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FATIGUE STRENGTH AND DURABILITY ANALYSIS BY NORMALISED EQUIVALENT STRESS FUNCTIONALS

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ABSTRACT

Generalised fatigue durability diagrams are considered for a material under multi-axial loading given by a (non-regularly) oscillating function of time. Phenomenological strength conditions under such loading are presented in terms of the normalised equivalent stress functionals both for quasi-ductile rupture and brittle fracture. Examples of the functionals are presented for some known and new durability models including those sensitive to the load sequence. Some complex strength conditions applicable to the durability description at fatigue, creep, dynamic loading and their combinations are presented.

INTRODUCTION

In the traditional local approach to cyclic strength of a body without cracks or other singular stress concentrators, the fatigue durability in a considered point *y* is evaluated from the inequality

$$\omega^N(\{\sigma(y)\};n) < 1, \qquad (1)$$

where the damage measure $\omega^N(\{\sigma(y)\};n)$ is a functional of the loading history $\{\sigma(y)\}=\{\sigma^c(y,m)\}_{m=1,2...}$ at the point *y*, and *m*,*n* are the cycle numbers. When the number of cycles becomes sufficiently large, such that inequality (1) is violated (transfers into the equality), fracture under cyclic loading occurs at the point *y*. A particular form of $\omega^N(\{\sigma(y)\};n)$ for a non-periodic cycling (i.e. with varying cycles $\sigma(m)$) is related to a particular damage accumulation law. Under the popular Palmgren-Miner hypothesis of linear damage accumulation,

$$\omega^N(\{\sigma(y)\};n) = \sum_{m=1}^{n} \frac{1}{n^{0*}[\sigma^c(m,y)]}, \qquad (2)$$

where $n^{0*}(\sigma)$ is the number of cycles to rupture in the process *periodic* in time and homogeneous in space, given by an appropriate S–N diagram with all the cycles equal to the cycle $\sigma^c(m,y)$. However the linear accumulation rule does not describe the durability dependence on the different cycle sequence and some attempts to improve it seem to be not sufficiently justified [13, 15]. Even more methodical problems appear when the process under consideration is presented by *random (with unclosed cycles)* oscillations where the known procedures for a reduction of such processes to cyclic ones (e.g. the rain flow method) give not accurate prediction or not applicable at all (say, for out-of-phase random multiaxial oscillations).

On the other hand, when a body with a crack is considered under high cyclic fatigue loading, the Paris type law of crack propagation at a crack front point y

$$\frac{da}{dn} = f(K_1(y,n), K_2(y,n), K_3(y,n)) \tag{3}$$

is usually used, where f is a material function.

A common practice of a body fatigue life prediction is to use first a condition of type (1) up to the instant (the cycle number $n = n^*\{\sigma(y)\}$) when the condition will be violated at a point y of a considered body Ω, assuming that a crack with a length $2a_0$ arises at that point. Then one uses equation (3) with the initial condition $a(n^*\{\sigma\}) = a_0$ for evaluation of the number of cycles $n^*(\{\sigma\};\Omega)$ to separation of the body Ω into pieces or to unstable crack growth. But the value a_0 being a key issue for the fatigue crack propagation prediction by (3) is often not clearly fixed or is connected with the measuring ability of an equipment available. For this reason it would be preferable to have a unique approach of the fatigue life prediction from a stage of a virgin material without cracks to the crack propagation through the material damaged on previous cycles, up to a size when the crack becomes unstable.

Both conditions (1) and relationship (3) are local, since they use the stresses $\sigma_{ij}(y)$ or the stress intensity factors $K_i(y)$ (i=1,2,3), characteristics of the stress field only at one point y, a smooth body point or a crack tip. Such conditions can not describe the scale effect (for short cracks) caused by materials micro-inhomogeneity. They can be separately used either for smooth bodies or for bodies with crack-like stress concentrators but can not be used for bodies with singular concentrators which give rise to stress singularities different from the square root, occurring e.g. in corner points, in bonded bodies and so on. Moreover, the damage functional ω, a material property for a body without crack, appears to be not connected in such approach with the material function f for the same body with a crack.

A way to solve the problems connected with the locality of the traditional approaches, for time and history independent, e.g., elastic materials under monotone loading, is an application of non-local strength conditions and fracture criteria (see e.g. [11, 3, 4, 7, 2]). The non-local conditions use stresses or strains obtained on the assumption of solid macro-homogeneity by solving usual local boundary value problems (e.g. of elasticity). All micro-heterogeneity is then implicitly taken into account by a corresponding non-locality of the strength condition. Non-local condition parameters should be obtained from macro-tests, although they could in principle be estimated also by developing adequate micro-mechanical models where local strength conditions operate. Some applications of the non-local strength conditions are presented in [1].

Several modifications of the "static" non-local approaches to fatigue before a crack initiation were used in [17, 14, 12, 16, 18].

LOCAL CYCLIC NORMALISED EQUIVALENT STRESS FUNCTIONAL AND STRENGTH CONDITION

To overcome the difficulties with analysing processes sufficiently general in time and to create a more robust and experimentally verifiable tool for durability analysis under creep and dynamic loadings, some theoretical backgrounds of a functional approach to durability description under a loading program (process) $\sigma_{ij}(\tau)$ varying in time τ, were presented in [6, 8].

However, the time dependence is not essential for the durability description of the materials, whose rupture depends only on the loading sequence but not on time itself or on the loading rate. The pure fatigue rupture (without creep and ageing) is an example of such behaviour. Although such processes may still be considered with respect to time as a natural parameter and the approach described in [6, 8] holds true, a special cyclic parameterisations seem to be more relevant for the cyclic fatigue and the approach will be adopted here, see also [9].

Let $\{\sigma\} = \{\sigma^c(m)\}_{m=1,2...}$ be a cyclic, generally non-periodic multiaxial process independent on the space coordinates, where $\sigma^c(m)\} = \{\sigma_{ij}(\tau); \tau_{m-1} \le \tau \le \tau_m\}$ is an *m*-th (quasi-)cycle and τ is time. Denoting the durability (number of cycles to rupture) by $n^*\{\sigma\}$, the notion of the generalised S–N diagram $\lambda \to n^*\{\lambda\sigma\}$ may be introduced, where $\lambda \ge 0$ is a constant number. Some schematic forms of the S–N diagram (the step-wise graphs) can be obtained by combining one of the curves *a*, *b* or *c* with one of the curves *d*, *e* or *f* of Fig.1(a); the smooth graphs concern corresponding durability diagrams in time *t*, [6, 8]. Then the cyclic functional safety factor, the cyclic normalised equivalent stress functional and the cyclic endurance normalised equivalent stress can be defined: *The cyclic functional safety factor* $\underline{\lambda}^N(\{\sigma\},n)$ *is supremum of* $\lambda \ge 0$ *such that* $n^*\{\lambda''\sigma\} > n$ *for any* $\lambda'' \in [0,\lambda]$*; if there is no such λ, we take* $\underline{\lambda}^N(\{\sigma\},n) := 0$*. The cyclic normalised equivalent stress functional, CNESF, is defined as* $\underline{\Lambda}^N(\{\sigma\},n) := 1/\underline{\lambda}^N(\{\sigma\},n)$ *if* $\underline{\lambda}^N(\{\sigma\},n) \ne 0$ *and* $\underline{\Lambda}^N(\{\sigma\},n) := \infty$ *otherwise. The endurance normalised equivalent stress functional is* $\underline{\Lambda}^N_{th}(\{\sigma\}) := \underline{\Lambda}^N(\{\sigma\},\infty)$.

For a loading process $\{\sigma\}$, options of possible dependence of $\underline{\lambda}^N(\{\sigma\},n)$ and $\underline{\Lambda}^N(\{\sigma\},n)$ on *n* are schematically plotted on Fig.1(b) and Fig.1(c), step–wise graphs. It follows from the definition that the functional $\underline{\Lambda}^N(\{\sigma\},n)$ is positively homogeneous in $\{\sigma\}$ and monotonously non-decreasing in *n*. The CNESF $\underline{\Lambda}^N$ is a material characteristic which is not necessary connected with a geometrical, stiffness-related or abstract damage measure. Moreover, unlike abstract damage measures, it can be uniquely identified from the cyclic durability tests under homogeneous stress process fields. The cyclic stable strength condition for a cycle *n* can be written in the form

$$\underline{\Lambda}^N(\{\sigma\},n) < 1. \tag{4}$$

Using the above definition, any cyclic strength condition written in terms of a damage measure can be expressed in terms of a corresponding $\underline{\Lambda}^N$, although not always analytically, and easily generalised to include e.g. instant overloading, creep or dynamic effects.

Suppose, e.g., a material strength under a periodic cyclic loading is described by a power S–N diagram $n^*\{\sigma\} = \|| \sigma^c / \sigma^* \||^{-b}$, where $\||\sigma^c\|| = \max(|\sigma_{\max}|,|\sigma_{\min}|)$, and $\sigma^*(R)$ and $b(R)$ are material characteristics depending generally on the asymmetry ratio *R*. Then for a uniaxial cyclic

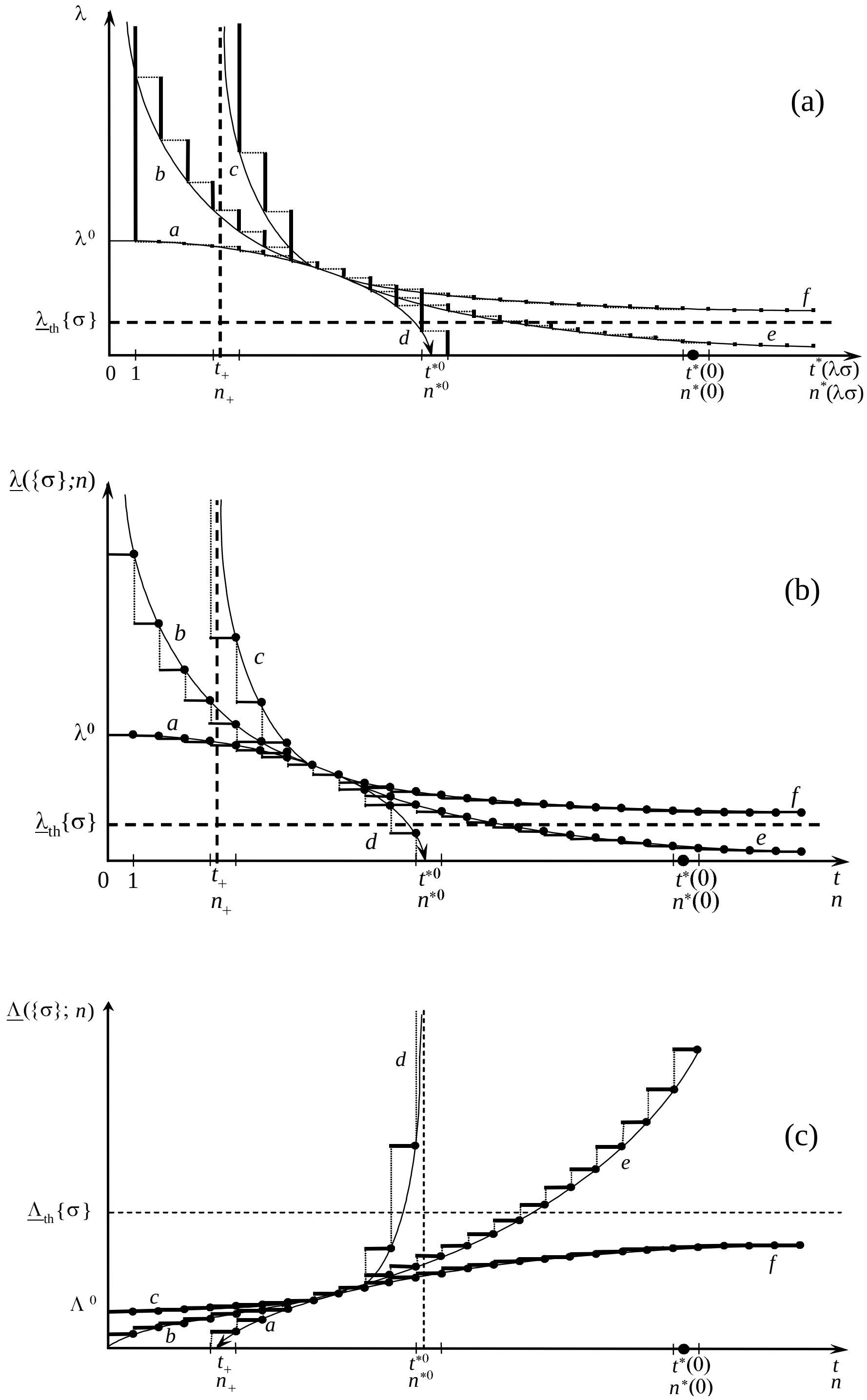

Figure 1: (a) S–N diagram for a process $\{\sigma\}$. (b) Safety factor vs. n and t for the process. (c) Normalised equivalent stress vs. n and t for the process.

(non-periodic) loading with a constant R or when b is independent on R, the Palmgren-Miner linear accumulation rule leads to the following CNESF and strength condition

$$\underline{\Lambda}_1^N(\{\sigma\},n)=\left[\sum_{m=1}^{n}\left(\frac{|||\sigma^c(m)|||}{\sigma^*(R(m))}\right)^b\right]^{1/b}=\left[\int_0^n\left(\frac{|||\sigma^c(m)|||}{\sigma^*(R(m))}\right)^b dm\right]^{1/b}<1, \tag{5}$$

where $|||\sigma^c(m)|||$ is a piece-wise constant function of the cycle number m. This expression is easily generalised to a multiaxial loading when either b is independent on the cycle characteristics or the cyclic loading is *self–similar*. To do this, one should take $|||\sigma^c(m)|||$ as a norm of the tensor function σ^c describing the stress tensor behaviour on the m-th cycle, for example, $|||\sigma^c(m)|||=\sup_{\sigma\in\sigma^c(m)}|\sigma|$, where $|\sigma|$ denotes a matrix norm of tensor σ_{ij}, for example, $|\sigma|=\sqrt{\sum_{i,j=1}^{3}\sigma_{ij}^2}$; $\sigma^*(R(m))=\sigma^*\left(\sigma^c(m)/|||\sigma^c(m)|||\right)$ and $b=b\left(\sigma^c/|||\sigma^c|||\right)$ should be taken as positive material characteristics in the power S–N diagram for the corresponding multiaxial *periodic* process depending generally on the cycle σ^c shape in the stress space but not on the cycle norm $|||\sigma^c|||$.

If one would like to take into account an influence of instantaneous overloads of material, one can introduce a new CNESF and arrive at the strength condition in the form

$$\underline{\Lambda}_1^{IN}(\{\sigma\},n(t))=\sup_{0\le t'\le t}\left\{\frac{\sigma_{eq}(\sigma(t'))}{\sigma_r}+\underline{\Lambda}_1^N(\{\sigma\},n(t'))\right\}<1, \tag{6}$$

where $\sigma_{eq}(\sigma)$ is e.g. von Mises, Tresca or other instantaneous equivalent stress; σ_r is a material strength under uniaxial monotone tensile loading; $n(t)$ is an integer-valued function of time t.

If one would like to take into account an influence of both the instantaneous overloads of material and creep durability, one can add a term $\underline{\Lambda}_1^T(\sigma,t')=\left[\frac{1}{\sigma_T^*}\int_0^{t'}|\sigma(\tau)|^{b_T}d\tau\right]^{1/b_T}$ connected with the Robinson rule of time-dependent damage accumulation and power time-durability diagram [8], and arrive at another one complex NESF and a local strength condition,

$$\underline{\Lambda}_1^{ITN}(\{\sigma\},n(t))=\sup_{0\le t'\le t}\left\{\frac{\sigma_{eq}(\sigma(t'))}{\sigma_r}+\underline{\Lambda}_1^T(\sigma,t')+\underline{\Lambda}_1^N(\{\sigma\},n(t'))\right\}<1. \tag{7}$$

Note that both strength conditions (6) and (7) lead to an accumulation rule sensitive to the loading sequence. One can create not only sums but also other homogeneous combinations of those three terms to get other possible simple forms of the NESF describing instant, time-dependent and cycle-dependent effects on the durability.

Let us consider another variant of an accumulation rule sensitive to the loading sequence. In contrast to the Palmgren-Miner linear accumulation rule for partial life–times, we can write a linear accumulation rule for partial increments of CNESF. Let $\sigma^*(R,n)$ be an arbitrary classical S–N diagram for a *periodic* uniaxial process with an asymmetry ratio R. Then the CNESF and strength condition for a cyclic non-periodic uniaxial process $\{\sigma\}$ can be taken in the following form [9],

$$\underline{\Lambda}_1^N(\{\sigma\},n) = \max_{1\le n'\le n}\left\{\frac{|||\,\sigma^c(n')\,|||}{\sigma^*(R(n'),1)} + \sum_{m=1}^{n'-1}|||\,\sigma^c(m)\,|||\left(\frac{1}{\sigma^*(R(m);n'-m+1)} - \frac{1}{\sigma^*(R(m);n'-m)}\right)\right\} < 1, \tag{8}$$

For a multiaxial cyclic loading, one should take $|||\,\sigma^c(m)\,|||$ as a norm of the tensor function σ^c on the m-th cycle and $\sigma^*(R(m),n) = \sigma^*(\sigma^c(m)/|||\,\sigma^c(m)\,|||, n)$. One could check that (8) interpolates the S–N diagram, that is, degenerates into the strength condition $|||\,\sigma^c\,||| < \sigma^*(R,n)$ for a *periodic* process $\sigma^c(n) = \sigma^c$. Note that the strength condition (8) is sensitive to the loading sequence in contrast to the Palmgren–Miner rule. A nonlinear CNESF increment accumulation rule is also presented in [9]. Both of the forms can be also combined with the creep strength conditions as above.

One or another CNESF (5)-(8) may better describe strength and durability of a particular material under a particular environment. If a chosen CNESF gives excessive deviations from experimental data, it can be considered as a first approximation and is to be refined by an identification procedure.

Local Quasi-Ductile Cyclic Strength Condition. For a stress field moderately inhomogeneous in the space coordinates, the so-called local approach can be employed, where it is supposed that rupture at a material point depends only on the stress (cyclic) history at the same point and does not depend on that at other points of the body. This leads to a *local* phenomenological cyclic stable strength condition for a point *y*,

$$\underline{\Lambda}^N(\{\sigma(y)\},n) < 1, \tag{9}$$

where $\underline{\Lambda}^N(\{\sigma(y)\},n)$ may be one of the NESFs (5)-(8). There is no rupture in the body Ω if inequality (9) is satisfied at all points $y \in \Omega$ but the body rupture initiates on the cycle $n^*\{\sigma(\Omega)\}$ when the inequality violates at least at one point y^*. During the next cycles, the rupture zone $Y^*(n)$ will propagate and the body will diminish from $\Omega(0) = \Omega$ to $\Omega(n) = \Omega \setminus Y^*(n)$ on a cycle *n*, if we consider the rupture points as excluded (crumbled out) of the body. One can track the zone $Y^*(n)$ taking into account the stress field change on each cycle due to the body shape change. Such scenario will evidently lead to a *volumetric (quasi-ductile)* cyclic rupture propagation.

Local Brittle Cyclic Strength Condition. To describe cyclic fracture, i.e. crack initiation and propagation under cycling loading, one have to analyse the brittle strength, that is strength at a particular point *y* along a particular infinitesimal plane with a normal vector $\vec{\zeta}$ at that point. The local brittle cyclic strength condition *for a plane* $\vec{\zeta}$ *at a point y* can be taken in the form

$$\underline{\Lambda}^N(\{\sigma^c(y)\};n,y,\vec{\zeta}) < 1, \tag{10}$$

where $\underline{\Lambda}^N(\{\sigma^c(y)\};n,y,\vec{\zeta})$ is a *local brittle* cyclic normalised equivalent stress functional defined similar to the above and is a material characteristics positively homogeneous in $\{\sigma_{ij}^c\}$ and non-decreasing in *n*. Particularly, the brittle CNESF can be taken in one of the forms (5)-(8), where the stress tensor loop sequence $\{\sigma_{ij}^c(m,y)\}_{m=1,2,..}$ may be replaced by the stress vector loop sequence $\{\vec{\sigma}^c(m;y,\vec{\zeta})\}_{m=1,2,..}$ on the plane $\vec{\zeta}$.

There is no fracture in the body Ω if inequality (10) is satisfied on all planes $\vec{\zeta}$ at all points $y\in\Omega$ but a crack initiates on the plane $\vec{\zeta}^*(y^*)$ at the point y^* during the cycle $n^*(\Omega)$, where and when the inequality violates. If the inequality is violated on a set $\tilde{Y}^*$ of points $\tilde{y}^*$ during the cycle, one can take the point y_0^* and the plane $\vec{\zeta}^*(y_0^*)$ where $\underline{\Lambda}^N(\{\sigma(\Omega)\};n,y_0^*,\vec{\zeta}(y_0^*))$ reaches maximum as the crack initiation point and plane. Then one can determine the crack $Y^*\in\tilde{Y}^*$, y^* ($y^*\in Y^*$) on the cycle $n^*(\Omega)$ as a surface passing through the point y_0^* and tangent to the planes $\vec{\zeta}^*(y^*)$ where $\underline{\Lambda}^N(\{\sigma(\Omega)\};n,y^*,\vec{\zeta})$ reaches maximum in $\vec{\zeta}$. During the next cycles, the crack set Y^* propagates and one can track it using the same procedure and taking into account the stress field redistribution on each cycle due to the body shape change when a new portion of Y^* becomes a part of the body boundary. This algorithm principally allows to describe the growth of the CNESF $\underline{\Lambda}^N(\{\sigma(\Omega)\};n,y^*,\vec{\zeta})$ on each plane at each point before and after the fracture initiation and the cyclic crack propagation through the damaged material points and planes.

As shown in [10], this algorithm together with local strength condition (10) and the CNESF given by (5) in the integral form can be used for crack initiation as well as crack propagation analysis. However, it works only for the S–N diagram power *b*<2 and the crack start delay is not predicted. The CNEFs like (6)-(8) with an instant terms are not applicable to fatigue crack analysis due to the stress singularity at the crack tip unless their corresponding non-local counterparts are used.

NON-LOCAL CYCLIC NORMALISED EQUIVALENT STRESS FUNCTIONAL AND STRENGTH CONDITION

We merge further the above functional approach to the cyclic strength with the non-local approach of [3], to analyse strength and durability under general cyclic in time and highly inhomogeneous stress fields and predict both the crack initiation and propagation as a united process.

Non-local Cyclic Quasi-Ductile Strength and Durability. *Let Ω be a body. We will suppose that cyclic strength of a point y depends not only on the cyclic stress tensor history at the point, $\{\sigma_{ij}^c(m,y)\}_{m=1,2,..}$, but also on the stress history in its neighbourhood and generally, in the whole of the body, $\{\sigma_{ij}^c(m,x)\}_{m=1,2,..}$, $x\in\Omega$.*

Then we can re-define for a point y the previous reasonings, replacing the cyclic process $\{\sigma\}=\{\sigma_{ij}^{c}(m)\}_{m=1,2,..}$ by the cyclic process field $\{\sigma^{c}(\Omega)\}=\{\sigma_{ij}^{c}(m,x)\}_{m=1,2,..}$, $x\in\Omega$ and arrive at the notions of non-local cyclic durability $n^{*\Theta}\{\sigma^{c}(\Omega);y\}$ and non-local CNESF $\underline{\Lambda}^{N\Theta}(\{\sigma^{c}(\Omega)\};n,y)$ *for a point* $y\in\Omega$. The non-local cyclic stabile strength condition on a cycle n at a point y can be written in the form

$$\underline{\Lambda}^{N\Theta}(\{\sigma^{c}(\Omega)\};n,y)<1. \tag{11}$$

The simplest examples of the point non-local CNESFs and strength conditions one can obtain by replacing the local stress $\sigma_{ij}(\tau,y)$ by its non-local counterpart $\sigma_{ij}^{\Theta}(\tau,y)$ in the local CNESFs, e.g. in (5)-(8). Here $\sigma_{ij}^{\Theta}(\tau,y)$ is a weighted average of $\sigma_{ij}(\tau,x)$ along some neighbourhood $\Omega^{\Theta}(\Omega;y)$ of y. That is, one can take,

$$\underline{\Lambda}^{N\Theta}(\{\sigma^{c}(\Omega)\};n,y)=\underline{\Lambda}^{N}(\{\sigma^{c\Theta}(\cdot,y);n),\qquad \sigma_{ij}^{\Theta}(\tau;y)=\int_{\Omega^{\Theta}(\Omega;y)}w(y,x)\sigma_{ij}(\tau;x)dx \tag{12}$$

and $w(x,y)$ as well as $\Omega^{\Theta}(\Omega,y)\subset\Omega$ are characteristics of material and (generally) of body Ω shape, such as $\int_{\Omega^{\Theta}(\Omega,y)}w(y,x)dx=1$. Particularly, if $w(y,x)$ equals to the Dirac delta-function $\delta(y-x)$, then the non-local stress $\sigma_{ij}^{\Theta}(y)$ degenerates into the local one, $\sigma_{ij}(y)$. In another particular case, $\Omega^{\Theta}(\Omega,y)$ can be taken as a ball of a radius d (material parameter) with the center in y (or as intersection of the ball with Ω for y near the boundary of Ω) and $\sigma_{ij}^{\Theta}(y)=\dfrac{1}{\left|\Omega^{\Theta}(\Omega;y)\right|}\int_{\Omega^{\Theta}(\Omega;y)}\sigma_{ij}(x)dx$ where $\left|\Omega^{\Theta}(\Omega;y)\right|$ is the volume of $\Omega^{\Theta}(\Omega,y)$. Note that (12) does not exhaust all possible forms of non-locality but can serve as a good first approximation. Its introduction smoothes sharp stress fields and makes possible to analyse the fatigue strength and durability of structure elements with singular stress concentrators and describe size effects.

Non-local Cyclic Brittle Strength and Durability. In the non-local version of brittle fracture, *we will suppose that cyclic strength of an infinitesimal plane* $\vec{\zeta}$ *at a point* $y\in\Omega$ *depends not only on the cyclic stress history at the point,* $\{\sigma_{ij}^{c}(m,y)\}_{m=1,2,..}$, *but also on the stress history in its neighbourhood and generally, in the whole of the body,* $\{\sigma_{ij}^{c}(m,x)\}_{m=1,2,..}$, $x\in\Omega$.

As above for a point, we then arrive at the notions of non-local cyclic durability $n^{*\Theta}\{\sigma(\Omega);y,\vec{\zeta}\}$, CNESF $\underline{\Lambda}^{N\Theta}(\{\sigma(\Omega)\};n,y,\vec{\zeta})$ and the non-local cyclic stable strength condition *for a plane* $\bar{\zeta}$ *at a point* $y\in\Omega$,

$$\underline{\Lambda}^{N\Theta}(\{\sigma(\Omega)\};n,y,\vec{\zeta})<1 \tag{13}$$

As before, one can obtain the simplest examples of the non-local CNESFs and strength conditions for a plane $\vec{\zeta}$ by replacing the local stress $\sigma_{ij}(\tau,y)$ by its non-local counterpart

$\sigma_{ij}^{\Theta}(\tau, y, \vec{\zeta})$ in the local CNESFs, e.g. in $\underline{\Lambda}_1^N(\{\sigma(y)\};n)$, $\underline{\Lambda}_1^{IN}(\{\sigma(y)\};n(t))$ or $\underline{\Lambda}_1^{INT}(\{\sigma(y)\};n(t))$ above. Here $\sigma_{ij}^{\Theta}(y, \vec{\zeta})$ is a weighted average of $\sigma_{ij}(x)$ along some neighbourhood (non-locality zone) $\Omega^{\Theta}(\Omega, y, \vec{\zeta})$ of y depending also on $\vec{\zeta}$. That is, one can take,

$$\underline{\Lambda}^{\Theta}(\{\sigma(\Omega)\};n,y,\vec{\zeta}) = \underline{\Lambda}(\{\sigma^{\Theta}(\cdot,y,\vec{\zeta})\};n),$$

$$\sigma_{ij}^{\Theta}(\tau;y,\vec{\zeta}) = \int_{\Omega^{\Theta}(\Omega;y,\vec{\zeta})} w(y,x,\vec{\zeta})\sigma_{ij}(\tau;x)dx \quad (14)$$

and $w(x,y,\vec{\zeta})$ as well as $\Omega^{\Theta}(\Omega, y, \vec{\zeta}) \subset \Omega$ are characteristics of material and (generally) of body Ω shape, such as $\int_{\Omega^{\Theta}(\Omega,y,\vec{\zeta})} w(y,x,\vec{\zeta})dx = 1$. Particularly, $\Omega^{\Theta}(\Omega, y, \vec{\zeta})$ can be taken as a disc of a diameter 2*d* (or a segment of a length 2*d* for a two-dimensional body Ω) in the plane $\vec{\zeta}$ with the centre in *y* (or as intersection of the disc/segment with Ω for *y* near the boundary of Ω), where *d* is considered as a material parameter, and $\sigma_{ij}^{\Theta}(y) = \frac{1}{\left|\Omega^{\Theta}(\Omega;y,\vec{\zeta})\right|}\int_{\Omega^{\Theta}(\Omega;y,\vec{\zeta})} \sigma_{ij}(x)dx$, where $|\Omega^{\Theta}(\Omega, y, \vec{\zeta})|$ is the area/length of $\Omega^{\Theta}(\Omega, y, \vec{\zeta})$.

Examples of the fatigue crack propagation numerical analysis using a brittle non-local CNESF (13) with $\underline{\Lambda}^N$ similar to (5) are presented in [10]. Such cyclic strength analysis is applicable for any power *b*>0 of the S–N diagram and predicts the crack start delay and short–crack effect.

CONCLUSION

Introduction of the notion of normalised equivalent stress functional and its use in the strength conditions allowed to unite the strength and durability description under static, fatigue, creep and dynamic loading and their combinations often appearing in practice. Explicit expressions of some new functionals are presented.

Unlike some damage measures usually used for such purposes, the functionals are mechanically meaningful material characteristics, which can be experimentally determined and verified. This makes a refinement of the CNESF approximations to reflect e.g. influence of the load sequence, more straightforward and justified than for damage measures.

The non-local versions of the CNESFs allow to unite the cyclic strength, durability and fatigue crack propagation analysis including crack initiation and propagation through the damaged material. It makes also possible to analyse the fatigue strength and durability of structure elements with singular stress concentrators and describe the short–crack effects and crack start delay, which was impossible by the local approach.

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