

Resilient Distributed Fusion Filtering for Nonlinear Systems With Dynamical Bias Subject to Dual-Channel Replay Attacks

Haijing Fu, Zidong Wang, and Qinyuan Liu

Abstract—This paper investigates the resilient distributed fusion filtering problem for nonlinear multi-sensor systems subject to dynamical bias and dual-channel replay attacks. Replay attacks are considered on both the sensor-to-local-filter and local-filter-to-fusion-filter communication channels, where current measurements and local estimates may be maliciously replaced by previously recorded data. To characterize such attacks more realistically, a unified probabilistic model involving Bernoulli stochastic variables and bounded replay delays is developed. A distributed fusion filtering scheme is then constructed, in which local filters are first designed to guarantee the exponential mean-square ultimate boundedness of the local filtering error dynamics. Sufficient conditions are established in terms of matrix inequalities, and the corresponding local filter gains are obtained by solving a constrained optimization problem. Based on the resulting local estimates, the fusion filtering performance is analyzed, and the fusion weights are further optimized through a convex optimization problem to minimize the ultimate upper bound of the fusion filtering error. Finally, a simulation example is provided to demonstrate the effectiveness and applicability of the proposed filtering method.

Index Terms—Distributed fusion filtering; nonlinear systems; dynamical bias; dual-channel replay attacks; resilient filtering; multi-sensor systems; exponential mean-square ultimate boundedness; convex optimization.

I. INTRODUCTION

With the rapid development of information technology, multi-sensor information fusion strategies have been successfully applied in various fields, including robot navigation, autonomous driving, and smart healthcare [11], [32], [40], [42], [44], [47]. A fundamental task in information fusion is multi-sensor fusion filtering/estimation, by which a reliable global estimate is obtained through the effective use of distributed sensor measurements and local state estimates. Fusion filtering frameworks are generally classified into centralized and distributed schemes [3], [15], [18], [52]. In centralized schemes, all sensor measurements are directly transmitted to a fusion center for global estimation, which usually leads

to high estimation accuracy but incurs considerable energy consumption. In distributed schemes, local state estimation is carried out at each sensor node, and the resulting local estimates are then transmitted to the fusion center, so that the communication burden can be reduced and system robustness can be enhanced. To date, the distributed fusion estimation problem for multi-sensor systems has been extensively investigated, and several representative fusion filtering methods have been developed, including distributed Kalman fusion filtering [14], [16], distributed set-membership fusion filtering [5], [17], and distributed H_∞ fusion filtering [28], [33].

In practical applications, system modeling and performance analysis are often complicated by unmodelable dynamics, unknown external disturbances, and environmental variations. Dynamical bias, as a typical form of unknown disturbance, is commonly described by stochastic dynamic equations driven by random noise. Such bias usually has strong randomness and a complicated structure, and a significant degradation in filtering performance may be caused if it is not properly handled. In recent years, considerable research attention has been devoted to the modeling and filtering problems associated with dynamical bias [21], [24], [34], [39], [43]. For instance, the cubature Kalman filtering problem has been investigated in [21] for systems subject to dynamical bias, where the bias was incorporated into the system state through an augmented state-bias modeling strategy. In [34], the maximum correntropy filtering problem has been studied for complex networks with uncertain bias evolution. Similarly, a quadratic filtering method has been proposed in [43] for relay networks subject to dynamical bias, where the bias evolution was modeled by stochastic equations driven by non-Gaussian noise. Existing studies on distributed fusion filtering in the presence of dynamical bias remain relatively fragmented, which motivates further investigation into this problem.

In multi-sensor systems, the communication channels from sensors to local filters and from local filters to the fusion filter are susceptible to cyber attacks owing to the openness and vulnerability of communication networks [2], [26]. In general, cyber attacks are categorized into denial-of-service (DoS) attacks, false data injection (FDI) attacks, and replay attacks [12], [36], [41]. Among these attacks, replay attacks refer to the malicious replacement of current measurements or estimates with previously recorded historical data. Recently, the modeling of cyber attacks in state estimation and filtering problems has received growing attention [8], [10], [27], [45], [50]. For example, energy-constrained DoS attacks have been

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characterized in terms of attack frequency and attack duration in [4], while Bernoulli-type packet-drop DoS attacks have been modeled in [45]. Conventional FDI attacks have also been described by energy-bounded or Gaussian-distributed signals in [27], [50]. Existing modeling methods are generally not applicable to replay attacks, since the attack durations and the timestamps of replayed data are unknown, and their dynamic behavior is therefore difficult to describe through existing mathematical forms. An analytical model capable of capturing the temporal and structural characteristics of replay attacks is still required, which forms one of the motivations of this paper.

With the widespread use of information technology, cyber security has become important in both engineering practice and theoretical research [1], [6], [37], [51]. As an important branch of cyber security, attack-tolerant estimation/filtering is concerned with the design of estimation algorithms that can mitigate the influence of cyber attacks on estimation performance. So far, the design of attack-tolerant estimation algorithms, including resilient estimation, robust estimation, and secure estimation, has attracted considerable attention [13], [19], [22], [46]. For example, in [19], the non-fragile state estimation problem under an event-triggered protocol has been studied for time-varying systems subject to deception attacks, where the estimation error system was shown to satisfy a finite-horizon H_∞ performance requirement. In [22], a fixed-time secure state estimation method has been developed against DoS attacks by using a secure transmission strategy with buffering. In [46], a resilient distributed Kalman filtering algorithm has been proposed by incorporating attack detection mechanisms under multiple cyber attacks. Distributed fusion filtering under replay attacks has not yet been sufficiently explored, especially for the more challenging case where dual-channel replay attacks and dynamical bias coexist. The design of a fusion filter capable of handling their combined effects constitutes another main motivation of this paper.

Motivated by the above discussions, the distributed fusion filtering problem is addressed in this paper for nonlinear multi-sensor systems subject to dual-channel replay attacks and dynamical bias. The considered problem is nontrivial due to three fundamental challenges: 1) how can a reasonable mathematical model be established to describe the dynamic behavior of dual-channel replay attacks? 2) how can the effects of dual-channel replay attacks and dynamical bias on local and fusion filtering performance be quantified? 3) how can the fusion weights be designed to optimize the fusion filtering performance in the presence of dual-channel replay attacks and dynamical bias? The main objective of this paper is to provide effective solutions to the above challenges. The primary contributions of this article are summarized as follows.

- 1) The distributed fusion filtering problem is, for the first time, investigated for nonlinear multi-sensor systems subject to dual-channel replay attacks and dynamical bias.
- 2) A comprehensive replay attack model dependent on stochastic variables and time-varying parameters is proposed to characterize the dynamic behavior of replay attacks.
- 3) Sufficient conditions are derived to ensure that the

local and fusion filtering error systems are exponentially mean-square ultimately bounded. In addition, an optimal set of fusion weights is obtained by solving a convex optimization problem.

The remainder of this article is organized as follows. In Section II, the system model and replay attack model are formulated, and the structures of the local and fusion filters are presented. In Section III, the exponential mean-square ultimate boundedness of the local and fusion filtering error systems is analyzed, and the local filter gain parameters are determined. The fusion weights are then designed by solving a corresponding optimization problem. In Section IV, a numerical example is provided to demonstrate the effectiveness of the proposed distributed fusion filtering approach. Section V concludes this paper.

Notation: Throughout this paper, M^T denotes the transpose of M . $\mathbb{R}^n$ stands for the n dimensional Euclidean space and $\mathbb{R}^{n \times m}$ represents the set of all $n \times m$ real matrices. 0 and I respectively denote the zero matrix and identity matrix. $\|x\|$ represents the Euclidean norm of a vector x . The symbol $*$ stands for the symmetric term in a matrix. $\mathbb{E}\{\cdot\}$ represents the mathematical expectation operator. $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ means the smallest and largest eigenvalue of X . The symbol $\otimes$ denotes the Kronecker product. The matrix $X > 0$ means X is real symmetric positive definite. $\text{Prob}\{\cdot\}$ denotes the occurrence probability of the event.

II. PROBLEM FORMULATION

A. System Description

Consider the following state-space model of discrete-time nonlinear systems:

$$x_{k+1} = Ax_k + g(x_k) + B\xi_k + Dw_k \quad (1)$$

where $x_k \in \mathbb{R}^{n_x}$ denotes the state vector, A , B and D are known matrices with compatible dimensions, and $w_k \in \mathbb{R}^{n_w}$ represents the process noise which is the bounded stochastic sequence with zero mean and covariance $\mathfrak{I}I_{n_w} > 0$. The dynamical bias is denoted by ξ_k and is governed by the following dynamics:

$$\xi_{k+1} = E\xi_k + \vartheta_k \quad (2)$$

where E denotes the known bias transition matrix, and ϑ_k represents the noise satisfying zero-mean stochastic sequence with covariance $\wp I_{n_\vartheta} > 0$, and is uncorrelated with the process noise w_k . The nonlinear function $g(\cdot)$ satisfies the following condition:

$$[g(x_1) - g(x_2) - U_1(x_1 - x_2)]^T \times [g(x_1) - g(x_2) - U_2(x_1 - x_2)] \leq 0 \quad (3)$$

for all $x_1, x_2 \in \mathbb{R}^{n_x}$, where U_1 and U_2 are known matrices with compatible dimensions.

For each sensor i ($i = 1, 2, \dots, N$), the measurement output is described by

$$y_{i,k} = H_i x_k + F_i v_k \quad (4)$$

where $y_{i,k} \in \mathbb{R}^{n_y}$ denotes the measurement of sensor i , $v_k \in \mathbb{R}^{n_v}$ denotes the measurement noise that satisfies zero-mean

stochastic sequence with covariance $\hbar I_{n_w} > 0$, and H_i and F_i are known matrices with compatible dimensions.

Defining $\bar{x}_k \triangleq [x_k^T \ \xi_k^T]^T$, the following augmented system is obtained as:

$$\begin{cases} \bar{x}_{k+1} = \bar{A}\bar{x}_k + g(\bar{x}_k) + \bar{D}\varpi_k \\ y_{i,k} = \bar{H}_i\bar{x}_k + F_i v_k \end{cases} \quad (5)$$

where

$$\bar{A} \triangleq \begin{bmatrix} A & B \\ 0 & E \end{bmatrix}, \quad g(\bar{x}_k) \triangleq \begin{bmatrix} g(x_k) \\ 0 \end{bmatrix}, \quad \bar{D} \triangleq \begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix},$$

$$\bar{H}_i \triangleq [H_i \ 0], \quad \varpi_k \triangleq [w_k^T \ \vartheta_k^T]^T.$$

B. Replay Attack Strategy

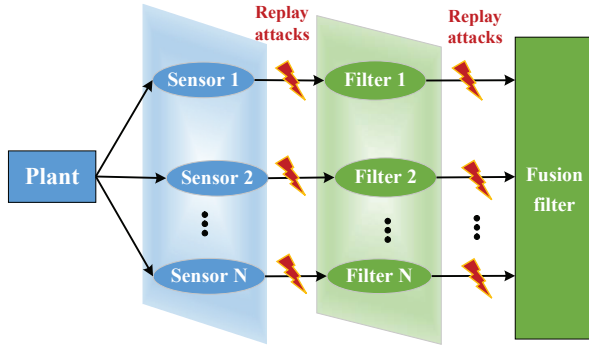


Fig. 1: The diagram of distributed fusion filtering for multi-sensor systems under dual-channel replay attacks.

Information between sensors and local filters is transmitted through a communication network, which is vulnerable to cyber attacks because of its openness. In this paper, replay attacks are considered, and the diagram of fusion filtering for multi-sensor systems under dual-channel replay attacks is shown in Fig. 1. Inspired by the replay attack modeling strategies in [30], the attack process is described as follows. The adversary is capable of eavesdropping on the measurement signal $y_{i,k}$ transmitted through the network and storing the intercepted signals in a finite-capacity memory. For an attacker targeting the i -th sensor, the memory capacity is assumed not to exceed $\bar{\sigma}$. The memory at time k is represented as $\mathcal{S}_{i,k} \triangleq \{y_{i,k(1)}, y_{i,k(2)}, \dots, y_{i,k(\bar{\sigma})}\}$, where $k^{(q)} < k$ ($q = 1, 2, \dots, \bar{\sigma}$) are indices of past instants. At each time k , the replay attack targeting the i -th sensor is performed according to the following rules:

- 1) **Attack-Occurrence** ($\alpha_{i,k} = 1$): The true measurement $y_{i,k}$ is discarded by the adversary, a historical datum $y_{i,k^{(q)}} \in \mathcal{S}_{i,k}$ is selected, and the replayed datum is transmitted to the local filter. The memory set remains unchanged, i.e., $\mathcal{S}_{i,k+1} = \mathcal{S}_{i,k}$.
- 2) **Attack-Free** ($\alpha_{i,k} = 0$): The true measurement $y_{i,k}$ is received by the filter. Meanwhile, $y_{i,k}$ is eavesdropped on by the adversary. If the memory set is not full ($|\mathcal{S}_{i,k}| < \bar{\sigma}$), $y_{i,k}$ is directly stored by the adversary; otherwise, the oldest datum in $\mathcal{S}_{i,k}$ is replaced by $y_{i,k}$.

It is noted that the memory set is empty at the initial time $k = 0$, i.e., $\mathcal{S}_{i,0} = \emptyset$. At this initial instant, only eavesdropping

is performed by the adversary, and no attack is launched; hence, $\alpha_{i,0} = 0$ is set.

Based on the above behavioral model, the actual measurement $\bar{y}_{i,k}$ received by the local filter is described as follows:

$$\bar{y}_{i,k} = y_{i,k} + \alpha_{i,k} (-y_{i,k} + y_{i,k^h}) \quad (6)$$

where $\alpha_{i,k}$ is the attack indicator variable, y_{i,k^h} represents the previous measurement signals with $k^h = k - h_{i,k}$. Here, $h_{i,k}$ denotes the attack duration for the i -th node, which is assumed to satisfy $\underline{h}_i \leq h_{i,k} \leq \bar{h}_i$. The lower bound is set to $\underline{h}_i = 1$, representing the minimum duration of a replay incident, while the upper bound $\bar{h}_i$ satisfies $1 \leq \bar{h}_i \leq \bar{\sigma}$, reflecting the constraint imposed by the attacker's finite memory capacity. In addition, the random variable $\alpha_{i,k}$ follows Bernoulli distributed sequences that takes values on 1 or 0 with the following probabilities:

$$\begin{cases} \text{Prob}\{\alpha_{i,k} = 1\} = \mathbb{E}\{\alpha_{i,k}\} = \bar{\alpha}_i \\ \text{Prob}\{\alpha_{i,k} = 0\} = 1 - \bar{\alpha}_i \end{cases} \quad (7)$$

where $\bar{\alpha}_i \in [0, 1)$ is a known constant.

C. Local Filter

Based on the measurement $\bar{y}_{i,k}$, the following local filter structure is considered:

$$\hat{x}_{i,k+1} = \bar{A}\hat{x}_{i,k} + g(\hat{x}_{i,k}) + K_i(\bar{y}_{i,k} - \bar{H}_i\hat{x}_{i,k}) \quad (8)$$

where $\hat{x}_{i,k}$ is the estimate of system state x_k , and K_i is filter gain to be designed.

Defining $e_{i,k} \triangleq \bar{x}_k - \hat{x}_{i,k}$ and $\tilde{g}_{i,k} \triangleq g(\bar{x}_k) - g(\hat{x}_{i,k})$, the following local filtering error dynamics is obtained:

$$\begin{cases} e_{i,k+1} = (\bar{A} - K_i\bar{H}_i)e_{i,k} + \tilde{g}_{i,k} + \bar{D}\varpi_k \\ \quad + \alpha_{i,k}K_i\bar{H}_i\bar{x}_k - \alpha_{i,k}K_i\bar{H}_i\bar{x}_{k^h} \\ \quad - (1 - \alpha_{i,k})K_iF_iv_k - \alpha_{i,k}K_iF_iv_{k^h} \\ e_{i,l} = \psi_{i,l}, \quad l = -\bar{h}_i, \dots, -1, 0. \end{cases} \quad (9)$$

Defining $\eta_{i,k} \triangleq [\bar{x}_k^T \ e_{i,k}^T]^T$, the augmented system is acquired as follows:

$$\begin{cases} \eta_{i,k+1} = (\mathcal{A}_i - \tilde{\alpha}_{i,k}\mathcal{H}_i)\eta_{i,k} + (\tilde{\alpha}_{i,k} + \bar{\alpha}_i)\mathcal{H}_i\eta_{i,k^h} \\ \quad + \mathcal{G}_{i,k} + ((1 - \bar{\alpha}_i) - \tilde{\alpha}_{i,k})\mathcal{F}_i v_k \\ \quad + \mathcal{D}\varpi_k + (\bar{\alpha}_i + \tilde{\alpha}_{i,k})\mathcal{F}_i v_{k^h} \\ \eta_{i,l} = \phi_{i,l}, \quad l = -\bar{h}_i, \dots, -1, 0 \end{cases} \quad (10)$$

where

$$\mathcal{A}_i \triangleq \begin{bmatrix} \bar{A} & 0 \\ \bar{\alpha}_i K_i \bar{H}_i & \bar{A} - K_i \bar{H}_i \end{bmatrix}, \quad \mathcal{H}_i \triangleq \begin{bmatrix} 0 & 0 \\ -K_i \bar{H}_i & 0 \end{bmatrix},$$

$$\mathcal{D} \triangleq \begin{bmatrix} \bar{D} \\ \bar{D} \end{bmatrix}, \quad \mathcal{F}_i \triangleq \begin{bmatrix} 0 \\ -K_i F_i \end{bmatrix}, \quad \mathcal{G}_{i,k} \triangleq \begin{bmatrix} g(\bar{x}_k) \\ \tilde{g}_{i,k} \end{bmatrix},$$

$$\tilde{\alpha}_{i,k} \triangleq \alpha_{i,k} - \bar{\alpha}_i, \quad \phi_{i,l} \triangleq [0 \ \psi_{i,l}^T]^T.$$

The definition of exponential mean-square ultimate boundedness is given below for the subsequent analysis.

Definition 1: The local filtering error dynamics (10) is said to be exponentially mean-square ultimately bounded (EM-SUB) if there exist constants $\kappa_i \in (0, 1)$, $\varepsilon_i > 0$ and $\theta_i > 0$ such that

$$\mathbb{E}\{\|\eta_{i,k}\|^2\} \leq \kappa_i^k \varepsilon_i + \theta_i \quad (11)$$

where θ_i is an ultimate upper bound of local filtering error.

D. Fusion Filter

As shown in Fig. 1, the fusion filter relies on the local state estimates transmitted from multiple local filters through a communication network. Owing to the openness of the network, the transmission of local estimates is also vulnerable to replay attacks. Specifically, the current local estimate transmitted by the i -th local filter is intermittently replaced by a previously recorded estimate by the adversary, whereby historical local information is injected into the fusion process.

Under replay attacks, the actual estimate received by the fusion filter is modeled as

$$\tilde{x}_{i,k} = \hat{x}_{i,k} + \beta_{i,k}(-\hat{x}_{i,k} + \hat{x}_{i,k-d_{i,k}}) \quad (12)$$

where $\beta_{i,k} \in \{0, 1\}$ is a Bernoulli stochastic variable indicating the occurrence of a replay attack. Specifically, when $\beta_{i,k} = 1$, the current estimate $\hat{x}_{i,k}$ is replaced by the previously recorded estimate $\hat{x}_{i,k-d_{i,k}}$; otherwise, the true local estimate is received by the fusion filter. The replay duration $d_{i,k}$ is assumed to be bounded as $1 \leq d_{i,k} \leq \bar{d}_i$, which reflects the attacker's limited storage capability and restricts the amount of historical local estimates that can be replayed. Moreover, the attack indicator $\beta_{i,k}$ satisfies

$$\begin{cases} \text{Prob}\{\beta_{i,k} = 1\} = \mathbb{E}\{\beta_{i,k}\} = \bar{\beta}_i \\ \text{Prob}\{\beta_{i,k} = 0\} = 1 - \bar{\beta}_i \end{cases} \quad (13)$$

where $\bar{\beta}_i \in [0, 1]$ is a known constant.

Remark 1: Compared with the existing replay attack models [23], [29], the replay attack models considered in this paper are described within a unified probabilistic framework. In both the sensor-local filter and local filter-fusion filter communication channels, replay attacks are characterized by Bernoulli random variables and bounded replay delays, which capture the intermittent nature of attacks and the attacker's limited memory capacity. The two replay attack scenarios differ only in the type of transmitted signals. In the sensor-local filter channel, historical measurement outputs are replayed, whereas, in the local filter-fusion filter channel, previously generated local state estimates are replayed. For brevity, the explicit modeling of the memory set is omitted for the fusion-level replay attacks, since it follows the same construction as that used in the sensor-level case and does not affect the subsequent analysis. Such a consistent modeling framework enables the two replay effects to be incorporated systematically into the local filtering and fusion filtering analyses.

Remark 2: The proposed model is applicable to intermittent replay attacks in which an adversary intermittently replaces current measurements or local estimates with previously recorded data subject to finite storage capacity. Unlike DoS attacks that impair information availability by disrupting data

transmission and FDI attacks that compromise data integrity through arbitrarily fabricated signals [4], [25], [38], replay attacks replace current measurements or estimates with previously recorded data. It should be noted that the present framework is developed for replay attacks with Bernoulli-type occurrences and bounded replay delays, and therefore does not directly accommodate DoS attacks, FDI attacks, or hybrid attacks. Addressing such scenarios would require extensions to both the attack model and the corresponding filtering performance analysis.

Based on (12), the following distributed fusion scheme is established to fuse the local estimates $\tilde{x}_{i,k}$ ($i = 1, 2, \dots, N$):

$$\hat{x}_k = \sum_{i=1}^N W_i \tilde{x}_{i,k} \quad (14)$$

where $\hat{x}_k$ is the fused estimate of $\tilde{x}_{i,k}$, and W_i represents the fusion weight matrix to be determined, which satisfies $\sum_{i=1}^N W_i = I$.

Defining $e_k \triangleq \bar{x}_k - \hat{x}_k$ and $\eta_k \triangleq [\bar{x}_k^T \ e_k^T]^T$, the following fusion filtering error dynamics is constructed as:

$$\begin{cases} e_k = \sum_{i=1}^N W_i (e_{i,k} + z_{i,k}) \\ \eta_k = \sum_{i=1}^N W_i (\eta_{i,k} + \tilde{z}_{i,k}) \end{cases} \quad (15)$$

where $z_{i,k} \triangleq \beta_{i,k}(-\hat{x}_{i,k} + \hat{x}_{i,k-d_{i,k}})$, $\tilde{z}_{i,k} \triangleq [0 \ z_{i,k}^T]^T$, $W_i \triangleq I_2 \otimes W_i$.

Definition 2: The fusion filtering error dynamics (15) is said to be EMSUB if there exist constants $\kappa \in (0, 1)$, $\varepsilon > 0$ and $\tilde{\theta} > 0$ such that

$$\mathbb{E}\{\|\eta_k\|^2\} \leq \kappa^k \varepsilon + \tilde{\theta} \quad (16)$$

where $\tilde{\theta}$ is the ultimate upper bound of the fusion filtering error.

The objectives of this article are summarized in the following three aspects:

- 1) analyze the exponential mean-square ultimate boundedness of the local filtering error dynamics and determine the local filter gain parameters;
- 2) fuse all the local estimates $\tilde{x}_{i,k}$ such that the fusion filtering error dynamics is EMSUB; and
- 3) design the fusion weighting matrix W_i such that the ultimate upper bound of the fusion filtering error dynamics is minimized.

Remark 3: The dual-channel replay attack scenario considered in this paper naturally arises in hierarchical IoT systems consisting of sensing devices, edge or local processing nodes, and a cloud-based or supervisory fusion center. The sensor-to-local-filter channel transmits measurement packets from sensing devices to edge nodes for local state estimation, whereas the local-filter-to-fusion-filter channel transmits local state estimates from edge nodes to the fusion center for global information fusion. Since both channels are implemented through networked communication infrastructures, either channel may become a target of replay attacks [7], [31].

III. MAIN RESULTS

In this section, a sufficient condition is first derived under which the local filtering error dynamics is exponentially mean-square ultimately bounded. The local filter gain matrices are then determined by solving a set of matrix inequalities. Based on the obtained local filters, the fusion filter design is formulated as an optimization problem to be solved.

The following lemma is introduced to facilitate the subsequent analysis.

Lemma 1: Under condition (3), one has

$$(\mathcal{G}_{i,k} - \mathcal{U}_1 \eta_{i,k})^T (\mathcal{G}_{i,k} - \mathcal{U}_2 \eta_{i,k}) \leq 0 \quad (17)$$

where

$$\begin{aligned} \mathcal{U}_1 &\triangleq \begin{bmatrix} \bar{U}_1 & 0 \\ 0 & \bar{U}_1 \end{bmatrix}, \quad \mathcal{U}_2 \triangleq \begin{bmatrix} \bar{U}_2 & 0 \\ 0 & \bar{U}_2 \end{bmatrix}, \\ \bar{U}_1 &\triangleq \begin{bmatrix} U_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{U}_2 \triangleq \begin{bmatrix} U_2 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

Proof: From (5), (9) and (10), together with the definitions of $e_{i,k}$ and $\eta_{i,k}$, one obtains

$$\begin{aligned} &(\mathcal{G}_{i,k} - \mathcal{U}_1 \eta_{i,k})^T (\mathcal{G}_{i,k} - \mathcal{U}_2 \eta_{i,k}) \\ &= (g(\bar{x}_k) - \bar{U}_1 \bar{x}_k)^T (g(\bar{x}_k) - \bar{U}_2 \bar{x}_k) \\ &\quad + (\tilde{g}_{i,k} - \bar{U}_1 e_{i,k})^T (\tilde{g}_{i,k} - \bar{U}_2 e_{i,k}) \\ &= (g(x_k) - U_1 x_k)^T (g(x_k) - U_2 x_k) \\ &\quad + (g(x_k) - g(\hat{x}_{i,k}^1) - U_1 (x_k - \hat{x}_{i,k}^1))^T \\ &\quad \times (g(x_k) - g(\hat{x}_{i,k}^1) - U_2 (x_k - \hat{x}_{i,k}^1)). \end{aligned} \quad (18)$$

According to (3), it follows that

$$\begin{aligned} &(g(x_k) - U_1 x_k)^T (g(x_k) - U_2 x_k) \leq 0, \\ &(g(x_k) - g(\hat{x}_{i,k}^1) - U_1 (x_k - \hat{x}_{i,k}^1))^T \\ &\quad \times (g(x_k) - g(\hat{x}_{i,k}^1) - U_2 (x_k - \hat{x}_{i,k}^1)) \leq 0. \end{aligned}$$

Hence, (17) holds, and the proof is complete. $\blacksquare$

A. Local Filtering Performance Analysis

The exponential mean-square ultimate boundedness of the local filtering error dynamics is analyzed below by constructing a Lyapunov-like functional.

Theorem 1: Let the filter gain parameters K_i be given. Suppose that there exist positive-definite matrices $P_i > 0$, $Q_i > 0$, and positive scalars $\pi > 0$ and $\delta > 0$ satisfying the following inequality:

$$\bar{\Theta}_i = \begin{bmatrix} \bar{\Theta}_{i,11} & \Theta_{i,12} & \bar{\Theta}_{i,13} \\ * & \Theta_{i,22} & \bar{\Theta}_{i,23} \\ * & * & \bar{\Theta}_{i,33} \end{bmatrix} < 0 \quad (19)$$

where

$$\begin{aligned} \bar{\Theta}_{i,11} &\triangleq \bar{\mathcal{A}}_i^T P \mathcal{A}_i + \bar{\alpha}^2 \mathcal{H}_i^T P_i \mathcal{H}_i - P_i + (1 + \bar{h}_i - \underline{h}_i) Q_i \\ &\quad - \pi (\mathcal{U}_1^T \mathcal{U}_2 + \mathcal{U}_2^T \mathcal{U}_1) / 2, \quad \bar{\alpha} \triangleq \sqrt{\bar{\alpha}(1 - \bar{\alpha})}, \\ \Theta_{i,12} &\triangleq \bar{\alpha} \mathcal{A}_i^T P_i \mathcal{H}_i - \bar{\alpha}^2 \mathcal{H}_i^T P_i \mathcal{H}_i, \\ \bar{\Theta}_{i,13} &\triangleq \bar{\mathcal{A}}_i^T P_i + \pi (\mathcal{U}_1^T + \mathcal{U}_2^T) / 2, \\ \Theta_{i,22} &\triangleq (\bar{\alpha}^2 + \bar{\alpha}^2) \mathcal{H}_i^T P_i \mathcal{H}_i - Q_i, \quad \Theta_{i,23} \triangleq \bar{\alpha} \mathcal{H}_i^T P_i, \\ \bar{\Theta}_{i,33} &\triangleq P_i - \pi I, \quad \underline{d} \triangleq \min\{\lambda_{\min}(P_i), \lambda_{\min}(Q_i)\}, \end{aligned}$$

$$\Psi \triangleq \text{diag}\{\Im I_{n_w}, \wp I_{n_\vartheta}\}, \quad \zeta_i \triangleq ((1 - \bar{\alpha}_i)^2 + \bar{\alpha}_i^2 + 2\bar{\alpha}_i^2) \bar{h}.$$

Then, the local filtering error dynamics (10) is EMSUB and the ultimate upper bound is given by:

$$\theta_i = \frac{\text{tr}\{\Psi \mathcal{D}^T P_i \mathcal{D} + \zeta_i \mathcal{F}_i^T P_i \mathcal{F}_i\}}{\delta \underline{d}}. \quad (20)$$

Proof: The following Lyapunov-like functional is established as:

$$V_k = V_{1,k} + V_{2,k} + V_{3,k} \quad (21)$$

where

$$\begin{aligned} V_{1,k} &\triangleq \eta_{i,k}^T P_i \eta_{i,k}, \quad V_{2,k} \triangleq \sum_{l=k-h_i,k}^{k-1} \eta_{i,l}^T Q_i \eta_{i,l}, \\ V_{3,k} &\triangleq \sum_{t=k-\bar{h}_i+1}^{k-h_i} \sum_{l=t}^{k-1} \eta_{i,l}^T Q_i \eta_{i,l}. \end{aligned}$$

First, the difference of $V_{1,k}$ along the local filtering error dynamics (10) is calculated as

$$\begin{aligned} &\mathbb{E}\{\Delta V_{1,k}\} = \mathbb{E}\{V_{1,k+1} - V_{1,k}\} \\ &= \mathbb{E}\{\eta_{i,k+1}^T P_i \eta_{i,k+1} - \eta_{i,k}^T P_i \eta_{i,k}\} \\ &= \mathbb{E}\left\{ \eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{A}_i \eta_{i,k} + \bar{\alpha}_{i,k}^2 \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k} \right. \\ &\quad + (\bar{\alpha}_{i,k}^2 + \bar{\alpha}_i^2) \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k} + \mathcal{G}_{i,k}^T P_i \mathcal{G}_{i,k} \\ &\quad + ((1 - \bar{\alpha}_i)^2 + \bar{\alpha}_i^2) v_k^T \mathcal{F}_i^T P_i \mathcal{F}_i v_k + \varpi_k^T \mathcal{D}^T P_i \mathcal{D} \varpi_k \\ &\quad + (\bar{\alpha}_{i,k}^2 + \bar{\alpha}_i^2) v_{k^h}^T \mathcal{F}_i^T P_i \mathcal{F}_i v_{k^h} + 2\bar{\alpha}_i \eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{H}_i \eta_{i,k^h} \\ &\quad - 2\bar{\alpha}_{i,k}^2 \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k^h} + 2\eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{G}_{i,k} \\ &\quad \left. + 2\bar{\alpha}_i \eta_{i,k^h}^T \mathcal{H}_i^T P_i \mathcal{G}_{i,k} - \eta_{i,k}^T P_i \eta_{i,k} \right\}. \end{aligned} \quad (22)$$

Based on the statistical properties of noise w_k , ϑ_k and v_k , we have:

$$\begin{aligned} \mathbb{E}\{w_k\} &= 0, \quad \mathbb{C}\{w_k\} = \Im I_{n_w}, \\ \mathbb{E}\{v_k\} &= 0, \quad \mathbb{C}\{v_k\} = \bar{h} I_{n_v}, \\ \mathbb{E}\{\vartheta_k\} &= 0, \quad \mathbb{C}\{\vartheta_k\} = \wp I_{n_\vartheta}, \\ \mathbb{E}\{\varpi_k\} &= 0, \quad \mathbb{C}\{\varpi_k\} = \Psi. \end{aligned} \quad (23)$$

Assuming that v_k and v_{k^h} have the same statistical properties, we further have

$$\begin{aligned} &\mathbb{E}\{\Delta V_{1,k}\} \\ &\leq \mathbb{E}\left\{ \eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{A}_i \eta_{i,k} + \bar{\alpha}_i^2 \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k} \right. \\ &\quad + (\bar{\alpha}_i^2 + \bar{\alpha}_i^2) \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k} + \mathcal{G}_{i,k}^T P_i \mathcal{G}_{i,k} \\ &\quad + 2\bar{\alpha}_i \eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{H}_i \eta_{i,k^h} - 2\bar{\alpha}_i^2 \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k^h} \\ &\quad + 2\eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{G}_{i,k} + 2\bar{\alpha}_i \eta_{i,k^h}^T \mathcal{H}_i^T P_i \mathcal{G}_{i,k} - \eta_{i,k}^T P_i \eta_{i,k} \\ &\quad + ((1 - \bar{\alpha}_i)^2 + \bar{\alpha}_i^2) \text{tr}\{\bar{h} \mathcal{F}_i^T P_i \mathcal{F}_i\} + \text{tr}\{\Psi \mathcal{D}^T P_i \mathcal{D}\} \\ &\quad \left. + (\bar{\alpha}_i^2 + \bar{\alpha}_i^2) \text{tr}\{\bar{h} \mathcal{F}_i^T P_i \mathcal{F}_i\} \right\}. \end{aligned} \quad (24)$$

Next, the differences of the second and third terms in (21) are calculated as

$$\begin{aligned} &\mathbb{E}\{\Delta V_{2,k}\} \\ &= \mathbb{E}\{V_{2,k+1} - V_{2,k}\} \end{aligned}$$

$$\begin{aligned}
&= \sum_{\iota=k-h_{k+1}+1}^k \eta_{i,\iota}^T Q_i \eta_{i,\iota} - \sum_{\iota=k-h_k}^k \eta_{i,\iota}^T Q_i \eta_{i,\iota} \\
&= \eta_{i,k}^T Q_i \eta_{i,k} - \eta_{i,k^h}^T Q_i \eta_{i,k^h} + \sum_{\iota=k-h_{k+1}+1}^{k-1} \eta_{i,\iota}^T Q_i \eta_{i,\iota} \\
&\quad - \sum_{\iota=k-h_k+1}^{k-1} \eta_{i,\iota}^T Q_i \eta_{i,\iota} \\
&\leq \eta_{i,k}^T Q_i \eta_{i,k} - \eta_{i,k^h}^T Q_i \eta_{i,k^h} + \sum_{\iota=k-\bar{h}_i+1}^{k-\underline{h}_i} \eta_{i,\iota}^T Q_i \eta_{i,\iota}, \quad (25)
\end{aligned}$$

and

$$\begin{aligned}
&\mathbb{E}\{\Delta V_{3,k}\} \\
&= \mathbb{E}\{V_{3,k+1} - V_{3,k}\} \\
&= \sum_{t=k-\bar{h}_i+2}^{k-\underline{h}_i+1} \sum_{\iota=t}^k \eta_{i,\iota}^T Q_i \eta_{i,\iota} - \sum_{t=k-\bar{h}_i+1}^{k-\underline{h}_i} \sum_{\iota=t}^{k-1} \eta_{i,\iota}^T Q_i \eta_{i,\iota} \\
&= (\bar{h}_i - \underline{h}_i) \eta_{i,k}^T Q_i \eta_{i,k} - \sum_{\iota=k-\bar{h}_i+1}^{k-\underline{h}_i} \eta_{i,\iota}^T Q_i \eta_{i,\iota}. \quad (26)
\end{aligned}$$

Combining (24)-(26) yields

$$\begin{aligned}
&\mathbb{E}\{\Delta V_k\} = \mathbb{E}\{\Delta V_{1,k}\} + \mathbb{E}\{\Delta V_{2,k}\} + \mathbb{E}\{\Delta V_{3,k}\} \\
&\leq \mathbb{E}\left\{ \eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{A}_i \eta_{i,k} + \check{\alpha}_i^2 \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k} \right. \\
&\quad + (\check{\alpha}_i^2 + \check{\alpha}_i^2) \eta_{i,k^h}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k^h} + \mathcal{G}_{i,k}^T P_i \mathcal{G}_{i,k} \\
&\quad + 2\bar{\alpha}_i \eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{H}_i \eta_{i,k^h} - 2\check{\alpha}_i^2 \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{H}_i \eta_{i,k^h} \\
&\quad + 2\eta_{i,k}^T \mathcal{A}_i^T P_i \mathcal{G}_{i,k} + 2\bar{\alpha}_i \eta_{i,k}^T \mathcal{H}_i^T P_i \mathcal{G}_{i,k} - \eta_{i,k}^T P_i \eta_{i,k} \\
&\quad + (1 + \bar{h}_i - \underline{h}_i) \eta_{i,k}^T Q_i \eta_{i,k} - \eta_{i,k^h}^T Q_i \eta_{i,k^h} \\
&\quad + ((1 - \bar{\alpha}_i)^2 + \check{\alpha}_i^2) \text{tr}\{\mathcal{H}_i^T P_i \mathcal{F}_i\} + \text{tr}\{\Psi \mathcal{D}^T P_i \mathcal{D}\} \\
&\quad \left. + (\check{\alpha}_i^2 + \bar{\alpha}_i^2) \text{tr}\{\mathcal{H}_i^T P_i \mathcal{F}_i\} \right\} \\
&= \mathbb{E}\{\varphi_{i,k}^T \Theta_i \varphi_{i,k}\} + \text{tr}\{\Phi_i\} \quad (27)
\end{aligned}$$

where

$$\begin{aligned}
\varphi_{i,k} &\triangleq \begin{bmatrix} \eta_{i,k} \\ \eta_{i,k^h} \\ \mathcal{G}_{i,k} \end{bmatrix}, \quad \Theta_i \triangleq \begin{bmatrix} \Theta_{i,11} & \Theta_{i,12} & \Theta_{i,13} \\ * & \Theta_{i,22} & \Theta_{i,23} \\ * & * & \Theta_{i,33} \end{bmatrix}, \\
\Theta_{i,11} &\triangleq \mathcal{A}_i^T P_i \mathcal{A}_i + \check{\alpha}^2 \mathcal{H}_i^T P_i \mathcal{H}_i - P_i + (1 + \bar{h}_i - \underline{h}_i) Q_i, \\
\Pi_{13} &\triangleq \mathcal{A}_i^T P_i, \quad \Pi_{33} \triangleq P_i, \quad \Phi_i \triangleq \text{diag}\{\Psi \mathcal{D}^T P_i \mathcal{D}, \zeta \mathcal{F}_i^T P_i \mathcal{F}_i\}.
\end{aligned}$$

In accordance with (17), we have

$$\begin{aligned}
&\frac{1}{2} (\mathcal{G}_{i,k} - \mathcal{U}_1 \eta_{i,k})^T (\mathcal{G}_{i,k} - \mathcal{U}_2 \eta_{i,k}) \\
&+ \frac{1}{2} (\mathcal{G}_{i,k} - \mathcal{U}_2 \eta_{i,k})^T (\mathcal{G}_{i,k} - \mathcal{U}_1 \eta_{i,k}) \leq 0, \quad (28)
\end{aligned}$$

which implies that

$$\begin{aligned}
&\mathcal{G}_{i,k}^T \mathcal{G}_{i,k} - \mathcal{G}_{i,k}^T \frac{\mathcal{U}_2 + \mathcal{U}_1}{2} \eta_{i,k} \\
&- \eta_{i,k}^T \frac{\mathcal{U}_1^T + \mathcal{U}_2^T}{2} \mathcal{G}_{i,k} + \eta_{i,k}^T \left(\frac{\mathcal{U}_1^T \mathcal{U}_2 + \mathcal{U}_2^T \mathcal{U}_1}{2} \right) \eta_{i,k} \leq 0. \quad (29)
\end{aligned}$$

Taking the nonlinear constraint (29) into account, one obtains

$$\begin{aligned}
&\mathbb{E}\{\Delta V_k\} \leq \mathbb{E}\left\{ \varphi_{i,k}^T \Theta_i \varphi_{i,k} - \pi \mathcal{G}_{i,k}^T \mathcal{G}_{i,k} + \pi \mathcal{G}_{i,k}^T \frac{\mathcal{U}_2 + \mathcal{U}_1}{2} \eta_{i,k} \right. \\
&\quad \left. + \pi \eta_{i,k}^T \frac{\mathcal{U}_1^T + \mathcal{U}_2^T}{2} \mathcal{G}_{i,k} - \pi \eta_{i,k}^T \left(\frac{\mathcal{U}_1^T \mathcal{U}_2 + \mathcal{U}_2^T \mathcal{U}_1}{2} \right) \eta_{i,k} \right\} \\
&\quad + \text{tr}\{\Phi_i\} \\
&\triangleq \mathbb{E}\{\varphi_{i,k}^T \bar{\Theta}_i \varphi_{i,k}\} + \text{tr}\{\Phi_i\}. \quad (30)
\end{aligned}$$

It follows from (19) and (30) that

$$\mathbb{E}\{\Delta V_k\} \leq -\delta \mathbb{E}\{\|\eta_{i,k}\|^2\} + \text{tr}\{\Phi_i\} \quad (31)$$

where $0 < \delta < 1$. From (21), it is obtained that

$$\begin{aligned}
V_k &\leq \lambda_{\max}(P_i) \mathbb{E}\{\|\eta_{i,k}\|^2\} \\
&\quad + (1 + \bar{h}_i - \underline{h}_i) \lambda_{\max}(Q_i) \sum_{t=k-\bar{h}_i}^{k-1} \mathbb{E}\{\|\eta_{i,t}\|^2\}. \quad (32)
\end{aligned}$$

For any $\gamma > 1$, it follows from (31) that

$$\begin{aligned}
&\mathbb{E}\{\gamma^{k+1} V_{k+1}\} - \mathbb{E}\{\gamma^k V_k\} \\
&= \gamma^{k+1} \mathbb{E}\{\Delta V_k\} + \gamma^{k+1} \mathbb{E}\{V_k\} - \gamma^k \mathbb{E}\{V_k\} \\
&= \gamma^{k+1} \mathbb{E}\{\Delta V_k\} + \gamma^k (\gamma - 1) \mathbb{E}\{V_k\} \\
&\leq \gamma^{k+1} (-\delta \mathbb{E}\{\|\eta_{i,k}\|^2\} + \text{tr}\{\Psi_i\}) + \gamma^k (\gamma - 1) \\
&\quad \times \left(\lambda_{\max}(P_i) \mathbb{E}\{\|\eta_{i,k}\|^2\} + (1 + \bar{h}_i - \underline{h}_i) \right. \\
&\quad \left. \times \lambda_{\max}(Q_i) \sum_{t=k-\bar{h}_i}^{k-1} \mathbb{E}\{\|\eta_{i,t}\|^2\} \right) \\
&\leq \varsigma_1(\gamma) \gamma^k \mathbb{E}\{\|\eta_{i,k}\|^2\} + \varsigma_2(\gamma) \gamma^k \sum_{t=k-\bar{h}_i}^{k-1} \mathbb{E}\{\|\eta_{i,t}\|^2\} \\
&\quad + \gamma^{k+1} \text{tr}\{\Phi_i\} \quad (33)
\end{aligned}$$

where

$$\begin{aligned}
\varsigma_1(\gamma) &\triangleq -\gamma\delta + (\gamma - 1) \lambda_{\max}(P_i), \\
\varsigma_2(\gamma) &\triangleq (\gamma - 1)(1 + \bar{h}_i - \underline{h}_i) \lambda_{\max}(Q_i).
\end{aligned}$$

Then, for any integer $\mu \geq \bar{h}_i + 1$, summing both sides of (33) with respect to k from 0 to $\mu - 1$ gives

$$\begin{aligned}
&\mathbb{E}\{\gamma^\mu V_\mu\} - \mathbb{E}\{V_0\} \\
&\leq \varsigma_1(\gamma) \sum_{k=0}^{\mu-1} \gamma^k \mathbb{E}\{\|\eta_{i,k}\|^2\} + \frac{\gamma(1 - \gamma^\mu)}{1 - \gamma} \text{tr}\{\Psi_i\} \\
&\quad + \varsigma_2(\gamma) \sum_{k=0}^{\mu-1} \sum_{t=k-\bar{h}_i}^{k-1} \gamma^k \mathbb{E}\{\|\eta_{i,t}\|^2\}. \quad (34)
\end{aligned}$$

Furthermore, the last term in (34) is computed as

$$\begin{aligned}
&\sum_{k=0}^{\mu-1} \sum_{t=k-\bar{h}_i}^{k-1} \gamma^k \mathbb{E}\{\|\eta_{i,t}\|^2\} \\
&\leq \left(\sum_{t=-\bar{h}_i}^{-1} \sum_{k=0}^{t+\bar{h}_i} + \sum_{t=0}^{\mu-\bar{h}_i-1} \sum_{k=t+1}^{t+\bar{h}_i} + \sum_{t=\mu-\bar{h}_i}^{\mu-1} \sum_{k=t+1}^{\mu-1} \right) \\
&\quad \times \gamma^k \mathbb{E}\{\|\eta_{i,t}\|^2\}
\end{aligned}$$

$$\begin{aligned}
&\leq \bar{h}_i \sum_{t=-\bar{h}_i}^{-1} \gamma^{t+\bar{h}_i} \mathbb{E}\{\|\eta_{i,t}\|^2\} + \bar{h}_i \sum_{t=0}^{\mu-\bar{h}_i-1} \gamma^{t+\bar{h}_i} \mathbb{E}\{\|\eta_{i,t}\|^2\} \\
&\quad + \bar{h}_i \sum_{t=\mu-\bar{h}_i}^{\mu-1} \gamma^{t+\bar{h}_i} \mathbb{E}\{\|\eta_{i,t}\|^2\} \\
&\leq \bar{h}_i \gamma^{\bar{h}_i} \sup_{l \in [-\bar{h}_i, 0]} \mathbb{E}\{\|\phi_{i,l}\|^2\} + \bar{h}_i \gamma^{\bar{h}_i} \sum_{t=0}^{\mu-1} \gamma^t \mathbb{E}\{\|\eta_{i,t}\|^2\}. \quad (35)
\end{aligned}$$

Combining (34) and (35), one has

$$\begin{aligned}
&\mathbb{E}\{\gamma^\mu V_\mu\} - \mathbb{E}\{V_0\} \\
&\leq \varsigma_3(\gamma) \sum_{k=0}^{\mu-1} \gamma^k \mathbb{E}\{\|\eta_{i,k}\|^2\} + \varsigma_4(\gamma) \sup_{l \in [-\bar{h}_i, 0]} \mathbb{E}\{\|\phi_{i,l}\|^2\} \\
&\quad + \frac{\gamma(1-\gamma^\mu)}{1-\gamma} \text{tr}\{\Phi_i\} \quad (36)
\end{aligned}$$

where

$$\varsigma_3(\gamma) \triangleq \varsigma_1(\gamma) + \bar{h}_i \gamma^{\bar{h}_i} \varsigma_2(\gamma), \quad \varsigma_4(\gamma) \triangleq \bar{h}_i \gamma^{\bar{h}_i} \varsigma_2(\gamma).$$

Since $\varsigma_3(1) = -\delta$ and $\lim_{\gamma \rightarrow \infty} \varsigma_3(\gamma) = +\infty$, there exists a scalar $\bar{\gamma}$ such that $\varsigma_3(\bar{\gamma}) = 0$. Thus, from (36), we have

$$\begin{aligned}
&\bar{\gamma}^\mu \mathbb{E}\{V_\mu\} - \mathbb{E}\{V_0\} \\
&\leq \frac{\bar{\gamma}(1-\bar{\gamma}^\mu)}{1-\bar{\gamma}} \text{tr}\{\Phi_i\} + \varsigma_4(\bar{\gamma}) \sup_{l \in [-\bar{h}_i, 0]} \mathbb{E}\{\|\phi_{i,l}\|^2\}, \quad (37)
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}\{V_\mu\} &\leq \frac{1}{\bar{\gamma}^\mu} \left(\mathbb{E}\{V_0\} + \varsigma_4(\bar{\gamma}) \sup_{l \in [-\bar{h}_i, 0]} \mathbb{E}\{\|\phi_{i,l}\|^2\} \right) \\
&\quad + \frac{\bar{\gamma}(\bar{\gamma}^\mu - 1)}{\bar{\gamma}^\mu(\bar{\gamma} - 1)} \text{tr}\{\Phi_i\} \quad (38)
\end{aligned}$$

which, together with (21), implies that

$$\begin{aligned}
\mathbb{E}\{\|\eta_{i,\mu}\|^2\} &\leq \frac{\mathbb{E}\{V_\mu\}}{d} \\
&\leq \frac{\left(\mathbb{E}\{V_0\} + \varsigma_4(\bar{\gamma}) \sup_{l \in [-\bar{h}_i, 0]} \mathbb{E}\{\|\phi_{i,l}\|^2\} \right)}{d\bar{\gamma}^\mu} \\
&\quad + \frac{\bar{\gamma}(\bar{\gamma}^\mu - 1)}{d\bar{\gamma}^\mu(\bar{\gamma} - 1)} \text{tr}\{\Phi_i\}. \quad (39)
\end{aligned}$$

Denote

$$\begin{aligned}
s &\triangleq \frac{\left(\mathbb{E}\{V_0\} + \varsigma_4(\bar{\gamma}) \sup_{l \in [-\bar{h}_i, 0]} \mathbb{E}\{\|\phi_{i,l}\|^2\} \right)}{d}, \\
\theta_{i,\mu} &\triangleq \frac{\bar{\gamma}(\bar{\gamma}^\mu - 1)}{d\bar{\gamma}^\mu(\bar{\gamma} - 1)} \text{tr}\{\Phi_i\}.
\end{aligned}$$

According to Definition 1, the local filtering error dynamics is EMSUB by setting $\varepsilon_i = s$ and $\kappa_i = \bar{\gamma}^{-1}$. Finally, the ultimate upper bound of the local filtering error dynamics is obtained as

$$\theta_i = \lim_{\mu \rightarrow \infty} \theta_{i,\mu} = \frac{\text{tr}\{\Phi_i\}}{\delta d}, \quad (40)$$

which concludes the proof. $\blacksquare$

B. Local Filter Design

The filter gain parameters K_i are calculated below by solving matrix inequalities.

Theorem 2: Let the scalar $0 < \delta < 1$ be given. Suppose that there exist positive-definite matrices $P_i > 0$, $P_{1i} > 0$, $P_{2i} > 0$, $Q_i > 0$, matrix X_i , and scalar $\pi > 0$ satisfying the following matrix inequalities

$$\begin{bmatrix} \tilde{\Theta}_{0i} & * \\ \hat{\Theta}_i & -\bar{P}_i \end{bmatrix} < 0 \quad (41)$$

$$P_i \geq I, \quad Q_i \geq I, \quad (42)$$

where

$$\begin{aligned}
\tilde{\Theta}_{0i} &\triangleq \begin{bmatrix} \Upsilon_i & 0 & \Psi \\ 0 & -Q_i & 0 \\ \Psi^T & 0 & -\pi I \end{bmatrix}, \quad \bar{\mathcal{H}}_i \triangleq \begin{bmatrix} 0 & 0 \\ -X_i \bar{H}_i & 0 \end{bmatrix}, \\
\hat{\Theta}_i &\triangleq \begin{bmatrix} P_i \bar{\mathcal{A}} + \bar{X}_i & \bar{\alpha} \bar{\mathcal{H}}_i & P_i \\ \bar{\alpha} \bar{\mathcal{H}}_i & -\bar{\alpha} \bar{\mathcal{H}}_i & 0 \end{bmatrix}, \quad \Psi \triangleq \pi(\mathcal{U}_1^T + \mathcal{U}_2^T)/2, \\
\bar{X}_i &\triangleq \begin{bmatrix} 0 & 0 \\ \bar{\alpha} X_i \bar{H}_i & -X_i \bar{H}_i \end{bmatrix}, \quad \bar{\mathcal{A}} \triangleq \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{A} \end{bmatrix}, \\
\bar{P}_i &\triangleq \text{diag}\{P_i, P_i\}, \quad P_i \triangleq \text{diag}\{P_{1i}, P_{2i}\}, \\
\Upsilon_i &\triangleq -P_i + (1 + \bar{h}_i - \underline{h}_i)Q_i - \pi(\mathcal{U}_1^T \mathcal{U}_2 + \mathcal{U}_2^T \mathcal{U}_1)/2.
\end{aligned}$$

Then, the local filtering error dynamics (10) is EMSUB, and the minimum ultimate upper bound of $\mathbb{E}\{\|\eta_{i,k}\|^2\}$ is obtained by solving the following optimization problem:

$$\min \{ \text{tr}\{\Psi \mathcal{D}^T P_i \mathcal{D} + \zeta \mathcal{F}_i^T P_i \mathcal{F}_i\} \} \quad (43)$$

subject to (41) and (42). Moreover, the filter gain parameter is determined by

$$K_i = P_{2i}^{-1} X_i. \quad (44)$$

Proof: First, (19) is rewritten as follows:

$$\bar{\Theta}_i = \tilde{\Theta}_{0i} + \tilde{\Theta}_{1i}^T \bar{P}_i \tilde{\Theta}_{1i} < 0 \quad (45)$$

where

$$\begin{aligned}
\tilde{\Theta}_{1i} &\triangleq [\check{\Theta}_{1i}^T \quad \check{\Theta}_{2i}^T]^T, \quad \check{\Theta}_{1i} \triangleq [\mathcal{A}_i \quad \bar{\alpha} \mathcal{H}_i \quad I], \\
\check{\Theta}_{2i} &\triangleq [\bar{\alpha} \mathcal{H}_i \quad \bar{\alpha} \mathcal{H}_i \quad 0].
\end{aligned}$$

By applying the Schur Complement Lemma to (45), the following inequality is obtained:

$$\Theta = \begin{bmatrix} \tilde{\Theta}_{0i} & * \\ \tilde{\Theta}_{1i} & -\bar{P}_i^{-1} \end{bmatrix} < 0. \quad (46)$$

Then, pre- and post-multiplying (46) by $\text{diag}\{I, I, I, P_i, P_i\}$ and its transpose, we derive

$$\begin{bmatrix} \tilde{\Theta}_{0i} & * \\ \bar{P}_i \tilde{\Theta}_{1i} & -\bar{P}_i \end{bmatrix} < 0. \quad (47)$$

By setting $X_i = P_{2i} K_i$, it is confirmed that (47) holds if and only (41) is true. Thus, the proof is complete. $\blacksquare$

C. Fusion Filter Design

In this subsection, based on the obtained minimum ultimate upper bound of the local filtering error, the fusion weights W_i are designed by solving an optimization problem.

Under replay attacks, the estimates received by the fusion filter correspond to a delayed state estimate with a finite lag. As a result, the fusion filtering error depends explicitly on the accumulated state variation over the replay interval. To facilitate the subsequent analysis, a finite-lag state increment bound is introduced, as formalized in the following Lemma.

Lemma 2: Given the system model (1-5), the scalars $\rho_{1i} > 0$ and $\rho_{2i} > 0$, if the augmented system state $\bar{x}_k$ is mean-square bounded, then the finite-lag increment exists with a uniform bound

$$\Delta_i \triangleq \sup_{k \geq 0} \sup_{1 \leq d_{i,k} \leq \bar{d}_i} \mathbb{E}\{\|\bar{x}_k - \bar{x}_{k-d_{i,k}}\|^2\}. \quad (48)$$

Then, the replay-induced term $\tilde{z}_{i,k}$ satisfies the following mean-square bound:

$$\mathbb{E}\{\|\tilde{z}_{i,k}\|^2\} \leq c_{1i}\Delta_i + c_{2i}\mathbb{E}\{\|\eta_{i,k}\|^2\} + c_{2i}\mathbb{E}\{\|\eta_{i,k-d_{i,k}}\|^2\}. \quad (49)$$

where $c_{1i} \triangleq (\check{\beta}_i^2 + \bar{\beta}_i^2)(1 + \rho_{1i})$, $c_{2i} \triangleq (\check{\beta}_i^2 + \bar{\beta}_i^2)(1 + \rho_{1i}^{-1})(1 + \rho_{2i})$, and $c_{3i} \triangleq (\check{\beta}_i^2 + \bar{\beta}_i^2)(1 + \rho_{1i}^{-1})(1 + \rho_{2i}^{-1})$.

Proof: According to $z_{i,k} = \beta_{i,k}(-\hat{x}_{i,k} + \hat{x}_{i,k-d_{i,k}})$, it is known that

$$\|z_{i,k}\|^2 = (\check{\beta}_{i,k} + \bar{\beta}_i)^2 \|\hat{x}_{i,k-d_{i,k}} - \hat{x}_{i,k}\|^2.$$

Then, taking the expectation of the above equation, we obtain

$$\mathbb{E}\{\|z_{i,k}\|^2\} = (\check{\beta}_i^2 + \bar{\beta}_i^2)\mathbb{E}\{\|\hat{x}_{i,k-d_{i,k}} - \hat{x}_{i,k}\|^2\}. \quad (50)$$

By definition $e_{i,k} = \bar{x}_k - \hat{x}_{i,k}$, for any integer $d_{i,k} \geq 1$, it is derived that

$$\begin{aligned} \hat{x}_{i,k-d_{i,k}} - \hat{x}_{i,k} &= (\bar{x}_{k-d_{i,k}} - e_{i,k-d_{i,k}}) - (\bar{x}_k - e_{i,k}) \\ &= (\bar{x}_{k-d_{i,k}} - \bar{x}_k) - (e_{i,k-d_{i,k}} - e_{i,k}). \end{aligned} \quad (51)$$

For the scalars ρ_{1i} and ρ_{2i} , applying Young's inequality Lemma to (51) yields

$$\begin{aligned} &\|\hat{x}_{i,k-d_{i,k}} - \hat{x}_{i,k}\|^2 \\ &\leq (1 + \rho_{1i})\|\bar{x}_k - \bar{x}_{k-d_{i,k}}\|^2 + (1 + \rho_{1i}^{-1})(1 + \rho_{2i})\|e_{i,k}\|^2 \\ &\quad + (1 + \rho_{1i}^{-1})(1 + \rho_{2i}^{-1})\|e_{i,k-d_{i,k}}\|^2 \end{aligned} \quad (52)$$

By definition $\eta_{i,k} \triangleq [\bar{x}_k^T \ e_{i,k}^T]^T$, it is known that $\|\eta_{i,k}\|^2 = \|\bar{x}_{i,k}\|^2 + \|e_{i,k}\|^2 \geq \|e_{i,k}\|^2$ and $\|\eta_{i,k-d_{i,k}}\|^2 \geq \|e_{i,k-d_{i,k}}\|^2$. Since $\|z_{i,k}\|^2 = \|\tilde{z}_{i,k}\|^2$, it follows that

$$\begin{aligned} &\mathbb{E}\{\|\tilde{z}_{i,k}\|^2\} \\ &\leq (\check{\beta}_i^2 + \bar{\beta}_i^2)(1 + \rho_{1i})\mathbb{E}\{\|\bar{x}_k - \bar{x}_{k-d_{i,k}}\|^2\} \\ &\quad + (\check{\beta}_i^2 + \bar{\beta}_i^2)(1 + \rho_{1i}^{-1})(1 + \rho_{2i})\mathbb{E}\{\|\eta_{i,k}\|^2\} \\ &\quad + (\check{\beta}_i^2 + \bar{\beta}_i^2)(1 + \rho_{1i}^{-1})(1 + \rho_{2i}^{-1})\mathbb{E}\{\|\eta_{i,k-d_{i,k}}\|^2\}. \end{aligned} \quad (53)$$

According to the definition of Δ_i in (48), one has $\mathbb{E}\{\|\bar{x}_k - \bar{x}_{k-d_{i,k}}\|^2\} \leq \Delta_i$. Thus, it is derived that (49) holds, and the proof is now complete. ■

Remark 4: Since a closed-form expression for Δ_i is generally unavailable for nonlinear stochastic systems, it is estimated offline by Monte Carlo simulation. Let $\bar{x}_k^r$ denote the

augmented state generated in the r -th independent realization. For M independent simulation runs and a finite simulation horizon T , define

$$\hat{m}(k, d) = \frac{1}{M} \sum_{r=1}^M \|\bar{x}_k^r - \bar{x}_{k-d_{i,k}}^r\|^2,$$

and

$$\hat{\Delta}_i = \max_{k_b \leq k \leq T} \max_{1 \leq d_{i,k} \leq \bar{d}_i} \hat{m}(k, d),$$

where k_b is a burn-in index introduced to reduce the effect of the initial transient. This procedure first approximates the expectation for each pair (k, d) and then takes the maximum, which is consistent with the definition in (48).

Based on Lemma 2 and the above remark, the replay-induced deviation can be incorporated into the fusion performance analysis. The following theorem establishes an ultimate upper bound on the fusion filtering error system.

Theorem 3: For the obtained estimates $\hat{x}_{i,k}$, the local minimized ultimate upper bounds θ_i and the given fusion weights W_i . The fusion filtering error dynamics (15) is EMSUB, and an ultimate upper bound is given by:

$$\bar{\theta} = 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 ((1 + c_{2i} + c_{3i})\theta_i + c_{1i}\Delta_i). \quad (54)$$

Proof: According to the definition of η_k and applying the norm triangle inequality, it is derived that

$$\begin{aligned} \|\eta_k\|^2 &= \left(\sum_{i=1}^N \mathcal{W}_i(\eta_{i,k} + \tilde{z}_{i,k}) \right)^T \sum_{i=1}^N \mathcal{W}_i(\eta_{i,k} + \tilde{z}_{i,k}) \\ &\leq N \sum_{i=1}^N \|\mathcal{W}_i(\eta_{i,k} + \tilde{z}_{i,k})\|^2 \\ &\leq N \sum_{i=1}^N \|\mathcal{W}_i\|^2 (\|\eta_{i,k}\|^2 + \|\tilde{z}_{i,k}\|^2). \end{aligned} \quad (55)$$

Then, taking the expectation on both sides of (55), we have

$$\mathbb{E}\{\|\eta_k\|^2\} \leq 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 \mathbb{E}\{(\|\eta_{i,k}\|^2 + \|\tilde{z}_{i,k}\|^2)\}. \quad (56)$$

Next, it is derived from Lemma 2 that

$$\begin{aligned} \mathbb{E}\{\|\eta_k\|^2\} &\leq 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 ((1 + c_{2i})\mathbb{E}\{\|\eta_{i,k}\|^2\} \\ &\quad + c_{3i}\mathbb{E}\{\|\eta_{i,k-d_{i,k}}\|^2\} + c_{1i}\Delta_i). \end{aligned} \quad (57)$$

Furthermore, it is known from Definition 1 that $\mathbb{E}\{\|\eta_{i,k}\|^2\} = \mathbb{E}\{\|\eta_{i,k-d_{i,k}}\|^2\} \leq \kappa_i^k \varepsilon_i + \theta_i$, we further have

$$\begin{aligned} &\mathbb{E}\{\|\eta_k\|^2\} \\ &\leq 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 ((1 + c_{2i} + c_{3i})\mathbb{E}\{\|\eta_{i,k}\|^2\} + c_{1i}\Delta_i) \\ &\leq 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 (1 + c_{2i} + c_{3i})\kappa_i^k \varepsilon_i \end{aligned}$$

$$+ 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 ((1 + c_{2i} + c_{3i})\theta_i + c_{1i}\Delta_i).$$

Let $\kappa = \max_{1 \leq i \leq N} \kappa_i$. Since $\kappa_i \in (0, 1)$, it follows that $\kappa \in (0, 1)$ and $\kappa_i^k \leq \kappa^k$ for all i . Therefore, one has

$$\begin{aligned} & \mathbb{E}\{\|\eta_k\|^2\} \\ & \leq \kappa^k 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 (1 + c_{2i} + c_{3i})\varepsilon_i \\ & \quad + 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 ((1 + c_{2i} + c_{3i})\theta_i + c_{1i}\Delta_i). \end{aligned} \quad (58)$$

Define the constant ε for the fusion filtering error dynamics as $\varepsilon = 2N \sum_{i=1}^N \|\mathcal{W}_i\|^2 (1 + c_{2i} + c_{3i})\varepsilon_i$. Given that $\varepsilon_i > 0$ and the weights satisfy $W_i > 0$ with $\sum_{i=1}^N W_i = I$, the constant ε is guaranteed to be positive and finite.

Based on the condition (54), we obtain

$$\mathbb{E}\{\|\eta_k\|^2\} \leq \kappa^k \varepsilon + \tilde{\theta}, \quad (59)$$

which confirms that the fusion filtering error dynamics (11) is EMSUB according to Definition 2. Thus, the proof is complete. ■

In the following Theorem, the fusion weights are designed by solving a convex optimization problem that minimizes the ultimate upper bound of the fusion filtering error.

Theorem 4: According to Theorem 3, minimizing the ultimate upper bound $\tilde{\theta}$ leads to the following convex optimization problem for fusion weight matrices $\{W_i\}$:

$$\begin{aligned} & \min_{\{W_i\}} \sum_{i=1}^N \|I_2 \otimes W_i\|^2 \gamma_i \\ & \text{s.t.} \quad \sum_{i=1}^N W_i = I, \end{aligned} \quad (60)$$

where $\gamma_i \triangleq (1 + c_{2i} + c_{3i})\theta_i + c_{1i}\Delta_i$.

Proof: Since $\theta_i > 0$, $\Delta_i \geq 0$, $c_{1i} > 0$, $c_{2i} > 0$ and $c_{3i} > 0$, it follows that $\gamma_i \geq 0$. The spectral norm is convex, and the scalar function $f \mapsto f^2$ is convex and nondecreasing for $f \geq 0$. Therefore, $\|I_2 \otimes W_i\|^2 \gamma_i$ is convex with respect to W_i . The objective function is consequently a nonnegative weighted sum of convex functions. Moreover, the constraint $\sum_{i=1}^N W_i = I$ is affine. Thus, the optimization problem (60) is convex. ■

Based on the theoretical results established in Theorem 2-4, the proposed distributed fusion filtering scheme under dual-channel replay attack can be implemented according to Algorithm 1.

Remark 5: The dual-channel replay attacks affect the distributed filtering architecture through distinct mechanisms. Sensor-level attacks introduce stochastic delays into the local filtering-error dynamics, coupling the current error with delayed augmented states and measurement noises. Fusion-level attacks replace current local estimates with delayed ones, causing the fusion error to depend on local errors at different instants and finite-lag state variations. These effects

Algorithm 1 Distributed Fusion Filtering Under Dual-Channel Replay Attack

Offline design

Input: System matrices A, B, D, E, H_i, F_i ; noise covariances $\mathfrak{X}, \bar{h}, \wp$; attack probabilities $\bar{\alpha}_i, \bar{\beta}_i$; bounds $\bar{h}_i, \bar{d}_i$; scalars ρ_{1i} and ρ_{2i} ; and the number of Monte Carlo trials M

- 1: **for** $i = 1$ **to** N **do**
- 2: Solve the optimization problem (43) subject to LMIs (41)–(42) to obtain matrices $P_{2i} > 0, X_i$;
- 3: Compute the filter gain $K_i = P_{2i}^{-1} X_i$;
- 4: Calculate the performance bound θ_i according to (20);
- 5: **end for**
- 6: For each $i = 1, \dots, N$, obtain the Monte Carlo estimate $\hat{\Delta}_i$ according to Remark 4;
- 7: Compute the coefficient $\gamma_i = (1 + c_{2i} + c_{3i})\theta_i + c_{1i}\Delta_i$;
- 8: Solve the optimization problem (60) subject to $\sum_{i=1}^N W_i = I$;
- 9: Deploy K_i to the corresponding local nodes and W_i to the fusion center;

Online implementation

- 10: Initialize $\hat{x}_{i,0}, i = 1, \dots, N$;
- 11: **for** each sampling instant k **do**
- 12: Each local filter receives the network-delivered measurement $\bar{y}_{i,k}$;
- 13: Each local filter updates its local state estimate as $\hat{x}_{i,k+1}$;
- 14: Each local node transmits $\hat{x}_{i,k+1}$ to the fusion center;
- 15: The fusion center receives the estimates $\tilde{x}_{i,k+1}$ and computes $\hat{x}_{k+1}$;
- 16: **end for**

Output: Online local state estimates $\hat{x}_{i,k}, i = 1, \dots, N$, and the fused state estimate $\hat{x}_k$.

must be analyzed while accounting for nonlinear dynamics, dynamical bias, and the convexity of the fusion-weight design. Accordingly, Theorems 1 and 2 establish the exponential mean-square ultimate boundedness of the local filtering errors and provide filter design conditions. Lemma 2 derives a finite-lag state-increment bound, based on which Theorems 3 and 4 obtain the fusion filtering error bound and the optimal fusion weights, respectively.

Remark 6: Compared with existing resilient distributed fusion filtering methods, the proposed approach has the following distinctive features.

- 1) A unified dual-channel replay attack model is developed using Bernoulli variables and bounded time-varying delays to capture intermittent attacks and finite-memory constraints at both communication channels.
- 2) A layer-dependent stochastic analysis framework is established to characterize the impact of dual-channel replay attacks on local and fusion estimation performance, and the ultimate bounds of local and fusion filtering errors are derived.
- 3) A bound-oriented design method is proposed, where local filter gains are obtained via matrix inequalities

and fusion weights are designed through a convex optimization incorporating local error bounds, replay probabilities, and finite-lag state increments.

IV. SIMULATION EXAMPLE

This section presents a simulation example to demonstrate the usefulness and effectiveness of the developed distributed fusion filter design algorithm.

Consider a class of nonlinear multi-sensor systems with dynamical bias, and the corresponding parameters are given as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0.95 & -0.62 \\ 0.65 & -0.45 \end{bmatrix}, \quad B = \begin{bmatrix} 0.11 & 0.15 \\ 0.14 & 0.20 \end{bmatrix}, \\ D &= \begin{bmatrix} 0.12 \\ -0.10 \end{bmatrix}, \quad E = \begin{bmatrix} 0.105 & -0.077 \\ 0.007 & -0.091 \end{bmatrix}, \\ U_1 &= \begin{bmatrix} -0.1 & 0.2 \\ 0 & 0.05 \end{bmatrix}, \quad U_2 = \begin{bmatrix} -0.2 & -0.15 \\ 0.2 & -0.1 \end{bmatrix}, \\ H_1 &= [1.0 \quad 0.1], \quad H_2 = [0.1 \quad 1], \quad F_1 = 0.19; \\ H_3 &= [0.8 \quad -0.6], \quad H_4 = [0.6 \quad 0.8], \quad F_2 = 0.14; \\ H_5 &= [-0.7 \quad 0.7], \quad F_3 = 0.15, \quad F_4 = 0.16, \quad F_5 = 0.11. \end{aligned}$$

Furthermore, the parameters $\alpha_{i,k}$, $\beta_{i,k}$, $\bar{h}_i$ and $\bar{d}_i$ are chosen as $\bar{\alpha}_1 = 0.16$, $\bar{\alpha}_2 = 0.11$, $\bar{\alpha}_3 = 0.15$, $\bar{\alpha}_4 = 0.18$, $\bar{\alpha}_5 = 0.21$, $\bar{\beta}_1 = 0.11$, $\bar{\beta}_2 = 0.14$, $\bar{\beta}_3 = 0.09$, $\bar{\beta}_4 = 0.18$, $\bar{\beta}_5 = 0.16$, $\bar{h}_1 = 3$, $\bar{h}_2 = 4$, $\bar{h}_3 = \bar{h}_4 = 5$, $\bar{h}_5 = 4$, $\bar{d}_1 = \bar{d}_3 = 3$, $\bar{d}_2 = \bar{d}_5 = 4$, $\bar{d}_4 = 5$. The noise covariance coefficients are given as $\mathfrak{S} = 0.1$, $\wp = 0.09$ and $h = 0.01$. For nonlinear function $g(x_k)$, we select

$$g(x_k) = \begin{bmatrix} -0.1x_k^1 + \tanh(0.4x_k^1) \\ 0.1x_k^2 + \tanh(0.4x_k^2) \end{bmatrix}$$

where x_k^u ($u = 1, 2$) represents the u -th element of x_k .

Given the number of Monte Carlo trails $M = 1000$, the finite-lag state increment bounds $\hat{\Delta}_i$ are estimated according to Remark 4, and their convergence is illustrated in Fig. 2. In the following, the mean-square estimation error (MSEE) is defined to evaluate the filtering performance for node i :

$$\text{MSEE}_i = \frac{1}{L} \sum_{u=1}^L \|\bar{x}_k - \hat{x}_{i,k}\|^2$$

where $L = 100$ denotes the number of simulation trials.

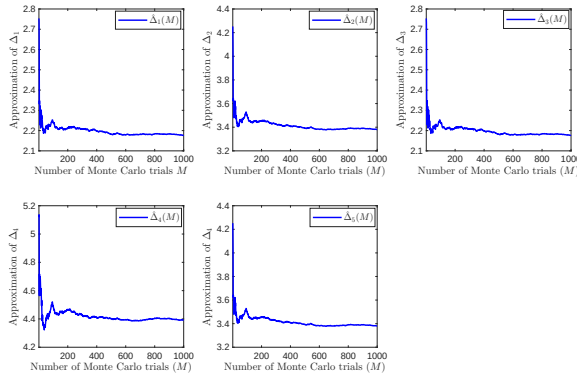


Fig. 2: Approximated increment bounds $\hat{\Delta}_i$ of sensor i .

By solving (41), (42) and (60), the gain parameters of local filter K_i and the weights of fusion filter W_i are obtained. The simulation results are shown in Figs. 3-9. More specifically, Figs. 3-7 describe the 1st, 2nd, 3rd, and 4th elements of the local filtering error of each sensor. Figs. 8 depicts the local MSEE and its upper bound for the five sensors. Figs. 9 describes the 1st, 2nd, 3rd, and 4th elements of the fusion filtering error. The simulation results indicate that satisfactory estimation performance has been achieved by the developed distributed fusion filtering method despite the effects of dual-channel replay attacks and dynamical bias.

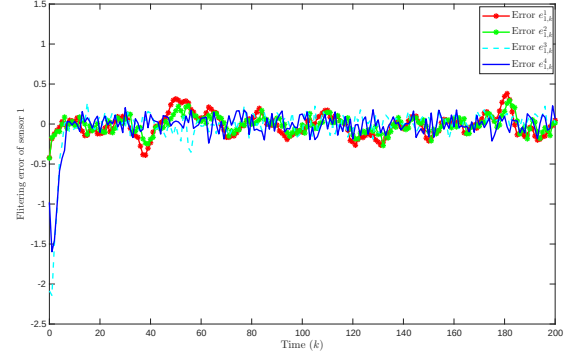


Fig. 3: Filtering error of sensor 1.

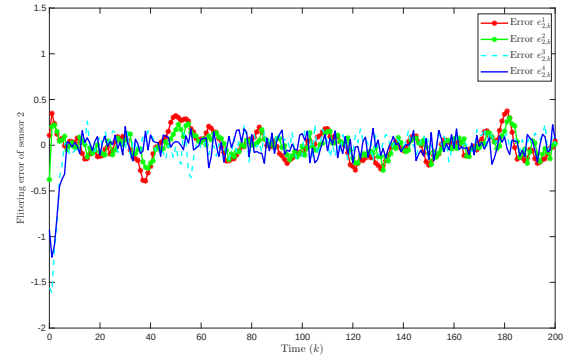


Fig. 4: Filtering error of sensor 2.

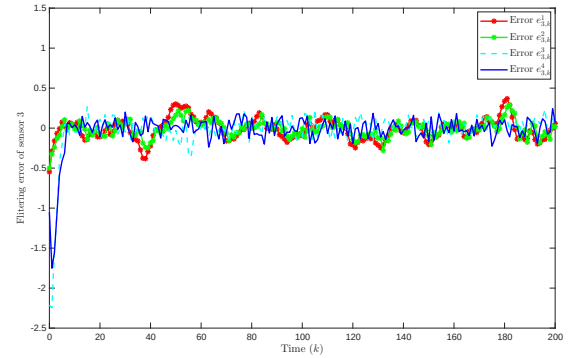


Fig. 5: Filtering error of sensor 3.

To quantitatively illustrate the impact of replay attacks on filtering performance at different communication layers, three representative scenarios are considered. Table I reports the ultimate upper bounds of the local and fusion filtering errors

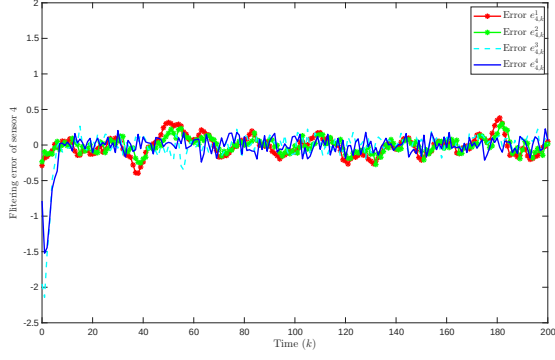


Fig. 6: Filtering error of sensor 4.

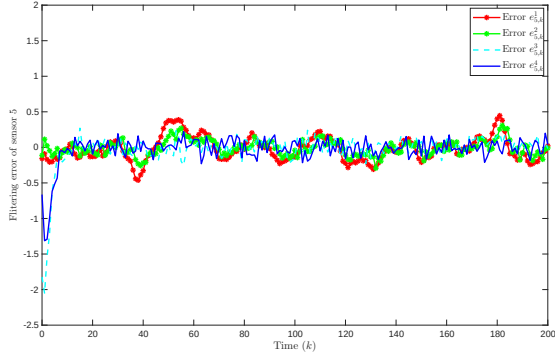


Fig. 7: Filtering error of sensor 5.

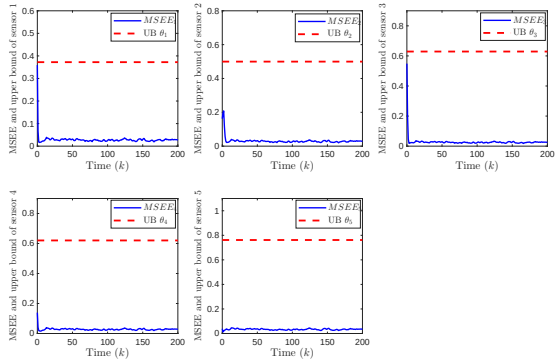


Fig. 8: MSEE and upper bound of sensor i.

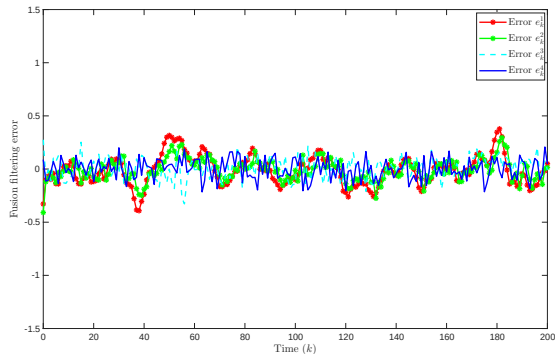


Fig. 9: Fusion filtering error.

under different replay attack scenarios. In Case 1, where no replay attacks are present ($\bar{\alpha}_i = 0, \bar{\beta}_i = 0$), both the local bounds θ_i and the fusion bound $\tilde{\theta}$ remain small, indicating satisfactory filtering performance. In Case 2, replay attacks are introduced only on the sensor-filter communication channel ($\bar{\alpha}_i \neq 0, \bar{\beta}_i = 0$), which leads to a noticeable increase in the local error bounds θ_i . Consequently, the fusion error bound $\tilde{\theta}$ also increases, reflecting the propagation of degraded local estimates into the fusion process. In Case 3, replay attacks simultaneously occur on both the sensor-local filter and local-fusion filter channels ($\bar{\alpha}_i \neq 0, \bar{\beta}_i \neq 0$). Although the local error bounds θ_i remain unchanged compared with Case 2, the fusion error bound $\tilde{\theta}$ increases significantly due to the additional perturbation caused by replayed local estimates. Such an observation is consistent with the theoretical results in Theorems 1–3, where fusion-level replay attacks are shown to further enlarge the fusion ultimate upper bound through replay-induced increment terms.

TABLE I: Ultimate upper bounds of local and fusion filtering errors under different replay attack scenarios

Case	θ_1	θ_2	θ_3	θ_4	θ_5	$\tilde{\theta}$
Case 1	0.104	0.105	0.104	0.105	0.104	1.669
Case 2	0.372	0.499	0.629	0.620	0.762	8.694
Case 3	0.372	0.499	0.629	0.620	0.762	9.609

To further illustrate the influence of different replay attack probabilities on the filtering performance, a sensitivity analysis is conducted by simultaneously varying α_i and β_i from 0.1 to 0.5, and the results are reported in Table II. It can be observed that the local filtering error bounds θ_i vary only slightly, whereas the fusion filtering error bound $\tilde{\theta}_i$ increases markedly from 8.3382 to 36.9208. These results indicate that the local filtering performance is relatively insensitive to the attack probabilities, while the fusion filtering performance deteriorates significantly as attack probabilities increase.

TABLE II: Ultimate upper bounds of local and fusion filtering errors under different attack probabilities

$\bar{\alpha}_i, \bar{\beta}_i$	θ_1	θ_2	θ_3	θ_4	θ_5	$\tilde{\theta}$
0.1	0.3722	0.4996	0.6285	0.6200	0.7599	8.3382
0.2	0.3724	0.5002	0.6297	0.6204	0.7616	15.4511
0.3	0.3727	0.5011	0.6312	0.6209	0.7623	22.5916
0.4	0.3731	0.5022	0.6327	0.6215	0.7602	29.7654
0.5	0.3736	0.5036	0.6330	0.6222	0.7577	36.9208

V. CONCLUSION

In this paper, the distributed fusion filtering problem has been investigated for nonlinear multi-sensor systems subject to dual-channel replay attacks and dynamical bias. Replay attacks on both the sensor-to-local-filter and local-filter-to-fusion-filter communication channels have been modeled within a unified probabilistic framework by using Bernoulli-type stochastic variables and bounded time-varying parameters. Sufficient conditions have been established to guarantee the exponential mean-square ultimate boundedness of both the local filtering error system and the fusion filtering error system. The local

filter gains have then been obtained by solving a set of matrix inequalities, while the fusion weights have been designed through a convex optimization problem aimed at minimizing the upper bound of the fusion filtering error. Simulation results have been provided to validate the theoretical analysis and demonstrate the effectiveness of the proposed distributed fusion filtering scheme under dual-channel replay attacks. Future research will focus on extending the developed framework to privacy-preserving filtering and stealthy attack detection strategies [9], [20], [35].

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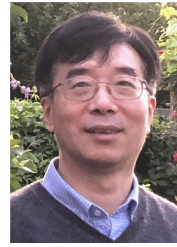
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