\max_{m\neq n}\{\|b_{nm}\Upsilon\|\}, \|C_n - b_{nn}\Upsilon\|\} + \|L_n F_n\| + \kappa$.

Following the same line in the proof of Theorem 1, it can be inferred that

$$
\begin{aligned}
\dot{V}(z) \le -\alpha V(z) + \delta N Z_1^2 \tilde{\mu}^2 \lambda_M(U^{-1}) \max_{\varrho\in[iXd,z)} V(\varrho), \\
z \in [iXd, iXd + Z_1). \quad (24)
\end{aligned}
$$

The rest of the proof can be easily accomplished by applying almost the same arguments as in the proof of Theorem 1. The proof is now complete. ■

Remark 3: In [27], a sampled-data controller/observer with a time-varying gain of the form $Ke^{-\eta(z-z_r)}$ was proposed, where $\eta > 0$ and K is a constant matrix of appropriate dimension. While several numerical simulations demonstrated that the introduction of time-varying gains could lead to a substantial enlargement of the maximum sampling intervals, no rigorous theoretical proof was given. In this paper, matrix exponential gains are integrated into the design of sampled-data state estimator. It is shown that the proposed matrix-exponential-gain-based sampled-data estimator is more efficient than the traditional ones in enlarging the MAB of sampling intervals. In addition, in [27], the MAB of sampling intervals was estimated by solving a number of matrix inequalities, and the MAB of sampling intervals in this paper is estimated by solving an algebraic inequality. Thus, the approach adopted in this paper can be implemented more easily.

Remark 4: In Corollary 1, we have established some sufficient conditions to guarantee that the ASD state estimator (17) without matrix exponential gains is also an exponential state estimator for the nonlinear CN (1). In light of Theorem 1 and Corollary 1, it is clear that for a given parameter $\alpha > 0$ and an estimator gain L, if the LMI (15) has a feasible solution, the MAB of the sampling intervals of the ASD state estimator (2) with matrix exponential gains is greater than that of the ASD state estimator (17) without matrix exponential gains. Such a fact indicates that the introduction of matrix exponential gains is indeed effective in terms of enlarging the MAB of the sampling intervals, especially when the term $\|L_nF_n\|$ is large.

Thus far, we have concentrated on analyzing the convergence and performance of the proposed ASD estimator for the nonlinear CN (1). Subsequently, the focus will be shifted towards the design problem of the state estimator (2).

Theorem 2: Let the positive constant α be given. Under Assumption 1, the ASD state estimator (2) with matrix exponential gains is an exponential state estimator for the nonlinear CN (1) if there exists a block diagonal positive definite matrix $U = \mathrm{diag}\{U_1, U_2, \ldots, U_N\}$, a diagonal positive definite matrix $\varPhi = \mathrm{diag}\{\phi_1, \phi_2, \ldots, \phi_N\}$, matrices $Y_n \in \mathbb{R}^{p\times q}, n \in \mathbb{I}_1^N$, and a positive constant δ such that the following inequality holds

$$\bar{\mathscr{D}} = \begin{bmatrix} \bar{\mathscr{D}}_0 & U+\varPhi\otimes\check{P}_2 & YF \\ * & -\varPhi\otimes I & 0 \\ * & * & -\delta I \end{bmatrix} < 0, \tag{25}$$

and the MAB of the sampling interval with respect to each node satisfies

$$Z_M < \frac{1}{\mu Q}\sqrt{\frac{\alpha}{\delta N\lambda_M(U^{-1})}}, \tag{26}$$

where $\bar{\mathscr{D}}_0 = \alpha U - \varPhi\otimes\check{P}_1 + U(C - B\otimes\Upsilon) + (C - B\otimes\Upsilon)^TU + YF + F^TY^T$, $Y = \mathrm{diag}\{Y_1, Y_2, \ldots, Y_N\}$, and the remaining parameters are defined as before. Accordingly, the estimator gain L_n can be designed as follows:

$$L_n = U_n^{-1}Y_n. \tag{27}$$

Proof: By resorting to Theorem 1, the proof is easily accessible and is therefore omitted here for brevity. ■

Remark 5: In Theorem 2, an effective scheme has been presented to determine the desired estimator gains by searching for the feasible solution of LMI (25). Meanwhile, it should be emphasized that by resorting to the algebraic inequality (26), Theorem 2 has also provided an explicit MAB of the sampling intervals. As we all know, in the context of sampled-data control systems, a larger sampling period is usually preferred for reducing both computational burden and communication frequency. Therefore, based on Theorem 2, we shall further utilize a convex optimization approach to design the estimator gain and maximize the MAB of the sampling intervals.

Theorem 3: Let the positive scalar α be given. Under Assumption 1, the ASD state estimator (2) with matrix exponential gains is an exponential state estimator for the nonlinear CN (1) if an optimization problem with the following linear objective

$$\min_{U>0,\Phi>0,\delta>0,\rho>0,Y} \quad \nu > 0 \tag{28}$$

and LMI constraints

$$\text{LMI (25)} \tag{28a}$$

$$\begin{bmatrix} -\rho I & I \\ * & -U \end{bmatrix} \le 0 \tag{28b}$$

$$\begin{bmatrix} -\frac{\nu}{2} & \delta & \rho \\ * & -1 & 0 \\ * & * & -1 \end{bmatrix} \le 0 \tag{28c}$$

has a set of solutions $U > 0$, $\Phi > 0$, $\delta > 0$, $\rho > 0$ and Y, and the MAB of the sampling interval with respect to each node satisfies

$$Z_M < \frac{1}{\mu Q}\sqrt{\frac{\alpha}{\nu N}}. \tag{29}$$

Accordingly, the estimator gain L_n can be designed as follows:

$$L_n = U_n^{-1}Y_n. \tag{30}$$

Proof: Applying the Schur complement lemma to (28b) yields

$$-\rho I + U^{-1} \le 0, \tag{31}$$

which means that

$$\lambda_M(U^{-1}) \le \rho. \tag{32}$$

Analogously, it follows from (28c) that

$$\delta^2 + \rho^2 \le \frac{\nu}{2}. \tag{33}$$

which, together with (31) and (32), gives rise to

$$\delta^2 + \lambda_M^2(U^{-1}) \le \frac{\nu}{2}, \tag{34}$$

implying that

$$\delta\lambda_M(U^{-1}) \le \nu. \tag{35}$$

Therefore, we are able to arrive at

$$Z_M < \frac{1}{\mu Q}\sqrt{\frac{\alpha}{\nu N}} \le \frac{1}{\mu Q}\sqrt{\frac{\alpha}{N\delta\lambda_M(U^{-1})}}. \tag{36}$$

Now, it can be concluded that all the conditions stated in Theorem 2 are satisfied, which completes the proof. ■

Remark 6: In Theorem 3, a novel convex optimization approach has been put forward to design the desired estimator gain and maximize the MAB of the sampling intervals. It is worth noting that we have introduced several additional inequalities within Theorem 3, which may lead to a certain degree of conservativeness in our results. When implementing Theorem 3, one can first solve the optimization problem (28) and then determine the estimator gain according to (30). Particularly, the MAB of the sampling intervals can be obtained by resorting to (16).

Next, we will present a design scheme for the synchronous sampled-data state estimator with matrix exponential gains. Specifically, let us construct the following state estimator for the nonlinear CN (1):

$$\begin{aligned}\dot{\hat{s}}_n(z) =& C_n\hat{s}_n(z) + h(\hat{s}_n(z)) + \sum_{m=1}^{N} a_{nm}\Upsilon(\hat{s}_m(z) - \hat{s}_n(z)) \\ &+ e^{L_nF_n(z-z_r)}L_n(y_n(z_r) - F_n\hat{s}_n(z_r)), \\ & z \in [z_r, z_{r+1}). \quad (37)\end{aligned}$$

In light of Theorems 1 and 2, one can easily obtain the following corollary.

Corollary 2: Let the positive constant α be given. Under Assumption 1, the synchronous sampled-data state estimator (37) with matrix exponential gains is an exponential state estimator for the nonlinear CN (1) if there exists a block diagonal positive definite matrix $U = \text{diag}\{U_1, U_2, \dots, U_N\}$, a diagonal positive definite matrix $\Phi = \text{diag}\{\phi_1, \phi_2, \dots, \phi_N\}$, matrices $Y_n \in \mathbb{R}^{p\times q}$, $n \in \mathbb{I}_1^N$, and a positive constant δ such that the following inequality holds

$$\bar{\mathscr{D}} = \begin{bmatrix} \bar{\mathscr{D}}_0 & U + \Phi\otimes\check{P}_2 & YF \\ * & -\Phi\otimes I & 0 \\ * & * & -\delta I\end{bmatrix} < 0, \quad (38)$$

and the MAB of the sampling interval with respect to each node satisfies

$$Z_M < \frac{1}{\mu}\sqrt{\frac{\alpha}{\delta N\lambda_M(U^{-1})}}, \quad (39)$$

where $\bar{\mathscr{D}}_0 = \alpha U - \Phi\otimes\check{P}_1 + U(C - B\otimes\Upsilon) + (C - B\otimes\Upsilon)^TU + YF + F^TY^T$, and $Y = \text{diag}\{Y_1, Y_2, \dots, Y_N\}$. Accordingly, the estimator gain L_n can be designed as follows:

$$L_n = U_n^{-1}Y_n. \quad (40)$$

Proof: This corollary follows directly from Theorem 2, and the proof is omitted here for brevity. ■

Remark 7: Up to now, we have investigated the ASD state estimation problems for a class of continuous-time nonlinear CNs. In comparison with the existing literature, the main results obtained in this paper stand out due mainly to the following five distinguishing features: 1) a novel ASD state estimator with matrix exponential gains for nonlinear CNs is proposed, which allows each network node to sample and transmit data independently, thereby accommodating diverse sampling intervals; 2) the use of matrix exponential gains significantly increases the MAB of the sampling intervals, reducing communication frequency and computational burden compared to existing methods; 3) a new modified Halanay-Like inequality is proposed to simplify the analysis of estimation error dynamics, which helps to reduce computational complexity while ensuring exponential stability; 4) a convex optimization framework is developed to maximize the MAB of the sampling intervals in order to optimize estimator gain for resource efficiency; and 5) the superiority of the proposed estimator is demonstrated over traditional ASD estimators without matrix exponential gains, highlighting its enhanced performance in terms of sampling intervals and stability.

IV. Simulation Examples

In this section, both numerical simulation and practical examples are given to verify the effectiveness of the proposed ASD state estimator (2) with matrix exponential gains.

Example 1: Consider a nonlinear CN (1) with six nonidentical nodes and the related parameters are listed as follows:

$$B = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ \Upsilon = 0.15I_2,$$

$$C_1 = C_2 = \begin{bmatrix} 0.15 & -0.2 \\ 0.2 & -0.1 \end{bmatrix}, \ C_3 = C_4 = \begin{bmatrix} 0.2 & -0.2 \\ 0.3 & 0 \end{bmatrix},$$

$$C_5 = C_6 = \begin{bmatrix} -0.1 & -0.2 \\ 0.2 & 0.1 \end{bmatrix}, F_n = [1\ 1], n \in \mathbb{I}_1^6,$$

$$h(s) = -0.2\begin{bmatrix} -0.8s_1 + 0.2(|s_1+10| - |s_1-10|) + 0.4s_2 \\ 0.8s_2 + \tanh(-0.6s_2) \end{bmatrix}$$

with $s = [s_1\ s_2]^T$.

It is easy to verify that the nonlinear function $h(s)$ satisfies Assumption 1 with the following parameters:

$$P_1 = -0.2\begin{bmatrix} -0.8 & 0.4 \\ 0 & 0.8 \end{bmatrix}, \ P_2 = -0.2\begin{bmatrix} -0.4 & 0.4 \\ 0 & 0.2 \end{bmatrix}.$$

With the aforementioned parameters and the initial states $s_1(0) = [3,\ -2.5]^T$, $s_2(0) = [-3\ 1]^T$, $s_3(0) = [3.5\ -0.5]^T$, $s_4(0) = [-2\ 2.75]^T$, $s_5(0) = [-3\ -1]^T$, and $s_6(0) = [-3.5\ 3.75]^T$, the state trajectories of the nonlinear CN are plotted in Figs. 1 and 2. Clearly, it is evident from Figs. 1 and 2 that the nonlinear CN is unstable, which implies that the considered state estimation task for such a nonlinear CN is non-trivial.

Choosing $\alpha = 0.3$ and employing the Matlab software with the YALMIP toolbox to solve the linear optimization problem (28), we can acquire a set of estimator gain matrices as follows:

$$L_1 = \begin{bmatrix} -1.1389 \\ -0.2044 \end{bmatrix}, L_2 = \begin{bmatrix} -1.0896 \\ -0.1496 \end{bmatrix}, L_3 = \begin{bmatrix} -1.0968 \\ -0.2263 \end{bmatrix},$$

$$L_4 = \begin{bmatrix} -1.0698 \\ -0.2043 \end{bmatrix}, L_5 = \begin{bmatrix} -0.7653 \\ -0.0139 \end{bmatrix}, L_6 = \begin{bmatrix} -0.7790 \\ -0.0167 \end{bmatrix},$$

and $Z_M < 0.2554$. According to Theorem 2, it can be concluded that the estimator (2) is an exponential state estimator for the nonlinear CN (1).

Let us set $d = 0.01$, $Z_1 = 0.24$, $Z_2 = 0.12$, $Z_3 = 0.12$, $Z_4 = 0.08$, $Z_5 = 0.08$, and $Z_6 = 0.24$. Figs. 1 and 2 show the state estimation results for the nonlinear CN under different initial estimated states $\hat{s}(0) = -3s(0)$ and $\hat{s}(0) = 2s(0)$, respectively. Evidently, the developed state estimator can effectively estimate the true state of the nonlinear CN. Denote by $\varpi(z) \triangleq \sum_{n=1}^{6}\|\xi_n(z)\|^2$ the total estimation error of the CN. The evolution of the estimation error is shown in Fig. 3, which is exactly consistent with the theoretical findings. Moreover, Fig. 3 shows the trajectories of the estimation errors for different sampling intervals with the initial condition

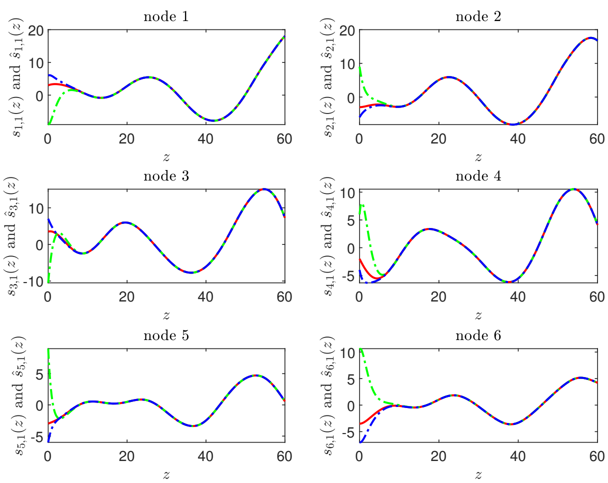

Fig. 1. Trajectories of $s_{n,1}(z)$ and $\hat{s}_{n,1}(z)$, $n \in \mathbb{I}_1^6$. The solid red lines denote the trajectories of true states, the green dash-dotted lines and blue dash-dotted lines show, respectively, the trajectories of estimated states under initial estimated states $\hat{s}_{n,1}(0) = -3s_{n,1}(0)$ and $\hat{s}_{n,1}(0) = 2s_{n,1}(0)$.

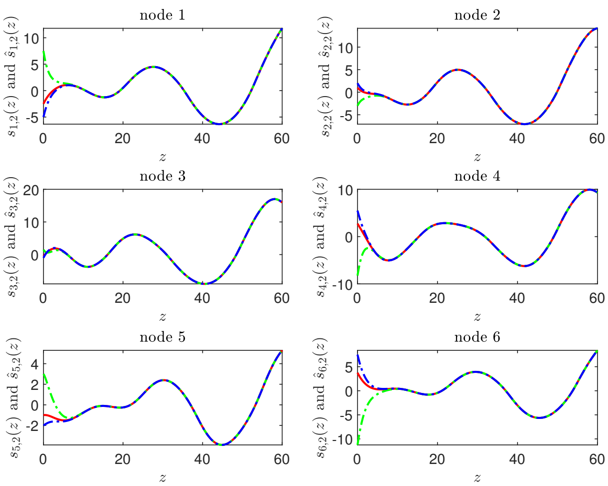

Fig. 2. Trajectories of $s_{n,2}(z)$ and $\hat{s}_{n,2}(z)$, $n \in \mathbb{I}_1^6$. The solid red lines denote the trajectories of true states, the green dash-dotted lines and blue dash-dotted lines show, respectively, the trajectories of estimated states under initial estimated states $\hat{s}_{n,2}(0) = -3s_{n,2}(0)$ and $\hat{s}_{n,2}(0) = 2s_{n,2}(0)$.

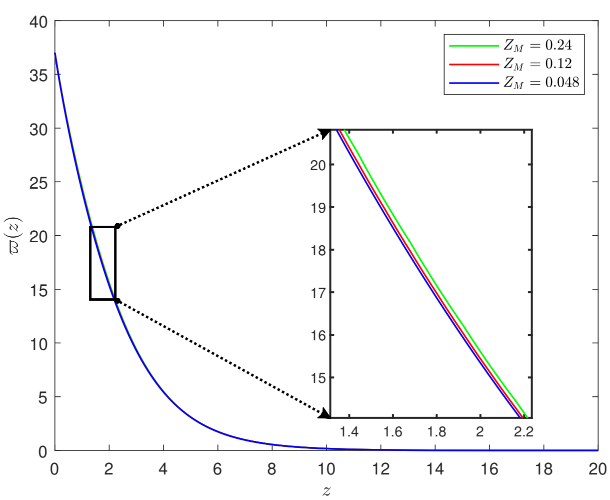

Fig. 3. Trajectories of estimation errors $\varpi(z)$ with different sampling intervals Z_M.

$\hat{s}(0) = -3s(0)$. It can be observed that a smaller sampling interval contributes to a faster convergence rate.

To explicitly demonstrate the advantages of the proposed method, Table I provides the MAB of sampling intervals for different values of α. It is evident from Table I that introducing matrix exponential gains significantly enlarges the MAB of sampling intervals, which conforms well to the theoretical analysis.

TABLE I
THE MAB OF SAMPLING INTERVALS FOR DIFFERENT VALUES OF α

α	0.05	0.1	0.2	0.3	0.4	0.5
Z_M	0.171	0.220	0.256	0.255	0.237	0.214
$\bar{Z}_M$	0.045	0.058	0.067	0.067	0.062	0.056

Example 2 (A Practical Example): Consider the standard IEEE 39-bus power system [43]–[45], which includes 10 generators, 29 loads, and 40 transmission lines. The IEEE 39-bus power system can be described by CN (1) with the following parameters:

$$s_n(z) = \begin{bmatrix} \Delta\omega_n(z) \\ \Delta P_{nm}(z) \\ \Delta P_{m_n}(z) \\ \Delta P_{v_n}(z) \end{bmatrix}, \ \Upsilon = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_n = \begin{bmatrix} -\frac{D_n}{2H_n} & -\frac{1}{2H_n} & \frac{1}{2H_n} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{\mathrm{ch}_n}} & 0 & \frac{1}{T_{\mathrm{ch}_n}} \\ -\frac{1}{R_n T_{\mathrm{g}_n}} & 0 & 0 & -\frac{1}{T_{\mathrm{g}_n}} \end{bmatrix}, \ a_{nm} = T_{nm},$$

where the physical meanings of the above parameters are explained in [43]–[45]. For the subsequent analysis, both the load-generation equilibrium point and the load deviation are assumed to be zero. The model parameters in above matrices are given in Table II, and suppose that the measurement matrix $F_n = \begin{bmatrix} 0.2 & -0.2 & 0 & 0 \end{bmatrix}, n \in \mathbb{I}_1^{10}$. In addition, set $T_{12} = 0.3, T_{13} = 0.1, T_{1,10} = 0.1, T_{21} = 0.3, T_{23} = 0.2, T_{31} = 0.1, T_{32} = 0.2, T_{34} = 0.15, T_{43} = 0.15, T_{45} = 0.1, T_{46} = 0.15, T_{47} = 0.25, T_{48} = 0.15, T_{54} = 0.1, T_{64} = 0.15, T_{67} = 0.16, T_{74} = 0.25, T_{76} = 0.16, T_{84} = 0.15, T_{89} = 0.1, T_{8,10} = 0.16, T_{98} = 0.1, T_{10,1} = 0.1, T_{10,8} = 0.16$.

By employing YALMIP to solve the optimization problem (28) with the given scalar $\alpha = 0.1$, a set of feasible solutions is obtained. According to Theorem 3, the ASD state estimator (2) is an exponential state estimator for the nonlinear CN (1). In addition, with the above parameters, we can calculate the MAB of the sampling period $Z_M < 0.0661$. Set $Z_1 = Z_2 = Z_3 = Z_4 = 0.066, Z_5 = Z_6 = 0.033, Z_7 = Z_8 = 0.022, Z_9 = Z_{10} = 0.011$. Figs. 4–7 illustrate the trajectories of true states $\Delta\omega_n(z)$, $\Delta P_{nm}(z)$, $\Delta P_{m_n}(z)$, $\Delta P_{v_n}(z)$ and their estimations, where $n = 2, 6, 8, 9$.

V. CONCLUSIONS

In this paper, the sampled-data state estimation issues have been studied for a class of continuous-time nonlinear CNs. In order to enlarge the MAB of the sampling intervals, a novel ASD state estimator has been proposed with matrix

TABLE II
PARAMETERS FOR THE IEEE 39-BUS SYSTEM

	D_n	T_{ch_n}	T_{g_n}	R_n	H_n
Area 1	5	4	2	0.75	2
Area 2	4	4	2	0.75	8
Area 3	7	4	2	0.75	5
Area 4	4	4	2	0.75	8
Area 5	6	4	2	0.75	10
Area 6	4	4	2	0.75	7
Area 7	4	4	2	0.75	4
Area 8	3	4	2	0.75	7
Area 9	6	4	2	0.75	6
Area 10	5	4	2	0.75	12

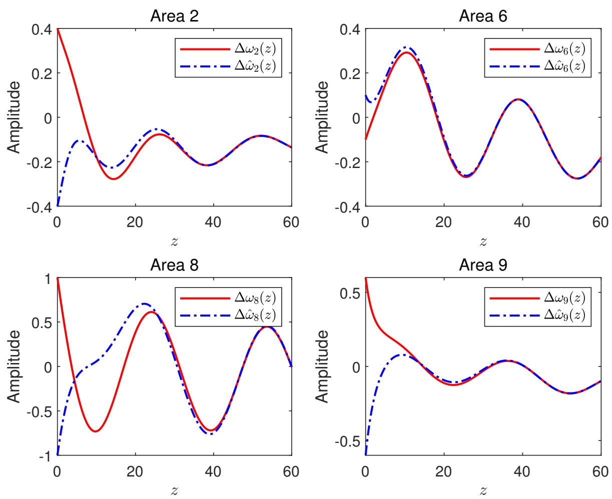

Fig. 4. True states $\Delta\omega_n(z)$ and their estimations $(n = 2, 6, 8, 9)$.

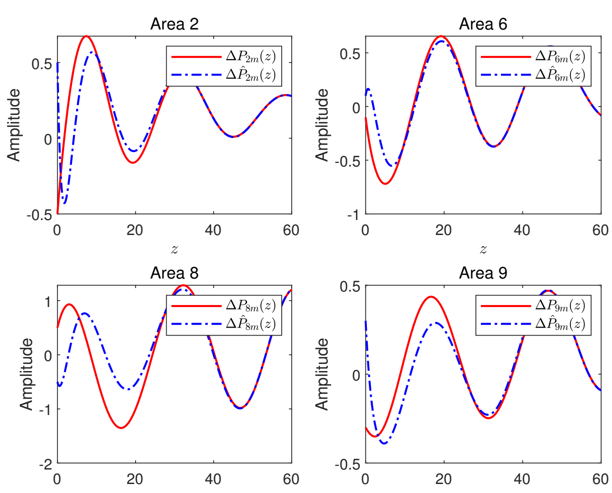

Fig. 5. True states $\Delta P_{nm}(z)$ and their estimations $(n = 2, 6, 8, 9)$.

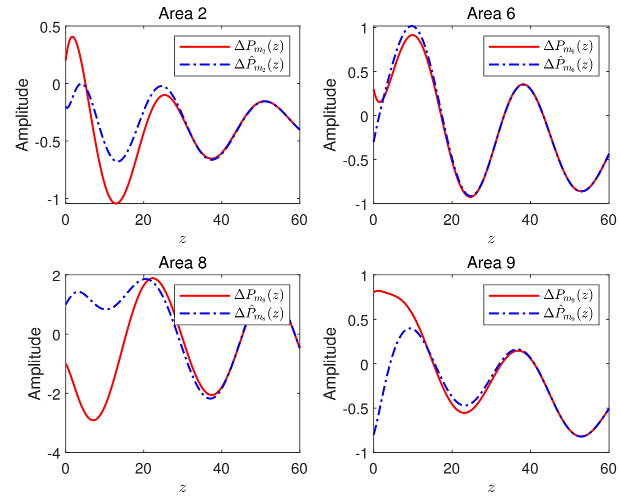

Fig. 6. True states $\Delta P_{m_n}(z)$ and their estimations $(n = 2, 6, 8, 9)$.

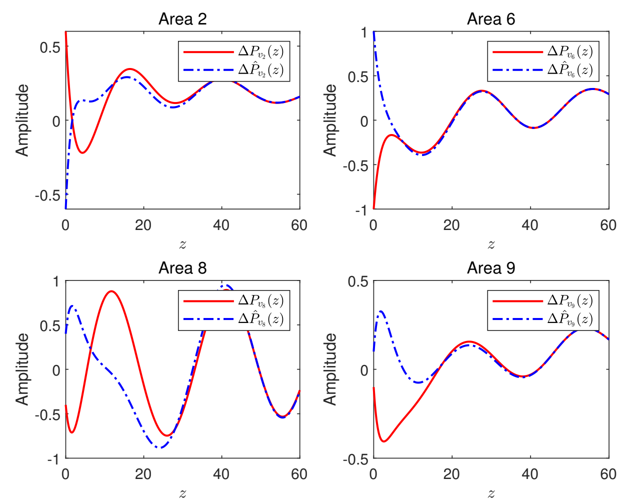

Fig. 7. True states $\Delta P_{v_n}(z)$ and their estimations $(n = 2, 6, 8, 9)$.

exponential gains. A modified Halanay-like inequality has also been established, based on which some easy-to-check sufficient conditions have been derived to ensure that the estimation error dynamics is exponentially stable. In addition, a new convex optimization approach has been employed to determine the desired estimator gain by maximizing the MAB of the sampling intervals. Finally, a numerical example and the standard IEEE 39-bus power system have been provided to illustrate the feasibility and effectiveness of the obtained main results. Future research directions would include i) extending the developed state estimation method to stochastic, time-delay, or hybrid networks; ii) improving robustness against uncertainties and disturbances; and iii) handling the scalability issue for large networks and experimental validation in practical systems [46]–[51].

Appendix

Proof of Theorem 1: To begin with, let us construct the following Lyapunov function

$$V(z)=\sum_{n=1}^{N}V_n(z)=\sum_{n=1}^{N}\xi_n^T(z)U_n\xi_n(z). \tag{41}$$

Without loss of generality, assume that $Z_1<Z_2<\cdots<Z_N$, namely, $T_1<T_2<\cdots<T_N$. Since X is the least common multiple of $T_1,T_2,\ldots,T_N$, it is clear that there exist positive integers k_n satisfying $X=T_nk_n$. Now, we consider the time domain $[iXd,(i+1)Xd)$, $i\in\mathbb{N}$.

Let us denote

$$\varepsilon_{n,j_n}(z)=\xi_n(z)-e^{L_nF_n(z-iXd-j_nZ_n)}\xi_n(iXd+j_nZ_n),$$

where $z\in[iXd+j_nZ_n,iXd+(j_n+1)Z_n)$, $j_n\in\mathbb{I}_0^{k_nT_n-1}$, and $n\in\mathbb{I}_1^N$.

When $z\in[iXd,iXd+Z_1)$, it follows from (3) that

$$\begin{aligned}\dot{\xi}_n(z)&=C_n\xi_n(z)-\sum_{m=1}^{N}b_{nm}\Upsilon\xi_m(z)\\&\quad+L_nF_n\Big(e^{L_nF_n(z-iXd)}\xi_n(iXd)-\xi_n(z)\Big)\\&\quad+\bar{h}(\xi_n(z))+L_nF_n\xi_n(z)\\&=(C_n+L_nF_n)\xi_n(z)-\sum_{m=1}^{N}b_{nm}\Upsilon\xi_m(z)\\&\quad+\bar{h}(\xi_n(z))-L_nF_n\varepsilon_{n,0}(z),\ \ n\in\mathbb{I}_1^N.\end{aligned} \tag{42}$$

Taking the time derivative of $V(z)$ along the error dynamics (42) yields

$$\begin{aligned}\dot{V}(z)=&\sum_{n=1}^{N}2\xi_n^T(z)U_n\Big[(C_n+L_nF_n)\xi_n(z)+\bar{h}(\xi_n(z))\\&-\sum_{m=1}^{N}b_{nm}\Upsilon\xi_m(z)-L_nF_n\varepsilon_{n,0}(z)\Big].\end{aligned} \tag{43}$$

By applying Young's inequality, it can be verified that

$$\begin{aligned}-2\sum_{n=1}^{N}\xi_n^T(z)U_nL_nF_n\varepsilon_{n,0}(z)\le\delta\sum_{n=1}^{N}\varepsilon_{n,0}^T(z)\varepsilon_{n,0}(z)\\+\delta^{-1}\sum_{n=1}^{N}\xi_n^T(z)U_nL_nF_nF_n^TL_n^TU_n\xi_n(z).\end{aligned} \tag{44}$$

Now, we are going to give an estimate for $\varepsilon_{n,0}(z)$. Noting that

$$\begin{aligned}\dot{\varepsilon}_{n,0}(z)&=\dot{\xi}_n(z)-L_nF_ne^{L_nF_n(z-iXd)}\xi_n(iXd)\\&=C_n\xi_n(z)+\bar{h}(\xi_n(z))-\sum_{m=1}^{N}b_{nm}\Upsilon\xi_m(z),\end{aligned} \tag{45}$$

and $\varepsilon_{n,0}(iXd)=0$, we can arrive at

$$\begin{aligned}\varepsilon_{n,0}(z)=&\int_{iXd}^{z}\Big(C_n\xi_n(\varrho)\\&+\bar{h}(\xi_n(\varrho))-\sum_{m=1}^{N}b_{nm}\Upsilon\xi_m(\varrho)\Big)d\varrho,\end{aligned} \tag{46}$$

and

$$\begin{aligned}\|\varepsilon_{n,0}(z)\|\le&\int_{iXd}^{z}\Big\|\Big(C_n\xi_n(\varrho)+\bar{h}(\xi_n(\varrho))\\&-\sum_{m=1}^{N}b_{nm}\Upsilon\xi_m(\varrho)\Big)\Big\|d\varrho\\\le&\int_{iXd}^{z}\Big(\Big\|C_n\xi_n(\varrho)-\sum_{m=1}^{N}b_{nm}\Upsilon\xi_m(\varrho)\Big\|\\&+\|\bar{h}(\xi_n(\varrho))\|\Big)d\varrho\\\le&Z_1x_n\max_{\varrho\in[iXd,z)}\|\xi(\varrho)\|\\\le&Z_1\mu\max_{\varrho\in[iXd,z)}\|\xi(\varrho)\|,\end{aligned} \tag{47}$$

where $x_n=\max\{\max_{m\neq n}\{\|b_{nm}\Upsilon\|\},\|C_n-b_{nn}\Upsilon\|\}+\kappa$.

Based on the above inequality, it is not difficult to see that

$$\sum_{n=1}^{N}\|\varepsilon_{n,0}(z)\|^2\le NZ_1^2\mu^2\max_{\varrho\in[iXd,z)}\|\xi(\varrho)\|^2, \tag{48}$$

which, together with Lemma 2, implies that

$$\sum_{n=1}^{N}\|\varepsilon_{n,0}(z)\|^2\le NZ_1^2\mu^2\lambda_M(U^{-1})\max_{\varrho\in[iXd,z)}V(\varrho). \tag{49}$$

On the other hand, in light of Assumption 1, one has

$$\begin{bmatrix}\xi_n(z)\\\bar{h}(\xi_n(z))\end{bmatrix}^T\begin{bmatrix}\check{P}_1&-\check{P}_2\\ *&I\end{bmatrix}\begin{bmatrix}\xi_n(z)\\\bar{h}(\xi_n(z))\end{bmatrix}\le0. \tag{50}$$

Then, it is easy to verify that

$$\sum_{n=1}^{N}\phi_n\begin{bmatrix}\xi_n(z)\\\bar{h}(\xi_n(z))\end{bmatrix}^T\begin{bmatrix}\check{P}_1&-\check{P}_2\\ *&I\end{bmatrix}\begin{bmatrix}\xi_n(z)\\\bar{h}(\xi_n(z))\end{bmatrix}\le0, \tag{51}$$

which further means that

$$\begin{bmatrix}\xi(z)\\H(\xi(z))\end{bmatrix}^T\begin{bmatrix}\Phi\otimes\check{P}_1&-\Phi\otimes\check{P}_2\\ *&\Phi\otimes I\end{bmatrix}\begin{bmatrix}\xi(z)\\H(\xi(z))\end{bmatrix}\le0, \tag{52}$$

namely,

$$\begin{aligned}\xi^T(z)(\Phi\otimes\check{P}_1)\xi(z)-2\xi^T(z)(\Phi\otimes\check{P}_2)H(\xi(z))\\+H^T(\xi(z))(\Phi\otimes I)H(\xi(z))\le0,\end{aligned} \tag{53}$$

where $H(\xi(z))=(\bar{h}^T(\xi_1(z)),\bar{h}^T(\xi_2(z)),\ldots,\bar{h}^T(\xi_N(z)))^T$.

Taking (43)-(53) into account, we arrive at

$$\begin{aligned}\dot{V}(z)\le&\,\xi^T(z)\Big(U(C+LF-B\otimes\Upsilon)+\delta^{-1}ULFF^TL^TU\\&+(C+LF-B\otimes\Upsilon)^TU\Big)\xi(z)\\&-\xi^T(z)(\Phi\otimes\check{P}_1)\xi(z)+2\xi^T(z)(\Phi\otimes\check{P}_2)H(\xi(z))\\&-H^T(\xi(z))(\Phi\otimes I)H(\xi(z))\\&+\delta NZ_1^2\mu^2\lambda_M(U^{-1})\max_{\varrho\in[iXd,z)}V(\varrho)\\=&\,\delta NZ_1^2\mu^2\lambda_M(U^{-1})\max_{\varrho\in[iXd,z)}V(\varrho)-\alpha V(z)\\&+\mathfrak{a}^T(z)\mathscr{G}\mathfrak{a}(z),\ \ z\in[iXd,iXd+Z_1),\end{aligned} \tag{54}$$

where $\mathfrak{a}(z)=(\xi^T(z),H^T(\xi(z)))^T$ and

$$\mathscr{G}=\begin{bmatrix}\mathscr{G}_0&U+\Phi\otimes\check{P}_2\\ *&-\Phi\otimes I\end{bmatrix} \tag{55}$$

with

$$\mathscr{G}_0 = \alpha U + \delta^{-1} ULFF^T L^T U - \varPhi \otimes \check{P}_1 + U(C + LF - B \otimes \Upsilon) + (C + LF - B \otimes \Upsilon)^T U.$$

Applying the Schur complement lemma to (15) gives rise to $\mathscr{G} < 0$, which, together with (54), implies that

$$\dot{V}(z) \leq -\alpha V(z) + \delta N Z_1^2 \mu^2 \lambda_M(U^{-1}) \max_{\varrho \in [iXd, z)} V(\varrho), \quad z \in [iXd, iXd + Z_1). \tag{56}$$

When $z \in [iXd + Z_1, iXd + 2Z_1)$ and $2Z_1 \leq Z_2$, similar to (42), we have

$$\dot{\xi}_1(z) = (C_1 + L_1 F_1)\xi_1(z) - \sum_{m=1}^{N} b_{1m} \Upsilon \xi_m(z) + \bar{h}(\xi_1(z)) - L_1 F_1 \varepsilon_{1,1}(z), \tag{57}$$

and

$$\dot{\xi}_n(z) = (C_n + L_n F_n)\xi_n(z) - \sum_{m=1}^{N} b_{nm} \Upsilon \xi_m(z) + \bar{h}(\xi_n(z)) - L_n F_n \varepsilon_{n,0}(z), \; n \in \mathbb{I}_2^N. \tag{58}$$

Repeating the same routine of (45)-(47) yields

$$\begin{aligned}\|\varepsilon_{1,1}(z)\| \leq & \int_{iXd+Z_1}^{z} \Big\| \Big(C_1 \xi_1(\varrho) + \bar{h}(\xi_1(\varrho)) - \sum_{m=1}^{N} b_{1m} \Upsilon \xi_m(\varrho) \Big) \Big\| d\varrho \\ \leq & \int_{iXd+Z_1}^{z} \Big(\Big\| C_1 \xi_1(\varrho) - \sum_{m=1}^{N} b_{1m} \Upsilon \xi_m(\varrho) \Big\| + \big\| \bar{h}(\xi_1(\varrho)) \big\| \Big) d\varrho \\ \leq & \, Z_1 x_1 \max_{\varrho \in [iXd+Z_1, z)} \|\xi(\varrho)\| \\ \leq & \, 2Z_1 \mu \max_{\varrho \in [iXd, z)} \|\xi(\varrho)\|,\end{aligned} \tag{59}$$

and, for $2 \leq n \leq N$,

$$\begin{aligned}\|\varepsilon_{n,0}(z)\| \leq & 2Z_1 x_n \max_{\varrho \in [iXd, z)} \|\xi(\varrho)\| \\ \leq & 2Z_1 \mu \max_{\varrho \in [iXd, z)} \|\xi(\varrho)\|.\end{aligned} \tag{60}$$

Thus, we can observe that

$$\begin{aligned}\|\varepsilon_{1,1}(z)\|^2 + \sum_{n=2}^{N} \|\varepsilon_{n,0}(z)\|^2 & \leq 4NZ_1^2 \mu^2 \max_{\varrho \in [iXd, z)} \|\xi(\varrho)\|^2 \\ & \leq 4NZ_1^2 \mu^2 \lambda_M(U^{-1}) \max_{\varrho \in [iXd, z)} V(\varrho).\end{aligned} \tag{61}$$

If $2Z_1 > Z_2$, for the sake of brevity in this proof, let us suppose that $2Z_1 \leq Z_3$. In fact, the proof for the case where $2Z_1 > Z_3$ follows a similar line, so it has been omitted here.

Now, let us consider two separate cases: $z \in [iXd + Z_1, iXd + Z_2)$ and $z \in [iXd + Z_2, iXd + 2Z_1)$.

When $z \in [iXd + Z_1, iXd + Z_2)$, similar to (57)-(61), one has

$$\|\varepsilon_{n,1}(z)\| \leq 2Z_1 \mu \max_{\varrho \in [iXd, z)} \|\xi(\varrho)\|, \; n \in \mathbb{I}_1^2, \tag{62}$$

and

$$\|\varepsilon_{n,0}(z)\| \leq 2Z_1 \mu \max_{\varrho \in [iXd, z)} \|\xi(\varrho)\|, \; n \in \mathbb{I}_3^N. \tag{63}$$

When $z \in [iXd + Z_2, iXd + 2Z_1)$, we can also arrive at

$$\|\varepsilon_{n,1}(z)\| \leq 2Z_1 \mu \max_{\varrho \in [iXd, z)} \|\xi(\varrho)\|, \; n \in \mathbb{I}_1^2, \tag{64}$$

and

$$\|\varepsilon_{n,0}(z)\| \leq 2Z_1 \mu \max_{\varrho \in [iXd, z)} \|\xi(\varrho)\|, \; n \in \mathbb{I}_3^N. \tag{65}$$

Therefore, it follows readily that

$$\dot{V}(z) \leq -\alpha V(z) + \delta 4NZ_1^2 \mu^2 \lambda_M(U^{-1}) \max_{\varrho \in [iXd, z)} V(\varrho), \quad z \in [iXd + Z_1, iXd + 2Z_1). \tag{66}$$

In general, for any $z \in [iXd, (i+1)Xd)$, there must exist a nonnegative number η such that $z \in [iXd + \eta Z_1, iXd + (\eta + 1)Z_1)$. Thus, it is easy to see that

$$\dot{V}(z) \leq \delta N(\eta+1)^2 Z_1^2 \mu^2 \lambda_M(U^{-1}) \max_{\varrho \in [iXd, z)} V(\varrho) - \alpha V(z), z \in [iXd + \eta Z_1, iXd + (\eta+1)Z_1). \tag{67}$$

In light of (16), it is not difficult to verify that $\alpha > \delta N X^2 d^2 \mu^2 \lambda_M(U^{-1}) > 0$. With the help of Lemma 1, one has

$$V(z) \leq e^{-\lambda(z - iXd)} V(iXd), \; z \in [iXd, (i+1)Xd), \tag{68}$$

where $\lambda > 0$ is the real root of the following equation

$$\lambda + \delta N X^2 d^2 \mu^2 \lambda_M(U^{-1}) e^{\lambda X d} = \alpha.$$

It is easy to verify that

$$\begin{aligned}V(z) \leq & e^{-\lambda(z - iXd)} V(iXd) \\ \leq & e^{-\lambda(z-(i-1)Xd)} V((i-1)Xd) \\ \leq & e^{-\lambda(z-(i-2)Xd)} V((i-2)Xd) \\ \leq & \dots \\ \leq & e^{-\lambda z} V(0), \; z \in [iXd, (i+1)Xd).\end{aligned} \tag{69}$$

Thus, for any $z \geq 0$, the following holds

$$V(z) \leq e^{-\lambda z} V(0), \tag{70}$$

which, together with (41), leads to

$$\|\xi(z)\| \leq \sqrt{\frac{V(0)}{\lambda_m(U)}} e^{-\frac{\lambda}{2} z}, \; z \geq 0. \tag{71}$$

Consequently, according to Definition 1, the ASD state estimator (2) with matrix exponential gains is an exponential state estimator for the nonlinear CN (1), which completes the proof. ■

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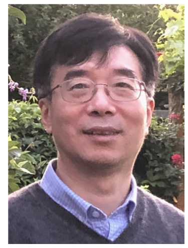

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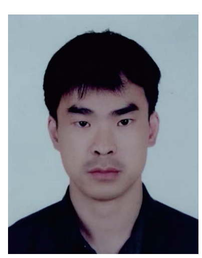

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