THE STOCHASTIC UNIT ROOT MODEL AND FRACTIONAL INTEGRATION: AN EXTENSION TO THE SEASONAL CASE

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Abstract

In a recent paper, Yoon (2003) shows that the Stochastic Unit Root (STUR) model is closely related to long memory processes, and, in particular, that it is a special case of an I(d) process, with d = 1.5. In this paper we further examine this issue by using parametric and semiparametric techniques for modelling long memory. In particular, we extend the analysis by considering both non-normality and seasonality, and shed light, theoretically and by means of Monte Carlo methods, on the relationship between the seasonal STUR and the seasonal I(d) models. The results show that, even in the case of I(1.5) underlying processes, the methods, which are specifically designed for testing I(d) statistical models are not appropriate for testing the STUR model. Moreover, they have in some cases very low power against STUR alternatives.

Keywords: *Stochastic Unit Roots; Long Memory; Seasonality*

JEL classification: *C22; C32*

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1. Introduction

It is widely agreed in the time series literature that many economic and financial time series exhibit strong autocorrelation that can be modelled as a long memory process. Examples are the studies of Diebold and Rudebusch (1989), Baillie (1996), Gil-Alana and Robinson (1997), *inter alia*. On the other hand, McCabe and Tremayne (1995), Leybourne, McCabe and Treymaine (LMT, 1996), Leybourne, McCabe and Mills (LMM, 1996) and Granger and Swanson (1997) introduced the Stochastic Unit Root (STUR) model, which is a non-linear process that has a unit root only on average. Empirical applications using STUR models include those of Wu and Chen (1997), Bleaney et al. (1999) and Sollis et al. (2000).

The literature on long memory and STUR models has evolved almost independently over the years. Granger (2000) examined the relationship between the two types of models, and, more recently, Yoon (2003) provided both theoretical and Monte Carlo evidence that the STUR model is a particular case of fractional integration with an order of integration equal to 1.5. This is an important result, which implies that taking first differences will not transform a STUR model into a stationary invertible process. Also, it is in stark contrast to the usual assumption that economic and financial series are I(1) (or possibly I(2)) processes. Yoon (2003) applied the tests of LMT (1996) to different I(d) processes, and showed that the highest rejection frequencies were those corresponding to d = 1.5. A similar experiment was also conducted by Taylor and van Dijk (2002).

The objective of this paper is three-fold. Firstly, we show by means of Monte Carlo methods that there is no direct link between the STUR model and various widely used techniques for modelling long memory. Moreover, these techniques for estimating and testing the fractionally differencing parameter can lead to serious bias in the inference about the stochastic nature of the process, especially in finite samples. Secondly, we investigate these issues in the context of non-normal disturbances. Finally, we extend the STUR model to the seasonal case, examining its relation to seasonal fractional integration. The outline of the paper is the following. Section 2 briefly describes the STUR and the I(d) statistical models. In Section 3 we present other parametric and semiparametric methods for testing I(d) statistical models, which have been extensively employed in the literature. Section 4 analyses the relationship between the two types of models by means of Monte Carlo experiments. In Section 5 we extend the Monte Carlo analysis to the case of non-Gaussian disturbances. Section 6 examines the case of seasonality, and more Monte Carlo evidence is provided on the relationship between the two types of model. Section 7 contains some concluding comments.

2. The STUR and I(d) models

A simple Stochastic Unit Root (STUR) model can be specified as follows:

$$x_t = (1 + \eta_t) x_{t-1} + \varepsilon_t, \qquad t = 1, 2, ..., \qquad (1)$$

$$x_t = 0, \qquad t \leq 0, \qquad (2)$$

where $\eta_t \approx$ i.i.N$(0, \sigma^2)$; $\varepsilon_t \approx$ i.i.N$(0, \sigma_\varepsilon^2)$, and η_t and ε_t are assumed to be independent of each other. Clearly, if $\sigma^2 = 0$, (1) becomes a standard unit root model and, given that $E\eta_t = 0$, x_t is stationary for some periods and mildly explosive for others. However, on average, x_t may seem to be I(1), according to standard tests. The STUR model can be thought of as a special case of the time-varying parameter processes discussed in Andel (1976), with the special feature that the variable is nonstationary. In this sense, STUR models combine unit roots and time-varying parameter characteristics which are relevant for economic time series. Thus, for example, Wu and Chen (1997) found evidence supporting a STUR specification in the case of monthly nominal exchange rates, Bleaney et al. (1999) for real exchange rates, and Sollis et al. (2000) for several stock market indices.

Next, we describe the I(d) model. For the moment, we define an I(0) process $\{u_t, t = 0, \pm 1, ...\}$ as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. In this context, we say that a given raw time series $\{x_t, t = 0, \pm 1, ...\}$ is I(d) if:

$$(1 - L)^d \; x_t \; = \; u_t, \qquad t \; = \; 1, 2, ..., \tag{3}$$

with $x_t = 0$ for $t \leq 0$[1], where u_t is I(0) and where L stands for the lag operator ($Lx_t = x_{t-1}$). Note that the polynomial above can be expressed in terms of its Binomial expansion, such that for all real d,

$$(1 \; - \; L)^d \; = \; \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j \, L^j \; = \; 1 \; - \; d\,L \; + \; \frac{d\,(d-1)}{2} L^2 \; - \; ....$$

The macroeconomic literature has stressed the cases of d = 0 and 1; however, d can be any real number. Clearly, if d = 0 in (1), $x_t = u_t$, and a “weakly autocorrelated” x_t is allowed for. However, if d > 0, x_t is said to be a long memory process, also called “strongly autocorrelated”, and so named because of the strong association between observations widely separated in time. As d increases above 0.5 and towards 1, x_t can be viewed as becoming “more nonstationary”, in the sense, for example, that the variance of the partial sums increases in magnitude. These processes were initially introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981) (though earlier work by Adenstedt, 1974, and Taqqu, 1975 shows an awareness of its representation), and were theoretically justified in terms of aggregation of ARMA processes with randomly varying coefficients by Robinson (1978) and Granger (1980). Similarly, Cioczek-Georges and Mandelbrot (1995), Taqqu et al (1997), Chambers (1998) and

[1] For an alternative definition of fractionally integrated process (the type I class), see Marinucci and Robinson (1999).

Lippi and Zaffaroni (1999) also use aggregation to motivate long memory processes, while Parke (1999) uses a closely related discrete time error duration model. Moreover, Diebold and Inoue (2001) report another source of long memory based on structural change/regime-switching. Empirical applications based on fractional models like (3) can be found, *inter alia*, in the studies of Diebold and Rudebusch (1989), Baillie and Bollerslev (1994), Gil-Alana and Robinson (1997) and Gil-Alana (2000).[2]

The I(d) model is a particular case of a wider class of models exhibiting long memory. The literature provides several definitions of long memory. The first two definitions are as follows. Given a discrete covariance stationary process, say $\{x_t\}$, with autocovariance function $E[(x_t - Ex_t)(x_{t-j} - Ex_t)] = \gamma_j$, according to McLeod and Hipel (1978), the process is a long memory one if:

$$\lim_{T\to\infty} \sum_{j=-T}^{j=T} \left|\gamma_j\right| \qquad (4)$$

is infinite.

A second way to characterise this process is in the frequency domain. For that purpose, suppose that $\{x_t\}$ has spectral density, denoted $f(\lambda)$, and defined as

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{j=\infty} \gamma_j \cos\lambda\, j, \qquad -\pi < \lambda \leq \pi. \qquad (5)$$

Then, we say that x_t displays the property of long memory if the spectral density function has a pole at some frequency λ in the interval $[0, \pi]$. A popular technique within this framework is the fractionally integrated model described by (3). It may be shown that this model satisfies:[3]

$$\gamma_j \approx c_1\, j^{2d-1} \quad \text{as} \quad j \to \infty, \qquad \text{for } \left|c_1\right| < \infty, \qquad (6)$$

and

[2] See also Baillie (1996) for an interesting review of I(d) models.
[3] Condition (6) is satisfied by the fractional ARIMA(0, d, 0) case. However, when including ARMA components, it is required that all γ_j be eventually non-negative.

$$f(\lambda) \approx c_2 \lambda^{-2d} \quad \text{as} \quad \lambda \to 0^+, \qquad \text{for} \quad 0 < c_2 < \infty, \qquad (7)$$

where the symbol $\approx$ means that the ratio of the left-hand-side and the right-hand-side tends to 1 as $j \to \infty$ in (6), and as $\lambda \to 0^+$ in (7). Conditions (6) and (7) are not always equivalent but Zygmund (1995, Cap.V Sect. 2) and Yong (1974) provide conditions under which both expressions are equivalent.

A final definition of long memory involves the rate of growth of variances of partial sums,

$$Var(S_T) = O(T^{2d+1}), \quad \text{with} \quad S_T = \sum_{t=1}^{T} x_t. \qquad (8)$$

There is a tight connection between this variance-of-partial-sum definition of long memory and the previous ones, since the spectral density at zero frequency is the limit of $(1/T)S_T$. Yoon (2003) related STUR and I(d) models using the third of the above definitions. He showed that if σ^2 in (1) is $O(T^{-2k})$ and $k > 0$, then the variance of the partial sums of x_t grows at a rate corresponding to an I(1.5-k) process. In the standard STUR model, $k = 0$, so that a STUR is I(1.5).

3. Estimation and testing of I(d) statistical models

There exist many approaches to estimating and testing the fractional differencing parameter d (see, e.g., Geweke and Porter-Hudak, 1983; Dahlhaus, 1989; Sowell, 1992; Tanaka, 1999; Dolado et al., 2002; etc.). In this paper we will use various parametric and semiparametric methods, already employed in the literature, which have several distinguishing features compared with alternative ones. First, we present a parametric testing procedure due to Robinson (1994a) which is the most efficient method for appropriate (fractional) alternatives. Then, we outline a semiparametric estimation procedure.

3.1 A parametric testing procedure

Robinson (1994a) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o : \; d \; = \; d_o, \tag{9}$$

in a model given by:

$$y_t \; = \; \beta' z_t \; + \; x_t, \qquad t \; = \; 1, 2, ..., \tag{10}$$

and (3), for any real value d_o, where y_t is the time series we observe; $\beta = (\beta_1, \ldots, \beta_k)'$ is a (kx1) vector of unknown parameters; and z_t is a (kx1) vector of deterministic regressors that may include, for example, an intercept, (e.g., $z_t \equiv 1$), or an intercept and a linear time trend (in case of $z_t = (1,t)'$). Clearly, x_t is the series that is filtered through the fractional differencing polynomial in (3). Specifically, the test statistic is given by:

$$\hat{R} \; = \; \hat{r}^2; \quad \hat{r} \; = \; \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \tag{11}$$

where T is the sample size and

$$\hat{a} \; = \; \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j)\, g(\lambda_j; \hat{\tau})^{-1}\, I(\lambda_j); \qquad \hat{\sigma}^2 \; = \; \sigma^2(\hat{\tau}) \; = \; \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1}\, I(\lambda_j);$$

$$\hat{A} \; = \; \frac{2}{T}\left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 \; - \sum_{j=1}^{T-1} \psi(\lambda_j)\, \hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j)\, \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \; \log\left| 2 \sin \frac{\lambda_j}{2} \right|; \qquad \hat{\varepsilon}(\lambda_j) \; = \; \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \qquad \lambda_j \; = \; \frac{2\pi \; j}{T}; \qquad \hat{\tau} \; = \; \arg\min \sigma^2(\tau).$$

$I(\lambda_j)$ is the periodogram of u_t evaluated under the null, i.e.,

$$\hat{u}_t \; = \; (1 \; - \; L)^{d_o} \, y_t \; - \; \hat{\beta}' w_t;$$

$$\hat{\beta} \; = \; \left(\sum_{t=1}^{T} w_t \, w_t' \right)^{-1} \sum_{t=1}^{T} w_t \, (1 \; - \; L)^{d_o} \, y_t; \qquad w_t \; = \; (1 \; - \; L)^{d_o} \, z_t,$$

and the function g above is a known function coming from the spectral density function of u_t,

$$f(\lambda;\sigma^2;\tau) \;=\; \frac{\sigma^2}{2\pi} g(\lambda;\tau), \qquad -\pi \;<\; \lambda \;\leq\; \pi.$$

Note that this test is purely parametric and, therefore, it requires specific modelling assumptions about the short memory specification of u_t. Thus, if u_t is white noise, $g \equiv 1$, and if it is an AR process of the form $\phi(L)u_t = \varepsilon_t$, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ.[4]

Based on the null hypothesis H_o (9), Robinson (1994a) established that under certain regularity conditions:[5]

$$\hat{R} \;\rightarrow_d\; \chi_1^2, \qquad as \quad T \rightarrow \infty. \qquad (12)$$

Thus, unlike in other procedures, we are in a classical large-sample testing situation for the reasons explained in Robinson (1994a), who also showed that the tests are efficient in the Pitman sense against local departures from the null. Because $\hat{R}$ involves a ratio of quadratic forms, its exact null distribution can be calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions: an approximate one-sided $100\alpha\%$ level test of H_o (9) against the alternative: H_a: $d > d_o$ ($d < d_o$) will be given by the rule: "Reject H_o if $\hat{r} > z_\alpha$ ($\hat{r} < -z_\alpha$)", where the probability that a standard normal variate exceeds z_α is α. This version of the tests of Robinson (1994a) was used in empirical applications in Gil-Alana and Robinson (1997) and Gil-Alana (2000), and other versions of his tests, based on seasonal (quarterly and monthly) and cyclical data respectively, can be found in Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001).

There exist other procedures for estimating and testing parametrically the fractionally differenced parameter, some of them also based on the likelihood function. As in other standard large-sample testing situations, Wald and LR test statistics against

4 If u_t is AR(1): $u_t = \alpha u_{t-1} + \varepsilon_t$, $g(\lambda;\tau) = |1 - \alpha e^{i\lambda}|^{-2}$, so that $\alpha = \tau$.

fractional alternatives will have the same null and local limit theory as the LM tests of Robinson (1994a). Sowell (1992) essentially employed such a Wald testing procedure, but his method requires an efficient estimate of d, and, while such estimates can be obtained, no closed-form formulae are available, and therefore the LM procedure of Robinson (1994a) seems computationally more attractive.

A problem with parametric procedures is that the model must be correctly specified; otherwise, the estimates can be inconsistent. In fact, misspecification of the short run components of the process may invalidate the estimation of the long run parameter d. This is the main reason for also using here the semiparametric procedure which we now describe.

3.2 A semiparametric estimation procedure

There exist several methods for estimating the fractional differencing parameter in a semiparametric way. Examples are the log-periodogram regression estimator (LPE), initially proposed by Geweke and Porter-Hudak (1983) and modified later by Künsch (1986) and Robinson (1995a), the average periodogram estimator (APE, Robinson, 1994b) and the quasi maximum likelihood estimator (QMLE, Robinson, 1995b). In this paper we use the QMLE of Robinson (1995b) which we now describe.

The QMLE is essentially a local "Whittle estimator" in the frequency domain, based on a band of frequencies that degenerates to zero. The estimator is implicitly defined by:

$$d_1 = \arg\min_d \left(\log \overline{C(d)} - 2\,d\,\frac{1}{m}\sum_{j=1}^{m} \log \lambda_j \right), \qquad (13)$$

[5] These conditions are very mild, regarding technical assumptions to be satisfied by $\psi(\lambda)$.

$$\overline{C(d)} = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j)\, \lambda_j^{2d}, \quad \lambda_j = \frac{2\pi j}{T}, \quad \frac{m}{T} \to 0.$$

where $I(\lambda_j)$ is the periodogram of the raw series, and d $\in$ (-0.5, 0.5).[6] Under finiteness of the fourth moment and other mild conditions, Robinson (1995a) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \to_d N(0, 1/4) \quad \text{as } T \to \infty,$$

where d_o is the true value of d, with the only additional requirement that m $\to \infty$ slower than T. Robinson (1995a) showed that m must be smaller than T/2 to avoid aliasing effects. A multivariate extension of this estimation procedure can be found in Lobato (1999).

Other methods also based on semiparametric models (like the APE and the LPE) have been applied to economic time series (see, e.g. Gil-Alana, 2002). However, in the present study we use the QMLE, primarily because of its computational simplicity. Note that this also means that we do not need to employ any additional user-chosen numbers in the estimation (which is instead required by the LPE and the APE). Also, we do not have to assume Gaussianity in order to obtain an asymptotic normal distribution, the QMLE being more efficient than the LPE.

4. Simulation results

Yoon (2003) showed that the STUR and the I(d) models are related, specifically that in the standard STUR model, d = 1.5. Several Monte Carlo experiments have confirmed this result. Taylor and van Dijk (2002) carried out an extensive simulation study of the performance of the STUR tests of LMT (1996) applied to I(d) processes. They noticed that the rejection frequencies of the tests initially increased with d, and then decreased as d $\rightarrow$ 2. New simulations were conducted by Yoon (2003), who obtained the highest frequencies at d = 1.5. In these two papers, I(d) processes are generated, and then the tests of LMM (1996) are performed. Here, the experimental setup is the opposite. We generate a STUR model, obtained from Gaussian series, using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986), and then perform the tests for long memory described in Section 3.

In Tables 1 – 4 we report the rejection frequencies of the version of the tests of Robinson (1994a) described in Section 3.1, testing H_o (9) in (3) for values d_o = 0, (0.25), 2, for sample sizes T = 100, (100), 500, 1000 and 2000. In Tables 1 and 3 the tests are performed assuming that u_t in (3) is white noise, while Tables 2 and 4 report the results assuming that u_t is an AR(1) process. First, in Tables 1 and 2, we assume that the true model is a simple STUR model as in (1). In Tables 3 and 4, the true model is given by (1) with

$$\eta_t \; = \; \alpha \, \eta_{t-1} \; + \; v_t, \qquad t \; = \; 1, 2, \ldots \qquad (14)$$

where α = 0.50 and v_t is i.i.d. (0, σ^2) independent of ε_t. Then, y_t is a first order random coefficient autoregressive (RCAR(1)) model with mean unit root, called STUR by LMT (1996) and Granger and Swanson (1997).

(Insert Tables 1 and 2 about here)

In Table 1, where u_t is assumed to be white noise, we notice, first of all, that the lowest rejection probabilities are obtained at d = 0.75 with T = 100, and at d = 0.50 for all the other sample sizes. Increasing T, the rejection frequencies also increase, and, if T $\geq$ 500, the values are exactly 1 for all d except 0.50. If we assume that u_t is AR(1), the rejection frequencies are relatively high in all cases, being higher than 0.9 for d $\geq$ 0.25 even for T = 100. Here, the lowest probabilities are obtained at d = 0. This might be due

[6] Velasco (1999a,b) has recently shown that the fractionally differencing parameter can also be

to the fact that the AR estimates are of the Yule-Walker type, which entails roots that, though automatically less than 1 in absolute value, can be arbitrarily close to unity, and thus might be competing with d in describing nonstationarity.

(Insert Tables 3 and 4 about here)

Tables 3 and 4 are analogous to Tables 1 and 2 but are based on the alternative model given by (1) and (14). The rejection frequencies for the case of white noise disturbances are given in Table 3. We can see that the values are much lower than in Table 1, and the lowest probabilities are obtained at d = 0.75 for all T, and, although the values increase with T, they only reach 1 for d = 1.75 and 2 even with T = 2000. If u_t is AR(1), the lowest probabilities are obtained with d = 0, which is consistent with the results in Table 2, and, although the values in this table are smaller, they are in all cases close to 1 for T = 2000.

The results reported in Tables 1 – 4 clearly contradict the findings of Yoon (2003), who argues that the STUR model corresponds to an I((1.5) model. Two points are noteworthy. The first is that Robinson's (1994a) procedure is fully parametric and, therefore, specifies the model in its complete form, i.e.,

$$(1 - L)^{d_o} x_t \;=\; \varepsilon_t,$$

in Tables 1 and 3, and

$$(1 - L)^{d_o} x_t \;=\; u_t, \quad u_t \;=\; \alpha u_{t-1} \;+\; \varepsilon_t,$$

in Tables 2 and 4, both of which are clearly different from the theoretical STUR model (1) (and (14)). The second is related to the definition of I(d) used by Yong (2003). This is based on the concept of the rate of growth of the variances of the partial sums, while the I(d) model used in Robinson's (1994a) tests is simply defined as a process that is I(0) once it has been d_o-differenced. Thus, though both processes share the same rate of

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consistently and semiparametrically estimated in nonstationary contexts by means of tapering.

decay of the autocorrelations, they are not identical, explaining why the tests reject the null with d = 1.5.

(Insert Tables 5 and 6 about here)

In order to solve the first of these two problems, we report, in Tables 5 and 6, the rejection probabilities using the semiparametric method presented in Section 3.2. Using the same d_o-values and sample sizes as in Tables 1 – 4, we calculate the rejection frequencies for values of the bandwidth parameter m = T/10, T/5, T/4, T/3 and (T/2)-1.[7] Table 5 uses as the true model the simple STUR model (1), while its more elaborated form (1) and (14)) is used in Table 6. We notice that the null of d = 1.5 is always rejected with a probability higher than 0.9, which is consistent with the results produced by the parametric procedure. Using the simple model (1), the lowest rejection probabilities are obtained at d = 1 (Table 5), implying that the STUR specification is easily confused with the I(1) process. Using the model with AR(1) components ((1) and (14)) the lowest values are obtained now at d = 0.5 (Table 6).

To sum up, both parametric and semiparametric methods tend to reject the hypothesis of I(1.5) processes for the STUR model. Moreover, they have in some cases very low power against the STUR alternatives and tend not to reject the null of d = 0.5 or d = 1. This is a serious problem for the practitioner, since if the true process follows a STUR model, the two methods described in Section 3 can produce spurious results about the order of integration of the series, with the implications that this might have in terms of theorising, modelling and/or forecasting.

5. The STUR model with non-Gaussian disturbances

[7] Some attempts to calculate the optimal bandwidth numbers have been examined in Delgado and Robinson (1996) and Robinson and Henry (1996). However, in the case of the Whittle estimator (QMLE), the use of optimal values has not been theoretically justified.

In this section, the Monte Carlo experiments conducted in Section 4 are extended to the case where the true model is a STUR one, but non-Gaussian disturbances are used. In particular, we assume a t_3 distribution in ε_t, in η_t, and in both of them. The results were very similar in all cases. Thus, we only report those corresponding to the t_3 distribution for ε_t.

(Insert Tables 7 – 10 about here)

Tables 7 - 10 are analogous to Tables 1 – 4, but based on non-Gaussian disturbances. The results are completely in line with those obtained in the case of Gaussian u_t. In Table 7 we consider the simple STUR model (1), and perform Robinson's (1994a) tests with a white noise u_t. (Note that Gaussianity is not required in the tests of Robinson, 1994a – the only requirement is a moment condition of order 2). We see that, similarly to Table 1, if T = 100, the lowest values occur at d = 0.50. When imposing AR(1) u_t in the specification of the tests (Table 8), the lowest values occur at d = 0, and the same pattern as in Tables 7 and 8 is obtained when using the more elaborated versions of the STUR (STUR*) model in Tables 9 and 10.

6. A seasonal STUR model and seasonal fractional integration

The STUR specification described in Section 2 can be easily extended to the seasonal case by considering the model,

$$x_t \;=\; (1 + \eta_t)\, x_{t-s} \;+\; \varepsilon_t, \qquad t \;=\; 1, 2, \ldots, \qquad (15)$$

where s again corresponds to the number of time periods within a year (s = 4 with quarterly data; s = 12, monthly; etc.), with $\eta_t \approx N(0, \sigma^2)$ and $\varepsilon_t \approx$ i.i.N$(0, \sigma_\varepsilon^2)$. If $\sigma^2 = 0$, x_t becomes a seasonal unit root model of the form advocated by Dickey, Hasza and Fuller (DHF, 1984), Hylleberg, Engle, Granger and Yoo (HEGY, 1990), Tam and Reinsel

(1997) and others. As in the non-seasonal case, given that $E\eta_t = 0$, on average x_t may appear to be a seasonal I(1) process.

On the other hand, in fractional contexts, the I(d) model presented in Section 2 can be extended to

$$(1 - L^s)^d \, x_t \;=\; u_t, \qquad t \;=\; 1, 2, \ldots, \tag{16}$$

where u_t is I(0), and d can be any real number. Here, the seasonal fractional polynomial can also be expressed in terms of its Binomial expansion, such that, for all real d,

$$(1 - L^s)^d \;=\; \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^{sj} \;=\; 1 \;-\; d\,L^s \;+\; \frac{d\,(d-1)}{2} L^{2s} \;-\; \ldots .$$

We can test the same null hypothesis (9) in the model given by (10) and (16) and the test statistic takes a similar form as $\hat{R}$ in (11), the only difference being in the estimated residuals, which are now:

$$\hat{u}_t \;=\; (1 \;-\; L^s)^{d_o}\, y_t \;-\; \hat{\beta}' w_t; \quad \hat{\beta} \;=\; \left(\sum_{t=1}^{T} w_t\, w_t' \right)^{-1} \sum_{t=1}^{T} w_t\,(1 \;-\; L^s)^{d_o}\, y_t; \qquad w_t \;=\; (1 \;-\; L^s)^{d_o}\, z_t,$$

and the functional form of $\psi(\lambda_j)$, which is, for the case of s = 4:

$$\psi(\lambda_j) \;=\; \log \left| \sin \frac{\lambda_j}{2} \right| + \log \left(2 \cos \frac{\lambda_j}{2} \right) + \log \left| 2 \cos \lambda_j \right| .$$

Robinson (1994a) showed that the test is still characterised by the same standard null limit distribution. Empirical applications of this version of Robinson’s (1994a) tests can be found in Gil-Alana and Robinson (2001) and Gil-Alana (2002), inter alia.

It can be easily proved that, after recursive substitutions, (15) becomes:

$$x_t \;=\; \sum_{j=0}^{\operatorname{int}\left[\frac{t-1}{s}\right]} \left[\prod_{k=0}^{\operatorname{int}\left[\frac{j-1}{s}\right]} (1 + \eta_{t-ks}) \right] \varepsilon_{t-sj},$$

implying that

$$(1 - L^s)\, x_t \;=\; \left(\eta_t \sum_{j=0}^{\mathrm{int}\left[\frac{t-2}{s}\right]} \left[\prod_{k=0}^{\mathrm{int}\left[\frac{j-1}{s}\right]} (1 + \eta_{t-ks-1} \right] \right) \;+\; \varepsilon_t .$$

Following Abadir (2003), it can be proved that $(1\text{-}L^s)x_t$ can be approximated by taking the linear terms in the product of the η_s, such that $(1\text{-}L^s)x_t$ can be expressed as $\eta_t S_{t-1} +$ ε_t, where $S_t = \sum_{i=1}^{t} x_i$, and some of the η_s will be exactly 0 for the non-seasonal values.

Using now the same proposition as in Yoon (2003), if $\sigma^2 = O(T^{-2k})$ and $k > 0$, the variance of the partial sums of x, $\mathrm{Var}\left(\sum_{i=1}^{T} x_i \right)$, grows at a rate corresponding to I(1.5 – k) behaviour. Thus, in the standard case, k = 0, so that the seasonal STUR model is I(1.5).

In Tables 11 - 14 we perform the same type of analysis as before, but focus on the seasonal case with s = 4. Thus, we compute the rejection frequencies of the new tests, for the same (d_o/T) combinations as in the previous cases, using the versions of Robinson's (1994a) tests for seasonality. In Tables 11 and 13 we assume that u_t is a white noise process, while Tables 12 and 14 report the results based on a seasonal AR(1) process of the form:

$$u_t \;=\; \alpha\, u_{t-4} \;+\; \varepsilon_t ,$$

with $\alpha = 0.50$ and white noise ε_t. Tables 12 and 14 consider a more elaborated version of the STUR model, consisting of (1) and

$$\eta_t \;=\; \alpha\, \eta_{t-4} \;+\; v_t , \qquad (17)$$

with white noise v_t, independent of ε_t.

(Insert Tables 11 -14 about here)

As expected, the same problem encountered before occurs again, and the null hypothesis of d = 1.5 is almost always rejected. Starting with the simple STUR model, the lowest rejection frequencies occur at d = 0.5 when white noise disturbances are assumed, and at d = 0.25 with AR(1) u_t (in Tables 11 and 12 respectively), and the same happens when the model with (16) and (17) is employed (Tables 13 and 14).

Thus, as in the non-seasonal case, we do not find evidence of I(1.5) in the seasonal STUR model when using the seasonal version of Robinson's (1994a) tests, and, more worryingly, if the sample size is not very large, the tests tend not to reject the null for d = 0.25 or d = 0.5 in many cases.

7. Conclusions

This paper makes a contribution to the recent literature by examining if long memory and stochastic unit root models, which have evolved independently, are in fact closely related as claimed by other authors as well (see, e.g., Granger, 2000). Specifically, in a recent study, Yoon (2003) shows that the Stochastic Unit Root (STUR) model is a special case of an I(d) process, with d = 1.5.[8] The present paper further examines this issue by means of parametric and semiparametric techniques for modelling long memory. Moreover, it extends the analysis by considering both non-normality and seasonality, and shows theoretically and by means of Monte Carlo simulations that the seasonal stochastic unit root model and seasonal long memory are closely related.

Our findings suggest that the current practice of assuming that most economic variables are I(1) or at most I(2) is not warranted, which obviously has important implications for economic modelling and policy-making. However, they also call into question the adequacy of STUR specifications for many economic and financial series. More precisely, both seasonal and non-seasonal STUR models imply I(1.5) processes, in the sense that the autocorrelation function of their first differences decays at a hyperbolic rate according to the law $O(T^{2d+1})$. Our analysis, which is based on various

[8] Other, more elaborate, STUR models have been proposed by Granger and Swanson (1997), Brandt (1996), Pourahmandi (1989), etc.

parametric and semiparametric techniques for estimating the fractional differencing parameter widely used in the literature, shows that, indeed, the same hyperbolic decay property characterises both types of models. However, it also implies a strong rejection of the null hypothesis represented by this particular law of motion. In fact, if one has reasons to assume that the process underlying the series is of a STUR type, we would highly recommend using at the outset procedures which are specifically designed for testing STUR specifications, such as those developed by McCabe and Tremayne (1995), Leybourne et al. (LMM, 1996), Leybourne et al. (LMT, 1996), Distaso (2003) etc. On the other hand, if the correlograms show some evidence of a hyperbolic decay in the autocorrelations and there is no prior evidence of a STUR process, standard approaches based on I(d) statistical models should be used.

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TABLE 1									
Rejection frequencies of Robinson's (1994a) tests (with white noise u_t) in a STUR model									
True model: STUR model (1)									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.857	0.517	0.129	0.126	0.386	0.726	0.887	0.942	0.960
200	0.918	0.632	0.239	0.330	0.694	0.893	0.969	0.987	0.994
300	0.933	0.721	0.302	0.420	0.790	0.945	0.988	0.996	0.997
400	0.945	0.737	0.384	0.495	0.825	0.957	0.991	0.998	0.999
500	1.000	1.000	0.666	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	0.726	1.000	1.000	1.000	1.000	1.000	1.000
2000	1.000	1.000	0.898	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 2									
Rejection frequencies of Robinson's (1994a) tests (with AR (1) u_t) in a STUR model									
True model: STUR model (1)									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.402	0.752	0.908	0.959	0.975	0.988	0.993	0.995	0.998
200	0.705	0.903	0.970	0.991	0.997	0.999	0.999	0.999	0.999
300	0.793	0.950	0.991	0.997	0.999	1.000	1.000	1.000	1.000
400	0.829	0.959	0.992	0.997	0.998	1.000	1.000	1.000	1.000
500	0.862	0.960	0.992	0.999	0.999	1.000	1.000	1.000	1.000
1000	0.943	0.991	0.998	1.000	1.000	1.000	1.000	1.000	1.000
2000	0.958	0.991	0.999	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 3									
Rejection frequencies of Robinson's (1994a) tests (with white noise u_t) in a STUR* model									
True model: STUR* model in (1) and (9)									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.848	0.583	0.309	0.259	0.410	0.579	0.758	0.861	0.902
200	0.903	0.712	0.411	0.382	0.573	0.774	0.894	0.938	0.965
300	0.941	0.773	0.477	0.422	0.650	0.848	0.942	0.967	0.984
400	0.941	0.789	0.525	0.500	0.739	0.887	0.952	0.978	0.992
500	0.953	0.789	0.589	0.546	0.743	0.889	0.952	0.980	0.999
1000	0.963	0.853	0.687	0.668	0.818	0.917	0.971	0.986	0.998
2000	0.966	0.883	0.754	0.746	0.862	0.941	0.999	1.000	1.000

TABLE 4									
Rejection frequencies of Robinson's (1994a) tests (with AR (1) u_t) in a STUR* model									
True model: STUR* model in (1) and (9)									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.385	0.571	0.768	0.867	0.920	0.949	0.967	0.974	0.979
200	0.556	0.772	0.894	0.942	0.965	0.980	0.981	0.982	0.987
300	0.632	0.836	0.937	0.967	0.987	0.991	0.992	0.992	0.994
400	0.728	0.880	0.951	0.975	0.988	0.991	0.993	0.994	0.996
500	0.735	0.883	0.956	0.988	0.990	0.999	0.999	1.000	1.000
1000	0.821	0.927	0.981	0.995	0.998	0.999	1.000	1.000	1.000
2000	0.881	0.959	0.987	0.999	0.999	1.000	1.000	1.000	1.000

TABLE 5

Rejection frequencies of Robinson (1995a) tests in a simple STUR model

m	T / d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
T/10	100	1.000	0.810	0.810	0.470	0.340	0.190	0.839	0.970	1.000
	200	1.000	1.000	0.949	1.000	0.150	0.720	0.980	1.000	1.000
	300	1.000	1.000	0.980	1.000	0.059	0.899	1.000	1.000	1.000
	400	1.000	1.000	0.990	1.000	0.079	0.910	1.000	1.000	1.000
	500	1.000	1.000	1.000	1.000	0.070	0.930	1.000	1.000	1.000
	1000	1.000	1.000	1.000	1.000	0.050	1.000	1.000	1.000	1.000
	2000	1.000	1.000	1.000	1.000	0.070	1.000	1.000	1.000	1.000
T/5	100	1.000	1.000	0.920	1.000	0.140	0.670	1.000	1.000	1.000
	200	1.000	1.000	0.970	1.000	0.079	0.930	1.000	1.000	1.000
	300	1.000	1.000	1.000	1.000	0.079	0.970	1.000	1.000	1.000
	400	1.000	1.000	1.000	1.000	0.079	0.990	1.000	1.000	1.000
	500	1.000	1.000	1.000	1.000	0.120	1.000	1.000	1.000	1.000
	1000	1.000	1.000	1.000	1.000	0.059	1.000	1.000	1.000	1.000
	2000	1.000	1.000	1.000	1.000	0.012	1.000	1.000	1.000	1.000
T/4	100	1.000	1.000	0.920	1.000	0.200	0.790	1.000	1.000	1.000
	200	1.000	1.000	0.970	1.000	0.090	0.949	1.000	1.000	1.000
	300	1.000	1.000	1.000	1.000	0.079	0.980	1.000	1.000	1.000
	400	1.000	1.000	1.000	1.000	0.079	1.000	1.000	1.000	1.000
	500	1.000	1.000	1.000	1.000	0.080	1.000	1.000	1.000	1.000
	1000	1.000	1.000	1.000	1.000	0.039	1.000	1.000	1.000	1.000
	2000	1.000	1.000	1.000	1.000	0.059	1.000	1.000	1.000	1.000
T/3	100	1.000	1.000	0.959	1.000	0.110	0.880	1.000	1.000	1.000
	200	1.000	1.000	1.000	1.000	0.051	0.970	1.000	1.000	1.000
	300	1.000	1.000	1.000	1.000	0.059	0.990	1.000	1.000	1.000
	400	1.000	1.000	1.000	1.000	0.070	1.000	1.000	1.000	1.000
	500	1.000	1.000	1.000	1.000	0.080	1.000	1.000	1.000	1.000
	1000	1.000	1.000	1.000	1.000	0.059	1.000	1.000	1.000	1.000
	2000	1.000	1.000	1.000	1.000	0.079	1.000	1.000	1.000	1.000
(T/2)-1	100	1.000	1.000	0.990	1.000	0.059	0.970	1.000	1.000	1.000
	200	1.000	1.000	1.000	1.000	0.039	1.000	1.000	1.000	1.000
	300	1.000	1.000	1.000	1.000	0.039	0.990	1.000	1.000	1.000
	400	1.000	1.000	1.000	1.000	0.070	1.000	1.000	1.000	1.000
	500	1.000	1.000	1.000	1.000	0.050	1.000	1.000	1.000	1.000
	1000	1.000	1.000	1.000	1.000	0.030	1.000	1.000	1.000	1.000
	2000	1.000	1.000	1.000	1.000	0.059	1.000	1.000	1.000	1.000

TABLE 6										
Rejection frequencies of Robinson (1995a) tests in a simple STUR* model										
m	T / d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
T/10	100	1.000	0.140	0.140	0.005	0.860	0.860	1.000	1.000	1.000
	200	1.000	1.000	0.051	1.000	0.949	0.990	1.000	1.000	1.000
	300	1.000	1.000	0.039	1.000	0.970	0.990	1.000	1.000	1.000
	400	1.000	1.000	0.029	1.000	0.970	1.000	1.000	1.000	1.000
	500	1.000	1.000	0.029	1.000	0.970	1.000	1.000	1.000	1.000
	1000	1.000	1.000	0.012	1.000	1.000	1.000	1.000	1.000	1.000
	2000	1.000	1.000	0.012	1.000	1.000	1.000	1.000	1.000	1.000
T/5	100	1.000	1.000	0.170	1.000	0.870	0.930	0.990	1.000	1.000
	200	1.000	1.000	0.100	1.000	0.930	0.980	0.990	1.000	1.000
	300	1.000	1.000	0.090	1.000	0.930	0.990	1.000	1.000	1.000
	400	1.000	1.000	0.079	1.000	0.971	0.990	1.000	1.000	1.000
	500	1.000	1.000	0.070	1.000	0.940	0.990	1.000	1.000	1.000
	1000	1.000	1.000	0.039	1.000	0.959	1.000	1.000	1.000	1.000
	2000	1.000	1.000	0.019	1.000	1.000	1.000	1.000	1.000	1.000
T/4	100	1.000	1.000	0.230	0.980	0.820	0.940	0.980	1.000	1.000
	200	1.000	1.000	0.160	1.000	0.880	0.970	0.990	1.000	1.000
	300	1.000	1.000	0.140	1.000	0.899	0.959	1.000	1.000	1.000
	400	1.000	1.000	0.090	1.000	0.940	0.990	1.000	1.000	1.000
	500	1.000	1.000	0.100	1.000	0.910	0.990	1.000	1.000	1.000
	1000	1.000	1.000	0.110	1.000	0.889	1.000	1.000	1.000	1.000
	2000	1.000	1.000	0.130	1.000	1.000	1.000	1.000	1.000	1.000
T/3	100	1.000	1.000	0.300	1.000	0.750	0.959	0.970	1.000	1.000
	200	1.000	1.000	0.259	1.000	0.790	0.979	1.000	1.000	1.000
	300	1.000	1.000	0.209	1.000	0.829	0.980	0.990	1.000	1.000
	400	1.000	1.000	0.190	0.990	0.850	0.970	1.000	1.000	1.000
	500	1.000	1.000	0.170	1.000	0.860	0.990	1.000	1.000	1.000
	1000	1.000	1.000	0.200	1.000	0.829	0.980	1.000	1.000	1.000
	2000	1.000	1.000	0.232	1.000	1.000	1.000	1.000	1.000	1.000
(T/2)-1	100	1.000	1.000	0.411	1.000	0.671	0.959	0.959	1.000	1.000
	200	1.000	1.000	0.479	1.000	0.630	0.959	0.980	1.000	1.000
	300	1.000	1.000	0.410	1.000	0.670	0.970	1.000	1.000	1.000
	400	1.000	1.000	0.360	0.990	0.730	0.970	0.990	1.000	1.000
	500	1.000	1.000	0.410	1.000	0.690	0.980	0.990	1.000	1.000
	1000	1.000	1.000	0.430	1.000	0.670	0.959	1.000	1.000	1.000
	2000	1.000	1.000	0.497	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 7									
Rejection frequencies of Robinson's (1994a) tests (with white noise u_t) in a STUR model									
True model: STUR model with a t_3 distribution for ε_t									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.853	0.557	0.155	0.116	0.378	0.707	0.874	0.941	0.977
200	0.911	0.676	0.240	0.237	0.679	0.875	0.965	0.984	0.993
300	0.914	0.731	0.352	0.414	0.766	0.927	0.970	0.988	0.996
400	0.966	0.749	0.339	0.511	0.837	0.957	0.986	0.997	0.997
500	0.964	0.793	0.418	0.523	0.868	0.973	0.994	0.999	0.999
1000	0.978	0.852	0.555	0.671	0.941	0.991	0.998	0.999	0.999
2000	1.000	0.999	0.997	0.999	1.000	1.000	1.000	1.000	1.000

TABLE 8									
Rejection frequencies of Robinson's (1994a) tests (with AR (1) u_t) in a STUR model									
True model: STUR model with a t_3 distribution for ε_t									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.3931	0.715	0.875	0.957	0.985	0.997	0.999	0.999	0.999
200	0.672	0.899	0.972	0.990	0.995	1.000	1.000	1.000	1.000
300	0.769	0.930	0.985	0.993	1.000	1.000	1.000	1.000	1.000
400	0.838	0.958	0.984	0.993	1.000	0.999	1.000	1.000	1.000
500	0.864	0.971	0.995	0.997	1.000	1.000	1.000	1.000	1.000
1000	0.945	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2000	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 9									
Rejection frequencies of Robinson's (1994a) tests (with white noise u_t) in a STUR* model									
True model: STUR* model with a t_3 distribution for ε_t									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.871	0.589	0.261	0.221	0.371	0.570	0.738	0.843	0.893
200	0.932	0.714	0.382	0.352	0.548	0.785	0.882	0.947	0.977
300	0.922	0.754	0.490	0.472	0.679	0.849	0.932	0.967	0.984
400	0.956	0.775	0.519	0.525	0.730	0.888	0.955	0.985	0.994
500	0.955	0.796	0.590	0.545	0.745	0.907	0.975	0.986	0.997
1000	0.966	0.876	0.683	0.677	0.835	0.929	0.975	0.993	0.998
2000	0.998	0.996	0.983	0.984	0.998	1.000	1.000	1.000	1.000

TABLE 10									
Rejection frequencies of Robinson's (1994a) tests (with AR (1) u_t) in a STUR* model									
True model: STUR* model with a t_3 distribution for ε_t									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.364	0.590	0.774	0.871	0.934	0.956	0.979	0.983	0.985
200	0.555	0.782	0.898	0.940	0.972	0.979	0.980	0.986	0.999
300	0.639	0.842	0.939	0.967	0.985	0.998	0.998	0.997	0.997
400	0.715	0.874	0.948	0.977	0.985	0.991	0.993	0.995	0.997
500	0.735	0.887	0.949	0.979	0.987	0.994	0.995	0.994	0.998
1000	0.819	0.937	0.983	0.997	0.998	1.000	0.999	0.999	0.999
2000	0.989	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 11									
Rejection frequencies of Robinson's (1994a) tests (with white noise u_t) in a STUR model									
True model: Seasonal STUR model (16)									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.351	0.150	0.046	0.071	0.229	0.482	0.683	0.801	0.874
200	0.792	0.459	0.188	0.241	0.526	0.786	0.897	0.944	0.963
300	0.898	0.610	0.258	0.333	0.662	0.868	0.947	0.973	0.982
400	0.930	0.667	0.300	0.399	0.740	0.926	0.968	0.989	0.995
500	0.931	0.705	0.356	0.454	0.784	0.934	0.977	0.987	0.996
1000	0.963	0.791	0.471	0.600	0.882	0.975	0.995	0.997	0.998
2000	0.999	0.976	0.945	0.869	0.991	0.993	0.999	1.000	1.000

TABLE 12									
Rejection frequencies of Robinson's (1994[a]) tests (with AR (1) u_t) in a STUR model									
True model: Seasonal STUR model (16)									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.083	0.157	0.339	0.654	0.852	0.934	0.973	0.988	0.991
200	0.289	0.232	0.488	0.800	0.935	0.975	0.987	0.993	0.998
300	0.508	0.282	0.546	0.872	0.964	0.981	0.989	0.995	0.997
400	0.638	0.370	0.618	0.914	0.979	0.996	0.999	0.999	0.999
500	0.684	0.410	0.666	0.928	0.984	0.995	0.998	0.999	0.999
1000	0.798	0.701	0.893	0.995	0.999	1.000	1.000	1.000	1.000
2000	0.988	0.982	0.994	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 13									
Rejection frequencies of Robinson's (1994a) tests (with white noise u_t) in a STUR* model									
True model: Seasonal STUR* model (16) and (17)									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.353	0.213	0.120	0.102	0.209	0.367	0.528	0.635	0.716
200	0.827	0.570	0.323	0.296	0.449	0.654	0.788	0.854	0.892
300	0.896	0-691	0.460	0.395	0.560	0.745	0.866	0.908	0.926
400	0.931	0.752	0.464	0.434	0.628	0.811	0.903	0.952	0.967
500	0.927	0.771	0.513	0.485	0.676	0.837	0.919	0.957	0.971
1000	0.954	0.912	0.893	0.877	0.934	0.995	1.000	1.000	1.000
2000	1.000	0.999	0.994	0.991	0.996	1.000	1.000	1.000	1.000

TABLE 14									
Rejection frequencies of Robinson's (1994a) tests (with AR (1) u_t) in a STUR* model									
True model: Seasonal STUR* model (16) and (17)									
T	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
100	0.105	0.133	0.285	0.490	0.688	0.803	0.861	0.895	0.912
200	0.306	0.227	0.379	0.655	0.822	0.893	0.920	0.930	0.942
300	0.587	0.397	0.476	0.728	0.870	0.924	0.946	0.959	0.965
400	0.677	0.424	0.528	0.807	0.916	0.956	0.972	0.978	0.981
500	0.722	0.502	0.564	0.821	0.934	0.960	0.978	0.983	0.988
1000	0.912	0.823	0.856	0.912	0.974	0.999	1.000	1.000	1.000
2000	0.994	0.946	0.957	0.996	1.000	1.000	1.000	1.000	1.000