A Novel Distributed Optimization Algorithm with Linear Convergence for Economic Dispatch under Compressed Communication

Wei Chen, Zidong Wang, Jimmy Chih-Hsien Peng, and Guo-Ping Liu

***Abstract*—This paper addresses the distributed economic dispatch issue of microgrids. The primary objective of this study is to derive a distributed optimization algorithm with a compressed communication scheme over directed networks. Specifically, the algorithm aims to solve the economic dispatch problem, where the total power generation of distributed energy resources (DERs) is dispatched to meet the overall demand at the minimum operational cost under DER capacity constraints. To improve communication efficiency, a novel data compressed transmission mechanism is introduced into the consensus-based distributed algorithm by constructing estimator-like equations. Furthermore, by resorting to the property of matrix norms and system theory, a sufficient condition is derived to ensure that the proposed algorithm *linearly* converge to the optimal solution under *arbitrary* compression rate. This condition explicitly depends on the communication topologies and the algorithm parameters but is independent of the compression rate. Finally, simulated examples are provided to validate the theoretical claims and demonstrate the performance of the proposed algorithm.**

***Index Terms*—Economic dispatch problem, distributed optimization, consensus-based algorithm, compressed communication, microgrids.**

I. Introduction

In recent years, electric grids have been witnessing the rapid expansion of distributed energy resources (DERs), particularly renewable energy and storage. The purpose of these resources is to offset carbon emissions in an effort to transform power systems into sustainable ones [30], [46], [51]. However, renewable generation resources are intermittent, imposing additional uncertainties and randomness into daily system operations. This issue is particularly pronounced in power distribution systems and community microgrids, where the majority of DERs are integrated.

In this context, to manage the high variability of renewable generation resources, DERs are required to regularly update their power and frequency references in the control loops. A real and significant challenge in the energy management is the rapid regulation of each DER to meet grid demand at minimal cost. Moreover, with the ever-growing scale of power grids, the traditional centralized strategy may become impracticable due to its high overhead in communication and computation. Consequently, extensive research has been conducted into distributed implementation, which offers advantages such as high decentralization, plug-and-play capability, autonomous decision-making, and low complexity [3], [13], [38], [54].

Unlike traditional centralized approaches [4], [14], [49], the primary concept of the DED algorithm is to decentralize the function of the dispatching center by introducing local agents in each DER. These local agents interact with their neighboring nodes, rather than with all nodes, to determine the optimal power output [50]. Recently, DED problems have garnered increasing research interest, and many distributed optimization schemes have been reported in the literature [5], [10], [18], [41]–[43], [50], [55]. For instance, the ED problem has been preliminarily addressed in a distributed manner using gradient descent approaches [18], [31], [42], where a diminishing step size is introduced to ensure accurate convergence. Furthermore, by adding an additional gradient term to previous algorithms, a decentralized exact first-order algorithm (EXTRA) has been developed to achieve fast convergence [11], [41]. With a constant step size, the distributed gradient tracking algorithm (DGT) has been explored in [43], [50], where an auxiliary equation is constructed to track the average gradient (i.e., power dispatch). Additionally, the distributed alternating direction method of multipliers (ADMM) algorithms have been validated in [10], [55] to achieve optimal ED.

In the context of the energy internet, the communication network has become an indispensable component of microgrids due to its flexible architecture and low installation cost. However, in practical applications, signal transmission over a limited network bandwidth may introduce undesired network-induced phenomena, which can significantly degrade algorithm performance. To date, considerable effort has been devoted to developing DED algorithms for practical scenarios, such as packet dropouts [47], communication delays [12], external disturbances [52], [56], and cyber-attacks [37], [53]. Note that these studies primarily focus on the tolerability and resilience of DED algorithms, while little attention has been given to designing an *active* communication protocol to alleviate network overload. This lack of focus is possibly due to the challenges in understanding the effects of non-

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Wei Chen and Jimmy Chih-Hsien Peng are with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117583, Singapore (Emails: chenw3@sustech.edu.cn; jpeng@nus.edu.sg).

Zidong Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk).

Guo-Ping Liu is with School of Automation and Intelligent Manufacturing, Southern University of Science and Technology, Shenzhen 518055, China, (Email: liugp@sustech.edu.cn).

original data, caused by communication-efficient protocols, on the convergence accuracy of the DED algorithm. This challenge serves as the motivation for our current study.

It is widely recognized that communication cost poses a significant obstacle to the efficiency of DED algorithms [16], [48]. To enhance efficiency, compressed communication has gained increasing popularity in recent times, leading to the development of various compressed methods, which can be broadly categorized into sparsification [7], [40] and quantization [6], [20]. Consequently, it is natural to consider integrating compressed schemes into the DED algorithm. However, a direct combination inevitably results in inaccurate convergence or even divergence of the algorithm due to compression error effects [2], [27]. To mitigate these effects, several techniques have been developed, including difference and extrapolation compression as well as error compensation [9], [32], [40]. For instance, a compressed distributed subgradient algorithm has been proposed in [20], [21], where the convergence rate is sublinear for strongly convex optimization problems. Subsequently, studies such as [19], [39], [48] have developed a quantized gradient tracking method with linear convergence. However, the algorithm entails a complex design dependent on a specific quantization scheme, and its linear convergence conditions are challenging to verify. It should be pointed out that for the DED algorithm under general compression framework, the corresponding results are rather scattered, not to mention the case with linear convergence. It is, therefore, our aim to bridge this gap.

In view of the preceding observations, we endeavor to develop a communication-efficient DED algorithm, which presents two fundamental difficulties, highlighted as follows. *1) How can a general class of compressed mechanism be integrated into a distributed optimization algorithm to achieve low communication burden while ensuring exact and linear convergence? 2) How can a systematic analysis framework be established to reveal the feasible region of algorithm parameters ensuring convergence to the optimal solution to ED problem?*

In response to the aforementioned difficulties, a novel compressed DED algorithm is developed, incorporating an estimator-like dynamic compressing/decompressing scheme. This approach enables optimal power dispatch without the adverse effects of compression errors while enhancing communication efficiency. The state of the art and the main contributions of this study are twofold.

1) A novel compressed distributed algorithm over directed networks is proposed for optimal economic dispatch without compression error effects. The asymmetric topology employed in our work is much weaker then the strongly connected topology assumed most previous works [43], [48], [50], which offers more flexibility in designing graphs. Compared with existing compressed schemes [6], [15], [22], the complexity of the proposed compressed algorithm is independent of the number of neighboring nodes, which has an advantage in device scalability.
2) With resorting to the spectral radius criterion for stability and the properties of matrix norm, the *linear* convergence condition, in terms of algorithm parameters and network topologies, is established in Theorem 1 under *arbitrary* compression rate, and thus the proposed algorithm is deployed friendly satisfying diverse application needs of data compression. Furthermore, the proposed compression operator is general, which covers mostly unbiased and biased compressors, quantizers, sparsification and norm-sign compressors [6], [20], [40].

Notation: $\mathbb{R}^m$ is an m-dimensional real vector. $\mathbb{Z}$ is the set of integers. I_N and $\mathbf{1}_N$ denote the N-dimensional identical matrix and column vector of ones, respectively. $\|\cdot\|$ and $\|\cdot\|_\infty$ are the Euclidean and infinite norms. $[a_{ij}]_N$, $\mathrm{col}_N\{a_i\}$, and $\mathrm{diag}_N\{a_i\}$ represent the N-dimensional matrix, vector, and diagonal matrix, whose elements are a_{ij}, a_i, and a_i, respectively. $\{a_1,\ldots,a_n\}^+$ and $\{a_1,\ldots,a_n\}^-$ refer to the largest and smallest value among $a_1,\ldots,a_n$, respectively.

II. Problem Formulation and Preliminaries

A. Graph Theory

For a microgrid of N-bus agents, the network is depicted by a directed graph $\mathscr{G} \triangleq \{\mathcal{V},\mathcal{E},W\}$, where $\mathcal{V} \triangleq \{1,2,\cdots,N\}$ represents the vertex set, $\mathcal{E} \triangleq \{(i,j)|i,j \in \mathcal{V}\}$ represents the edge set, and $W \triangleq [w_{ij}]_N$ is the weight matrix with nonnegative elements. $(i,j) \in \mathcal{E}$ means that a directed edge from parent node j to child node i. Only the parent node can transmit information to the child node. The in-neighborhood (out-neighborhood) set of agent i is denoted as $\mathcal{N}_i^{in} \triangleq \{j|(i,j)\in\mathcal{E}\}$ ($\mathcal{N}_i^{out} \triangleq \{j|(i,j)\in\mathcal{E}\}$) where node i can receive (send) data from (to) node j. Agent i can be an in-neighbor or out-neighbor of itself. The element $w_{ij} > 0$ if and only if the agent $j \in \mathcal{N}_i^{in}$. $|\mathcal{N}_i^{in}|$ refers to the number of neighbor nodes of agent i. Under the directed graph $\mathscr{G}$, the weight matrix $W = [w_{ij}]_N$ can be column stochastic with its elements satisfying $\sum_{j\in\mathcal{N}_i^{in}} w_{ij} = 1$, or row-stochastic with $\sum_{j\in\mathcal{N}_i^{out}} w_{ji} = 1, \forall i,j \in \mathcal{V}$. A directed graph $\mathscr{G}$ is strongly connected if there is a directed path between any two nodes. A directed graph $\mathscr{G}$ contains a spanning tree if there exists a node that can reach all other nodes via a directed path.

B. Optimization Problem

In microgrids, the economic dispatch (ED) problem is essentially an optimization issue, with the primary aim of achieving optimal power dispatch to completely eliminate supply-demand mismatch while minimizing total generation costs under DER capacity constraints. More specifically, the ED problem can be formulated as follows [43], [50]:

$$\begin{aligned}&\underset{\{p_1,\ldots,p_N\}}{\arg\min}\ \sum\nolimits_{i=1}^{N} f_i(p_i)\\ &\text{s.t.}\quad \sum\nolimits_{i=1}^{N} p_i = \sum\nolimits_{i=1}^{N} p_i^d, \quad p_i^{\min} \le p_i \le p_i^{\max}, \quad (1)\end{aligned}$$

where p_i and p_i^d represent the local power supply and demand at agent i, respectively; $p_i^{\min}$ and $p_i^{\max}$ denote the bounds of the generation capacity; and $f_i(p_i)$ is the local cost function which takes the following form $f_i(p_i) = a_i p_i^2 + b_i p_i + c_i,\ i \in$

$\mathcal{V}$, where a_i, b_i, c_i refer to the appropriate cost coefficients. To ensure the existence of a solution to issue (1), we assume that

$$\sum\nolimits_{i=1}^{N} p_i^{\min} < \sum\nolimits_{i=1}^{N} p_i^{d} < \sum\nolimits_{i=1}^{N} p_i^{\max}. \tag{2}$$

For ease of analysis later, the local incremental cost is denoted as follows:

$$\lambda_i \triangleq df_i(p_i)/dp_i = 2a_i p_i + b_i. \tag{3}$$

C. Compression Mechanism

To improve efficiency, a compressor $C_\varphi(x) : \mathbb{R}^n \to \mathbb{R}^n$ is employed, which satisfies the following condition:

$$\|C_\varphi(x) - x\|_2 \leq \varphi \|x\|_2, \tag{4}$$

where $\varphi \in [0, 1)$ is the compression rate to be determined.

Remark 1: Generally speaking, the compressor $C_\varphi(x)$ can be regarded as a mapping where the compressed output is represented with significantly fewer bits than the input (e.g., through quantization and sparsification). Note that the proposed compressor (4) is very general, and some common compressors are listed as follows.

1) *Biased quantization [20]:* Denote the element-wise absolute, the sign function, the Hadamard product, and the floor function as $|\cdot|$, $\mathrm{sgn}(\cdot)$, $\circ$, and $\lfloor\cdot\rfloor$, respectively. The compressed mapping relationship can be expressed as

$$C_\varphi(x) = \frac{\|x\|_\infty \mathrm{sgn}(x)}{\sigma} \circ \left\lfloor \frac{\sigma|x|}{\|x\|_\infty} + 0.5\mathbf{1}_n \right\rfloor,$$

where $\sigma = 2^{l-1}$. It has been verified that $\varphi = \sqrt{\frac{n}{4\sigma^2+n}}$.

2) *Logarithmic quantization [6]:* Denote the set of quantization levels as $Q_\varphi \triangleq \{\pm\Theta^s\}_{s\in\mathbb{Z}} \cup \{0\}$ with $\Theta \triangleq \frac{1+\varphi}{1-\varphi}$, where $\mathbb{Z}$ is the integer set. The logarithmic quantizer $C_\varphi(x) : \mathbb{R} \to Q_\varphi$ is defined as

$$C_\varphi(x) \triangleq \begin{cases} \Theta^s, & \text{if } x \in \left[\frac{\Theta^s}{1+\varphi}, \frac{\Theta^s}{1-\varphi}\right) \\ 0, & \text{if } x = 0 \\ -C_\varphi(-x), & \text{if } x < 0. \end{cases}$$

3) *Sparsification [40]:* The greedy (Top-l) quantizer is constructed as

$$C_\varphi(x) \triangleq \sum\nolimits_{i=1}^{n} x^{(i_s)} e_{i_s},$$

where $x^{(i_s)}$ is the i_s-th element of x, $i_1, i_2, \cdots, i_l$ are the largest l $(l < n)$ elements of x, and $e_1, e_2, \cdots, e_n$ are the standard unit basis vectors in $\mathbb{R}^n$. It is derived that $\varphi = \sqrt{1 - l/n}$.

D. Objective

Before presenting the major goal of this work, we first outline the key arguments of the ED problem (1) as follows [50]:

$$\begin{cases} \lambda^\star = \dfrac{\sum_{i=1}^{N} p_i^d - \sum_{i\in\mathcal{D}_1} p_i^\star + \sum_{i\in\mathcal{D}_2} \frac{b_i}{2a_i}}{\sum_{i\in\mathcal{D}_2} \frac{1}{2a_i}} \\ p_i^\star = \begin{cases} p_i^{\min}, & \lambda^\star < \lambda_i^{\min}, \\ \frac{1}{2a_i}(\lambda^\star - b_i), & \lambda_i^{\min} \leq \lambda^\star \leq \lambda_i^{\max}, \\ p_i^{\max}, & \lambda^\star > \lambda_i^{\max}, \end{cases} \end{cases} \tag{5}$$

where $\lambda^\star$ and $p_i^\star$ $(i \in \mathcal{V})$ are the optimal incremental cost and power outputs, respectively. Here, $\lambda_i^{\min} \triangleq 2a_i p_i^{\min} + b_i$, $\lambda_i^{\max} \triangleq 2a_i p_i^{\max} + b_i$, $\mathcal{D}_1 \triangleq \{i | p_i^\star = p_i^{\min} \text{ or } p_i^{\max}\}$, and $\mathcal{D}_2 \triangleq \{i | p_i^{\min} < p_i^\star < p_i^{\max}\}$. Note that $\mathcal{D}_1, \mathcal{D}_2 \subset \mathcal{D}$ satisfy $\mathcal{D}_1 \cup \mathcal{D}_2 = \mathcal{D}$, $\mathcal{D}_1 \cap \mathcal{D}_2 = \emptyset$, and $\mathcal{D}_2 \neq \emptyset$.

The main purpose of this work is to derive a distributed algorithm under compressed communication (4) that linearly converges to $\lambda^\star$ and $p_i^\star$ $(i \in \mathcal{V})$ in (5) for the ED issue (1).

III. Distributed Algorithm Design and Analysis

In this section, a compressed communication scheme is first incorporated into the distributed optimization algorithm to enhance communication efficiency. Then, a sufficient condition is established under which the developed algorithm can achieve optimal power dispatch under arbitrary compression rate with a geometric convergence rate.

A. Compressed Distributed Optimization Algorithm

The proposed distributed algorithm involves two iterates via communication with adjacent nodes over two different directed networks named as $\mathscr{G}_{W^a}$ and $\mathscr{G}_{W^b}$. An assumption on the communication topology is given as follows.

Assumption 1: [34], [44] The graphs $\mathscr{G}_{W^a}$ and $\mathscr{G}_{(W^b)^T}$ 1) both have at least one spanning tree; and 2) have at least one common root node of spanning trees, where $\mathscr{G}_{W^b}$ and $\mathscr{G}_{(W^b)^T}$ are identical but with edges in opposite directions.

In this paper, Assumption 1 is weaker than strong connectivity of direct graphs [43], [48], [50].

To achieve optimal power dispatch in a distributed implementation, the basic distributed algorithm framework developed in [43], [48], [50] is presented as follows:

$$\begin{cases} \lambda_{i,k+1} = \lambda_{i,k} + \sum\limits_{j\in\mathcal{N}_i^{in}} w_{ij}^a(\lambda_{j,k} - \lambda_{i,k}) + \epsilon y_{i,k} \\ y_{i,k+1} = y_{i,k} + \sum\limits_{j\in\mathcal{N}_i^{in}} w_{ij}^b(y_{j,k} - y_{i,k}) - (p_{i,k+1} - p_{i,k}) \\ p_{i,k+1} \triangleq p_i(\lambda_{i,k+1}) = \left\{\left\{\dfrac{\lambda_{i,k+1} - b_i}{2a_i},\ p_i^{\min}\right\}^+,\ p_i^{\max}\right\}^-, \end{cases} \tag{6}$$

where $i \in \mathcal{V}$, $\epsilon > 0$ is the gain scalar to be determined, $y_{i,k}$ is the local power mismatch concerning the supply and demand, and weight matrices $W^a \triangleq [w_{ij}^a]_N$ and $W^b \triangleq [w_{ij}^b]_N$ are column and row stochastic. Note that $\lambda_{i,k}$ and $y_{i,k}$ are transmitted over two different networks. The initial value is selected as

$$\lambda_{i,0} = 2a_i p_{i,0} + b_i,\ y_{i,0} = p_i^d - p_{i,0},\ p_{i,0} \in \left[p_i^{\min},\ p_i^{\max}\right]. \tag{7}$$

To improve communication efficiency, the distributed optimization algorithm with compressed communication scheme is constructed by

$$\begin{cases} \lambda_{i,k+1} = \lambda_{i,k} + \alpha z_{i,k}^a + \epsilon y_{i,k} \\ y_{i,k+1} = y_{i,k} + \alpha z_{i,k}^b - (p_{i,k+1} - p_{i,k}) \\ p_{i,k+1} = p_i(\lambda_{i,k+1}),\ i \in \mathcal{V}, \end{cases} \tag{8}$$

where

$$\begin{aligned}
z^a_{i,k} &= z^a_{i,k-1} + \beta \sum\nolimits_{j\in\mathcal{N}_i^{in}} w^a_{ij}(\mathcal{C}^a_{j,k} - \mathcal{C}^a_{i,k}),\\
z^b_{i,k} &= z^b_{i,k-1} + \beta \sum\nolimits_{j\in\mathcal{N}_i^{in}} w^b_{ij}(\mathcal{C}^b_{j,k} - \mathcal{C}^b_{i,k}),\\
\hat{\lambda}_{i,k} &= \hat{\lambda}_{i,k-1} + \beta\mathcal{C}^a_{i,k},\ \hat{y}_{i,k} = \hat{y}_{i,k-1} + \beta\mathcal{C}^b_{i,k},\\
\mathcal{C}^a_{i,k} &= C_\varphi(\lambda_{i,k} - \hat{\lambda}_{i,k-1}),\ \mathcal{C}^b_{i,k} = C_\varphi(y_{i,k} - \hat{y}_{i,k-1}). \qquad (9)
\end{aligned}$$

Here, $\hat{\lambda}_{i,k}$ and $\hat{y}_{i,k}$ can be considered as the estimates of $\lambda_{i,k}$ and $y_{i,k}$, respectively. $\alpha \in (0,1]$ and $\beta \in (0,1]$ are two adjustable parameters, and the initial values are set as $\hat{\lambda}_{i,-1} = \hat{y}_{i,-1} = z^a_{i,-1} = z^b_{i,-1} = 0$. Furthermore, by simple calculation, we can obtain

$$z^a_{i,k} = \sum_{j\in\mathcal{N}_i^{in}} w^a_{ij}(\hat{\lambda}_{j,k} - \hat{\lambda}_{i,k}),\ z^b_{i,k} = \sum_{j\in\mathcal{N}_i^{in}} w^b_{ij}(\hat{y}_{j,k} - \hat{y}_{i,k}),$$

which implies that the compressed communication scheme (9) seems similar to the schemes in [6], [15], [22]. Note that in [6], [15], [22], $\hat{\lambda}_{j,k}$ is obtained via $\hat{\lambda}_{j,k} = \hat{\lambda}_{j,k-1} + \beta\mathcal{C}^a_{j,k}$ for each agent i. However, our compressed communication scheme does not require additional updating and maintaining of $|\mathcal{N}_i^{in}|$ auxiliary equations for each agent i and hence is highly resource-saving.

In what follows, we first define the estimation error as:

$$e^\lambda_{i,k} \triangleq \hat{\lambda}_{i,k} - \lambda_{i,k},\quad e^y_{i,k} \triangleq \hat{y}_{i,k} - y_{i,k}. \qquad (10)$$

In light of (8), (9), one has

$$\begin{aligned}
e^\lambda_{i,k} &= \beta C_\varphi(\lambda_{i,k} - \hat{\lambda}_{i,k-1}) - (\lambda_{i,k} - \hat{\lambda}_{i,k-1})\\
&= \beta\big(C_\varphi(\lambda_{i,k} - \hat{\lambda}_{i,k-1}) - (\lambda_{i,k} - \hat{\lambda}_{i,k-1})\big)\\
&\quad + (\beta - 1)(\lambda_{i,k} - \hat{\lambda}_{i,k-1}),\\
e^y_{i,k} &= \beta C_\varphi(y_{i,k} - \hat{y}_{i,k-1}) - (y_{i,k} - \hat{y}_{i,k-1})\\
&= \beta\big(C_\varphi(y_{i,k} - \hat{y}_{i,k-1}) - (y_{i,k} - \hat{y}_{i,k-1})\big)\\
&\quad + (\beta - 1)(y_{i,k} - \hat{y}_{i,k-1}). \qquad (11)
\end{aligned}$$

It follows from (8)-(11) that

$$\begin{cases}
\lambda_{i,k+1} = \lambda_{i,k} + \alpha \displaystyle\sum_{j\in\mathcal{N}_i^{in}} w^a_{ij}(\lambda_{j,k} - \lambda_{i,k}) + \epsilon y_{i,k}\\
\qquad + \alpha \displaystyle\sum_{j\in\mathcal{N}_i^{in}} w^a_{ij}(e^\lambda_{j,k} - e^\lambda_{i,k})\\
y_{i,k+1} = y_{i,k} + \alpha \displaystyle\sum_{j\in\mathcal{N}_i^{in}} w^b_{ij}(y_{j,k} - y_{i,k})\\
\qquad - (p_{i,k+1} - p_{i,k}) + \alpha \displaystyle\sum_{j\in\mathcal{N}_i^{in}} w^b_{ij}(e^y_{j,k} - e^y_{i,k})\\
p_{i,k+1} = p_i(\lambda_{i,k+1}),\ i\in\mathcal{V}.
\end{cases} \qquad (12)$$

For the sake of convenience, we denote

$$\begin{aligned}
&\lambda_k \triangleq \mathrm{col}_N\{\lambda_{i,k}\},\ y_k \triangleq \mathrm{col}_N\{y_{i,k}\},\ p_k \triangleq \mathrm{col}_N\{p_{i,k}\},\\
&e^\lambda_k \triangleq \mathrm{col}_N\{e^\lambda_{i,k}\},\ e^y_k \triangleq \mathrm{col}_N\{e^y_{i,k}\}, \qquad (13)\\
&W^s_\alpha \triangleq \alpha W^s + (1-\alpha)I_N, \bar{W}^s_\alpha \triangleq W^s_\alpha - I_N, s\in\{a,b\}.
\end{aligned}$$

Then, the augmented form of (12) is written as

$$\lambda_{k+1} = W^a_\alpha\lambda_k + \epsilon y_k + \bar{W}^a_\alpha e^\lambda_k,$$

$$y_{k+1} = W^b_\alpha y_k - (p_{k+1} - p_k) + \bar{W}^b_\alpha e^y_k. \qquad (14)$$

To be more specific, the proposed compressed distributed optimization (CDO) algorithm is presented in Algorithm 1.

Algorithm 1: CDO Algorithm

► **Initialization**:
1. Given the local supply $p_{i,0} \in [p_i^{\min}, p_i^{\max}]$, the local demand p_i^d, scalars α, ϵ, φ, and β, where $i\in\mathcal{V}$;
2. Calculate the incremental cost $\lambda_{i,0}$, and the mismatch $y_{i,0}$ by (7). Set $\hat{\lambda}_{i,-1}, \hat{y}_{i,-1}, z^a_{i,-1}, z^b_{i,-1}$ as 0.

► **Loop**:
1. Agent i generates compressed signals $\mathcal{C}^a_{i,k}$ and $\mathcal{C}^b_{i,k}$, and transforms them into codewords;
2. Agent i sends codewords to its neighboring nodes, and receives information from its neighboring nodes, simultaneously;
3. Agent i calculates $z^a_{i,k}$ and $z^b_{i,k}$ by (9); updates $\lambda_{i,k+1}$ and $y_{i,k+1}$ by (8);
4. If $|\lambda_{i,k+1} - \lambda_{i,k}| < \chi_1$ and $|y_{i,k+1}| < \chi_2$ for $\forall i\in\mathcal{V}$, where χ_1, χ_2 are the accuracy tolerance, break.

► **Output**:
1. $\lambda_{i,k}$ and $p_{i,k}$.

Remark 2: In (9), agent i is equipped with a pair of estimator-like dynamic equations, which is independent of the number of neighbors. It should be emphasized that, in (9), the signal $\lambda_{i,k} - \hat{\lambda}_{i,k-1}$ can be regarded as "innovation". Intuitively, the amplitude of the innovation is much smaller than that of the original signal $\lambda_{i,k}$, leading to reduced data transmission. Therefore, compared to direct variable compression, innovation compression is more efficient. Furthermore, it follows from (4) and (11) that if the innovation eventually approaches zero, the estimation error in (10) will also decrease to zero. The convergence analysis of $e^\lambda_{i,k}$ and $e^y_{i,k}$ in (10) will be shown in Section III-C. Consequently, the proposed CDO algorithm can ensure accurate convergence without the effects of compression errors.

B. Supporting Lemmas

Lemma 1: [44] Under Assumption 1, the matrices W^a and W^b both have a single eigenvalue of 1, and the corresponding unique left and right eigenvectors are u^T and v satisfying $u^T\mathbf{1}_N = N$ and $\mathbf{1}_N^T v = N$. Moreover, the i-th element of vector $u\in\mathbb{R}^n$ (or $v\in\mathbb{R}^n$) is nonzero only when the corresponding i-th node is the root node, and thus $u^Tv > 0$.

Lemma 2: [44] Under Assumption 1, the spectral radii of matrices $W_a - 1/N\mathbf{1}_N u^T$ and $W_b - 1/N\mathbf{1}_N v^T$ are $\rho(W_a - 1/N\mathbf{1}_N u^T) \triangleq \rho_a < 1$ and $\rho(W_b - 1/N\mathbf{1}_N v^T) \triangleq \rho_b < 1$. Moreover, there exist two norms $\|\cdot\|_a$, and $\|\cdot\|_b$ satisfying $\tau_a \triangleq \|W_a - 1/N\mathbf{1}_N u^T\|_a < 1$, $\tau_b \triangleq \|W_b - 1/N\mathbf{1}_N v\mathbf{1}_N^T\|_b < 1$, and $\tau_a = \rho_a + \varpi_a$, $\tau_b = \rho_b + \varpi_b$, where $\varpi_a > 0$, $\varpi_b > 0$ are arbitrarily close to 0.

Lemma 3: [34] For all $A\in\mathbb{R}^{n\times p}$, there exist two constants $\delta_a, \delta_b > 1$ such that $\|A\|_2 \le \|A\|_a \le \delta_a\|A\|_2$, $\|A\|_2 \le \|A\|_b \le \delta_b\|A\|_2$.

C. Convergence Analysis

Denote

$$
\begin{aligned}
&\bar{\lambda}_k \triangleq \frac{1}{N}u^T\lambda_k,\ \bar{y}_k \triangleq \frac{1}{N}\mathbf{1}_N^T y_k,\ g_k \triangleq \frac{1}{N}\sum_{i=1}^{N}(p_i(\bar{\lambda}_k)-p_i^d),\\
&\tilde{\lambda}_k \triangleq \lambda_k - \mathbf{1}_N\bar{\lambda}_k = H_a\lambda_k,\ H_a = I - \frac{1}{N}\mathbf{1}_N u^T,\\
&\tilde{y}_k \triangleq y_k - v\bar{y}_k = H_b y_k,\ H_b = I - \frac{1}{N}v\mathbf{1}_N^T,\ \vec{\lambda}_k = \bar{\lambda}_k - \lambda^\star,\\
&\underline{\gamma} \triangleq \left\{\frac{1}{2a_1},\ldots,\frac{1}{2a_N}\right\}^-,\ \bar{\gamma} \triangleq \left\{\frac{1}{2a_1},\ldots,\frac{1}{2a_N}\right\}^+. \quad (15)
\end{aligned}
$$

In light of Lemma 1, we have from (14) that

$$
\begin{aligned}
\bar{\lambda}_{k+1} &= \frac{1}{N}u^T(W_\alpha^a\lambda_k + \epsilon y_k + \bar{W}_\alpha^a e_k^\lambda)\\
&= \bar{\lambda}_k + \frac{\epsilon}{N}u^T y_k\\
&= \bar{\lambda}_k - \frac{\epsilon}{N}u^T v g_k + \frac{\epsilon}{N}u^T v(\bar{y}_k + g_k)\\
&\quad + \frac{\epsilon}{N}u^T(y_k - v\bar{y}_k),\\
\bar{y}_{k+1} &= \frac{1}{N}\mathbf{1}_N^T(W_\alpha^b y_k - (p_{k+1}-p_k) + \bar{W}_\alpha^b e_k^y)\\
&= \bar{y}_k - \frac{1}{N}\sum_{i=1}^{N}(p_{i,k+1}-p_{i,k}), \quad (16)
\end{aligned}
$$

where $u^T W_\alpha^a = u^T$, $u^T\bar{W}_\alpha^a = \mathbf{0}_N^T$, $\mathbf{1}_N^T W_\alpha^b = \mathbf{1}_N^T$, and $\mathbf{1}_N^T\bar{W}_\alpha^b = \mathbf{0}_N^T$. For given $y_{i,0}$ in (7), we have

$$
\begin{aligned}
\bar{y}_{k+1} + \frac{1}{N}\sum_{i=1}^{N}(p_{i,k+1}-p_i^d) &= \bar{y}_k + \frac{1}{N}\sum_{i=1}^{N}(p_{i,k}-p_i^d)\\
&= \frac{1}{N}\sum_{i=1}^{N}(y_{i,0}+p_{i,0}-p_i^d)\\
&= 0,
\end{aligned}
$$

and derive

$$
\bar{y}_k = -\frac{1}{N}\sum\nolimits_{i=1}^{N}(p_{i,k+1}-p_i^d). \quad (17)
$$

In view of (14), (16), we have

$$
\begin{aligned}
\tilde{\lambda}_{k+1} &= W_\alpha^a\lambda_k - \mathbf{1}_N\bar{\lambda}_k + \epsilon(I - \frac{1}{N}\mathbf{1}_N u^T)y_k + \bar{W}_\alpha^a e_k^\lambda\\
&= \Big(W_\alpha^a - \frac{1}{N}\mathbf{1}_N u^T\Big)\tilde{\lambda}_k + \epsilon H_a y_k + \bar{W}_\alpha^a e_k^\lambda, \quad (18)\\
\tilde{y}_{k+1} &= W_\alpha^b y_k - v\bar{y}_k - (I_N - \frac{1}{N}v\mathbf{1}_N^T)(p_{k+1}-p_k) + \bar{W}_\alpha^b e_k^y\\
&= \Big(W_\alpha^b - \frac{1}{N}v\mathbf{1}_N^T\Big)\tilde{y}_k - H_b(p_{k+1}-p_k) + \bar{W}_\alpha^b e_k^y. \quad (19)
\end{aligned}
$$

In the following, a crucial lemma is given by constructing a linear system of inequalities that is related to the terms $\|\vec{\lambda}_k\|_2$, $\|\tilde{\lambda}_k\|_a$, $\|\tilde{y}_k\|_b$, $\|e_k^\lambda\|_2$, and $\|e_k^y\|_2$.

Lemma 4: Under Assumption 1, if $\epsilon \le \frac{2N^2}{(N\bar{\gamma}+\underline{\gamma})(u^T v)}$, then we have

$$
\psi_{k+1} \preceq \Pi\psi_k \quad (20)
$$

where "$\preceq$" is element-wisely less than or equal to, $\psi_k \triangleq \begin{bmatrix} \|\vec{\lambda}_k\|_2 & \|\tilde{\lambda}_k\|_a & \|\tilde{y}_k\|_b & \|e_k^\lambda\|_2 & \|e_k^y\|_2 \end{bmatrix}^T$, and the transition matrix $\Pi \triangleq [\pi_{ij}]_5$ is

$$
\Pi = \begin{bmatrix}
1-\frac{\underline{\gamma}u^T v}{N^2}\epsilon & \frac{\bar{\gamma}u^T v}{N\sqrt{N}}\epsilon & \frac{\|u\|_2}{N}\epsilon & 0 & 0\\
\bar{\gamma}\delta_a\|v\|_2\epsilon & \pi_{22} & \delta_a\epsilon & 2\delta_a\alpha & 0\\
\bar{\gamma}^2\delta_b\|v\|_2\epsilon & \pi_{32} & \pi_{33} & 2\bar{\gamma}\delta_b\alpha & 2\delta_b\alpha\\
\bar{\gamma}\|v\|_2\varphi_\beta\epsilon & \pi_{42} & \varphi_\beta\epsilon & \pi_{44} & 0\\
\bar{\gamma}^2\|v\|_2\varphi_\beta\epsilon & \pi_{52} & \pi_{53} & 0 & \pi_{55}
\end{bmatrix}
$$

with

$$
\begin{aligned}
\pi_{22} &= 1-(1-\tau_a)\alpha + \frac{\delta_a\bar{\gamma}\|v\|_2}{\sqrt{N}}\epsilon, \pi_{32} = \Big(2\alpha + \frac{\bar{\gamma}\|v\|_2}{\sqrt{N}}\epsilon\Big)\bar{\gamma}\delta_b,\\
\pi_{33} &= 1-(1-\tau_b)\alpha + \bar{\gamma}\delta_b\epsilon,\ \pi_{42} = \Big(2\alpha + \frac{\bar{\gamma}\|v\|_2}{\sqrt{N}}\epsilon\Big)\varphi_\beta,\\
\pi_{44} &= (2\alpha+1)\varphi_\beta,\ \pi_{52} = \bar{\gamma}\pi_{42},\ \pi_{53} = \bar{\gamma}\varphi_\beta\epsilon + 2\varphi_\beta\alpha,\\
\pi_{55} &= (2\alpha+1)\varphi_\beta,\ \varphi_\beta = 1-(1-\varphi)\beta.
\end{aligned}
$$

Proof: See Appendix A. ∎

It is observed from (20) that ψ_k linearly converge to 0 at least at rate $\mathcal{O}(\rho(\Pi)^k)$ if the spectral radius of Π satisfies $\rho(\Pi) < 1$. The next lemma presents a sufficient condition under which $\rho(\Pi) < 1$.

Lemma 5: [17] For a nonnegative matrix $\Pi \in \mathbb{R}^{m\times m}$ and an element-wisely positive vector $\zeta \in \mathbb{R}^m$, if $\Pi\zeta \le \varrho\zeta$, then $\rho(\Pi) \le \varrho$ with $\varrho \in (0,1)$.

Theorem 1: Under Assumption 1, if $\beta \in (0,1]$, $\varphi \in [0,1)$,

$$
\begin{aligned}
\epsilon &\le \left\{\frac{(1-\tau_a)\alpha\zeta_2}{2m_1},\ \frac{(1-\tau_b)\alpha\zeta_3}{2m_2},\ \frac{m_4}{m_3},\ \frac{m_6}{m_5}\right\}^-,\\
\alpha &\le \left\{1,\ \frac{(1-\varphi)\beta\zeta_4}{2\varphi_\beta(\zeta_2+\zeta_4)},\ \frac{(1-\varphi)\beta\zeta_5}{2\varphi_\beta(\bar{\gamma}\zeta_2+\zeta_3+\zeta_5)}\right\}^-, \quad (21)
\end{aligned}
$$

where $m_1 - m_6$ are given in (31)-(34) and $\zeta_1 - \zeta_5$ satisfy (36), then the spectral radius of Π satisfies $\rho(\Pi) \le 1 - \frac{\underline{\gamma}u^T v}{2N^2}\epsilon$, and thus the proposed algorithm (8), (9) linearly converges to $\lambda^\star$ at least with the rate $\mathcal{O}\big((1-\frac{\underline{\gamma}u^T v}{2N^2}\epsilon)^k\big)$.

Proof: See Appendix B. ∎

Remark 3: Up to now, we have derived a distributed optimization algorithm under compressed communication that effectively solves the ED problem with an exponential convergence rate while enhancing communication efficiency. Compared with the numerous existing studies, our work presents four distinctive novelties as follows. 1) Unlike the static compressed scheme, the CDO algorithm can achieve exact convergence to $\lambda^\star$ in absence of compression error effects; 2) Compared to the existing dynamic compressed schemes in [6], [15], [22], our scheme does not require additional updating and maintaining of $|\mathcal{N}_i^{in}|$ auxiliary equations for each agent i and hence is highly resource-saving; 3) In contrast to the strongly connected condition in [43], [48], [50], our assumption is much weaker; and 4) Compared with the existing results [29], [42], [54], our algorithm has an advantage in fast convergence at a geometric rate due to the constant step size. In summary, our paper provides an efficient distributed scheme for addressing the DED issue in microgrids. Future research can explore adaptive compression schemes that dynamically

TABLE I
COEFFICIENTS OF 10 DERS [43]

DERi	$a_i(\$/kW^2h)$	$b_i(\$/kWh)$	$c_i(\$)$	$p_i^{\min}(kW)$	$p_i^{\max}(kW)$
$1,2$	0.074	3.17	62	4.2	18
$3,4$	0.082	4.02	42	5.4	45
$5,6$	0.105	2.53	78	3.8	40
$7,8$	0.094	1.22	51	10	80
$9,10$	0.078	3.41	31	8	60

balance communication efficiency and algorithm performance [23], [24], [26], [28], [39]. Integrating renewable generation forecasting into the DO framework could enhance dispatch accuracy under uncertainty [1], [8], [25], [36], [45].

IV. NUMERICAL EXAMPLES

In this section, we evaluate the validity of the derived results through numerical examples. As depicted in Fig. 1, we consider the IEEE 33-bus distribution network with 10 DERs, and each DER is governed by an agent. In Fig. 1, the communication network is represented by blue dashed lines. Since each agent updates two iterates over two different communication networks in Algorithm 1, the second communication network in this study is identical to the first network in Fig. 1 with the directions of edges reversed. The DER coefficients are provided in Table I.

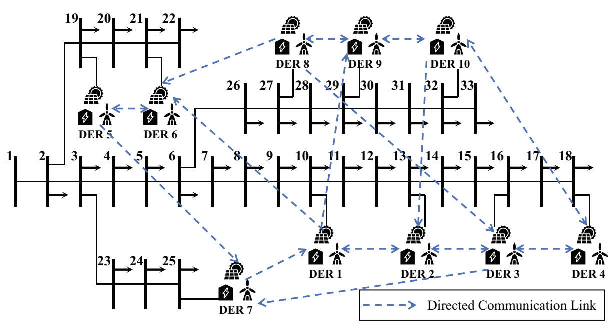

Fig. 1. Developed microgrid model based on a modified IEEE 33-bus distribution network

A. Case Study 1: Convergence Test

In this case, the local demand p_i^d $(i \in \mathcal{N}_i)$ is set as $15kW$, $16kW$, $30kW$, $18kW$, $40kW$, $10kW$, $28kW$, $42kW$, $20kW$, and $21kW$, respectively, and the total demand is $\sum_{i=1}^{10} P_i^d = 240kW$. The local power outputs $p_{i,0}(i \in \mathcal{N}_i)$ are chosen as $10kW$, $15kW$, $18kW$, $14kW$, $17kW$, $16kW$, $16kW$, $15kW$, $22kW$, and $26kW$, respectively. In light of the expression in (5), the optimal solution is calculated as $\lambda^\star = 7.39$, and $p_1^\star = p_2^\star = 18kW$, $p_3^\star = p_4^\star = 20.54kW$, $P_5^\star = P_6^\star = 23.14kW$, $P_7^\star = P_8^\star = 32.81kW$, and $P_9^\star = P_{10}^\star = 25.51kW$, respectively. The logarithmic quantization scheme [6] is considered, as described in Remark 1.

We set $\beta = 1$, $\alpha = 0.3$, $\epsilon = 10^{-4}$, and $\varphi = 0.1$ in light of Theorem 1. The simulation results are plotted in Figs. 2-4. Fig. 2 depicts the trajectories of the incremental cost $\lambda_{i,k}$, local mismatch $y_{i,k}$, and the local supply $p_{i,k}$, and total supply $\sum_{i=1}^{N} p_{i,k}$. It is observed that the incremental cost achieves consensus and converges to the optimal value $\lambda^\star$, the mismatch decreases to zero, the local power reaches the optimal solution $p_i^\star$, and the total supply-demand mismatch of the microgrids is eliminated, demonstrating the validity of the derived results.

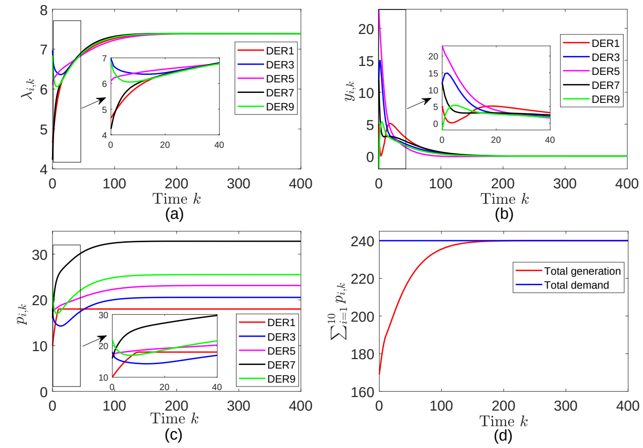

Fig. 2. Numerical results showcasing the following items. (a) Incremental cost $\lambda_{i,k}$; (b) Local mismatch $y_{i,k}$; (c) Local supply $p_{i,k}$; (d) Supply-demand balance.

Fig. 3 shows the dynamics of compressed signals $r_{i,k}^\lambda$ and $r_{i,k}^y$, where $r_{i,k}^\lambda \triangleq C_\varphi(\lambda_{i,k} - \hat{\lambda}_{i,k-1})$, $r_{i,k}^y \triangleq C_\varphi(y_{i,k} - \hat{y}_{i,k-1})$, along with the estimation errors $e_{i,k}^\lambda$, $e_{i,k}^y$. Notably, $r_{i,k}^\lambda$, $r_{i,k}^y$, $e_{i,k}^\lambda$ and $e_{i,k}^y$ all converge to zero, indicating that the constructed distributed algorithm can achieve exact convergence without the effects of compression errors.

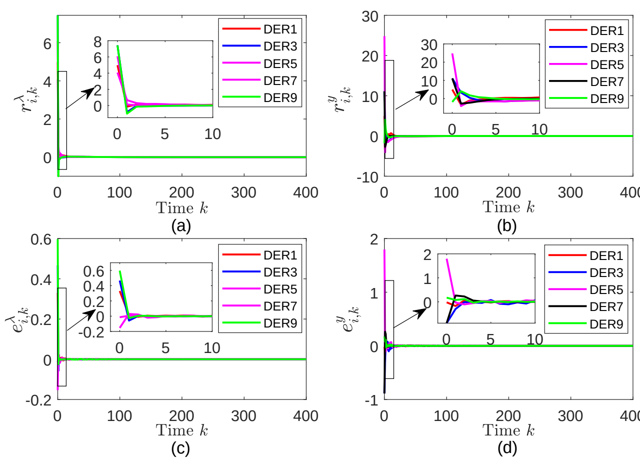

Fig. 3. Numerical results showing the following performance indices. (a) Compressed signal $r_{i,k}^\lambda$; (b) Compressed signal $r_{i,k}^y$; (c) Estimation error $e_{i,k}^\lambda$; (d) Estimation error $e_{i,k}^y$.

Next, we use the value of s, defined in Remark 1, to describe the amount of transmitted data, and define the total size of transmitted data at time instant k as $\sum_{i=0}^{10} s_{i,k}$. Note that the larger the absolute value of $\sum_{i=0}^{10} s_{i,k}$, the more data need to be transmitted. In Fig. 4, we test different compressed rates φ

corresponding to the total amount of transmitted data at time instant k. It is found that a larger compression rate leads to less data being transmitted.

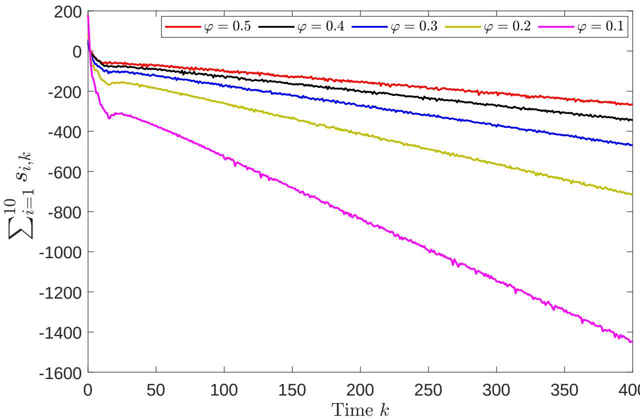

Fig. 4. Comparison of the amount of transmitted data for different compressed rate φ.

Denote the total mismatch as $\Delta p_k \triangleq \sum_{i=1}^{10}(p_{i,k} - p_i^d)$. Fig. 5 plots the convergence rate (speed) of the proposed algorithm under different compression rates φ and different initial local supply $p_{i,0}(i \in \mathcal{V})$. Intuitively, the convergence rate corresponds to the slope of the curve on a log scale. It is observed that all the lines become parallel after a certain point, indicating that the convergence rate remains unchanged across different compression rates and different initial conditions, which is consistent with our theoretical results.

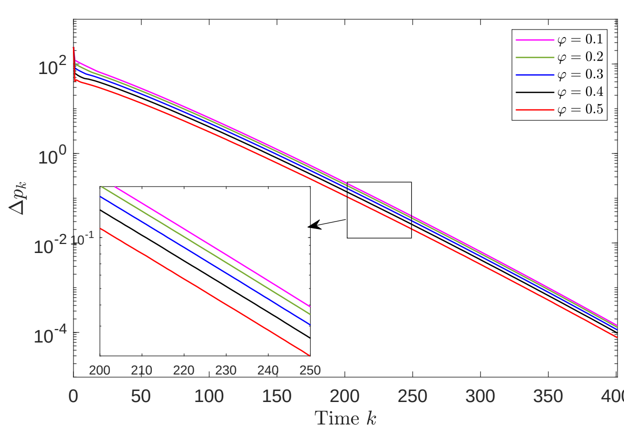

Fig. 5. Comparison of the convergence rate for different compression rates φ and different initial conditions.

In this following, we test the case of time-varying demand. Assume that the total demand is changed from $240kW$ to $190kW$ at time instant $k = 400$, and then to $280kW$ at time instant $k = 800$. As shown in Fig. 6, the results are the same as in Fig. 2 before 400 iterations, after that the incremental cost and local supply converge to the new optimal values as demand changes. In Fig. 6, the output power of DER 1 is always $18kW$ due to generation capacity constraints. Simulation results verify that the proposed algorithm is capable of rapidly adapting to demand variations, and such a feature is of practical significance in the context of high renewable energy penetration.

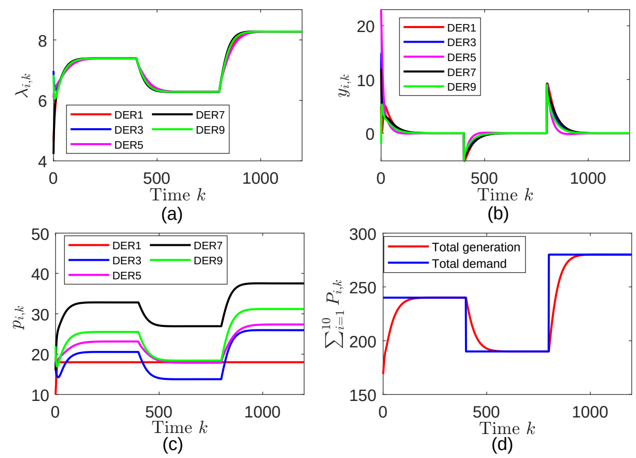

Fig. 6. Numerical results with time-varying demand. (a) Incremental cost $\lambda_{i,k}$; (b) Local mismatch $y_{i,k}$; (c) Local supply $p_{i,k}$; (d) Supply-demand balance.

To verify that the proposed CDO algorithm can solve the large-scale ED problem, we consider a test system with 50 nodes. Fig. 7 plots the convergence rate between 50-node and 10-node test systems. In this simulation, the 50-node and 10-node communication topologies contain 122 and 21 edges, respectively. To reach the same level of error $\Delta p_k \leq 10^{-3}$, the total communication overhead of the 50-node network is 5.8 times that of the 10-node network, which shows good scalability of the proposed algorithm.

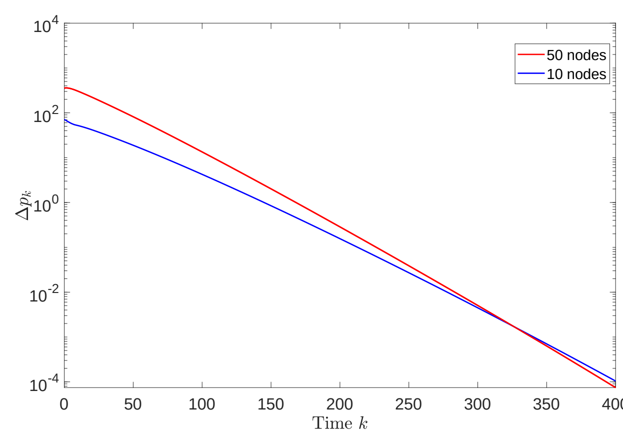

Fig. 7. Comparison of the convergence rate between 50-node and 10-node test systems.

B. Case Study 2: Communication Overhead Test

In this case, we consider a 3-dimensional ED problem. To verify the theoretical results, we test the CDO algorithm under different compressors (i.e., the biased, logarithmic, and greedy (Top-l) quantization schemes as shown in Remark

1). The simulation results are shown in Fig. 8. In order to avoid the complete overlap of the four curves, their vertical values are multiplied by 1, 0.5, 0.25, and 2, respectively. This adjustment is performed solely for visualization clarity. In Fig. 8, it is observed that all curves are parallel and linearly converge to 0, which confirms that the convergence rate of the algorithm is independent of compression operators, provided that the other parameters are identical and feasible. Furthermore, in this simulation, a scalar is set as $b = 32$ bits to be transmitted with sufficient precision. Transmitting $C_\varphi(x)$ needs 41, 21, and 32 bits under biased, logarithmic, and greedy (Top-l) quantization, respectively. Hence, the biased, logarithmic, and greedy (Top-l) quantization schemes need, respectively, 42.7%, 21.9%, and 33.3% of the bits used by the regular distributed algorithms without compression to reach the same level of error.

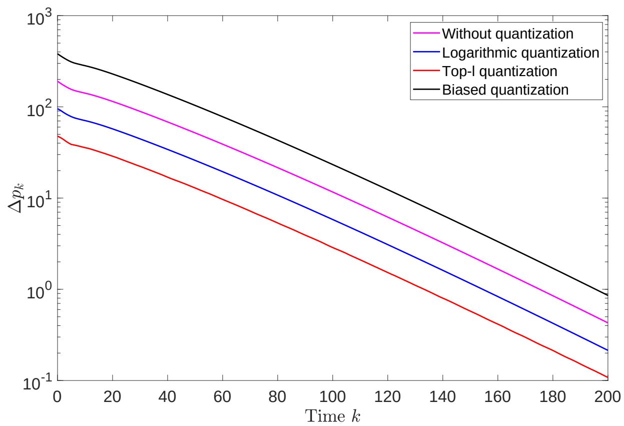

Fig. 8. Comparison of the convergence rate between uncompressed and compressed communication with different compressors.

C. Case Study 3: Comparisons with Existing Results

In this case study, we first compare the convergence performance of the proposed CDO algorithm with that of distributed ED algorithm based on static compressed communication (i.e., using $C_\varphi(\lambda_{i,k})$, $C_\varphi(y_{i,k})$) to replace $\lambda_{i,k}$, $y_{i,k}$ in (6)) under the same compression rate. Fig. 9 plots the dynamic evolutions of incremental cost, power mismatch, and corresponding compression errors under the static compression scheme, where $\hat{e}^{\lambda}_{i,k} \triangleq C_\varphi(\lambda_{i,k}) - \lambda_{i,k}$, $\hat{e}^{y}_{i,k} \triangleq C_\varphi(y_{i,k}) - y_{i,k}$ are defined as compression errors. Due to compression error effects, all incremental costs $\lambda_{i,k}$ neither achieve consensus nor converge to the optimal value $\lambda^\star$, and the compression error $\hat{e}^{\lambda}_{i,k}$ cannot approach 0. Compared to static compressed scheme in Fig. 9, the proposed CDO algorithm has a merit in achieving exact convergence to $\lambda^\star$ without compression error effects.

In the following, to emphasize the convergence rate, a comparative analysis is conducted among four DED algorithms as follows: 1) CDO: the proposed compressed distributed optimization algorithm; 2) PDPD: the push-based distributed primal-dual algorithm [54]; 3) DPG: the distributed push-gradient method [29]; and 4) PDDA: the push-based distributed dual averaging algorithm [42]. The evolutions of the total mismatch between supply and demand are presented in Fig. 10. Among four algorithms, the proposed algorithm shows an advantage in fast convergence.

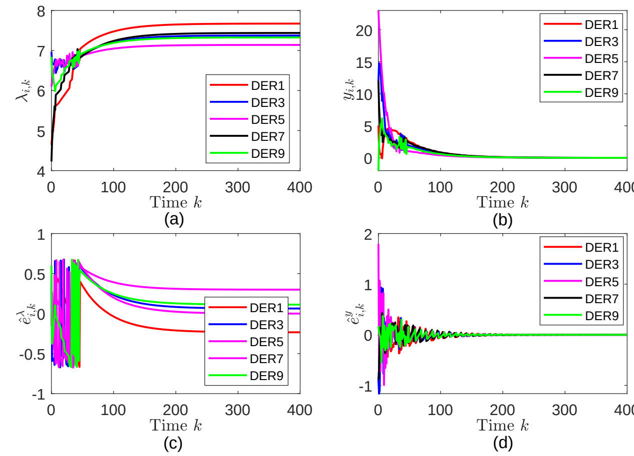

Fig. 9. Numerical results of the static compression scheme. (a) Incremental cost $\lambda_{i,k}$; (b) Local mismatch $y_{i,k}$; (c) Compression error $\hat{e}^{\lambda}_{i,k}$; (d) Compression error $\hat{e}^{y}_{i,k}$.

In light of the theoretical results, the convergence rates of the four algorithms are at least $\mathcal{O}\big(o^k\big)$ $(o = (1 - \frac{\overline{\gamma} u^T v}{2N^2}\epsilon) \in (0,1))$, $\mathcal{O}\big(\frac{\ln(k)}{\sqrt{k}}\big)$, $\mathcal{O}\big(\frac{1}{\sqrt{k}}\big)$, and $\mathcal{O}\big(\frac{1}{\sqrt{k}}\big)$, respectively. Note that the existing algorithms [29], [42], [54] introduce a *diminishing* step size to ensure accurate convergence, while the diminishing step size reduces the convergence speed of the algorithm. In our work, a constant step size combined with an auxiliary equation is used to ensure the linear convergence of the proposed algorithm by virtue of the spectral radius criterion for stability and the properties of matrix norm.

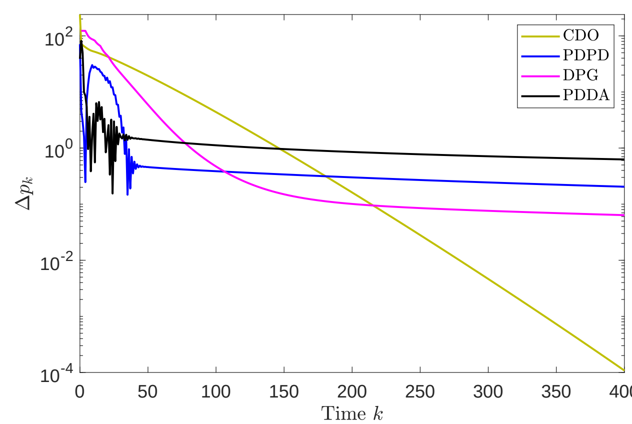

Fig. 10. Comparisons of existing DED algorithms.

V. Conclusion

In this work, the DED issue of under communication constraints has been solved. A novel CDO algorithm has been developed to achieve optimal power dispatch by utilizing an estimator-like dynamic compression scheme that effectively eliminates the impact of compression errors on convergence accuracy. Furthermore, a sufficient condition has been derived concerning algorithm parameters and communication topologies, under which the proposed algorithm converges linearly to the optimal solution under *arbitrary* compression rate. Finally, numerical studies have validated the fast and exact convergence as well as communication efficiency of the developed algorithm. Future research topics include the extension of the main results to more complex ED model with valve-point effects, transmission losses, or security constraints under time-varying or switching topologies, and other types of compressors, such as those with bounded absolute errors or adaptive and learning-based mechanisms [39].

Appendix A
Proof of Lemma 4

A. Supporting Lemma

Lemma 6: If $\epsilon \leq \frac{2N^2}{(N\overline{\gamma}+\underline{\gamma})(u^Tv)}$, the following inequalities hold:

$$\|\bar{y}_k+g_k\|_2 \leq \frac{\overline{\gamma}}{\sqrt{N}}\|\tilde{\lambda}_k\|_2,\ \ \|g_k\|_2 \leq \overline{\gamma}\|\vec{\lambda}_k\|_2,$$
$$\Big\|\vec{\lambda}_k-\frac{\epsilon}{N}u^Tvg_k\Big\|_2 \leq \Big(1-\frac{\epsilon\underline{\gamma}}{N^2}u^Tv\Big)\|\vec{\lambda}_k\|_2.$$

Proof: In light of the definition of $p_i(\cdot)$ in (6), we have from (15), (17) that

$$\begin{aligned}\|\bar{y}_k+g_k\|_2 =& \left\|\frac{1}{N}\sum_{i=1}^{N}\big(p_i(\bar{\lambda}_k)-p_i(\lambda_{i,k})\big)\right\|_2\\ \leq& \frac{\overline{\gamma}}{N}\sum_{i=1}^{N}\|\lambda_{i,k}-\bar{\lambda}_k\|_2\\ \leq& \frac{\overline{\gamma}}{\sqrt{N}}\|\lambda_k-\mathbf{1}_N\bar{\lambda}_k\|_2=\frac{\overline{\gamma}}{\sqrt{N}}\|\tilde{\lambda}_k\|_2,\end{aligned}$$

where the first "$\leq$" holds based on the non-expansiveness of projection operators [33].

Due to the fact that $\sum_{i=1}^{N}p_i^d=\sum_{i=1}^{N}p_i(\lambda^\star)$, we obtain

$$\begin{aligned}\|g_k\|_2 =& \left\|\frac{1}{N}\sum_{i=1}^{N}\big(p_i(\bar{\lambda}_k)-p_i^d\big)\right\|_2\\ =& \left\|\frac{1}{N}\sum_{i=1}^{N}\big(p_i(\bar{\lambda}_k)-p_i(\lambda^\star)\big)\right\|_2\\ \leq& \frac{1}{N}\sum_{i=1}^{N}\|p_i(\bar{\lambda}_k)-p_i(\lambda^\star)\|_2 \leq \overline{\gamma}\|\vec{\lambda}_k\|_2.\end{aligned}$$

Based on the assumption in (2), there exists at least a $p_i(\bar{\lambda}_k)\in(p_i^{\min},\ p_i^{\max})$. In light of the definition of $p_i(\cdot)$ in (6), we have for $\bar{\lambda}_k$ and $\bar{\lambda}_k'$ ($\bar{\lambda}_k\neq\bar{\lambda}_k'$) that

$$\begin{aligned}(g_k-g_k')(\bar{\lambda}_k-\bar{\lambda}_k') =&\frac{1}{N}\sum_{i=1}^{N}\big(p_i(\bar{\lambda}_k)-p_i(\bar{\lambda}_k')\big)(\bar{\lambda}_k-\bar{\lambda}_k')\\ \geq&\frac{\underline{\gamma}}{N}(\bar{\lambda}_k-\bar{\lambda}_k')^2.\end{aligned}\tag{22}$$

In light of the non-expansiveness of projection operators, we have

$$\begin{aligned}\|g_k-g_k'\| =& \frac{1}{N}\Big\|\sum_{i=1}^{N}\big(p_i(\bar{\lambda}_k)-p_i(\bar{\lambda}_k')\big)\Big\|\\ \leq& \frac{1}{N}\sum_{i=1}^{N}\Big\|\big(p_i(\bar{\lambda}_k)-p_i(\bar{\lambda}_k')\big)\Big\|\\ \leq& \frac{1}{N}\sum_{i=1}^{N}\Big\|\frac{1}{2a_i}(\bar{\lambda}_k-\bar{\lambda}_k')\Big\|\\ \leq& \overline{\gamma}\|\bar{\lambda}_k-\bar{\lambda}_k'\|.\end{aligned}\tag{23}$$

Based on (22), (23), it follows from Lemma 10 of [35] that if $\frac{\epsilon}{N}u^Tv\leq\frac{2}{\overline{\gamma}+\underline{\gamma}/N}$ (i.e., $\epsilon\leq\frac{2N^2}{(N\overline{\gamma}+\underline{\gamma})u^Tv}$), we have

$$\Big\|\vec{\lambda}_k-\frac{\epsilon}{N}u^Tvg_k\Big\|_2 \leq \Big(1-\frac{\epsilon\underline{\gamma}}{N^2}u^Tv\Big)\|\vec{\lambda}_k\|_2.$$

■

B. Proof of Lemma 4

Proof: In light of Lemmas 1, 3, and 6, we have from (15), (16) that

$$\begin{aligned}\|\vec{\lambda}_{k+1}\|_2 =& \Big\|\vec{\lambda}_k-\frac{\epsilon}{N}u^Tvg_k+\frac{\epsilon}{N}u^Tv(\bar{y}_k+g_k)\\ &+\frac{\epsilon}{N}u^T(y_k-v\bar{y}_k)\Big\|_2\\ \leq& \Big\|\vec{\lambda}_k-\frac{\epsilon}{N}u^Tvg_k\Big\|_2+\frac{\epsilon u^Tv}{N}\Big\|\bar{y}_k+g_k\Big\|_2\\ &+\frac{\epsilon\|u\|_2}{N}\Big\|y_k-v\bar{y}_k\Big\|_2\\ \leq& \Big(1-\frac{\underline{\gamma}u^Tv}{N^2}\epsilon\Big)\|\vec{\lambda}_k\|_2+\frac{\overline{\gamma}u^Tv}{N\sqrt{N}}\epsilon\|\tilde{\lambda}_k\|_a\\ &+\frac{\|u\|_2}{N}\epsilon\|\tilde{y}_k\|_b.\end{aligned}\tag{24}$$

Based on Lemmas 2, 3, and 6, it follows from (18) that

$$\begin{aligned}&\|\tilde{\lambda}_{k+1}\|_a\\ \leq& \Big\|(W_\alpha^a-\frac{1}{N}\mathbf{1}_Nu^T)\tilde{\lambda}_k\Big\|_a+\epsilon\|H_ay_k\|_a+\delta_a\|\bar{W}_\alpha^ae_k^\lambda\|_2\\ \leq& \big(1-(1-\tau_a)\alpha\big)\|\tilde{\lambda}_k\|_a+\delta_a\epsilon\|y_k-v\bar{y}_k\|_b\\ &+\delta_a\epsilon\|v\bar{y}_k+vg_k\|_2+\delta_a\epsilon\|vg_k\|_2+2\delta_a\alpha\|e_k^\lambda\|_2\\ \leq& \Big(1-(1-\tau_a)\alpha+\frac{\delta_a\overline{\gamma}\|v\|_2}{\sqrt{N}}\epsilon\Big)\|\tilde{\lambda}_k\|_a+\overline{\gamma}\delta_a\|v\|_2\epsilon\|\vec{\lambda}_k\|_2\\ &+\delta_a\epsilon\|\tilde{y}_k\|_b+2\delta_a\alpha\|e_k^\lambda\|_2.\end{aligned}\tag{25}$$

Due to $\bar{W}_\alpha^a\lambda_k=\bar{W}_\alpha^a\tilde{\lambda}_k$, we have (14) that

$$\begin{aligned}&\|\lambda_{k+1}-\lambda_k\|_2=\|\bar{W}_\alpha^a\tilde{\lambda}_k+\epsilon y_k+\bar{W}_\alpha^ae_k^\lambda\|_2\\ \leq& 2\alpha\|\tilde{\lambda}_k\|_a+\epsilon\|y_k-v\bar{y}_k+v\bar{y}_k+vg_k-vg_k\|_2+2\alpha\|e_k^\lambda\|_2\\ \leq& \Big(2\alpha+\frac{\overline{\gamma}\|v\|_2}{\sqrt{N}}\epsilon\Big)\|\tilde{\lambda}_k\|_a+\overline{\gamma}\|v\|_2\epsilon\|\vec{\lambda}_k\|_2+\epsilon\|\tilde{y}_k\|_b\\ &+2\alpha\|e_k^\lambda\|_2.\end{aligned}\tag{26}$$

Recalling Lemmas 2, 3, and 6, we have from (19), (26) that

$$\|\tilde{y}_{k+1}\|_b$$

$$\leq \left\|(W_\alpha^b - \frac{1}{N}v\mathbf{1}_N^T)\tilde{y}_k\right\|_b + \|H_b(p_{k+1}-p_k)\|_b + \delta_b\|\bar{W}_\alpha^b e_k^y\|_2$$
$$\leq \big(1-(1-\tau_b)\alpha\big)\|\tilde{y}_k\|_b + \overline{\gamma}\delta_b\|\lambda_{k+1}-\lambda_k\|_2 + 2\delta_b\alpha\|e_k^y\|_2$$
$$\leq \big(1-(1-\tau_b)\alpha + \overline{\gamma}\delta_b\epsilon\big)\|\tilde{y}_k\|_b + \overline{\gamma}^2\delta_b\|v\|_2\epsilon\|\vec{\lambda}_k\|_2 + \Big(2\alpha + \frac{\overline{\gamma}\|v\|_2}{\sqrt{N}}\epsilon\Big)\overline{\gamma}\delta_b\|\tilde{\lambda}_k\|_a + 2\overline{\gamma}\delta_b\alpha\|e_k^\lambda\|_2 + 2\delta_b\alpha\|e_k^y\|_2. \tag{27}$$

It follows from (11), (26) that

$$\begin{aligned}\|e_{k+1}^\lambda\|_2 &\leq \varphi_\beta\|\lambda_{k+1}-\hat{\lambda}_k\|_2\\ &\leq \varphi_\beta\|\lambda_{k+1}-\lambda_k+e_k^\lambda\|_2\\ &\leq (2\alpha+1)\varphi_\beta\|e_k^\lambda\|_2 + \Big(2\alpha + \frac{\overline{\gamma}\|v\|_2}{\sqrt{N}}\epsilon\Big)\varphi_\beta\|\tilde{\lambda}_k\|_a\\ &\quad + \overline{\gamma}\|v\|_2\epsilon\varphi_\beta\|\vec{\lambda}_k\|_2 + \epsilon\varphi_\beta\|\tilde{y}_k\|_b. \end{aligned}\tag{28}$$

It follows from (11), (14) , (26) that

$$\begin{aligned}&\|e_{k+1}^y\|_2\\ &\leq \varphi_\beta\|y_{k+1}-\hat{y}_k\|_2\\ &\leq \varphi_\beta\|y_{k+1}-y_k+e_k^y\|_2\\ &\leq \varphi_\beta\|\bar{W}_\alpha^b\tilde{y}_k-(p_{k+1}-p_k)+(\bar{W}_\alpha^b+I_N)e_k^y\|_2\\ &\leq (2\alpha+1)\varphi_\beta\|e_k^y\|_2 + 2\alpha\varphi_\beta\|\tilde{y}_k\|_b + \overline{\gamma}\varphi_\beta\|\lambda_{k+1}-\lambda_k\|_2\\ &\leq (2\alpha+1)\varphi_\beta\|e_k^y\|_2 + \overline{\gamma}^2\|v\|_2\varphi_\beta\epsilon\|\vec{\lambda}_k\|_2\\ &\quad + \Big(2\alpha + \frac{\overline{\gamma}\|v\|_2}{\sqrt{N}}\epsilon\Big)\overline{\gamma}\varphi_\beta\|\tilde{\lambda}_k\|_a + (\overline{\gamma}\varphi_\beta\epsilon + 2\alpha\varphi_\beta)\|\tilde{y}_k\|_b.\end{aligned}\tag{29}$$

Then, the elements of Π are the coefficients in (24)-(25), (27)-(29). The proof is ended. ■

Appendix B
Proof of Theorem 1

Proof: From Lemma 5, it is enough to show

$$\Pi\zeta \leq \Big(1-\frac{\underline{\gamma}u^Tv}{2N^2}\epsilon\Big)\zeta \tag{30}$$

where $\zeta = \begin{bmatrix}\zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5\end{bmatrix}^T$ with $\zeta_1,\ldots,\zeta_5 > 0$.

1) First inequality in (30)*:*

$$\Big(1-\frac{\underline{\gamma}u^Tv}{N^2}\epsilon\Big)\zeta_1 + \frac{\overline{\gamma}u^Tv}{N\sqrt{N}}\epsilon\zeta_2 + \frac{\|u\|_2}{N}\epsilon\zeta_3 \leq \Big(1-\frac{\underline{\gamma}u^Tv}{2N^2}\epsilon\Big)\zeta_1$$

holds iff $\zeta_1 \geq \frac{2\overline{\gamma}\sqrt{N}}{\underline{\gamma}}\zeta_2 + \frac{2N\|u\|_2}{\underline{\gamma}u^Tv}\zeta_3$.

2) Second inequality in (30)*:*

$$\overline{\gamma}\delta_a\|v\|_2\epsilon\zeta_1 + \pi_{22}\zeta_2 + \delta_a\epsilon\zeta_3 + 2\delta_a\alpha\zeta_4 \leq \Big(1-\frac{\underline{\gamma}u^Tv}{2N^2}\epsilon\Big)\zeta_2$$

holds iff $m_1\epsilon + 2\delta_a\alpha\zeta_4 \leq (1-\tau_a)\alpha\zeta_2$ where

$$m_1 = \overline{\gamma}\delta_a\|v\|_2\zeta_1 + \Big(\frac{\delta_a\overline{\gamma}\|v\|_2}{\sqrt{N}} + \frac{\underline{\gamma}u^Tv}{2N^2}\Big)\zeta_2 + \delta_a\zeta_3. \tag{31}$$

It is sufficient that $4\delta_a\alpha\zeta_4 \leq (1-\tau_a)\alpha\zeta_2$ and $2m_1\epsilon \leq (1-\tau_a)\alpha\zeta_2$. Thus, we have $\zeta_2 \geq \frac{4\delta_a\zeta_4}{1-\tau_a}$ and $\epsilon \leq \frac{(1-\tau_a)\alpha\zeta_2}{2m_1}$.

3) Third inequality in (30)*:*

$$\overline{\gamma}^2\delta_b\|v\|_2\epsilon\zeta_1 + \pi_{32}\zeta_2 + \pi_{33}\zeta_3 + 2\overline{\gamma}\delta_b\alpha\zeta_4 + 2\delta_b\alpha\zeta_5 \leq \Big(1-\frac{\underline{\gamma}u^Tv}{2N^2}\epsilon\Big)\zeta_3$$

holds iff $m_2\epsilon + 2\delta_b\alpha(\overline{\gamma}\zeta_2 + \overline{\gamma}\zeta_4 + \zeta_5) \leq (1-\tau_b)\alpha\zeta_3$ where

$$m_2 = \overline{\gamma}^2\delta_b\|v\|_2\zeta_1 + \frac{\overline{\gamma}^2\delta_b\|v\|_2}{\sqrt{N}}\zeta_2 + \Big(\overline{\gamma}\delta_b + \frac{\underline{\gamma}u^Tv}{2N^2}\Big)\zeta_3. \tag{32}$$

It is sufficient that $4\delta_b\alpha(\overline{\gamma}\zeta_2 + \overline{\gamma}\zeta_4 + \zeta_5) \leq (1-\tau_b)\alpha\zeta_3$ and $2m_2\epsilon \leq (1-\tau_b)\alpha\zeta_3$. Hence, we have $\zeta_3 \geq \frac{4\delta_b(\overline{\gamma}\zeta_2+\overline{\gamma}\zeta_4+\zeta_5)}{1-\tau_b}$ and $\epsilon \leq \frac{(1-\tau_b)\alpha\zeta_3}{2m_2}$.

4) Fourth inequality in (30)*:*

$$\overline{\gamma}\|v\|_2\varphi_\beta\epsilon\zeta_1 + \pi_{42}\zeta_2 + \varphi_\beta\epsilon\zeta_3 + \pi_{44}\zeta_4 \leq \Big(1-\frac{\underline{\gamma}u^Tv}{2N^2}\epsilon\Big)\zeta_4$$

holds iff $m_3\epsilon \leq m_4$ where

$$\begin{aligned}m_3 &= \overline{\gamma}\|v\|_2\varphi_\beta\zeta_1 + \frac{\overline{\gamma}\|v\|_2\varphi_\beta}{\sqrt{N}}\zeta_2 + \varphi_\beta\zeta_3 + \frac{\underline{\gamma}u^Tv}{2N^2}\zeta_4,\\ m_4 &= (1-\varphi)\beta\zeta_4 - 2\varphi_\beta(\zeta_2+\zeta_4)\alpha.\end{aligned}\tag{33}$$

It is sufficient that $m_4 \geq 0$. Thus, we have $\alpha \leq \frac{(1-\varphi)\beta\zeta_4}{2\varphi_\beta(\zeta_2+\zeta_4)}$ and $\epsilon \leq \frac{m_4}{m_3}$.

5) Fifth inequality in (30)*:*

$$\overline{\gamma}^2\|v\|_2\varphi_\beta\epsilon\zeta_1 + \pi_{52}\zeta_2 + \pi_{53}\zeta_3 + \pi_{55}\zeta_5 \leq \Big(1-\frac{\underline{\gamma}u^Tv}{2N^2}\epsilon\Big)\zeta_5$$

holds iff $m_5\epsilon \leq (1-\varphi)\beta\zeta_5 - 2\varphi_\beta(\overline{\gamma}\zeta_2+\zeta_3+\zeta_5)\alpha$ where

$$\begin{aligned}m_5 &= \overline{\gamma}^2\|v\|_2\varphi_\beta\zeta_1 + \frac{\overline{\gamma}^2\|v\|_2\varphi_\beta}{\sqrt{N}}\zeta_2 + \overline{\gamma}\varphi_\beta\zeta_3 + \frac{\underline{\gamma}u^Tv}{2N^2}\zeta_5,\\ m_6 &= (1-\varphi)\beta\zeta_5 - 2\varphi_\beta(\overline{\gamma}\zeta_2+\zeta_3+\zeta_5)\alpha.\end{aligned}\tag{34}$$

It is sufficient that $m_6 \geq 0$. Thus, we have $\alpha \leq \frac{(1-\varphi)\beta\zeta_5}{2\varphi_\beta(\overline{\gamma}\zeta_2+\zeta_3+\zeta_5)}$ and $\epsilon \leq \frac{m_6}{m_5}$.

Note that $\frac{(1-\tau_b)\alpha\zeta_3}{2m_2} < \frac{2N^2}{(N\overline{\gamma}+\underline{\gamma})(u^Tv)}$. Hence, $\rho(\Pi) < 1$ if

$$\begin{aligned}\epsilon &\leq \Big\{\frac{(1-\tau_a)\alpha\zeta_2}{2m_1},\ \frac{(1-\tau_b)\alpha\zeta_3}{2m_2},\ \frac{m_4}{m_3},\ \frac{m_6}{m_5}\Big\}^-,\\ \alpha &\leq \Big\{1,\ \frac{(1-\varphi)\beta\zeta_4}{2\varphi_\beta(\zeta_2+\zeta_4)},\ \frac{(1-\varphi)\beta\zeta_5}{2\varphi_\beta(\overline{\gamma}\zeta_2+\zeta_3+\zeta_5)}\Big\}^-,\end{aligned}\tag{35}$$

where $\zeta_1 - \zeta_5$ satisfy

$$\begin{aligned}&\zeta_1 \geq \frac{2\overline{\gamma}\sqrt{N}}{\underline{\gamma}}\zeta_2 + \frac{2N\|u\|_2}{\underline{\gamma}u^Tv}\zeta_3,\ \zeta_2 \geq \frac{4\delta_a\zeta_4}{1-\tau_a},\\ &\zeta_3 \geq \frac{4\delta_b(\overline{\gamma}\zeta_2+\overline{\gamma}\zeta_4+\zeta_5)}{1-\tau_b},\ \zeta_4 > 0,\ \zeta_5 > 0.\end{aligned}\tag{36}$$

■

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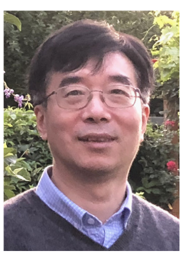
Wei Chen (Member, IEEE) received the Ph.D. degree in Control Science and Engineering from University of Shanghai for Science and Technology, Shanghai, China, in 2021.

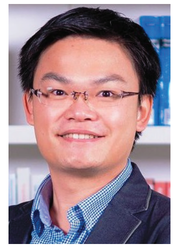
From 2019 to 2020, he was a visiting Ph.D. student with the Brunel University London, Uxbridge, U.K. From 2021 to 2022, he was a Research Associate with the City University of Hong Kong Shenzhen Research Institute, Shenzhen, China. From 2022 to 2024, he was a Post-Doctoral Research Fellow with the Southern University of Science and Technology, Shenzhen, China. He is currently a Research Fellow with the National University of Singapore, Singapore. He is a recipient of an Alexander von Humboldt Fellowship in 2025. His current research interests include distributed optimization/control, privacy preservation, energy management of smart grids. He is a very active reviewer for many international journals.



Zidong Wang (Fellow, IEEE) received the B.Sc. degree in mathematics from Suzhou University, Suzhou, China, in 1986, and the M.Sc. degree in applied mathematics and the Ph.D. degree in electrical engineering from the Nanjing University of Science and Technology, Nanjing, China, in 1990 and 1994, respectively.

He is currently a Professor in dynamical systems and computing with the Department of Computer Science, Brunel University London, Uxbridge, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany, and the U.K. He has published a number of articles in international journals. His research interests include dynamical systems, signal processing, bioinformatics, control theory and applications.

Prof. Wang is a Holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, and William Mong Visiting Research Fellowship of Hong Kong. He serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, *Neurocomputing*, and *Systems Science & Control Engineering*; and an Associate Editor for 12 international journals, including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a member of the Academia Europaea and European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a fellow of the Royal Statistical Society, and a member of program committee for many international conferences.



Jimmy Chih-Hsien Peng (Senior Member, IEEE) received the B.E. and Ph.D. degrees in electrical and computer engineering from the University of Auckland, Auckland, New Zealand, in 2008 and 2012, respectively. He is currently an Associate Professor with the Electrical and Computer Engineering, National University of Singapore, Singapore. Previously, he was with the Masdar Institute (now part of the Khalifa University), Abu Dhabi, UAE. In 2013, he was appointed a Visiting Scientist with the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA, USA, where he became a Visiting Assistant Professor in 2014.

His research interests include power system stability, grid resilience, cyber security, microgrids, and high-performance computing. Dr. Peng is currently a Member of the Electrical and Electronic Standards Committee, under the Singapore Standards Council. He is also an Associate Editor of the IEEE Transaction on Power Systems.

Guo-Ping Liu (Fellow, IEEE) received the Ph.D. degree in control engineering from the University of Manchester, Manchester, U.K., in 1992.

He is a professor with the Southern University of Science and Technology, Shenzhen, China. He has authored/co-authored over 400 journal papers and 10 books on control systems. His current research interests include the Internet of Things for renewable energy integration, networked control systems and advanced control of industrial systems. He is a Member of the Academy of Europe, a Fellow of IET and a Fellow of CAA.