



CRFPA: A Novel Rock Fracture Process Analysis Method Coupled with Cohesive Zone Model

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Abstract

To enhance the effectiveness of fracture process simulation within the Rock Fracture Process Analysis (RFPA) framework, this study proposes a cohesive-enhanced RFPA (CRFPA) method. Cohesive elements are pre-inserted along potential fracture interfaces, either globally or locally, providing modeling flexibility tailored to different simulation objectives. A dual constitutive strategy is employed: cohesive elements adopt both brittle and quasi-brittle constitutive models to accommodate a wider range of material behaviors while solid elements retain the brittle damage model of conventional RFPA. This allows accurate distinction between matrix cracking and interfacial degradation. The method is formulated within a variational framework that ensures thermodynamic consistency, guaranteeing proper energy dissipation and irreversible damage evolution throughout the entire loading history. Leveraging RFPA's full-step iteration scheme, CRFPA achieves stable convergence while capturing progressive interface and matrix failure. Three benchmark examples, a masonry wallette shear test, a Brazilian disc splitting test, and flaw-driven crack propagation, are used to validate CRFPA method. Simulation results show strong agreement with experimental and analytical references, highlighting improvements in crack path realism, stress field resolution, and failure mode prediction compared to traditional RFPA. This study establishes CRFPA as a robust and versatile numerical framework for multi-scale fracture analysis in quasi-brittle materials.

Highlights

- A cohesive-enhanced RFPA (CRFPA) framework is proposed for simulating fracture in brittle and quasi-brittle materials.
- CRFPA integrates cohesive zone modeling into RFPA and inherits its full-step iteration scheme, ensuring stable convergence while enabling simultaneous simulation of bulk and interface failure.
- A variational formulation guarantees thermodynamic consistency and damage irreversibility throughout the entire loading process.
- A dual cohesive law of brittle and quasi-brittle enhances model adaptability across materials, validated through three benchmark cases: masonry shear, Brazilian disc, and mortar–granite crack propagation.

Keywords CZM · RFPA · CRFPA · Full-step iterative · Crack growth

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1 Introduction

The RFPA (Rock Fracture Process Analysis) method was first proposed by Tang et al. (Tang et al. 1998; Zhu and Tang 2002) in the 1990s, with the aim of numerically simulating the fracture and failure processes of materials such as rocks and concrete. Compared to traditional finite element methods, RFPA focuses on material heterogeneity and the gradual evolution of cracks, allowing for a more accurate representation of material failure behavior (Tang and Kou 1998). As a result, it has been widely applied in geotechnical engineering, materials science, and related fields (Yu et al. 2025a; Feng et al. 2022; Zhang et al. 2024). The core feature of the RFPA method is its ability to simulate the gradual damage, crack propagation, and interaction between cracks in materials. By discretizing the material into several elements and introducing local stress, strain, and failure criteria for each element, RFPA can accurately describe the material's damage evolution during the loading process. One of the key advantages of RFPA is its ability to account for material heterogeneity, utilizing statistical distribution models (such as the Weibull distribution) to model parameters such as strength and toughness, thereby capturing the randomness and statistical nature of material failure. The method can simulate crack propagation, coalescence, and the final failure mode, providing a more realistic simulation platform for the fracture process of complex materials.

In recent years, with improvements in computational power and algorithm optimization, the application and research of the RFPA method have made significant progress. On one hand, researchers have continuously improved the computational models based on RFPA to enhance its simulation accuracy and efficiency. For example, by employing parallel computing strategies, RFPA is now capable of handling larger and more complex problems, meeting the demands of industrial applications (Li et al. 2017a; Li and Tang 2015). The scope of RFPA's applications has gradually expanded, covering not only fracture analysis of traditional materials such as rock and concrete but also the fracture and damage research of complex systems such as composite materials, thin films, and bonding interfaces. Gong et al. (2018) proposed the Discontinuous Deformation Displacement (DDD) method, which couples RFPA with Discontinuous Deformation Analysis (DDA). In this approach, the RFPA module simulates crack initiation and propagation, and when the displacement of an element exceeds a threshold, it is automatically switched to a DDA block to handle contact separation, Gong and Tang (2017) successfully reproduced the Italian Alpetto Mine landslide. This method adopts a dual-domain partitioning structure of RFPA and DDA. In order to enhance the processing capability of block

motion, DDD does not inherit the iterative method of RFPA. Therefore, when calculating the contact force between separated blocks, open and closed iterations are required, which limits the computational efficiency and scale of the DDD method (Gong et al. 2019).

The traditional finite element method uses element deletion techniques to simulate crack propagation; however, this approach has inherent flaws in terms of physical validity. On the one hand, the instantaneous disappearance of elements violates the law of energy conservation, leading to energy distortion in the damage evolution process. On the other hand, the geometric size limitations of solid elements force crack widths to be discretized at the element boundaries, making it difficult to reflect the continuous topological features of real cracks (Wei et al. 2025; Zhang et al. 2026). The traditional RFPA method shares these shortcomings. The Cohesive element method, developed in the 1990s, provides an innovative solution to this problem. This method is based on the cohesive zone model (CZM) proposed by Goodman (1968) and Ghaboussi (1973), and uses interface elements along predefined crack paths to represent the material separation process. The fictitious crack model (FCM) proposed by Hillerborg et al. (Hillerborg et al. 1976; Hillerborg 1983) further introduced this concept to concrete fracture analysis, laying the theoretical foundation for modern cohesive element methods. Its core advantages include: the natural description of the interaction between crack surfaces through a traction–separation constitutive relationship; the continuous simulation of crack initiation and propagation using a damage mechanics framework; and the avoidance of stress singularity problems inherent in traditional methods (Camanho and Dávila 2002; Bosch et al. 2006). In recent years, with the development of multi-scale computational mechanics, the cohesive element method has achieved significant progress in fields such as composite material interface failure (Spring and Paulino 2015; Kumar et al. 2024), biological tissue layering (Wu et al. 2025), and hydraulic fracturing in rocks (Li et al. 2017b). Yu et al. (2025b) extended the dynamic fracture simulation capabilities by introducing rate-dependent constitutive models, while Choi et al. (2025a) developed cohesive force adaptive mesh techniques, improving the prediction accuracy of complex crack paths. However, existing research still faces two major bottlenecks: firstly, most models assume uniform distribution of interface material properties, making it difficult to reflect the inherent heterogeneity of natural materials (*e.g.*, rocks, concrete); secondly, traditional implicit solving methods face convergence challenges in quasi-static problems, limiting their application to engineering scales.

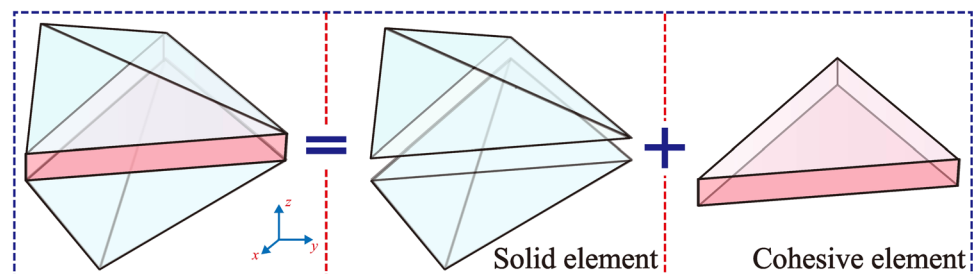
To overcome the aforementioned limitations of conventional RFPA and cohesive zone approaches, this study proposes an enhanced Realistic Fracture Process Analysis method incorporating cohesive elements, termed the

Cohesive-augmented RFPA (CRFPA). The CRFPA framework retains the core strengths of RFPA, such as full-field heterogeneity representation, progressive failure, and statistical variability, while integrating cohesive elements to explicitly simulate interfacial debonding and crack evolution. CRFPA embeds cohesive elements either globally or locally within the finite element mesh, enabling adaptive representation of interface separation and interaction with bulk material cracking. Within CRFPA, solid elements follow the brittle damage model consistent with standard RFPA formulations, while the interface behavior is governed by a dual constitutive strategy: both brittle and quasi-brittle cohesive laws are employed to reflect a broader spectrum of material responses. This cohesive modeling approach is framed within a variational formulation that ensures thermodynamic consistency and damage irreversibility. Moreover, CRFPA inherits RFPA's full-step iterative scheme, which updates stiffness and damage fields at each global time step. This global iteration strategy enhances numerical stability and allows for accurate tracking of both bulk and interface damage without relying on incremental remeshing or element deletion. By combining these features, CRFPA provides a unified and robust computational platform for modeling complex fracture processes in heterogeneous materials.

2 Methodology

In the traditional RFPA method, only the failure behavior of solid elements is considered. Building upon this, the CRFPA method integrates the Cohesive Zone Model (CZM), enhancing the realism of crack simulation in brittle material failure processes. In CRFPA, three-dimensional four-node solid elements are used, with a brittle material constitutive model consistent with that of RFPA. To match the solid elements, three-dimensional six-node cohesive elements are employed, as shown in Fig. 1. In order to broaden the applicability, both brittle and quasi-brittle constitutive models are adopted for the cohesive elements. These cohesive elements are inserted locally or globally during the preprocessing stage according to requirements. The numerical solution scheme of CRFPA follows the same full-step iteration strategy as RFPA, thereby avoiding the convergence issues often encountered in traditional incremental finite element methods.

Fig. 1 Element types in CRFPA



2.1 The Formulation of Solid and Cohesive Elements

In CRFPA, the solid element type is chosen to be a tetrahedral shape, as shown in Fig. 1, the displacement field of the element is expressed as shown in Eq. (1):

$$\mathbf{u}_{(3 \times 1)}(x, y, z) = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 \end{bmatrix} \cdot \mathbf{q}_{(12 \times 1)} = \mathbf{N}_{(3 \times 12)} \cdot \mathbf{q}_{(12 \times 1)} \quad (1)$$

where N_i denotes the element shape function, as defined in Eq. (2):

$$N_i = \frac{1}{6V}(a_i + b_i x + c_i y + d_i z) \quad (2)$$

where V is the element volume, a_i , b_i , c_i , and d_i are coefficients related to the geometric positions of element nodal.

The strain field of the element is expressed as shown in Eq. (3):

$$\epsilon_{(6 \times 1)}(x, y, z) = [\partial] \mathbf{u}_{(3 \times 1)} = [\partial] \mathbf{N}_{(6 \times 3)} \cdot \mathbf{q}_{(12 \times 1)} = \mathbf{B}_{(6 \times 12)} \cdot \mathbf{q}_{(12 \times 1)} \quad (3)$$

The expression of the $\mathbf{B}$ matrix is given in Eqs. (4) and (5).

$$\mathbf{B}_{(6 \times 12)} = [\partial] \mathbf{N}_{(6 \times 3)} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} \quad (4)$$

$$\mathbf{B}_i = [\partial] \begin{bmatrix} N_i & 0 & 0 \\ 0 & N_i & 0 \\ 0 & 0 & N_i \end{bmatrix} = \frac{1}{6V} \begin{bmatrix} b_i & 0 & 0 \\ 0 & c_i & 0 \\ 0 & 0 & d_i \\ c_i & b_i & 0 \\ 0 & d_i & c_i \\ d_i & 0 & b_i \end{bmatrix}, \quad i = 1, 2, 3, 4 \quad (5)$$

According to the $\mathbf{B}$ matrix, the material stiffness matrix $\mathbf{D}$ and the strain tensor ϵ at the integration points, the stress

tensor, element stiffness matrix and nodal forces can be obtained, as shown in Eqs. (6) to (8).

$$\underset{(6 \times 1)}{\boldsymbol{\sigma}} = \underset{(6 \times 6)}{\mathbf{D}} \underset{(6 \times 1)}{\boldsymbol{\epsilon}} = \underset{(6 \times 6)}{\mathbf{D}} \underset{(6 \times 12)}{\mathbf{B}} \underset{(12 \times 1)}{\mathbf{q}} \quad (6)$$

$$\underset{(12 \times 12)}{\mathbf{K}} = \int_{\Omega} \underset{(12 \times 6)}{\mathbf{B}^T} \underset{(6 \times 6)}{\mathbf{D}} \underset{(6 \times 12)}{\mathbf{B}} d\Omega \quad (7)$$

$$\underset{(12 \times 1)}{\mathbf{f}} = \int_{\Omega} \underset{(12 \times 3)}{\mathbf{B}^T} \underset{(3 \times 3)}{\boldsymbol{\sigma}} d\Omega \quad (8)$$

Compared to solid elements, cohesive elements are more specialized, with their constitutive model primarily describing the relationship between the crack separation displacement and the traction. To match the tetrahedral solid elements, cohesive elements are designed with 6 nodes, as shown in Fig. 2.

The shape function and Jacobian matrix of the cohesive element are given in Eqs. (9) and (10).

$$N_1 = r; N_2 = s; N_3 = 1 - s - r \quad (9)$$

$$\begin{aligned} \underset{(2 \times 3)}{\mathbf{J}} &= \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \end{bmatrix} \\ &= \begin{bmatrix} \sum_{k=1}^3 \frac{\partial N_k}{\partial r} \cdot x_{m,k} & \sum_{k=1}^3 \frac{\partial N_k}{\partial r} \cdot y_{m,k} & \sum_{k=1}^3 \frac{\partial N_k}{\partial r} \cdot z_{m,k} \\ \sum_{k=1}^3 \frac{\partial N_k}{\partial s} \cdot x_{m,k} & \sum_{k=1}^3 \frac{\partial N_k}{\partial s} \cdot y_{m,k} & \sum_{k=1}^3 \frac{\partial N_k}{\partial s} \cdot z_{m,k} \end{bmatrix} \\ &= \begin{bmatrix} \left(\frac{\partial \mathbf{X}_{JP}}{\partial r} \right)^T \\ \left(\frac{\partial \mathbf{X}_{JP}}{\partial s} \right)^T \end{bmatrix} \end{aligned} \quad (10)$$

Based on the Jacobian matrix, the transformation matrix $\mathbf{R}$ can be derived. $\mathbf{R}$ converts the global coordinates of nodes into the local coordinates, as shown in Eq. (11) and (12).

$$\begin{cases} \underset{(3 \times 1)}{\mathbf{n}} = \frac{1}{\left\| \frac{\partial \mathbf{X}_{JP}}{\partial r} \times \frac{\partial \mathbf{X}_{JP}}{\partial s} \right\|} \left(\frac{\partial \mathbf{X}_{JP}}{\partial r} \times \frac{\partial \mathbf{X}_{JP}}{\partial s} \right) \\ \underset{(3 \times 1)}{\mathbf{t}_1} = \frac{1}{\left\| \frac{\partial \mathbf{X}_{JP}}{\partial r} \right\|} \left(\frac{\partial \mathbf{X}_{JP}}{\partial r} \right) \\ \underset{(3 \times 1)}{\mathbf{t}_2} = \frac{1}{\left\| \mathbf{n} \times \mathbf{t}_1 \right\|} (\mathbf{n} \times \mathbf{t}_1) \end{cases} \quad (11)$$

$$\underset{(3 \times 3)}{\mathbf{R}} = [\underset{(3 \times 1)}{\mathbf{t}_1} \underset{(3 \times 1)}{\mathbf{t}_2} \underset{(3 \times 1)}{\mathbf{n}}]^T \quad (12)$$

In CRFPA, all cohesive-element kinematics are evaluated with respect to the initial configuration. Therefore, the local basis $(\mathbf{t}_1, \mathbf{t}_2, \mathbf{n})$ and the transformation matrix $\mathbf{R}$ are computed once from the initial facet mapping as defined in Eq. (11) and remain unchanged during the full-step iterations. Provided that the initial facet is non-degenerate, $\mathbf{R}$ is orthonormal and satisfies $\mathbf{R}^T \mathbf{R} = \mathbf{I}$, thus the robustness of the transformation is ensured by basic initial mesh-quality checks.

Finally, the local coordinates for each nodes can be obtained, as shown in Eq. (13).

$$\underset{(3 \times 6)}{\mathbf{X}_L} = \underset{(3 \times 3)}{\mathbf{R}} \underset{(3 \times 6)}{\mathbf{X}_D} \quad (13)$$

where $\mathbf{X}_D$ represents the global nodal coordinates, and $\mathbf{X}_L$ represents the local nodal coordinates. $\mathbf{X}_L$ is shown in Eq. (14).

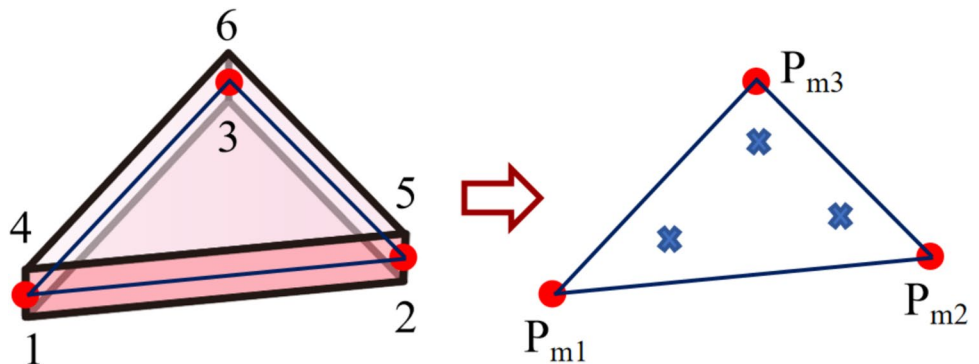
$$\underset{(3 \times 3)}{\Delta_L} = [\Delta x_i \ \Delta y_i \ \Delta z_i]^T \quad (14)$$

Finally, the separation displacement at the integration points can be obtained based on Eq. (14) and the shape functions, as shown in Eq. (15).

$$\begin{aligned} \Delta I (3 \times 1) &= [\Delta I, t1 \ \Delta I, t2 \ \Delta I, n]^T \\ &= \left[\sum_{i=1}^3 \Delta x_i N_i \ \sum_{i=1}^3 \Delta y_i N_i \ \sum_{i=1}^3 \Delta z_i N_i \right]^T \end{aligned} \quad (15)$$

According to Eqs. (9) to (15), the separation displacements at the three integration points can be calculated. Based on the current separation displacements, the constitutive elasticity matrix corresponding to the full iteration

Fig. 2 Configuration and integration points of the 6-Node cohesive element



can be obtained. The constitutive elasticity matrices at the three integration points can be used to form the element stiffness matrix, as shown in Eq. (16).

$$\mathbf{K}_{(18 \times 18)} = \int_{\Omega} \mathbf{T}_r^T \mathbf{B}^T \mathbf{D} \mathbf{B} \mathbf{T}_r d\Omega \quad (16)$$

The traction force at each integration point, obtained from the constitutive model, can be used to calculate the equivalent nodal force, as expressed in Eq. (17).

$$\mathbf{f}_{(18 \times 1)} = \int_{\Omega} \mathbf{T}_r^T \mathbf{B}^T \mathbf{T} d\Omega \quad (17)$$

where, $\mathbf{T}$ is traction in integration point of cohesive element. The $\mathbf{T}_r$ matrix is composed of the $\mathbf{R}$ matrix, and the expressions for the $\mathbf{T}_r$ and $\mathbf{B}$ matrices are given in Eqs. (18) and (19).

$$\mathbf{T}_r = \begin{bmatrix} \mathbf{R}_{3 \times 3} & & & & \\ & \mathbf{R}_{3 \times 3} & & & \\ & & \mathbf{R}_{3 \times 3} & & \\ & & & \mathbf{R}_{3 \times 3} & \\ & & & & \mathbf{R}_{3 \times 3} \end{bmatrix} \quad (18)$$

$$\mathbf{B}_{(3 \times 18)} = \begin{bmatrix} N_1 & & N_2 & & N_3 & & -N_1 & & -N_2 & & -N_3 & & & & & & \\ & N_1 & & N_2 & & N_3 & & -N_1 & & -N_2 & & -N_3 & & & & & \\ & & N_1 & & N_2 & & -N_1 & & -N_2 & & -N_3 & & & & & \\ & & & N_1 & & N_2 & & -N_1 & & -N_2 & & -N_3 & & & & \\ & & & & N_1 & & -N_1 & & -N_2 & & -N_3 & & & & & \end{bmatrix} \quad (19)$$

2.2 The Constitutive Model of Cohesive Element

RFPA focuses on addressing the simulation of failure processes in brittle and quasi-brittle materials. The interface constitutive models in CRFPA are classified into two types: brittle and quasi-brittle. Figure 3 depicts the normal behavior, tangential behavior, and mixed-mode schematic of the brittle cohesive constitutive model.

In Fig. 3, σ_{max} and τ_{max} are the normal and tangential strengths of the interface, respectively; The subscripts “n” and “t” represent the normal and tangential directions, respectively; k is interface stiffness; r is residual strength ratio; μ is friction coefficient; Δ is failure crack separation displacement; δ is interface separation displacement; T is interface traction. As shown in Fig. 3a, cohesive elements will be failure under tension, but do not fail under compression and maintaining linear elastic behavior throughout. The traction after failure is controlled by the residual strength

ratio. The tangential behavior is shown in Fig. 3b, the stress state considering the compressive-shear coupling is taken into account. As a result, the peak strength and residual strength of the interface are influenced by the frictional force. If the current cohesive element is under compression and shear slip occurs, the peak shear strength is $\tau_{max} + \mu T_n$, and the residual strength is μT_n . When the cohesive element is in a tension-shear coupling state, its strength composition is as shown in Fig. 3c. The specific strength criterion is given by Eq. (20):

$$F_{mix} = \sqrt{\left(\frac{T_n}{\sigma_{max}}\right)^2 + \left(\frac{T_t}{\tau_{max}}\right)^2} \quad (20)$$

When F_{mix} is greater than 1.0, it can be determined that the cohesive element has failed.

Figure 4 depicts the normal behavior, tangential behavior, and mixed-mode schematic of the quasi-brittle cohesive constitutive model.

The PPR cohesive model, originally developed by Park et al. (Park et al. 2009; Park 2009), offers a comprehensive framework for characterizing the mechanical response of cohesive interfaces. This model is based on a smooth and continuous potential energy function, from which the normal and tangential cohesive traction (T_n and T_t) are obtained through first-order derivatives with respect to the

corresponding displacement components. Such a formulation inherently links traction to the interfacial separation, ensuring a consistent and physically meaningful response. Both the normal and tangential tractions are influenced by the combined effect of opening displacements in their respective directions, capturing mode coupling within the cohesive zone.

The unified potential function is shown in Eq. (21):

$$\begin{aligned} \psi(\delta_n, \delta_t) = & \min(\phi_n, \phi_t) \\ & + \left[\Gamma_n \left(1 - \frac{\delta_n}{\Delta_n}\right)^\alpha \left(\frac{m}{\alpha} + \frac{\delta_n}{\Delta_n}\right)^m + \langle \phi_n - \phi_t \rangle \right] \\ & \times \left[\Gamma_t \left(1 - \frac{|\delta_t|}{\Delta_t}\right)^\beta \left(\frac{n}{\beta} + \frac{|\delta_t|}{\Delta_t}\right)^n + \langle \phi_t - \phi_n \rangle \right] \end{aligned} \quad (21)$$

The normal and tangential cohesive traction are shown in Eq. (22):

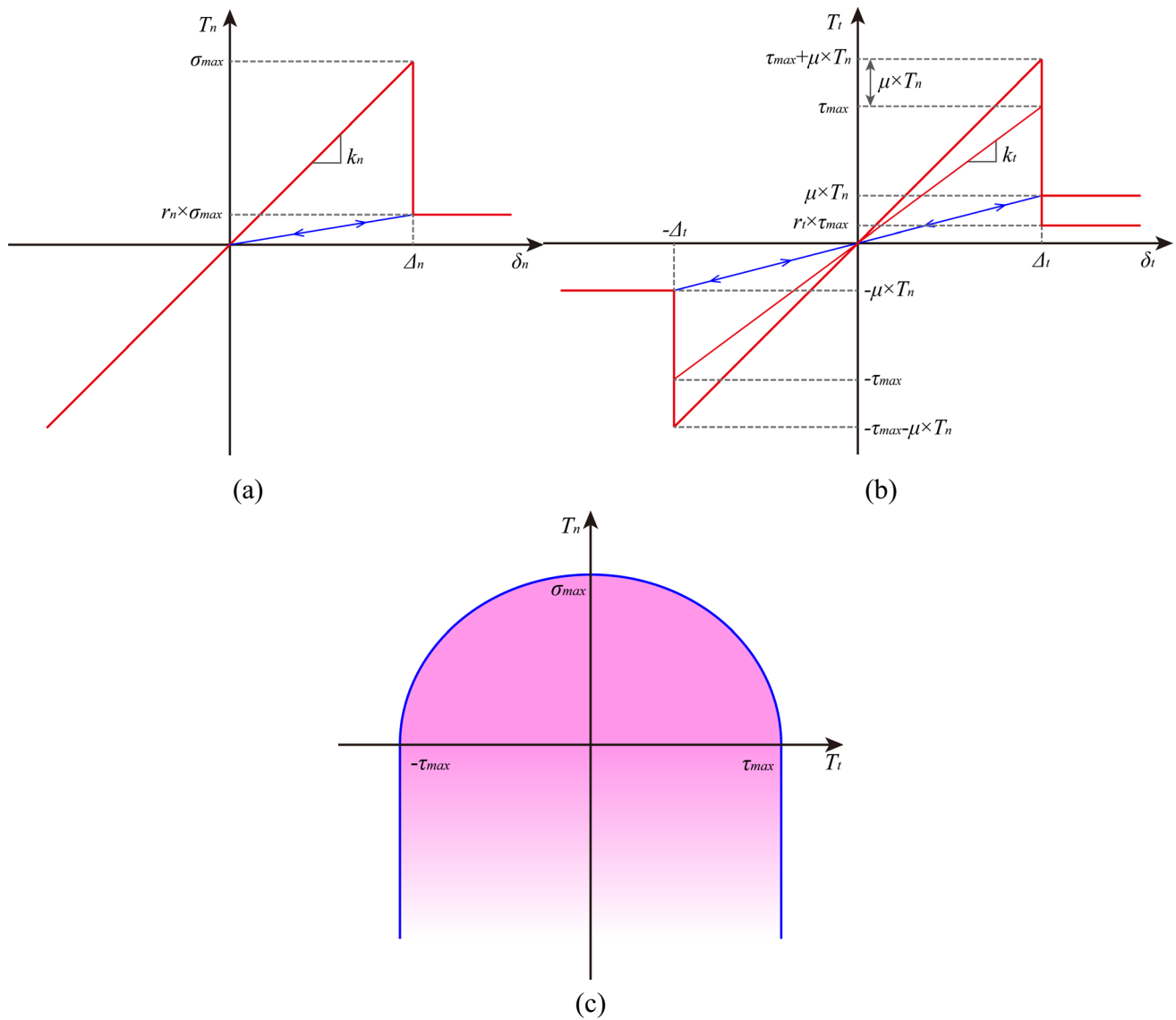


Fig. 3 Schematic diagram of the brittle cohesive constitutive model **a** Normal behavior, **b** Tangential behavior, **c** Mixed mode

$$\left\{ \begin{aligned} T_n(\delta_n, \delta_t) &= \frac{\partial \psi_c}{\partial \delta_n} = \frac{\Gamma_n}{\Delta_n} \left[m \left(1 - \frac{\delta_n}{\Delta_n} \right)^\alpha \left(\frac{m}{\alpha} + \frac{\delta_n}{\Delta_n} \right)^{m-1} - \alpha \left(1 - \frac{\delta_n}{\Delta_n} \right)^{\alpha-1} \left(\frac{m}{\alpha} + \frac{\delta_n}{\Delta_n} \right)^m \right] \\ &\quad \times \left[\Gamma_t \left(1 - \frac{|\delta_t|}{\Delta_t} \right)^\beta \left(\frac{n}{\beta} + \frac{|\delta_t|}{\Delta_t} \right)^n + \langle \phi_t - \phi_n \rangle \right] \\ T_t(\delta_n, \delta_t) &= \frac{\partial \psi_c}{\partial \delta_t} = \frac{\Gamma_t}{\Delta_t} \left[n \left(1 - \frac{|\delta_t|}{\Delta_t} \right)^\beta \left(\frac{n}{\beta} + \frac{|\delta_t|}{\Delta_t} \right)^{n-1} - \beta \left(1 - \frac{|\delta_t|}{\Delta_t} \right)^{\beta-1} \left(\frac{n}{\beta} + \frac{|\delta_t|}{\Delta_t} \right)^n \right] \\ &\quad \times \left[\Gamma_n \left(1 - \frac{\delta_n}{\Delta_n} \right)^\alpha \left(\frac{m}{\alpha} + \frac{\delta_n}{\Delta_n} \right)^m + \langle \phi_n - \phi_t \rangle \right] \frac{\delta_t}{|\delta_t|} \end{aligned} \right. \quad (22)$$

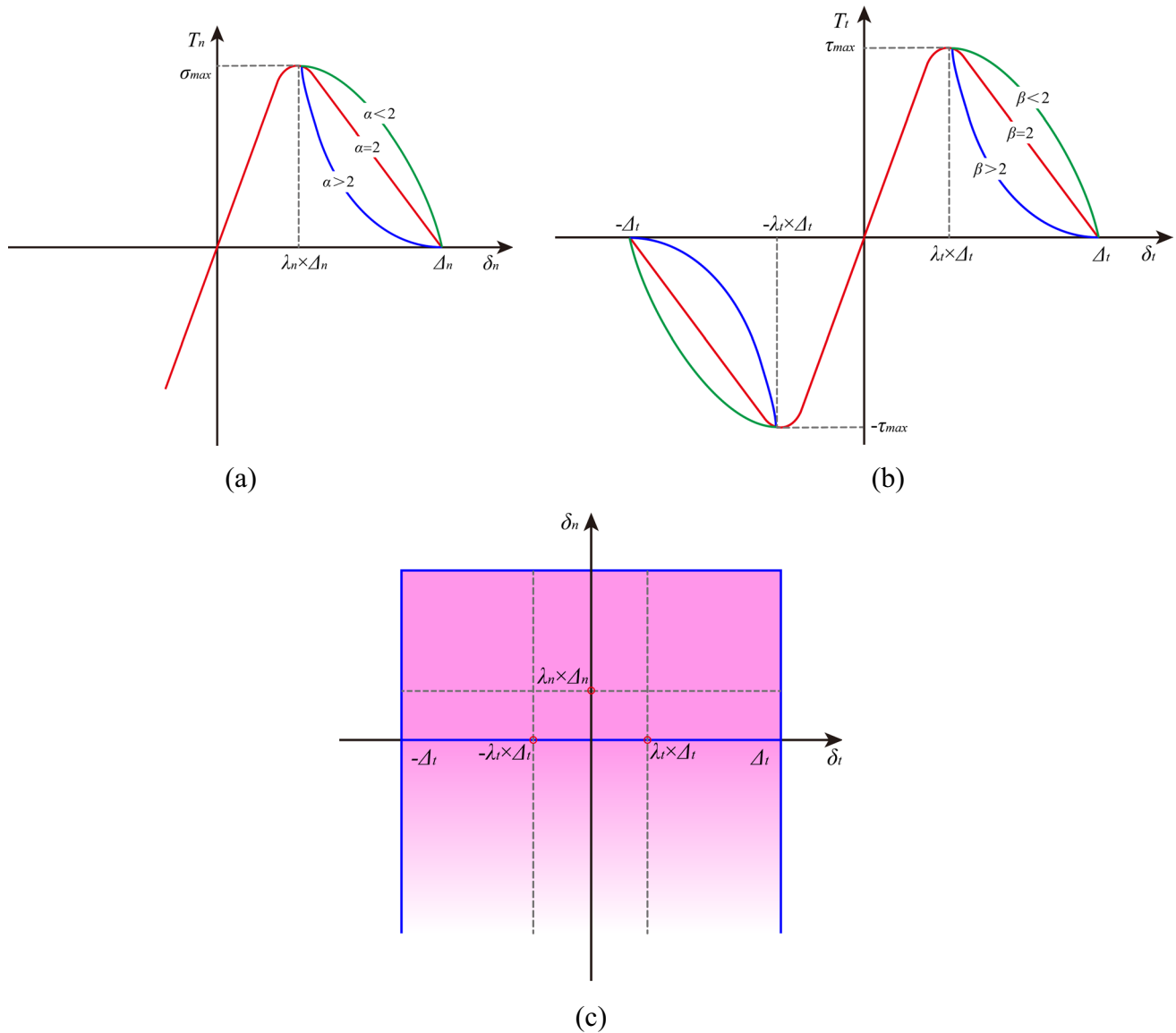


Fig. 4 Schematic diagram of the brittle cohesive constitutive model **a** Normal behavior, **b** Tangential behavior, **c** Mixed mode

where ϕ is the fracture energies; Δ is failure crack separation displacement; m and n are process variables, while Γ is energy constants. Their specific expressions are given in Eqs. (23) to (25).

$$\begin{cases} \Delta_n = \frac{\phi_n}{\sigma_{\max}} \alpha \lambda_n (1 - \lambda_n)^{\alpha-1} \left(\frac{\alpha}{m} + 1 \right) \left(\frac{\alpha}{m} \lambda_n + 1 \right)^{m-1} \\ \Delta_t = \frac{\phi_t}{\tau_{\max}} \beta \lambda_t (1 - \lambda_t)^{\beta-1} \left(\frac{\beta}{n} + 1 \right) \left(\frac{\beta}{n} \lambda_t + 1 \right)^{n-1} \end{cases} \quad (23)$$

$$\begin{cases} \Gamma_n = (-\phi_n)^{\langle \phi_n - \phi_t \rangle / (\phi_n - \phi_t)} \left(\frac{\alpha}{m} \right)^m \\ \Gamma_t = (-\phi_t)^{\langle \phi_t - \phi_n \rangle / (\phi_t - \phi_n)} \left(\frac{\beta}{n} \right)^n \end{cases} \quad (24)$$

$$\begin{cases} m = \frac{\alpha(\alpha-1)\lambda_n^2}{(1-\alpha\lambda_n^2)} \\ n = \frac{\beta(\beta-1)\lambda_t^2}{(1-\beta\lambda_t^2)} \end{cases} \quad (25)$$

Under compressive loading, when opposing elements are driven into contact, interfacial resistance is activated due to element interpenetration. This resistance depends primarily on two factors: the extent of penetration and the normal stiffness of the interface under compression. Notably, the compressive stiffness is assumed to be equivalent to that associated with minimal tensile separation. In the current model, interfacial compression is accompanied by frictional effects, which induce shear slip between adjacent surfaces.

As the tangential displacement increases, the magnitude of this slip grows progressively (Yu et al. 2024, 2025b). The resulting frictional behavior is quantitatively expressed in Eq. (26).

$$T_f = \mu \times K(\delta_t) \times |T_n|, T_n < 0 \text{ and } |\delta_t| > 0 \quad (26)$$

Here, μ denotes the friction coefficient, and K represents the frictional response factor, which evolves continuously and monotonically from 0 to 1 as shear displacement increases. This relationship is described mathematically in Eq. (27).

$$K(\delta_t) = \begin{cases} \left(\frac{T_i(0, \delta_t)}{\tau_{\max}} \right)^\eta, & |\delta_t| \in [0, \lambda_t \Delta_t) \\ 1, & |\delta_t| \in [\lambda_t \Delta_t, +\infty) \end{cases} \quad (27)$$

The proposed model requires eight primary inputs: ϕ_n , ϕ_r , $\sigma_{\max}$, $\tau_{\max}$, λ_n , λ_t , α , and β . In addition, the friction-growth shape parameter η and the friction coefficient μ must be specified. The friction shape parameter η critically influences the evolution profile of the frictional response. Its effect on the response factor K is illustrated in Fig. 5a, highlighting how different values of η alter the growth behavior. The resulting T_t curve generated by the model is shown in Fig. 5b. In three-dimensional problems, δ_2 and δ_3 represent the crack openings along the two tangential directions ($\delta_t = \sqrt{\delta_2^2 + \delta_3^2}$). The interaction between frictional resistance and tangential cohesive tractions along these directions is visualized in Fig. 6.

Cohesion Weakening Friction Strengthening (CWFS) emphasizes that, during brittle fracture evolution, the dominant load-bearing mechanism transitions from bonded

cohesion to frictional sliding along newly formed fracture surfaces (Goodman et al. 1968; Ghaboussi et al. 1973). In the interface constitutive model shown in Fig. 6, the progressive degradation of cohesive traction driven by interface damage corresponds to cohesion weakening, whereas once the interface enters compressive contact, the shear resistance becomes governed primarily by the frictional term through friction mobilization, reflecting the late-stage friction-dominated mechanism. It is also important to note that our model realizes this transition at the cohesive element level via traction–separation relations coupled with contact and friction, while CWFS is commonly formulated at the continuum scale by prescribing the evolution of T_i and T_f as functions of damage. Despite the difference in scale and mathematical representation, both descriptions are physically consistent in capturing the shift from cohesion-controlled to friction-controlled resistance during brittle failure.

2.3 The Constitutive Model of Solid Elements

The Rock Failure Process Analysis (RFPA) method, originally proposed by Tang et al. (Tang et al. 1998; Zhu and Tang 2002), is capable of simulating the full evolution of deformation and fracture in brittle materials. By incorporating statistical damage mechanics, RFPA effectively captures the influence of material heterogeneity during crack initiation and propagation. In the element scale, failure is governed by the Mohr–Coulomb criterion supplemented with a tensile strength cutoff. To accurately resolve crack evolution across scales, the method employs a finer mesh than conventional finite element approaches. The use of an elastic-damage formulation under small deformation assumptions

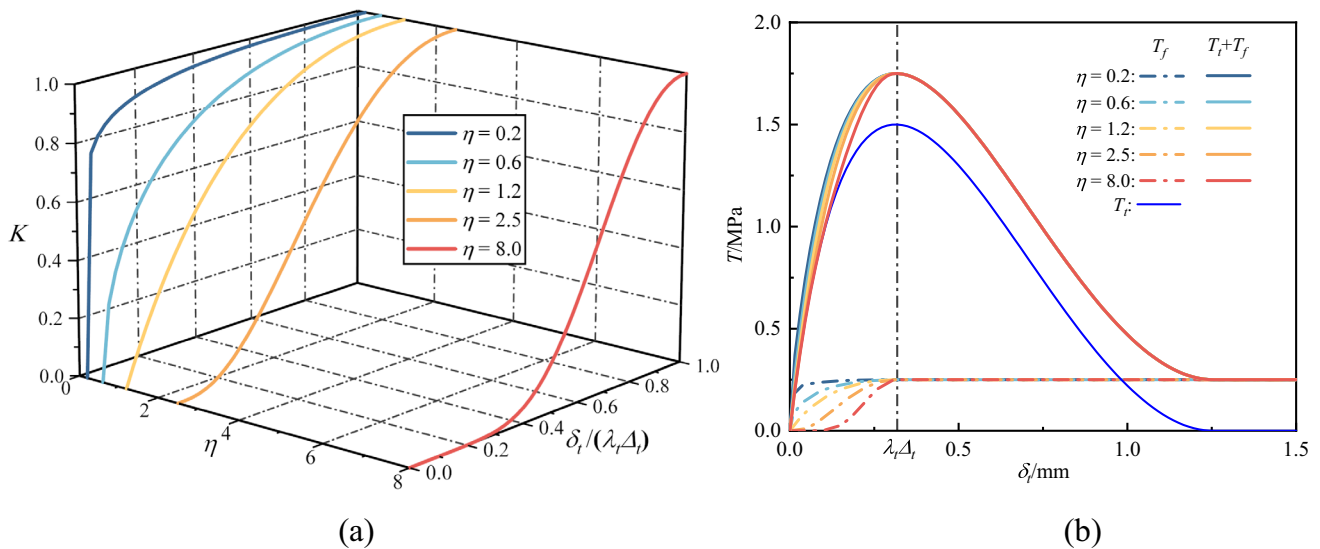


Fig. 5 Coupling method between friction and tangent traction ($\phi_t = 1 \text{ MPa} \cdot \text{mm}$, $\tau_{\max} = 1.5 \text{ MPa}$, $\lambda_t = 0.25$, $\beta = 3.0$, $\mu = 0.25$) **a** Effect of the friction growth shape parameter η on the response factor K , **b** Relationship between the tangential cohesion and the tangential displacement

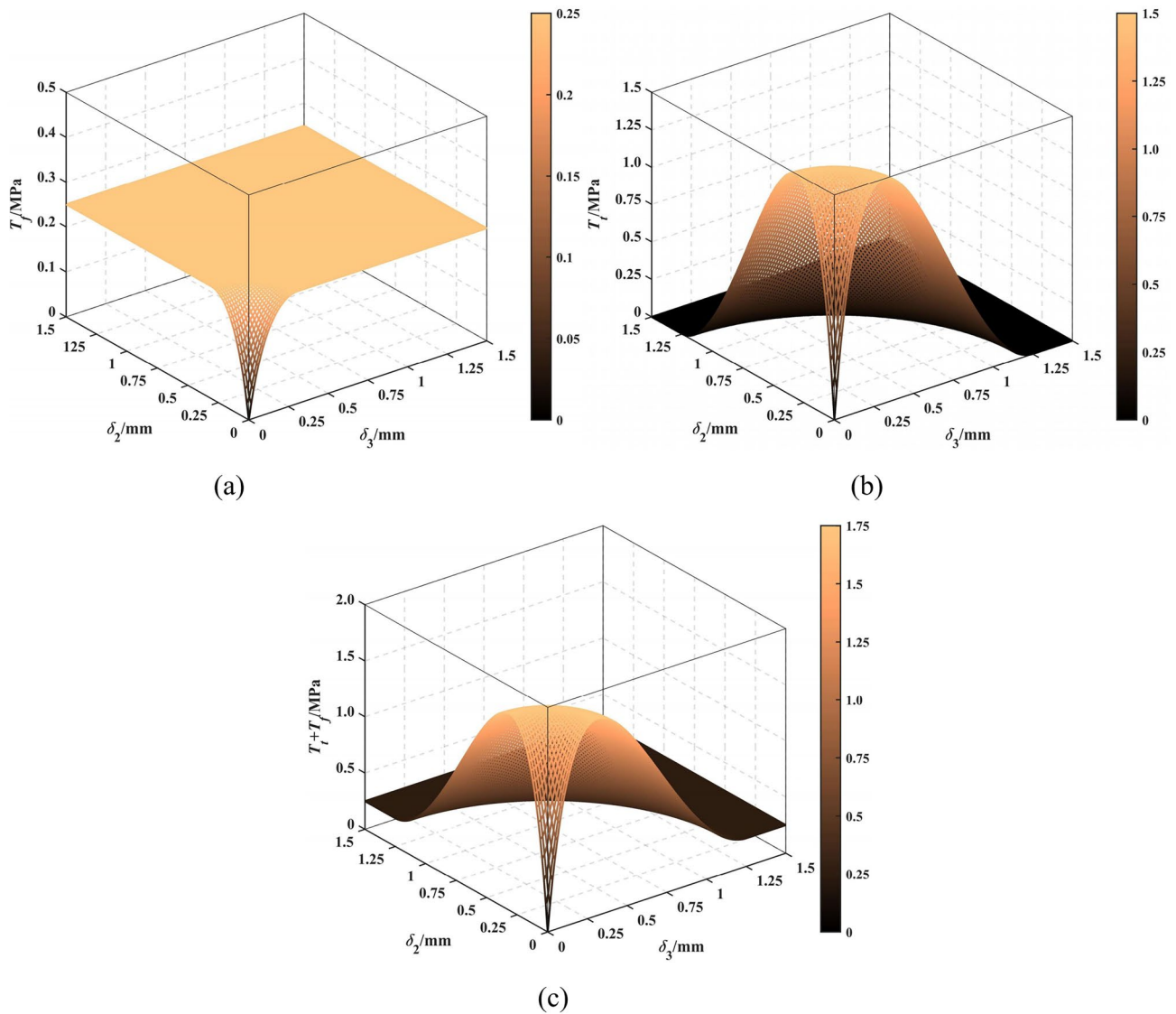


Fig. 6 Interactive influence of friction and cohesion ($\phi_f=1 \text{ MPa}\cdot\text{mm}$, $\tau_{max}=1.5 \text{ MPa}$, $\lambda_t=0.25$, $\beta=3.0$, $\mu=0.25$) **a** Friction along δ_2 and δ_3 directions, **b** Cohesion along δ_2 and δ_3 directions, **c** Coupled friction and cohesion along δ_2 and δ_3 directions (modified after (Yu et al. 2024)).

enhances numerical stability and computational efficiency. The applied failure criterion, constitutive response, and statistical heterogeneity framework are illustrated in Fig. 7.

The M-C strength criterion incorporating a tensile cut-off is depicted in Fig. 7a. According to the prevailing stress conditions, the failure behavior of mesoscopic elements is categorized into three types: compression-shear failure, shear-dominated tensile failure, and purely tensile failure. Each failure type corresponds to a distinct residual strength envelope. In the event of pure tensile failure, the element is assumed to retain compressive and shear load-bearing capacity. For compression-shear failure, tensile resistance is preserved. However, in the case of tension-shear failure, the element is considered fully degraded, with no remaining

structural capacity. Figure 7b presents the uniaxial stress vs. strain response of meso-elements, defining the compressive and tensile strength thresholds (σ_c and σ_t , respectively). Once these thresholds are exceeded, the material enters a residual strength regime. Under tensile loading, if the strain surpasses the ultimate limit ϵ_{tu} , further strength degradation occurs.

From the diagrams shown in Fig. 7a and b, the strength boundaries for various failure modes can be derived. The corresponding failure surfaces in principal stress space are given by Eqs. (28) to (30):

$$\sigma_3 \leq \sigma_t, \text{ Tension failure surface} \quad (28)$$

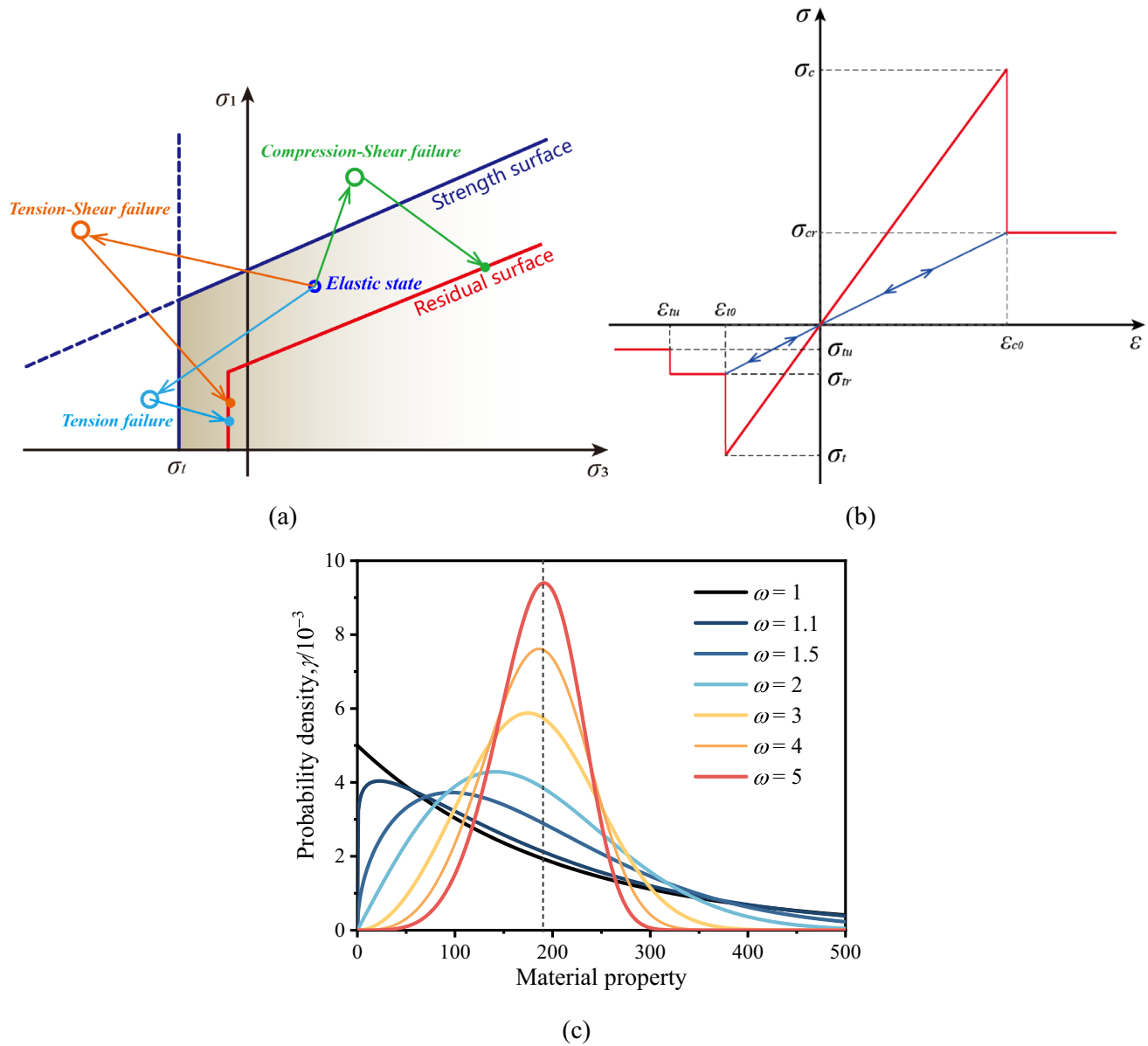


Fig. 7 The method of meso-elements' failure **a** Tension cut-off M-C strength criterion, **b** stress-strain relationship under uniaxial loading, **c** Statistical distribution theory

$$\sigma_1 - \frac{1 + \sin \phi}{1 - \sin \phi} \sigma_3 - \sigma_c \geq 0, \text{ Compression - shear failure surface} \quad (29)$$

$$\sigma_3 \leq \sigma_t \text{ and } \sigma_1 - \frac{1 + \sin \phi}{1 - \sin \phi} \sigma_3 - \sigma_c \geq 0, \quad (30)$$

Tension - shear failure surface

In Eqs. (28) to (30), σ_1 and σ_3 denote the major and minor principal stresses, respectively, while ϕ refers to the internal friction angle. The parameters σ_c and σ_t correspond to the uniaxial compressive strength (UCS) and uniaxial tensile strength. A dimensionless coefficient λ is introduced

to characterize the ratio between tensile and compressive strengths, thereby establishing a link between the two failure limits, as shown in Eq. (31).

$$\lambda \sigma_c = \sigma_t \quad (31)$$

The damage factor D is introduced to quantify the extent of stiffness degradation following material failure, as formulated in Eq. (32).

$$E = E_0(1 - D) \quad (32)$$

The evaluation of the damage factor D depends on the specific failure mode of the element. For tensile and compression failure, D is computed using the formulation provided in Eqs. (33) and (34).

$$D = \begin{cases} 0, \varepsilon > \varepsilon_{t0} \\ 1 - \frac{\lambda_{tr} \varepsilon_{t0}}{\varepsilon}, \varepsilon_{tu} < \varepsilon \leq \varepsilon_{t0} \\ 1, \varepsilon < \varepsilon_{tu} \end{cases} \quad (33)$$

$$D = \begin{cases} 0, \varepsilon < \varepsilon_{c0} \\ 1 - \frac{\lambda_{cr} \varepsilon_{c0}}{\varepsilon}, \varepsilon \geq \varepsilon_{c0} \end{cases} \quad (34)$$

where, ε_{t0} denotes the strain at the onset of tensile failure, while ε_{tu} is the ultimate tensile strain. The residual strength ratio under tension is given by $\lambda_{tr} = \sigma_t / \sigma_{tr}$, and the ultimate strength ratio is defined as $\lambda_{tu} = \varepsilon_{t0} / \varepsilon_{tu}$. In the compressive regime, ε_{c0} is the strain corresponding to compressive failure, and the residual strength ratio is expressed as $\lambda_{cr} = \sigma_c / \sigma_{cr}$.

To capture the spatial variability of material properties, a heterogeneity coefficient ω is introduced within the Weibull distribution framework, as formulated in Eq. (35). Figure 7c illustrates the probability density function, where the x-axis denotes a given material parameter and the y-axis represents its distribution density. A higher value of ω indicates increased material uniformity, whereas a smaller ω reflects greater heterogeneity across the element ensemble.

$$\gamma(x) = \frac{\omega}{x_0} \left(\frac{x}{x_0} \right)^{\omega-1} \exp \left(- \left(\frac{x}{x_0} \right)^{\omega} \right) \quad (35)$$

In Eq. (35), ω is the heterogeneity coefficient, x is the value of the distribution parameter, x_0 is the scale parameter.

2.4 The Numerical Iteration of CRFPA.

The CRFPA method, while inheriting the basic characteristics of the original RFPA for solving brittle and quasi-brittle materials, introduces Cohesive elements and adopts a full-scale iteration strategy, maintaining RFPA's advantages in computational accuracy and efficiency (Li and Tang 2015). The content in the Fig. 8a clearly demonstrates this iteration process, where each time step (*Step*—*i*) is further subdivided into multiple sub-steps, with each sub-step corresponding to a same increment, and the full-scale iteration progressively approaches the final solution. During this process, the stiffness matrix (**K**) is updated in each *Sub-step* based on the current displacement and failure state, accompanied by the stiffness degradation treatment of the elements, ensuring that the nonlinear behavior of the solid and cohesive elements is accurately captured. When the number of new

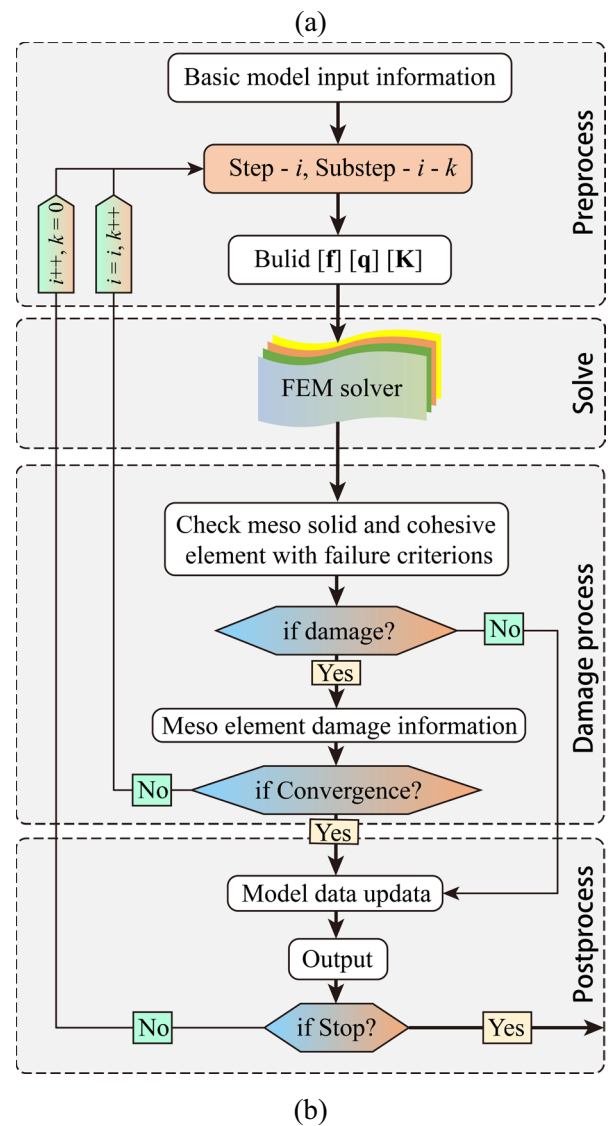
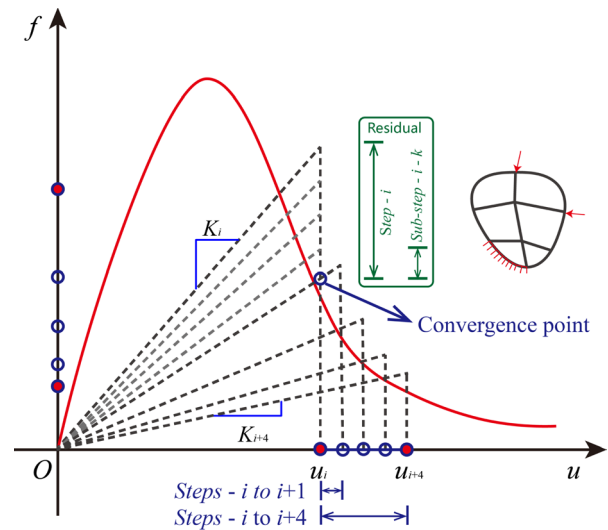


Fig. 8 The full-scale iteration scheme of CRFPA **a** Iteration path and residual minimization, **b** Stepwise iteration flowchart

damaged elements in a step is detected to be greater than a certain proportion (r_f) of the total elements, the sub-steps of the CRFPA iteration are activated. Each sub-step selects the elements with the most severe damage degree among the newly entered damaged elements for stiffness reduction processing. When the number of newly added damaged elements detected in current sub-step is less than r_f , convergence is considered to have been achieved and the next step is entered. An independent analysis step follows the end of each substep to adjust the opening and closing state of cohesive elements, and no stiffness reduction is performed within this step. The residual force of each time step gradually decreases as the number of predefined sub-steps increases, allowing the system to quickly converge to a relatively accurate solution. In addition to increasing the number of sub-steps, reducing the size of each analysis step can also decrease the residual force, which helps with convergence. The curve in the figure represents the relationship between traction and displacement, with the system progressively updating from the initial state (O) to the new equilibrium point (U_x, f) as the iteration progresses. Each update reflects the real-time changes in the mechanical properties of both the solid and Cohesive elements, actively searching for and reducing the stiffness of the elements. Under this full-scale iteration, shown in Fig. 8(b), the constitutive model of the cohesive elements combines perfectly with the advantages of RFPA, enabling CRFPA to better handle complex crack propagation and interface behavior, ensuring the stability and convergence of the algorithm while improving computational accuracy and efficiency.

It is crucial to calculate the elastic matrix ($\mathbf{D}$) of each integer point in cohesive element. Supplying an appropriate elastic matrix is highly demand for forming the global stiffness matrix. This can be expressed in matrix format as shown in Eq. (36).

$$\mathbf{D} = \begin{bmatrix} D_{nn} & D_{nt} \\ D_{tn} & D_{tt} \end{bmatrix} \text{ in 2D problem, } \mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \text{ in 3D problem} \quad (36)$$

$$\begin{cases} D_{nn} = \frac{T_n(\delta_n, \delta_t)}{\delta_n} = \frac{\Gamma_n}{\Delta_n} \left[m \left(1 - \frac{\delta_n}{\Delta_n} \right)^\alpha \left(\frac{m}{\alpha} + \frac{\delta_n}{\Delta_n} \right)^{m-1} - \alpha \left(1 - \frac{\delta_n}{\Delta_n} \right)^{\alpha-1} \left(\frac{m}{\alpha} + \frac{\delta_n}{\Delta_n} \right)^m \right] \\ \quad \times \left[\Gamma_t \left(1 - \frac{|\delta_t|}{\Delta_t} \right)^\beta \left(\frac{n}{\beta} + \frac{|\delta_t|}{\Delta_t} \right)^n + \langle \phi_t - \phi_n \rangle \right] \frac{1}{\delta_n} \\ D_{tt} = \frac{T_t(\delta_n, \delta_t)}{\delta_t} = \frac{\Gamma_t}{\Delta_t} \left[n \left(1 - \frac{|\delta_t|}{\Delta_t} \right)^\beta \left(\frac{n}{\beta} + \frac{|\delta_t|}{\Delta_t} \right)^{n-1} - \beta \left(1 - \frac{|\delta_t|}{\Delta_t} \right)^{\beta-1} \left(\frac{n}{\beta} + \frac{|\delta_t|}{\Delta_t} \right)^n \right] \\ \quad \times \left[\Gamma_n \left(1 - \frac{\delta_n}{\Delta_n} \right)^\alpha \left(\frac{m}{\alpha} + \frac{\delta_n}{\Delta_n} \right)^m + \langle \phi_n - \phi_t \rangle \right] \frac{1}{|\delta_t|} \\ D_{nt} = D_{tn} = 0 \end{cases} \quad (37)$$

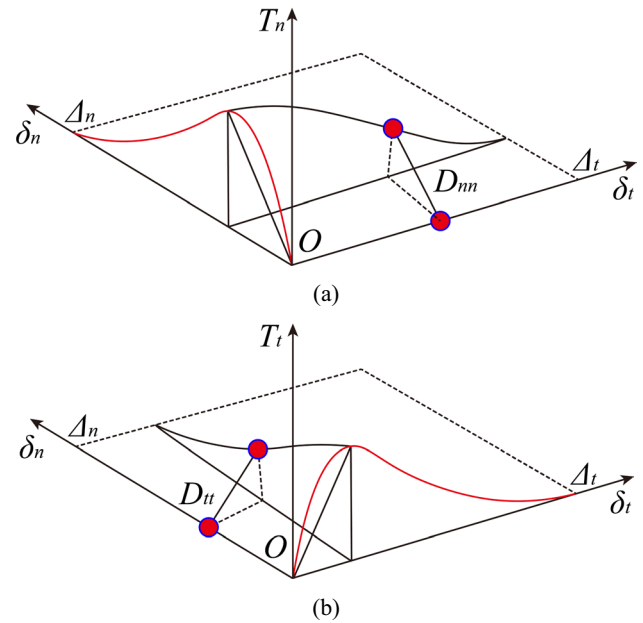


Fig. 9 Schematic of interface stiffness calculation for the PPR cohesive model a D_{nn} , b D_{tt}

The iteration process of the brittle cohesive constitutive model in CRFPA is consistent with that of the solid elements. When the interface is undamaged, the initial interface stiffness is used; when the interface enters the damaged state, the interface stiffness is weakened. We can get the interface stiffness by making the interface traction equal to the residual traction. The PPR cohesive constitutive model for quasi-brittle materials requires further explanation. Since the stiffness of the PPR cohesive model is depend on δ_n and δ_t , and T_n and T_t are also influenced by the coupling of δ_n and δ_t , as shown in Fig. 9, the interface stiffness needs to be updated for every increment during the solution process.

The interface separation displacement obtained from the solution can be used to calculate the current interface traction using Eq. (37).

The improved PPR cohesive model takes into account the effect of friction (Yu et al. 2024). When the interface is under a compressive-shear coupled stress state, the friction term introduces an additional effect (D_{ft}) on D_{tt} . Based on Eqs. (26) and (27), the expression for D_{tt} is shown in Eq. (38):

$$D_{tt} = \frac{T_t}{\delta_t} + \frac{T_f}{\delta_t} = \frac{T_t}{\delta_t} + D_{ft}$$

$$= \begin{cases} \frac{T_t}{\delta_t} + \mu |T_n| \left(\frac{T_t(0, \delta_t)}{\tau_{\max}} \right)^n / \delta_t, & |\delta_t| \in [0, \lambda_t \Delta_t) \\ \frac{T_t}{\delta_t} + \frac{\mu |T_n|}{\delta_t}, & |\delta_t| \in [\lambda_t \Delta_t, +\infty) \end{cases} \quad (38)$$

It should be noted that, Eqs. (37) to (38) represent matrix elements in the context of two-dimensional finite element modeling. Referring to the approach by Park et al. (Park and Paulino 2012; Yu et al. 2024), the 2-D elastic matrix can be extended to a 3-D format, as shown in Eq. (39).

$$\begin{cases} D_{11} = D_{nn}; D_{12} = D_{nt} \frac{\delta_2}{\delta_t}; D_{13} = D_{nt} \frac{\delta_3}{\delta_t} \\ D_{21} = D_{nt} \frac{\delta_2}{\delta_t}; D_{31} = D_{nt} \frac{\delta_3}{\delta_t} \\ D_{22} = \left(D_{tt} \delta_2 + D_{ft} |\delta_2| \frac{\delta_2}{|\delta_2|} \right) \frac{\delta_2}{\delta_t^2} + \left(T_t + T_f \frac{\delta_2}{|\delta_2|} \frac{\delta_2}{|\delta_2|} \right) \frac{\delta_2^2}{\delta_t^3} \\ D_{23} = \left(D_{tt} \delta_2 + D_{ft} |\delta_2| \frac{\delta_2}{|\delta_2|} \right) \frac{\delta_3}{\delta_t^2} - \left(T_t \delta_2 + T_f |\delta_2| \frac{\delta_2}{|\delta_2|} \right) \frac{\delta_3}{\delta_t^3} \\ D_{32} = \left(D_{tt} \delta_3 + D_{ft} |\delta_3| \frac{\delta_3}{|\delta_3|} \right) \frac{\delta_2}{\delta_t^2} - \left(\hat{T}_t \delta_3 + T_f |\delta_3| \frac{\delta_3}{|\delta_3|} \right) \frac{\delta_2}{\delta_t^3} \\ D_{33} = \left(D_{tt} \delta_3 + D_{ft} |\delta_3| \frac{\delta_3}{|\delta_3|} \right) \frac{\delta_3}{\delta_t^2} + \left(\hat{T}_t + T_f \frac{\delta_3}{|\delta_3|} \frac{\delta_3}{|\delta_3|} \right) \frac{\delta_3^2}{\delta_t^3} \end{cases} \quad (39)$$

2.5 Variational Formulation and Thermodynamic Consistency

The CRFPA method is rooted in continuum damage mechanics and thermodynamic principles. It couples a damage model defined on solid elements with a cohesive interface representation formulated within an energy-minimization framework.

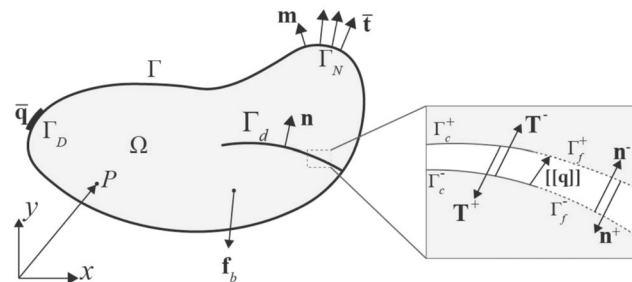


Fig. 10 Schematic representation of the equilibrium and boundary conditions of a cracked continuum body

To clarify the mechanical equilibrium conditions of CRFPA under the combined action of continuum damage and cohesive interfaces, this section first presents a global force-balance schematic of a cracked continuum, as shown in Fig. 10. The solid domain Ω may contain pre-existing or evolving crack interfaces Γ_d . Fully traction-free crack interfaces are denoted by Γ_f , while crack interfaces that still sustain tractions are denoted by Γ_c , such that $\Gamma_f \cup \Gamma_c = \Gamma_d$. The interface tractions on the two crack faces must satisfy local equilibrium. The external boundary of the domain consists of Γ_D , where displacement constraints are prescribed, and Γ_N , where external surface tractions are applied. The figure also illustrates the body force $\mathbf{f}_b$, surface tractions, the interface displacement jump $[[\mathbf{q}]]$, and the tractions $\mathbf{T}^\pm$ acting on the positive and negative crack faces, which together provide the fundamental setting for the subsequent derivation of the variational equilibrium equations and the thermodynamic consistency conditions.

Based on the force and boundary-condition layout of the cracked continuum shown in Fig. 10, the governing mechanical equilibrium equations of the system can be written directly. The body forces applied in the interior of the domain, the surface tractions and displacement constraints prescribed on the external boundary, and the cohesive-traction conditions imposed on the cohesive interface segments are collectively expressed in Eq. (40).

$$\begin{cases} \text{div } \boldsymbol{\sigma} + \mathbf{f}_b = 0 & \forall \mathbf{P}(\mathbf{X}) \in \Omega \\ \boldsymbol{\sigma} \mathbf{n} = \bar{\mathbf{t}} & \forall \mathbf{P}(\mathbf{X}) \in \Gamma_N \\ \boldsymbol{\sigma} \mathbf{n} = \mathbf{T}(\boldsymbol{\delta}) & \forall \mathbf{P}(\mathbf{X}) \in \Gamma_c \\ \boldsymbol{\sigma} \mathbf{n} = 0 & \forall \mathbf{P}(\mathbf{X}) \in \Gamma_f \end{cases}, \quad \begin{cases} \boldsymbol{\epsilon} = \nabla^{sym} \mathbf{q} & \forall \mathbf{P}(\mathbf{X}) \in \Omega \\ \mathbf{q} = \bar{\mathbf{q}} & \forall \mathbf{P}(\mathbf{X}) \in \Gamma_D \\ \boldsymbol{\delta} = [[\mathbf{q}]] = \mathbf{q}^+ - \mathbf{q}^- & \forall \mathbf{P}(\mathbf{X}) \in \Gamma_d \end{cases} \quad (40)$$

According to Eq. (40), for a solid domain Ω containing potential crack surfaces Γ_c , the total potential energy functional Π of the CRFPA system can be written as Eq. (41) (Miehe et al. 2010; Pham et al. 2010; Xu et al. 2024):

$$\begin{aligned} \Pi[\mathbf{q}, d] &= \int_{\Omega} \psi_b(\boldsymbol{\epsilon}(\mathbf{q}), d) dV + \int_{\Gamma_c} \psi_c(\boldsymbol{\delta}) dA \\ &\quad - \int_{\Omega} \mathbf{f}_b \cdot \mathbf{q} dV - \int_{\Gamma_N} \bar{\mathbf{t}} \cdot \mathbf{q} dA \end{aligned} \quad (41)$$

where $\mathbf{q}$ denotes the displacement vector, $\boldsymbol{\epsilon}(\mathbf{q})$ is the small-strain tensor, and $d \in [0, 1]$ is the scalar damage variable of the bulk material. The function $\psi_b(\boldsymbol{\epsilon}, d)$ represents the elastic strain-energy density of the degraded solid elements, and can be written as $(1-d)\psi_b(\boldsymbol{\epsilon}, 0)$, where $\psi_b(\boldsymbol{\epsilon}, 0)$ is the strain-energy density of the undamaged material. The term $\psi_c(\boldsymbol{\delta})$ denotes the cohesive energy potential per unit area, where $\boldsymbol{\delta}$ is the displacement jump across Γ_c . Its derivative $\mathbf{T} = \partial \psi_c / \partial \boldsymbol{\delta}$ defines the cohesive traction. By applying the principle of minimum potential energy and setting the first variation of

Π to zero, the weak form of equilibrium is obtained, as given in Eq. (42).

$$\begin{aligned}\delta\Pi = 0 &\Rightarrow \int_{\Omega} \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}} dV + \int_{\Gamma_c} \mathbf{T} \cdot \dot{\boldsymbol{\delta}} dA \\ &= \int_{\partial\Omega} \mathbf{f}_b \cdot \dot{\mathbf{q}} dV + \int_{\Gamma_N} \bar{\mathbf{t}} \cdot \dot{\mathbf{q}} dA\end{aligned}\quad (42)$$

where $\dot{\boldsymbol{\epsilon}}$ and $\dot{\mathbf{q}}$ denote the variations of the strain and displacement fields, respectively. To ensure that the CRFPA model satisfies the second law of thermodynamics, all internal damage and fracture processes must dissipate energy. The increment of the total energy dissipation is given in Eq. (43).

$$\dot{D} = \int_{\Omega} Y \dot{d} dV + \int_{\Gamma_c} \mathbf{T} \cdot \dot{\boldsymbol{\delta}} dA \geq 0 \quad (43)$$

where $Y = -\partial\psi_c/\partial d$ is the energy release rate that drives bulk damage, and $\dot{d} > 0$ ensures the irreversibility of damage. Figure 8a illustrates the iterative path on the load–displacement curve. At each global analysis step i , the solution is updated progressively through multiple sub-step k . The equilibrium system is given by Eq. (44) (Li et al. 2017a; Li and Tang 2015).

$$\mathbf{K}(\mathbf{q}^{i,k}, d^{i,k}) \mathbf{q}^{i,k} = \mathbf{f}^{i,k} \quad (44)$$

where $\mathbf{f}$ is the external force vector and $\mathbf{K}$ is the global stiffness matrix. The convergence criterion is not defined directly in terms of force or displacement residuals; instead, it is constrained by the number of elements that enter the damaged state, as specified in Eq. (45).

$$\begin{cases} N_d^{i,k} \geq r_f N_{total} \rightarrow \text{Step} - i = i, \text{Substep} - k + + \\ N_d^{i,k} < r_f N_{total} \rightarrow \text{Step} - i + +, \text{Substep} - k = 0 \end{cases} \quad (45)$$

If the number of newly damaged elements in sub-step k exceeds the threshold $r_f \times N_{total}$, the iteration continues. Conversely, if the number of newly damaged elements is less than $r_f \times N_{total}$, the system is assumed to have converged at sub-step k and proceeds to the next global step $i + 1$. This damage-based convergence rule is consistent with energy convergence, because each newly damaged element implies energy dissipation, as expressed in Eq. (46).

$$\dot{D} = \sum_{\Omega} [\psi_b(\boldsymbol{\epsilon}, 0) - (1 - d)\psi_b(\boldsymbol{\epsilon}, 0)] + \sum_{\Gamma_c} \psi_c(\boldsymbol{\delta}) \geq 0 \quad (46)$$

In Fig. 8a, the reduction in residual force reflects a decreasing energy release rate at each substep; thus, convergence in terms of damage saturation corresponds to stabilization of the energy dissipation rate, which is required for energy balance and thermodynamic consistency. At each substep, stiffness is irreversibly degraded for a prescribed

fraction of the most severely newly failed elements. In CRFPA, this irreversibility is enforced for both solid and cohesive by storing history variables that record the maximum strain for solid elements and the maximum opening or sliding for cohesive elements, and by updating the stiffness-reduction factors accordingly; the specific degradation laws are introduced later together with the constitutive models. By tracking the most unfavorable history, CRFPA prevents damage reversal even when subsequent load levels decrease. The scalar damage variable in solid elements, $d \in [0, 1]$, evolves according to a history-dependent criterion, as given in Eq. (47).

$$d = \mathcal{F}(\gamma), \gamma(t) = \max_{\zeta \leq t} \varepsilon_p(\zeta) \quad (47)$$

where γ denotes the maximum principal strain ever attained at the integration point. Since γ is non-decreasing with time, the damage variable d can only increase or remain constant. This mechanism ensures that unloading or reloading does not reverse damage evolution, which is a key thermodynamic requirement. For cohesive elements, the traction–separation response is governed by a softening law derived from the potential function $\psi_c(\boldsymbol{\delta})$. When the external load decreases, the separation component δ in a given direction may reduce; however, the corresponding traction T follows the residual slope along the unloading path and does not recover the interface strength, as expressed in Eq. (48).

$$\dot{\delta}(t) < 0 \Rightarrow T(t) \leq T(\delta_{max}) \quad (48)$$

When the reduced load is increased again, the system continues to evolve along the currently degraded traction–displacement path rather than returning to the undamaged elastic path. This reflects the permanence of cohesive degradation, until the separation in that direction reaches its historical maximum value, δ_{max} . The CRFPA variational formulation based on minimizing the total potential energy functional $\Pi[\mathbf{q}, d]$ provides a natural foundation for enforcing mechanical equilibrium and thermodynamic consistency. This minimization problem is subject to irreversibility constraints on the internal damage variables, which intrinsically enforces positive energy dissipation throughout the loading process. Considering the incremental quasi-static solution procedure, at each loading step $t^i \rightarrow t^{i+1}$, CRFPA seeks the state $(\mathbf{q}^{i+1}, d^{i+1})$ that satisfies Eq. (49).

$$(\mathbf{q}^{i+1}, d^{i+1}) = \arg \min_{\substack{\mathbf{q} \in \mathcal{V} \\ d \in [d^i, 1]}} \Pi[\mathbf{q}, d] \quad (49)$$

Accordingly, Eq. (50) follows.

$$d^{i+1}(\mathbf{x}) \geq d^i(\mathbf{x}), \forall \mathbf{x} \in \Omega \cup \Gamma_c \quad (50)$$

Where ν denotes the kinematically admissible space of displacement fields. This constrained minimization framework guarantees the following: the equilibrium condition is enforced through the stationarity of the energy functional Π with respect to the displacement field $\mathbf{q}$; meanwhile, the update of irreversible damage leads to a reduction of stored energy and thus naturally produces energy dissipation. More specifically, any positive damage increment $\dot{d} = d^{i+1} - d^i \geq 0$ causes stiffness degradation and releases part of the stored elastic energy. Since this released energy cannot be recovered because the damage rate satisfies $\dot{d} \geq 0$, it can be interpreted as internal dissipation, which is consistent with the Clausius–Duhem inequality, as expressed in Eq. (51) (Truesdell and Noll 2004).

$$\dot{D} = \Pi[\mathbf{q}^i, d^i] - \Pi[\mathbf{q}^{i+1}, d^{i+1}] \geq 0 \quad (51)$$

Equations (41)–(51) formalizes the link between the minimization of potential energy and the thermodynamic requirement that damage evolution must not reduce the total energy of the system. The damage field evolves only when it lowers Π , and only in directions allowed by the irreversibility constraint. Thus, the variational framework of CRFPA not only defines equilibrium but also rigorously governs how damage evolves to dissipate energy, ensuring that all simulated fracture processes remain thermodynamically consistent, mechanically balanced, and energetically admissible.

3 Validation

To verify the usability of the CRFPA method proposed in this study, we need to test the interface under pure tension, pure shear, tensile-shear coupling, and compressive-shear coupling stress states.

3.1 Mode I and II Fracture

To highlight the impact of the material properties of the cohesive elements on the fracture process, we choose to set the solid elements as homogeneous and assign them a relatively large elastic modulus, ensuring that the deformation of the specimen is primarily concentrated in the cohesive

elements. To verify that both the brittle and quasi-brittle cohesive constitutive models are applicable to CRFPA, we assign the brittle constitutive model and the PPR cohesive constitutive model to the cohesive elements during the simulation process. This also facilitates a comparison between the brittle and quasi-brittle models. To control for variables, we set the strength of the brittle cohesive constitutive model equal to that of the PPR Cohesive constitutive model and make the damage initiation displacement of the PPR cohesive constitutive model equal to the failure displacement of the brittle cohesive constitutive model. The specific material parameters are shown in Table 1.

In addition, proper fitting of input parameters for the PPR model is essential. For Mode II fracture under quasi-static conditions, key parameters such as the tangential strength (τ_{max}), the tangential displacement at peak strength (λ_t), and the ultimate failure displacement (Δ_t) can be extracted from experimental data. The stiffness-related parameter λ_t is computed using the ratio $\lambda_t \Delta_t / \Delta_t$. The tangential fracture energy is obtained by integrating the area under the traction vs. displacement curve. For the quasi-brittle cohesive model, this energy can be estimated as the area of a triangle. To ensure higher numerical fidelity, the fracture energy of the PPR model is set to 0.015 MPa•mm, which is three times that of the brittle model. The shape parameter β , however, lacks a direct analytical expression due to the complexity of Eq. (21). Therefore, numerical root-finding techniques, such as MATLAB's "fzero" function, are employed to determine its value.

As illustrated in Fig. 11a, the direct tensile and shear test consisted of two blocks. These two blocks are bonded together, with the bottom surface of block fixed. In Fig. 11c, a vertically upward displacement load is applied to the upper surface of the block. In Fig. 11d, a horizontal displacement load is applied to the upper surface of the upper block, while the vertical displacement on this surface is constrained. A fixed displacement constraint is applied to the side surface of the lower block. To focus on the constitutive model validation at the element level, the number of elements should be minimized to ensure that the obtained data closely represents the behavior of a single element. The model contains a total of 24 solid elements and 2 cohesive elements, as shown in Fig. 11b.

Table 1 Material parameters of the verification model

Solid element	$E = 200,000.0 \text{ MPa}$; $\nu = 0.2$; $\sigma_c = 1000 \text{ MPa}$; $\lambda = 1.0$
Brittle cohesive element	$k_n = 100.0 \text{ MPa/mm}$; $k_t = k_s = 100.0 \text{ MPa/mm}$; $\sigma_{max} = 1.0 \text{ MPa}$; $\tau_{max} = 1.0 \text{ MPa}$; $r_n = 0.1$; $r_t = 0.2$; $\Delta_n = 0.01 \text{ mm}$; $\Delta_t = 0.01 \text{ mm}$
Quasi-brittle cohesive element	$\phi_n = 0.015 \text{ MPa}\cdot\text{mm}$; $\phi_t = 0.015 \text{ MPa}\cdot\text{mm}$; $\sigma_{max} = 1.0 \text{ MPa}$; $\tau_{max} = 1.0 \text{ MPa}$; $\lambda_n = 0.333$; $\lambda_t = 0.333$; $\alpha = 4.318$; $\beta = 4.318$; $\eta = 3.0$; $\mu = 0.25$

Fig. 11 Setup of the model of single fracture mode **a** Size, **b** Element model, **c** Mode I, **d** Mode II

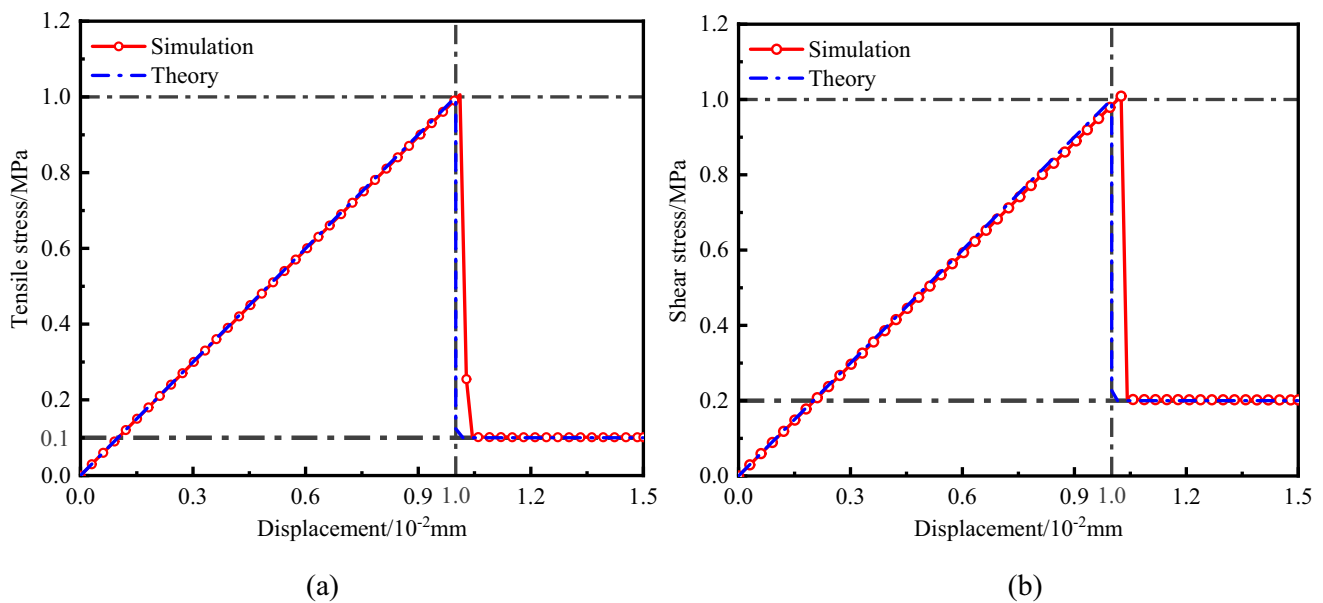
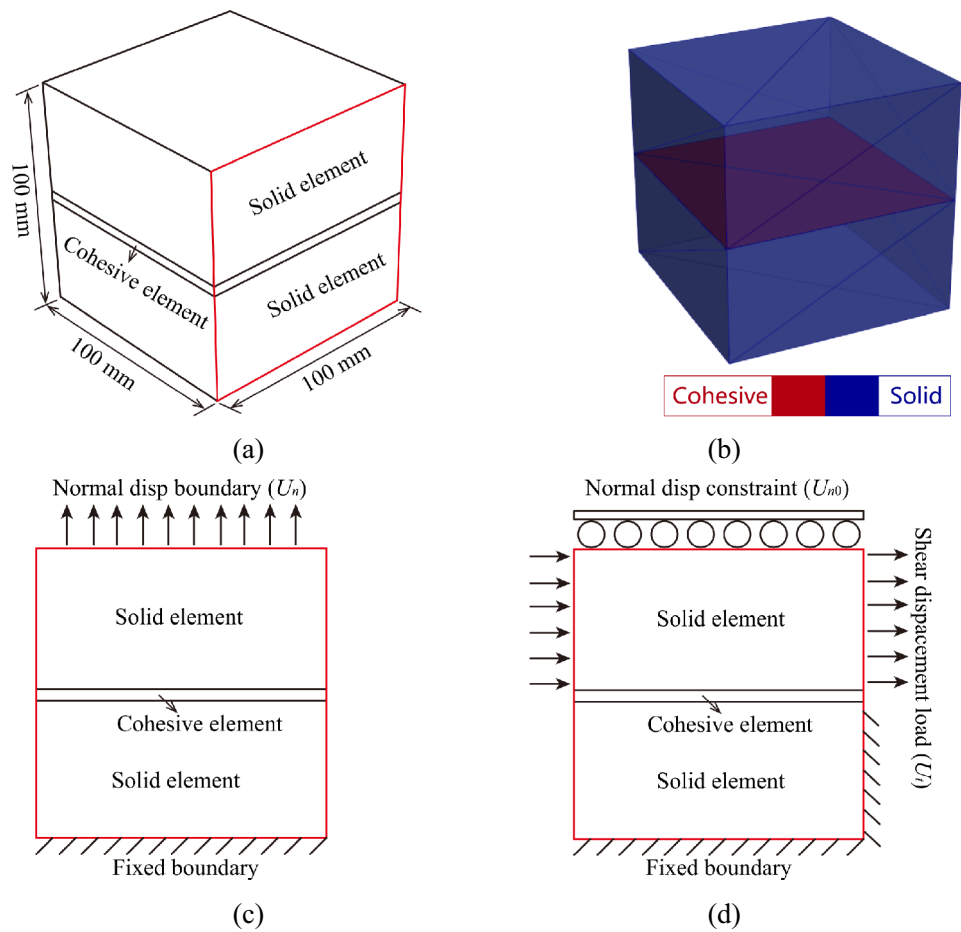


Fig. 12 Stress vs. crack separation displacement of brittle cohesive model **a** Mode I, **b** Mode II

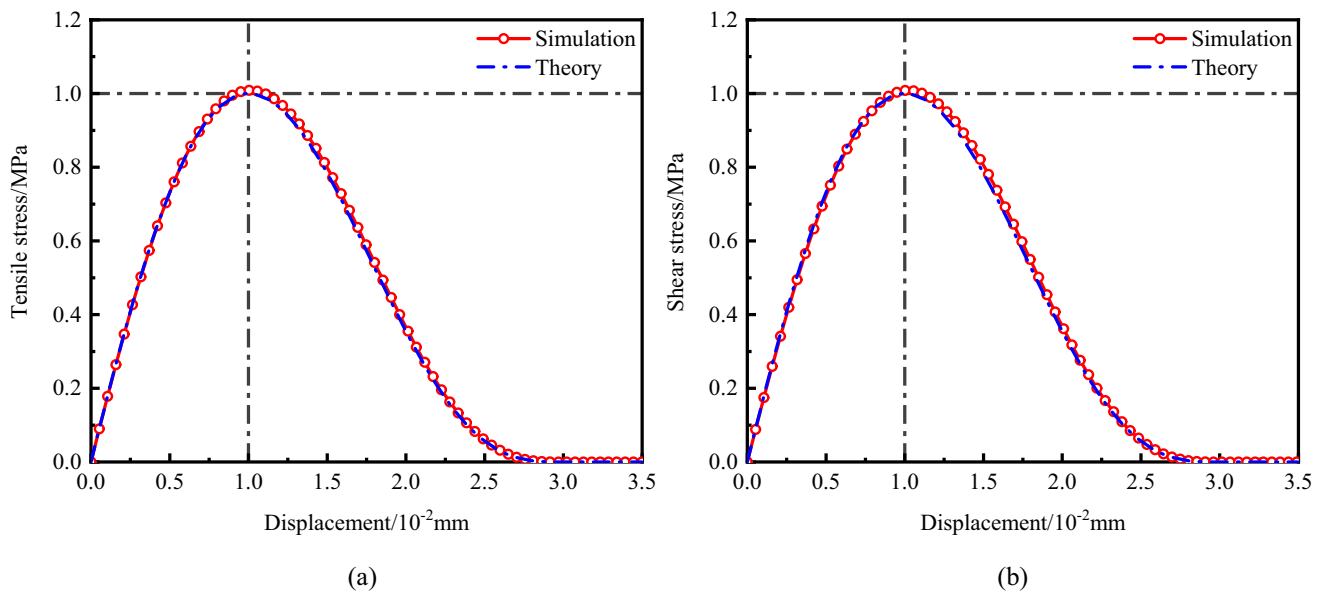


Fig. 13 Stress vs. crack separation displacement of PPR cohesive model **a** Mode I, **b** Mode II

Set the total displacement to 0.015 mm, with 100 analysis steps, each step being 0.00015 mm. After the calculations, the results of the brittle cohesive constitutive model for uniaxial tension and shear are shown in Fig. 12.

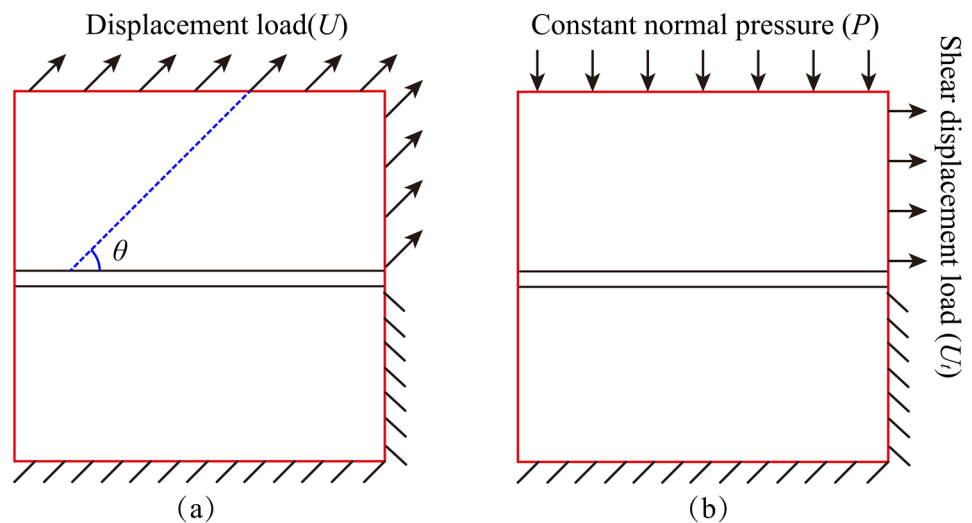
As shown in Fig. 12, the theoretical curve is almost identical to the curve obtained from the CRPFA calculation. The CRPFA simulation accurately predicts the strength and residual strength, especially with the normal residual strength (r_n) coefficient of 0.1 and the tangential coefficient (r_t) of 0.2, the simulation results clearly reflect the impact of the different residual strength coefficient. The stiffness of the simulation curve is slightly lower than that of the theoretical curve, which is due to the fact that the solid elements also

contribute to some deformation. Therefore, when the load reaches 0.01 mm, the interface separation displacement has not yet reached 0.01 mm, resulting in a slight delay in the final fracture displacement, which in turn causes the simulated stiffness to be slightly lower than the theoretical value.

The results of the PPR cohesive constitutive model for uniaxial tension and shear are shown in Fig. 13.

As shown in Fig. 13, the theoretical curve of PPR cohesive model is almost identical to the curve obtained using the CRPFA simulation. The CRPFA simulation accurately predicts the strength and displacements at peak traction and failure, the simulation results clearly predict mechanical behavior of quasi-brittle interface. Similar to

Fig. 14 Setup of the CRPFA model of mixed-mode fracture **a** Tensile-shear, **b** Compressive-shear



the interface of brittle materials, the stiffness of the simulation curve is slightly lower than that of the theoretical curve, and the slight lag in the descending stage, which is due to the solid elements also contribute to some deformation. The comparison between the simulation results of the brittle and quasi-brittle (PPR) interface constitutive models and the theoretical results proves that the CRPFA correctly predicts the Mode I and Mode II fracture behavior of the interface.

3.2 Mixed Fracture Mode

The loading conditions for the simulations are designed to investigate the accuracy of the CRPFA in predicting mixed-mode fracture behavior at interfaces, as shown in Fig. 14.

As shown in Fig. 14a, the loading involves a coupling of tensile and shear forces, with the loading angle α being the primary variable. The loading angles considered in this case are 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, and 90°. These angles allow for the evaluation of how varying shear and normal stress components influence the fracture behavior at the interface under different loading configurations. In Fig. 14b, the loading consists of a coupling between compressive and shear, where the normal pressure P is the key variable. The values for P in this simulation range from 0 to 2.0 MPa, with increments of 0.2 MPa. The purpose of these tests is to capture the effects of different loading on the mixed-mode fracture, where the interface is subjected to both normal and shear stress simultaneously.

The peak normal and tangential traction results under mixed-mode fracture, calculated using the brittle cohesive model in CRPFA, are shown in Fig. 15. The theoretical critical traction boundaries are also included for comparison with the CRPFA simulation results.

As shown in Fig. 15, we divided the strength envelope curve into two regions: the compressive-shear coupling region and the tensile-shear coupling region. The CRPFA simulation results in both regions are in high agreement with the theoretical results, indicating that the use of the brittle interface constitutive model in CRPFA for predicting mixed-mode fracture strength at the interface is accurate. The traction vs. crack displacement relationship in the PPR cohesive model is nonlinear, and the traction is controlled by both normal and tangential crack displacements. Therefore, to verify the accuracy of the simulation, it is necessary to extract all the features of the traction vs. crack opening displacement curve from the simulation. For comparison with the theoretical results, in the quasi-brittle constitutive model tests, we first apply normal displacement loading and then control the tangential displacement loading to gradually increase. In the tensile-shear coupling test case, normal displacement loading values are set to 0 mm, 0.005 mm, 0.01 mm, 0.015 mm, 0.02 mm, and 0.025 mm. In the compressive-shear coupling test case, the applied normal pressure is consistent with the brittle case, ranging from 0 to 1.5 MPa with an increment of 0.25 MPa.

The traction-displacement curve morphology of the quasi-brittle interface constitutive model demonstrates heightened complexity, and merely extracting the peak

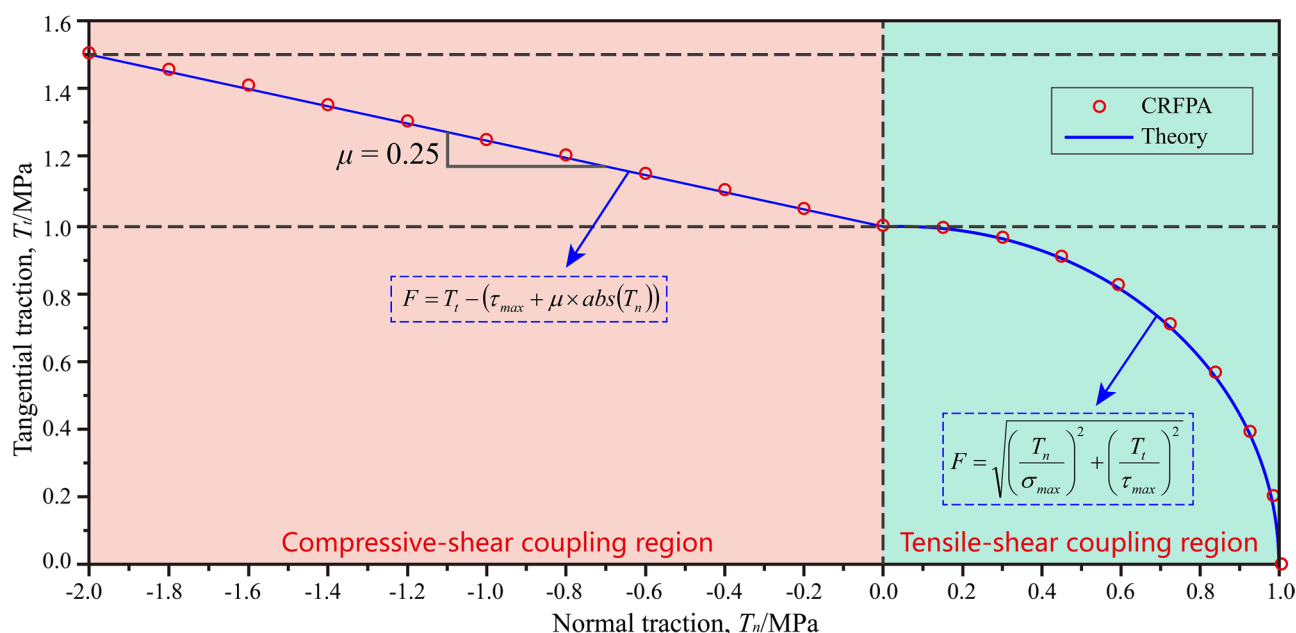


Fig. 15 Peak normal and tangential traction in mixed mode fracture of brittle cohesive model

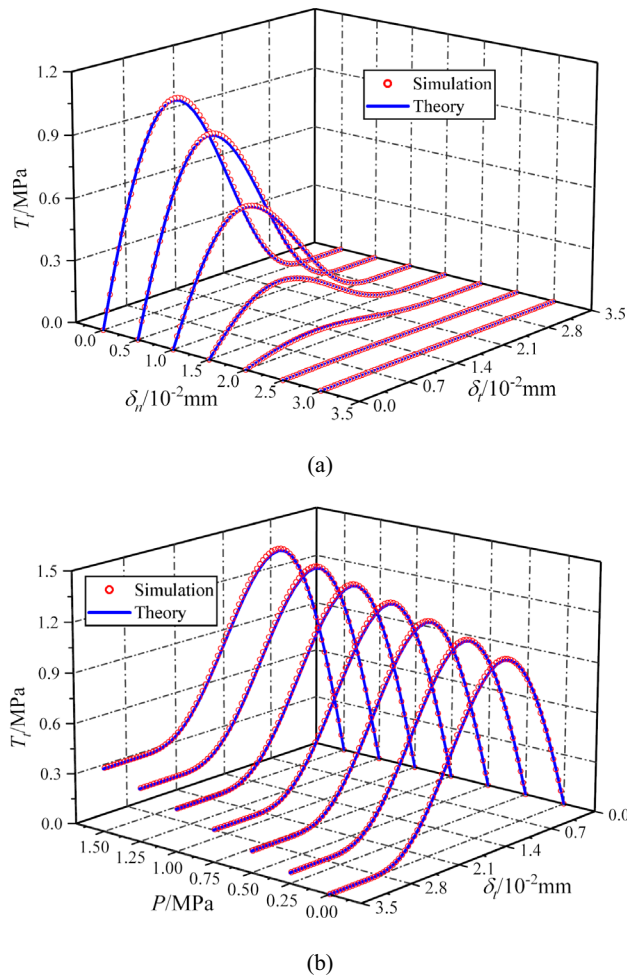
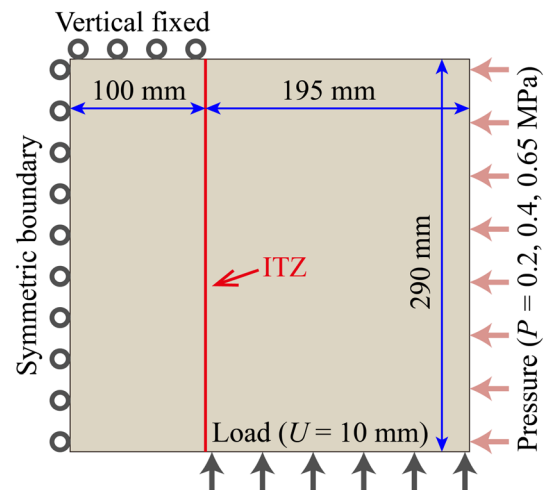


Fig. 16 Traction vs. crack displacement in mixed mode fracture of PPR cohesive model **a** Tensile-shear, **b** Compressive-shear

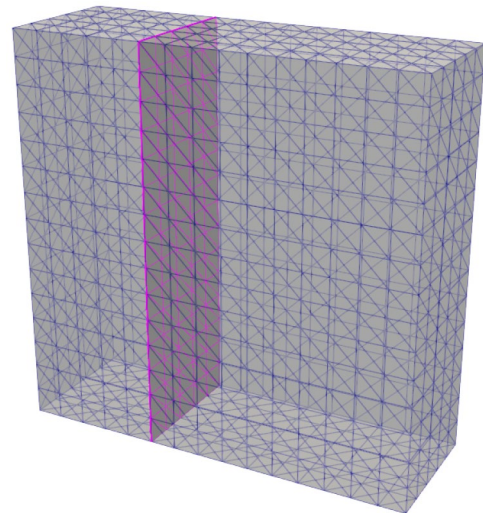
traction cannot fully validate the accuracy of CRFPA simulations. Therefore, the complete traction-displacement curves during loading were analyzed, as shown in Fig. 16, where Fig. 16a represents the tension-shear coupling mode simulation results, while Fig. 16b corresponds to the compression-shear coupling mode. For both tension-shear and compression-shear mixed-mode fracture scenarios, CRFPA achieves results exhibiting strong agreement with theoretical curves. In tension-shear coupling, the tangential traction curve degrades entirely as the prescribed normal displacement increases. Similarly, in compression-shear coupling, the tangential traction curve strengthens with rising normal pressure, accompanied by increases in both peak and residual traction. These observations conclusively assert the accuracy of CRFPA in simulating mixed-mode fracture behavior at quasi-brittle interfaces.



(a)



(b)



(c)

Fig. 17 Setup of the model of masonry wallette shear test **a** Test setup with instrumentation of the Wallette, **b** Size and loading, **c** CRFPA model

4 Application

4.1 Shear Test of The Masonry Wallette

Masonry wallettes are commonly employed in civil engineering for structural assessment and experimental validation. In this study, the proposed CRFPA approach is evaluated against the experimental results reported by Beyer et al. (2012, 2010), which are widely referenced for verifying interface constitutive laws and numerical methods. As depicted in Fig. 17a, the test specimen comprises three brick units connected by mortar, where the central brick is supported on its top surface by two rigid bricks. Vertical displacement loads are applied to the lower surfaces of the outer bricks to generate a shear response, while lateral compressive loads are imposed on the vertical sides to activate frictional resistance. Due to the full symmetry of the boundary conditions, the numerical model is reduced to half of

the specimen with symmetric constraints applied along the central plane, as illustrated in Fig. 17b. The CRFPA representation incorporating cohesive elements is presented in Fig. 17c. The solid blocks are modeled with an elastic modulus of 14,000 MPa and a Poisson's ratio of 0.15. The interface is governed by a quasi-brittle PPR cohesive-friction model, with parameter values drawn from the study by Yu et al. (2024). ($\phi_n = 1 \text{ MPa}\cdot\text{mm}$, $\phi_t = 1 \text{ MPa}\cdot\text{mm}$, $\sigma_{max} = 1.5 \text{ MPa}$, $\tau_{max} = 1.5 \text{ MPa}$, $\lambda_n = 0.06$, $\lambda_t = 0.06$, $\alpha = 5$, $\beta = 5$, $\eta = 1$, $\mu = 0.77$).

Based on the boundary conditions and model parameters described in Fig. 17, the shear stress versus displacement curves presented in Fig. 18 are analyzed to validate the accuracy of the CRFPA model when applying the quasi-brittle interface constitutive model. The figure compares the experimental results from Beyer et al. (2012, 2010) with various numerical simulation results, including previous studies (Yu et al. 2024; Snozzi and Molinari 2013; Baek and Park 2018)

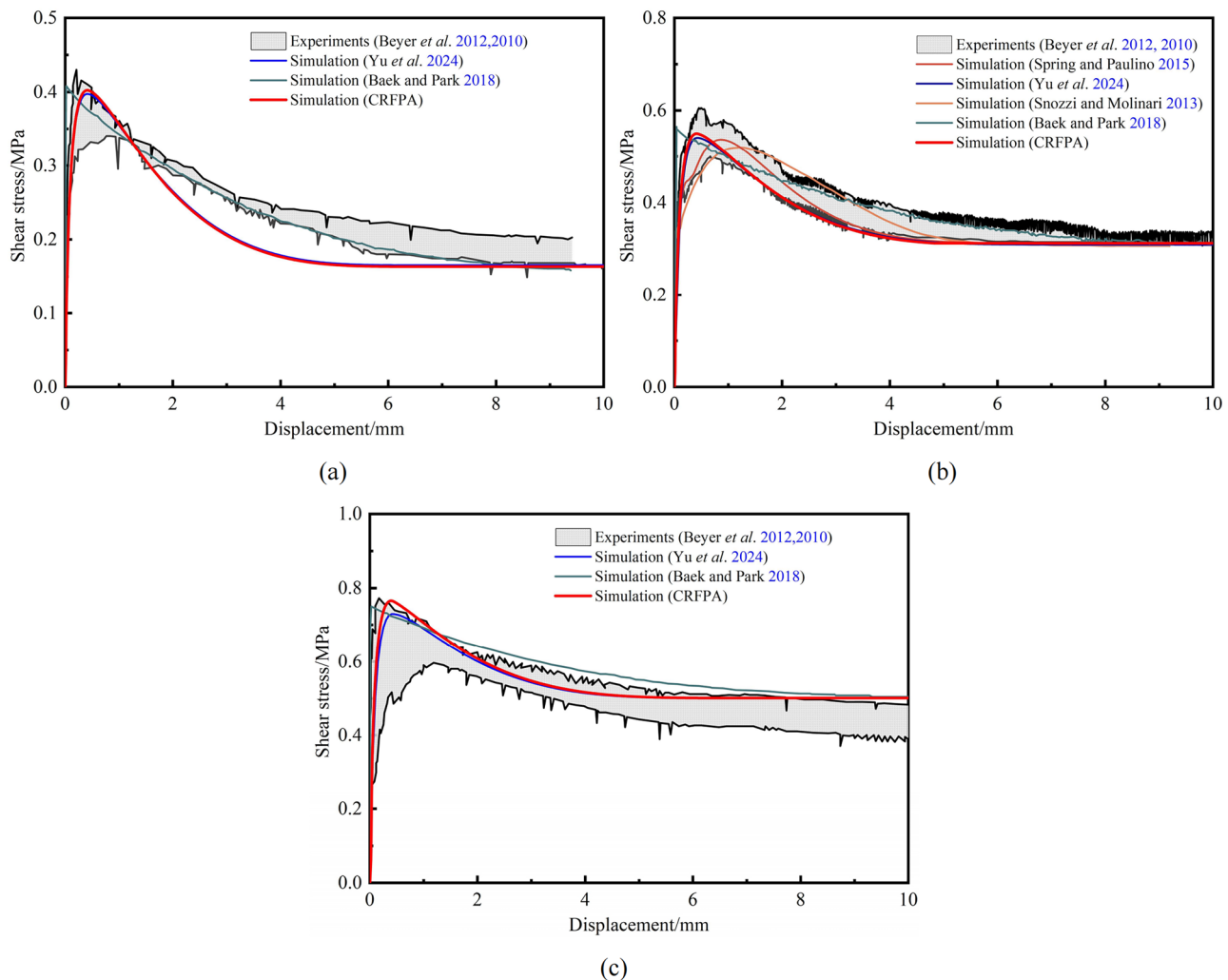


Fig. 18 Shear stress and load displacement curves of the masonry wallette **a** $P = 0.2 \text{ MPa}$, **b** $P = 0.4 \text{ MPa}$, **c** $P = 0.65 \text{ MPa}$

and the proposed CRFPA model. Three different applied pressures ($P=0.2, 0.4$, and 0.65 MPa) are considered to evaluate the effect of normal stress on shear strength and deformation behavior.

The comparison in Fig. 18 demonstrates that the CRFPA model effectively captures the peak shear strength and post-peak softening behavior observed in the experiments. The peak shear stress values predicted by the CRFPA model closely match the experimental results, indicating the CRFPA's ability to accurately predict failure initiation. Like the previous numerical models, the CRFPA model provides a balanced estimation that falls within the experimental range. Additionally, the model successfully represents the progressive interface degradation after peak stress, which is essential for accurately capturing the frictional slip and cohesive weakening occurring in masonry joints.

Another significant advantage of the CRFPA model is its improved consistency across different pressure levels. As the applied normal pressure increases, both the experimental and CRFPA curves show an increase in peak shear stress, demonstrating the influence of friction on shear resistance. The cohesive-frictional constitutive formulation in CRFPA allows it to reliably simulate this effect, resulting in more accurate predictions compared to traditional quasi-brittle cohesive models. Yu et al. (2024) also employed the PPR cohesive constitutive model with friction effects; however, their solution framework was based on an incremental finite element method. In comparison, the CRFPA simulation not only ensures the accuracy of the results, but also benefits from the full-step iteration scheme of RFPA, which effectively eliminates convergence issues. Overall, the results in Fig. 18 confirm that the CRFPA model, incorporating a quasi-brittle cohesive-friction interface, achieves high accuracy in predicting shear stress-displacement relationships in masonry joints. By accurately capturing both bulk material degradation and interface failure mechanisms, the model provides a more realistic representation of crack evolution under shear loading. These findings validate the effectiveness of CRFPA as a robust numerical approach for simulating interface failure in masonry structures and other quasi-brittle interface materials.

4.2 Brazilian Disc Test

The Brazilian disc test is used to validate the ability of CRFPA to simulate complex problem scenarios, as illustrated in Fig. 19a, is a widely used method in geotechnical engineering and rock mechanics to determine the tensile strength of brittle materials such as rocks and concrete. The model depicts a circular disc specimen with a diameter (d) of 50 mm and a thickness (l) of 15 mm. In the test, compressive forces are applied vertically along the disc's diameter, as shown by the displacement arrows in the front and side

views, while the disc is supported at the bottom on fixed supports. This loading configuration induces tensile stress along the vertical diameter (indicated by the dashed line), causing the disc to split along this line. The angle ($\theta=14^\circ$) may represent the orientation or distribution of stress or crack propagation. The tensile strength is calculated using the formula $\sigma_t = 2 \times P / (\pi dl)$, where P is the maximum load at failure, providing critical data for evaluating material behavior in applications such as civil engineering and mining. In the CRFPA model of the Brazilian disc splitting test, cohesive elements are globally embedded within solid elements, meaning that cohesive elements exist between any two solid elements, as specifically shown in Fig. 19b.

Following the procedure proposed by Liang et al. (2013) and Meng et al. (2022), the heterogeneity parameters in RFPA were converted into equivalent homogeneous parameters to construct the CRFPA model. Accordingly, the solid elements were treated as a purely elastic medium with an elastic modulus of 28,800.0 MPa and a Poisson's ratio of 0.25. The cohesive elements employed a purely brittle constitutive law, with the normal and tangential stiffnesses set to 7000.0 MPa/mm and 5000.0 MPa/mm, respectively; the corresponding normal and tangential strengths were taken as 4.2 MPa and 12.6 MPa. A friction angle of 30° and a residual strength ratio of 0.01.

The CRFPA method demonstrates reasonable computational results in simulating the mechanical response of a complete Brazilian disc specimen in the elastic stage, as shown in Fig. 20. Firstly, from the displacement field distribution in Fig. 20a, the horizontal displacement field exhibits symmetry during vertical loading, and the opening deformation in the central region conforms to the classic failure mode observed in Brazilian disc experiments. Secondly, Fig. 20b presents the crack morphology, where the crack propagates along the loading direction and penetrates the entire specimen, indicating that tensile stress dominates the failure process. Specifically, in the stress contour of σ_{xx} shown in Fig. 20c, the stress distribution primarily reflects the characteristics of the elastic stress field. The distribution shows that compressive stress is mainly concentrated at the loading ends and rapidly transitions to tensile stress as the distance from the loading ends increases, subsequently remaining relatively stable along the diameter. This is consistent with the theoretical expectation that tensile stress dominates in the central region of the Brazilian disc specimen. Finally, Fig. 20d further compares the stress distribution obtained from CRFPA with the analytical solution of Hondros (Hondros 1959). The results indicate that the variation trends of circumferential stress σ_{xx} and radial stress σ_{yy} agree well with the theoretical solution, demonstrating a high level of consistency between the numerical simulation and the analytical solution as Eq. (52).

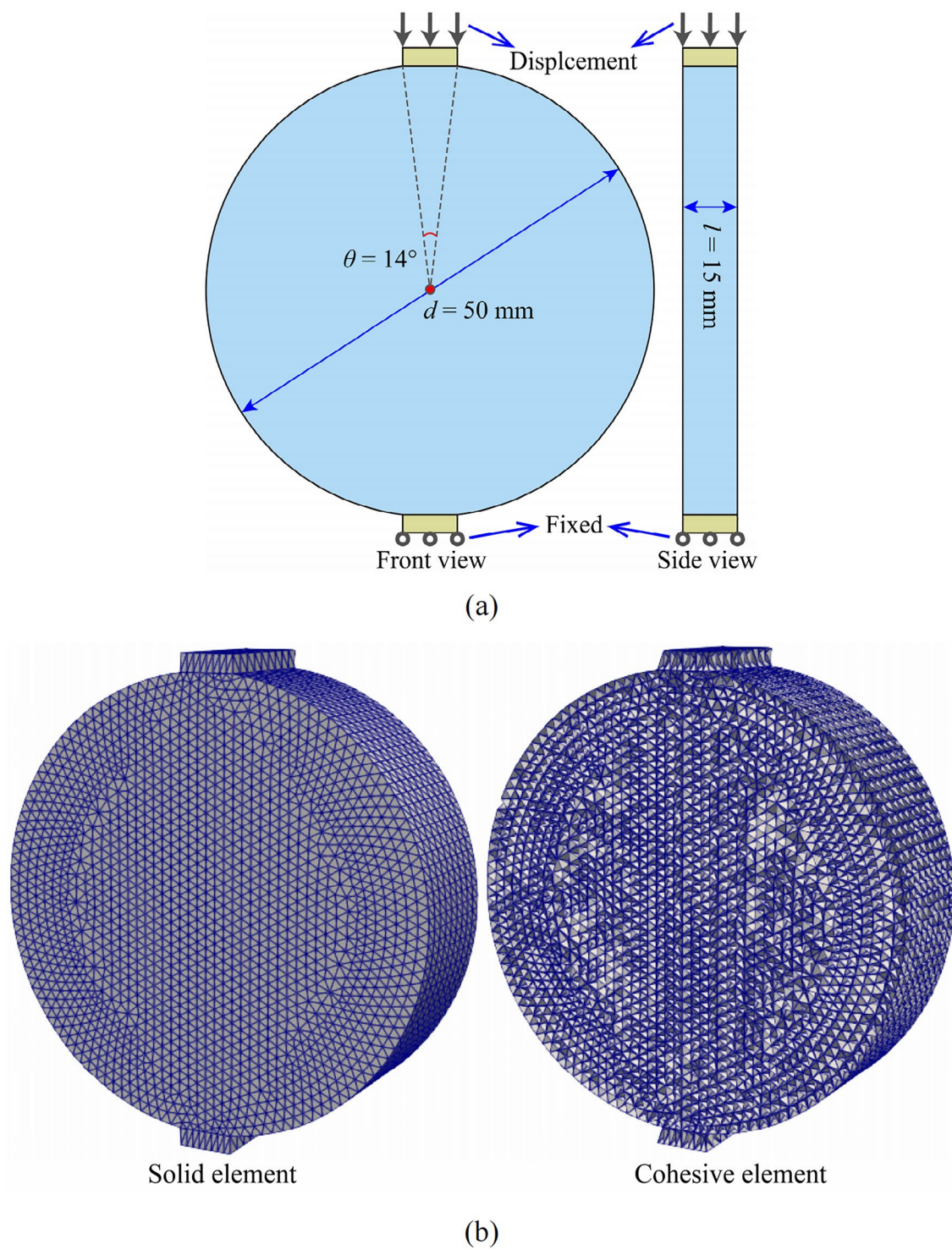


Fig. 19 Setup of the model of Brazilian disc splitting test **a** Size and loading, **b** CRFPA model

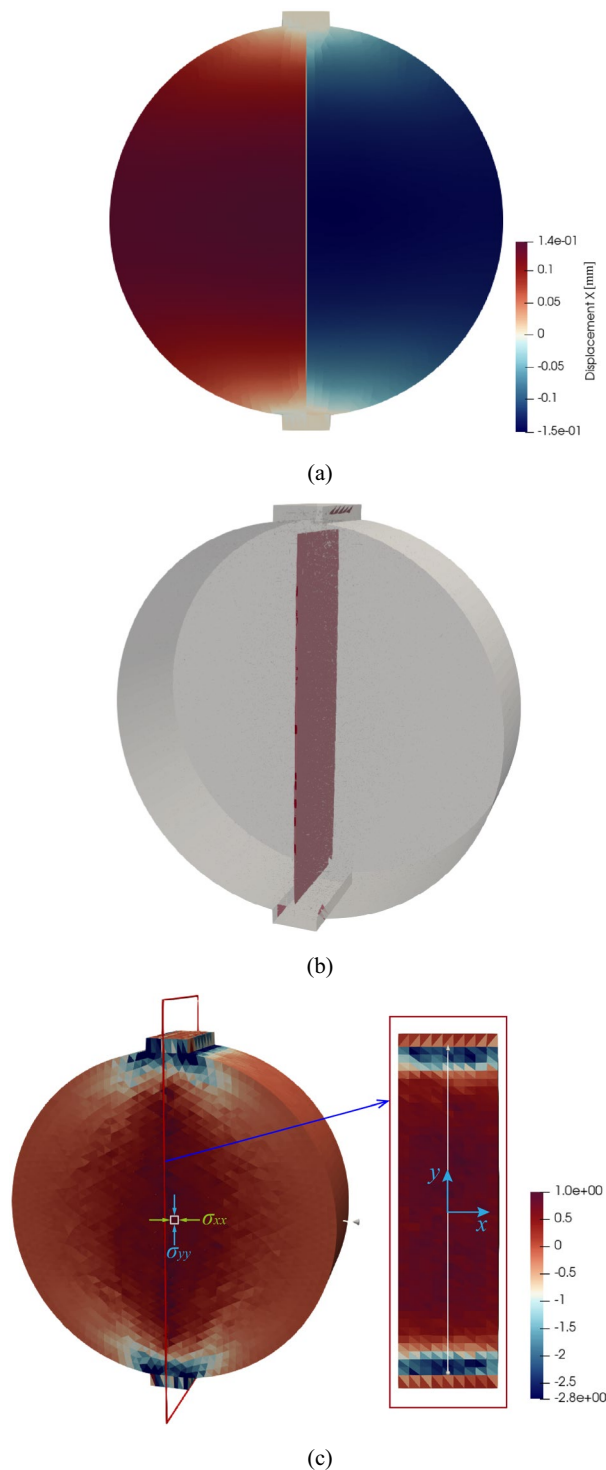


Fig. 20 Simulation results of intact Brazilian disc specimens **a** Displacement field, **b** Crack shape, **c** σ_{xx} field, **d** Elastic stress verification

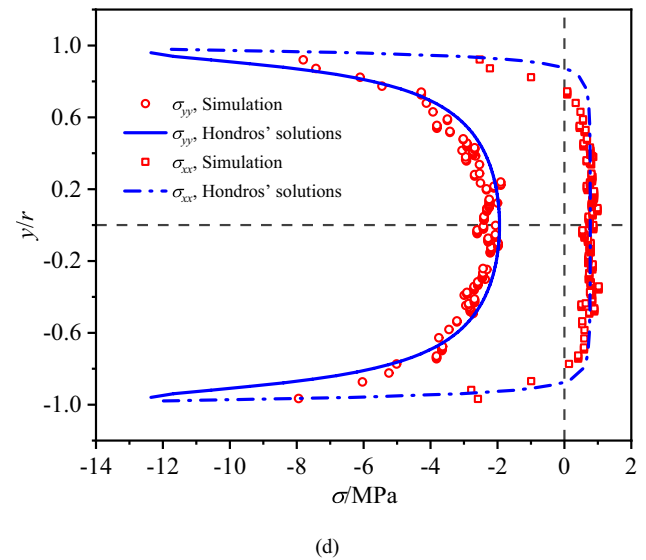


Fig. 20 (continued)

$$\begin{cases} \sigma_{xx} = \frac{P}{\pi r t \theta} \left\{ \frac{[1-(y/r)^2] \sin \theta}{1-2(y/r)^2 \cos \theta + (y/r)^4} - \tan^{-1} \left[\frac{1+(y/r)^2}{1-(y/r)^2} \tan(\theta/2) \right] \right\} \\ \sigma_{yy} = -\frac{P}{\pi r t \theta} \left\{ \frac{[1-(y/r)^2] \sin \theta}{1-2(y/r)^2 \cos \theta + (y/r)^4} + \tan^{-1} \left[\frac{1+(y/r)^2}{1-(y/r)^2} \tan(\theta/2) \right] \right\} \end{cases} \quad (52)$$

Therefore, based on the comparison of the displacement field, stress distribution, and analytical solution, it can be concluded that CRFPA can reasonably simulate the stress evolution law of the Brazilian disc specimen in the elastic stage, and the computational results are highly reliable.

To demonstrate the accuracy of CRFPA in describing crack morphology, a splitting test of a defective Brazilian disc specimen was simulated using CRFPA and compared with experimental results and RFPA simulation results (Wang et al. 2024). The external dimensions of the model are consistent with those in Fig. 19a, while the internal defect dimensions are shown in Fig. 21a. The initial defect has a width (b) of 2 mm and a length ($2 \times a$) of 20 mm. The defect deflection angle (β) is set to 0° , 30° , 60° , and 90° , respectively.

As shown in Fig. 21b, the simulation results confirm the accuracy of the CRFPA approach, as its predicted crack propagation patterns closely match experimental observations for different initial crack angles (0° , 30° , 60° , and 90°). The displacement field contours in CRFPA simulations also exhibit a clear correlation with crack evolution, further validating its correctness. Compared to RFPA, which models fracture based on element damage evolution, CRFPA introduces cohesive elements between all solid elements, significantly improving the representation of crack initiation and propagation (Wang et al. 2024). This enhancement

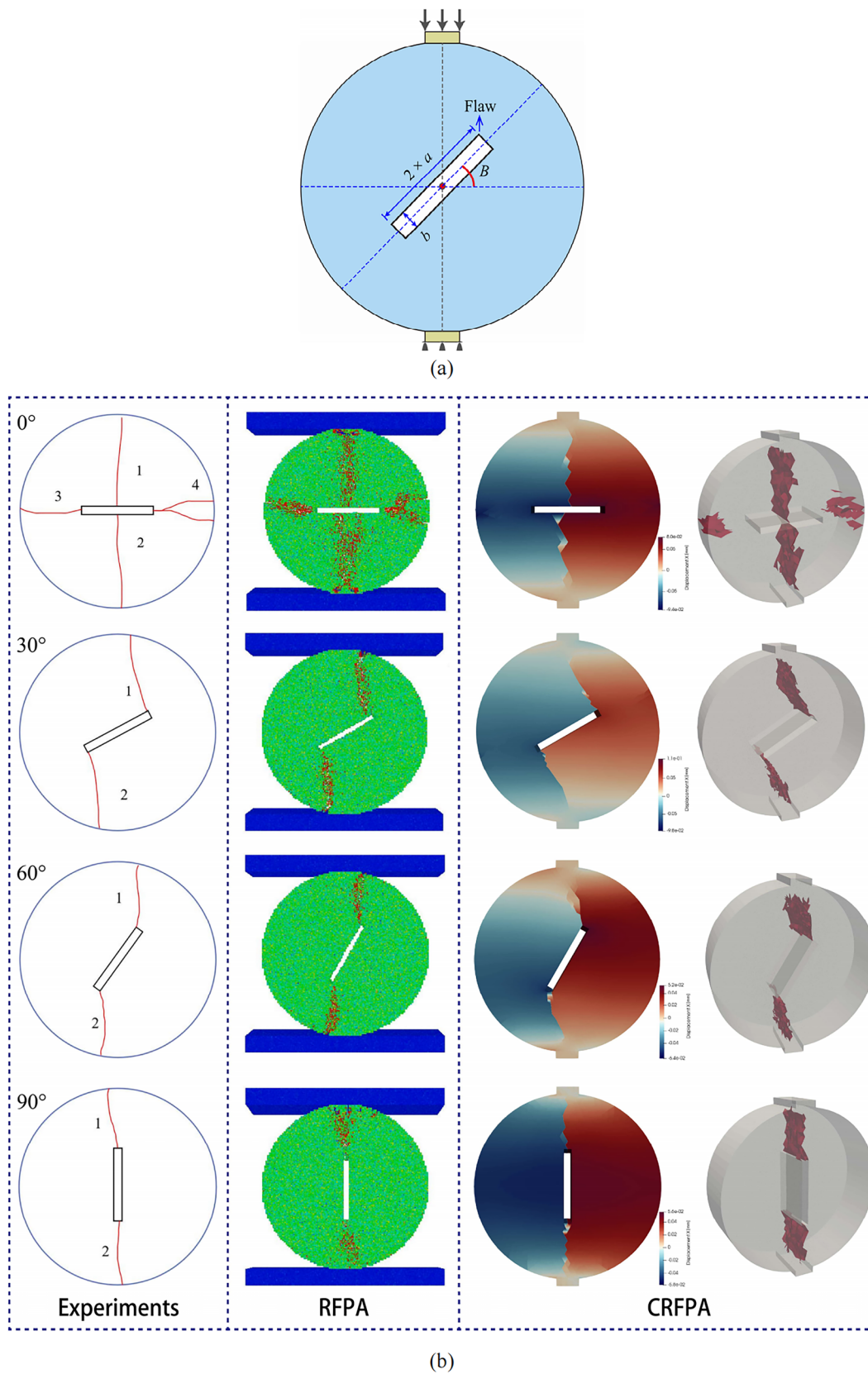


Fig. 21 Comparison of splitting failure effects in Brazilian disc specimens with flaw **a** Size and loading, **b** Failure patterns, **c** Comparison of tensile strength

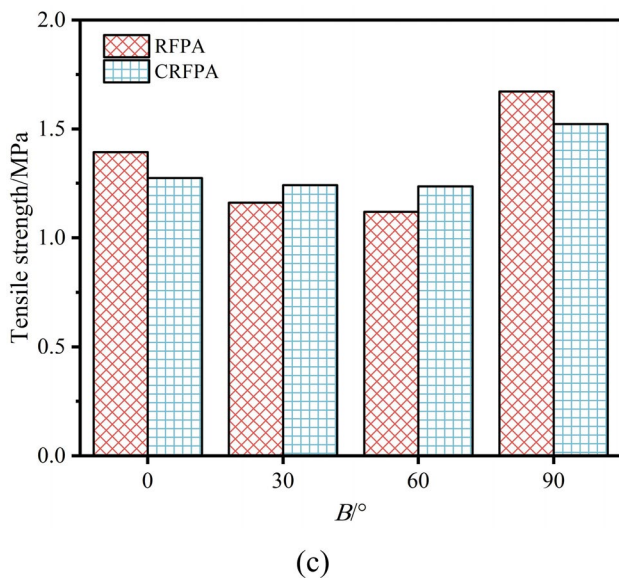
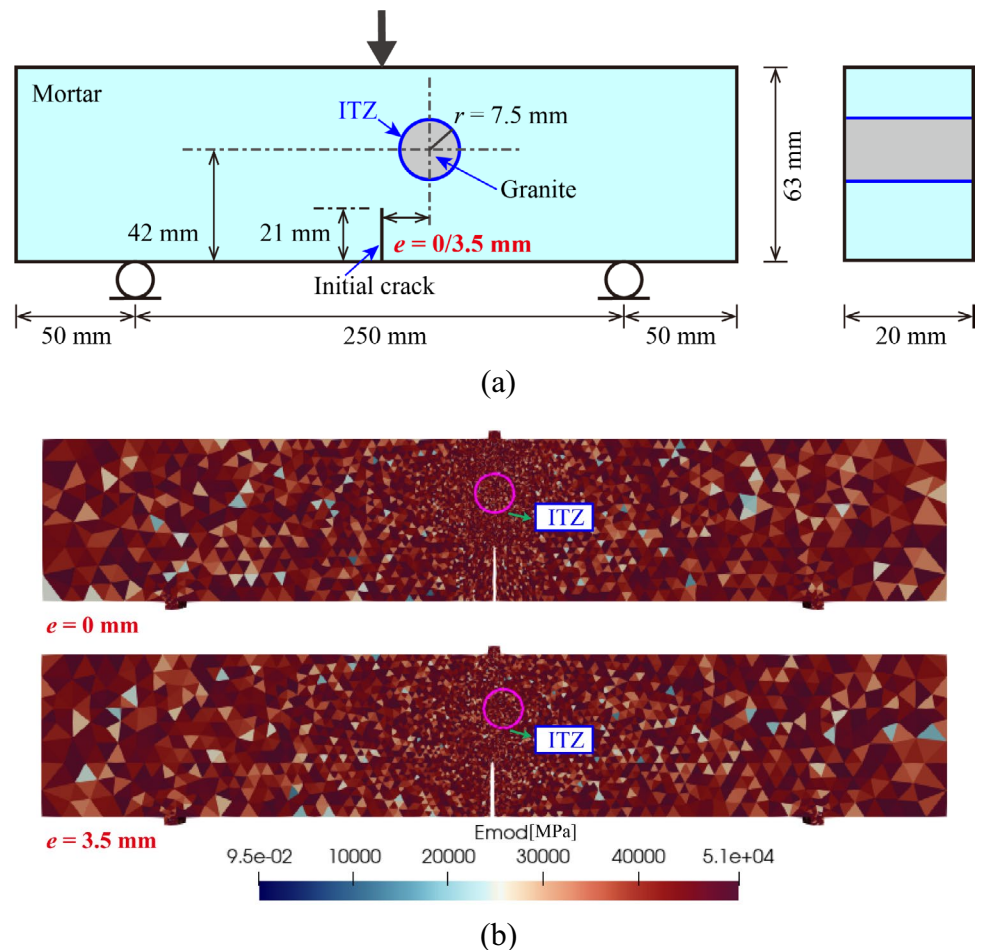


Fig. 21 (continued)

allows CRFPA to capture more realistic crack morphologies and failure mechanisms, making it more consistent with experimental results. Additionally, the displacement field in CRFPA provides a more detailed and physically meaningful depiction of deformation around the crack, demonstrating its superiority over RFPA in accurately describing fracture processes and stress redistribution. The incorporation of cohesive elements enhances the predictive capability of CRFPA, making it a more reliable and robust approach for fracture simulation. A comparison of the tensile strength is presented in Fig. 21c. In terms of strength prediction, the results of CRFPA exhibit good consistency with those of RFPA (Wang et al. 2024), and the trend of their strength variation with the change of the bate angle is similar. The small discrepancy in strength between the two approaches arises because the inherent heterogeneity parameters in RFPA lead to variability in the predicted strength, while the strength parameters used in ICRFPA are equivalent homogenized values converted from RFPA's heterogeneous inputs; in addition, the mismatch in the α angle between ICRFPA and RFPA also affects the predicted strength. Despite these influences, the maximum error is limited to 9.8%. This demonstrates that

Fig. 22 Configuration and FEM of TPB test specimen **a** Size and boundary conditions, **b** FEM



ICRFPA not only enhances the crack expression capability of RFPA but also possesses strength prediction ability.

4.3 Three-Point Bending Test of Mortar Beams with Granite Inclusion

To validate the capability of the CRFPA model in simulating scenarios involving both solid element failure and interface element failure, a three-point bending (TPB) test model was selected, as shown in Fig. 22a (Choi et al. 2025a, b; Razakamandimby et al. 2025). The specimen has dimensions of 250 mm in length, 63 mm in height, and 20 mm in width, consisting of a granite block embedded within a mortar matrix. The granite block, with a radius of 7.5 mm, is centrally positioned and separated from the mortar by an interface transition zone (ITZ) with a thickness of 0 mm, which is discretized using cohesive elements. An initial crack, 21 mm in length, is introduced at the bottom center of the specimen, directly beneath the granite block, to investigate crack propagation behavior. The model considers two configurations: one with zero eccentricity ($e = 0$ mm) and another with an eccentricity of 3.5 mm ($e = 3.5$ mm), where the granite block is offset from the specimen's centerline. Material heterogeneity is characterized by a coefficient $m = 5$, with both the mortar and granite assigned an elastic modulus of 25,300 MPa and a Poisson's ratio of 0.2. In the zero-eccentricity model, the tensile strength of the mortar is 3.8 MPa, while in the 3.5 mm eccentricity model, it increases to 4.3 MPa. The elastic parameters of the granite are assumed to be identical to mortar, with a cohesive strength of 6.5 MPa and a friction angle of 40° for concrete, and 10 MPa and 45° for granite, respectively. The normal and tangential strengths of the ITZ are both 1.2 MPa, while the normal and tangential stiffnesses are 400 MPa/mm, respectively, the friction coefficient is 0.5. The residual strength ratio of mortar and ITZ is set to 0.2. A vertical downward displacement load is applied at the top center of the specimen, with a loading increment of 0.001 mm/step.

The finite element model, as depicted in Fig. 22b, employs tetrahedral elements to discretize the mortar and granite regions, with the ITZ explicitly modeled as a distinct layer to capture interface failure behavior. This setup ensures that the model can effectively simulate the interaction between solid element failure and interface element failure under controlled loading conditions. Based on common knowledge, the crack in a TPB test tends to concentrate near the centerline of the beam. Therefore, the mesh is refined at the midspan position of the beam to enhance the accuracy of the simulation. At the top of the model, a vertical downward displacement load is applied, with each analysis step having a loading increment set at 0.001 mm.

Figure 23 presents a comparison between the failure modes observed in experiments and those predicted by the

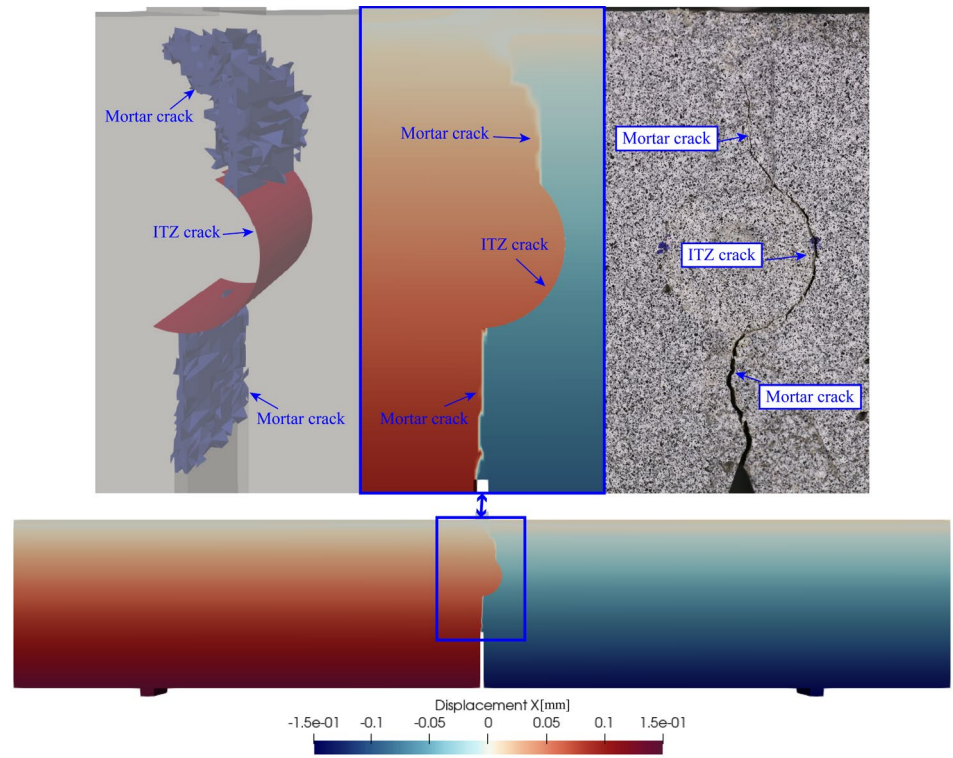
CRFPA model for TPB specimens with different granite inclusions (Choi et al. 2025a, b; Razakamandimby et al. 2025). The figure includes numerical simulation results (left), displacement contour plots highlighting the failure regions (middle), and experimental failure patterns (right). Two cases are considered: (a) a specimen with a centrally positioned granite inclusion ($e = 0.0$ mm) and (b) a specimen with an off-centered granite inclusion ($e = 3.5$ mm). The cracks in the mortar and ITZ are marked in both the simulations and the experimental images to facilitate direct comparison.

In Fig. 23a, where the granite inclusion is centrally located, both the numerical model and experimental results indicate that the primary failure mode involves a combination of mortar cracking and ITZ failure (Choi et al. 2025a, b; Razakamandimby et al. 2025). The crack initiates from the initial notch and propagates through the mortar, with partial crack deflection along the ITZ, leading to a mixed fracture pattern. The crack path in the simulation closely resembles the experimentally observed crack trajectory, demonstrating the accuracy of the CRFPA model in capturing the interaction between the mortar matrix and the ITZ. In Fig. 23b, where the granite inclusion is offset by 3.5 mm, the failure mode exhibits a more complex fracture pattern. The crack initially propagates through the mortar, but due to the asymmetry introduced by the granite inclusion, it deviates and forms a curved trajectory around the inclusion. The simulation successfully predicts the mortar fracture and crack deflection along the ITZ, which are also evident in the experimental specimen. Additionally, the granite particle influences the local stress distribution, causing secondary cracks in the mortar, which are observed in both the numerical and experimental results.

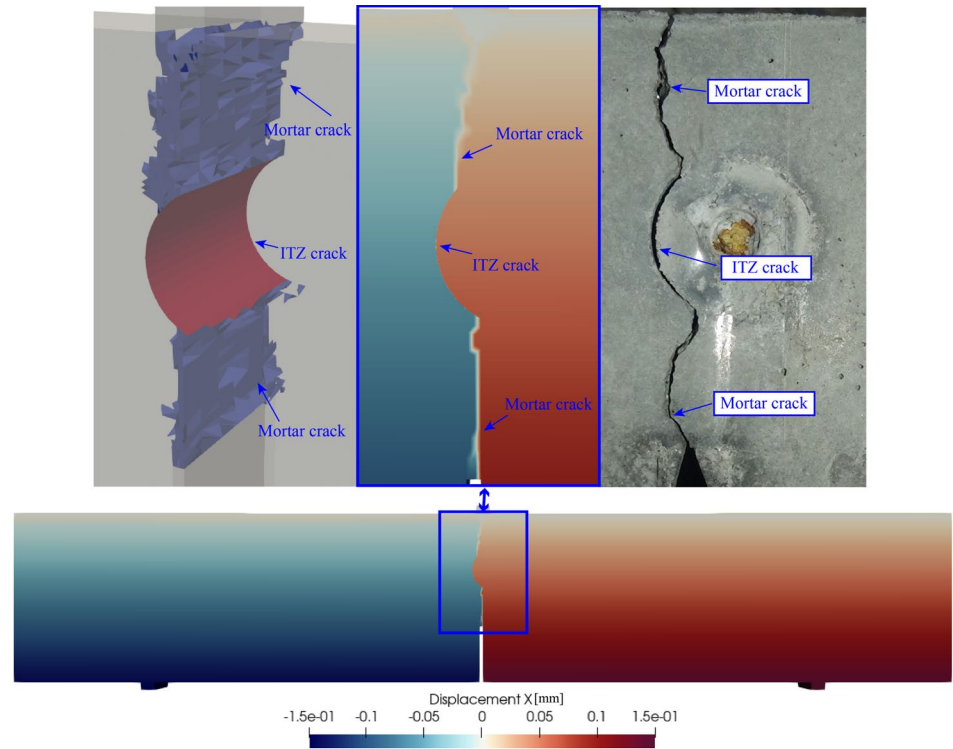
Figure 24 presents the comparison of the Load-CMOD curves obtained from TPB experiments and CRFPA simulations for specimens with different granite inclusion positions. The blue curves represent the experimental results, while the red curves correspond to the numerical simulations. Overall, the simulation results exhibit a similar trend to the experimental data, demonstrating the capability of the CRFPA model in capturing the fracture behavior of heterogeneous materials.

At the initial loading stage, both the experimental and numerical Load-CMOD curves exhibit a linear increase, indicating the elastic response of the material. The initial stiffness of the simulated curves closely matches that of the experimental results, suggesting that the mechanical properties and boundary conditions are well represented in the numerical model. However, a slight underestimation of the peak load is observed in the simulations compared to the experiments. This discrepancy may be attributed to differences in local heterogeneity, microstructural imperfections, or simplifications in the representation of the ITZ and mortar fracture properties. Following the peak load, both experimental and numerical

Fig. 23 Comparison of the failure modes of TPB beams between the related experiments and simulations (Choi et al. 2025a, b; Razakamandimby et al. 2025) **a** $e=0.0$ mm; **b** $e=3.5$ mm



(a)



(b)

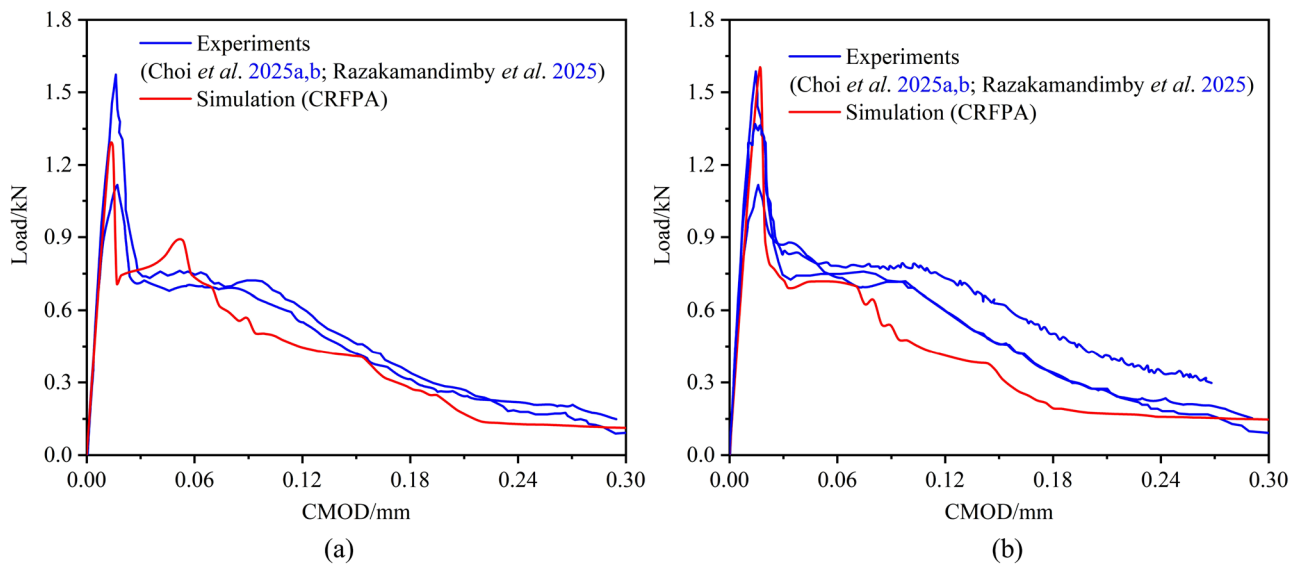


Fig. 24 Comparison of the load and CMOD relationship of TPB tests between experiments and simulations **a** $e = 0.0$ mm, **b** $e = 3.5$ mm

curves display a softening response, characterized by a gradual decrease in load with increasing CMOD. The CRFPA model successfully captures this softening trend; however, the simulated post-peak load reduction is more pronounced compared to the experimental curves. This discrepancy may be due to the simplified damage evolution law or differences in the crack propagation process between the simulation and physical experiments. Overall, the qualitative agreement between experimental and numerical results validates the CRFPA model's effectiveness in predicting the fracture behavior of TPB specimens with granite inclusions. While minor discrepancies exist, particularly in the peak load and post-peak behavior, the model successfully captures the influence of inclusion position on failure patterns. These results demonstrate that the CRFPA approach can effectively simulate both bulk material failure and interface failure, providing a reliable tool for analyzing fracture processes in heterogeneous materials.

5 Discussion

5.1 Parameter Determination and Material Dependence

The CRFPA framework is a quasi-static numerical scheme that integrates cohesive-interface mechanics into an RFPA-style full-step equilibrium iteration. In this context, CRFPA is material dependent through the constitutive inputs of both solid and cohesive elements, while the procedure for parameter determination remains consistent with established quasi-static modelling practice. Specifically, the solid-element parameters are identified from conventional intact-rock tests, whereas the cohesive-element parameters follow the

same calibration principles as those used in cohesive-zone modelling in incremental FEM. The cohesion of weak plane $c_j = 5$ MPa, the friction angle $\varphi_j = 30^\circ$, and the dip angle of the weak plane $\theta = 45^\circ$. An independent post-failure friction coefficient $\mu = 0.3$ is set to investigate the residual bearing capacity.

To further demonstrate that the cohesive and solid inputs can reproduce macroscale strength parameters in an interface-controlled setting, a confined compression benchmark with a single through-going pre-defined weak plane is introduced. The bulk material is assumed to remain intact, and the peak strength is governed by sliding along the weak plane. The specimen geometry, weak-plane configuration, and mesh are illustrated in Fig. 25a.

Under axisymmetric triaxial loading, the stresses resolved on a plane with inclination angle θ relative to the major principal stress are shown in Eq. (53).

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta, \tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \quad (53)$$

If sliding follows a Mohr–Coulomb criterion $\tau = c_j + \sigma_n \tan \varphi_j$, a closed-form relation between σ_1 and σ_3 is obtained. For $\theta = 45^\circ$, the peak axial stress reads as Eq. (54).

$$\sigma_1 = \sigma_3 + \frac{2(c_j + \sigma_3 \tan \varphi_j)}{1 - \tan \varphi_j} \quad (54)$$

Figure 25b shows the axial stress–strain responses predicted by CRFPA under different confining pressures, where the peak strength increases monotonically with σ_3 and the post-peak behavior remains stable due to the prescribed frictional cohesive-interface law. In Fig. 25c, the peak $\sigma_1 - \sigma_3$ data from CRFPA are fitted by a linear envelope,

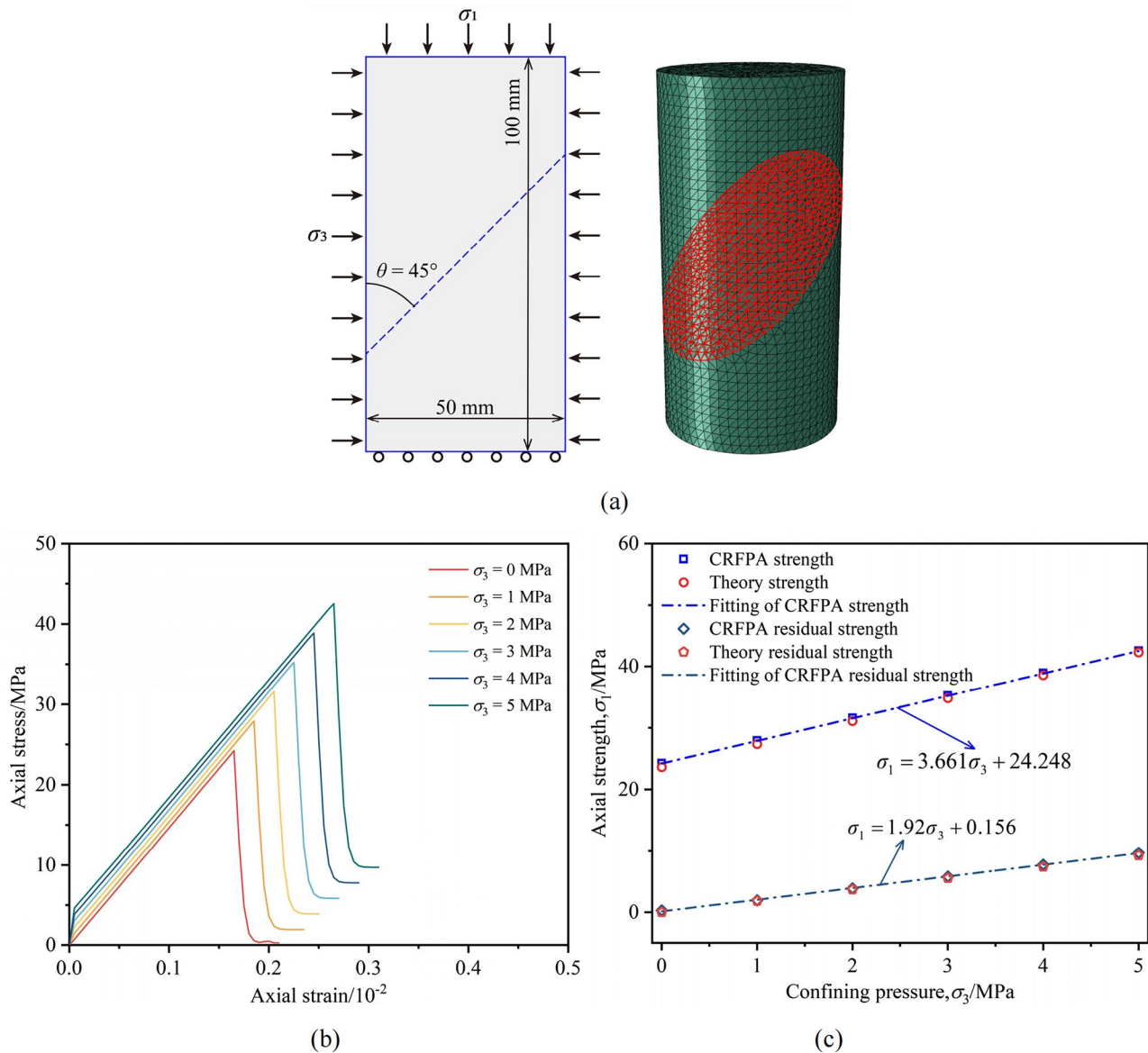


Fig. 25 Confined compression benchmark with a single weak plane for validating macroscale strength parameters **a** Geometry, weak-plane, and mesh configuration; **b** Axial stress–strain curves predicted by CRFPA, **c** Comparison between analytical prediction and CRFPA results

$\sigma_1 = 3.661\sigma_3 + 24.248$ MPa, and then converted to an equivalent triaxial Mohr–Coulomb representation, yielding macroscale parameters $c_{\text{CRFPA}} \approx 6.34$ MPa and $\varphi_{\text{CRFPA}} \approx 24.248^\circ$. These values agree closely with the analytical prediction $c_{\text{Theory}} \approx 6.12$ MPa, $\varphi_{\text{Theory}} \approx 25.36^\circ$, confirming that CRFPA can accurately reproduce the expected macroscale strength envelope in the rock failure process. Figure 25c also compares the theoretical residual strength derived from the friction coefficient in the residual stage with the residual strength obtained by the CRFPA method, and it is found that the two are basically consistent. The macroscopic residual friction angle of the specimen back-calculated from the residual strength of CRFPA is 17.46° , while the

theoretical macroscopic residual friction angle is 18.12° . It can be concluded that CRFPA can not only realize reasonable prediction of strength in interface problems, but also appropriately characterize the residual bearing behavior after interface failure.

5.2 Advantages of CRFPA for Interface-Dominated Failure

To further verify the effectiveness of the proposed CRFPA method in improving the description of interfacial mechanical responses, a case with an applied normal pressure of 0.2

MPa was considered, and the numerical performance of the conventional RFPA model was compared with that of the CRFPA model during the shear failure process. Consistent with the CRFPA configuration, the elastic modulus and Poisson's ratio of the mortar blocks in the RFPA model were set to 14.0 GPa and 0.15, respectively. The interface was represented by a weakened layer of solid elements, with an elastic modulus of 0.1 MPa and a Poisson's ratio of 0.25, while both the tensile strength and cohesion were taken as 0.2295 MPa. In addition, the homogeneity index of the elements in both the blocks and the interface was set to 10,000 to approximate an almost fully homogeneous material and thereby reduce the influence of other factors.

In the RFPA model, two normal-loading schemes were considered: (1) directly applying a normal pressure of 0.2 MPa; and (2) imposing a normal displacement constraint that is equivalent to this pressure. The resulting shear stress-displacement curves obtained from the two schemes are shown in Fig. 26a. The results indicate that neither loading scheme can reasonably capture the post-failure mechanical behavior of the interface. For the model with directly applied pressure, once the mesoscopic elements at the interface fail, the abrupt reduction in elastic modulus leads to a pronounced loss of local stiffness, so that the right-end pressure can no longer be sustained and a drastic instability occurs. As a consequence, the right block disappears completely, as shown in Fig. 26b and c. Meanwhile, the shear stress rapidly drops to nearly 0 MPa after failure, implying that the residual frictional resistance at the interface is entirely lost, as illustrated in Fig. 26b. For the displacement-controlled model, although geometric interpenetration can be avoided, the post-failure shear stress still instantaneously decreases to 0 MPa, such that a physically meaningful softening stage cannot be formed and the energy dissipation associated with frictional sliding cannot be represented, as shown in Fig. 26c. In addition, regardless of the loading mode, RFPA directly deletes the failed mesoscopic interface elements after interfacial failure, making key quantities such as interfacial slip, residual stress, and softening-stage responses unobservable. Moreover, the solid elements in RFPA cannot represent the interfacial slip and stresses. This numerical mechanism is therefore inadequate for simulating interface-dominated failure and cannot reproduce the coupled degradation process of cohesion and friction observed in reality.

In contrast, the CRFPA approach introduces cohesive elements along the failed interface and adopts a friction-involved PPR quasi-brittle interfacial constitutive law, thereby maintaining a finite residual stiffness and frictional load-carrying capacity after interface opening or sliding. This effectively prevents interfacial interpenetration and enables an accurate reproduction of the stress-displacement evolution during the softening stage. Figure 27 further presents the contour plots of interfacial shear stress and shear

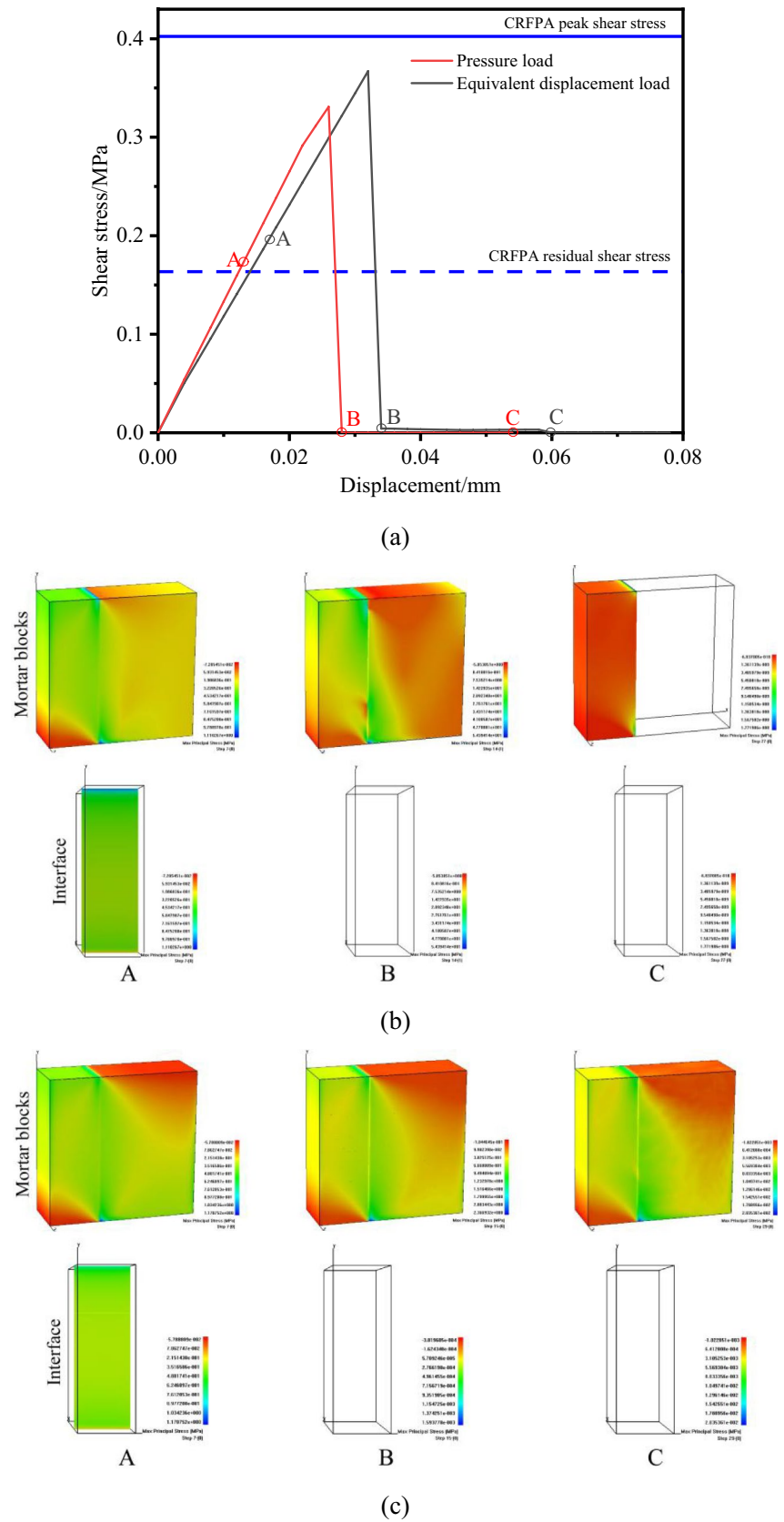
slip at the peak shear stress under a normal pressure of 0.2 MPa for the CRFPA model. It can be seen that the CRFPA model can continuously capture the evolution of shear stress from the peak to the residual level, whereas RFPA cannot track such changes once the elements are deleted. Therefore, describing crack opening and sliding through cohesive elements provides a clear advantage over RFPA.

6 Conclusions

This study presents a Cohesive-enhanced Rock Fracture Process Analysis (CRFPA) method for simulating multi-scale fracture behavior in brittle and quasi-brittle materials. The main contributions of this work can be summarized as follows:

- (a) The CRFPA method integrates the cohesive zone model (CZM) into the traditional RFPA framework, enabling the simulation of interfacial fracture alongside bulk material failure. Cohesive elements are embedded globally or locally between solid elements, allowing for explicit modeling of debonding, shear slip, and mixed-mode interface behavior. More importantly, CRFPA inherits the full-step global iteration scheme from RFPA, which improves numerical stability and avoids the convergence difficulties typically encountered in incremental finite element methods. This inheritance ensures consistent energy updates across iterations and a clearer representation of crack development. Furthermore, the thermodynamic consistency and damage irreversibility of the method are rigorously validated within a variational framework, ensuring that the system satisfies the Clausius–Duhem inequality throughout loading and unloading.
- (b) A dual constitutive control strategy is introduced to coordinate damage evolution between the matrix and the interface. Solid elements follow the original brittle damage model used in RFPA, while interface elements adopt either a brittle or quasi-brittle cohesive-frictional model. This allows the method to adaptively represent different fracture modes, including sudden rupture and gradual delamination, within a unified computational framework.
- (c) CRFPA supports multiple crack representation mechanisms, including solid element degradation, cohesive element damage, and their hybrid interaction. This versatility enables the model to simulate complex crack initiation, propagation, and coalescence with enhanced fidelity, improving upon the limitations of traditional RFPA, which considers only bulk failure.
- (d) The proposed CRFPA model is validated through three benchmark tests: shear failure in masonry wal-

Fig. 26 Shear failure process of the RFP model under different loading methods **a** Shear stress-displacement curve, **b** Stress field under compressive load, **c** Stress field under displacement load



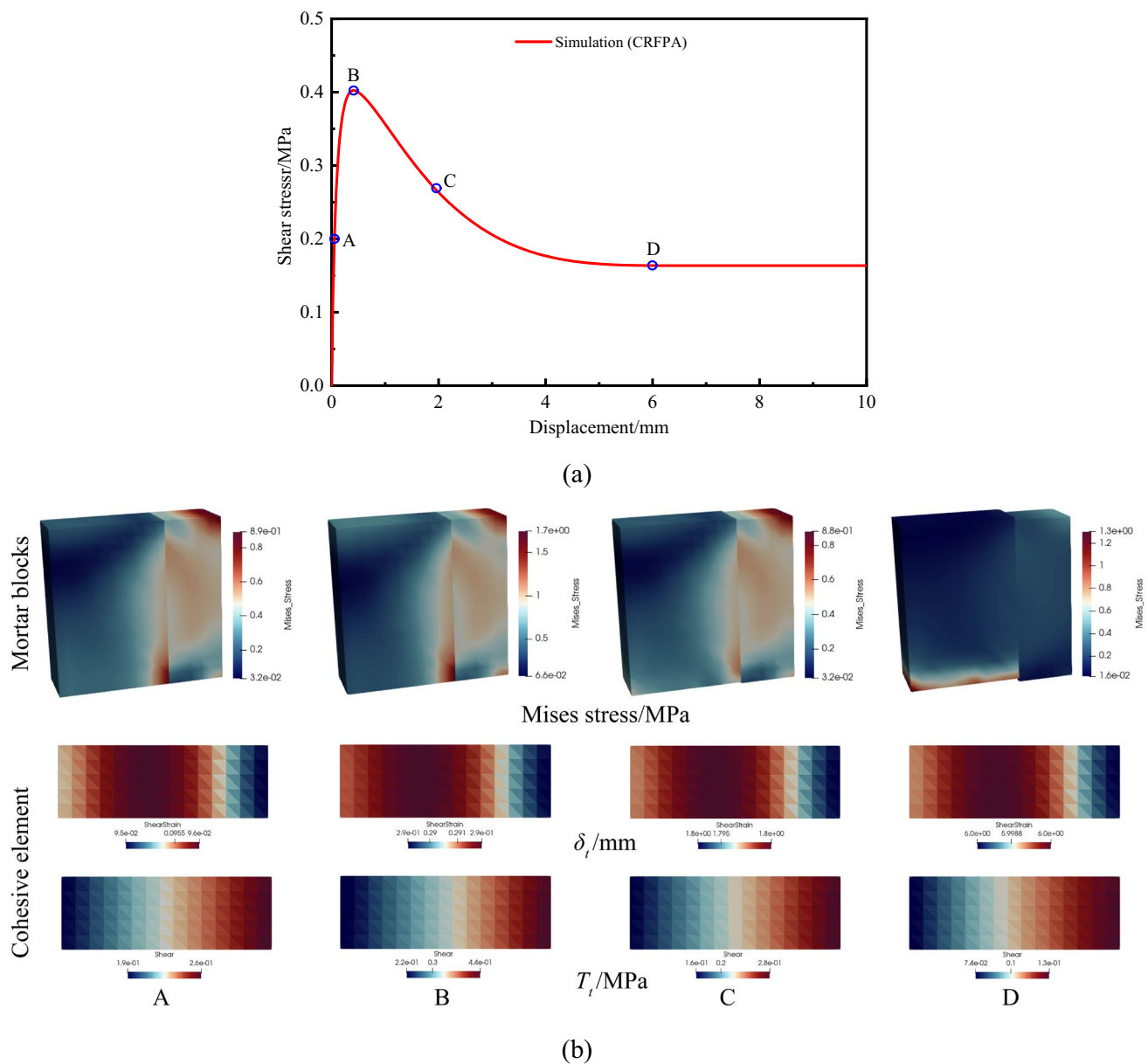


Fig. 27 Shear failure process of the intrinsic CRFPA model under different loading methods **a** Shear stress-displacement curve, **b** Field variable contour of CRFPA

lettes, Brazilian disc splitting, and three-point bending of mortar beams with granite inclusions. In all cases, the model shows good agreement with experimental and analytical results, confirming its effectiveness in capturing both bulk and interfacial fracture phenomena in heterogeneous materials. In addition, the Discussion introduces a confined compression benchmark with a predefined weak plane to verify the macroscale strength envelope under confining pressure, where the

macroscopic cohesion and friction angle extracted from CRFPA agree closely with the analytical prediction. The Discussion also highlights the advantage of CRFPA for interface-dominated compression-shear problems, showing that cohesive-interface modeling can continuously capture post-peak softening and residual frictional resistance, which are difficult to maintain in RFPA due to interface element deletion.

Appendix 1

This pseudocode implements the CRFPA solution scheme, incorporating a full-step iteration strategy to ensure numerical stability. It iteratively evaluates the number of damaged elements and weakens the top rate of newly

damaged elements for property degradation. The process continues until convergence criteria based on damage evolution are met, ensuring accurate fracture simulation. The pseudocode is presented in Algorithm 1.

Algorithm 1 Pseudocode for the CRFPA solution process

```

for each Step in Steps do
    Steptime = 0
    Timestep = 0
    Time = Step.total_time
    while Steptime < Time do
        Cycflag = true
        Cycnum = 0
        // Full-step iteration loop
        while Cycflag do
            WritePVD(filename)
            Solver() // Solve global FEM equations
            Feles2 = Feles1
            StressVar() // Update stress and damage states
            failrates = (Feles1 - Feles2) / N_total
            failincrates = Felesinc / N_total
            if (failrates  $\geq$   $d_1$  and Cycnum = 0) or (failincrates  $\geq$   $d_2$  and Cycnum > 0) then
                Cycflag = true
                Sort Faildata by damage index
                failnum =  $b_3 \times \text{size}(\text{Faildata})$ 
                for i = 1 to (size(Faildata) - failnum) do
                    id = Faildata[i].ID
                    Degrade stiffness of element id
                end for
                Clear Faildata
                Cycnum = Cycnum + 1
            else
                Cycflag = false
            end if
            Felesinc = 0
        end while
        Timestep = Timestep + 1
        Steptime = Steptime +  $\Delta t$ 
    end while
end for
return

```

Appendix 2

During the finite element solution process, parallelization using OpenMP can significantly improve the efficiency of stiffness matrix assembly and linear system solving. The core idea is to leverage multithreading to decompose computationally intensive tasks. The following presents the specific implementation approach and technical details within the CRFPA framework. The pseudocode for parallel processing of element-level data and assembling the global stiffness matrix is provided in Algorithm 2.

Appendix 3

To efficiently solve the large-scale sparse linear systems constructed in the CRFPA framework, this study introduces the Pardiso sparse direct solver from the Intel MKL library. The solver is further accelerated using the OpenMP multi-threading mechanism. Pardiso employs a multithreaded LU/LDL^t factorization algorithm, optimized for both symmetric and non-symmetric sparse matrices, and demonstrates excellent performance and accuracy, particularly for medium-sized systems. The detailed implementation is illustrated in Algorithm 3.

Algorithm 2 Parallel assembly of the stiffness matrix and RHS vector in CRFPA

```

Initialize thread number: omp_set_num_threads(n_threads)
for each element group e in elens do
    if element type = SOLID then
        Get number of elements en = lens[e].Eleinfo.size()
        Parallel for i = 1 to en do
            Compute element matrix C3D4LG and vector C3D4U
            Store results in local triplet list tripletList_local
        end for
        In critical region:
            Append tripletList_local to global tripletList
    else if element type = COH then
        Get number of elements en = elens[e].Eleinfo.size()
        Parallel for i = 1 to en do
            Compute element matrix CohLG and vector CohU
            Store results in local triplet list tripletList_local
        end for
        In critical region:
            Append tripletList_local to global tripletList
    end if
end for
Construct global sparse matrix:
    K.setFromTriplets(tripletList.begin(), tripletList.end())
return

```

Algorithm 3 Pardiso-based sparse solver with OpenMP acceleration

```

1: Set OpenMP and MKL threads
    omp_set_num_threads(n_threads)
    mkl_set_num_threads(n_threads)
    mkl_set_dynamic(false)
2: Convert Eigen sparse matrix K to CSR format
    [ia], [ja], [a] = ConvertSparseMatrixToCSR(K)
3: Prepare vectors
    b = RHS, x = zero vector
4: Initialize Pardiso solver
    pt[], iparm[] = InitializePardisoParameters()
5: Perform symbolic factorization
    phase = 11
    Call PARDISO(pt, ..., phase, a, ia, ja, b, x, ...)
6: Perform numerical factorization
    phase = 22
    Call PARDISO(pt, ..., phase, a, ia, ja, b, x, ...)
7: Solve the linear system
    phase = 33
    Call PARDISO(pt, ..., phase, a, ia, ja, b, x, ...)
8: Write result to global solution
    q = x
9: Release internal Pardiso memory
    phase = -1
    Call PARDISO(pt, ..., phase, ...)

```

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Data Availability The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Declarations

Conflict of Interest The authors declare that they have no known competing financial interests or personal relationships that could appear to influence the work reported here.

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