Diffraction of sound by a rigid screen with an absorbent edge

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Abstract

A solution is obtained for the problem of diffraction of a plane wave sound source by a semi-infinite plane. A finite region in the vicinity of the edge has an impedance boundary condition; the remaining part of the half plane is rigid.

The Problem which is solved is a mathematical model for a rigid barrier with an absorbing edge. It is found that the absorbing material, that comprises the edge, need only be of the order of a wavelength long to have approximately the same effect on the sound attenuation in the shadow region of the barrier, as a completely absorbent barrier. Also the softer the absorbent lining the greater the attenuation in the shadow of the barrier.

In the illuminated region a reduction in the sound intensity level can be achieved by a suitable choice of the absorptive material of the strip and its length. It is found that the effect of the absorptive strip is lost if its length is less than two wavelengths long. For a strip length of six wavelengths or more the system is equivalent to a wholly absorbing half plane.

Introduction

Noise from motorways, railways and airports can be shielded by a barrier which intercepts the line of hearing from the noise source to the receiver. The acoustic field in the shadow region of a barrier, (when transmission through the barrier is negligible) is due to diffraction at the edge alone. For this reason Butler [1] suggested that the region in the immediate vicinity of the edge should be lined with absorbent material to reduce the sound level in the shadow region. This technique has potential applications in engine noise shielding by aircraft wings. A question of some importance, therefore, is what effect does the absorptive quality and length of the absorbent material of the edge have on the sound attentuation in the shadow region?

The presence of an acoustically absorbing lining on a surface is usually described by an impedence relationship between the pressure (p) and the normal velocity fluctuation on the lining surface, see Morse and Ingard [2]. This gives rise to a boundary condition on the obsorbing lining of the form $\partial p/\partial n = ik\beta p$, ($\mathrm{Re}\ (\beta) > 0$) where the sound wave has harmonic time variation $e^{-i\omega t}$ and $k = \omega/c$, c is the velocity of sound, n the normal pointing into the lining, and β the complex specific admittance of the acoustic lining. The limiting case when the surface is ideally soft (pressure fluctuation vanishing on surface) is given by $|\beta| \to \infty$.

In a previous paper Rawlins [3] the situation where the edge region of a rigid barrier was connected to a soft ($|\beta| \to \infty$) strip was analysed. The analysis showed that the strip need only be the order of a wavelength long to have the same effect, on the sound attenuation in the shadow region as a soft half plane. By using the concept of a perfectly absorbing strip it was shown in a qualitative sense that the same was true for an absorbing strip. In the present work we shall consider the more general and practical case where β is finite.

If the wave length of the sound is much smaller than the length scale associated with the barrier, the diffraction process is governed to all intents and purposes by the solution to the canonical problem of diffraction by a semi-infinite rigid plane with an absorbent edge. We propose to solve this mixed boundary value problem.

In section one the canonical boundary value problem is formulated. In section two a solution to the boundary value problem is obtained in terms of two Fredholm integral equations of the second kind. The mathematical method used to obtain these Fredholm integral equations is Jones' method and the Wiener-Hopf technique, Noble [4]. In sections three and four approximate solutions of the integral equations are obtained. Section five consists of asymptotic expressions for the far field which are suitable for plotting graphs. These expressions are also conceptually easily related to the physical problem. Sections six and seven are the graphical results and the conclusions derived from them respectively.

1. Formulation of the boundary value problem

A semi-infinite plane is assumed to occupy $y = 0$, $x \leq 0$; see fig. 1. The half plane is assumed to be infinitely thin, and over the interval $-l < x < 0$ there / ...

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In a previous paper Rawlins [2] the situation where the edge region of the barrier was soft $(|\beta| \rightarrow \infty)$ was analysed. The analysis showed that in a qualitative sense the absorbing surface need only be the order of a wavelength long to have the same effect, on sound attenuation in the shadow region, as a completely absorbing half plane. In the present work we shall consider the more general and practical case where β is finite.

If the wave length of the sound is much smaller than the length scale associated with the barrier, the diffraction process is governed to all intents and purposes by the solution to the canonical problem of diffraction by a semi-infinite rigid plane with an absorbent edge. We propose to solve this mixed boundary value problem.

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1. Formulation of the boundary value problem

A semi-infinite plane is assumed to occupy $y = 0$, $x \leqslant 0$; see fig 1. The half plane is assume to be infinitely thin, and over the interval $-\ell < x < 0$

there is an absorbing substance, the remainder $-\infty < x < -l$ of the half plane is rigid. The perturbation velocity $\underset{\sim}{u}$ of the irrotational sound wave can be expressed in terms of a velocity potential $\chi(x,y)$ by $\underset{\sim}{u} = \operatorname{grad}\chi(x,y)$. The resulting pressure (p) in the sound field is given by $p = -\rho_0\, \partial\chi/\partial t$, where ρ_0 is the density of the ambient medium.

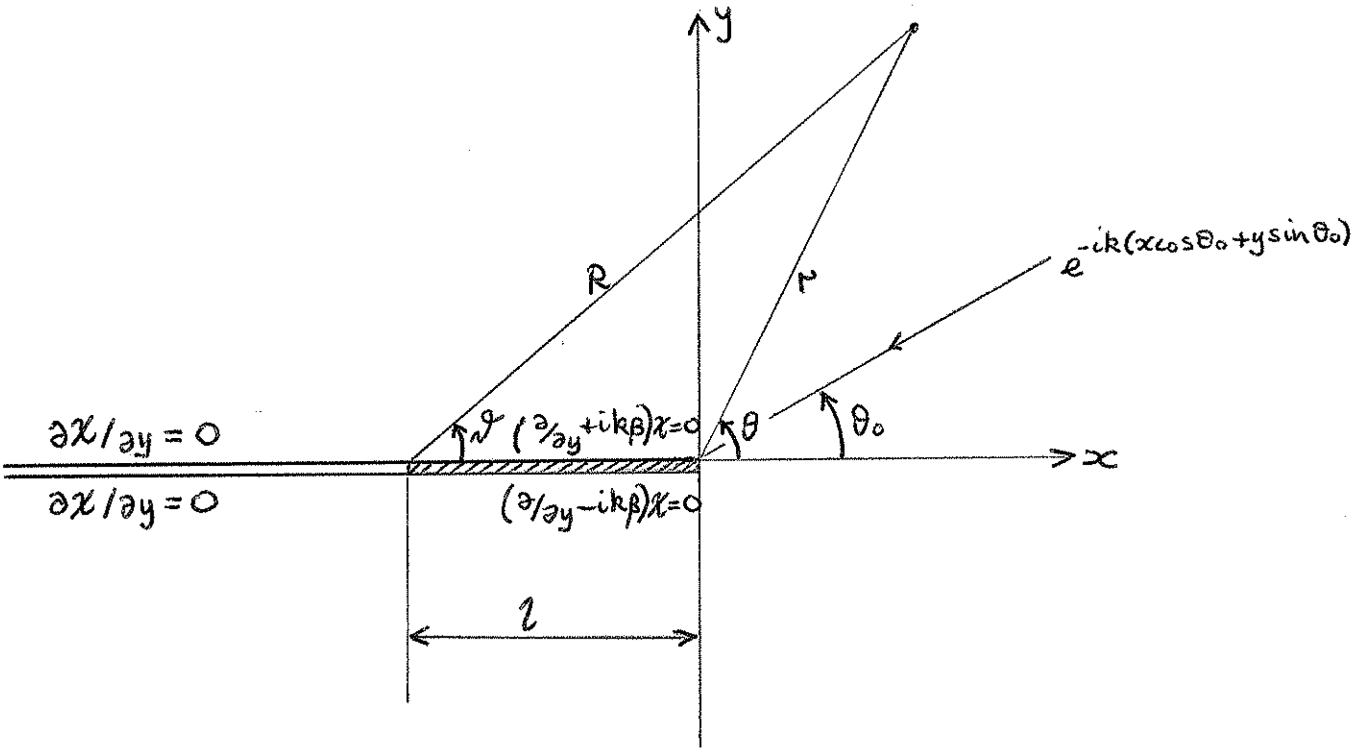

figure 1.

We shall assume that a given incident sound source potential field $\phi_0(x,y)\,e^{-i\omega t}$ is diffracted by the semi-infinite plane. In future work we shall drop the time harmonic variation factor $e^{-i\omega t}$. Then the boundary value problem becomes one of solving the wave equation

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right\}\chi(x,y) = 0; \tag{1}$$

subject to the boundary conditions

$$\frac{\partial\chi}{\partial y}(x, 0^\pm) = 0, \qquad (x < -l); \tag{2}$$

$$\left(\frac{\partial}{\partial y} \pm ik\beta\right)\chi(x, 0^\pm) = 0 \qquad (-l < x < 0); \tag{3}$$

$$\chi(x,0^+) = \chi(x,0^-), \quad \frac{\partial \chi}{\partial y}(x,0^+) = \frac{\partial \chi}{\partial y}(x,0^-), \quad (x>0); \tag{4}$$

where $\beta = \rho_0 c / Z$, $\mathrm{Re}(Z) > 0$, $k = \omega/c$, and c is the velocity of sound.

We shall assume that a solution can be written in the following form

$$\chi(x,y) = \phi_0(x,y) + \phi(x,y), \tag{5}$$

where $\phi(x,y)$ represents the perturbed field due to the presence of the half plane. For analytic convenience we let $k = k_r + ik_i$ $(k_r, k_i > 0)$, in which case for a unique solution of the boundary value problem (1) to (5), (see Peters and Stoker [4] we also require the satisfaction of the radiation condition

$$\phi(x,y) = O\left(\frac{e^{-k_i r}}{\sqrt{r}}\right), \quad \text{as} \quad r = \sqrt{(x^2+y^2)} \to \infty; \tag{6}$$

and also the "edge condition" see Jones [5]

$$\left.\begin{aligned} &\chi(x,0) = O(1) \quad ; \text{ and } \quad \frac{\partial \chi}{\partial y}(x,0) = O(x^{-1/2}) \quad , \text{ as } \quad x \to 0^+, \\ &\chi(x,0) = O(1) \quad , \text{ and } \quad \frac{\partial \chi}{\partial y}(x,0) = O((x+l)^{-1/2}), \text{ as } \quad x \to -l \end{aligned}\right\} \tag{7}$$

2. Solution of the boundary value problem

We introduce the Fourier transform

$$\Phi(\alpha,y) = \int_{-\infty}^{\infty} \phi(x,y)\, e^{i\alpha x}\, dx \tag{8}$$

and its inverse

$$\phi(x,y) = \frac{1}{2\pi} \int_{-\infty+i\tau}^{\infty+i\tau} \Phi(\alpha,y)\, e^{-i\alpha x}\, d\alpha, \tag{9}$$

where $\alpha = \sigma + i\tau$. The transform (8) and its inverse (9) will exist provided $-k_i < \tau < k_i$; this follows from the radiation condition (6).

Applying (8) to the equation (1) gives

$$\Phi(\alpha,y) = A(\alpha)\, e^{i\kappa y}, \quad (y>0), \tag{10}$$

$$= B(\alpha)\, e^{-i\kappa y}, \quad (y<0); \tag{11}$$

where $\kappa = \sqrt{(k^2-\alpha^2)}$ is defined on the cut sheet for which $Im(\kappa) > 0$ when $|Im(\alpha)| < k_i$. From the equations (10) and (11) we obtain

$$e^{-i\alpha l}\Phi_-(\alpha,0^+) + \Phi_1(\alpha,0^+) + \Phi_+(\alpha,0) = A(\alpha), \tag{12a}$$

$$e^{-i\alpha l}\Phi_-'(\alpha,0) + \Phi_1'(\alpha,0^+) + \Phi_+'(\alpha,0) = i\kappa A(\alpha) \tag{12b}$$

$$e^{-i\alpha l}\Phi_-(\alpha,0^-) + \Phi_1(\alpha,0^-) + \Phi_+(\alpha,0) = B(\alpha), \tag{13a}$$

$$e^{-i\alpha l}\Phi_-'(\alpha,0) + \Phi_1'(\alpha,0^-) + \Phi_+'(\alpha,0) = -i\kappa B(\alpha); \tag{13b}$$

where

$$\left.\begin{aligned} \Phi_+(\alpha,y) &= \int_0^\infty \phi(x,y)\, e^{i\alpha x}\,dx \\ \Phi_-(\alpha,y) &= \int_{-\infty}^{-l} \phi(x,y)\, e^{i\alpha(x+l)}\,dx \\ \Phi_1(\alpha,y) &= \int_{-l}^{0} \phi(x,y)\, e^{i\alpha x}\,dx \end{aligned}\right\} \tag{14}$$

The primes denote differentiation with respect to y, and the $\pm$ subscripts denote functions which are regular and analytic in the upper $(Im(\alpha) > -k_i)$ and lower $(Im(\alpha) < k_i)$ α-plane. Eliminating $A(\alpha)$ from equations (12a,b), and $B(\alpha)$ from (13a,b) and using the boundary condition (2) to obtain an expression for $\Phi_-'(\alpha,0)$ gives the following two equations

$$-e^{-i\alpha l}\Xi_-'(\alpha,0) + \Phi_1'(\alpha,0\pm) + \Phi_+'(\alpha,0) = \pm i\kappa\left[e^{-i\alpha l}\Phi_-(\alpha,0\pm) + \Phi_1(\alpha,0\pm) + \Phi_+(\alpha,0)\right], \tag{15a,b}$$

where

$$\Xi_-'(\alpha,0) = \int_{-\infty}^{-l} \frac{\partial\phi_0}{\partial y}(x,y)\, e^{i(x+l)}\,dx \tag{16}$$

Applying the Fourier transform to the boundary condition (3) gives

$$\Phi_1'(\alpha,0\pm) \pm ik\beta\,\Phi_1(\alpha,0\pm) = -\left[\Xi_1'(\alpha,0) \pm ik\beta\,\Xi_1(\alpha,0)\right], \tag{17a,b}$$

where

$$\Xi_1(\alpha,y)=\int_{-l}^{0}\phi_0(x,y)e^{i\alpha x}dx\,, \quad \Xi_1'(\alpha,y)=\frac{\partial}{\partial y}\Xi_1(\alpha,y)\,. \tag{18}$$

Eliminating $\Phi_1(\alpha,0^+)$ from (15a) and (17a) and $\Phi_1(\alpha,0^-)$ from (15b) and (17b) gives

$$\pm i\kappa\left[e^{-i\alpha l}\Phi_-(\alpha,0^\pm)\mp\{\Phi_1'(\alpha,0^\pm)+(\Xi_1'(\alpha,0)\pm ik\beta\,\Xi_1(\alpha,0))\}/(ik\beta)+\Phi_+(\alpha,0)\right]=-e^{-i\alpha l}\Xi_-'(\alpha,0)+\Phi_1'(\alpha,0^\pm)+\Phi_+'(\alpha,0)\,. \tag{19a,b}$$

Eliminating $\Phi_1'(\alpha,0^+)$ from (15a) and (17a) and $\Phi_1'(\alpha,0^-)$ from (15b) and (17b) gives

$$-e^{-i\alpha l}\Xi_-'(\alpha,0)\mp ik\beta\,\Phi_1(\alpha,0^\pm)-\left[\Xi_1'(\alpha,0)\pm ik\beta\,\Xi_1(\alpha,0)\right]+\Phi_+'(\alpha,0)=\pm i\kappa\left[e^{-i\alpha l}\Phi_-(\alpha,0^\pm)+\Phi_1(\alpha,0^\pm)+\Phi_+(\alpha,0)\right]\,. \tag{20a,b}$$

Subtracting equation (19a) from (19b) and adding the equation (20a) to (20b) gives on a slight rearrangement of the resulting expressions the following two equations:

$$e^{-i\alpha l}\Psi_-(\alpha)+L(\alpha)\Psi_1(\alpha)+\Psi_+(\alpha)=S(\alpha)\,, \tag{21}$$

$$e^{-i\alpha l}W_-(\alpha)+\sqrt{(k-\alpha)}\,L(\alpha)W_1(\alpha)+W_+(\alpha)=N(\alpha)\,, \tag{22}$$

where

$$\Psi_-(\alpha)=\left[\Phi_-(\alpha,0^+)+\Phi_-(\alpha,0^-)\right]/2\,, \quad \Psi_+(\alpha)=\Phi_+(\alpha,0)\,,$$

$$\Psi_1(\alpha)=i\left[\Phi_1'(\alpha,0^+)-\Phi_1'(\alpha,0^-)\right]/2\,, \quad S(\alpha)=\Xi_1(\alpha,0)\,,$$

$$W_-(\alpha)=-i\sqrt{(k-\alpha)}\left[\Phi_-(\alpha,0^+)-\Phi_-(\alpha,0^-)\right]/2\,, \quad W_+(\alpha)=\Phi_+(\alpha,0)/\sqrt{(k+\alpha)}$$

$$W_1(\alpha)=-ik\beta\left[\Phi_1(\alpha,0^+)+\Phi_1(\alpha,0^-)\right]/2\,, \quad N(\alpha)=\left[\Xi_1'(\alpha,0)+e^{-i\alpha l}\Xi_-'(\alpha,0)\right]/\sqrt{(k+\alpha)}$$

$$L(\alpha)=\left(1/k\beta+1/\kappa\right)=L_+(\alpha)L_-(\alpha)\,. \tag{23}$$

Before proceeding further with equations (21) and (22) it is necessary to know how the various quantities in (23) grow as $|\alpha|\to\infty$. The edge condition (7) means that the transformed functions satisfy the following growth estimates as $|\alpha|\to\infty$

$$\Phi_+(\alpha,0)\sim O(|\alpha|^{-1}),\quad \Phi_+'(\alpha,0)\sim O(|\alpha|^{-1/2}),\qquad \text{in}\quad Im(\alpha)>-ki$$

$$\Phi_-(\alpha,0)\sim O(|\alpha|^{-1}),\quad \Phi_-'(\alpha,0)\sim O(|\alpha|^{-1/2}),\qquad \text{in}\quad Im(\alpha)<ki$$

$$e^{i\alpha l}\Phi_1(\alpha,0)\sim O(|\alpha|^{-1}),\quad e^{i\alpha l}\Phi_1'(\alpha,0)\sim O(|\alpha|^{-1/2}),\qquad \text{in}\quad Im(\alpha)>-ki$$

$$\Phi_1(\alpha,0)\sim O(|\alpha|^{-1}),\quad \Phi_1'(\alpha,0)\sim O(|\alpha|^{-1/2}),\qquad \text{in}\quad Im(\alpha)<ki$$

Using the above asymptotic estimates in the expressions (23) gives as $|\alpha|\to\infty$

$$\Psi_-(\alpha)\sim O(|\alpha|^{-1}),\quad N_-(\alpha)\sim O(|\alpha|^{-1/2}),\quad \Psi_1(\alpha)\sim O(|\alpha|^{-1/2})$$

$$N_1(\alpha)\sim O(|\alpha|^{-1}),\quad L_-(\alpha)\sim O((k\beta)^{-1}),\qquad \text{in}\quad Im(\alpha)<ki;$$

$$\Psi_+(\alpha)\sim O(|\alpha|^{-1}),\quad N_+(\alpha)\sim O(|\alpha|^{-1}),\quad \Psi_1(\alpha)\sim O(|\alpha|^{-1}e^{-i\alpha l})$$

$$N_1(\alpha)\sim O(|\alpha|^{-1}e^{-i\alpha l}),\quad L_+(\alpha)\sim O((k\beta)^{-1})\qquad \text{in}\quad Im(\alpha)>-ki.$$

The equations (21) and (22) cannot be split by the standard Wiener-Hopf argument because of the second term on the left hand side of the equations. However using the above asymptotic estimates, and the technique used in Noble's book [3] pp196-199, these equations can be split and yield Fredholm integral equations of the second kind. Although in the equations (21) and (22) the right hand sides of the equality sign are of a more general form than that considered by Noble [3] the basic technique used [3] follows through, mutatis mutandis, and so we omit the exact details. Thus for equation (21), we obtain, noting that the coefficient of $\Psi_1(\alpha)$ is an even function of α (see Noble [3])

$$\frac{\Gamma_+(\alpha)}{L_+(\alpha)}=-\frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{[S(-t)+e^{itl}S(t)]}{L_-(t)(t+\alpha)}\,dt+\frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{e^{itl}\Gamma_+(t)}{L_-(t)(t+\alpha)}\,dt, \tag{24}$$

$$\frac{\gamma_+(\alpha)}{L_+(\alpha)}=-\frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{[S(-t)-e^{itl}S(t)]}{L_-(t)(t+\alpha)}\,dt-\frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{e^{itl}\gamma_+(t)}{L_-(t)(t+\alpha)}\,dt, \tag{25}$$

$$Im(\alpha)>-a>-ki;$$

where $\Gamma_+(\alpha) = \mathcal{Y}_+(\alpha) + \mathcal{Y}_-(-\alpha)$, $\quad \gamma_+(\alpha) = \mathcal{Y}_+(\alpha) - \mathcal{Y}_-(-\alpha)$. (26)

The coefficient of $\mathcal{N}_1(\alpha)$ in the equation (22) is not an even function of α and therefore the above asymptotic estimates and the technique of Noble [3] yield the slightly different expressions

$$\frac{\mathcal{N}_+(\alpha)}{L_+(\alpha)} = \frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic} \frac{e^{-itl}\mathcal{N}_-(t)\,dt}{L_+(t)(t-\alpha)} - \frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic} \frac{N(t)}{L_+(t)(\alpha-t)}\,dt, \tag{27}$$

$$Im(\alpha) > c > -ki;$$

$$\frac{\mathcal{N}_-(\alpha)}{\sqrt{(k-\alpha)}L_-(\alpha)} = -\frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id} \frac{e^{itl}\mathcal{N}_+(t)}{\sqrt{(k-t)}L_-(t)(\alpha-t)}\,dt - \frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id} \frac{e^{itl}N(t)}{\sqrt{(k-t)}L_-(t)(t-\alpha)}\,dt, \tag{28}$$

$$Im(\alpha) < d < ki.$$

The solution of the equations (24) to (28) ultimately gives the solution of the boundary value problem (1) to (5). An exact solution of these integral equations is too difficult, and therefore approximate solutions will be obtained by asymptotic methods. To be specific we shall assume that the incident sound field is given by the plane wave expression

$$\phi_0(x,y) = e^{-ik[x\cos\theta_0 + y\sin\theta_0]}, \qquad 0 < \theta_0 < \pi.$$

Hence (16) and (18) become

$$\mathcal{E}'_-(\alpha,0) = -k\sin\theta_0(\alpha - k\cos\theta_0)^{-1}e^{ikl\cos\theta_0},$$

$$\mathcal{E}_1(\alpha,0) = i\left[e^{-i[\alpha-k\cos\theta_0]l} - 1\right](\alpha - k\cos\theta_0)^{-1},$$

$$\mathcal{E}'_1(\alpha,0) = k\sin\theta_0\left[e^{-i(\alpha-k\cos\theta_0)l} - 1\right](\alpha - k\cos\theta_0)^{-1}, \tag{29}$$

so that

$$S(\alpha) = i\left[e^{-i(\alpha-k\cos\theta_0)l} - 1\right]/(\alpha - k\cos\theta_0), \tag{30}$$

$$N(\alpha) = -k\sin\theta_0/\{\sqrt{(k+\alpha)}(\alpha - k\cos\theta_0)\}. \tag{31}$$

3. Approximate solution of equations (24) and (25) for $kl \gg 1$.

REstricting the path of integration in the expression (24) to the band $k_i\cos\theta_0 < a < k_i$, see figure 2, and then substituting the expression for $S(\alpha)$, given by (30), into (24) and making the further substitution

$$\Gamma_+(\alpha) = G_+(\alpha) - \frac{i e^{ikl\cos\theta_0}}{(\alpha + k\cos\theta_0)} - \frac{i}{(\alpha - k\cos\theta_0)}, \tag{32}$$

gives

$$\frac{G_+(\alpha)}{L_+(\alpha)} = \frac{i e^{ikl\cos\theta_0}}{L_+(\alpha)(\alpha+k\cos\theta_0)} + \frac{i}{L_+(\alpha)(\alpha - k\cos\theta_0)} - \frac{1}{2\pi}\int_{-\infty+ia}^{\infty+ia} \frac{dt}{L_-(t)(t+\alpha)}$$

$$- \frac{e^{ikl\cos\theta_0}}{2\pi}\int_{-\infty+ia}^{\infty+ia} \frac{dt}{L_-(t)(t+\alpha)(t-k\cos\theta_0)} + \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia} \frac{e^{itl}\, G_+(t)}{L_-(t)(t+\alpha)}\, dt, \quad k_i\cos\theta_0 < a < k_i.$$

The first two integrals of the above expression can be evaluated by distorting the path of integration into the lower half of the t-plane. The only poles captured will be $t = -\alpha$ and $t = -k\cos\theta_0$, see figure 2. Thus

$$\frac{G_+(\alpha)}{L_+(\alpha)} = \frac{i}{L_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{i e^{ikl\cos\theta_0}}{L_-(k\cos\theta_0)(\alpha + k\cos\theta_0)}$$

$$+ \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia} \frac{e^{itl}\, L_+(t)\, G_+(t)\, dt}{L(t)(t+\alpha)}, \quad k_i\cos\theta_0 < a < k_i. \tag{33}$$

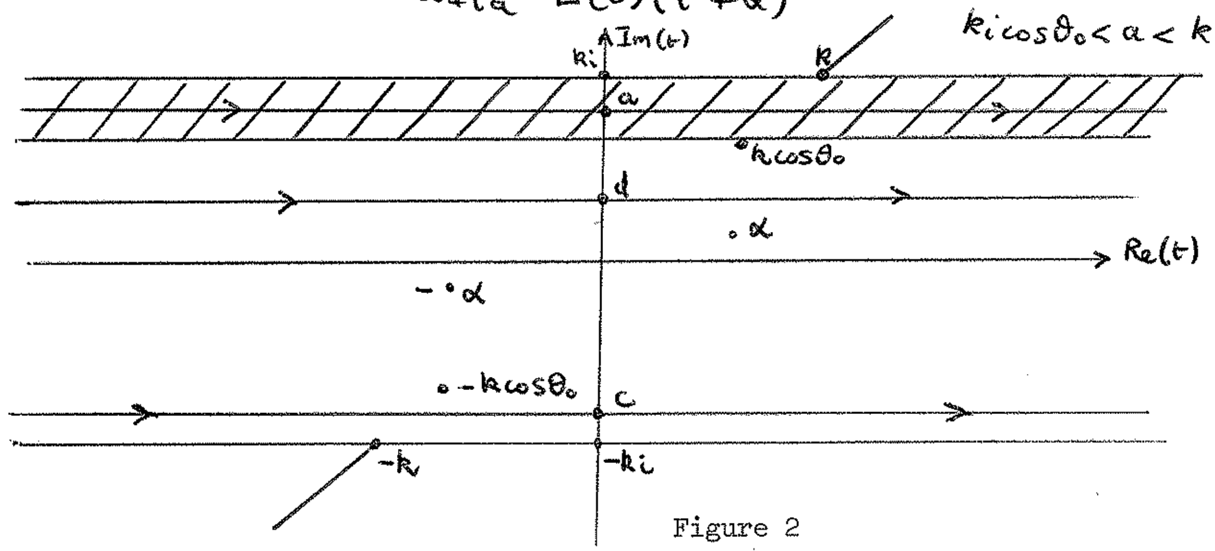

Figure 2

Equation (25) can be dealt with in a similar manner by substituting $S(\alpha)$, given by (30), into (25), where $ki\cos\theta_0 < a < ki$, and making the substitution

$$\gamma_+(\alpha) = g_+(\alpha) + \frac{i\,e^{ikl\cos\theta_0}}{(\alpha + k\cos\theta_0)} - \frac{i}{(\alpha - k\cos\theta_0)} . \tag{34}$$

Thus one obtains eventually

$$\frac{g_+(\alpha)}{L_+(\alpha)} = \frac{i}{L_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} - \frac{i\,e^{ikl\cos\theta_0}}{(\alpha + k\cos\theta_0)L_-(k\cos\theta_0)}$$
$$- \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia} \frac{e^{itl}L_+(t)\,g_+(t)\,dt}{L(t)(t+\alpha)}, \quad ki\cos\theta_0 < a < ki . \tag{35}$$

The equations (33) and (35) are too difficult to solve exactly and therefore an asymptotic technique, for $kl \gg 1$, is employed to obtain approximate solutions. We use the technique given by Jones [6], who considered the problem of diffraction by a strip.

We may write (33) in the form

$$G_+(\alpha) = \{S_1(\alpha) + \mathcal{G}_+(\alpha)\}L_+(\alpha) \tag{36}$$

where

$$S_1(\alpha) = \frac{i}{L_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{i\,e^{ikl\cos\theta_0}}{L_-(k\cos\theta_0)(\alpha + k\cos\theta_0)} \tag{37}$$

$$\mathcal{G}_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia} \frac{e^{itl}L_+(t)G_+(t)}{L(t)(t+\alpha)}\,dt . \tag{38}$$

Substituting (36) into (33) gives

$$\mathcal{G}_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia} \frac{e^{itl}}{L(t)(t+\alpha)}\left\{L_+^2(t)\left(S_1(t) + \mathcal{G}_+(t)\right)\right\}dt , \tag{39}$$
$$ki\cos\theta_0 < a < ki .$$

If the contour of integration in (39) is distorted into the region $\mathrm{Im}(\alpha) > a$ then the integral can be asymptotically approximated, for $kl \gg 1$, by the integral with its path of integration wrapped around the branch cut $t = k$.

The part of the integrand of (39) within the curly bracket is regular and analytic in this region and provided $\theta_0 \neq 0, \pi$ this term will vary slowly in the vicinity of t = k. Thus since the dominant part of the integrand comes from the region t = k the term in the curly bracket can be removed from under the integral sign and t can be replaced by k. The remaining integral can be replaced by the asymptotic approximation given by (2A) in appendix A. Hence

$$\mathcal{G}_+(\alpha) \simeq \frac{\sqrt{2k}\, L_+^2(k)}{(1-\lambda^2(\alpha+k))}\left\{ S_1(k) + \mathcal{G}_+(k) \right\} W(\alpha)$$

where $\mathcal{G}_+(k)$ is obtained by putting $\alpha = k$ in (40) and solving the resulting equation for $\mathcal{G}_+(k)$.

Similarly we may write (35) in the form

$$g_+(\alpha) = \left\{ S_2(\alpha) - \mathcal{g}_+(\alpha) \right\} L_+(\alpha) \tag{41}$$

where

$$S_2(\alpha) = \frac{i}{L_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} - \frac{i e^{ikl\cos\theta_0}}{L_-(k\cos\theta_0)(\alpha + k\cos\theta_0)}, \tag{42}$$

and

$$\mathcal{g}_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia} \frac{e^{itl} L_+(t)\, g_+(t)}{L(t)(t+\alpha)}\, dt. \tag{43}$$

Substituting (41) into (35) and using an argument similar to that of the last paragraph, in conjuction with the result (2A) of appendix A gives

$$\mathcal{g}_+(\alpha) = \frac{\sqrt{2k}\, L_+^2(k)}{(1-\lambda^2(\alpha+k))}\left\{ S_2(k) - \mathcal{g}_+(k) \right\} W(\alpha). \tag{44}$$

$\mathcal{g}_+(k)$ can be determined by letting = k in the expression (44) and solving for $\mathcal{g}_+(k)$. Using the expressions (26), (32), (34), (36) (40), (41) and (44) and some simple manipulation we obtain

$$\begin{aligned} Y_+(\alpha) = {} & \frac{-i}{(\alpha - k\cos\theta_0)}\left(1 - \frac{L_+(\alpha)}{L_+(k\cos\theta_0)}\right) \\ & + \sqrt{\frac{k}{2}} \cdot \frac{L_+^2(k) L_+(\alpha)}{(1-\lambda^2(\alpha+k))}\left\{ S_1(k) - S_2(k) + \mathcal{G}_+(k) + \mathcal{g}_+(k) \right\} W(\alpha). \end{aligned} \tag{45}$$

$$y_{-}(\alpha) = \frac{i e^{ikl\cos\theta_0}}{(\alpha - k\cos\theta_0)}\left(1 - \frac{L_{-}(\alpha)}{L_{-}(k\cos\theta_0)}\right)$$

$$+ \sqrt{\frac{k}{2}} \cdot \frac{L_+^2(k)L_{-}(\alpha)}{(1-\lambda^2(k-\alpha))}\left\{S_1(k) + S_2(k) + G_+(k) - ig_+(k)\right\} W(-\alpha). \tag{46}$$

4. Approximate solution of equations (27) and (28) for $kl \gg 1$

The method of obtaining an approximate solution for the equations (27) and (28) is slightly different from the method used in the last section. Substituting the plane wave expression for $N(\alpha)$, given by (31), into the expressions (27) and (28) gives

$$\frac{W_+(\alpha)}{L_+(\alpha)} = \frac{k\sin\theta_0}{2\pi i}\int_{-\infty+ic}^{\infty+ic} \frac{dt}{L_+(t)\sqrt{(k+t)}(t-k\cos\theta_0)(\alpha-t)} - \frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic} \frac{e^{-itl}W_{-}(t)}{(t-\alpha)L_+(t)}\,dt,$$

$$\operatorname{Im}(\alpha) > c > -k_i; \tag{47}$$

$$\frac{W_{-}(\alpha)}{\sqrt{(k-\alpha)}L_{-}(\alpha)} = \frac{k\sin\theta_0}{2\pi i}\int_{-\infty+id}^{\infty+id} \frac{e^{itl}}{L_{-}(t)(t-\alpha)(t-k\cos\theta_0)\sqrt{(k^2-t^2)}}\,dt$$

$$- \frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id} \frac{e^{itl}W_+(t)}{L_{-}(t)\sqrt{(k-t)}(\alpha-t)}\,dt, \qquad \operatorname{Im}(\alpha) < d < k_i\cos\theta_0. \tag{48}$$

The first integral of the expression (47) can be evaluated by distorting the path of integration into the upper half t-plane. Since $\operatorname{Im}(\alpha) > c > -k_i$ then the pole $t = \alpha$, and $t = k\cos\theta_0$ will give rise to residue contributions, see figure 2. Hence

$$\frac{W_+(\alpha)}{L_+(\alpha)} = \frac{-k\sin\theta_0}{L_+(\alpha)\sqrt{(k+\alpha)}(\alpha-k\cos\theta_0)} + \frac{\sqrt{(k(1-\cos\theta_0))}}{L_+(k\cos\theta_0)(\alpha-k\cos\theta_0)}$$

$$- \frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic} \frac{e^{itl}\{\sqrt{(k-t)}L_{-}(t)W_{-}(t)\}}{(t-\alpha)\sqrt{(k-t)}\,L(t)}\,dt, \qquad \operatorname{Im}(\alpha) > c > -k_i. \tag{49}$$

The evaluation of the first integral in the expression (48) is best achieved by distorting the path of integration into the upper half of the t-plane. However this requires a knowledge of the singularities of $L_{-}(t)$ in $\operatorname{Im}(t) > -k_i$.

It can be shown, by a method used in Rawlins [6], that the only singularities of $L(t)$ are the branch points at $t = \pm k$; no poles occur in the cut plane. Hence moving the path of integration vertically until it crosses the pole $t = k\cos\theta_0$, see figure 2, but not the branch point $t = k$ gives

$$\frac{\mathcal{W}_-(\alpha)}{\sqrt{(k-\alpha)}\,L_-(\alpha)} = \frac{-e^{ikl\cos\theta_0}}{L_-(k\cos\theta_0)(\alpha-k\cos\theta_0)}$$

$$+\frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id}\left\{\frac{k\sin\theta_0 L_+(t)}{\sqrt{(k+t)}(t-k\cos\theta_0)}+\mathcal{W}_+(t)L_+(t)\right\}\frac{e^{itl}}{L(t)\sqrt{(k-t)}(t-\alpha)}\,dt, \qquad (50)$$

$$k_i\cos\theta_0<d<k_i.$$

For $kl \gg 1$, the dominant contribution of the integral (49) comes from the region $t = -k$; and in the integral (50) from the region $t = k$. Provided $\theta_0 \neq 0$ the term in the curly bracket of the integrands of (49) and (50) are slowly varying in the vicinity of $t = \pm k$. We can therefore replace t by $-k$ in this part of the integrand of (49) and remove it from under the integral sign. Similarly we can replace t by k in the curly part of the integrand of (50) and remove it from under the integral sign. The integrals remaining can be replaced by the asymptotic approximations given by (6A) and (8A) in appendix A. Thus

$$\frac{\mathcal{W}_+(\alpha)}{L_+(\alpha)} = \frac{-k\sin\theta_0}{\sqrt{(k+\alpha)}\,L_+(\alpha)(\alpha-k\cos\theta_0)}+\frac{\sqrt{(k(1-\cos\theta_0))}}{L_+(k\cos\theta_0)(\alpha-k\cos\theta_0)}$$

$$+\sqrt{2k}\,L_+(k)\mathcal{W}_-(-k)\left(1-\lambda^2(k+\alpha)\right)^{-1}W(\alpha). \qquad (51)$$

$$\frac{\mathcal{W}_-(\alpha)}{\sqrt{(k-\alpha)}\,L_-(\alpha)} = \frac{-e^{ikl\cos\theta_0}}{L_-(k\cos\theta_0)(\alpha-k\cos\theta_0)}-\frac{\lambda\sqrt{2k}\,L_+(k)}{(1-\lambda^2(k-\alpha))}\left\{\frac{\sin\theta_0}{\sqrt{2k}\,(1-\cos\theta_0)}+\mathcal{W}_+(k)\right\}W(-\alpha) \qquad (52)$$

The constants $\mathcal{W}_\pm(\pm k)$ are obtained by putting $\alpha = k$ in (51) and $\alpha = -k$ in (52) and solving the resulting two equations for the two unknowns $\mathcal{W}_\pm(\pm k)$.

The expressions (45), (46), (51) and (52) in conjuction with the expressions (23) will now give explicit expressions for $\Phi_+(\alpha, 0)$, $\Phi_+'(\alpha, 0)$, $\Phi_-(\alpha, 0^+)$ and $\Phi_-(\alpha, 0^-)$. $A(\alpha)$ and $B(\alpha)$ can now be given in terms of these known quantities. For example multiply the equation (12a) throughout by $ik\beta$ and adding the resulting expression to (12b) and using the equations (16) and (18a) gives an explicit expression for $A(\alpha)$. Similarly an explicit expression for $B(\alpha)$ is obtained by multiplying the equation (13a) throughout by $ik\beta$ and subtracting the resulting expression from (13b) and using the expressions (16) and (18b). Thus in carrying out the above it is not difficult to show that

$$
\begin{aligned}
A(\alpha) = \frac{1}{\kappa L(\alpha)} \Bigg[& \frac{i}{(\alpha - k\cos\theta_0)} \left(\frac{L_+(\alpha)}{L_+(k\cos\theta_0)} - \frac{2L_-(\alpha) e^{-i(\alpha - k\cos\theta_0)l}}{L_-(k\cos\theta_0)} - \frac{\sqrt{(k(1-\cos\theta_0))}\sqrt{(k+\alpha)}L_+(\alpha)}{k\beta L_+(k\cos\theta_0)} \right) \\
& + e^{-i\alpha l} \frac{L_+(k)L_-(\alpha)}{(1-\lambda^2(k-\alpha))} \Bigg\{ \sqrt{\tfrac{k}{2}}\, L_+(k)\left(S_1(k) + S_2(k) + G_+(k) - i g_+(k)\right) - i\sqrt{2k}\,\lambda \left(\frac{\sin\theta_0}{\sqrt{2k}(1-\cos\theta_0)} \right. \\
& \left. + N_+(k) \right) \Bigg\} W(-\alpha) + \sqrt{\tfrac{k}{2}}\, \frac{L_+^2(k)L_+(\alpha)}{(1-\lambda^2(k+\alpha))} \left\{ S_1(k) - S_2(k) + G_+(k) + i g_+(k) \right\} W(\alpha) \\
& + \frac{\sqrt{2k}\, L_+(\alpha) L_+(k) \sqrt{(k+\alpha)}\, N_-(-k)}{ik\beta\, (1-\lambda^2(k+\alpha))} W(\alpha) \Bigg] .
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
B(\alpha) = \frac{1}{\kappa L(\alpha)} \Bigg[& \frac{i}{(\alpha - k\cos\theta_0)} \left(\frac{L_+(\alpha)}{L_+(k\cos\theta_0)} + \frac{\sqrt{(k(1-\cos\theta_0))}\sqrt{(k+\alpha)}L_+(\alpha)}{k\beta L_+(k\cos\theta_0)} \right) \\
& + e^{-i\alpha l} \frac{L_+(k)L_-(\alpha)}{(1-\lambda^2(k-\alpha))} \Bigg\{ \sqrt{\tfrac{k}{2}}\, L_+(k)\left(S_1(k) + S_2(k) + G_+(k) - i g_+(k)\right) + i\sqrt{2k}\,\lambda \left(\frac{\sin\theta_0}{\sqrt{2k}(1-\cos\theta_0)} + \right. \\
& \left. + N_+(k) \right) \Bigg\} W(-\alpha) + \sqrt{\tfrac{k}{2}}\, \frac{L_+^2(k)L_+(\alpha)}{(1-\lambda^2(k+\alpha))} \left\{ S_1(k) - S_2(k) + G_+(k) + i g_+(k) \right\} W(\alpha) \\
& - \frac{\sqrt{2k}\, L_+(k) L_+(\alpha) \sqrt{(k+\alpha)}\, N_-(-k)}{ik\beta\, (1-\lambda^2(k+\alpha))} W(\alpha) \Bigg] .
\end{aligned}
\tag{54}
$$

The solution to the boundary value problem is now known and is given by

$$\chi(x,y) = \phi_0(x,y) + \frac{1}{2\pi}\int_{-\infty+i\tau}^{\infty+i\tau} A(\alpha)\, e^{-i\alpha x + i\kappa y}\, d\alpha, \qquad (y>0) \tag{55}$$

$$= \phi_0(x,y) + \frac{1}{2\pi}\int_{-\infty+i\tau}^{\infty+i\tau} B(\alpha)\, e^{-i\alpha x - i\kappa y}\, d\alpha, \qquad (y<0), \tag{56}$$

$$-ki < \tau < ki\cos\theta_0 .$$

5. Asymptotic expressions for the far field, kr $\gg$ 1

For the purposes of plotting the polar diagram of the scattered far field the expressions (55) and (56) are approximated asymptotically for kr $\to\infty$.

This is achieved by using the result

$$\frac{1}{2\pi}\int_{-\infty+i\tau}^{\infty+i\tau} \frac{f(\alpha)\, e^{-i(\alpha X - \kappa|Y|)}}{\kappa(\alpha - k\cos\theta_0)}\, d\alpha \sim \frac{f(-k\cos\Theta)}{k\sqrt{(2\pi k\rho)}}\left[\frac{2|N|F(|N|)}{(\cos\Theta+\cos\theta_0)}\right] e^{i(k\rho+\pi/4)}$$

$$+ \operatorname{sgn}(\cos\theta_0)\, H\left[-\cos\theta_0(\cos\theta_0+\cos\Theta)\right] \frac{f(k\cos\theta_0)}{k|\sin\theta_0|}\, e^{-ik(X\cos\theta_0 - |Y|\sin\theta_0)} \tag{57}$$

as $k\rho\to\infty$;

where

$$X = \rho\cos\Theta, \quad Y = \rho\sin\Theta, \quad 0<\Theta<\pi,$$

$$F(|N|) = e^{-iN^2}\int_{|N|}^{\infty} e^{it^2}\, dt, \quad |N| = \sqrt{\frac{k\rho}{2}}\left|\frac{\cos\Theta+\cos\theta_0}{\sin\Theta}\right| .$$

The above result is obtained by the saddle point method, modified slightly because of the presence of the pole $\alpha = k\cos\theta_0$, see Rawlins [7] . When N is large the above result (57) can be simplified by using the asymptotic expansion for the Fresnel integral, viz

$$F(|N|) \sim i/(2|N|) + O(|N|^{-3}) \quad \text{as } |N|\to\infty . \tag{58}$$

The expression (58) can be used in (57) if no pole occurs at $\alpha = k\cos\theta_0$ in the integrand.

Thus using the result (57) and (58), for $0 < \theta_0 < \pi$, the expression (55) and (56) yield

$$
\begin{aligned}
\chi(r,\theta) &= I + RF_1 + D_1, && (\pi-\theta_0<\vartheta<\pi),\\
&= I + RF_2 + D_1, && (\vartheta<\pi-\theta_0<\theta),\\
&= I + D_1, && (0<\theta<\pi-\theta_0),\\
&= I + D_2, && (\theta_0-\pi<\theta<0),\\
&= D_2, && (-\pi<\theta<\theta_0-\pi);
\end{aligned}
\tag{59}
$$

where $x = r\cos\theta$, $y = r\sin\theta$, $-\pi \leq \theta \leq \pi$,

$$I = e^{-ikr\cos(\theta-\theta_0)} \tag{60}$$

$$RF_1 = e^{-ikr\cos(\theta+\theta_0)} \tag{61}$$

$$RF_2 = -(\beta-|\sin\theta_0|)(\beta+|\sin\theta_0|)^{-1}e^{-ikr\cos(\theta+\theta_0)} \tag{62}$$

$$
\begin{aligned}
D_1 = &-\frac{e^{i(kr-\pi/4)}}{\sqrt{(2\pi kr)}}\cdot\frac{2|Q|F(|Q|)}{(\cos\theta+\cos\theta_0)}\cdot\frac{\{L_+(k\cos\theta_0)\}^{-1}}{kL_+(k\cos\theta)}\left\{1-\frac{\sqrt{(1-\cos\theta_0)}\sqrt{(1-\cos\theta)}}{\beta}\right\}\\
&+\frac{e^{i(kR-\pi/4)}}{\sqrt{(2\pi kR)}}\cdot\frac{2|Q'|F(|Q'|)}{(\cos\vartheta+\cos\theta_0)}\cdot\frac{2\,e^{ikl\cos\theta_0}}{kL_-(k\cos\theta_0)L_-(k\cos\vartheta)}\\
&+\frac{e^{i(kr-\pi/4)}}{\sqrt{(2\pi kr)}}G_1(\theta,\theta_0)+\frac{e^{i(kR-\pi/4)}}{\sqrt{(2\pi kR)}}G_1(\vartheta,\theta_0);
\end{aligned}
\tag{63}
$$

$$
\begin{aligned}
D_2 = &-\frac{e^{i(kr-\pi/4)}}{\sqrt{(2\pi kr)}}\cdot\frac{2|Q|F(|Q|)}{(\cos\theta+\cos\theta_0)}\cdot\frac{\{L_+(k\cos\theta_0)\}^{-1}}{kL_+(k\cos\theta)}\left\{1+\frac{\sqrt{(1-\cos\theta_0)}\sqrt{(1-\cos\theta)}}{\beta}\right\}\\
&+\frac{e^{i(kr-\pi/4)}}{\sqrt{(2\pi kr)}}\cdot G_2(\theta,\theta_0)+\frac{e^{i(kR-\pi/4)}}{\sqrt{(2\pi kR)}}G_2(\vartheta,\theta_0).
\end{aligned}
\tag{64}
$$

In the above expressions for D_1 and D_2 the various quantities appearing are given by

$$Q = \sqrt{\frac{kr}{2}}\left(\frac{\cos\theta+\cos\theta_0}{\sin\theta}\right), \quad Q' = \sqrt{\frac{kR}{2}}\left(\frac{\cos\vartheta+\cos\theta}{\sin\vartheta}\right),$$

$$R^2 = r^2+l^2+2rl\cos\theta, \qquad \vartheta = \mathrm{sgn}(\theta)\cos^{-1}((l+r\cos\theta)/R),$$

$$G_{1,2}(\theta,\theta_0) = \frac{k^{3/2}|\sin\theta|L_+^2(k)L_-(k\cos\theta)}{(1-2(1-\cos\theta)\beta^{-2})}\left\{S_1(k)-S_2(k)+G_+(k)+ig_+(k)\right\}W(-k\cos\theta)$$

$$\pm\ \frac{k\sqrt{2}|\sin\theta|L_+(k)L_-(k\cos\theta)\sqrt{(1-\cos\theta)}W_-(-k)}{i\beta(1-2(1-\cos\theta)\beta^{-2})}W(-k\cos\theta) \qquad (65)$$

$$G_{1,2}(\vartheta,\theta_0) = \frac{k^{1/2}|\sin\vartheta|L_+(k)L_+(k\cos\vartheta)}{(1-2(1+\cos\vartheta)\beta^{-2})}\left\{\frac{k}{\sqrt{2}}L_+(k)(S_1(k)+S_2(k)+G_+(k)-ig_+(k))\right.$$

$$\left.\mp\ \frac{2i}{\beta}\left(\frac{\sin\theta_0}{\sqrt{2}(1-\cos\theta_0)}+k^{1/2}W_+(k)\right)\right\}W(k\cos\vartheta),$$

(66)

where the upper sign corresponds to the subscript 1 and the lower sign to the subscript 2. The expression for the total acoustic far field, given by the expressions (59) to (66), together with figure 2 give a picture of the physical fields. I represents the incident plane wave. The terms D_1 and D_2 represent the fields diffracted from the edges $x = 0$ and $x = -l$. The terms involving r and θ represent the contribution to the diffracted field from the edge $x = 0$. The terms involving R and ϑ give the field diffracted by the joint $x = -l$. The terms RF_1 and RF_2 represent the reflected field from the rigid part and the absorbing strip of the half plane respectively. These reflected fields will only exist in certain angular regions, which are defined in (59).

6. Graphical results

The expressions (59) were used to give praphical polar diagrams of the modulus of the far field, i.e. $|\chi(r,\theta)|$, for kr = 10π. The values of kl that were chosen were 0, 5π, 15π, and ∞ . The case for which $kl = 0$ and $kl = \infty$ correspond to the hard and absorbing half plane solutions respectively. The hard half plane diagrams were obtained from the known solution for this situation, Rawlins [7]. In the numerical calculation of $D_{1,2}$ the last two terms involving $G_{1,2}$ were dropped. This was done because of the considerable amount of numerical calculation required for these terms, which have a negligible effect on the far field for $kl > 1$, see Rawlins [2].

For an absorbing material the following values of specific impedance $\zeta = \xi + i\eta (= 1/\beta)$ seem to be of practical importance:

fibrous sheet $\xi = 0.5, \quad -1 < \eta < 3,$

perforated steel $0 < \xi < 2, \quad -1 < \eta < 3.$

Values of specific impedance which fall in the above range were used for the graphical plots. These polar diagrams are given in graphs (1) to (4) for $\theta_0 = 90^\circ, \xi = 1, \eta = 0.5$; and in graphs (5) to (8) for $\theta_0 = 45^\circ, \xi = 1.5, \eta = 0$. Graphs (9) to (11) give the attenuation of the sound field, i.e. $20 \log_{10} |\chi(r,\theta)|$, in the shadow region of the screen for $\theta_0 = 90^\circ$, $kl = 0$ (hard screen) $kl = 2\pi$, and various values of ζ.

From the graphs (9) to (11) it can be seen that the level of attenuation, compared to a hard half plane, in the shadow region is greater the larger $|\beta| (= |\zeta|^{-1})$ is. It has already been shown in [2] that provided $kk > 1$ the field in the shadow of the screen is to all intents and purposes the same as for $kl = \infty$.

The sound field level and orientation on the illuminated side of the screen are more acutely affected by the magnitude of kl . However it can be seen from graphs (3), (4) and (7), (8) that provided $k\ell \geq 15\pi$ the sound field level in this region is approximately the same as that for $k\ell \to \infty$.

7. Conclusions

From the graphical results we conclude that the sound attenuation in the shadow region increases as $|\beta|$ increases, i.e. the softer the absorbent material becomes, which makes up the edge, the greater the attenuation. It is only necessary to apply the absorbent material to within a wavelength of the edge of the regid screen to have the same effect, on the sound attenuation in the shadow region, as a screen covered completely with absorbent material. Experimental work by Butler [8] would seem to fully support the above conclusions.

The radiated sound intensity in the illuminated region $0 < \theta < \pi$ is due to the constructive/destructive interference between the incident wave; the diffracted fields from the edge $(0,0)$, and the joint $(0,-\ell)$ between the absorptive strip and rigid region of the screen; and the reflected waves from the absorptive and rigid regions of the screen. For a given value of θ_0, a value of the absorptive parameter given by $\beta = |\sin\theta_0|$ can make the reflected field RF_2 vanish identically in the region $\vartheta < \pi - \theta_0 < \theta$. This will reduce the maximum intensity of sound in this region. This choice of $\beta (= |\sin\theta_0|)$ will also make the first term of D_1, given by (63), vanish at $\theta = \pi - \theta_0$, thus further reducing the interference effect between the incident and diffracted fields in $0 < \theta < \pi$. The criterion that the reflected wave should vanish means physically that the strip absorbs all the energy incident upon it and does not reflect any. The dominant effect of RF_2 can be seen in the graphs (3) and (4) where $RF_2 = 0,23$, for $= 1+ i0.5$, $\theta_0 = 90^\circ$. For the graphs (7) and (8) $RF_2 = 0.03$ where $= 1.5 + i0$, $\theta_0 = 45^\circ$. The interference effects give greater sound intensity levels in graphs (3) and (4) than in graphs (7) and (8). It can also be seen from the graphs (1) to (8) that for $k\ell \geq 15$

the absorptive strip has the same effect on the sound level intensity in $0 < \theta < \pi$ as a completely absorbent screen ($kl = \infty$). The effectiveness of the absorptive strip is lost for $kl \leq 5\pi$, in the illuminated region.

The above conclusions suggest a possible design for a practical barrier which would reduce noise both in the illuminated and shadow region. The lengths and absorptive properties of the strip would need to be different for the lining on the illuminated and shadow side of the barrier. On the illuminated surface of the barrier we choose the length l_i to fall in the range $5\pi < kl_i < 15\pi$; and on the shadow side of the barrier we choose the length $_s$ such that $kl_s \simeq 2\pi$. The impedance ζ_s of the lining on the shadow side of the screen is chosen to give the softess lining mechanically possible. The admittance β_i of the lining on the illuminated side of the screen is chosen, for a given angle of incidence θ_0 , to minimise the reflection coefficient $|(\beta_i - |\sin\theta_0|)(\beta_i + |\sin\theta_0|)^{-1}|$.

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Appendix

Consider the integral

$$I_1 = \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia} \frac{e^{itl}}{L(t)(t+\alpha)}\,dt \tag{1A}$$

$$\mathrm{Im}(\alpha) > -a, \quad k i\cos\vartheta_0 < a < ki.$$

From the way $\mathcal{K}$ has been defined it has been shown, see Rawlins [6] that for $\mathrm{Re}(\beta) > 0$, $L(t)$ has no poles or zeros in the cut plane. Thus in the region $\mathrm{Im}(\alpha) > a$ the only singularity is a branch cut at $t = k$. Distorting the path of integration in (1A) into the upper t-plane until it runs around the branch cut $t = k$, see figure 2, gives

$$I_1 = \frac{\sqrt{2k}}{2\pi i}\left\{-i\int_{\infty k}^{k} \frac{e^{itl}\sqrt{(t-k)}\,dt}{(t+\alpha)(1-i\lambda\sqrt{(t-k)})} + i\int_{k}^{\infty k} \frac{e^{itl}\sqrt{(t-k)}\,dt}{(t+\alpha)(1+i\lambda\sqrt{(t-k)})}\right\},$$

where $\lambda = \sqrt{2k}/(k\beta)$ is obtained by replacing the smoothly varying function $\sqrt{(k+\alpha)}$ by $\sqrt{2k}$. Making an obvious change of variable we obtain

$$I_1 = \frac{\sqrt{2k}\,e^{ikl}}{\pi(1-\lambda^2(\alpha+k))}\int_0^\infty e^{iul}u^{1/2}\left\{\frac{1}{u+k+\alpha} - \frac{1}{u+\lambda^{-2}}\right\}du$$

$$= \frac{\sqrt{2k}}{(1-\lambda^2(\alpha+k))}\left\{W_0[\sqrt{(l(k+\alpha))}] - W_0[\sqrt{l}/\lambda]\right\} = \frac{\sqrt{2k}\,W(\alpha)}{(1-\lambda^2(\alpha+k))} \tag{2A}$$

W_0 can be expressed in terms of the Fresnel integral $F(z)$, see (57), (58), by

$$W_0[\sqrt{zl}] = \frac{e^{i(kl+\pi/4)}}{\sqrt{\pi l}}\left\{1 + 2i\sqrt{zl}\,F(\sqrt{zl})\right\} \tag{3A}$$

$$l > 0, \quad |\arg(z)| < \pi.$$

We also note the asymptotic expansion, see (58)

$$W_0[\sqrt{zl}] \sim -\frac{e^{i(kl-\pi/4)}}{2\pi^{1/2}l^{3/2}z}. \tag{4A}$$

Consider the integral

$$I_2 = \frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id} \frac{e^{it\ell}}{L(t)\sqrt{(k-t)(t-\alpha)}}\,dt \tag{5A}$$

$$Im(\alpha) < d < k_i$$

Anticipating that the major contribution from the smooth functions under the integral sign occurs at t = k, we obtain on distorting the path of integration round the branch cut t = k

$$I_2 = \frac{\sqrt{2k}}{2\pi i}\left\{\int_{\infty k}^{k} \frac{e^{it\ell}}{(1-i\lambda\sqrt{(t-k)})(t-\alpha)}\,dt + \int_{k}^{\infty k} \frac{e^{it\ell}}{(1+i\lambda\sqrt{(t-k)})(t-\alpha)}\,dt\right\}$$

$$= \frac{-\lambda\sqrt{2k}\,e^{ik\ell}}{(1+\lambda^2(\alpha-k))}\int_0^\infty e^{iu\ell}u^{1/2}\left\{\frac{1}{u+k-\alpha} - \frac{1}{u+\lambda^{-2}}\right\}du$$

$$= \frac{-\lambda\sqrt{2k}}{(1+\lambda^2(\alpha-k))}\left\{W_0\left[\sqrt{(\ell(k-\alpha))}\right] - W_0\left[\sqrt{\ell}/\lambda\right]\right\} = \frac{-\lambda\sqrt{2k}}{(1+\lambda^2(\alpha-k))}W(-\alpha) \tag{6A}$$

$$\ell > 0,\ |\arg(k-\alpha)| < \pi,\ |\arg(\lambda^{-1})| < \pi$$

Consider the integral

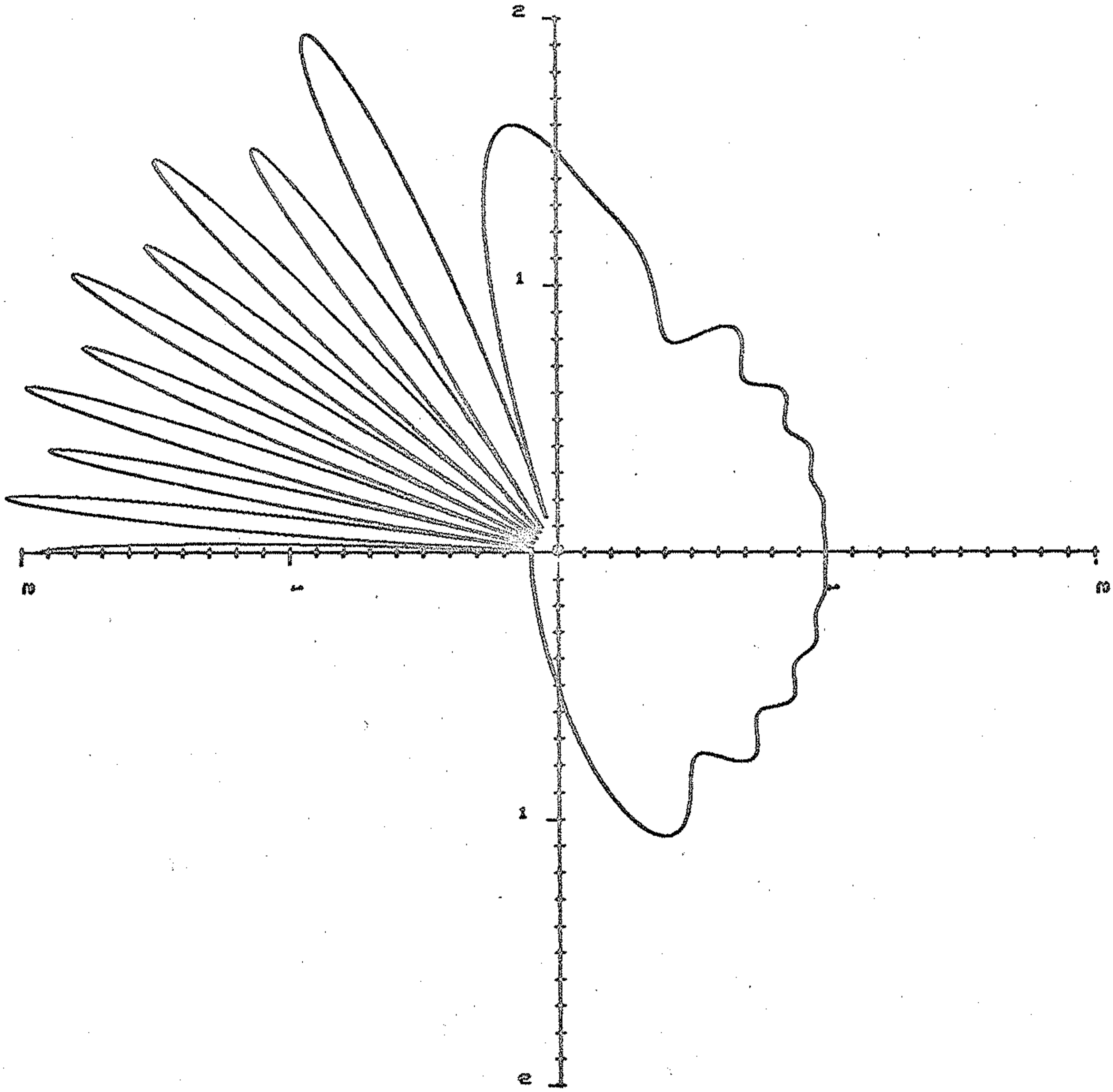
$$I_3 = \frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic} \frac{e^{-it\ell}}{L(t)\sqrt{(k-t)(t-\alpha)}}\,dt \qquad Im(\alpha) > c > k_i. \tag{7A}$$

Let t be replaced by (-t), c by (-a), and using the fact that L(-t) = L(t), $\sqrt{(k+t)} \simeq \sqrt{2k}$ gives

$$I_3 = \frac{-1}{2\pi i\sqrt{2k}}\int_{-\infty+ia}^{\infty+ia} \frac{e^{it\ell}}{L(t)(t+\alpha)}\,dt = -I_1/\sqrt{2k}\ . \tag{8A}$$

HARD HALF PLANE FIELD

$\theta_0 = 90°$ $kr = 10\pi$ $kl =$



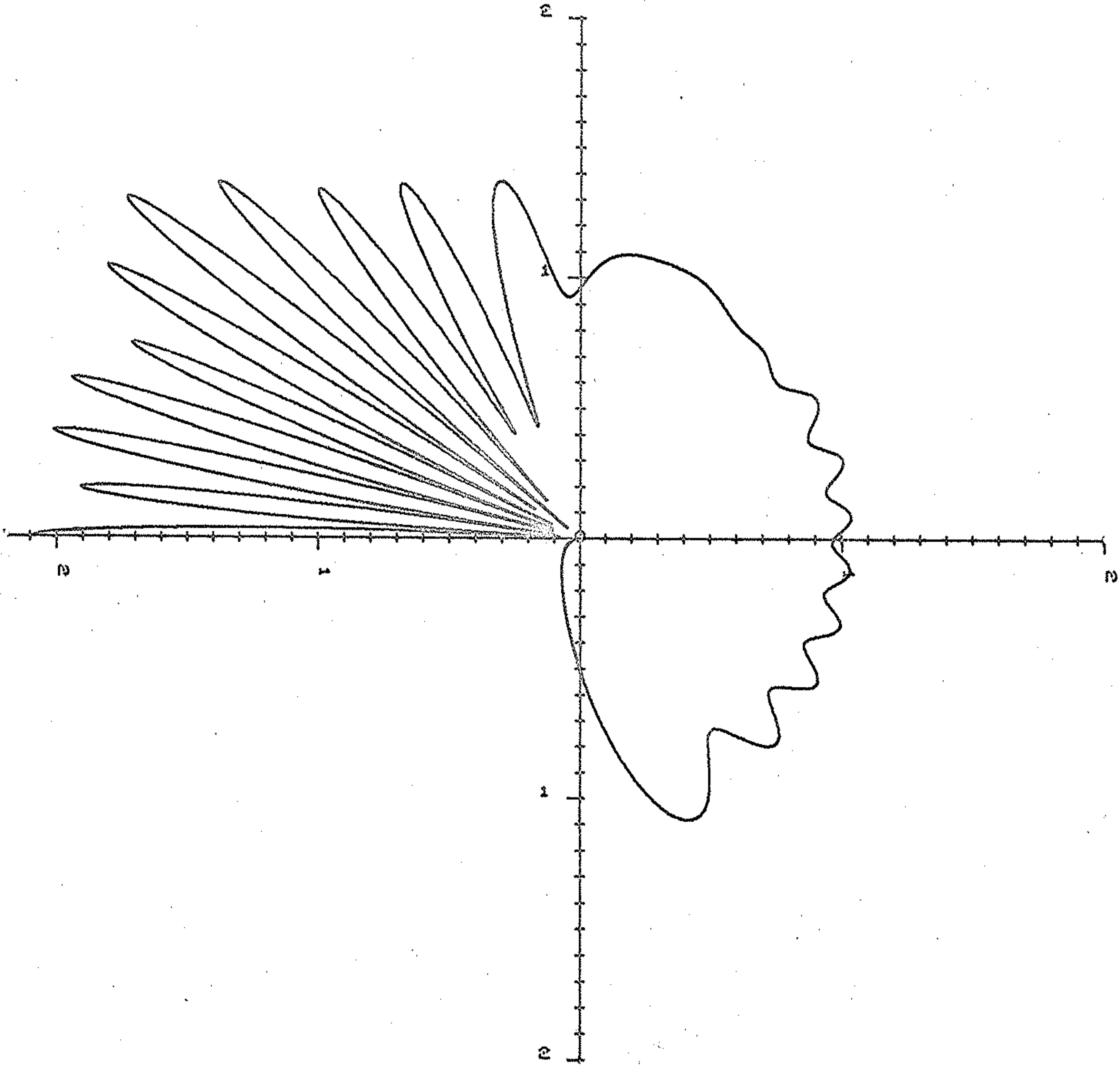
graph of hard half plane field for $\theta_0 = 90°$, $kr = 10\pi$ see reference [7].

graph 1

HARD HALF PLANE ABSORBING STRIP FIEL

$\theta_0 = 90°$ $kr = 10\pi$. $kl =$

$\zeta = 1. + i0.5$

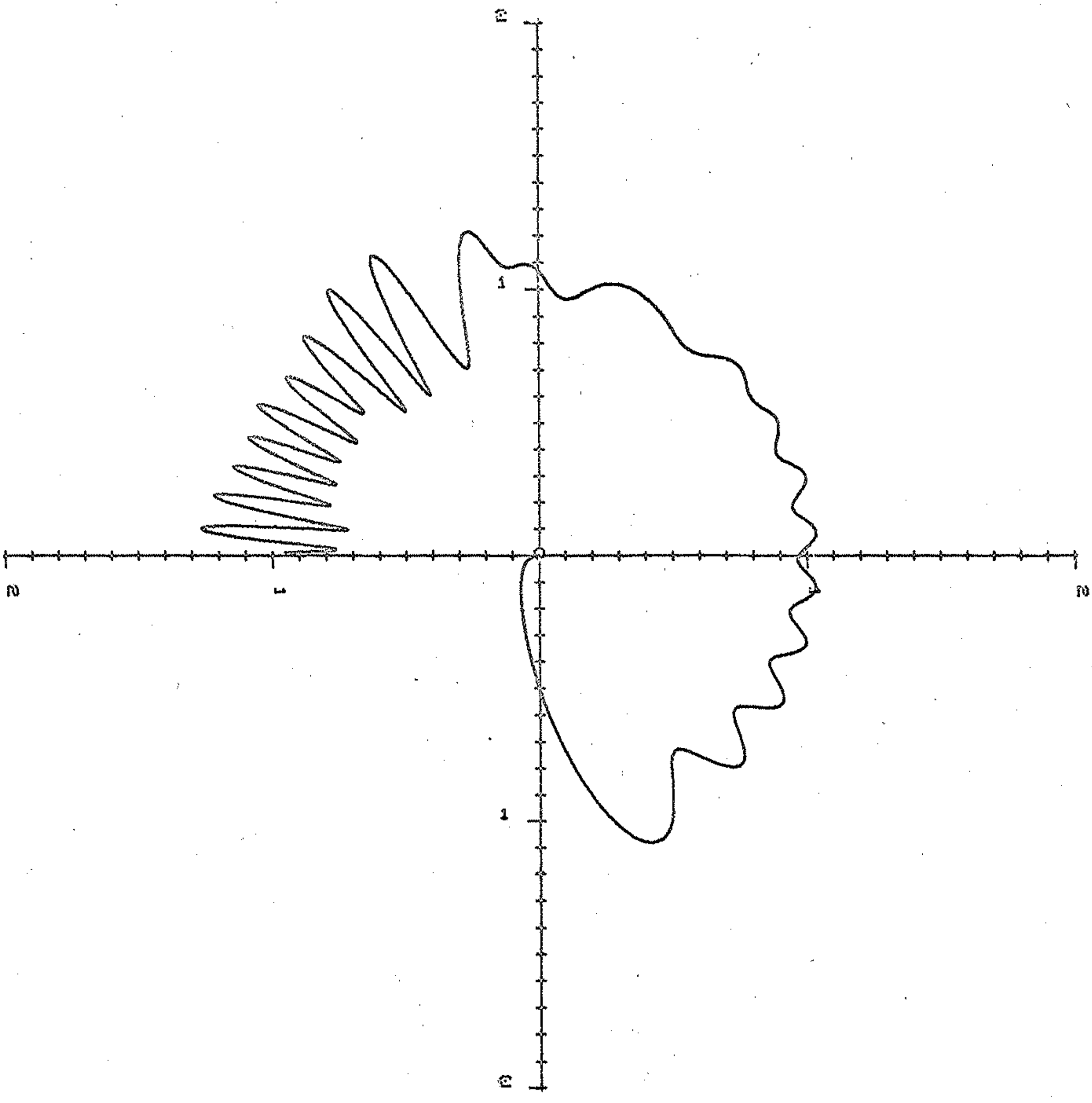


graph of $|X(10\pi, \theta)|$ for $\theta_0 = 90°$, $kl = 5\pi$, $\zeta = 1 + i0.5$
i.e. hard half plane absorbing strip. graph 2

HARD HALF PLANE ABSORBING STRIP FIELD

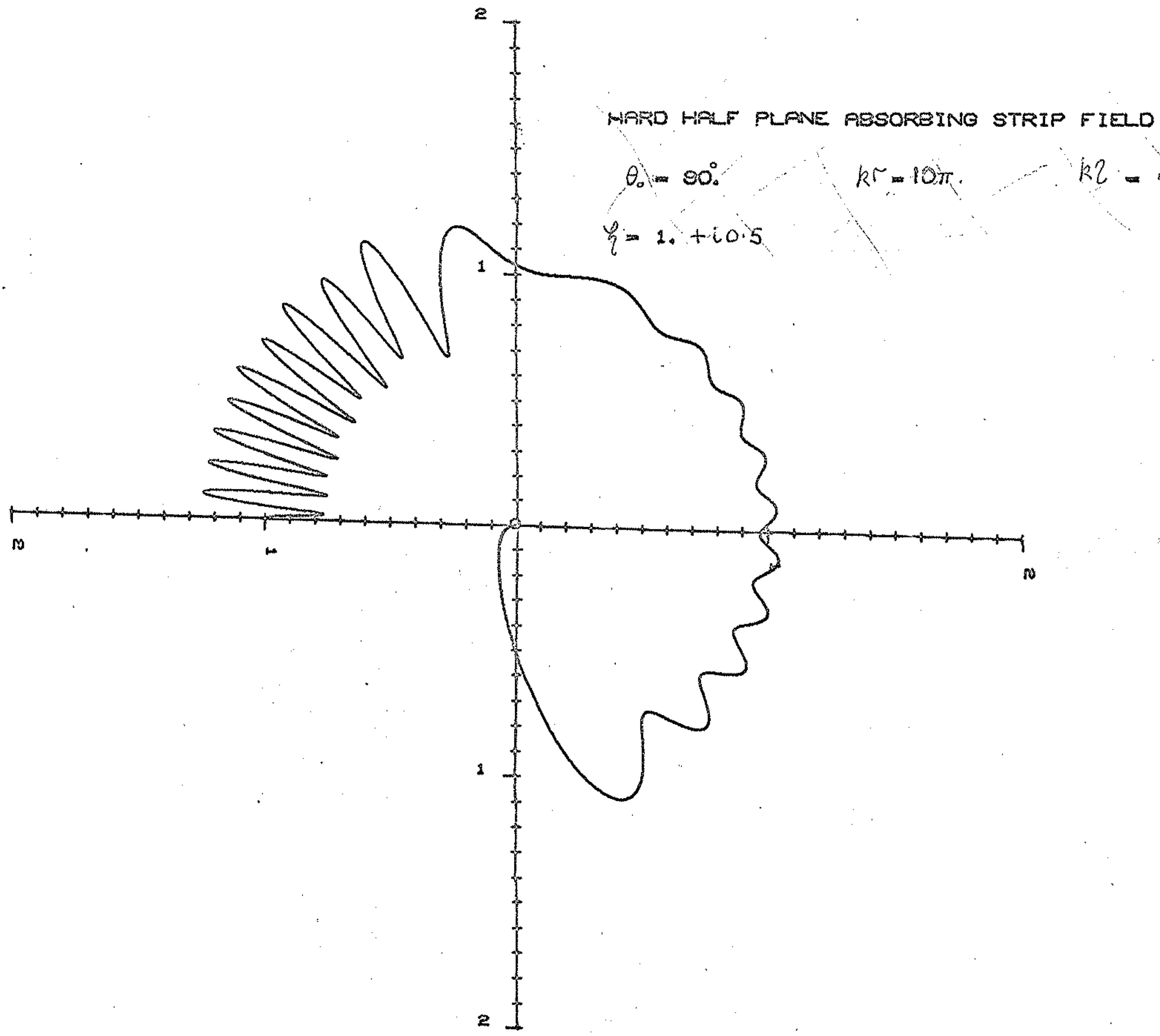
$\theta_0 = 90.$ $kr = 10\pi.$ $kl = 1$

$\eta = 1. + i0.5$



graph of $|\chi(10\pi, \theta)|$ for $\theta_0 = 90°$, $kl = 15\pi$, $\eta = 1 + i0.5$
ie hard half plane absorbing strip.

graph 3

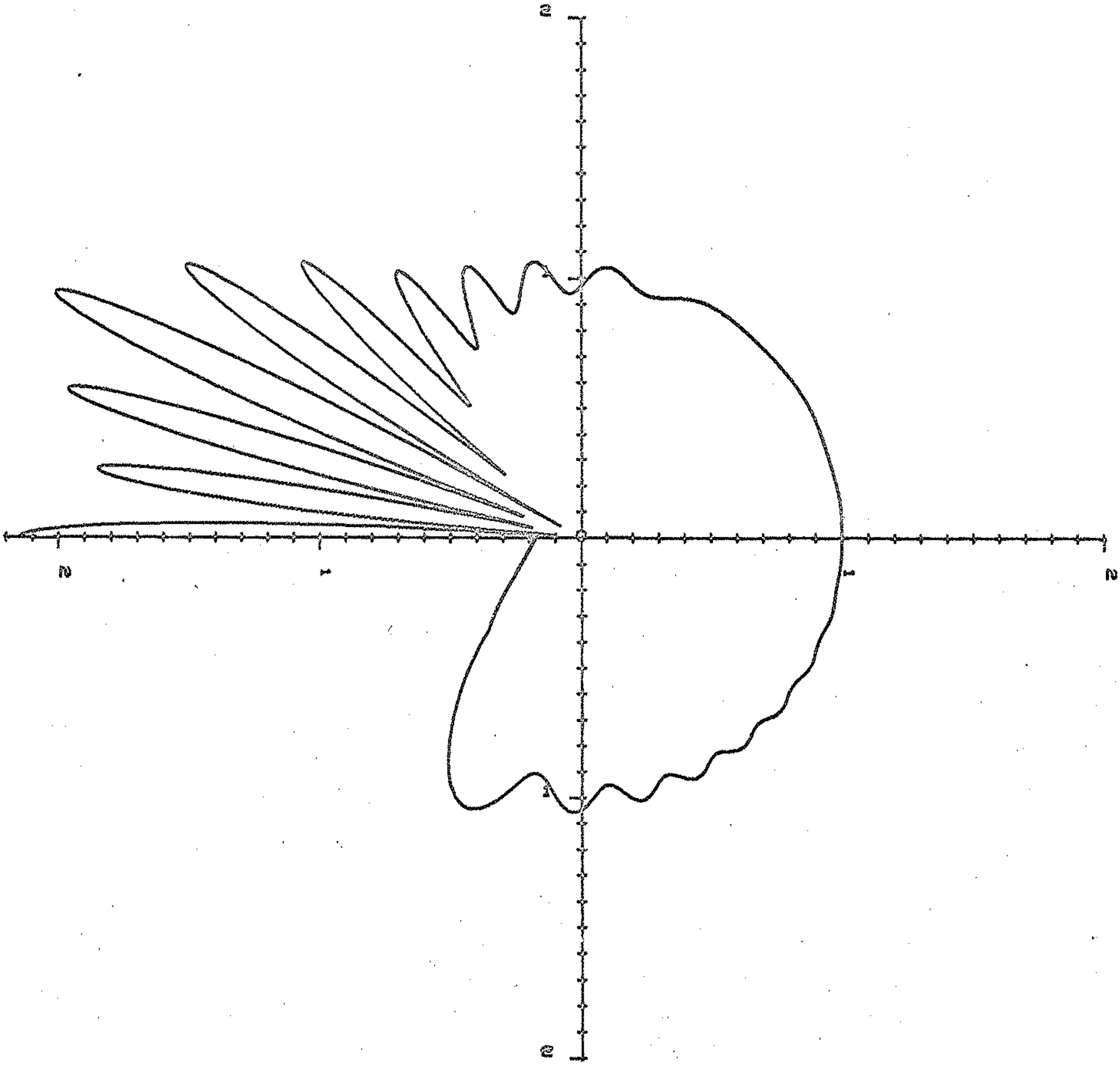


graph of $|\chi(10\pi, \theta)|$ for $\theta_0 = 90°$, $kl = \infty$, $\zeta = 1 + i0.5$
i.e. absorbing half plane

graph 4

HARD HALF PLANE FIELD

$\theta_0 = 45°$ $kr = 10\pi$.



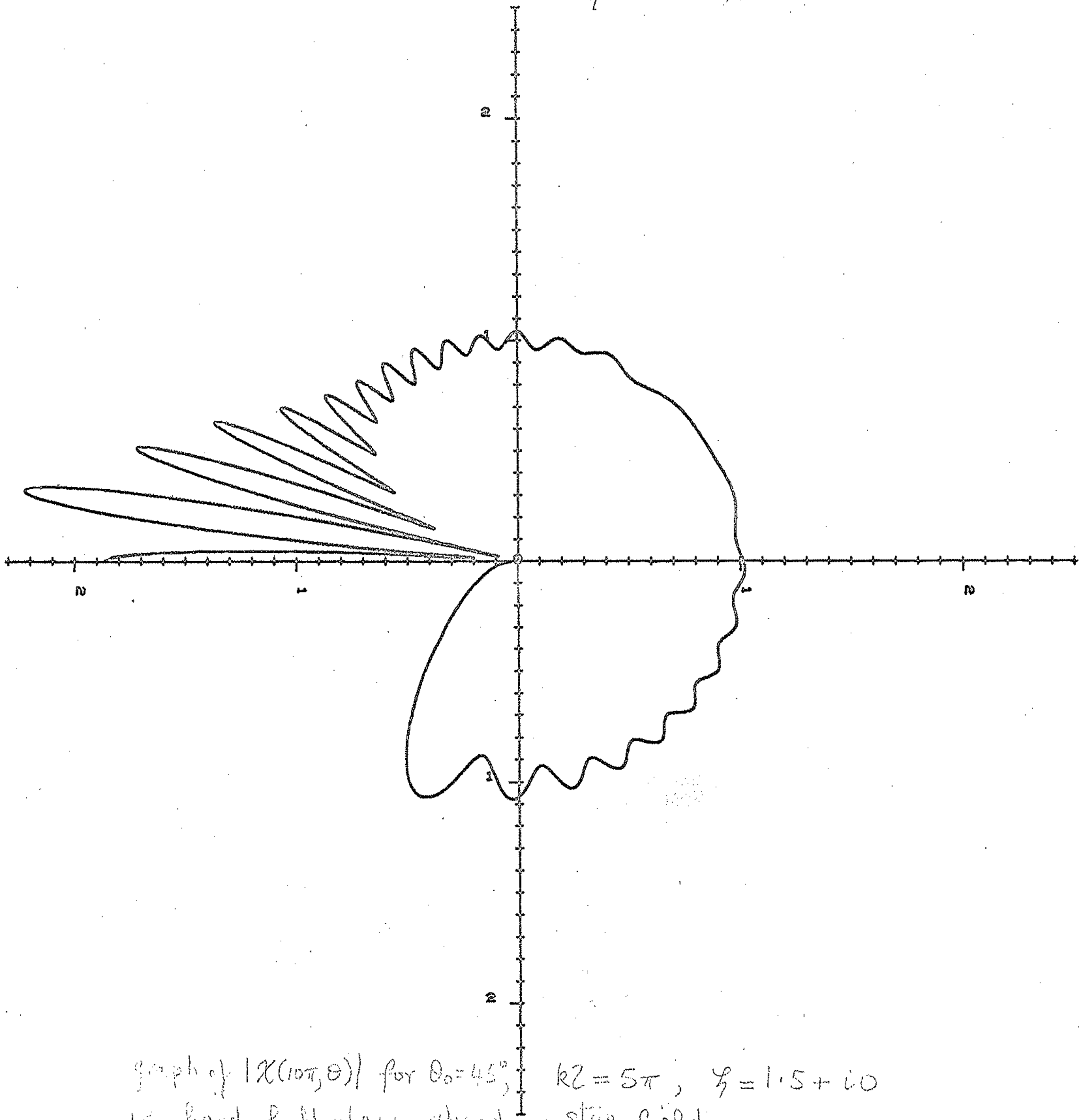
graph of hard half plane field for $\theta_0 = 45°$, $kr = 10\pi$, See reference [7].

graph 5

HARD HALF PLANE ABSORBING STRIP FIELD

$\theta_0 = 45^\circ$ $kr = 10\pi$ $kl =$

$\zeta = 1.5 + i0$



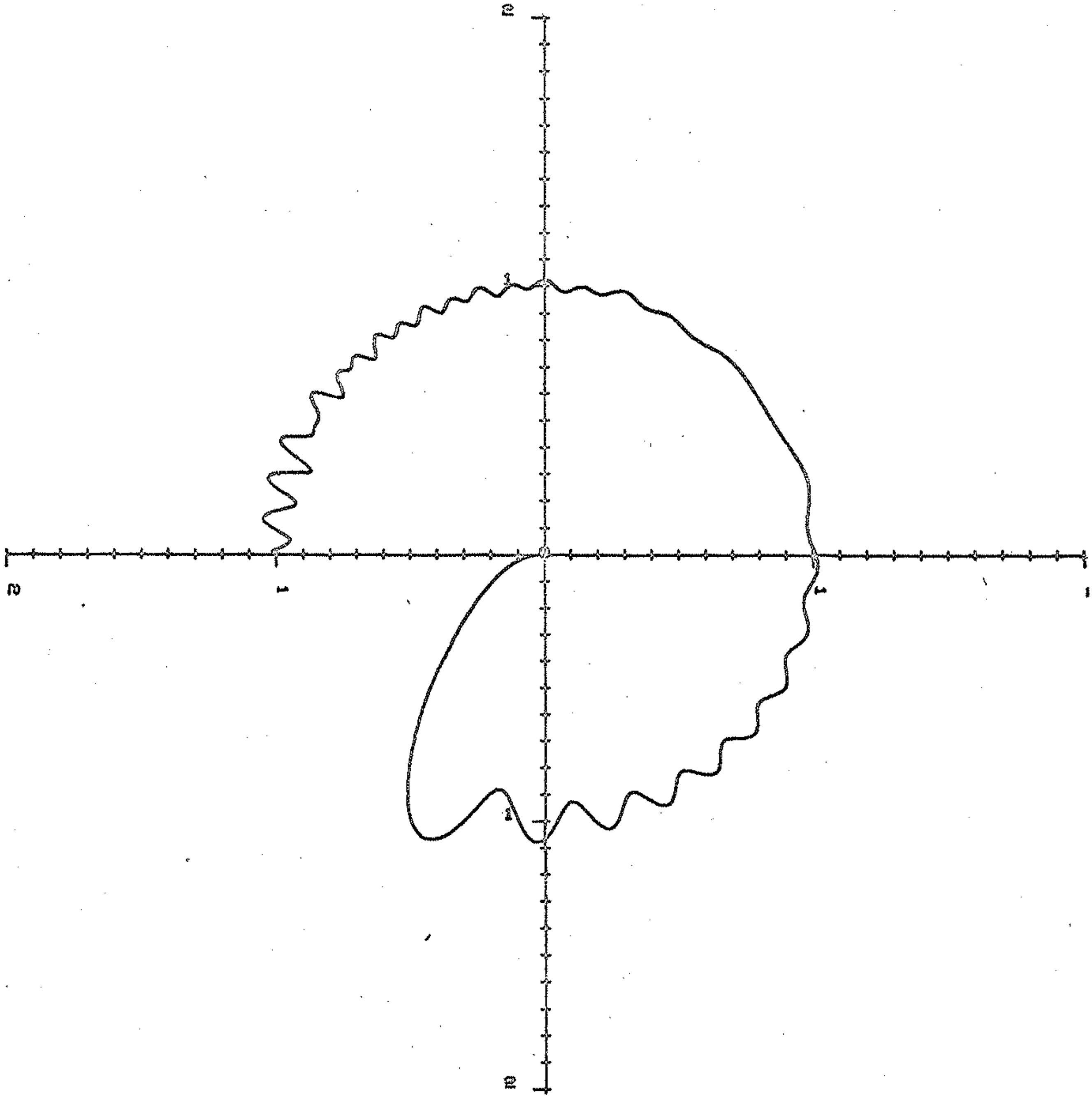
graph of $|\chi(10\pi, \theta)|$ for $\theta_0 = 45^\circ$, $kl = 5\pi$, $\zeta = 1{\cdot}5 + i0$
i.e. hard half plane absorbing strip field

graph 6

HARD HALF PLANE ABSORBING STRIP FIELD

$\theta_0 = 45°$ $kr = 10\pi$ $kl =$

$\eta = 1.5 + i0$

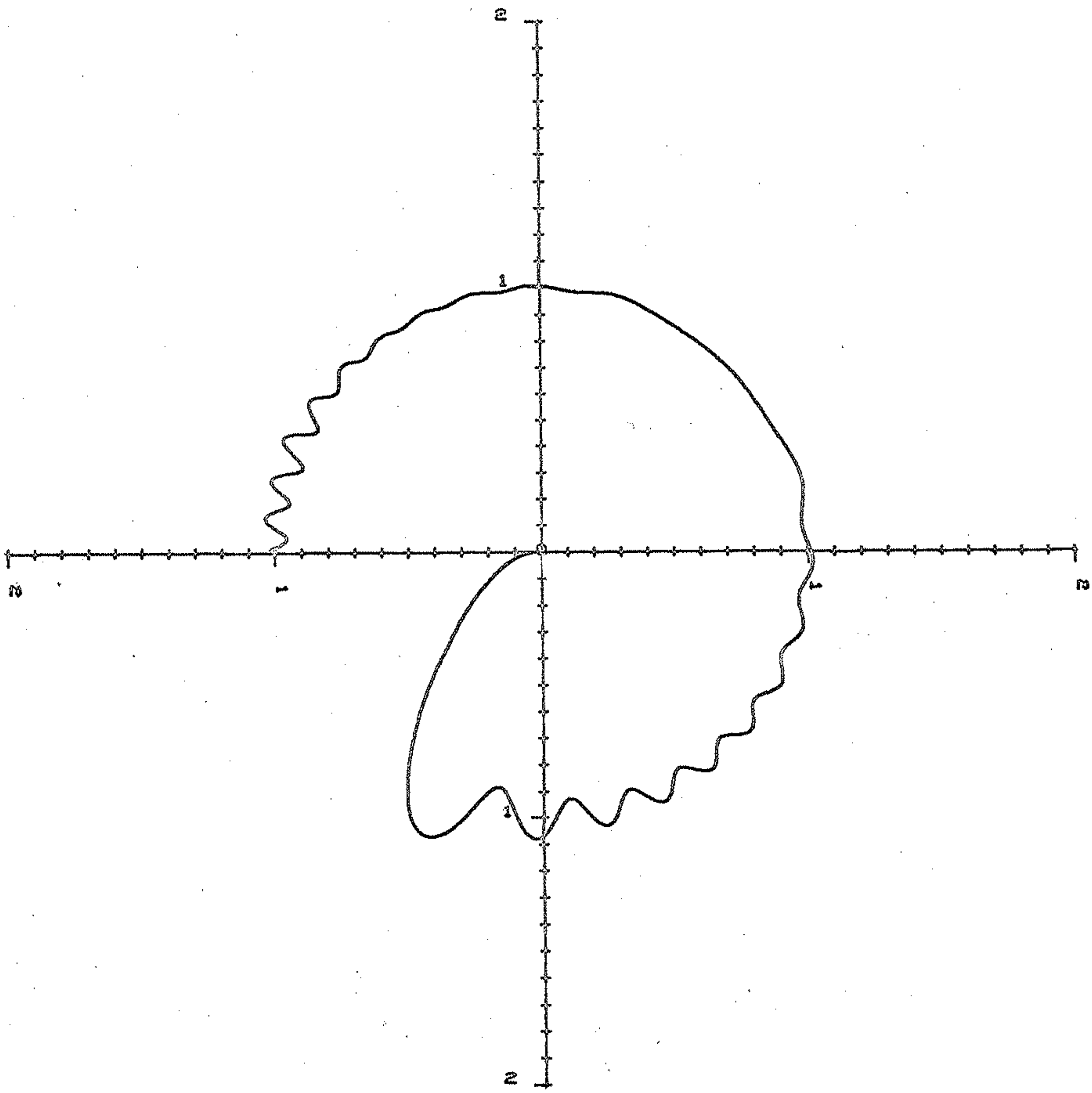


graph of $|\chi(10\pi, \theta)|$ for $\theta_0 = 45°$, $kl = 15\pi$, $\eta = 1.5 + i0$
i.e. hard half plane absorbing strip field
graph 7

HARD HALF PLANE ABSORBING STRIP FIEL

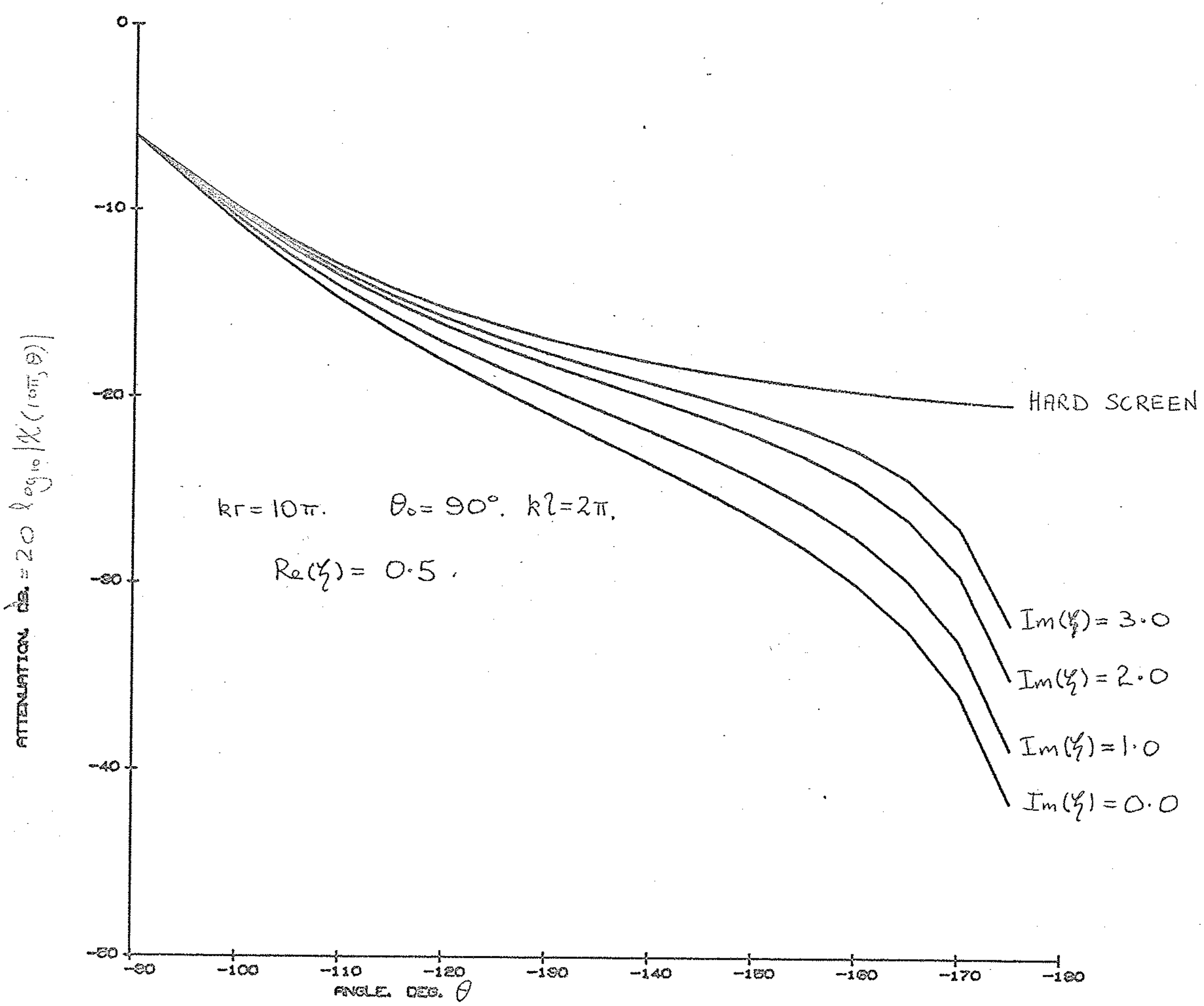
$\theta_0 = 45°$ $kr = 10\pi$ $kl =$

$\zeta = 1.5 + i0$



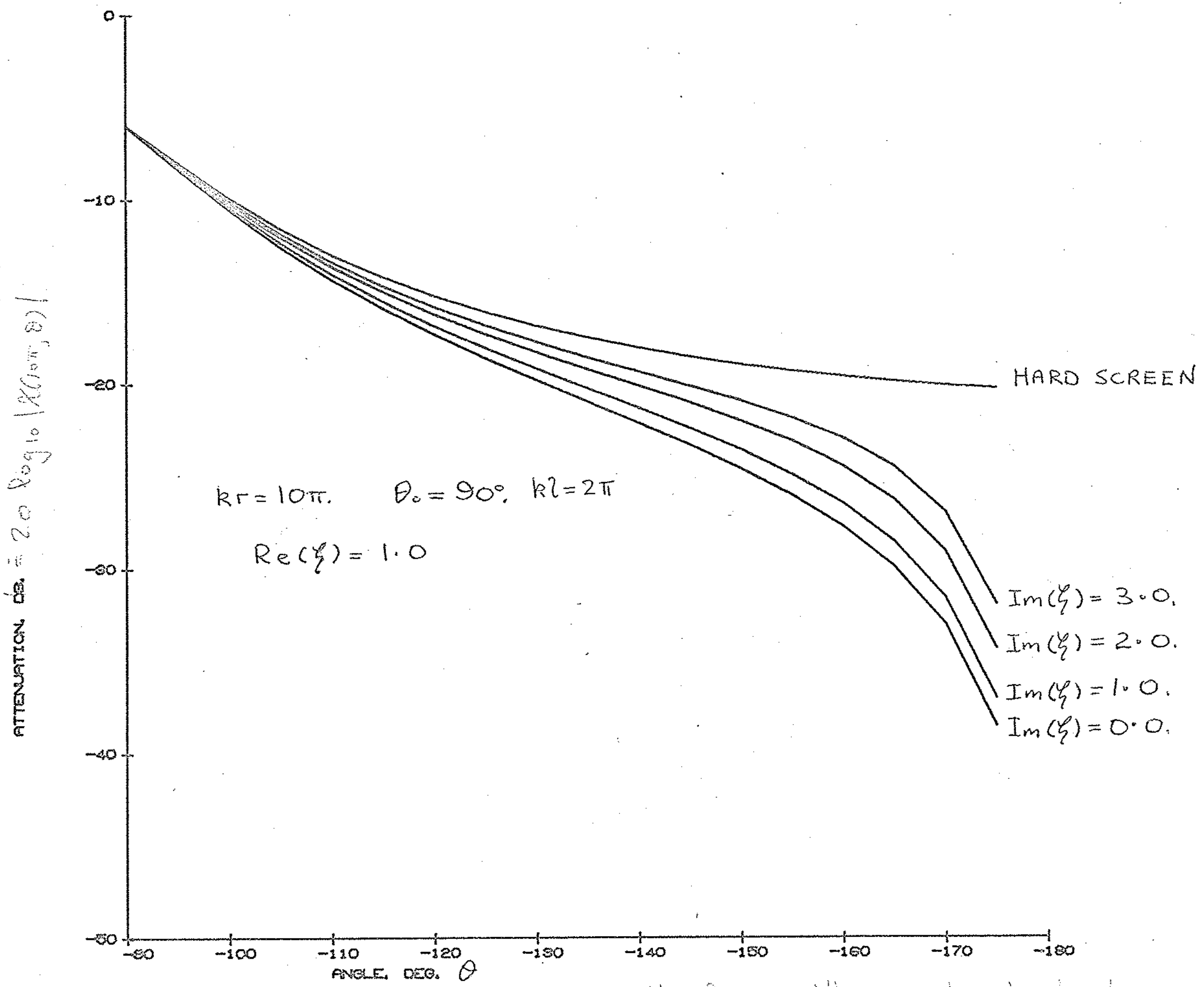
graph of $|\chi(10\pi, \theta)|$ for $\theta_0 = 45°$, $kl = \infty$, $\zeta = 1\cdot5 + i0$
i.e. absorbing half plane field.

graph 8



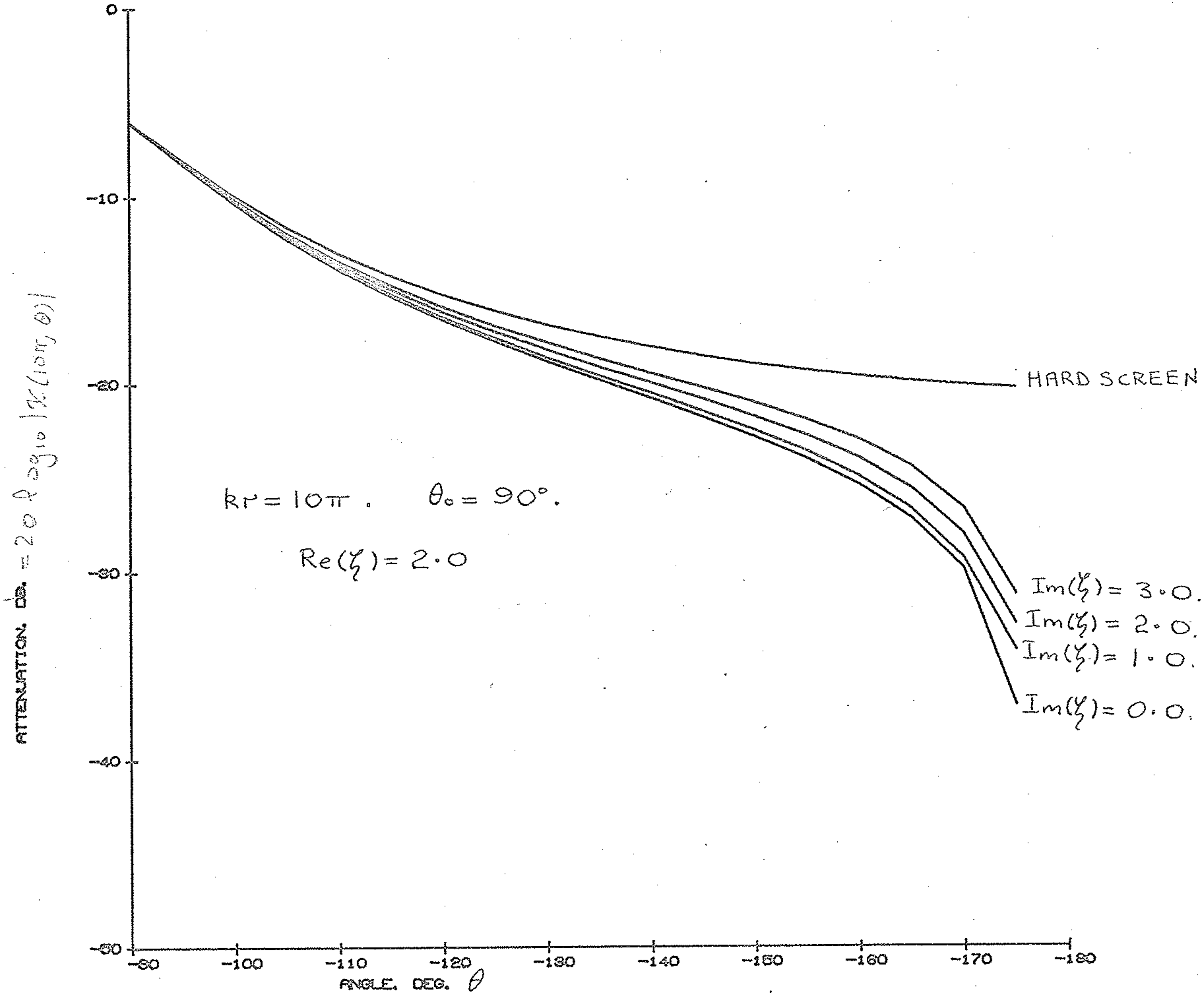
ATTENUATION dB. = 20 log10 |χ(10π, θ)|
ANGLE. DEG. θ
0
-10
-20
-30
-40
-50
-90
-100
-110
-120
-130
-140
-150
-160
-170
-180
HARD SCREEN
Im(ζ) = 3·0
Im(ζ) = 2·0
Im(ζ) = 1·0
Im(ζ) = 0·0
kr = 10π. θ₀ = 90°. kl = 2π.
Re(ζ) = 0·5.

RE Z = 0·5



Attenuation given by a hard half plane with an absorbent edge

RF2=



attenuation given by a hard half plane with an absorbent edge.

References

1 P. M. Morse and K. V. Ingard, Encyclopedia of Physics Acoustics I, Springer Verlag, (1961), chapter 1.

2 A. D. Rawlins, Diffraction of sound by a rigid screen with a soft or perfectly absorbent edge. J. Sound. Vib. (1976).

3 B. Noble, The Wiener-Hopf technique, London, Pergamon (1968).

4 A. S. Peters and J. J. Stoker, A uniqueness theorem and a new solution for Sommerfeld and other diffraction problems, Comm. Pure App. Math 7(1954) pp 565-585.

5 D. S. Jones, The theory of electromagnetism, London, Pergamon (1964).

6 A. D. Rawlins, Acoustic diffraction by an absorbing semi-infinite plane in a moving fluid, Proc. Roy. Soc. Edin. 72 (1975) pp 337-357

7 A. D. Rawlins, The solution of a mixed boundary value problem in the theory of diffraction by a semi-infinite plane, Proc. Roy. Soc. London A346 (1975) pp 469-484

8 G. F. Butler, Diffraction of sound by a rigid screen with an absorbent edge (1974) internal report of RAE Farnborough.