

Square vs. Hexagonal OWC Performance

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Abstract—Accurate performance evaluation is essential for unlocking the full potential of optical wireless communication (OWC) systems, particularly under different network deployment strategies. This paper presents a comprehensive analytical framework for evaluating OWC system performance that explicitly accounts for key parameters, including inter-node distance, height difference, and full cell geometry, thereby overcoming limitations introduced by simplifying assumptions commonly adopted in the literature. Using the proposed framework, a systematic and balanced comparison between the two most widely used network configurations, namely hexagonal and square deployments, is conducted. Two deployment scenarios are considered. In the first scenario, both deployments employ the same inter-node distance. In the second scenario, the inter-node distance in the square deployment is reduced to account for differences in node density and to enable a fair comparison. Simulation results validate the accuracy of the proposed analytical model and demonstrate that disc-shaped cell approximations can lead to misleading performance predictions. The results further show that, when identical inter-node distances are used, the square deployment generally achieves higher coverage probability across most signal-to-interference-plus-noise ratio (SINR) thresholds. When node density is equalised, the hexagonal deployment provides comparable performance at low and medium SINR thresholds for larger inter-node distances, while the square deployment consistently achieves higher coverage probability at high SINR thresholds. These findings provide practical insights into the impact of network geometry on OWC system design.

I. INTRODUCTION

Optical wireless communication (OWC) has emerged as a strong alternative to radio-frequency (RF) systems, offering higher bandwidth and data rates, enhanced physical-layer security, and immunity to RF interference. To fully exploit these advantages, accurate modelling of OWC performance is essential, as factors such as network geometry have a significant impact on coverage and reliability. In this context, the choice of deployment configuration—such as irregular Poisson point process (PPP) deployments or regular square and hexagonal layouts—plays a critical role in interference management and coverage optimisation.

Although PPP-based deployments are widely used in RF systems [1], comparative studies have shown that regular deployments can outperform irregular PPP-based ones [2]. Identifying the most suitable regular configuration—hexagonal or square—is therefore of practical importance for supporting diverse OWC applications while accounting for system constraints and deployment limitations.

The performance of regular OWC deployments has been investigated in the literature. However, for analytical tractability, many studies rely on simplifying assumptions regarding network geometry. A common assumption is that the coverage areas of regular cells are circular, even for square or hexagonal layouts. Such approximations can significantly affect the accuracy of performance evaluation and may obscure important edge effects [3]. Moreover, to the best of the authors' knowledge, no prior work has explicitly compared the performance of square and hexagonal deployments, despite the well-known influence of network geometry on system behaviour.

In [2], the performance of visible light communication (VLC) systems was analysed in terms of outage probability for both regular and irregular deployments, demonstrating the superiority of regular layouts. However, interference was approximated by considering only the first layer of hexagonal interferers, neglecting the impact of more distant nodes. In addition, although a hexagonal layout was assumed, circular-shaped cells were used in the analysis, thereby overlooking the true hexagonal cell boundaries [3]. Similar geometric simplifications were adopted in [4], where angle diversity receivers (ADRs) were shown to improve coverage and mitigate interference in hexagonal VLC networks. Likewise, [5] employed a two-region approximation consisting of a central disk and an outer ring to model interference in hexagonal layouts; however, this approach does not strictly confine user equipment (UE) to the actual hexagonal cell and may include regions belonging to neighbouring cells. In [6], the signal-to-interference-plus-noise ratio (SINR) distribution in multi-cell ADR scenarios was characterised using extreme UE locations, which does not fully capture the dependence of SINR on the exact spatial relationship between UEs and all interfering nodes. In [9], outage probability and average bit-error rate were analysed for light fidelity (Li-Fi) systems. Nevertheless, the regular deployment was again simplified using circular conic coverage regions, and interference was neglected by focusing solely on signal-to-noise ratio (SNR)-based performance.

Motivated by these limitations, this paper presents a comprehensive analytical framework to evaluate the performance of OWC systems deployed in hexagonal and square layouts. Unlike existing work, the proposed framework accurately models both desired and interfering signals by explicitly accounting for true cell boundaries, 3D distance, and edge effects. Furthermore, the cell areas are rigorously analysed,

avoiding simplifying assumptions that may overlook regions within the cell. To the best of the authors' knowledge, this is the first work to model and compare the coverage performance of hexagonal and square OWC deployments under two distinct scenarios: i) identical inter-node distances, resulting in different node densities, and ii) identical node densities, achieved by adjusting the inter-node distances.

The main contributions are summarised as follows:

- An analytical framework is developed to model the coverage probability of OWC systems deployed in hexagonal and square layouts, explicitly capturing layout-dependent geometric characteristics and system parameters.
- The impact of cell boundaries is incorporated by separately modelling the desired signal and interference statistics at the cell centre and edge, with the corresponding distance distributions derived for each case.
- A comparative coverage analysis of hexagonal and square layouts is conducted under two deployment scenarios. In the first scenario, both layouts use the same inter-node distance ($d_h = d_s$), leading to different node densities.
- In the second scenario, a fair comparison is ensured by adjusting the inter-node distance of the square layout to match the node density of the hexagonal layout.

II. SYSTEM MODEL

Consider two regular OWC deployment configurations, namely square and hexagonal layouts, where the nodes are mounted on the ceiling with inter-node distances of d_h for the hexagonal and d_s for the square, as shown in Fig. 1. The maximum height difference between UE and the plane of the nodes is denoted by Z . A downlink transmission is considered in which only line-of-sight (LOS) links are taken into account. This assumption is justified by the fact that the received power of non-line-of-sight (NLOS) components is significantly lower than that of the LOS component [7], [8]. It is assumed that all nodes reuse the same resources and operate under full-load conditions. Consequently, interference may be generated by any node in the system, provided it lies within the field of view (FOV) of UE. The coverage probability is defined as the probability that the received SINR at a given UE location exceeds a predefined threshold γ :

$$\mathcal{P}_a(\gamma) = \mathbb{P}[\text{SINR}_a > \gamma] = \int_{\mathbb{D}_a} \mathbb{P}\left[\frac{S_a}{I_a + \sigma^2} > \gamma\right] f_a d\mathbb{D}_a \quad (1)$$

where $a \in \{h, s\}$ denotes the hexagonal and square configurations, respectively, $\mathbb{D}_a$ denotes the bounded spatial domain of the cell in the a th configuration, $\sigma^2 = \eta B$ denotes the noise variance, η is the noise power spectral density, B is the system bandwidth, f_a is the joint probability density function (PDF) of the distance parameters, S_a represents the desired received signal from the serving node, and I_a denotes the interference caused by the remaining nodes. S_a can be expressed as:

$$S_a = \frac{p_t(\mu+1)A_{\text{det}}\cos^\mu(\phi_t)\cos(\phi_r)G_{\text{con}}T_s}{2\pi(r^2+z^2)} \quad (2)$$

where p_t denotes the transmit power, $\mu = -\log_{10}(2)/\log_{10}(\cos(\theta_t))$ is the Lambertian order of

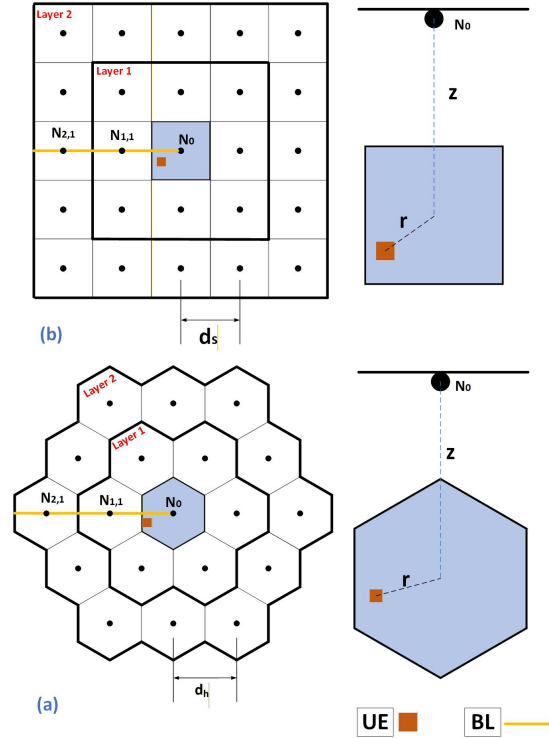


Fig. 1. System Model. (a) Hexagonal (b) Square.

emission, θ_t is the transmitter semi-angle at half-power, A_{det} is the physical area of the detector, ϕ_t and ϕ_r denote the transmitter and receiver alignment angles, G_{con} is the optical concentrator gain, and T_s is the optical filter gain. $\sqrt{r^2+z^2}$ represents the 3D distance, where r is the horizontal node-UE distance and z is the vertical separation. It is assumed that both the nodes and UE are similarly aligned, with the nodes pointing downward at 90° relative to the ceiling and UE pointing upward at 90° relative to the floor. Consequently, $\cos(\phi_t) = \cos(\phi_r) = \frac{z}{\sqrt{r^2+z^2}}$. Therefore, Eq. (2) becomes:

$$S_a = \frac{p_t(\mu+1)A_{\text{det}}G_{\text{con}}T_s z^{\mu+1}}{2\pi(r^2+z^2)^{\frac{\mu+3}{2}}} \quad (3)$$

A. Hexagonal Configuration

The aggregate interference experienced by UE is caused by multiple nodes in the system. These interfering nodes may belong to different layers, defined as concentric rings of cells surrounding the serving node. For example, the first layer consists of the six nodes that immediately neighbour the serving node, the second layer includes the next ring of nodes, and so on, as illustrated in Fig. 1(a). Mathematically, the interference in this configuration is expressed as:

$$I_h = \sum_{l=1}^{L_h} \sum_{i=1}^{l6} K z^{\mu+1} (z^2 + r_{l,i}^2)^{-\frac{\mu+3}{2}} \Gamma(r_{l,i}) \quad (4)$$

where $K = \frac{p_t(\mu+1)A_{\text{det}}G_{\text{con}}T_s}{2\pi}$, L_h denotes the number of interfering layers experienced by UE in the hexagonal configuration, and $r_{l,i} = \sqrt{r^2 + D_{l,i}^2 - 2rD_{l,i}\cos(\phi_{l,i})}$ represents

the horizontal distance between UE and the i th node in the l th layer. The term $D_{l,i}$ is the distance between the serving node (N_0) and the i th node in the l th layer ($N_{l,i}$), $\phi_{l,i}$ denotes the angle between r and the straight line connecting N_0 and $N_{l,i}$, and $\Gamma(r_{l,i})$ is the FOV constraint function:

$$\Gamma(r_{l,i}) = \begin{cases} 1 & r_{l,i} \leq z \tan(\theta_r) \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where θ_r is the photodetector FOV, representing the maximum angle at which the signal can be received. Interference may be generated by nodes from multiple layers in the system, depending on d_h and the FOV. For instance, UE at height z may experience interference from all or some nodes in the first layer if the closest interferer lies within a horizontal distance of $z \tan(\theta_r)$. Specifically, when $z \tan(\theta_r) > \frac{d_h}{2}$, a portion of the cell-of-interest area is subject to interference from at least one node in the first layer. To receive interference from at least one node in the second layer, $z \tan(\theta_r)$ must be greater than or equal to $2R$, where $R = \frac{d_h}{\sqrt{3}}$ is the distance from the cell centre to any vertex (i.e. circumradius). In general, the number of interfering layers L_h can be expressed as:

$$L_h = \begin{cases} 1 & 2R > z \tan(\theta_r) \geq \frac{d_h}{2} \\ \lceil \frac{z \tan(\theta_r)}{R} \rceil & z \tan(\theta_r) \geq 2R \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

where $\lceil \cdot \rceil$ denotes the ceiling function. Assume that the straight line originating from N_0 and passing through $N_{1,1}$ is defined as the baseline BL . Due to the network symmetry and the random location of UE within the system, the triangular region 012 in Fig. 2 is considered for analyzing the aggregate interference. α is defined as the angle between r and the baseline BL , and it can take values in the range $[0, \pi/6]$. The distance between UE and $N_{l,i}$ can be rewritten as:

$$r_{l,i} = \sqrt{r^2 + D_{l,i}^2 - 2rD_{l,i} \cos(\hat{\phi}_{l,i} - \alpha)} \quad (7)$$

where $\hat{\phi}_{l,i}$ is the counter-clockwise angle between the baseline BL and the line connecting N_0 to $N_{l,i}$. The joint PDF of the distance parameters, f_h , must include r , z , and α , as these variables capture the randomness of the 3D distances to different nodes in the system. These distance parameters follow different distributions in the centre and edge areas of the cell. In the centre area, UE is located at a distance of $\frac{d_h}{2}$ or less from N_0 , as illustrated in Fig. 2(a). Following [3], the PDF of r in the cell centre is given by:

$$f_h^{cr}(r) = \frac{d}{dr} F_h^{cr}(r) = \frac{d}{dr} \frac{A_{034}}{A_{012}} = \frac{4\pi r}{\sqrt{3}d_h^2} \quad (8)$$

where $F_h^{cr}(r)$ is the probability of UE being located at a distance r from N_0 , $A_{034} = \frac{\pi r^2}{12}$ represents the area of sector 034 centred at the serving cell, and $A_{012} = \frac{Rd_h}{8}$ represents the area of triangle 012. The distribution of interference also depends on the random variable α . From observation, α can take any value in the range $[0, \frac{\pi}{6}]$. Since UE is uniformly distributed and the values of r and z do not affect the

distribution of α in the centre region, α is uniformly distributed over $[0, \frac{\pi}{6}]$, and $f_h^{ca}(\alpha) = \frac{6}{\pi}$. Since UE can be located at any height between 0 and Z , $f_h^{cz}(z) = \frac{1}{Z}$. Note that there may be some correlation between certain random variables (e.g., r and α) at the cell edge. The distributions of the main parameters at the cell edge differ from those in the centre, making careful modelling essential for accurately capturing interference. In the cell edge, UE is located at a distance between $\frac{d_h}{2}$ and $R = \frac{d_h}{\sqrt{3}}$ from the serving node, as illustrated in Fig. 2(b). Similar to the centre region, $f_h^{er}(r)$ is derived as:

$$\begin{aligned} f_h^{er}(r) &= \frac{d}{dr} F_h^{er}(r) \\ &= \frac{d}{dr} \frac{A_{1654}(r)}{A_{021}} \\ &= \frac{d}{dr} \frac{A_{1653}(r) - A_{143}(r)}{A_{021}} \\ &= \frac{1}{A_{021}} \frac{d}{dr} ((A_{035}(r) - A_{016}) - (A_{043}(r) - A_{041}(r))) \\ &= \frac{1}{A_{021}} \frac{d}{dr} \left(\underbrace{\left(\frac{r^2 \pi}{12} \right)}_{A_{053}(r)} - \underbrace{\left(\frac{d_h^2 \pi}{48} \right)}_{A_{016}} - \right. \\ &\quad \left. \underbrace{\left(\frac{r^2 \cos^{-1}(d_h/2r)}{2} \right)}_{A_{043}(r)} + \underbrace{\left(\frac{d_h \sqrt{r^2 - d_h^2/4}}{4} \right)}_{A_{041}(r)} \right) \\ &= \frac{16r}{\sqrt{3}d_h^2} \left(\frac{\pi}{6} - \cos^{-1}\left(\frac{d_h}{2r}\right) \right) \end{aligned} \quad (9)$$

where $F_h^{er}(r) = \frac{A_{1654}(r)}{A_{021}}$ is the probability of UE located at distance r or less, $A_{1654}(r) = A_{1653}(r) - A_{143}(r)$ is the total edge area, $A_{021} = \sqrt{3}d_h^2/16$ is the total area of interest, $A_{1653}(r)$ is the area of annulus sector 1653 and obtained by subtracting the area of sector 016 (A_{016}) from the area of sector 053 ($A_{053}(r)$), $A_{143}(r)$ is the area located in annulus sector 1653 but part of the neighbouring cell and can be obtained by subtracting the area of triangle 041 (A_{041}) from the area of sector 043 ($A_{043}(r)$).

Unlike in the centre region, the angle α in the edge region follows a different distribution due to the cell boundaries. Specifically, α depends on the value of r . For instance, when $r > \frac{d_h}{2}$, α cannot take the value 0. Furthermore, when $r = \frac{d_h}{\sqrt{3}}$, α cannot be smaller than $\frac{\pi}{6}$. In general, α is uniformly distributed over the range $[\alpha_{\min}, \alpha_{\max}]$, where $\alpha_{\min} = \cos^{-1}(\frac{d_h}{2r})$ is the minimum value that α can take for a given r , and $\alpha_{\max} = \frac{\pi}{6}$. Accordingly, the conditional PDF of α is expressed as:

$$f_h^{e\alpha|r}(\alpha|r) = \frac{1}{\alpha_{\max} - \alpha_{\min}} \quad (10)$$

Since UE can be located at any height between 0 and Z , the PDF of z in the edge region is given by $f_h^{ez}(z) = \frac{1}{Z}$.

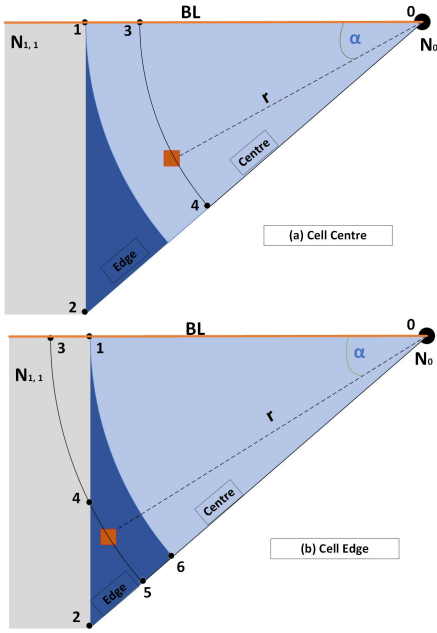


Fig. 2. Cell Centre and Edge.

Therefore, the joint PDF of the distance parameters in the hexagonal configuration becomes:

$$f_h = \begin{cases} f_h^{cr}(r) f_h^{c\alpha}(\alpha) f_h^{cz}(z) & 0 \leq r \leq \frac{d_h}{2} \\ f_h^{er}(r) f_h^{e\alpha|r}(\alpha|r) f_h^{ez}(z) & \frac{d_h}{2} < r \leq \frac{R}{\sqrt{3}} \end{cases} \quad (11)$$

where $f_h^{cr}(r)$, $f_h^{er}(r)$ and $f_h^{e\alpha|r}(\alpha|r)$ are obtained from Eqs. (8), (9) and (10) respectively, $f_h^{c\alpha}(\alpha) = \frac{6}{\pi}$, and $f_h^{cz}(z) = f_h^{ez}(z) = \frac{1}{Z}$. The coverage probability $\mathcal{P}_h(\gamma)$ is obtained by substituting Eqs. (11), (4), and (3) into Eq. (1).

B. Square Configuration

In this subsection, the system performance of the square configuration is investigated, as shown in Fig. 1(b). The distributions of interference, desired signal, and coverage probability differ from those in the hexagonal configuration due to the distinct network layout. In particular, the distance parameters follow different distributions, which directly affect system performance. For instance, the interference experienced by UE can be expressed as:

$$I_s = \sum_{l=1}^{L_s} \sum_{i=1}^{l8} K z^{\mu+1} (z^2 + r_{l,i}^2)^{-\frac{\mu+3}{2}} \Gamma(r_{l,i}) \quad (12)$$

where L_s denotes the total number of interfering layers affecting the cell of interest. Note that the number of nodes in each layer differs from that in the hexagonal configuration. Specifically, each layer in the square configuration contains $8l$ nodes, where l denotes the layer index. Determining the number of interfering layers in the square configuration differs due to the distinct network layout. Since neighbouring nodes are separated by d_s , the minimum horizontal distance between UE and $N_{1,1}$ is $\frac{d_s}{2}$. Similarly, interference from the second layer occurs when $\min(r_{2,1}) \leq z \tan(\theta_r)$, where $\min(r_{2,1}) = \frac{3d_s}{2}$.

Consequently, the maximum number of layers contributing to interference at the cell of interest occurs when UE is located at the maximum height, $z = Z$.

$$L_s = 2 \left\lfloor \frac{\frac{2Z \tan(\theta_r)}{d_s} - 1}{2} \right\rfloor + 1 \quad (13)$$

where $\lfloor \cdot \rfloor$ is the floor function.

Similar to the hexagonal layout, the distance parameters follow different distributions in the cell centre and edge regions. In the centre region, UE is located at a distance of at most $\frac{d_s}{2}$ from the serving node, as shown in Fig. 2(a). The PDF of r in the cell centre is given by:

$$f_s^{cr}(r) = \frac{d}{dr} F_s^{cr}(r) = \frac{d}{dr} \frac{A_{034}}{A_{012}} = \frac{2\pi r}{d_s^2} \quad (14)$$

where $F_s^{cr}(r)$ represents the probability that UE is located at a distance r from its serving node, $A_{034} = \frac{\pi r^2}{8}$ represents the area of sector 034 centred at the serving cell, and $A_{012} = \frac{d_s^2}{8}$ represents the area of triangle 012. Since UE is uniformly distributed and the values of r and z do not affect the distribution of α in the centre region, α can be assumed to be independent and uniformly distributed over the range $[0, \frac{\pi}{4}]$. Therefore, $f_s^{c\alpha}(\alpha) = \frac{4}{\pi}$. Similar to the hexagonal layout, z is assumed to be independent and uniformly distributed over the range $[0, Z]$. Therefore, the PDF of r in the cell edge can be obtained similarly to (9) and Fig. 2. By taking into account $\frac{d_s}{2} < r \leq \frac{d_s}{\sqrt{2}}$, $f_s^{er}(r)$ becomes:

$$f_s^{er}(r) = \frac{2r}{d_s^2} \left(\pi - 4 \cos^{-1} \left(\frac{d_s}{2r} \right) \right) \quad (15)$$

In the edge region, the distribution of α is similar to that in the hexagonal layout. Specifically, α is uniformly distributed over the range $[\alpha_{\min}, \alpha_{\max}]$, where $\alpha_{\min} = \cos^{-1} \left(\frac{d_s}{2r} \right)$ represents the minimum value that α can take for a given r , and $\alpha_{\max} = \frac{\pi}{4}$. Therefore, the conditional PDF of α is expressed as:

$$f_s^{e\alpha|r}(\alpha|r) = \frac{1}{\alpha_{\max} - \alpha_{\min}} \quad (16)$$

Similar to the centre region, $f_s(z) = \frac{1}{Z}$. Therefore,

$$f_s = \begin{cases} f_s^{cr}(r) f_s^{c\alpha}(\alpha) f_s^{cz}(z) & 0 \leq r \leq \frac{d_s}{2} \\ f_s^{er}(r) f_s^{e\alpha|r}(\alpha|r) f_s^{ez}(z) & \frac{d_s}{2} < r \leq \frac{d_s}{\sqrt{2}} \end{cases} \quad (17)$$

where $f_s^{cr}(r)$, $f_s^{er}(r)$ and $f_s^{e\alpha|r}(\alpha|r)$ are obtained from Eqs. (14), (15) and (16), respectively, $f_s^{c\alpha}(\alpha) = \frac{4}{\pi}$ and $f_s^{cz}(z) = f_s^{ez}(z) = \frac{1}{Z}$. The coverage probability $\mathcal{P}_s(\gamma)$ is obtained by substituting Eqs. (17), (12), and (3) into Eq. (1).

Two comparison scenarios are considered. In the first scenario, the inter-node distance is assumed to be the same in both configurations, i.e., $d_h = d_s$. The second scenario considers different inter-node distances by accounting for the node density in the system, thereby ensuring a balanced comparison between the two configurations. The total area of a hexagonal cell is given by $\mathcal{CA}_h = d_h^2 \frac{\sqrt{3}}{2}$. When $d_h = d_s$, the area of the square cell is larger than that of the hexagonal cell. Consequently, the density of cells in the hexagonal

configuration is higher. Scenario 2 accounts for this difference to ensure a fair comparison between the two systems. To achieve equal cell density, the inter-node distance d_s must be smaller than that in the hexagonal configuration, d_h . By equating the cell areas, $\mathcal{CA}_h = \mathcal{CA}_s$, d_s is obtained as

$$d_s = d_h \sqrt{\frac{\sqrt{3}}{2}}. \quad (18)$$

III. RESULTS

The results presented in this section demonstrate the accuracy of the proposed analysis, as well as the impact of different system parameters. The system parameters used in the simulations are listed in the table below. Fig. 3 illustrates

Simulation Parameters	
Parameters	Value
Noise power spectral density (η)	$10^{-20} \text{ A}^2/\text{Hz}$
Bandwidth (B)	20 MHz
Semi-angle at half power (θ_t)	70°
Transmitted optical power (p_t)	2 Watt
Detector physical area (A_{det})	10^{-4} m^2
Gain of optical filter (T_s)	1
Gain of optical concentrator (G_{con})	1
Height difference (Z)	2 m

the coverage probability for different scenarios and deployment types when $d_h = d_s$. The first two characters in the legend denote the deployment type and the inter-node distance. For example, $H2 - \star$ and $S3 - \star$ correspond to a hexagonal deployment with an inter-node distance of 2 m and a square deployment with an inter-node distance of 3 m, respectively. The last part of the label indicates the method used to obtain the results. Specifically, $\star - S$, $\star - N$, and $\star - DA$ refer to simulation, numerical results obtained from the proposed analysis, and results based on the disc assumption (similar to [4], [5]), respectively.

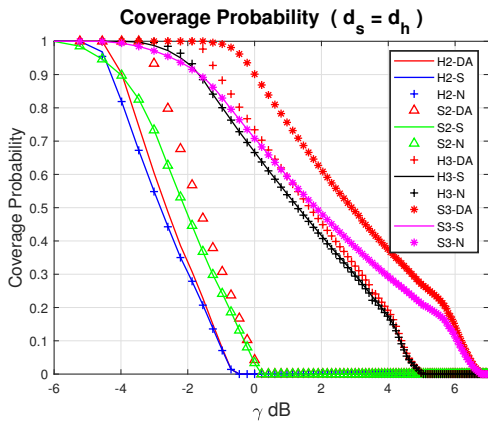


Fig. 3. Scenario 1—Coverage probability comparison for $d_s = d_h$.

The figure confirms the accuracy of the proposed analytical framework, as the simulation results ($\star - S$) closely match the numerical results ($\star - N$) for both deployment types and all inter-node distances considered. In contrast, the widely used disc assumption ($\star - DA$) yields inaccurate performance

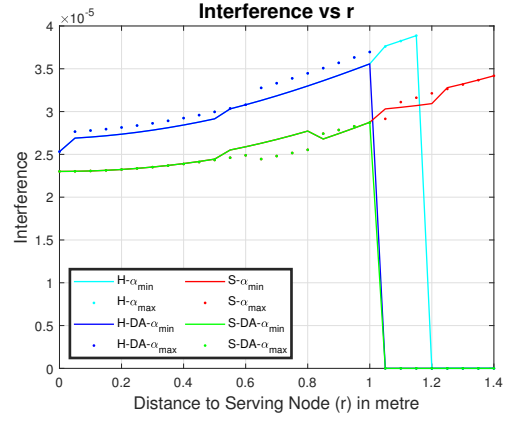


Fig. 4. Interference vs. distance r for $d_s = d_h = 2\text{m}$ and two values of α .

estimates for both deployments. This inaccuracy arises from neglecting the interference experienced at the cell edges. For instance, the curves $H2 - DA$, $H3 - DA$, $S2 - DA$, and $S3 - DA$ provide misleading insights into system performance. The disc assumption overlooks regions near the cell boundaries where UEs experience the highest interference from neighbouring nodes and the lowest desired signal power from the serving node. Notably, this inaccuracy is more pronounced in square deployments, as a larger portion of the cell area is neglected compared to hexagonal deployments.

Furthermore, Fig. 3 shows that the square deployment generally provides better performance. This behaviour is primarily attributed to network geometry. Although the number of interfering nodes in each layer is smaller in the hexagonal deployment, the distances between UE and interfering nodes are shorter. Specifically, in hexagonal deployments, all first-layer interfering nodes are located at a distance of d_h from the node of interest. In contrast, in square deployments, four out of the eight first-layer interfering nodes are located at a distance of d_s , while the remaining four are located at a distance of $\sqrt{2}d_s$. Fig. 3 also indicates that, although the square deployment outperforms the hexagonal deployment overall, the hexagonal deployment can achieve better performance in the low system-performance regime. For example, when $d_s = d_h = 3\text{ m}$, the performance of the hexagonal deployment exceeds that of the square deployment for $\gamma \leq -2\text{ dB}$. A similar trend is observed when $d_s = d_h = 2\text{ m}$.

Fig. 4 illustrates the interference experienced by UE located at different distances from its serving node for two values of α in both deployment types. The labels $H - \alpha_{min}$, $H - \alpha_{max}$, $S - \alpha_{min}$, and $S - \alpha_{max}$ denote the interference in hexagonal and square deployments corresponding to the minimum and maximum values of α obtained using the framework proposed. Similarly, $H - DA - \alpha_{min}$, $H - DA - \alpha_{max}$, $S - DA - \alpha_{min}$, and $S - DA - \alpha_{max}$ represent the interference in hexagonal and square deployments for the minimum and maximum values of α obtained using the disc-shaped assumption. Note that the minimum and maximum values of α vary across the cell area in each deployment. For example, at the cell centre, $\theta_{min} = 0$ in both hexagonal and square deployments, while $\theta_{max} = \frac{\pi}{4}$

in the square deployment and $\theta_{max} = \frac{\pi}{6}$ in the hexagonal deployment. At the cell edge, $\theta_{min} = \cos^{-1}(\frac{d_a}{2r})$ in both deployments, whereas $\theta_{max} = \frac{\pi}{4}$ in the square deployment and $\theta_{max} = \frac{\pi}{6}$ in the hexagonal deployment.

Fig. 4 clearly demonstrates the inaccuracy of the DA assumption, which effectively considers only the cell centre where the interference becomes zero when $r > d_a/2$. In practice, however, the cell edge also experiences significant interference, comparable to that observed near the cell centre, as illustrated by the other curves in the figure. The figure also explains why the square deployment generally outperforms the hexagonal deployment. The interference in the hexagonal deployment is higher than that in the square deployment for most UE locations. Additionally, interference generally increases as UE moves farther away from its serving node in both deployments. This behaviour occurs because, within the core zone, UE moves closer to nearby interfering nodes as its distance from the serving node increases. Another observation from Fig. 4 concerns the impact of the angular position α . In the hexagonal deployment, interference is maximized at θ_{max} for most UE distances, both near the cell centre and at the cell edge. In contrast, in the square deployment, this behaviour is observed only at certain distances, primarily near the cell edge. This difference arises from the distinct geometry and node distribution of each deployment. Fig. 5 illustrates a comparison

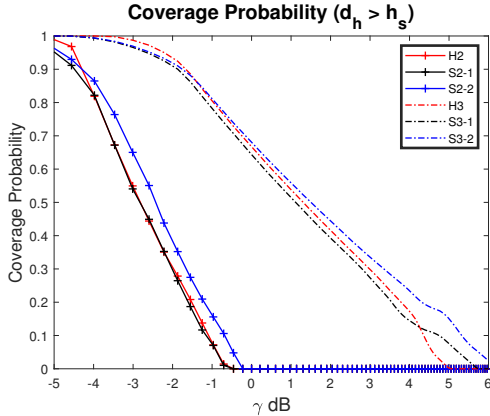


Fig. 5. Scenario 2—Coverage probability comparison for $d_s < d_h$.

between the hexagonal and square deployments in terms of coverage probability for $d_h = 2$ m (H2) and $d_h = 3$ m (H3). Unlike Fig. 3, the inter-node distance in the square deployment is smaller than that in the hexagonal deployment. Specifically, in S2-1 and S3-1, $d_s = d_h \sqrt{\frac{\sqrt{3}}{2}}$. In contrast, in S2-2 and S3-2, $d_s = \frac{d_h(1 + \sqrt{\frac{\sqrt{3}}{2}})}{2}$. The latter case is included for comparison purposes, as it represents the mean value of d_s across the two scenarios: the first scenario with $d_s = d_h$ (Fig. 3) and the second scenario with $d_s = d_h \sqrt{\frac{\sqrt{3}}{2}}$ (Fig. 5).

The figure confirms that the hexagonal deployment offers only limited performance improvement compared to the square deployment when $d_s = d_h \sqrt{\frac{\sqrt{3}}{2}}$, for different values of d_h . For example, the deployments H2 and S2-1 exhibit nearly

identical performance. A slight performance improvement is observed for the hexagonal deployment with a larger d_h (H3) compared to the square deployment S3-1. Furthermore, although the hexagonal deployment provides better performance for low and medium SINR thresholds, the square deployment achieves higher coverage probability at high SINR values, i.e., $\gamma \geq 4$ dB. Fig. 5 also indicates that the square deployment outperforms the hexagonal deployment when larger values of d_s are considered, as illustrated by S2-2 and S3-2.

IV. CONCLUSION

This paper analyses the performance of OWC networks under hexagonal and square deployments using a unified analytical and simulation framework. The proposed model captures inter-node distance, height variation, and full cell geometry, and is validated via simulations, demonstrating that common disc-based approximations can introduce notable inaccuracies. Two scenarios are examined for fair comparison. With equal inter-node distances ($d_s = d_h$), square deployments generally achieve higher coverage, while hexagonal layouts perform better at low SINR due to differing interference patterns. When node density is equalised ($d_s < d_h$), both topologies exhibit similar performance at small distances, but square deployments maintain an advantage at high SINR. Overall, the results demonstrate that the perceived benefits of hexagonal layouts diminish when node density is considered. Therefore, deployment design should be guided by SINR targets, node density, and practical constraints rather than idealised geometric assumptions.

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