Common trends, Cointegration and Competitive Price Behaviour

Simon P Burke
Department Economics, University of Reading

John Hunter*
Department of Economics and Finance, Brunel University, Uxbridge, Middlesex, UB8 3PH, England

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Abstract

This article describes a characterisation of competitive market behaviour using the concepts of cointegration analysis. It requires all (n) firms to set prices to follow a single stochastic trend (equivalently the vector of n prices should have cointegrating rank $n-1$). This implies that, in the long run, prices are driven by the shocks that impact on all companies, ruling out the possibility that the price set by any one firm is weakly exogenous.

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1 Introduction

In this article we define statistical criteria for determining competitive behaviour from the long-run decomposition of prices. Regulatory authorities and firms have exploited tests of stationarity and cointegration to attempt to determine non-competitive behaviour (Forni, 2004, London Economics, 2002). Here, tests for stationary relative prices are seen as a special case of cointegration. When market prices are sufficiently inter-related in the long-run via cointegration, then the market is viewed as having a broad definition or being more competitive.

We generalize the approach outlined by Hendry and Juselius (2001) to the case of multi-product price comparisons for a competitive market with n commodities. It is assumed that all prices are integrated of the same order. In the bivariate case competitive behaviour can often be seen as being consistent with parallel pricing (Buccarossi, 2006 and Forni, 2004) and this proposition might be appropriately tested by determining whether in their natural logarithm (log) price proportions are stationary.

Here, n price responses are consistent with competitive behaviour when all prices are (I(1)), there are $n-1$ cointegrating relationships or a single common trend, and the common trend is driven by a combination of shocks to all n prices. The test of cointegration is a primary test of the proposition that all series are driven by a single common trend and thus a weighted average of the price shocks of all firms, but in the multiproduct case this does not imply parallel pricing (Buccarossi, 2006). Pure parallel pricing only arises when $n-1$ prices respond to a single price and this price is then weakly exogenous for the vector of cointegrating relationships (Johansen, 1992). In the latter case the price set by one firm defines the stochastic trend and all firms respond to the prices set by that firm. The price that is weakly exogenous responds only to past values of that price and more generally to the shocks that apply to that firm's price. In this article, the common stochastic trend is not restricted to being generated in the above manner.

2 The Stochastic Trend, Long-run Equilibrium Price Targeting (LEPT) and Cointegration.

Consider a market consisting of n firms. These firms are viewed as being competitive when they all respond to a single common stochastic trend, itself consisting of a linear combination of the vector of shocks to individual firms (ϵ_t). This common trend we refer to as an Equilibrium Price Target (EPT) when each of the firms responds to it in the same way and the relationship between each firm's price and this trend defines a set of restrictions on the $n-1$ cointegrating relations (β), sufficient to exactly identify the $n \times (n-1)$ matrix of cointegrating vectors, β.[1]

[1] We use information that derives from the long-run inter-action of prices, because: we believe that arbitrage is likely to require firms to respond to the forces of competition, and this defines an informationaly efficient starting point from which to detect anomalous pricing

Competitive firms are viewed as correcting their price behaviour in response to some equilibrium price target. The underlying target to which the competitive firm responds is a weighted average of the vector of all firms prices $x_t' = \begin{bmatrix} p_{1t} & ... & p_{nt} \end{bmatrix}$ and for series that are all $I(1)$ is defined by the non-stationary component of a single common trend.

Let us consider the case where prices have a p^{th} order Vector Error Correction form:

$$\Gamma(L)\Delta x_t = \alpha\beta' x_{t-1} + \epsilon_t$$

where $\Gamma(L) = I - \Gamma_1 L - \Gamma_2 L^2 ... - \Gamma_{p-1} L^2$ and we define $\Gamma = -(I - \Gamma_1 - \Gamma_2 ... - \Gamma_{p-1})$.The following common trends definition of the equilibrium price target derives from Theorem 4.2 in Johansen (1995) that gives rise to a cointegrating rank of $n-1$.

Definition *Let* $\exists\ p_t^*$ *where:*

$$\begin{aligned} p_t^* &= w'x_t = w'Cx_0 + w'C(\sum_{i=1}^{t} \epsilon_i + \mu) \\ &\quad + w'\alpha(\beta'\alpha)^{-1} \sum_{i=0}^{\infty} (I + \beta'\alpha)^i \beta'(\epsilon_i + \mu) \end{aligned} \tag{1}$$

Where the price weights are $w' = \begin{bmatrix} w_1 & ... & w_n \end{bmatrix}$, $C = \beta_\perp(\alpha_\perp' \Gamma \beta_\perp)^{-1}\alpha_\perp'$, $\alpha\alpha_\perp' = 0$, $\beta_\perp'\beta = 0$, x_0 *are initial values and* μ *is the drift. Then for* $p_{it} \sim I(1)$, $\forall\ i = 1, \cdots n$, *Long-run Equilibrium Price Targeting(LEPT) implies that:*

$$p_{it} - p_t^* \sim I(0).$$

A case of special interest is *where the price weights sum to one* ($w'\iota = 1$, $\iota' = [1, ..., 1]$) *or prices are homogenous of degree zero. Then:*

$$\begin{aligned} p_{it} - p_t^* &= p_{it} - w'x_t \\ &= (w'\iota j_i - w')x_t = w'(\iota j_i - I_n)x_t. \end{aligned}$$

Where j_i is the transpose of the i^{th} unit vector. When $(\iota j_i - I_n) = R_i$ *then there are* n *cointegrating vectors of the form* $\beta_{.i} = w'(\iota j_i - I_n)$ *that are dependent, when all prices have the same order of integration.*

Here we consider a trivariate system[2] with $w' = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}$ and:

$$\begin{aligned} \beta_n' &= \begin{bmatrix} \beta_{.1} \\ \beta_{.2} \\ \beta_{.3} \end{bmatrix} = \begin{bmatrix} w'R_1 \\ w'R_2 \\ w'R_3 \end{bmatrix} \\ &= \begin{bmatrix} w_2 + w_3 & -w_2 & -w_3 \\ -w_1 & w_1 + w_3 & -w_3 \\ -w_1 & -w_2 & w_1 + w_2 \end{bmatrix}, \end{aligned}$$

behaviour. There are alternative measures of competitive behaviour (for example, Froeb and Werden, 1998), but they are informationaly burdensome and sensitive to the nature of the uncertainty (Hunter, Ioannidis, Iossa and Skerratt, 2001).

[2] The n variable case can be easily imputed from the case where $n = 3$.

where

$$R_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \ R_2 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \text{ and } R_3 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

As $rank(\beta_n') < n$ we consider $n-1$ cointegrating vectors:

$$\beta' = \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \end{bmatrix} = \begin{bmatrix} w_2 + w_3 & -w_2 & -w_3 \\ -w_1 & w_1 + w_3 & -w_3 \end{bmatrix}.$$

In general the unrestricted cointegrating relationships are not identified. This is often something ignored by practitioners, but by comparison of the restricted and unrestricted forms of β, LEPT gives rise to:

$$r^2 = (n-1)^2 = (n-1)(n-2) + n - 1, \tag{2}$$

restrictions that are necessary and sufficient to identify β. Firstly, economic theory suggests $n-1=2$ price homogeneity restrictions[3] that fix the first column of β':

$$\beta_{11} + \beta_{21} + \beta_{31} = 0 \text{ and } \beta_{12} + \beta_{22} + \beta_{32} = 0.$$

Secondly there are $(n-1)(n-2) = 2$ restrictions that fix $n-2=1$ elements in the remaining $n-1=2$ rows:

$$\beta_{31} - \beta_{32} = 0 \text{ and } \beta_{22} - \beta_{21} - 1 = 0.$$

Generic identification (see Burke and Hunter, 2005, Chapter 5) follows, because LEPT imposes just enough restrictions to satisfy an order condition (2). Now the formulae above can be used to solve the $r^2 = 4$ equations in terms of $n-1=2$ identified parameters:

$$\begin{aligned} \beta_{32} &= \beta_{31} = -w_3 \\ \beta_{22} &= \beta_{21} + 1 = 1 - w_2 \\ \beta_{11} &= -\beta_{21} - \beta_{31} = w_2 + w_3 \\ \beta_{12} &= -\beta_{22} - \beta_{32} = -\beta_{21} - 1 - \beta_{31} = w_2 + w_3 - 1 = -w_1. \end{aligned}$$

Although, the above criterion are necessary and sufficient for generic identification, for empirical identification we require $\beta_{21} \neq 0$ and $\beta_{31} \neq 0$.

There are a number of different ways by which both α and β can be identified, Burke and Hunter (2005) present a sufficient condition for the generic identification that is implicit in being able to solve for the structural parameters from a long-run reduced form:

$$\beta' = \begin{bmatrix} 1 & 0 & \beta_{31} \\ 0 & 1 & \beta_{32} \end{bmatrix}.$$

[3] Notice, that price homogeneity is a long-run property of LEPT. This means that in the short-run agents may mistake relative and absolute price movements. However, long-run pricing that does not satisfy this property would not appear to be consistent with competitive behaviour.

This parameterization of β' is termed a Normalization Rule by Boswijk (1996) and it also implies the imposition of r^2 exactly identifying restrictions. Consider, an orientation that operates on the first two columns of β' :

$$B_{1,2} = \begin{bmatrix} w_2 + w_3 & -w_2 \\ -w_1 & w_1 + w_3 \end{bmatrix}.$$

A necessary condition for the long-run reduced form to exist is:

$$\det(B_{1,2}) = \det\left(\begin{bmatrix} w_2 + w_3 & -w_2 \\ -w_1 & w_1 + w_3 \end{bmatrix}\right) = (w_1 + w_2 + w_3)w_3 \neq 0.$$

However, empirical identification according to Theorem 3 in Boswijk (1996) implies that identification is not sensitive to the columns selected to generically identify β. This implies for the normalization associated with columns i and j:

$$\beta'_{ij} = \begin{bmatrix} I_{n-1} & B_{i,j}^{-1} b_{\neq ij} \end{bmatrix},$$

for the unrestricted vector of parameters $b_{\neq i,j}$ related to the remaining price. Were $b_{\neq ij} = 0$, then one of the prices is long-run excluded and for the case considered here, when $r = n-1$ this implies $\beta'_{ij} = \begin{bmatrix} I_{n-1} & 0 \end{bmatrix}$. If $r = n-1$ then $b_{\neq ij} = 0$ contradicts the notion that all the series are $I(1)$.

Hence for generic and empirical identification of β' via the normalization rule of Boswijk for the trivariate case where $i = 1$ and $j = 2$, we require an ordering such that:

$$\det(B_{i,j}) = \det\left(\begin{bmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{bmatrix}\right) = \beta_{11}\beta_{22} - \beta_{12}\beta_{21} \neq 0$$

and

$$b_{\neq ij} = \begin{bmatrix} \beta_{31} \\ \beta_{32} \end{bmatrix} \neq 0.$$

In our case empirical identification follows when $\beta_{31} \neq 0$ and $\beta_{12} \neq 0$ and this is consistent with Theorem 2 and 3 in Boswijk (1996). Firstly, when $\beta_{31} \neq 0$, theorem 2 must hold as:

$$b'_{\neq 12} = \begin{bmatrix} \beta_{31} & \beta_{32} \end{bmatrix} = \begin{bmatrix} \beta_{31} & \beta_{31} \end{bmatrix} \neq 0.$$

Secondly, Theorem 3 is satisfied when $\det(B_{1,1}) = (w_1 + w_2 + w_3)w_3 \neq 0$ that follows from LEPT as $w_3 \neq 0$ when $\beta_{31} \neq 0$ and $(w_1 + w_2 + w_3) = -\beta_{21} - \beta_{31} + \beta_{21} + 1 + \beta_{31} \neq 0$.

Notice, that identification may be sensitive to the ordering of the system and this may occur, because the loadings on the common trend depend on the impact that shocks to that company price have on the market. Also LEPT can be linked back to a number of normalized long-run reduced forms, but the restrictions do not apriori fix the long-run to be:

$$\beta'_{ij} = \begin{bmatrix} I_{n-1} & -\iota \end{bmatrix}.$$

Notice, that the form of β given above implies two further over-identifying restrictions not needed for LEPT, though LEPT might imply them.

To draw out the key aspects of the concept, consider the special case of the first order VECM ($p = 1$) and $w = \alpha_\perp$, so that $C = \beta_\perp(\alpha'_\perp\beta_\perp)^{-1}\alpha'_\perp$, $\alpha'_\perp\alpha = 0$.[4] We can isolate the trend component by multiplying (1) by $\alpha'_\perp$.

Therefore:

$$\alpha'_\perp x_t = \alpha'_\perp x_0 + \alpha'_\perp(\sum_{i=1}^{t}\epsilon_i + \mu) = \alpha'_\perp \sum_{i=1}^{t}\epsilon_i,$$

where the initial condition is set to zero.

From the definition of LEPT, for a broad market[5] all series must follow the same order of integration otherwise different market segments may respond to different trends as $rank(\Pi) \leq n-2$. However, this type of relation is only consistent with competitive behaviour when the cointegrating relations depend on all prices or we preclude the case where, by any simple re-ordering, $\alpha'_\perp \neq \begin{bmatrix} 0 & 0 & \alpha_{3\perp} \end{bmatrix}$. More specifically the identifying cointegrating combination negates the possibility that $n-1$ prices depend exactly on a single price; this is the case where one of the prices is long-run weakly exogenous and all prices react to this price. If there are $n-1$ cointegrating vectors and α is an $n \times r$ matrix of loadings, then it follows from Johansen (1992) for WE of a variable for the parameters of interest (β) that a row of α is set to zero. With $rank(\alpha) = n-1$, then only one price can be weakly exogenous as otherwise $rank(\alpha) < n-1$ and there is more than one common trend.

If there is a single common trend, a single weakly exogenous variable and $w = \alpha_\perp$, then the following Theorem applies.

Theorem *$rank(\beta) = rank(\Pi) = n-1$ and $\alpha' = \begin{bmatrix} \alpha'_{n-1} & 0 \end{bmatrix}$ for some ordering of the p_i, $i = 1, ...n$, implies a broad market as all prices interact, but there is non-competitive behaviour as* p_i for $i = 1, ..., n-1$ *follow* p_n.

Proof. *In general, $\alpha'_\perp = \begin{bmatrix} \alpha_{1\perp} & \alpha_{2\perp} & ... & \alpha_{n\perp} \end{bmatrix}$ and the common trend drives all prices:*

$$p_t^* = w'x_t = \alpha'_\perp x_t = \alpha'_\perp x_0 + \alpha'_\perp(\sum_{i=1}^{t}\epsilon_i + \mu).$$

For WE $\alpha' = \begin{bmatrix} \alpha'_{n-1} & 0 \end{bmatrix}$ and with price homogeneity:

$$\alpha'_\perp = \begin{bmatrix} 0 & 0 & ... & \alpha_{n\perp} \end{bmatrix}.$$

[4] In the first order VECM case when the initial conditions are removed empirically using a procedure, such as that described by Taylor(1999), then the common trend is a weighted average of the prices. More generaly, this does not hold though the non-stationarity in the price series is still driven by $\alpha'_\perp(\sum_{i=1}^{t}\epsilon_i)$.

[5] The term broad market is used by Forni(2004) to consider cases where all prices in a market or market segment interact.

Therefore:

$$\begin{bmatrix} 0 & ... & \alpha_{n\perp} \end{bmatrix} \begin{bmatrix} p_{1t} \\ \vdots \\ p_{nt} \end{bmatrix} = \alpha_{n\perp} p_{nt} = \alpha_{n\perp} p_{n0} + \alpha_{n\perp}(\sum_{i=1}^{t} \epsilon_{ni} + \mu_n).^6$$

In the trivariate case under LEPT:

$$\beta' = \begin{bmatrix} \alpha_{3\perp} & 0 & -\alpha_{3\perp} \\ 0 & \alpha_{3\perp} & -\alpha_{3\perp} \end{bmatrix}$$

and:

$$\beta' x_t = \begin{bmatrix} \alpha_{3\perp} & 0 & -\alpha_{3\perp} \\ 0 & \alpha_{3\perp} & -\alpha_{3\perp} \end{bmatrix} \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} = \begin{bmatrix} \alpha_{3\perp}(p_{1t} - p_{3t}) \\ \alpha_{3\perp}(p_{2t} - p_{3t}) \end{bmatrix}.$$

From price homogeneity, $\alpha'_{\perp}\iota = 1$ and so $\alpha_{3\perp} = 1$. Therefore $p^*_t = p_{nt}$ and all prices are driven by the stochastic behaviour that underlies p_{nt}.■

When all firms prices are conditioned on p_{nt}, then firm n is the long-run price leader and LEPT implies:

$$\beta = \begin{bmatrix} I_{n-1} & -\iota_{n-1} \end{bmatrix} \text{ and } \iota'_{n-1} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}.$$

Therefore, we have a broad market in the sense that firms follow the common trend, but when the common trend is driven by a single firm without reference to other firms or more pertinently without reference to the direct shocks associated with miss-pricing by these other firms, then the firm must hold a dominant position in the market place or that firm must define the barometer to which all other firms respond. However, a barometer should not normally behave without reference to the other firms. It follows, with one price being weakly exogenous for the parameters of interest, that the n^{th} firms price can be viewed as driving all the other firms prices. This, we would argue is a form of price leadership as the long-run is conditioned only on the behaviour of the n^{th} firm price. In this case, under the restrictions associated with LEPT all firms respond to those of the n^{th} firm, but in the long-run the n^{th} firm does not respond to any of the other firms prices. Hence, although there are $n-1$ long-run price relations and β satisfies the restrictions this is not a competitive case. Hence, for competitive behaviour, we have a further requirement that the common trend is not defined by a single firms price or that none of the prices are weakly exogenous for β.

A number of side issues arise from $rank(\Pi) < n - 1$, there being at least two common trends. Firstly, individual prices may follow linear combinations of the common trends that happen to be different. In this case, one trend may eventually come to dominate. Secondly, the market may be partitioned, so a

[6] In the case of the p^{th} order VECM:
$p^*_t = w'x_t = \alpha'_{\perp}x_t = \alpha'_{\perp}x_0 + \widetilde{\beta}\alpha'_{\perp}(\sum_{i=1}^{t}\epsilon_i + \mu)$ and $\widetilde{\beta} = \alpha'_{\perp}\beta_{\perp}(\alpha'_{\perp}\Gamma\beta_{\perp})^{-1}$.

block of firms follow one price and another group responds to one or both prices, the latter case occurs when we have cointegrating exogeneity (Hunter, 1990). If they follow different linear combinations of the common trends, this may not be consistent with equilibrium in the very long-run as such a divergence of prices is likely in the end to imply death or dominance.

3 Conclusion

In this article we considered the conditions required for competitive behaviour using cointegration analysis. We argue that pricing is consistent with competitive behaviour when: i) there are $n-1$ cointegrating relationships, ii) the restrictions associated with LEPT are satisfied, iii) non of the price series are WE. Beyond the bivariate case the restrictions associated with LEPT are not in general simple price or log price differentials often applied in the literature. This has the implication that tests of stationarity on the price differentials in an $n > 2$ system will not be appropriate.

It is also feasible to extend this analysis to the multi-product case via panel cointegration, allow for shifting short-run dynamics (Kurita and Nielsen, 2005) and long memory processes with fractional cointegration (Robinson, 2006).

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