

Non-Fragile Distributed Fusion Filtering for Time-Varying Systems With Sensor Saturations: A Decode-and-Forward Relay-Aided Approach

Xueyang Meng, Zidong Wang, Fan Wang, and Yun Chen

Abstract—In this paper, the problem of distributed fusion filtering has been studied for discrete-time, time-varying systems affected by sensor saturations, packet losses, and stochastic gain perturbations. The transmission process between sensors and remote filters is carried out via relay channels subject to packet losses. To enhance communication quality and ensure transmission reliability, a decode-and-forward relay-aided mechanism is introduced. Random perturbations in the local filter gains are incorporated to model potential implementation uncertainties and parameter fluctuations. The objective of this work is to design an appropriate distributed fusion filter that ensures specific performance constraints are satisfied for both local and global filtering error systems. For each sensor node, a sufficient condition, derived from stochastic analysis theory, is first established to guarantee the existence of a desired local H_∞ filter. The associated filter gains are then computed by solving a set of coupled difference equations. Building on these local designs, a distributed fusion filter is constructed, and the corresponding fusion parameters are determined through the solution of a convex optimization problem. Finally, the effectiveness of the proposed fusion filtering framework is demonstrated through a numerical example.

Index Terms—Distributed fusion filtering, sensor saturations, filter gain perturbations, packet losses, decode-and-forward relay-aided mechanism, convex optimization.

I. INTRODUCTION

In recent decades, multi-sensor fusion filtering (MSFF) has been increasingly recognized as a promising research focus, owing to its proven effectiveness across a wide range of applications, including environmental monitoring, target tracking, and autonomous driving [16], [20], [41], [53], [54]. The basic idea of MSFF is that local measurement outputs (or state estimates) are properly integrated to produce fusion estimates

with enhanced reliability and accuracy, in comparison with the single-sensor counterpart. To date, tremendous effort has been devoted to MSFF due to its significant application potential, and numerous efficient fusion filtering algorithms have been proposed, which can be roughly categorized into centralized and distributed approaches [7], [17], [33], [36], [51]. It should be noted that, in distributed fusion filtering (DFF), sensor outputs are processed separately and the resulting local estimates are transmitted to the fusion center (FC) for further handling, whereby distinctive merits are demonstrated in reducing the computational burden and enhancing network reliability [47], [48], [52], [55].

In practical engineering, inherent physical and technical constraints inevitably lead to amplitude saturation in sensor outputs. This phenomenon, known as sensor saturation, may seriously degrade system performance. In recent years, significant attention has been devoted to the filtering problem under sensor saturations [1], [30], [43]. For instance, the H_∞ filtering problem has been investigated in [43] for a class of nonlinear systems subject to randomly occurring sensor saturations and missing measurements. In most filtering algorithms, an implicit assumption is made that the designed filters can be implemented precisely. However, the actual realization of a desired filter is often affected by random gain perturbations arising from undesirable implementation errors, such as rounding and programming errors [37]. Accordingly, the impact of stochastic gain variations (SGVs) has been regarded as both theoretically relevant and practically significant in the study of MSFF problems [12], [18]. Nevertheless, only limited results have been reported concerning MSFF problems under the joint effects of sensor saturations and SGVs, particularly within the DFF framework, which motivates the present study.

In networked environments, limited bandwidth is often recognized as a primary contributor to data congestion and collision, thereby giving rise to various network-induced complexities such as packet loss, channel fading, and transmission delay [3], [5], [9], [14], [15], [22], [32]. These complexities impose significant challenges and practical difficulties on the reliable and efficient implementation of filtering tasks. Among them, packet loss, commonly modeled as a random process, has emerged as one of the most widely investigated phenomena in the literature [21], [29], [34], [35]. In particular, once packet loss occurs, the transmitted information cannot reach the receiver, which may seriously deteriorate the resulting filtering performance. To date, substantial progress has been made in addressing filtering problems for networked systems

This work was supported in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LQN25F030012, in part by the Fundamental Research Funds for the Provincial Universities of Zhejiang under Grant GK269910299001-020, in part by the National Natural Science Foundation of China under Grant 62403175 and Grant U22A2044, in part by the Royal Society of U.K., and in part by the Alexander von Humboldt Foundation of Germany. (Corresponding author: Yun Chen.)

Xueyang Meng and Yun Chen are with the School of Automation, Hangzhou Dianzi University, Hangzhou 310018, China; Yun Chen is also with the School of Information and Electrical Engineering, Hangzhou City University, Hangzhou 310015, China (Emails: mengxy@hdu.edu.cn; yunchen@hdu.edu.cn).

Zidong Wang is with the Department of Computer Science, Brunel University of London, Middlesex, UB8 3PH Uxbridge, U.K. (Email: Zidong.Wang@brunel.ac.uk).

Fan Wang is with the School of Automation, Nanjing University of Information Science and Technology, Nanjing 210044, China, and also with Jiangsu Provincial University Key Laboratory of Big Data Analysis and Intelligent Systems, Nanjing 210044, China (Email: wangfan.92128@gmail.com).

subject to packet losses through various robust and adaptive filtering strategies [13], [26], [28].

In wireless communication systems, the transmission capability of the sensor-to-receiver channel is inherently limited, which often restricts the transmission distance and degrades communication quality. To enable reliable long-distance transmission, an effective solution is to deploy one or more relays to assist signal transmissions. Typical relay types include decode-and-forward (DaF), compress-and-forward (CaF), amplify-and-forward (AaF), and filter-and-forward (FaF) relays [2], [19], [25], [40], [46], [56]. Among these, the DaF relay relies on a predefined encoding-decoding scheme, where the received signal is demodulated and decoded before the restored data are forwarded to the receiver. In this regard, the main advantage of the DaF relay over other relay types lies in its ability to mitigate limited bandwidth and enhance data security. While effectively extending the transmission distance, the DaF relay may also increase system complexity and introduce decoding errors, thereby posing additional challenges for the analysis and design of networked systems. To date, initial efforts have been directed toward the analysis and synthesis problems involving DaF relays [24], [38], [44]. For example, a particle filtering scheme has been developed in [24] for a class of stochastic systems incorporating DaF relays. Unfortunately, limited attention has been given to DFF problems involving DaF relays, particularly under the simultaneous occurrence of sensor saturations, packet losses, and SGVs, which constitutes the principal motivation for the current study.

Following the preceding discussions, the objective of this study is to address the non-fragile DFF problem for time-varying systems affected by sensor saturations and packet losses over DaF relay channels. In pursuing this objective, three significant and nontrivial challenges need to be addressed: 1) how to formulate a reliable DaF relay-aided mechanism for governing signal transmissions across sensor-to-filter communication channels? 2) how to overcome the inherent complexities arising from sensor saturations, stochastic gain variations (SGVs), packet losses, and decoding errors in the transient analysis of filtering error systems? and 3) how to design appropriate local filters and subsequently establish an effective and practically implementable DFF algorithm? Addressing these challenges is essential for improving the flexibility, robustness, and reliability of signal transmissions in relay-aided networked environments.

The main contributions of this paper are outlined as follows.

- 1) A comprehensive study is conducted on the relay-based DFF problem with sensor saturations and SGVs, which has not been previously addressed in the literature and remains largely unexplored in practical scenarios.
- 2) A novel and unified measurement model is constructed to accommodate the joint effects of sensor saturations, DaF relays, randomly occurring packet losses, and additive channel noises, thereby capturing a more realistic networked sensing environment and providing a solid foundation for tackling remote DFF problems.
- 3) The local filter gains are recursively computed by solving two coupled backward Riccati difference equations

(RDEs), offering a numerically tractable and efficient solution suitable for real-time implementation.

- 4) A new DFF algorithm is parameterized, and sufficient analytical conditions are rigorously derived to guarantee prescribed finite-horizon H_∞ performance constraints for both the local and fusion filtering error dynamics in resource-constrained networked systems.

The structure of this paper is organized as follows. Section II presents the problem formulation of the non-fragile DFF issue for time-varying systems affected by sensor saturations under DaF relay channels. In Section III, the design of the local filter gain is carried out to ensure that a prescribed performance index is satisfied, after which the appropriate fusion parameters are determined through the solution of a relevant optimization problem. Section IV provides a numerical example to demonstrate the effectiveness and applicability of the proposed DFF algorithm. Finally, several concluding remarks and potential future directions are outlined in Section V.

Notations: $\mathbb{R}^m$ denotes the m -dimensional Euclidean space. I and 0 represent, respectively, the identity and zero matrices with appropriate dimensions. $\text{col}_m\{x_l\}$ denotes a column vector expressed as $[x_1^T \ x_2^T \ \cdots \ x_m^T]^T$. $\text{diag}_m\{M_l\}$ designates a block-diagonal matrix written as $\text{diag}\{M_1, M_2, \dots, M_m\}$. For a symmetric matrix C , the notation $C > 0$ implies that C is positive definite. The space $l_2([0, T], \mathbb{R}^m)$ refers to the set of square-summable m -dimensional vector functions defined over the time interval $[0, T]$. $\|B\|_F$ denotes the Frobenius norm of a matrix B . $A^\dagger$ represents the Moore-Penrose pseudoinverse of a matrix A . The notation “sign” refers to the signum function.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following class of time-varying systems with sensor saturations operating over the interval $[0, T]$ (T is a known positive integer):

$$\begin{cases} x_{t+1} = A_t x_t + B_t v_t \\ y_{i,t} = \sigma_i(C_{i,t} x_t) + D_{i,t} v_t \\ z_t = M_t x_t \end{cases} \quad (1)$$

where $x_t \in \mathbb{R}^{n_x}$ is the system state, $y_{i,t} \in \mathbb{R}^{n_y}$ is the measurement output of sensor i ($i \in \mathbb{D}_1 \triangleq \{1, 2, \dots, N\}$), and $z_t \in \mathbb{R}^{n_z}$ is the target output signal to be estimated. The disturbance signal $v_t \in l_2([0, T], \mathbb{R}^{n_v})$ belongs to the space of square-summable external disturbances. A_t , B_t , $C_{i,t}$, $D_{i,t}$ and M_t are known constant matrices of proper dimensions.

For any given vector $\bar{h} \triangleq [\bar{h}_1 \ \bar{h}_2 \ \cdots \ \bar{h}_{n_y}]^T$, the saturation function $\sigma_i(\cdot) : \mathbb{R}^{n_y} \mapsto \mathbb{R}^{n_y}$ is defined as

$$\sigma_i(\bar{h}) \triangleq [\sigma_{i1}(\bar{h}_1) \ \sigma_{i2}(\bar{h}_2) \ \cdots \ \sigma_{in_y}(\bar{h}_{n_y})]^T \quad (2)$$

where the component function $\sigma_{ij}(\bar{h}_j)$ is given by $\sigma_{ij}(\bar{h}_j) \triangleq \text{sign}(\bar{h}_j) \cdot \min\{\mu_{ij}, |\bar{h}_j|\}$ ($i \in \mathbb{D}_1, j \in \mathbb{D}_2 \triangleq \{1, 2, \dots, n_y\}$) with $\mu_{ij} > 0$ denoting the corresponding saturation level.

Remark 1: In practice, sensor outputs often encounter sensor saturations due to hardware limitations. In this case, the measurement model $y_{i,t}$ presented in (1) incorporates the effect of sensor saturations described by (2), which poses additional challenges for the subsequent performance analysis

and filter design. Furthermore, the component function $\sigma_{ij}(\bar{h}_j)$ can be written as

$$\sigma_{ij}(\bar{h}_j) = \begin{cases} \bar{h}_j, & 0 \leq \bar{h}_j < \mu_{ij} \\ \mu_{ij}, & \bar{h}_j > \mu_{ij} \\ -\sigma_{ij}(-\bar{h}_j), & \bar{h}_j < 0. \end{cases}$$

However, the DFF problem involving sensor saturations within a finite-horizon framework remains an open and challenging research topic that merits further investigation.

In order to enhance communication quality, N relays are deployed across distinct communication channels to regulate signal transmissions from sensors to local filters. The structural diagram of the considered target plant under DaF relays is illustrated in Fig. 1. Within this framework, each sensor communicates with its corresponding relay via a sensor-to-relay (STR) channel, while the relay communicates with the filter through a relay-to-filter (RTF) channel. Specifically, the sensor observation $y_{i,t}$ is first encoded into a compressed codeword with reduced bit length by encoder i , and this codeword is subsequently transmitted to relay i via the STR channel. At the relay, the received signal is demodulated and decoded before being retransmitted through the RTF channel. Moreover, the packet loss phenomenon is taken into account throughout the entire transmission process.

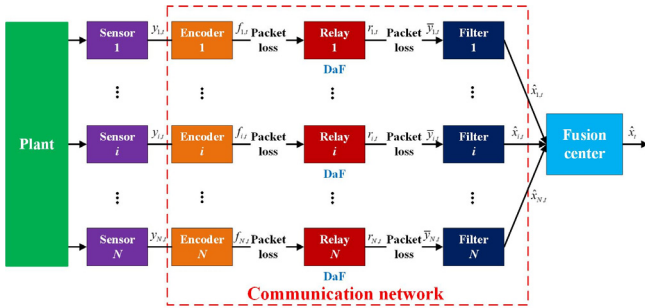


Fig. 1: Distributed fusion filtering under DaF relay channels.

1) Encoder:

Let $f_{i,t} \triangleq \text{col}_{n_y}\{f_{ij,t}\}$ denote the codeword generated by encoder i , whose component $f_{ij,t}$ ($i \in \mathbb{D}_1, j \in \mathbb{D}_2$) is given as

$$f_{ij,t} = \mathbf{E}_{ij}(y_{ij,t}) \quad (3)$$

with $y_{ij,t}$ being the j -th component of the sensor measurement $y_{i,t}$. The encoding function $\mathbf{E}_{ij}(\cdot)$ is characterized by

$$\mathbf{E}_{ij}(y_{ij,t}) = \tau_{ij}\left(Q_{ij}\left(\frac{y_{ij,t}}{\psi_{ij,t}}\right)\right) \quad (4)$$

where $\psi_{ij,t}$ is a known positive scalar, and $\tau_{ij}(\cdot)$ is a scalar-valued mapping defined as

$$\tau_{ij}(Q_{ij}(m)) = \begin{cases} 0, & Q_{ij}(m) = 0 \\ \text{sign}(m)\epsilon_{ij,t}, & Q_{ij}(m) = \text{sign}(m)\epsilon_{ij,t}b_{ij,t} \\ \text{sign}(m)d, & Q_{ij}(m) = \text{sign}(m)db_{ij,t}. \end{cases} \quad (5)$$

Here, $Q_{ij}(\cdot)$ denotes a prescribed quantization function given by

$$Q_{ij}(m) \triangleq \begin{cases} 0, & -0.5b_{ij,t} \leq m < 0.5b_{ij,t} \\ \epsilon_{ij,t}b_{ij,t}, & \underline{u}_{ij,t} \leq m < \bar{u}_{ij,t}, \quad \epsilon_{ij,t} = 1, 2, \dots, d-1 \\ db_{ij,t}, & m \geq 0.5(2d-1)b_{ij,t} \\ -Q_{ij}(-m), & m \leq -0.5b_{ij,t} \end{cases} \quad (6)$$

with $\underline{u}_{ij,t} \triangleq (2\epsilon_{ij,t} - 1)b_{ij,t}/2$ and $\bar{u}_{ij,t} \triangleq (2\epsilon_{ij,t} + 1)b_{ij,t}/2$. The parameter $db_{ij,t}$ represents the amplitude range of $Q_{ij}(\cdot)$, d is a known positive integer, and $b_{ij,t} > 0$ is a time-varying scalar updated according to

$$b_{ij,t+1} = \varsigma_{ij,t}b_{ij,t} + y_{ij,t}^T \mathbf{Y}_{ij,t} y_{ij,t} \quad (7)$$

with an initial value $b_{ij,0}$, a positive scalar $\varsigma_{ij,t} \in (0, 1)$, and a given weighting matrix $\mathbf{Y}_{ij,t} > 0$.

2) Decoder:

Let $r_{i,t} \triangleq \text{col}_{n_y}\{r_{ij,t}\}$ denote the decoded signal at relay i . The component $r_{ij,t}$ ($i \in \mathbb{D}_1, j \in \mathbb{D}_2$) is obtained from the received codeword $f_{ij,t}$ as follows:

$$r_{ij,t} = \alpha_{1i,t} \psi_{ij,t} \hat{Q}_{ij}(f_{ij,t}) \quad (8)$$

where the function $\hat{Q}_{ij}(\cdot)$ is defined by

$$\hat{Q}_{ij}(f_{ij,t}) = f_{ij,t} b_{ij,t}. \quad (9)$$

The random variables $\alpha_{1i,t}$ ($i \in \mathbb{D}_1$) are mutually independent and are introduced to capture the occurrence of packet losses over the STR channel. Their distribution is specified as

$$\text{Prob}\{\alpha_{1i,t} = 1\} = \bar{\alpha}_{1i}, \quad \text{Prob}\{\alpha_{1i,t} = 0\} = 1 - \bar{\alpha}_{1i} \quad (10)$$

where $\bar{\alpha}_{1i} > 0$ ($i \in \mathbb{D}_1$) are known scalars.

By defining the decoding error as $\delta_{ij,t} \triangleq \psi_{ij,t} \hat{Q}_{ij}(f_{ij,t}) - y_{ij,t}$ ($i \in \mathbb{D}_1, j \in \mathbb{D}_2$), we infer readily from (5), (6) and (9) that

$$|\delta_{ij,t}| = \psi_{ij,t} \left| Q_{ij}\left(\frac{y_{ij,t}}{\psi_{ij,t}}\right) - \frac{y_{ij,t}}{\psi_{ij,t}} \right| \leq 0.5\psi_{ij,t}b_{ij,t}. \quad (11)$$

Then, the decoded signal $r_{i,t}$ is rewritten as

$$r_{i,t} = \alpha_{1i,t}(y_{i,t} + \delta_{i,t}) \quad (12)$$

with $\delta_{i,t} \triangleq \text{col}_{n_y}\{\delta_{ij,t}\}$.

Next, the decoded signal $r_{i,t}$ is transmitted to the corresponding filter through the RTF channel. Let $\bar{y}_{i,t}$ denote the input to filter i , whose dynamics is modeled as

$$\bar{y}_{i,t} = \sqrt{p_i}\alpha_{2i,t}r_{i,t} + L_{i,t}\varphi_{i,t} \quad (13)$$

where p_i is a known average signal energy, $\varphi_{i,t} \in l_2([0, T], \mathbb{R})$ is the channel noise, and $L_{i,t}$ is a given matrix with compatible dimensions. The random variables $\alpha_{2i,t}$ ($i \in \mathbb{D}_1$) are also mutually independent and model the packet loss over the RTF channel, with their distribution specified by

$$\text{Prob}\{\alpha_{2i,t} = 1\} = \bar{\alpha}_{2i}, \quad \text{Prob}\{\alpha_{2i,t} = 0\} = 1 - \bar{\alpha}_{2i} \quad (14)$$

where $\bar{\alpha}_{2i} > 0$ ($i \in \mathbb{D}_1$) are known constants.

Remark 2: In long-distance signal transmissions, communication quality tends to degrade with increasing distance

because of limited transmission capacity, which may corrupt measurement outputs and further deteriorate the resulting filtering performance. To enable reliable signal transmissions, a DaF relay-aided mechanism is employed, aiming to extend the transmission range while maintaining the desired communication quality. Randomly occurring packet losses are considered in both the STR and RTF channels, characterized by Bernoulli sequences $\alpha_{1i,t}$ and $\alpha_{2i,t}$ ($i \in \mathbb{D}_1$) with statistical properties given in (10) and (14), respectively. Taking into account the joint effects of DaF relay-based communication, packet losses, and channel noise, the received signal at the remote filter i is modeled in (13), indicating that the introduced relay-aided mechanism significantly complicates the overall data transmission process.

Remark 3: In this study, the dynamic quantizer described in (6)-(7) is used to generate the codeword $f_{ij,t}$ ($i \in \mathbb{D}_1, j \in \mathbb{D}_2$). Compared with the commonly employed static quantizer [8], the dynamic quantizer offers greater adaptability by incorporating the adjustable parameter $b_{ij,t}$ in (7). Furthermore, the scaling factor $\psi_{ij,t}$ ensures that $\frac{y_{ij,t}}{\psi_{ij,t}}$ stays within the quantization interval $[-db_{ij,t}, db_{ij,t}]$. Based on the boundedness condition (11) for the decoding error $\delta_{ij,t}$, a smaller value of $\psi_{ij,t}$ reduces the decoding error. However, this may increase the risk of quantizer saturation (i.e. $|\frac{y_{ij,t}}{\psi_{ij,t}}| > db_{ij,t}$). Therefore, selecting an appropriate value of $\psi_{ij,t}$ is of critical importance for achieving a trade-off between reducing decoding error and avoiding quantizer saturation.

For presentation convenience, the following notations are introduced:

$$\begin{aligned} \bar{p}_i &\triangleq \sqrt{p_i} \bar{\beta}_i, & \sigma_{i,t} &\triangleq \sigma_i(C_{i,t}x_t), & \beta_{i,t} &\triangleq \alpha_{1i,t} \alpha_{2i,t}, \\ \bar{\beta}_i &\triangleq \bar{\alpha}_{1i} \bar{\alpha}_{2i}, & \tilde{\beta}_{i,t} &\triangleq \beta_{i,t} - \bar{\beta}_i. \end{aligned}$$

Based on the received measurement $\bar{y}_{i,t}$ ($i \in \mathbb{D}_1$), the structure of the local filter is specified as

$$\begin{cases} \hat{x}_{i,t+1} = A_t \hat{x}_{i,t} + \tilde{K}_{i,t}(\bar{y}_{i,t} - \bar{p}_i C_{i,t} \hat{x}_{i,t}) \\ \hat{z}_{i,t} = M_t \hat{x}_{i,t} \end{cases} \quad (15)$$

with $\tilde{K}_{i,t} \triangleq K_{i,t} + g_{i,t} \bar{K}_{i,t}$, where $\hat{x}_{i,t} \in \mathbb{R}^{n_x}$ and $\hat{z}_{i,t} \in \mathbb{R}^{n_z}$ are, respectively, the estimates of $x_{i,t}$ and $z_{i,t}$; $K_{i,t}$ is the filter gain to be designed; $\bar{K}_{i,t}$ is a known matrix of appropriate dimensions; and $g_{i,t} \in \mathbb{R}$ is a stochastic perturbation having zero mean and covariance $G_{i,t} > 0$.

Remark 4: The equation (15) reveals that the effects of sensor saturations, noise disturbances, packet losses, and SGVs are comprehensively incorporated into the local filtering framework. In particular, SGVs, arising from unexpected implementation errors (e.g. rounding or programming inaccuracies), are characterized by the term $g_{i,t} \bar{K}_{i,t}$ ($i \in \mathbb{D}_1$). Consequently, the filter gain is given by $K_{i,t} + g_{i,t} \bar{K}_{i,t}$, representing a randomly varying parameter. This formulation reduces to the conventional deterministic case when $\bar{K}_{i,t} = 0$.

Let $e_{i,t} \triangleq x_t - \hat{x}_{i,t}$ and $\tilde{z}_{i,t} \triangleq z_t - \hat{z}_{i,t}$ ($i \in \mathbb{D}_1$) denote the local estimation errors of the state and output, respectively.

Then, the corresponding error dynamics can be expressed as

$$\begin{cases} e_{i,t+1} = (A_t - \bar{p}_i \tilde{K}_{i,t} C_{i,t}) e_{i,t} + \bar{p}_i \tilde{K}_{i,t} C_{i,t} x_t \\ \quad + (B_t - \sqrt{p_i} \beta_{i,t} \tilde{K}_{i,t} D_{i,t}) v_t \\ \quad - \sqrt{p_i} \beta_{i,t} \tilde{K}_{i,t} (\sigma_{i,t} + \delta_{i,t}) - \tilde{K}_{i,t} L_{i,t} \varphi_{i,t} \\ \tilde{z}_{i,t} = M_t e_{i,t}. \end{cases} \quad (16)$$

By defining the augmented vector $\eta_{i,t} \triangleq [x_t^T \ e_{i,t}^T]^T$ ($i \in \mathbb{D}_1$), the following augmented filtering error system (AFES) is obtained:

$$\begin{cases} \eta_{i,t+1} = \mathbb{R}_{1i,t} \eta_{i,t} + \mathbb{R}_{2i,t} \omega_{i,t} + \mathbb{R}_{3i,t} (\sigma_{i,t} + \delta_{i,t}) \\ \tilde{z}_{i,t} = \mathcal{M}_t \eta_{i,t} \end{cases} \quad (17)$$

where

$$\begin{aligned} \mathbb{R}_{1i,t} &\triangleq R_{11i,t} + g_{i,t} R_{12i,t}, & \omega_{i,t} &\triangleq [v_t^T \ \varphi_{i,t}^T]^T, \\ \mathbb{R}_{2i,t} &\triangleq R_{21i,t} + \tilde{\beta}_{i,t} R_{22i,t} + g_{i,t} R_{23i,t} + g_{i,t} \tilde{\beta}_{i,t} R_{24i,t}, \\ \mathbb{R}_{3i,t} &\triangleq R_{31i,t} + \tilde{\beta}_{i,t} R_{32i,t} + g_{i,t} R_{33i,t} + g_{i,t} \tilde{\beta}_{i,t} R_{34i,t}, \\ R_{11i,t} &\triangleq \begin{bmatrix} A_t & 0 \\ \bar{p}_i K_{i,t} C_{i,t} & A_t - \bar{p}_i K_{i,t} C_{i,t} \end{bmatrix}, \\ R_{12i,t} &\triangleq \begin{bmatrix} 0 & 0 \\ \bar{p}_i \bar{K}_{i,t} C_{i,t} & -\bar{p}_i \bar{K}_{i,t} C_{i,t} \end{bmatrix}, \\ R_{21i,t} &\triangleq \begin{bmatrix} B_t & 0 \\ B_t - \bar{p}_i K_{i,t} D_{i,t} & -K_{i,t} L_{i,t} \end{bmatrix}, \\ R_{22i,t} &\triangleq \begin{bmatrix} 0 & 0 \\ -\sqrt{p_i} K_{i,t} D_{i,t} & 0 \end{bmatrix}, \\ R_{23i,t} &\triangleq \begin{bmatrix} 0 & 0 \\ -\bar{p}_i \bar{K}_{i,t} D_{i,t} & -\bar{K}_{i,t} L_{i,t} \end{bmatrix}, \\ R_{24i,t} &\triangleq \begin{bmatrix} 0 & 0 \\ -\sqrt{p_i} \bar{K}_{i,t} D_{i,t} & 0 \end{bmatrix}, & \mathcal{M}_t &\triangleq [0 \ M_t], \\ R_{31i,t} &\triangleq [0 \ -\bar{p}_i K_{i,t}^T]^T, & R_{32i,t} &\triangleq [0 \ -\sqrt{p_i} K_{i,t}^T]^T, \\ R_{33i,t} &\triangleq [0 \ -\bar{p}_i \bar{K}_{i,t}^T]^T, & R_{34i,t} &\triangleq [0 \ -\sqrt{p_i} \bar{K}_{i,t}^T]^T. \end{aligned}$$

To improve estimation accuracy, the local state estimates are next propagated to the FC and subsequently combined according to a diagonal matrix-weighted fusion (DMWF) criterion. Based on the available local estimates $\hat{x}_{i,t}$ ($i \in \mathbb{D}_1$), the following distributed fusion filter is constructed:

$$\begin{cases} \hat{x}_t = \sum_{i=1}^N \Theta_i \hat{x}_{i,t} \\ \hat{z}_t = M_t \hat{x}_t \end{cases} \quad (18)$$

where $\Theta_i \triangleq \text{diag}\{\theta_{i1}, \theta_{i2}, \dots, \theta_{in_x}\}$ is the fusion parameter matrix to be designed. Each component θ_{il} ($l \in \mathbb{D}_3 \triangleq \{1, 2, \dots, n_x\}$) is constrained to satisfy $\theta_{il} \in (0, 1)$ and $\sum_{i=1}^N \theta_{il} = 1$, ensuring proper convex weighting across all sensors for each state component.

Let $\bar{e}_t \triangleq x_t - \hat{x}_t$ and $\tilde{z}_t \triangleq z_t - \hat{z}_t$ denote the fusion estimation errors for the system state and output, respectively.

Based on (1) and (18), the following fusion filtering error system can be derived:

$$\begin{cases} \bar{e}_t = \sum_{i=1}^N \Theta_i e_{i,t} \\ \tilde{z}_t = \sum_{i=1}^N \Theta_i \tilde{z}_{i,t}. \end{cases} \quad (19)$$

The objective of this paper is two-fold and stated as follows.

$\mathbb{O}_1$: For the system (1) operating under the DaF relay-based communication mechanism, design a suitable local filter for each node $i \in \mathbb{D}_1$ such that the following H_∞ performance index is satisfied:

$$\begin{aligned} \mathbb{J}_{1i} &\triangleq \sum_{t=0}^T E\{\|\tilde{z}_{i,t}\|^2 - \gamma_i^2(\|\omega_{i,t}\|^2 + \bar{\Delta}_{i,t})\} - \gamma_i^2 \eta_{i,0}^T \mathbb{W}_i \eta_{i,0} \\ &< 0 \end{aligned} \quad (20)$$

over the finite horizon $[0, T]$ with

$$\bar{\Delta}_{i,t} \triangleq 0.25 \sum_{j=1}^{n_y} \psi_{ij,t}^2 b_{ij,t}^2$$

where $\eta_{i,0}$ is a given initial value, $\gamma_i > 0$ is a prescribed scalar, and $\mathbb{W}_i > 0$ is a known weighting matrix.

$\mathbb{O}_2$: Determine appropriate fusion parameter matrices Θ_i ($i \in \mathbb{D}_1$) such that the fusion error system (19) satisfies the global H_∞ performance constraint:

$$\mathbb{J}_2 \triangleq \sum_{t=0}^T E\{\|\tilde{z}_t\|^2 - \tilde{\gamma}^2(\|\bar{\omega}_t\|^2 + \bar{\Delta}_t)\} - \tilde{\gamma}^2 \eta_0^T \bar{\mathbb{W}} \eta_0 < 0 \quad (21)$$

where $\tilde{\gamma} > 0$ is the desired disturbance rejection level, and the auxiliary terms are defined by:

$$\begin{aligned} \bar{\omega}_t &\triangleq [v_t^T \quad \varphi_{1,t}^T \quad \varphi_{2,t}^T \quad \cdots \quad \varphi_{N,t}^T]^T, \quad \bar{\Delta}_t \triangleq \sum_{i=1}^N \bar{\Delta}_{i,t}, \\ \bar{\eta}_0 &\triangleq \text{col}_N\{\eta_{i,0}\}, \quad \bar{\mathbb{W}} \triangleq \text{diag}_N\{\mathbb{W}_i\}. \end{aligned}$$

Before proceeding with the main results, the following lemmas are introduced to support the design of the filter gain $K_{i,t}$ ($i \in \mathbb{D}_1$) in the subsequent analysis.

Lemma 1: [43] For the scalar-valued function $\sigma_{ij}(y_j)$ ($i \in \mathbb{D}_1, j \in \mathbb{D}_2$), there exists a scalar $\lambda_{ij} \in (0, 1)$ such that the following inequality holds:

$$[\sigma_{ij}(\bar{h}_j) - \lambda_{ij} \bar{h}_j][\sigma_{ij}(\bar{h}_j) - \bar{h}_j] \leq 0.$$

Lemma 2: [27] Let M, N and S be nonzero matrices with compatible dimensions. Then, the matrix $K = M^\dagger S N^\dagger$ is the solution to the following optimization problem:

$$\min_K \|MKN - S\|_F.$$

III. MAIN RESULTS

In this section, the design of a non-fragile distributed fusion filter is carried out for the target plant described in (1), taking into account sensor saturations and packet losses under DaF relay framework. First, a sufficient condition is established to ensure that the performance constraint (20) is satisfied. Based on this result, the desired filter gain for each local filter (15) is computed by solving a set of coupled backward RDEs. Subsequently, an appropriate distributed fusion filter is constructed by solving a convex optimization problem, ensuring the satisfaction of the global performance criterion.

A. Local H_∞ filter design

To begin with, a sufficient condition is established in the following theorem to guarantee the performance index (20).

Theorem 1: Let the scalar $\gamma_i > 0$, weighting matrix $\mathbb{W}_i > 0$ and filter gain $K_{i,t}$ ($i \in \mathbb{D}_1, t \in [0, T]$) be given. The performance constraint $\mathbb{J}_{1i} < 0$ is achieved if there exists a positive definite matrix $P_{i,t+1} \triangleq \text{diag}\{P_{1i,t+1}, P_{2i,t+1}\}$ satisfying the following RDE

$$P_{i,t} = \Xi_{1i,t} + \mathcal{M}_t^T \mathcal{M}_t + \gamma_i^2 \Xi_{5i,t} + \Xi_{4i,t} \Upsilon_{1i,t}^{-1} \Xi_{4i,t}^T \quad (22)$$

under conditions

$$\begin{cases} P_{i,T+1} = 0, \quad P_{i,0} < \gamma_i^2 \mathbb{W}_i \\ \Upsilon_{1i,t} \triangleq \gamma_i^2 I - \Xi_{2i,t} > 0 \\ \Xi_{3i,t} < \gamma_i^2 (I - U_{i,t}) \end{cases} \quad (23)$$

where

$$\begin{aligned} \Xi_{1i,t} &\triangleq 3(R_{11i,t}^T P_{i,t+1} R_{11i,t} + G_{i,t} R_{12i,t}^T P_{i,t+1} R_{12i,t}), \\ \Xi_{2i,t} &\triangleq 3R_{21i,t}^T P_{i,t+1} R_{21i,t} + 3\beta_i^* R_{22i,t}^T P_{i,t+1} R_{22i,t} \\ &\quad + 3G_{i,t} R_{23i,t}^T P_{i,t+1} R_{23i,t} \\ &\quad + 3\beta_i^* G_{i,t} R_{24i,t}^T P_{i,t+1} R_{24i,t}, \\ \Xi_{3i,t} &\triangleq 4R_{31i,t}^T P_{i,t+1} R_{31i,t} + 4G_{i,t} R_{33i,t}^T P_{i,t+1} R_{33i,t} \\ &\quad + 3\beta_i^* R_{32i,t}^T P_{i,t+1} R_{32i,t} \\ &\quad + 3\beta_i^* G_{i,t} R_{34i,t}^T P_{i,t+1} R_{34i,t}, \\ \Xi_{4i,t} &\triangleq R_{11i,t}^T P_{i,t+1} R_{21i,t} + G_{i,t} R_{12i,t}^T P_{i,t+1} R_{23i,t}, \\ \Xi_{5i,t} &\triangleq \text{diag}\{C_{i,t}^T (I - U_{i,t}) C_{i,t}, 0\}, \quad \beta_i^* \triangleq \bar{\beta}_i - \bar{\beta}_i^2, \\ \Lambda_{i,t} &\triangleq \text{diag}_{n_y}\{\lambda_{ij,t}\}, \quad U_{i,t} \triangleq 0.5(\Lambda_{i,t} + I). \end{aligned}$$

Proof: Selecting the Lyapunov-like function

$$V_{i,t} \triangleq \eta_{i,t}^T P_{i,t} \eta_{i,t},$$

we calculate from (17) to obtain

$$\begin{aligned} &E\{\Delta V_{i,t}\} \\ &\triangleq E\{V_{i,t+1} - V_{i,t}\} \\ &= E\{[\mathbb{R}_{1i,t} \eta_{i,t} + \mathbb{R}_{2i,t} \omega_{i,t} + \mathbb{R}_{3i,t}(\sigma_{i,t} + \delta_{i,t})]^T P_{i,t+1} \\ &\quad \times [\mathbb{R}_{1i,t} \eta_{i,t} + \mathbb{R}_{2i,t} \omega_{i,t} + \mathbb{R}_{3i,t}(\sigma_{i,t} + \delta_{i,t})] \\ &\quad - \eta_{i,t}^T P_{i,t} \eta_{i,t}\}. \end{aligned} \quad (24)$$

According to the definitions of $\beta_{i,t}$ and $g_{i,t}$, one has from (10) and (14) that

$$E\{\Delta V_{i,t}\}$$

$$\begin{aligned}
&= E\{\eta_{i,t}^T \mathbb{R}_{1i,t} P_{i,t+1} \mathbb{R}_{1i,t} \eta_{i,t} + \omega_{i,t}^T \mathbb{R}_{2i,t}^T P_{i,t+1} \mathbb{R}_{2i,t} \omega_{i,t} \\
&\quad + \sigma_{i,t}^T \mathbb{R}_{3i,t}^T P_{i,t+1} \mathbb{R}_{3i,t} \sigma_{i,t} + \delta_{i,t}^T \mathbb{R}_{3i,t}^T P_{i,t+1} \mathbb{R}_{3i,t} \delta_{i,t} \\
&\quad + 2\eta_{i,t}^T \mathbb{R}_{1i,t} P_{i,t+1} (\mathbb{R}_{2i,t} \omega_{i,t} + \mathbb{R}_{3i,t} \sigma_{i,t} + \mathbb{R}_{3i,t} \delta_{i,t}) \\
&\quad + 2\omega_{i,t}^T \mathbb{R}_{2i,t}^T P_{i,t+1} (\mathbb{R}_{3i,t} \sigma_{i,t} + \mathbb{R}_{3i,t} \delta_{i,t}) \\
&\quad + 2\sigma_{i,t}^T \mathbb{R}_{3i,t}^T P_{i,t+1} \mathbb{R}_{3i,t} \delta_{i,t} - \eta_{i,t}^T P_{i,t} \eta_{i,t}\} \\
&\leq E\{\eta_{i,t}^T (\Xi_{1i,t} - P_{i,t}) \eta_{i,t} + \omega_{i,t}^T \Xi_{2i,t} \omega_{i,t} + \sigma_{i,t}^T \Xi_{3i,t} \sigma_{i,t} \\
&\quad + \delta_{i,t}^T \Xi_{3i,t} \delta_{i,t} + 2\eta_{i,t}^T \Xi_{4i,t} \omega_{i,t}\}. \tag{25}
\end{aligned}$$

Noting $\Lambda_{i,t} = \text{diag}_{n_y}\{\lambda_{ij,t}\}$, $U_{i,t} = 0.5(\Lambda_{i,t} + I)$, and $\sigma_i(\bar{h}) = \text{col}_{n_y}\{\sigma_{ij}(\bar{h}_j)\}$, the inequality in Lemma 1 yields

$$\sigma_{i,t}^T \sigma_{i,t} + x_t^T C_{i,t}^T (2U_{i,t} - I) C_{i,t} x_t - 2x_t^T C_{i,t}^T U_{i,t} \sigma_{i,t} \leq 0 \tag{26}$$

which further implies

$$\begin{aligned}
&\sigma_{i,t}^T \sigma_{i,t} + x_t^T C_{i,t}^T (2U_{i,t} - I) C_{i,t} x_t \\
&\leq x_t^T C_{i,t}^T U_{i,t} C_{i,t} x_t + \sigma_{i,t}^T U_{i,t} \sigma_{i,t} \tag{27}
\end{aligned}$$

and

$$\begin{aligned}
\sigma_{i,t}^T (I - U_{i,t}) \sigma_{i,t} &\leq x_t^T C_{i,t}^T (I - U_{i,t}) C_{i,t} x_t \\
&= \eta_{i,t}^T \Xi_{5i,t} \eta_{i,t}. \tag{28}
\end{aligned}$$

On the basis of (11), we arrive at

$$\delta_{i,t}^T \delta_{i,t} = \sum_{j=1}^{n_y} \delta_{ij,t}^T \delta_{ij,t} \leq \bar{\Delta}_{i,t}. \tag{29}$$

Substituting (28) and (29) into (25) gives

$$\begin{aligned}
&E\{\Delta V_{i,t}\} \\
&\leq E\{\eta_{i,t}^T (\Xi_{1i,t} - P_{i,t}) \eta_{i,t} + \omega_{i,t}^T \Xi_{2i,t} \omega_{i,t} + \sigma_{i,t}^T \Xi_{3i,t} \sigma_{i,t} \\
&\quad + \delta_{i,t}^T \Xi_{3i,t} \delta_{i,t} + 2\eta_{i,t}^T \Xi_{4i,t} \omega_{i,t} - \gamma_i^2 (\delta_{i,t}^T \delta_{i,t} - \bar{\Delta}_{i,t}) \\
&\quad - \gamma_i^2 [\sigma_{i,t}^T (I - U_{i,t}) \sigma_{i,t} - \eta_{i,t}^T \Xi_{5i,t} \eta_{i,t}] \\
&\quad + (\|\tilde{z}_{i,t}\|^2 - \gamma_i^2 \|\omega_{i,t}\|^2) - (\|\tilde{z}_{i,t}\|^2 - \gamma_i^2 \|\omega_{i,t}\|^2)\} \\
&= E\{\eta_{i,t}^T (\Xi_{1i,t} + \mathcal{M}_t^T \mathcal{M}_t + \gamma_i^2 \Xi_{5i,t} - P_{i,t}) \eta_{i,t} \\
&\quad - \omega_{i,t}^T (\gamma_i^2 I - \Xi_{2i,t}) \omega_{i,t} + 2\eta_{i,t}^T \Xi_{4i,t} \omega_{i,t} \\
&\quad - \delta_{i,t}^T (\gamma_i^2 I - \Xi_{3i,t}) \delta_{i,t} - \sigma_{i,t}^T (\gamma_i^2 I - \gamma_i^2 U_{i,t} - \Xi_{3i,t}) \sigma_{i,t} \\
&\quad - [\|\tilde{z}_{i,t}\|^2 - \gamma_i^2 (\|\omega_{i,t}\|^2 + \bar{\Delta}_{i,t})]\}. \tag{30}
\end{aligned}$$

By letting $\omega_{i,t}^* \triangleq \Upsilon_{1i,t}^{-1} \Xi_{4i,t}^T \eta_{i,t}$, one obtains

$$\begin{aligned}
&-\omega_{i,t}^T (\gamma_i^2 I - \Xi_{2i,t}) \omega_{i,t} + 2\eta_{i,t}^T \Xi_{4i,t} \omega_{i,t} \\
&= (\omega_{i,t}^*)^T \Upsilon_{1i,t} \omega_{i,t}^* - (\omega_{i,t} - \omega_{i,t}^*)^T \Upsilon_{1i,t} (\omega_{i,t} - \omega_{i,t}^*). \tag{31}
\end{aligned}$$

Applying (22) and (31) to (30) leads to

$$\begin{aligned}
&E\{\Delta V_{i,t}\} \\
&\leq E\{\eta_{i,t}^T (\Xi_{1i,t} + \mathcal{M}_t^T \mathcal{M}_t + \gamma_i^2 \Xi_{5i,t} + \Xi_{4i,t} \Upsilon_{1i,t}^{-1} \Xi_{4i,t}^T \\
&\quad - P_{i,t}) \eta_{i,t} - (\omega_{i,t} - \omega_{i,t}^*)^T \Upsilon_{1i,t} (\omega_{i,t} - \omega_{i,t}^*) \\
&\quad - \delta_{i,t}^T (\gamma_i^2 I - \Xi_{3i,t}) \delta_{i,t} - \sigma_{i,t}^T (\gamma_i^2 I - \gamma_i^2 U_{i,t} - \Xi_{3i,t}) \sigma_{i,t} \\
&\quad - [\|\tilde{z}_{i,t}\|^2 - \gamma_i^2 (\|\omega_{i,t}\|^2 + \bar{\Delta}_{i,t})]\} \\
&\leq -E\{(\omega_{i,t} - \omega_{i,t}^*)^T \Upsilon_{1i,t} (\omega_{i,t} - \omega_{i,t}^*) \\
&\quad - [\|\tilde{z}_{i,t}\|^2 - \gamma_i^2 (\|\omega_{i,t}\|^2 + \bar{\Delta}_{i,t})]\}. \tag{32}
\end{aligned}$$

Noticing $\Upsilon_{1i,t} > 0$ and summing up both sides of (32) from $t = 0$ to $t = T$, one obtains

$$\begin{aligned}
&E\{V_{i,T+1} - V_{i,0}\} + \sum_{t=0}^T E\{\|\tilde{z}_{i,t}\|^2 - \gamma_i^2 (\|\omega_{i,t}\|^2 + \bar{\Delta}_{i,t})\} \\
&\leq -\sum_{t=0}^T E\{(\omega_{i,t} - \omega_{i,t}^*)^T \Upsilon_{1i,t} (\omega_{i,t} - \omega_{i,t}^*)\} \leq 0. \tag{33}
\end{aligned}$$

Combining $P_{i,T+1} = 0$ and $P_{i,0} < \gamma_i^2 \mathbb{W}_i$, it follows that

$$\begin{aligned}
\mathbb{J}_{1i} &= \sum_{t=0}^T E\{\|\tilde{z}_{i,t}\|^2 - \gamma_i^2 (\|\omega_{i,t}\|^2 + \bar{\Delta}_{i,t})\} - \gamma_i^2 \eta_{i,0}^T \mathbb{W}_i \eta_{i,0} \\
&\leq \eta_{i,0}^T (P_{i,0} - \gamma_i^2 \mathbb{W}_i) \eta_{i,0} < 0
\end{aligned}$$

which ends the proof of this theorem. $\blacksquare$

Now, let us design the filter gain for each sensor node under the worst-case scenario specified by

$$\omega_{i,t} = \omega_{i,t}^* = \Upsilon_{1i,t}^{-1} \Xi_{4i,t}^T \eta_{i,t}. \tag{34}$$

Under this scenario, the AFES in (17) is reformulated as

$$\begin{cases} \eta_{i,t+1} = \bar{\mathbb{R}}_{1i,t} \eta_{i,t} + \mathbb{R}_{3i,t} (\sigma_{i,t} + \delta_{i,t}) - \bar{\mathbb{I}} \ell_{i,t} \\ \tilde{z}_{i,t} = \mathcal{M}_t \eta_{i,t} \end{cases} \tag{35}$$

where

$$\begin{aligned}
\bar{\mathbb{R}}_{1i,t} &\triangleq \bar{R}_{11,t} + g_{i,t} R_{12i,t} + R_{13i,t} + \mathbb{R}_{2i,t} \Psi_{i,t}, \\
\bar{R}_{11,t} &\triangleq \text{diag}\{A_t, A_t\}, \quad \Psi_{i,t} \triangleq \Upsilon_{1i,t}^{-1} \Xi_{4i,t}^T, \\
R_{13i,t} &\triangleq \begin{bmatrix} 0 & 0 \\ \bar{p}_i K_{i,t} C_{i,t} & 0 \end{bmatrix}, \quad \bar{\mathbb{I}} \triangleq \begin{bmatrix} 0 \\ I \end{bmatrix}, \\
\ell_{i,t} &\triangleq \bar{p}_i K_{i,t} \bar{C}_{i,t} \eta_{i,t}, \quad \bar{C}_{i,t} \triangleq [0 \quad C_{i,t}].
\end{aligned}$$

Define the cost function

$$\mathbb{J}_{3i} \triangleq \sum_{t=0}^T E\{\|\tilde{z}_{i,t}\|^2 + \|\ell_{i,t}\|^2\}. \tag{36}$$

In the following theorem, the filter gain $K_{i,t}$ ($i \in \mathbb{D}_1$) is designed by minimizing such a function.

Theorem 2: Let the scalar $\gamma_i > 0$ and matrix $\mathbb{W}_i > 0$ ($i \in \mathbb{D}_1$) be given. Suppose that there exist matrices $P_{i,t+1} \triangleq \text{diag}\{P_{1i,t+1}, P_{2i,t+1}\}$, $O_{i,t+1} \triangleq \text{diag}\{O_{1i,t+1}, O_{2i,t+1}\}$ and $K_{i,t}$ ($t \in [0, T]$) such that the RDEs (22) and

$$O_{i,t} = \Pi_{1i,t} + \mathcal{M}_t^T \mathcal{M}_t + \Xi_{5i,t} - \Upsilon_{2i,t} \Xi_{6i,t}^{-1} \Upsilon_{2i,t}^T \tag{37}$$

are feasible subject to conditions (23) and

$$\begin{cases} O_{i,T+1} = 0 \\ \Xi_{6i,t} \triangleq 5\bar{\mathbf{I}}^T O_{i,t+1} \bar{\mathbf{I}} + I > 0 \\ \Pi_{2i,t} < I - U_{i,t} \end{cases} \tag{38}$$

where

$$\begin{aligned}
\Pi_{1i,t} &\triangleq 3(\bar{R}_{11,t}^T O_{i,t+1} \bar{R}_{11,t} + G_{i,t} R_{12i,t}^T O_{i,t+1} R_{12i,t}) \\
&\quad + 4R_{13i,t}^T O_{i,t+1} R_{13i,t} \\
&\quad + 4\Psi_{i,t}^T R_{21i,t}^T O_{i,t+1} R_{21i,t} \Psi_{i,t} \\
&\quad + 3\Psi_{i,t}^T \beta_i^* R_{22i,t}^T O_{i,t+1} R_{22i,t} \Psi_{i,t} \\
&\quad + 3\Psi_{i,t}^T G_{i,t} R_{23i,t}^T O_{i,t+1} R_{23i,t} \Psi_{i,t}
\end{aligned}$$

$$\begin{aligned}
& + 3\Psi_{i,t}^T \beta_i^* G_{i,t} R_{24i,t}^T O_{i,t+1} R_{24i,t} \Psi_{i,t} \\
& + 2[\bar{R}_{11,t}^T O_{i,t+1} R_{13i,t} + (G_{i,t} R_{12i,t}^T O_{i,t+1} R_{23i,t} \\
& + \bar{R}_{11,t}^T O_{i,t+1} R_{21i,t} + R_{13i,t}^T O_{i,t+1} R_{21i,t}) \Psi_{i,t}], \\
\Pi_{2i,t} & \triangleq 6R_{31i,t}^T O_{i,t+1} R_{31i,t} + 4G_{i,t} R_{33i,t}^T O_{i,t+1} R_{33i,t} \\
& + 3\beta_i^* R_{32i,t}^T O_{i,t+1} R_{32i,t} \\
& + 3\beta_i^* G_{i,t} R_{34i,t}^T O_{i,t+1} R_{34i,t}, \\
\Xi_{6i,t} & \triangleq 5\bar{\mathbf{I}}^T O_{i,t+1} \bar{\mathbf{I}} + I, \quad \Upsilon_{2i,t} \triangleq \bar{R}_{11,t}^T O_{i,t+1} \bar{\mathbf{I}}, \\
\Upsilon_{3i,t} & \triangleq \Xi_{6i,t}^{-1} \Upsilon_{2i,t}^T.
\end{aligned}$$

Then, the performance constraint $\mathbb{J}_{1i} < 0$ is satisfied, and the optimal filter gain $K_{i,t}$ is determined by the following Frobenius norm minimization problem:

$$K_{i,t} = \arg \min_{K_{i,t}} \|\bar{p}_i K_{i,t} \bar{C}_{i,t} - \Upsilon_{3i,t}\|_F. \quad (39)$$

Proof: Letting $\bar{V}_{i,t} \triangleq \eta_{i,t}^T O_{i,t} \eta_{i,t}$, we derive from (35) that

$$\begin{aligned}
& E\{\Delta \bar{V}_{i,t}\} \\
& \triangleq E\{\bar{V}_{i,t+1} - \bar{V}_{i,t}\} \\
& = E\{[\bar{\mathbb{R}}_{1i,t} \eta_{i,t} + \mathbb{R}_{3i,t}(\sigma_{i,t} + \delta_{i,t}) - \bar{\mathbf{I}} \ell_{i,t}]^T O_{i,t+1} \\
& \quad \times [\bar{\mathbb{R}}_{1i,t} \eta_{i,t} + \mathbb{R}_{3i,t}(\sigma_{i,t} + \delta_{i,t}) - \bar{\mathbf{I}} \ell_{i,t}] \\
& \quad - \eta_{i,t}^T O_{i,t} \eta_{i,t}\}. \quad (40)
\end{aligned}$$

From the expression of $\bar{\mathbb{R}}_{1i,t}$, one has

$$\begin{aligned}
& E\{\eta_{i,t}^T \bar{\mathbb{R}}_{1i,t}^T O_{i,t+1} \bar{\mathbb{R}}_{1i,t} \eta_{i,t}\} \\
& = E\{\eta_{i,t}^T [\bar{R}_{11,t}^T O_{i,t+1} \bar{R}_{11,t} + G_{i,t} R_{12i,t}^T O_{i,t+1} R_{12i,t} \\
& \quad + R_{13i,t}^T O_{i,t+1} R_{13i,t} + \Psi_{i,t}^T (R_{21i,t}^T O_{i,t+1} R_{21i,t} \\
& \quad + \beta_i^* R_{22i,t}^T O_{i,t+1} R_{22i,t} + G_{i,t} R_{23i,t}^T O_{i,t+1} R_{23i,t} \\
& \quad + \beta_i^* G_{i,t} R_{24i,t}^T O_{i,t+1} R_{24i,t}) \Psi_{i,t} \\
& \quad + 2(\bar{R}_{11,t}^T O_{i,t+1} R_{13i,t} + G_{i,t} R_{12i,t}^T O_{i,t+1} R_{23i,t} \Psi_{i,t} \\
& \quad + (\bar{R}_{11,t} + R_{13i,t})^T O_{i,t+1} R_{21i,t} \Psi_{i,t}) \eta_{i,t}]\}. \quad (41)
\end{aligned}$$

By utilizing the fundamental inequality technique, we derive

$$\begin{aligned}
& -2E\{\eta_{i,t}^T \bar{\mathbb{R}}_{1i,t}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t}\} \\
& = -2E\{\eta_{i,t}^T \bar{R}_{11,t}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t} + \eta_{i,t}^T R_{13i,t}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t} \\
& \quad + \eta_{i,t}^T \Psi_{i,t}^T R_{21i,t}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t}\} \\
& \leq E\{2\ell_{i,t}^T \bar{\mathbf{I}}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t} - 2\eta_{i,t}^T \bar{R}_{11,t}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t} \\
& \quad + \eta_{i,t}^T (\Psi_{i,t}^T R_{21i,t}^T O_{i,t+1} R_{21i,t} \Psi_{i,t} \\
& \quad + R_{13i,t}^T O_{i,t+1} R_{13i,t}) \eta_{i,t}\}. \quad (42)
\end{aligned}$$

It is easily checked from (40)-(42) that

$$\begin{aligned}
& E\{\Delta \bar{V}_{i,t}\} \\
& = E\{\eta_{i,t}^T (\bar{\mathbb{R}}_{1i,t}^T O_{i,t+1} \bar{\mathbb{R}}_{1i,t} - O_{i,t}) \eta_{i,t} + \ell_{i,t}^T \bar{\mathbf{I}}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t} \\
& \quad + (\sigma_{i,t} + \delta_{i,t})^T \mathbb{R}_{3i,t}^T O_{i,t+1} \mathbb{R}_{3i,t} (\sigma_{i,t} + \delta_{i,t}) \\
& \quad + 2\eta_{i,t}^T \bar{\mathbb{R}}_{1i,t}^T O_{i,t+1} [\mathbb{R}_{3i,t} (\sigma_{i,t} + \delta_{i,t}) - \bar{\mathbf{I}} \ell_{i,t}] \\
& \quad - 2(\sigma_{i,t} + \delta_{i,t})^T \mathbb{R}_{3i,t}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t}\} \\
& \leq E\{\eta_{i,t}^T (\Pi_{1i,t} - O_{i,t}) \eta_{i,t} + \sigma_{i,t}^T \Pi_{2i,t} \sigma_{i,t} + \delta_{i,t}^T \Pi_{2i,t} \delta_{i,t} \\
& \quad + 5\ell_{i,t}^T \bar{\mathbf{I}}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t} - 2\eta_{i,t}^T \bar{R}_{11,t}^T O_{i,t+1} \bar{\mathbf{I}} \ell_{i,t}\}. \quad (43)
\end{aligned}$$

Denoting $\ell_{i,t}^* \triangleq \Upsilon_{3i,t} \eta_{i,t}$, we have

$$\begin{aligned}
& \ell_{i,t}^T \Xi_{6i,t} \ell_{i,t} - 2\eta_{i,t}^T \Upsilon_{2i,t} \ell_{i,t} \\
& = (\ell_{i,t} - \ell_{i,t}^*)^T \Xi_{6i,t} (\ell_{i,t} - \ell_{i,t}^*) - (\ell_{i,t}^*)^T \Xi_{6i,t} \ell_{i,t}^*. \quad (44)
\end{aligned}$$

Then, combining (28), (29), (37), (43) and (44) leads to

$$\begin{aligned}
& E\{\Delta \bar{V}_{i,t}\} \\
& \leq E\{\eta_{i,t}^T (\Pi_{1i,t} + \mathcal{M}_t^T \mathcal{M}_t + \Xi_{5i,t} - \Upsilon_{2i,t} \Xi_{6i,t}^{-1} \Upsilon_{2i,t}^T \\
& \quad - O_{i,t}) \eta_{i,t} - \sigma_{i,t}^T (I - U_{i,t} - \Pi_{2i,t}) \sigma_{i,t} \\
& \quad - \delta_{i,t}^T (I - \Pi_{2i,t}) \delta_{i,t} + (\ell_{i,t} - \ell_{i,t}^*)^T \Xi_{6i,t} (\ell_{i,t} - \ell_{i,t}^*) \\
& \quad - (\|\tilde{z}_{i,t}\|^2 + \|\ell_{i,t}\|^2 + \bar{\Delta}_{i,t})\} \\
& \leq E\{(\ell_{i,t} - \ell_{i,t}^*)^T \Xi_{6i,t} (\ell_{i,t} - \ell_{i,t}^*) \\
& \quad - (\|\tilde{z}_{i,t}\|^2 + \|\ell_{i,t}\|^2 + \bar{\Delta}_{i,t})\}. \quad (45)
\end{aligned}$$

Recalling $O_{i,T+1} = 0$ and taking summation on both sides of (45) from $t = 0$ to $t = T$, we obtain

$$\begin{aligned}
\mathbb{J}_{3i} & = \sum_{t=0}^T E\{\|\tilde{z}_{i,t}\|^2 + \|\ell_{i,t}\|^2\} \\
& \leq \sum_{t=0}^T E\{(\ell_{i,t} - \ell_{i,t}^*)^T \Xi_{6i,t} (\ell_{i,t} - \ell_{i,t}^*)\} - \bar{\Delta}_{i,t} \\
& \quad + \eta_{i,0}^T O_{i,0} \eta_{i,0} \\
& = \sum_{t=0}^T E\{(\bar{p}_i K_{i,t} \bar{C}_{i,t} \eta_{i,t} - \Upsilon_{3i,t} \eta_{i,t})^T \Xi_{6i,t} \\
& \quad \times (\bar{p}_i K_{i,t} \bar{C}_{i,t} \eta_{i,t} - \Upsilon_{3i,t} \eta_{i,t})\} + \eta_{i,0}^T O_{i,0} \eta_{i,0} - \bar{\Delta}_{i,t} \\
& \leq \sum_{t=0}^T E\{\|\bar{p}_i K_{i,t} \bar{C}_{i,t} - \Upsilon_{3i,t}\|_F^2 \|\Xi_{6i,t}\|_F \|\eta_{i,t}\|^2\} \\
& \quad + \eta_{i,0}^T O_{i,0} \eta_{i,0} - \bar{\Delta}_{i,t} \quad (46)
\end{aligned}$$

and, consequently, the filter gain $K_{i,t}$ can be acquired by solving (39) in order to minimize the cost function $\mathbb{J}_{3i}$. The proof is complete. $\blacksquare$

Keeping in mind $\Upsilon_{3i,t} = \Xi_{6i,t}^{-1} \Upsilon_{2i,t}^T$, we further set

$$\bar{\Upsilon}_{3i,t} \triangleq \Xi_{6i,t}^{-1} O_{2i,t+1} A_t. \quad (47)$$

By invoking Lemma 2 and Theorem 2, the local filter design problem is addressed in the following result.

Theorem 3: Let the scalar $\gamma_i > 0$ and the weighting matrix $\mathbb{W}_i > 0$ ($i \in \mathbb{D}_1$) be given. Suppose that there exist matrices $P_{i,t+1} = \text{diag}\{P_{1i,t+1}, P_{2i,t+1}\}$, $O_{i,t+1} = \text{diag}\{O_{1i,t+1}, O_{2i,t+1}\}$, along with a filter gain $K_{i,t}$ ($t \in [0, T]$), such that the coupled backward RDEs (22) and (37) are feasible under conditions (23) and (38). Then, the performance constraint (20) is guaranteed to hold, and the corresponding filter gain $K_{i,t}$ is explicitly given by

$$K_{i,t} = \bar{p}_i^{-1} \bar{\Upsilon}_{3i,t} C_{i,t}^\dagger \quad (48)$$

where $\bar{\Upsilon}_{3i,t}$ is defined in (47) and $C_{i,t}^\dagger$ denotes the Moore-Penrose pseudoinverse of $C_{i,t}$.

B. Distributed fusion filter design

Having obtained the local filter for each node $i \in \mathbb{D}_1$, we endeavor to design suitable fusion parameters Θ_i for the distributed fusion filter (18) such that the performance constraint (21) is achieved.

Theorem 4: Let the scalar $\gamma_i > 0$ and the weighting matrix $\mathbb{W}_i > 0$ ($i \in \mathbb{D}_1$) be given. Suppose that there exist the matrix $\Theta_i = \text{diag}\{\theta_{i1}, \theta_{i2}, \dots, \theta_{in_x}\}$ and a positive scalar π satisfying the following inequality

$$\begin{bmatrix} -\frac{\pi}{N} & \Pi_1 & \Pi_2 & \cdots & \Pi_N \\ \Pi_1^T & -\mathbb{I} & 0 & \cdots & 0 \\ \Pi_2^T & 0 & -\mathbb{I} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Pi_N^T & 0 & 0 & \cdots & -\mathbb{I} \end{bmatrix} < 0 \quad (49)$$

with $\pi \triangleq \tilde{\gamma}^2$ and $\sum_{i=1}^N \theta_{il} = 1$ ($l \in \mathbb{D}_3$), where

$$\Pi_i \triangleq [\theta_{i1}\gamma_i \quad \theta_{i2}\gamma_i \quad \cdots \quad \theta_{in_x}\gamma_i], \quad \mathbb{I} \triangleq \text{diag}_{n_x}\{1\}.$$

Then, the prescribed performance constraint is satisfied for the fusion filtering error system.

Proof: From (19), one calculates that

$$\begin{aligned} \tilde{z}_t^T \tilde{z}_t &= \left[\sum_{i=1}^N \Theta_i \tilde{z}_{i,t} \right]^T \left[\sum_{i=1}^N \Theta_i \tilde{z}_{i,t} \right] \\ &\leq N \sum_{i=1}^N \tilde{z}_{i,t}^T \Theta_i^T \Theta_i \tilde{z}_{i,t} = N \sum_{i=1}^N \text{tr}\{\Theta_i^T \Theta_i \tilde{z}_{i,t} \tilde{z}_{i,t}^T\} \\ &\leq N \sum_{i=1}^N \text{tr}\{\Theta_i^T \Theta_i\} \tilde{z}_{i,t}^T \tilde{z}_{i,t}. \end{aligned} \quad (50)$$

By using Schur Complement Lemma, the inequality (49) implies that $\sum_{i=1}^N N \text{tr}\{\Theta_i^T \Theta_i\} \gamma_i^2 < \tilde{\gamma}^2$, then we have from (20) and (50) that

$$\begin{aligned} &\sum_{t=0}^T E\{\tilde{z}_t^T \tilde{z}_t\} \\ &\leq \sum_{t=0}^T \sum_{i=1}^N N \text{tr}\{\Theta_i^T \Theta_i\} E\{\tilde{z}_{i,t}^T \tilde{z}_{i,t}\} \\ &< \sum_{i=1}^N N \text{tr}\{\Theta_i^T \Theta_i\} \gamma_i^2 \left(\sum_{t=0}^T E\{\|\omega_{i,t}\|^2 + \bar{\Delta}_{i,t}\} \right. \\ &\quad \left. + \eta_{i,0}^T \mathbb{W}_i \eta_{i,0} \right) \\ &< \sum_{t=0}^T \tilde{\gamma}^2 E\{\|\bar{\omega}_t\|^2 + \bar{\Delta}_t\} + \tilde{\gamma}^2 \eta_0^T \bar{\mathbb{W}} \eta_0. \end{aligned} \quad (51)$$

Thus, the performance index (21) is satisfied for the system dynamics (19), which completes the proof. ■

By solving a certain convex optimization problem, suitable fusion parameters are designed in the following theorem to achieve the performance constraint $\mathbb{J}_2 < 0$ with the minimum disturbance rejection level $\tilde{\gamma}$ (namely, $\tilde{\gamma}_{\min}$).

Theorem 5: Based on Theorem 4, there exist optimal parameter matrices Θ_i ($i \in \mathbb{D}_1$) satisfying the performance

constraint (21) with the minimum value $\tilde{\gamma}_{\min}$ if the following optimization problem

$$\tilde{\gamma}_{\min} = \min_{\Theta_1, \Theta_2, \dots, \Theta_N} \tilde{\gamma} \quad \text{subject to (49)} \quad (52)$$

is feasible.

Remark 5: Thus far, a systematic investigation has been conducted on the non-fragile DFF problem for time-varying systems subject to sensor saturations and packet losses over relay-assisted communication channels. A major emphasis has been placed on the development of a DaF relay-aided mechanism, which enhances both communication reliability and data security in long-distance signal transmissions. A central contribution of this work is the establishment of a sufficient condition in Theorem 1, which ensures that the specified H_∞ performance constraint is satisfied for the AFES (17). Building on this result, Theorems 2–3 present a design methodology for the local filter at each node $i \in \mathbb{D}_1$ based on the solutions to a set of coupled RDEs. In addition, Theorem 4 provides a sufficient condition for the existence of feasible fusion parameters. These parameters are subsequently determined by solving the convex optimization problem described in Theorem 5, ensuring that the fusion-level performance index (21) is satisfied with the minimum possible disturbance rejection level. Collectively, the derived results contribute to the advancement of robust and efficient non-fragile DFF schemes, particularly in DaF relay network environments affected by sensor saturation and packet loss phenomena.

Remark 6: This paper distinguishes itself from the existing literature by addressing a challenging and practically relevant problem that has not been thoroughly explored. The distinctive features are summarized as follows:

- 1) the addressed problem represents one of the first systematic studies on the DFF problem for time-varying systems subject to SGVs, sensor saturations, and packet losses over relay channels;
- 2) a novel DaF relay-aided mechanism, relying on an integrated encoding-decoding strategy and a dynamic quantization scheme, is employed to govern signal transmissions over sensor-to-filter channels, thereby enhancing communication quality, prolonging transmission distance, and improving system resilience;
- 3) the desired local filter is designed for each node by recursively solving two coupled backward RDEs, which are numerically tractable and efficient for real-time implementation; and
- 4) the optimal fusion parameter matrices are obtained by formulating and solving a dedicated convex optimization problem, thereby guaranteeing the global filtering performance under finite-horizon constraints.

IV. A NUMERICAL EXAMPLE

In this section, a numerical example is put forward to verify the effectiveness of the proposed DFF scheme for the underlying system with DaF relays.

Consider the system (1) with the following parameters

$$A_t = \begin{bmatrix} 0.1 & 0.8 \\ -0.1 & -0.65 - 0.3\sin(1.6 + 0.05t) \end{bmatrix},$$

$$\begin{aligned}
B_t &= [-0.2 \ 0.1]^T, \quad C_{1,t} = C_{3,t} = [0.45 \ 0.45], \\
C_{2,t} &= C_{4,t} = [0.5 \ 0.5], \quad C_{5,t} = [-0.3\cos(3k) \ 0.3], \\
C_{6,t} &= [0.4 \ 0.4\cos(2k)], \quad C_{7,t} = [0.6 \ 0.6], \\
C_{8,t} &= [0.35 \ -0.35\cos(3k)], \quad M_t = [0.02 \ 0.02], \\
D_{1,t} &= D_{3,t} = D_{5,t} = D_{7,t} = 0.3, \\
D_{2,t} &= D_{4,t} = D_{6,t} = D_{8,t} = 0.2, \quad t \in [0, 20].
\end{aligned}$$

The initial system state and its estimates are set as

$$\begin{aligned}
x_0 &= [0.3 \ -0.3]^T, \quad \hat{x}_{1,0} = [0.34 \ -0.28]^T, \\
\hat{x}_{2,0} &= [0.32 \ -0.27]^T, \quad \hat{x}_{3,0} = [0.28 \ -0.29]^T, \\
\hat{x}_{4,0} &= [0.27 \ -0.28]^T, \quad \hat{x}_{5,0} = [0.31 \ -0.24]^T, \\
\hat{x}_{6,0} &= [0.35 \ -0.32]^T, \quad \hat{x}_{7,0} = [0.26 \ -0.33]^T, \\
\hat{x}_{8,0} &= [0.25 \ -0.23]^T.
\end{aligned}$$

The noise disturbances are considered to be

$$v_t = \frac{0.1 + 0.5\sin(2t)}{t+1}, \quad \varphi_{i,t} = \frac{0.1 + 0.03\cos(t)}{t+1},$$

and the other parameters are selected as

$$\begin{aligned}
\bar{\alpha}_{1i} &= \bar{\alpha}_{2i} = 0.85, \quad \gamma_1 = 1.8, \quad \gamma_2 = 2, \\
\gamma_3 &= 2.5, \quad \gamma_4 = 1.5, \quad \gamma_5 = 1.6, \\
\gamma_6 &= 1.2, \quad \gamma_7 = 2.2, \quad \gamma_8 = 1.9, \\
b_{ij,0} &= 0.005, \quad \varsigma_{ij,t} = 0.8, \quad \mathbf{Y}_{ij,t} = 0.1I, \\
d &= 20, \quad \psi_{ij} = 1, \quad \lambda_{ij} = 0.6, \quad p_i = 1, \\
\mu_{ij} &= 1, \quad \bar{K}_{i,t} = [0.3 \ 0.5]^T, \quad G_{i,t} = 0.1, \\
T &= 20, \quad \mathbb{W}_i = 10I, \quad P_{i,T+1} = O_{i,T+1} = 0.
\end{aligned}$$

Based on the presented DFF algorithm, the local filter gains $K_{i,t}$ ($i = 1, 2, \dots, 8$) are easily computed at each time step $t \in [0, 20]$. The fusion parameters Θ_i and the scalar $\tilde{\gamma}_{\min}$ can be obtained through the Matlab software that

$$\begin{aligned}
\Theta_1 &= \text{diag}\{0.1086, 0.1172\}, \quad \Theta_2 = \text{diag}\{0.0918, 0.0909\}, \\
\Theta_3 &= \text{diag}\{0.0591, 0.0585\}, \quad \Theta_4 = \text{diag}\{0.1630, 0.1618\}, \\
\Theta_5 &= \text{diag}\{0.1435, 0.1420\}, \quad \Theta_6 = \text{diag}\{0.2563, 0.2538\}, \\
\Theta_7 &= \text{diag}\{0.0761, 0.0753\}, \quad \Theta_8 = \text{diag}\{0.1016, 0.1005\}, \\
\tilde{\gamma}_{\min} &= 1.712.
\end{aligned}$$

Define $E_t \triangleq \|\bar{e}_t\|$ as the Euclidean norm of the fusion filtering error $\bar{e}_t$. Based on the designed DFF scheme, the simulation results are illustrated in Figs. 2–13. Figs. 2–3 show the occurrences of packet losses in the STR and RTF channels, respectively. Figs. 4–11 depict the actual system states and their corresponding local estimates for nodes $i = 1, 2, \dots, 8$, where the local estimates can track the true states despite sensor saturations, packet losses, SGVs, and DaF relay channels. Figs. 12–13 present the trajectories of the system state x_t , its fusion estimate $\hat{x}_t$, and E_t , demonstrating the effectiveness of the proposed DFF method.

To further evaluate the filtering performance and explore the effects of packet loss probabilities, DaF relays, and fusion methods, a comparative study is conducted under three scenarios.

Case 1: The mathematical expectations of $\bar{\alpha}_{1i}$ and $\bar{\alpha}_{2i}$ are reset as $\bar{\alpha}_{1i} = \bar{\alpha}_{2i} = 0.3$ while keeping the other parameters

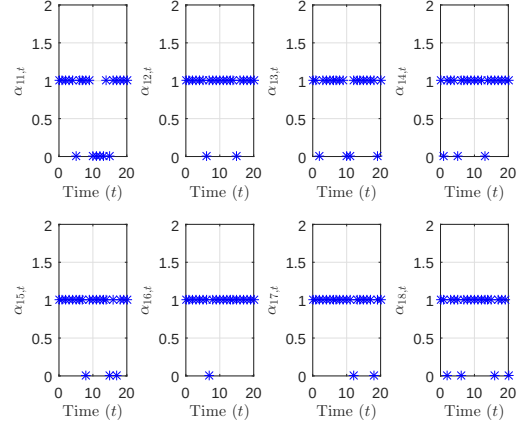


Fig. 2: The values of $\alpha_{1i,t}$ ($i = 1, 2, \dots, 8$).

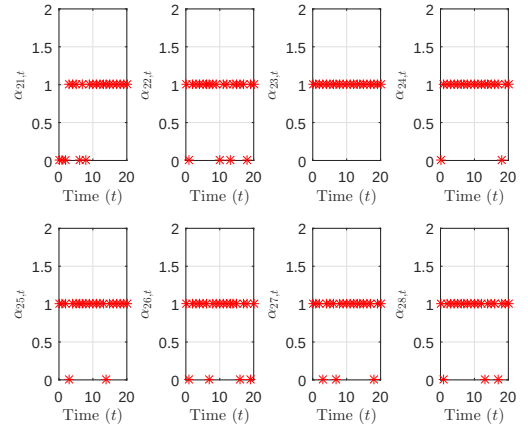


Fig. 3: The values of $\alpha_{2i,t}$ ($i = 1, 2, \dots, 8$).

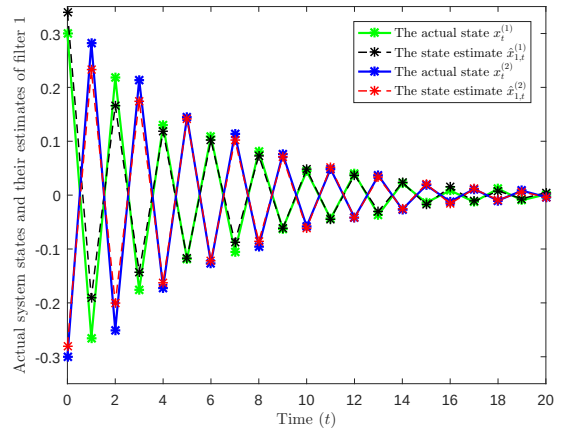
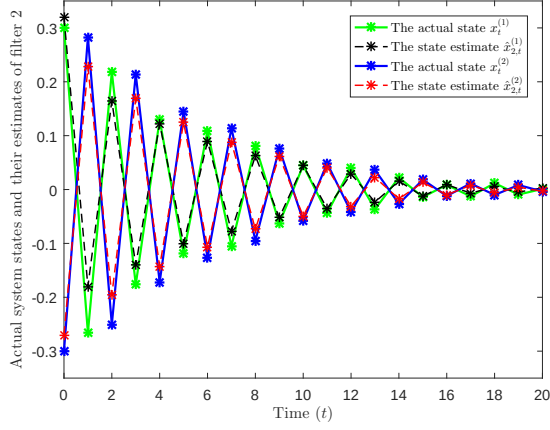
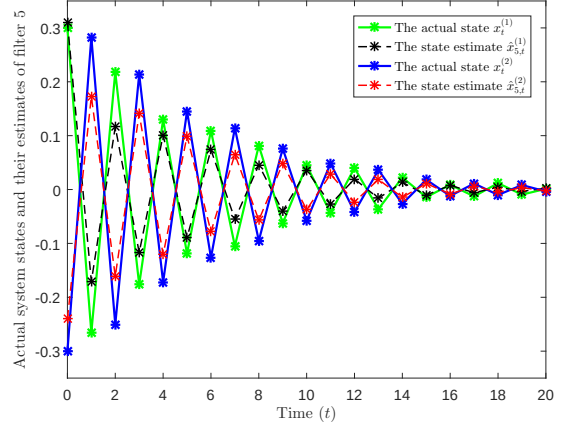
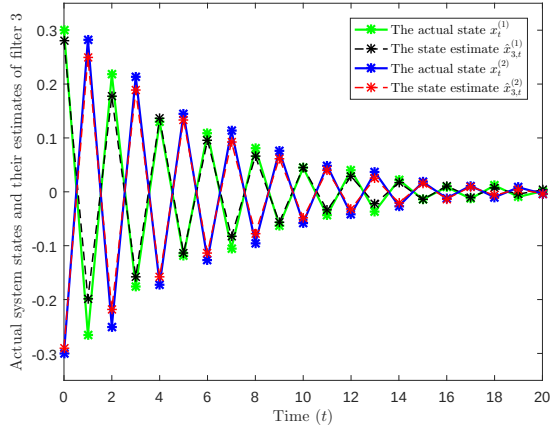
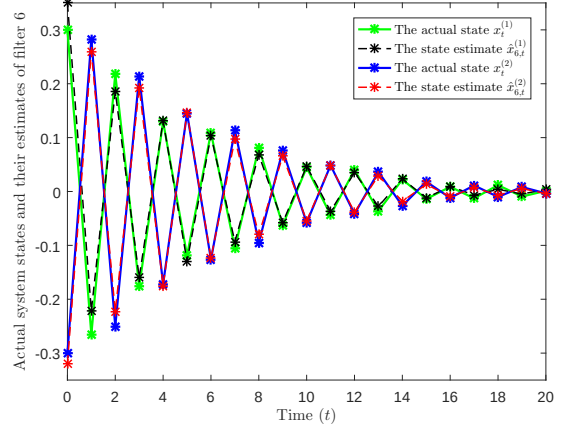
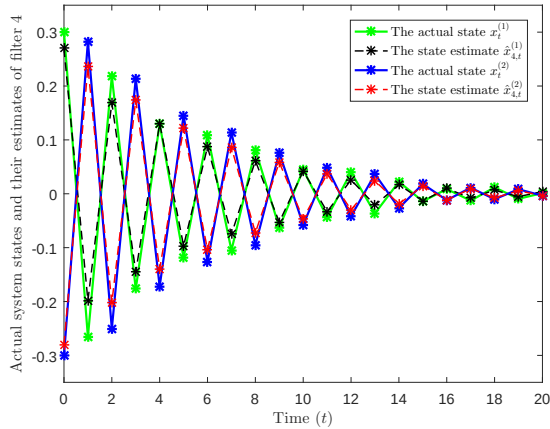
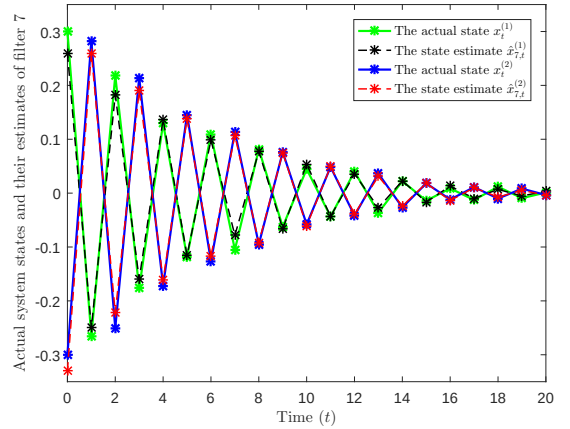


Fig. 4: x_t and its estimate $\hat{x}_{1,t}$.

unchanged. The obtained trajectory of E_t is presented in Fig. 14. A comparison between Fig. 13 and Fig. 14 shows that the fusion filtering error decreases as $\bar{\alpha}_{1i}$ and $\bar{\alpha}_{2i}$ increase, indicating that the estimation accuracy improves in scenarios with fewer packet losses.

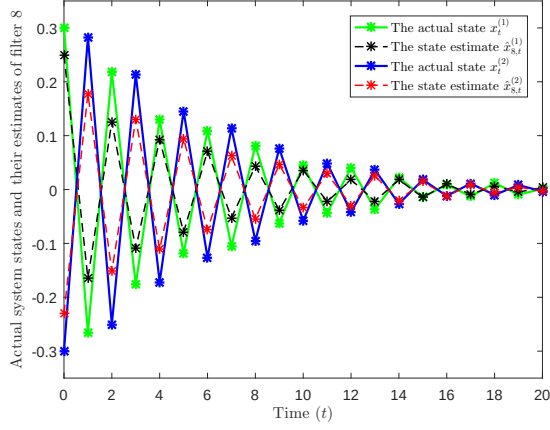
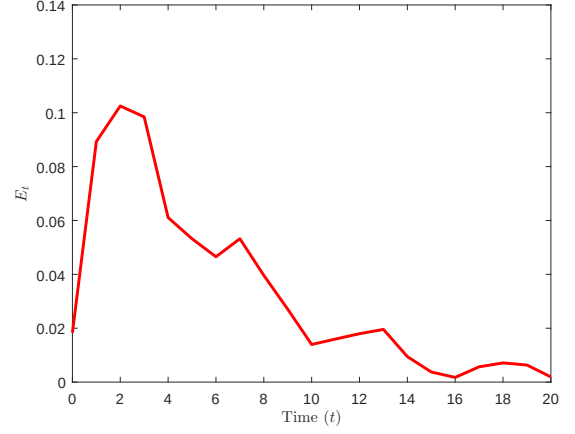
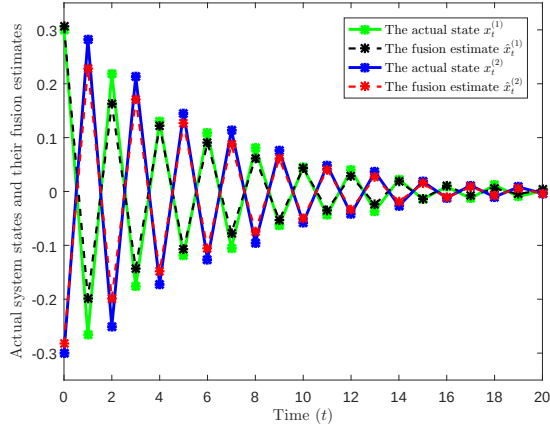
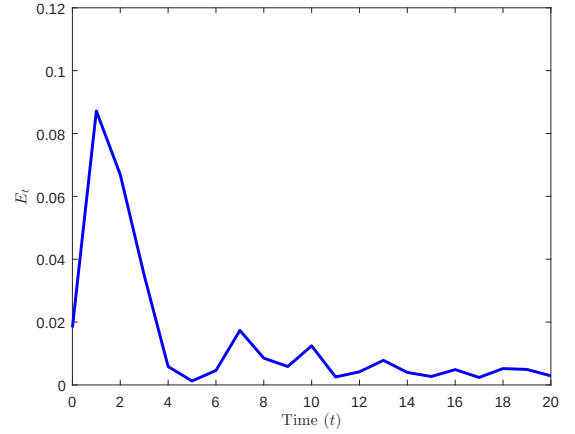
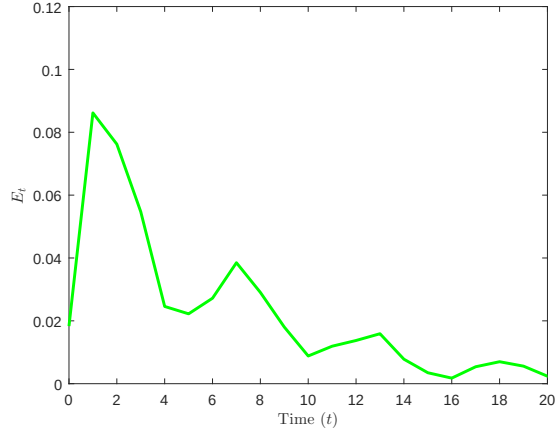
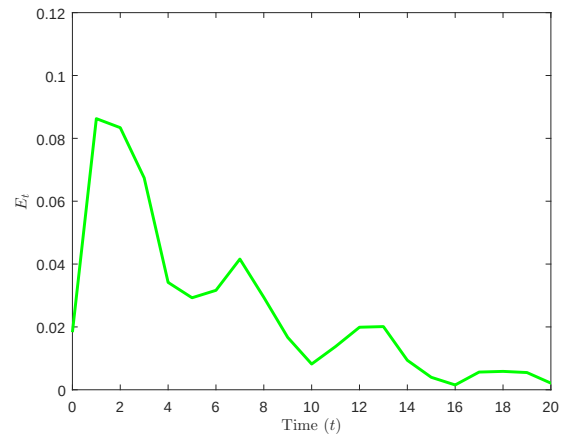
Case 2: Consider a scenario where no relay is deployed in

Fig. 5: x_t and its estimate $\hat{x}_{2,t}$.Fig. 8: x_t and its estimate $\hat{x}_{5,t}$.Fig. 6: x_t and its estimate $\hat{x}_{3,t}$.Fig. 9: x_t and its estimate $\hat{x}_{6,t}$.Fig. 7: x_t and its estimate $\hat{x}_{4,t}$.Fig. 10: x_t and its estimate $\hat{x}_{7,t}$.

each sensor-to-filter channel, and the packet loss probabilities and channel noises remain unchanged. The resulting trajectory of E_t is shown in Fig. 15, which exhibits lower filtering error compared with Fig. 13. This indicates that although the DaF relay enhances communication reliability and data security, it may slightly degrade filtering performance due to decoding

errors introduced during the encoding-decoding process.

Case 3: To further demonstrate the superiority of the adopted DMWF method, a comparison with the scalar-weighted fusion method is implemented, and the corresponding trajectory of E_t is shown in Fig. 16. As observed in Fig. 13 and Fig. 16, the distributed fusion filter based on the DMWF

Fig. 11: x_t and its estimate $\hat{x}_{8,t}$.Fig. 14: The trajectory of E_t with $\bar{\alpha}_{1i} = \bar{\alpha}_{2i} = 0.3$.Fig. 12: x_t and its estimate of fusion filter.Fig. 15: The trajectory of E_t without relays.Fig. 13: The trajectory of E_t with $\bar{\alpha}_{1i} = \bar{\alpha}_{2i} = 0.85$.Fig. 16: The trajectory of E_t with scalar-weighted fusion criterion.

criterion achieves higher estimation accuracy than that based on the scalar-weighted fusion criterion.

In summary, the results indicate that: 1) the designed distributed fusion filter can provide accurate state estimates for the considered system; 2) the presented DaF relay-based filtering approach shows slightly inferior performance compared

to the approach without relays; 3) lower values of $\{\bar{\alpha}_{1i}, \bar{\alpha}_{2i}\}$ (namely, more packet losses) lead to reduced estimation accuracy; and 4) the proposed DMWF-based DFF algorithm achieves superior performance over the DFF algorithm with the scalar-weighted fusion criterion.

V. CONCLUSION

This paper has investigated the DFF problem for time-varying systems subject to sensor saturations, packet losses, and SGVs over relay-assisted communication networks. A DaF relay-aided mechanism has been incorporated into the sensor-to-filter communication channels to enhance the reliability and efficiency of data transmission. Sufficient conditions have been rigorously derived to ensure the prescribed H_∞ performance of the AFES, and the corresponding local filter gains have been computed by solving a set of coupled RDEs. Based on the designed local filters, appropriate fusion parameters have been determined via the solution of a convex optimization problem. The effectiveness and applicability of the proposed non-fragile DFF scheme have been validated through a representative numerical example. In the future, several promising research directions can be considered, including 1) extending the proposed framework to more general communication topologies such as multi-hop or mesh relay networks [6], [45], [50]; 2) developing event-triggered or adaptive sampling strategies to reduce the communication burden [10], [23], [42], [49]; and 3) enhancing resilience against malicious attacks and data injection under cyber-physical security constraints [4], [11], [31], [39]. Moreover, it would be interesting to incorporate model uncertainties and explore learning-based filter adaptation in dynamic environments in order to further enhance the robustness and practicality of the DFF designs.

REFERENCES

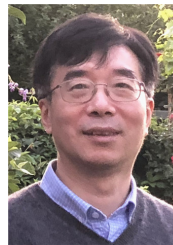
- [1] A. Basit, M. Tufail, and M. Rehan, Event-triggered distributed state estimation under unknown parameters and sensor saturations over wireless sensor networks, *IEEE Transactions on Circuits and Systems-II: Express Briefs*, vol. 69, no. 3, pp. 1772–1776, 2022.
- [2] S. Cai and J. Liang, Recursive filtering for nonlinear systems with relay communication, energy harvesting and correlated noises, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 3, art. no. 100021, 2025.
- [3] W. Chen, J. Hu, Z. Wu, and S. Ma, A survey on fault detection for networked systems under communication constraints, *Systems Science & Control Engineering*, vol. 13, no. 1, art. no. 2460434, 2025.
- [4] W. Chen, Z. Wang, Q. Ge, H. Dong, and G.-P. Liu, Quantized distributed economic dispatch for microgrids: Paillier encryption-decryption scheme, *IEEE Transactions on Industrial Informatics*, vol. 20, no. 4, pp. 6552–6562, 2024.
- [5] Y. Chen, W. Wang, and J. Xu, Robust H_∞ estimation for 2D FMLSS systems under asynchronous multi-channel delays: A partition reconstruction approach, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 3, art. no. 100017, 2025.
- [6] D. Dai, J. Li, J. Zhang, B. Jiang and H. Dong, Energy-harvesting recursive filtering for delayed systems with amplify-and-forward relays: Cooperative communication manner, *Journal of the Franklin Institute*, vol. 361, no. 18, art. no. 107265, 2024.
- [7] X. Feng, C. Wu, and Q. Ge, Cauchy kernel minimum error entropy centralized fusion filter, *Signal Processing*, vol. 220, art. no. 109465, 2024.
- [8] H. Fu, X. Meng, and Y. Chen, Finite-time distributed filtering of time-varying switched stochastic systems with DoS attacks based on encoding-decoding scheme, *IEEE Systems Journal*, vol. 18, no. 1, pp. 268–277, 2024.
- [9] H. Ge, L. Zou, H. Chen, and J. Guo, Finite-time bounded state estimation for time-varying systems over FlexRay networks with probabilistic bit flips, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 4, art. no. 100023, 2025.
- [10] Y. Guo, Z. Wang, J.-Y. Li and Y. Xu, An impulsive approach to state estimation for multirate singularly perturbed complex networks under bit rate constraints, *IEEE Transactions on Cybernetics*, vol. 55, no. 3, pp. 1197–1209, 2025.
- [11] A. Hasan, I. Kuncara, A. Widyotriatmo, O. Osen, and R. T. Bye, Secure state estimation and control for autonomous ships under cyberattacks, *Systems Science & Control Engineering*, vol. 13, no. 1, art. no. 2518964, 2025.
- [12] J. Hu, Z. Hu, R. Caballero-Águila, C. Chen, S. Fan, and X. Yi, Distributed resilient fusion filtering for nonlinear systems with multiple missing measurements via dynamic event-triggered mechanism, *Information Sciences*, vol. 637, art. no. 118950, 2023.
- [13] J. Hu, C. Wang, R. Caballero-Águila, and H. Liu, Distributed optimal fusion filtering for singular systems with random transmission delays and packet dropout compensations, *Communications in Nonlinear Science and Numerical Simulation*, vol. 119, art. no. 107093, 2023.
- [14] J. Jacob and S. Dharani, Projective lag synchronization for quaternion neural networks with semi-Markov switching and mixed delays via non-fragile sliding mode control, *Systems Science & Control Engineering*, vol. 13, no. 1, art. no. 2600780, 2025.
- [15] J. D. Jiménez-López, R. M. Fernández-Alcalá, F. Gutiérrez-Trujillo, E. Grassucci, and D. Communiello, Optimal fusion β -quaternion filtering from delayed observations and packet losses with compensation strategy under properness, *Systems Science & Control Engineering*, vol. 13, no. 1, art. no. 2481941, 2025.
- [16] S. Kianoush, S. Savazzi, V. Rampa, L. Costa, and D. Tolochenko, A random forest approach to body motion detection: Multi-sensory fusion and edge processing, *IEEE Sensors Journal*, vol. 23, no. 4, pp. 3801–3814, 2023.
- [17] L. Lan, G. Wei, D. Ding, and J. Zhang, Zonotopic distributed fusion over binary sensor networks with bit rate allocation: A coding-decoding approach, *IEEE Transactions on Cybernetics*, vol. 54, no. 11, pp. 6855–6866, 2024.
- [18] C. Li, Z. Wang, W. Song, S. Zhao, J. Wang, and J. Shan, Resilient unscented Kalman filtering fusion with dynamic event-triggered scheme: Applications to multiple unmanned aerial vehicles, *IEEE Transactions on Control Systems Technology*, vol. 31, no. 1, pp. 370–381, 2023.
- [19] N. Li, W. Mei, P. Wu, B. Ning, and L. Zhu, Movable antenna enhanced DF and AF relaying systems: Performance analysis and optimization, *IEEE Transactions on Communications*, vol. 73, no. 12, pp. 13239–13255, 2025.
- [20] J. Liu, Z. Yang, S. Li, S. Zlatanova, B. Yang, G. Li, B. Yu, and Q. Zhang, MMVP: Multiscene indoor localization for smartphones by using collaborative visual localization and multisensor fusion, *IEEE Internet of Things Journal*, vol. 13, no. 6, pp. 12076–12092, 2026.
- [21] Q. Liu, Z. Wang, H. Dong and C. Jiang, Recursive Bayesian estimation for discrete-time systems with state-dependent packet dropouts: a cross-coupled method, *IEEE Transactions on Automatic Control*, vol. 69, no. 6, pp. 3705–3716, Jun. 2024.
- [22] S. Liu, L. Wang, Y. Zhang, Y.-A. Wang, and H. Dong, Recursive filtering of networked systems with communication protocol scheduling: A survey, *International Journal of Systems Science*, vol. 56, no. 11, pp. 2499–2516, 2025.
- [23] X. Liu, Z. Ling, and Y. Zhang, Torus-event-based sliding mode control for networked interval type-2 fuzzy systems under deception attacks, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 1, art. no. 100003, 2025.
- [24] Y. Liu, Z. Wang, C. Liu, M. Coombes, and W.-H. Chen, Auxiliary particle filtering over sensor networks under protocols of amplify-and-forward and decode-and-forward relays, *IEEE Transactions on Signal and Information Processing over Networks*, vol. 8, pp. 883–893, 2022.
- [25] X. Meng, Z. Wang, F. Wang and Y. Chen, State estimation for nonlinear complex dynamical networks with random coupling strengths: a decode-and-forward relay-based strategy, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 54, no. 8, pp. 4749–4760, Aug. 2024.
- [26] Y. Nie, Z. Wang and Q. Liu, Expectation-maximization-based remote state estimation under state-dependent packet dropouts: from maximum a posteriori perspective, *IEEE Transactions on Automatic Control*, vol. 70, no. 3, pp. 1608–1622, Mar. 2025.
- [27] R. Penrose and J. A. Todd, On best approximate solutions of linear matrix equations, *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 52, no. 1, pp. 17–19, 1956.
- [28] Q. Qi and Y. Feng, Remote robust optimal state estimation for discrete-time systems over Markovian lossy channels, *IEEE Transactions on Automatic Control*, vol. 71, no. 4, pp. 2242–2257, 2026.
- [29] A. Rajagopal and S. Chitraganti, Distributed state estimation and optimal control for networked systems with correlated data dropouts over slow-fading channels: An experimental evaluation, *International Journal of Systems Science*, vol. 57, no. 7, pp. 2058–2077, 2026.
- [30] X. Shao, J. Zhang, M. Lyu, and Y. Lu, Event-based nonfragile state estimation for memristive neural networks with multiple time-delays

- and sensor saturations, *International Journal of Systems Science*, vol. 56, no. 3, pp. 618–637, 2025.
- [31] H. Shen, Y. Wang, J. Wu, J. H. Park, and J. Wang, Secure control for Markov jump cyber-physical systems subject to malicious attacks: A resilient hybrid learning scheme, *IEEE Transactions on Cybernetics*, vol. 54, no. 11, pp. 7068–7079, 2024.
- [32] M. Shi, L. Ma, and C. Gao, Distributed filtering over sensor networks with Byzantine attacks: A token bucket protocol, *IEEE Transactions on Cybernetics*, vol. 56, no. 6, pp. 3579–3591, 2026.
- [33] W. Song, Z. Wang, Z. Li, and H. Dong, Moving-horizon estimation for multisensor systems under probabilistic caching mechanism: A co-design scheme, *IEEE Transactions on Automatic Control*, in press, DOI: 10.1109/TAC.2026.3673082.
- [34] W. Song, Z. Wang, Z. Li, and H. Dong, Multisensor particle filtering for nonlinear complex networks with heterogeneous measurements under non-Gaussian noises, *IEEE Transactions on Cybernetics*, vol. 56, no. 3, pp. 1241–1254, 2026.
- [35] Z. Sun and C. Han, Linear state estimation for multi-rate NCSs with multi-channel observation delays and unknown Markov packet losses, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 1, art. no. 100005, 2025.
- [36] T. Tian and S. Sun, Fusion estimation against mixed network attacks for systems with random parameter matrices, correlated noises, and quantized measurements, *Digital Signal Processing*, vol. 150, art. no. 104523, 2024.
- [37] F. Wang, J. Liang, J. Lam, J. Yang, and C. Zhao, Robust filtering for 2-D systems with uncertain-variance noises and weighted try-once-discard protocols, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 53, no. 5, pp. 2914–2924, 2023.
- [38] F. Wang, Z. Wang, J. Liang, Q. Ge and S. X. Ding, Recursive state estimation for two-dimensional systems over decode-and-forward relay channels: a local minimum-variance approach, *Information Sciences*, vol. 678, art. no. 120928, Sept. 2024.
- [39] J. Wang, D. Wang, H. Yan, and H. Shen, Composite antidisturbance H_∞ control for hidden Markov jump systems with multi-sensor against replay attacks, *IEEE Transactions on Automatic Control*, vol. 69, no. 3, pp. 1760–1766, 2024.
- [40] L. Wang and Z. Wang, Event-based probability-guaranteed tracking control with amplify-and-forward relay schemes, *Automatica*, vol. 185, art. no. 112761, Mar. 2026.
- [41] X. Wang, K. Li, and A. Chehri, Multi-sensor fusion technology for 3D object detection in autonomous driving: A review, *IEEE Transactions on Intelligent Transportation Systems*, vol. 25, no. 2, pp. 1148–1165, 2024.
- [42] X. Wang, M. Zhang, S. Yue, K. Li and D. Ding, Platooning control of connected automated vehicles under communication constraints: advances and challenges, *Systems Science & Control Engineering*, vol. 14, no. 1, art. no. 2672175, 2026.
- [43] Z. Wang, B. Shen, and X. Liu, H_∞ filtering with randomly occurring sensor saturations and missing measurements, *Automatica*, vol. 48, pp. 556–562, 2012.
- [44] R.-Y. Wei, W.-Y. Chin, and T.-Y. Feng, Noncoherent two-way relaying with decode and forward: Two versus three phases, *IEEE Transactions on Vehicular Technology*, vol. 71, no. 3, pp. 3350–3354, 2022.
- [45] M. Xie, D. Ding, X. Ge, Q.-L. Han, H. Dong, and Y. Song, Distributed platooning control of automated vehicles subject to replay attacks based on proportional integral observers, *IEEE/CAA Journal of Automatica Sinica*, vol. 11, no. 9, pp. 1954–1966, 2024.
- [46] C. Yang and J. Liang, Probability-guaranteed filtering for 2-D nonlinear systems over full-duplex relay network with energy harvesting constraint, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 55, no. 9, pp. 5812–5826, 2025.
- [47] C. Yang, J. Liang, and Z. Zhao, Zonotopic distributed fusion filtering for 2-D nonlinear systems over sensor networks: A channel-based bit rate constraint, *IEEE Transactions on Signal Processing*, vol. 73, pp. 4654–4668, 2025.
- [48] D. Yu, Y. Xia, D.-H. Zhai, and Y. Zhan, On distributed fusion estimation with stochastic scheduling over sensor networks, *Automatica*, vol. 142, art. no. 110406, 2022.
- [49] B. Zhang, D. Liu and B. Wang, Fully distributed event-triggered containment control for complex networks, *International Journal of Systems Science*, vol. 56, no. 15, pp. 3689–3701, 2025.
- [50] W. Zhang, Y. Zhao, H. Wang and Z. Yang, Robust model predictive control for polyhedral uncertain systems under relay and redundancy protocol, *International Journal of Systems Science*, vol. 56, no. 8, pp. 1617–1632, 2025.
- [51] Z. Zhao, Z. Wang, J. Liang, and W. Xu, Zonotopic set-membership fusion estimation for complex networks: A buffer-aided strategy, *IEEE Transactions on Cybernetics*, vol. 56, no. 3, pp. 1465–1478, 2026.
- [52] Z. Zhao, Z. Wang, L. Zou, Y. Chen, and W. Sheng, Zonotopic distributed fusion for nonlinear networked systems with bit rate constraint, *Information Fusion*, vol. 90, pp. 174–184, 2023.
- [53] Z. Zhao, Z. Wang, L. Zou, Y. Chen and W. Sheng, Zonotopic non-fragile set-membership fusion estimation for nonlinear systems under sensor resolution effects: boundedness and monotonicity, *Information Fusion*, vol. 105, art. no. 102232, May. 2024.
- [54] D. Zheng, Y. Zhao, W. Tian, F. Zhang, G. Gui, and W. Gui, Self-supervised absolute-scale multi-sensor fusion odometry via multi-layer feature fusion and pose refinement for autonomous driving, *IEEE Transactions on Automation Science and Engineering*, vol. 23, pp. 995–1008, 2026.
- [55] H. Zhu, M. J. V. Amuri, J. Shen, and X. Li, Mean-shift-based robust distributed set-membership fusion filtering for sensor network systems with outliers, *Automatica*, vol. 177, art. no. 112345, 2025.
- [56] L. Zou, Z. Wang, B. Shen, and H. Dong, Moving horizon estimation over relay channels: Dealing with packet losses, *Automatica*, vol. 155, art. no. 111079, 2023.



Xuexiang Meng received the B.E. degree in electrical engineering and automation from Luoyang Institute of Science and Technology, Luoyang, China, in 2017, and the Ph.D. degree in control science and engineering from Hangzhou Dianzi University, Hangzhou, China, in 2023.

In 2023, he joined Hangzhou Dianzi University. His research interests include networked control systems and state estimation.



Zidong Wang (Fellow IEEE) received the B.Sc. degree in mathematics from Suzhou University, Suzhou, China, in 1986, and the M.Sc. degree in applied mathematics and the Ph.D. degree in electrical engineering from Nanjing University of Science and Technology, Nanjing, China, in 1990 and 1994, respectively.

From 1990 to 2002, he held teaching and research appointments in universities in China, Germany, and U.K. He is currently a Professor of dynamical systems and computing with the Department of Computer Science, Brunel University of London, Uxbridge, U.K. He is the holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, and the William Mong Visiting Research Fellowship of Hong Kong. He has published a number of articles in international journals. His research interests include dynamical systems, signal processing, bioinformatics, control theory, and applications.

Prof. Wang is a member of the Academia Europaea, a member of the European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a fellow of the Royal Statistical Society, and a member of the program committee for many international conferences. He serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, the Editor-in-Chief for *Systems Science & Control Engineering*, and an Associate Editor for 12 international journals including *IEEE TRANSACTIONS ON AUTOMATIC CONTROL*, *IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY*, *IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS*, *IEEE TRANSACTIONS ON SIGNAL PROCESSING*, and *IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART C: APPLICATIONS AND REVIEWS*.



Fan Wang (Member, IEEE) received the Ph.D. degree in applied mathematics from Southeast University, Nanjing, China, in 2018.

She is currently a Professor with the School of Automation, Nanjing University of Information Science and Technology, Nanjing, China. From 2016 to 2018, she was a Visiting Ph.D. Student with the Department of Computer Science, Brunel University of London, Uxbridge, U.K. From 2020 to 2021, she was a Post-Doctoral Fellow with the Department of Electrical and Computer Engineering, University of Macau, Macau, China. From 2022 to 2024, She was a Humboldt Research Fellow with the University of Duisburg-Essen, Duisburg, Germany. Her research interests include 2-D systems, networked control systems, recursive filtering, and optimal control.



Yun Chen received the B.E. degree in thermal engineering from Central South University of Technology, Changsha, China, in 1999, and the M.E. degree in engineering thermal physics and the Ph.D. degree in control science and engineering from Zhejiang University, Hangzhou, China, in 2002 and 2008, respectively.

In 2002, he joined Hangzhou Dianzi University, Hangzhou, where he is currently a Professor. From 2009 to 2010, he was a Visiting Fellow with the School of Computing, Engineering and Mathematics, Western Sydney University, Penrith, NSW, Australia. From 2016 to 2017, he was an Academic Visitor with the Department of Mathematics, Brunel University of London, Uxbridge, U.K. His research interests include stochastic systems, robust control, and filtering.