Engine-over-the-wing noise problem

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1. ABSTRACT

The problem of diffraction of a sound wave by a strip in a moving fluid is investigated. The sound source is a line source which is at a fixed finite distance from the strip. This system is in a moving subsonic fluid, and a vortex sheet is assumed to be attached to the trailing edge.

The above problem is supposed to be a theoretical model for the situation when an engine is at a fixed distance and orientation above an aircraft wing, the aircraft being in flight.

2. INTRODUCTION

In order to provide high speed transportation between city centres, powered lift aircraft such as STOL (Short Take Off and Landing and RTOL (Reduced Take Off and Landing) vehicles using shorter runways have been proposed. However, the extra lift device necessary for powered lift generates and redirects noise, which must be compatible with existing noise level requirements. A possible solution to the CTOL (Conventional Take Off and Landing), STOL and RTOL noise problem is to place the engine over the wing, see the review articles by Hubbard & Magglieri [1] and Brown and Blythe [2]. With such a configuration, the wing shields the ground from some of the engine noise and redirects it above the aircraft. It has been shown experiementally by Reshotko et al [3] that such a configuration does indeed reduce the noise below the wing by as much as 30 PN dB.

The object of the present work is to give a simple theoretical model for the engine over the wing situation. The noise field of a jet engine can be replaced by an equivalent distribution of point sources. At present a research program is being carried out to locate the position and magnitude of these equivalent sound sources. The magnitude, phase and position of the sources will depend on the physical dimensions of the jet and its speed. For the sake of clarity and to reduce the mathematical formulae in the present work the engine is assumed to be represented by a single line source which is fixed relative to a strip which represents the wing. The system is assumed to be in a moving subsonic fluid, this simulates flight conditions. A wake is assumed to be attached to the trailing edge of the strip. The sound field below the wing is investigated for different source orientations and flow Mach number. An extension to the situation where more than one source is above the strip is easily carried out by the principle of superposition.

For a unique solution to the theoretical model it is necessary that we impose different "edge conditions" at the leading and trailing edge of the strip, see Rawlins [5], Jones [6]. At the leading edge we impose the usual edge condition associated with acoustic diffraction theory. This requires that the sound energy is bounded in a finite region around the leading edge of the strip. Thus the velocity can be unbounded at the leading edge but the velocity singularity must be integrable. At the trailing edge, the attached wake, requires the imposition of a Kutta-Joukowski edge condition, see Jones [6]. This required that the velocity must be bounded at the trailing edge.

In section two the boundary value problem for the model is set up. In section three the boundary value problem is reformulated using the Wiener-Hopf technique and Jones' method (see Noble [7]), to give a system of Fredholm integral equations. The integral equations are too difficult to solve exactly and, therefore, for sufficiently large strip length an asymptotic solution to the integral equation is derived in section four. In section five some expressions for the far field are derived. These results are used to give some graphical plots of the attenuation in the shadow of the strip for various physical parameter values. Section six gives some conclusions from the graphical results and the model used.

FORMULATION OF THE BOUNDARY VALUE PROBLEM

Consider a small amplitude sound wave on a main stream moving with velocity U parallel to the x-axis. A finite strip is assumed to occupy $y = 0$, $-d < x < d$, $|z| < \infty$ (see figure 1). The strip is assumed to be infinitely thin and rigid, with a wake attached to it trailing edge. The effects of viscosity, gravity, and thermal conductivity will be neglected. Then on linearising the equations of motion the perturbation velocity $\underline{u}$ of the irrotational sound wave can be expressed in terms of the velocity potential $\chi(x,y)$ by $\underline{u} = \text{grad}\ \chi(x,y)$. The resulting pressure in the sound field is given by

$$p = -\rho_0 \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\chi(x,y) ,$$

where ρ_0 is the density of the undisturbed stream.

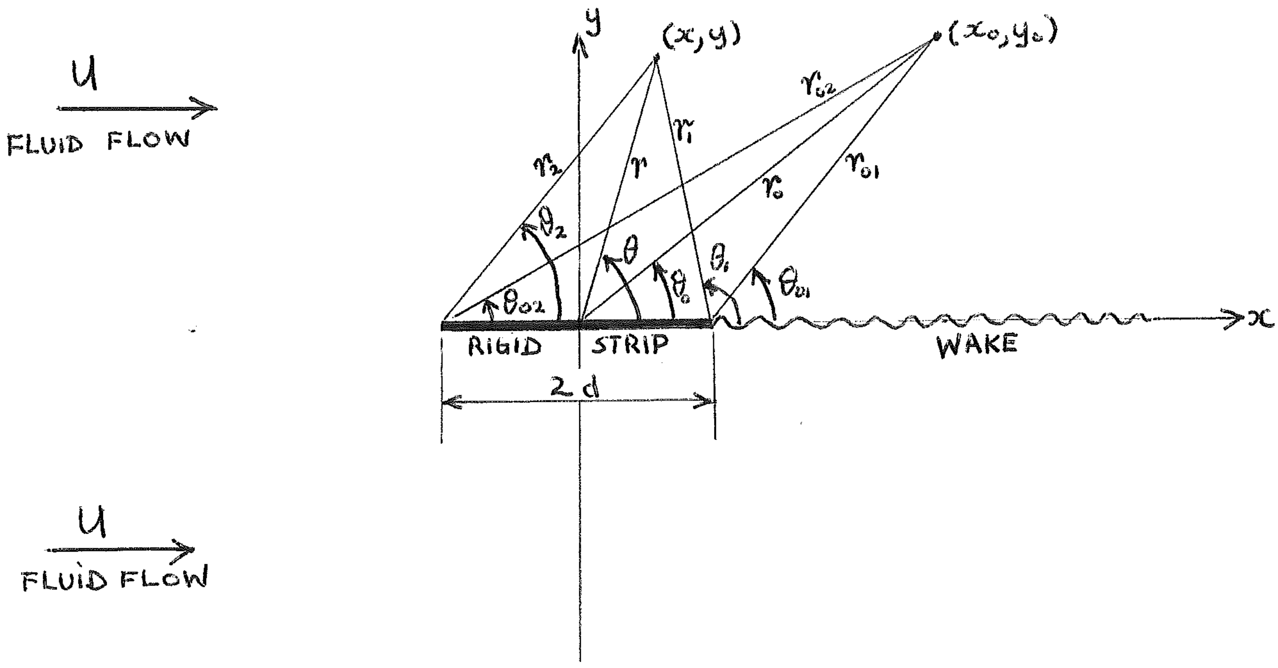


fig 1.

We shall assume a line source parallel to the edges exists at (x_0, y_0), $y_0 > 0$, and is fixed in space relative to the rigid strip. The line source is assumed to have time harmonic variation $e^{-i\omega t}$; this factor will be suppressed in future work. Then the problem becomes one of solving the convective wave equation

$$\left\{(1-M^2)\frac{\partial^2}{\partial x^2} + 2ikM\frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} + k^2\right\}\chi(x,y) = \delta(x-x_0)\,\delta(y-y_0), \tag{1}$$

subject to the boundary conditions

$$\frac{\partial}{\partial y}\chi(x,0^{\pm}) = 0, \qquad (-d \leq x \leq d), \quad (2)$$

$$\frac{\partial \chi}{\partial y}(x,0^{+}) = \frac{\partial \chi}{\partial y}(x,0^{-}), \quad \chi(x,0^{+}) = \chi(x,0^{-}), \qquad (x < -d), \quad (3)$$

$$\frac{\partial \chi}{\partial y}(x,0^{+}) = \frac{\partial \chi}{\partial y}(x,0^{-}), \; \chi(x,0^{+}) - \chi(x,0^{-}) = \delta \exp[ikx/M], (x > d), \quad (4)$$

where $k = \omega/c$, c is the velocity of sound and M $(=U/c)$ is the Mach number. The boundary condition (4) is the appropriate form which expresses a wake, which is assumed not to spread, trailing off the edge $x = d$ of the strip, for subsonic flow, viz $0 < M < 1$, Rawlins [4]. δ is an unknown constant which is determined by the field behaviour at the trailing edge. The second expression of (4) is obtained by direct integration of the boundary condition

$$\left(-ik + M\frac{\partial}{\partial x}\right)\phi(x,0^{+}) = \left(-ik + M\frac{\partial}{\partial x}\right)\phi(x,0^{-}), \quad (x > d),$$

which ensures the continuity of the pressure across the wake.

3. SOLUTION OF THE BOUNDARY VALUE PROBLEM

Since we are dealing with subsonic flow we can make the real substitutions

$$x = (1-M^2)^{1/2} X, \qquad x_0 = (1-M^2)^{1/2} X_0, \quad y = Y, \quad y_0 = Y_0$$

$$k = (1-M^2)^{1/2} K, \qquad d = (1-M^2)^{1/2} D, \tag{5}$$

which together with the substitution

$$\chi(x,y) = \Psi(X,Y)\exp[-iKMX], \tag{6}$$

reduce the boundary value problem (1) to (5) to

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + K^2\right)\Psi(X,Y) = \frac{\delta(X-X_0)\delta(Y-Y_0)e^{iKMX_0}}{(1-M^2)^{1/2}};$$

$$\frac{\partial \Psi}{\partial Y}(X, 0^{\pm}) = 0, \qquad (-D<X<D); \tag{8}$$

$$\frac{\partial \Psi}{\partial Y}(X, 0^{+}) = \frac{\partial \Psi}{\partial Y}(X, 0^{-}), \quad \Psi(X,0^+) = \Psi(X,0^-), \qquad (X<-D); \tag{9}$$

$$\frac{\partial \Psi}{\partial Y}(X, 0^{+}) = \frac{\partial \Psi}{\partial Y}(X, 0^{-}), \quad \Psi(X,0^+) - \Psi(X,0^-) = \delta\exp[iKX/M], \; (X>D) \tag{10}$$

We shall assume that a solution of (7) to (10) can be written in the form

$$\Psi(X,Y) = \phi_0(X,Y) + \phi(X,Y), \tag{11}$$

where

$$\phi_0(X,Y) = \frac{a}{4i} H_0^{(1)}\left(K\sqrt{\{(X-X_0)^2+(Y-Y_0)^2\}}\right) =$$

$$= \frac{a}{4\pi i}\int_{-\infty}^{\infty} \frac{\exp[i\{u(X-X_0)+\sqrt{(K^2-u^2)}(Y_0-Y)\}]\,du}{\sqrt{(K^2-u^2)}},$$

$a = (1-M^2)^{-1/2}\exp[iKMX_0]$

represents the line source field in the absence of the rigid strip. The term $\phi(X,Y)$ represents the perturbed field due to the presence of the strip.

that $k = k_r + ik_i$ $(k_r, k_i > 0)$, so that

For analytic convenience we shall assume $K = K_r + iK_i$ $(K_r, K_i > 0)$, in which case for a unique solution of the boundary value problem (7) to (11) requires the satisfaction of the radiation condition

$$\lim_{R \to \infty} |\phi(x,y)| < \infty, \qquad \text{where} \quad R = \sqrt{(x^2+y^2)}, \qquad (12)$$

and also the edge conditions

$$\phi(x,0) = O(1), \quad \text{and} \quad \frac{\partial \phi}{\partial y}(x,0) = O((x+D)^{-1/2}), \text{ as } x \to -D, \qquad (13)$$

$$\phi(x,0) = O(1), \quad \text{and} \quad \frac{\partial \phi}{\partial y}(x,0) = O((x-D)^{1/2}), \text{ as } x \to D, \qquad (14)$$

see Jones [6] and Peters and Stoker [9].

The edge condition (13) is the appropriate form for the leading edge, where there is no wake, and is the usual form used in diffraction theory which requires that the edge does not behave like a source. The edge condition (14) is a Kutta-Joukowski edge condition applied to the trailing edge, at which is attached a wake. This condition requires that the velocity be finite at the edge. It is necessary, for a unique solution, to impose a Kutta-Joukowski condition, rather than the edge condition associated with diffraction theory, because of the wake, see Jones [6]. However, we shall assume initially that the edge condition at x = D is the usual one associated with diffraction theory viz

$$\phi(x,0) = O(1), \quad \text{and} \quad \frac{\partial \phi}{\partial y}(x,0) = O((x-D)^{-1/2}), \text{ as } x \to D, \qquad (15)$$

and in the end result we shall impose the condition (14) which will enable us to determine δ, which occurs in the wake boundary condition (10).

We introduce the Fourier transform

$$\Phi(\alpha, y) = \int_{-\infty}^{\infty} \phi(x, y)\, e^{i\alpha x}\, dx\,, \tag{16}$$

and its inverse

$$\phi(x, y) = \frac{1}{2\pi} \int_{-\infty+i\tau}^{\infty+i\tau} \Phi(\alpha, y)\, e^{-i\alpha x}\, d\alpha\,, \tag{17}$$

where $\alpha = \sigma + i\tau$. The transform (15) and its inverse (17) will exist provided $-K_i < \tau < K_i$; this follows from the radiation condition (12) and the form of the incident field.

Applying (16) to equation (7) gives on solving the ordinary differential equation

$$\Phi(\alpha, y) = A(\alpha)\, e^{i\kappa y}\,, \qquad (y > 0)\,, \tag{18}$$

$$= B(\alpha)\, e^{-i\kappa y}\,, \qquad (y < 0)\,, \tag{19}$$

where $\kappa = \sqrt{(K^2 - \alpha^2)}$ is defined on the cut sheet for which $\mathrm{Im}(\kappa) > 0$ when $|\mathrm{Im}(\alpha)| < K_i$. From the equations (18) and (19) and the boundary conditions (8) to (10) we obtain using Jones' method, Noble [7],

$$e^{-i\alpha D}\Phi_-(\alpha, 0) + \Phi_1(\alpha, 0^+) + e^{i\alpha D}\Phi_+(\alpha, 0^+) = A(\alpha)\,, \tag{20a}$$

$$e^{-i\alpha D}\Phi_-'(\alpha, 0) + \xi_1'(\alpha, 0) + e^{i\alpha D}\Phi_+'(\alpha, 0) = i\kappa A(\alpha)\,, \tag{20b}$$

$$e^{-i\alpha D}\Phi_-(\alpha, 0) + \Phi_1(\alpha, 0^-) + e^{i\alpha D}\Phi_+(\alpha, 0^-) = B(\alpha)\,, \tag{21a}$$

$$e^{-i\alpha D}\Phi_-'(\alpha, 0) + \xi_1'(\alpha, 0) + e^{i\alpha D}\Phi_+'(\alpha, 0) = -i\kappa B(\alpha)\,; \tag{21b}$$

where

$$\Phi_+(\alpha, Y) = \int_D^\infty \phi(x,y)\, e^{i\alpha(x-D)}\, dx \,, \tag{22}$$

$$\Phi_-(\alpha, Y) = \int_{-\infty}^{-D} \phi(x,y)\, e^{i\alpha(x+D)}\, dx \,, \tag{23}$$

$$\Phi_1(\alpha, Y) = \int_{-D}^{D} \phi(x,y)\, e^{i\alpha x}\, dx \,, \tag{24}$$

$$\xi_1'(\alpha, 0) = -\frac{ia}{4\pi} \int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[ux_0+\kappa y_0]}}{(\alpha-u)} \left\{ e^{i(\alpha-u)D} - e^{-i(\alpha-u)D} \right\} du, \quad |b'|<ki \,. \tag{25}$$

and the dashes denote derivatives with respect to Y.

Eliminating A(α) from (20) and B(α) from (21) and adding the resulting equations gives

$$e^{-i\alpha D}\Phi_-'(\alpha,0) + \xi_1'(\alpha,0) + e^{i\alpha D}\Phi_+'(\alpha,0) =$$

$$= i\kappa\{\Phi_1(\alpha,0^+) - \Phi_1(\alpha,0^-)\}/2 + i\kappa\{\Phi_+(\alpha,0^+) - \Phi_+(\alpha,0^-)\}e^{i\alpha D}/2 \,. \tag{26}$$

From the boundary condition (10) we also have

$$e^{i\alpha D}\{\Phi_+(\alpha,0^+) - \Phi_+(\alpha,0^-)\} = i\delta \exp[iD(\alpha+K/M)]/(\alpha+K/M) \,.$$

Hence we can write (26) in the form

$$e^{-i\alpha D}\Phi_-'(\alpha,0) + \kappa N_1(\alpha) + e^{i\alpha D}\Phi_+'(\alpha,0) = S(\alpha) \,, \tag{27}$$

where
$$\mathcal{N}_1(\alpha) = -i\{\Phi_1(\alpha,0^+) - \Phi_1(\alpha,0^-)\}/2\,, \tag{28}$$

$$S(\alpha) = \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[uX_0+\kappa_i Y_0]}}{(\alpha-u)}\left\{e^{i(\alpha-u)D} - e^{-i(\alpha-u)D}\right\}du$$
$$- \kappa_i\,\delta\exp[i(\alpha+K/M)D]/\{2(\alpha+K/M)\}, \tag{29}$$

and from (22) and (24) the terms with subscripts $\pm$ denote functions analytic and regular in $\mathrm{Im}(\alpha) > K_i$ and $\mathrm{Im}(\alpha) < K_i$ respectively. The function $\mathcal{N}_1(\alpha)$ is an entire function in the whole α-plane. Equations of the form (27) have been considered in Rawlins [10] and it can be shown that the edge conditions (13) and (15) can be used to reform it into a system of Fredholm integral equations viz

$$\frac{\Gamma_+(\alpha)}{\sqrt{(k+\alpha)}} = \frac{-1}{2\pi i}\int_{-\infty+ib}^{\infty+ib} \frac{e^{itD}[S(-t)+S(t)]}{\sqrt{(k-t)}\,(t+\alpha)}dt + \frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib} \frac{e^{i2tD}\,\Gamma_+(t)\,dt}{\sqrt{(k-t)}(t+\alpha)}, \tag{30}$$

$$\frac{\gamma_+(\alpha)}{\sqrt{(k+\alpha)}} = \frac{-1}{2\pi i}\int_{-\infty+ib}^{\infty+ib} \frac{e^{itD}[S(-t)-S(t)]}{\sqrt{(k-t)}(t+\alpha)}dt - \frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib} \frac{e^{i2tD}\,\gamma_+(t)}{\sqrt{(k-t)}(t+\alpha)}dt, \tag{31}$$
$$\mathrm{Im}(\alpha) > -b > -K_i\,,$$

where $\Gamma_+(\alpha) = \Phi'_+(\alpha,0) + \Phi'_-(-\alpha,0)\,,\quad \gamma_+(\alpha) = \Phi'_+(\alpha,0) - \Phi'_-(-\alpha,0).$ (32)

The exact solution of the integral equations (30) and (31) is too difficult and, therefore, we shall make some asymptotic approximations.

4. APPROXIMATE SOLUTION OF EQUATIONS (30) and (31) FOR $2KD \geq 1$.

Substituting (29) into (30) and (31) and restricting the path of integration in the t-plane to $b' < b < K_i$ gives

$$\frac{\Gamma_+(\alpha)}{\sqrt{(K+\alpha)}} = \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} e^{i[(X_0-D)u+\kappa Y_0]}\,du\,.\,\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{dt}{\sqrt{(K-t)(t+\alpha)(t+u)}}$$

$$+\frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} e^{i[(X_0+D)u+\kappa Y_0]}\,du\,.\,\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{dt}{\sqrt{(K-t)(t+\alpha)(t-u)}}$$

$$-\frac{\delta e^{iKD/M}}{2}\,.\,\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{\sqrt{(K+t)}\,dt}{(t-K/M)(t+\alpha)}+\frac{\delta e^{iKD/M}}{2}\,.\,\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{\sqrt{(K+t)}\,e^{i2Dt}\,dt}{(t+K/M)(t+\alpha)}$$

$$+\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{e^{i2Dt}}{\sqrt{(K-t)(t+\alpha)}}\left\{\Gamma_+(t)-\frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'}\frac{e^{i[(X_0+D)u+\kappa Y_0]}}{(t+u)}\,du\right.$$

$$\left.-\frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'}\frac{e^{i[(X_0-D)u+\kappa Y_0]}}{(t-u)}\,du\right\}dt,\quad -Im(\alpha)<b,\quad b'<b<K_i; \tag{33}$$

$$\frac{\gamma_+(\alpha)}{\sqrt{(K+\alpha)}} = \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} e^{i[(X_0-D)u+\kappa Y_0]}\,du\,.\,\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{dt}{\sqrt{(K-t)(t+\alpha)(t+u)}}$$

$$-\frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} e^{i[(X_0+D)u+\kappa Y_0]}\,du\,.\,\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{dt}{\sqrt{(K-t)(t+\alpha)(t-u)}}$$

$$-\frac{\delta e^{iKD/M}}{2}\,.\,\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{\sqrt{(K+t)}\,dt}{(t-K/M)(t+\alpha)}-\frac{\delta e^{iKD/M}}{2}\,.\,\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{\sqrt{(K+t)}\,e^{i2Dt}\,dt}{(t+K/M)(t+\alpha)}$$

$$-\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{e^{i2Dt}}{\sqrt{(K-t)(t+\alpha)}}\left\{\gamma_+(t)+\frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'}\frac{e^{i[(X_0+D)u+\kappa Y_0]}}{(t+u)}\,du\right.$$

$$\left.-\frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'}\frac{e^{i[(X_0-D)u+\kappa Y_0]}}{(t-u)}\,du\right\}dt,\quad -Im(\alpha)<b,\quad b'<b<K_i \tag{34}$$

In the expressions (33) and (34) the first four integrals, integrated with respect to t, can be evaluated explicitly for $b' < b < K_i$, $\mathrm{Im}(-\alpha) < b$. In the first two terms of (33) and (34) we close that path of integration in the lower t-plane, thus avoiding branch cuts. The only poles captured are $t = -\alpha$ and $t = \pm u$. To avoid branch cuts we close the path of integration, in the third and fourth term of (33) and (34), in the upper t-plane. The only pole captured is $t = K/M$, and therefore it will be seen that the fourth term is identically zero. Carrying out the above calculations and letting

$$G_+(t) = \Gamma_+(t) - \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0+D)u+\kappa Y_0]}}{(t+u)}\,du - \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0-D)u+\kappa Y_0]}}{(t-u)}\,du, \tag{35}$$

$$g_+(t) = \gamma_+(t) + \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0+D)u+\kappa Y_0]}}{(t+u)}\,du - \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0-D)u+\kappa Y_0]}}{(t-u)}\,du, \tag{36}$$

gives eventually

$$G_+(\alpha) = \{S_1(\alpha) + \mathcal{G}_+(\alpha)\}\sqrt{(K+\alpha)} \tag{37}$$

$$g_+(\alpha) = \{S_2(\alpha) - i\mathcal{g}_+(\alpha)\}\sqrt{(K+\alpha)} \tag{38}$$

where

$$S_1(\alpha) = -\frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0-D)u+\kappa Y_0]}}{(\alpha-u)\sqrt{(k+u)}}\,du - \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0+D)u+\kappa Y_0]}}{(\alpha+u)\sqrt{(k-u)}}\,du$$

$$- \,\delta\sqrt{(k+K/M)}\exp[iKD/M]\,/\{2(\alpha+K/M)\}, \tag{39}$$

$$S_2(\alpha) = -\frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0-D)u+\kappa Y_0]}}{\sqrt{(k+u)}(\alpha-u)}\,du + \frac{ia}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0+D)u+\kappa Y_0]}}{\sqrt{(k-u)}(\alpha+u)}\,du$$

$$- \,\delta\sqrt{(k+K/M)}\exp[iKD/M]\,/\{2(\alpha+K/M)\}, \tag{40}$$

$$\mathcal{G}_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib} \frac{e^{i2Dt}\,G_+(t)}{\sqrt{(k-t)}(t+\alpha)}\,dt, \tag{41}$$

$$\mathcal{g}_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib} \frac{e^{i2Dt}\,g_+(t)\,dt}{\sqrt{(k-t)}(t+\alpha)}. \tag{42}$$

Now substituting (37) into (41) and (38) into (42) gives

$$\mathcal{G}_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib} \frac{e^{i2Dt}}{\sqrt{(k-t)}(t+\alpha)}\left\{(S_1(t)+\mathcal{G}_+(t))\sqrt{(k+t)}\right\}dt, \tag{43}$$

$$\mathcal{g}_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib} \frac{e^{i2Dt}}{\sqrt{(k-t)}(t+\alpha)}\left\{(S_2(t)-\mathcal{g}_+(t))\sqrt{(k+t)}\right\}dt. \tag{44}$$

So far no approximation has been made, we have simply put the integral equations (33) and (34) in a more convenient form amenable to asymptotic methods. If the contour of integration in (43) is distorted into the region $\mathrm{Im}(\alpha) > b$ no poles are captured and the expression can be asymptotically approximated, for $2KD \geq 1$, by the integral with its path of integration wrapped around the branch cut $t = K$. The part of the integrand of (43) within the curly bracket is regular and analytic in this region and will vary slowly in the vicinity of $t = K$. Thus since the dominant part of the integrand comes from the region $t = K$ the term in the curly bracket can be removed from under the integral sign and t be replaced by K. Thus

$$\mathcal{G}_+(\alpha) \simeq \sqrt{2K}\left(S_1(K) + \mathcal{G}_+(K)\right) . \frac{1}{2\pi i} \int_{\text{branch cut}} \frac{e^{i2Dt}\,dt}{\sqrt{(K-t)(t+\alpha)}},$$

and it is not difficult to show that

$$\frac{1}{2\pi i} \int_{\text{branch cut}} \frac{e^{i2Dt}\,dt}{\sqrt{(K-t)(t+\alpha)}} = -\frac{e^{i2DK}}{\pi} \int_0^\infty \frac{e^{i2Du}\,du}{u^{1/2}(u+\alpha+K)}$$

$$= -\frac{2\,e^{i(2KD-\pi/4)}}{\sqrt{(\pi(K+\alpha))}} F\left[\sqrt{(2D(K+\alpha))}\right] = W_{-1}\left[\sqrt{(2D(K+\alpha))}\right]$$

where $|\arg(K+\alpha)| < \pi$, $D > 0$,

$F(z) \left(= e^{-iz^2} \int_z^\infty e^{it^2}\,dt \right)$ is the Fresnel integral.

Thus

$$\mathcal{G}_+(\alpha) \simeq \sqrt{2K}\left(S_1(K) + \mathcal{G}_+(K)\right) W_{-1}\left[\sqrt{(2D(K+\alpha))}\right]; \tag{45}$$

and by a completely analogous method

$$\mathcal{J}_+(\alpha) = \sqrt{2K}\left(S_2(K) - \mathcal{J}_+(K)\right) W_{-1}\left[\sqrt{(2D(K+\alpha))}\right]. \tag{46}$$

In the expressions (45) and (46) the unknowns $\mathcal{G}_+(K)$ and $g_+(K)$ are determined by putting $\alpha = K$ into (45) and (46) and solving the resulting equations for $\mathcal{G}_+(K)$ and $g_+(K)$.

We can now obtain explicit expressions for $\phi'_{\pm}(\alpha,0)$ by using the expressions (32), (35) to (40), (45) and (46), viz

$$\Phi'_+(\alpha,0) = \frac{i\alpha}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0-D)u+\kappa Y_0]}}{(\alpha-u)}\left\{1-\sqrt{\left(\frac{k+\alpha}{k+u}\right)}\right\}du$$

$$-\ \delta \exp[ikD/M]\sqrt{(k+k/M)}\sqrt{(k+\alpha)}/\{2(\alpha+k/M)\}$$

$$+\sqrt{(k+\alpha)}\sqrt{k}\,(S_1(k)-S_2(k)+\mathcal{G}_+(k)+g_+(k))W_{-1}[\sqrt{(2D(k+\alpha))}]/\sqrt{2}\ . \quad (47)$$

$$\Phi'_-(\alpha,0) = -\frac{i\alpha}{4\pi}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0+D)u+\kappa Y_0]}}{(\alpha-u)}\left\{1-\sqrt{\left(\frac{k-\alpha}{k-u}\right)}\right\}du$$

$$+\sqrt{(k-\alpha)}\sqrt{k}\,(S_1(k)+S_2(k)+\mathcal{G}_+(k)-g_+(k))W_{-1}[\sqrt{(2D(k-\alpha))}]/\sqrt{2}\ . \quad (48)$$

Thus $A(\alpha)$ and $B(\alpha)$ are given by (20b) and (21b) as

$$A(\alpha) = -B(\alpha) = \frac{-\alpha e^{i\alpha D}}{4\pi\sqrt{(k-\alpha)}}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0-D)u+\kappa Y_0]}}{(\alpha-u)\sqrt{(k+u)}}du$$

$$+\ \frac{\alpha e^{-i\alpha D}}{4\pi\sqrt{(k+\alpha)}}\int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0+D)u+\kappa Y_0]}}{(\alpha-u)\sqrt{(k-u)}}du$$

$$+\ i\delta\sqrt{(k+k/M)}\exp[i(\alpha+k/M)D]/\{2\sqrt{(k-\alpha)}(\alpha+k/M)\}$$

$$-\ i(k-\alpha)^{-1/2}e^{i\alpha D}\sqrt{k}\,(S_1(k)-S_2(k)+\mathcal{G}_+(k)+g_+(k))W_{-1}[\sqrt{(2D(k+\alpha))}]/\sqrt{2}$$

$$-\ i(k+\alpha)^{-1/2}e^{-i\alpha D}\sqrt{k}\,(S_1(k)+S_2(k)+\mathcal{G}_+(k)-g_+(k))W_{-1}[\sqrt{(2D(k-\alpha))}]/\sqrt{2}\ .$$

(49)

If we let M = 0 and δ = 0 in (49) we obtain the solution of the problem of diffraction of a line source by a rigid strip in a fluid at rest. We now impose the Kutta-Joukowski edge condition (14) to account for the wake when 0 < M < 1. The edge condition (14) requires that in the expression for $A(\alpha)$ and $B(\alpha)$ the terms of order $O(e^{i\alpha D}|\alpha|^{-3/2})$ must vanish as $|\alpha| \to \infty$. Hence

$$\delta = \frac{a e^{-iKD/M}}{\sqrt{(K+K/M)}} \cdot \frac{1}{2\pi i} \int_{-\infty+ib'}^{\infty+ib'} \frac{e^{i[(X_0-D)u+\kappa Y_0]}}{\sqrt{(K+u)}}\, du$$
$$-\sqrt{2K}\exp[i\{(2-1/M)KD+\pi/4\}]\big(S_1(K)-S_2(K)+G_+(K)+\mathcal{G}_+(K)\big)\Big/\{\sqrt{(2\pi KD)}\sqrt{(1+1/M)}\} \tag{50}$$

where the last term of (50) is obtained by using the dominant term in the asymptotic expansion of $W_{-1}[\sqrt{(2D(K+\alpha))}]$, viz

$$W_{-1}[\sqrt{(2D(K+\alpha))}] \sim -\frac{\exp[i(2KD+\pi/4)]}{\sqrt{(2\pi D)}\;(\alpha+K)} + O(|\alpha|^{-2}), \text{ as } |\alpha| \to \infty \tag{51}$$

The solution to the original boundary value problem is now known and is given by

$$\chi(x,y) = -i H_0^{(1)}(K\sqrt{\{(X-X_0)^2+(Y-Y_0)^2\}})\exp[-iKM(X-X_0)](1-M^2)^{-1/2}/4$$
$$\pm \frac{\exp[-iKM(X-X_0)]}{8\pi^2\sqrt{(1-M^2)}} \int_{-\infty+i\tau}^{\infty+i\tau}\int_{-\infty+ib'}^{\infty+ib'} \frac{\exp[i\{-\alpha(X+D)\pm\kappa Y+(X_0+D)u+\sqrt{(K^2-u^2)}Y_0\}]\,du\,d\alpha}{(\alpha-u)\sqrt{(K+\alpha)}\sqrt{(K-u)}}$$
$$\mp \frac{\exp[-iKM(X-X_0)]}{8\pi^2\sqrt{(1-M^2)}} \int_{-\infty+i\tau}^{\infty+i\tau}\int_{-\infty+ib'}^{\infty+ib'} \frac{\exp[i\{-\alpha(X-D)\pm\kappa Y+(X_0-D)u+\sqrt{(K^2-u^2)}Y_0\}]\,du\,d\alpha}{\sqrt{(K-\alpha)}\sqrt{(K+u)}\left((\alpha-u)^{-1}-(\alpha+K/M)^{-1}\right)^{-1}}$$
$$\pm \frac{\exp[-iKM(X-X_0)]}{2\pi i} \int_{-\infty+i\tau}^{\infty+i\tau} \exp[i\{-\alpha(X+D)\pm\kappa Y\}]\left(\sqrt{\left(\frac{K}{2(K+\alpha)}\right)}\{S_1(K)+S_2(K)\right.$$
$$\left.+G_+(K)-\mathcal{G}_+(K)\}W_{-1}[\sqrt{(2D(K-\alpha))}]\right)d\alpha \;\pm$$

$$
\pm \frac{\exp[-ikM(X-X_0)]}{2\pi i}\int_{-\infty+i\tau}^{\infty+i\tau} \exp[i\{-\alpha(X-D)\pm\kappa_1 Y\}]\left(\sqrt{\frac{k}{2(k-\alpha)}}\,\{S_1(k)-S_2(k)\right.
$$

$$
\left. +\,\mathcal{G}_+(k)+ig_+(k)\}\left\langle W_{-1}[\sqrt{(2D(k+\alpha))}]+\frac{\sqrt{k}\,\exp[i(2kD+\pi/4)]}{\sqrt{(2\pi kD)}(\alpha+k/M)}\right\rangle\right)d\alpha
$$

(52)

where the upper sign corresponds to y > 0 and the lower sign to y < 0.

5. ASYMPTOTIC EXPRESSIONS FOR THE FAR FIELD IN THE SHADOW OF THE STRIP

We shall assume that the line source is placed such that $kr_{01}(1 - M^2)^{-1} = KR_{01} \geq 1$, $kr_{02}(1 - M^2)^{-1} = KR_{02} \geq 1$ where $X_o - D = R_{01} \cos \Theta_{01}$, $Y_o = R_{01} \sin \Theta_{01}$; $X_o + D = R_{02} \cos \Theta_{02}$, $Y_o = R_{02} \sin \Theta_{02}$. This assumption means that the source is not too near the edges. We shall also assume that the observation point is not too near the edges, i.e. $kr_1(1 - M^2)^{-1} = KR_1 \geq 1$, $kr_2(1 - M^2)^{-1} = KR_2 \geq 1$, where $X - D = R_1 \cos \Theta_1$, $Y = R_1 \sin \Theta_1$, $X + D = R_2 \cos \Theta_2$, and $Y = R_2 \sin \Theta_2$. Making these transformations the last two integrals of (52) can be evaluated asymptotically by the standard method of stationary phase. In the second double integral of (52) the integrand with the pole $\alpha = -K/M$ is separable so that the α and u integration can be carried out independently of each other. This integral is evaluated by using the standard method of stationary phase for the u-integration. The α-integration also uses the stationary phase method, due allowance being made for the wake pole $\alpha = -K/M$, see Rawlins [5]. In the remaining double integrals the integrands are not separable because of the pole $u = \alpha$. However by using a technique of Clemmow's [11] the following result is not difficult to derive

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{f(\nu,\mu) \exp\left[i\left\{\nu\xi + \sqrt{(k^2-\nu^2)}|\eta| + \mu\xi_0 + \sqrt{(k^2-\mu^2)}|\eta_0|\right\}\right] d\nu\, d\mu}{\sqrt{(k^2-\nu^2)}\sqrt{(k^2-\mu^2)}\,(\nu+\mu)}$$

$$\sim -\sqrt{\frac{2\pi}{k\rho}}\, e^{i(k\rho+\pi/4)} \cdot \sqrt{\frac{2\pi}{k\rho_0}}\, e^{i(k\rho_0-\pi/4)} \cdot f(k\cos\vartheta, k\cos\vartheta_0) \cdot \frac{2|N|F(|N|)}{k(\cos\vartheta+\cos\vartheta_0)}$$

+ POLE CONTRIBUTIONS,

where $\xi = \rho\cos\vartheta$, $\eta = \rho\sin\vartheta$, $\xi_0 = \rho_0\cos\vartheta_0$, $\eta_0 = \rho_0\sin\vartheta_0$,

$$0 \leq \vartheta \leq \pi, \quad 0 \leq \vartheta_0 \leq \pi, \quad N = \sqrt{\frac{k\rho\rho_0}{2}}\left(\frac{\cos\vartheta + \cos\vartheta_0}{\sqrt{(\rho_0|\sin\vartheta|^2 + \rho|\sin\vartheta_0|^2)}}\right).$$

Thus carrying out the above asymptotic evaluations and using (53) the expression for the far field is given by

$$\chi(r,\theta) = \chi_0(r,\theta) + \chi_d(r,\theta), \quad \text{in} \left\{ \begin{array}{c} \Theta_{01} - \pi < \Theta_1 < \pi - \Theta_{01} \\ \Theta_{02} < \Theta_2 < -\Theta_{02} \end{array} \right\}$$

$$= \chi_o(r,\theta) - \chi_r(r,\theta) + \chi_d(r,\theta), \text{ in } \begin{cases} \pi - \Theta_{01} < \Theta_1 < \pi \\ 0 < \Theta_2 < \Theta_{02} \end{cases}$$

$$= \chi_d(r,\theta) \qquad \text{in } \begin{cases} -\pi < \Theta_1 < \Theta_{01} - \pi \\ -\Theta_{02} < \Theta_2 < 0 \end{cases}$$

where

$$\chi_o(r,\theta) = -ia \exp[-ikMX] H_0^{(1)}(k\sqrt{\{(X-X_0)^2+(Y-Y_0)^2\}})/4$$

$$\chi_r(r,\theta) = -ia \exp[-ikMX] H_0^{(1)}(k\sqrt{\{(X-X_0)^2+(Y+Y_0)^2\}})/4$$

and

$$\chi_d(r,\theta) = \frac{\exp[ik\{(R_{01}+R_1)-MX\}]}{\sqrt{(2\pi kR_{01})}\sqrt{(2\pi kR_1)}} \cdot \frac{a\sqrt{(1-\cos\Theta_{01})}\sqrt{(1-\cos\Theta_1)}}{2}\left\{\frac{2|Q_1|F(|Q_1|)}{(\cos\Theta_1+\cos\Theta_{01})} + \frac{2|Q'|F(|Q'|)}{(1/M-\cos\Theta_1)}\right\}$$

$$- \frac{\exp[ik\{(R_{02}+R_2)-MX\}]}{\sqrt{(2\pi kR_{02})}\sqrt{(2\pi kR_2)}} \cdot \frac{a\sqrt{(1+\cos\Theta_{02})}\sqrt{(1+\cos\Theta_2)}}{2} \cdot \frac{2|Q_2|F(|Q_2|)}{(\cos\Theta_2+\cos\Theta_{02})}$$

$$+ \frac{\exp[i\{(kR_2+\pi/4)-kMX\}]}{\sqrt{(2\pi kR_2)}} W_{-1}[\sqrt{(2kD(1+\cos\Theta_2))}]\sqrt{(1+\cos\Theta_2)}\Big\{ K(S_1(K)+S_2(K)+G_+(K)-\mathcal{G}_+(K))/\sqrt{2}\Big\}$$

$$+ \frac{\exp[i\{(kR_1+\pi/4)-kMX\}]}{\sqrt{(2\pi kR_1)}}\left\langle W_{-1}[\sqrt{(2kD(1-\cos\Theta_1))}] + \frac{k^{-1/2}\exp[i\{2kD+\pi/4\}]}{(1/M-\cos\Theta_1)\sqrt{(2\pi kD)}}\right\rangle$$

$$\times \sqrt{(1-\cos\Theta_1)}\,K(S_1(K)-S_2(K)+G_+(K)+\mathcal{G}_+(K))/2 \quad , \qquad (36)$$

where

$$Q_1 = \sqrt{\frac{kR_1 kR_{01}}{2}}\left(\frac{\cos\Theta_1 + \cos\Theta_{01}}{\sqrt{(kR_{01}|\sin\Theta_1|^2 + kR_1|\sin\Theta_{01}|^2)}}\right),$$

$$Q_2 = \sqrt{\frac{kR_2 kR_{02}}{2}}\left(\frac{\cos\Theta_2 + \cos\Theta_{02}}{\sqrt{(kR_{02}|\sin\Theta_2|^2 + kR_2|\sin\Theta_{02}|^2)}}\right),$$

$$Q' = \sqrt{\frac{kR_1}{2}}\left(\frac{\cos\Theta_1 - 1/M}{\sin\Theta_1}\right),$$

$$S_1(k) = \frac{a}{2ik^{1/2}\sqrt{(1-\cos\Theta_{01})}}\cdot\frac{\exp[i(kR_{01}-\pi/4)]}{\sqrt{(2\pi kR_{01})}}$$
$$+ \frac{a}{2ik^{1/2}\sqrt{(1+\cos\Theta_{02})}}\cdot\frac{\exp[i(kR_{02}-\pi/4)]}{\sqrt{(2\pi kR_{02})}} - \frac{a\sqrt{(1-\cos\Theta_{01})}}{2ik^{1/2}(1+1/M)}\cdot\frac{\exp[i(kR_{01}-\pi/4)]}{\sqrt{(2\pi kR_{01})}},$$

$$S_2(k) = \frac{a}{2ik^{1/2}\sqrt{(1-\cos\Theta_{01})}}\cdot\frac{\exp[i(kR_{01}-\pi/4)]}{\sqrt{(2\pi kR_{01})}}$$
$$- \frac{a}{2ik^{1/2}\sqrt{(1+\cos\Theta_{02})}}\cdot\frac{\exp[i(kR_{02}-\pi/4)]}{\sqrt{(2\pi kR_{02})}} - \frac{a\sqrt{(1-\cos\Theta_{01})}}{2ik^{1/2}(1+1/M)}\cdot\frac{\exp[i(kR_{01}-\pi/4)]}{\sqrt{(2\pi kR_{01})}},$$

$$g_+(k) = \frac{\sqrt{2k}\,S_1(k)\,W_{-1}[\sqrt{(4kD)}]}{1-\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}]},$$

$$\mathcal{g}_+(k) = \frac{\sqrt{2k}\,S_2(k)\,W_{-1}[\sqrt{(4kD)}]}{1+\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}]}.$$

$x = r\cos\theta$, $y = r\sin\theta$, $x_0 = r_0\cos\theta_0$, $y_0 = r_0\sin\theta_0$,

$R_{01} = \sqrt{\{(r_0\cos\theta_0 - d)^2(1-M^2)^{-1} + r_0^2\sin^2\theta_0\}}$, $\Theta_{01} = \cos^{-1}[(r_0\cos\theta_0 - d)/\{\sqrt{(1-M^2)}R_{01}\}]$,

$R_{02} = \sqrt{\{(r_0\cos\theta_0 + d)^2(1-M^2)^{-1} + r_0^2\sin^2\theta_0\}}$, $\Theta_{02} = \cos^{-1}[(r_0\cos\theta_0 + d)/\{\sqrt{(1-M^2)}R_{02}\}]$,

$R_1 = \sqrt{\{(r\cos\theta - d)^2(1-M^2)^{-1} + r^2\sin^2\theta\}}$, $\Theta_1 = \cos^{-1}[(r\cos\theta - d)/\{\sqrt{(1-M^2)}R_1\}]$,

$R_2 = \sqrt{\{(r\cos\theta + d)^2(1-M^2)^{-1} + r^2\sin^2\theta\}}$, $\Theta_2 = \cos^{-1}[(r\cos\theta + d)/\{\sqrt{(1-M^2)}R_2\}]$.

A more useful quantity is the attenuation in the shadow region. The attenuation is given by 20 $\log_{10}|\chi_d(r,\theta)/\phi_0(r,\theta)|$, which is a measure of the effective shielding provided, and is independent of the type of source. More specifically the attenuation is given by

$$20\log_{10}\Bigg[\{(KR_2)^2 + (KR_{02})^2 - 2KR_2.KR_{02}\cos(\Theta_{02} - \Theta_2)\}^{1/4}.(2\pi)^{-1/2} \times$$

$$\times\Bigg|\frac{\sqrt{(1-\cos\Theta_{01})}\sqrt{(1-\cos\Theta_1)}}{\sqrt{(kR_{01}.KR_1)}}\left\{\frac{2|Q_1|F(|Q_1|)}{(\cos\Theta_1 + \cos\Theta_{01})} + \frac{2|Q'|F(|Q'|)}{(1/M - \cos\Theta_1)}\right\}e^{ik(R_{01}+R_1)}$$

$$-\frac{\sqrt{(1+\cos\Theta_{02})}\sqrt{(1+\cos\Theta_2)}}{\sqrt{(KR_{02}.KR_2)}}.\frac{2|Q_2|F(|Q_2|)}{(\cos\Theta_2 + \cos\Theta_{02})}e^{iK(R_{02}+R_2)}$$

$$+\frac{\sqrt{(1+\cos\Theta_2)}}{\sqrt{(KR_2)}}\{\sqrt{2k'}W_{-1}[\sqrt{(2KD(1+\cos\Theta_2))}]\}F_2(\Theta_{01},\Theta_{02})e^{iKR_2}$$

$$+\frac{\sqrt{(1-\cos\Theta_1)}}{\sqrt{(kR_1)}}\left\{\sqrt{2k'}W_{-1}[\sqrt{(2KD(1-\cos\Theta_1))}] + \frac{e^{i(2KD+\pi/4)}}{\sqrt{(\pi KD)}(1/M - \cos\Theta_1)}\right\}F_1(\Theta_{01},\Theta_{02})e^{ikR_1}\Bigg|\Bigg] \quad (37)$$

where

$$F_1(\Theta_{01},\Theta_{02}) = -\frac{i\exp[ikR_{02}]}{\sqrt{(1+\cos\Theta_{02})}\sqrt{(kR_{02})}} - \frac{i\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}]}{2(1-\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}])}\left\{\frac{\exp[ikR_{02}]}{\sqrt{(1+\cos\Theta_{02})}\sqrt{(kR_{02})}} + \frac{\exp[ikR_{01}]}{\sqrt{(1-\cos\Theta_{01})}\sqrt{(kR_{01})}} - \frac{\sqrt{(1-\cos\Theta_{01})}\exp[ikR_{01}]}{(1+1/M)\sqrt{(kR_{01})}}\right\}$$

$$-\frac{i\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}]}{2(1+\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}])}\left\{\frac{\exp[ikR_{01}]}{\sqrt{(1-\cos\Theta_{01})}\sqrt{(kR_{01})}} - \frac{\exp[ikR_{02}]}{\sqrt{(1+\cos\Theta_{02})}\sqrt{(kR_{02})}} - \frac{\sqrt{(1-\cos\Theta_{01})}\exp[ikR_{01}]}{(1+1/M)\sqrt{(kR_{01})}}\right\},$$

$$F_2(\Theta_{01},\Theta_{02}) = -\frac{i\exp[ikR_{01}]}{\sqrt{(1-\cos\Theta_{01})}\sqrt{(kR_{01})}} + \frac{i\sqrt{(1-\cos\Theta_{01})}\exp[ikR_{01}]}{(1+1/M)\sqrt{(kR_{01})}}$$

$$-\frac{i\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}]}{2(1-\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}])}\left\{\frac{\exp[ikR_{02}]}{\sqrt{(1+\cos\Theta_{02})}\sqrt{(kR_{02})}} + \frac{\exp[ikR_{01}]}{\sqrt{(1-\cos\Theta_{01})}\sqrt{(kR_{01})}} - \frac{\sqrt{(1-\cos\Theta_{01})}\exp[ikR_{01}]}{(1+1/M)\sqrt{(kR_{01})}}\right\} + \frac{i\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}]}{2(1+\sqrt{2k}\,W_{-1}[\sqrt{(4kD)}])}\left\{\frac{\exp[ikR_{01}]}{\sqrt{(1-\cos\Theta_{01})}\sqrt{(kR_{01})}}\right.$$

$$\left. - \frac{\exp[ikR_{02}]}{\sqrt{(1+\cos\Theta_{02})}\sqrt{(kR_{02})}} - \frac{\sqrt{(1-\cos\Theta_{01})}\exp[ikR_{01}]}{(1+1/M)\sqrt{(kR_{01})}}\right\}.$$

6. GRAPHICAL RESULTS

As a practical numerical example the expression (37) is used to give graphs of the attenuation in the shadow on an aeroplane wing. The wing chord is taken as 10 ft., and the observation point is assumed to be 1000 ft. below the wing. The sound source, of frequency 1.128 KHz, is assumed to be positioned 1 ft. above the wing and (a) 1 ft. from the trailing edge, (b) at the centre of the wing, and (c) 1 ft. from the leading edge (see fig. 2). The speed of the plane is assumed to be such that the Mach number can be 0.1, 0.5 and 0.9. In terms of the variables used in this paper:

$f = 1.128$ KHz; $k = 2\pi$ ft.$^{-1}$; $M = 0.1, 0.5$, and 0.9;

$kd = 10\pi$; $kr = 2000\pi$;

(a) $kr_0 = 2\sqrt{17}\,\pi$, $\theta_0 = \tan^{-1}(1/4) \simeq 14°$

(b) $kr_0 = 2\pi$, $\theta_0 = \pi/2 = 90°$

(c) $kr_0 = 2\sqrt{17}\,\pi$, $\theta_0 = \pi - \tan^{-1}(1/4) \simeq 166°$

Figures 3 to 8 are plots of the attenuation given by the expression (37) in the shadow region of the stip for Mach number M = 0.5 and the three source positions. The oscillations of the attenuation are due to the constructive/destructive interaction of the diffracted fields from the edges of the strip. The number of oscillations increase as M increases because the apparent wavelength is proportional to the source wavelength multiplied by $(1 - M^2)^{\frac{1}{2}}$. Thus the apparent wavelength decreases as $|M|$ increases. The important part of the to from the point of noise measurement is the upper envelope, i.e. the profile of the maxima of these graphs. This profile gives the least, or worst, possible attenuation in the shadow of the wing. Thus these profiles are given in figures 6 to 8 for various Mach numbers and source positions. It should be remarked that the results given here are for the situation where the source, shield and observer are moving. In the flyover situation, the source and shield are moving but the observer is stationary.

It is possible that the present work, together with a Lorentz transformation technique used by Cooke [12], can be combined to give theoretical results for the flyover situation.

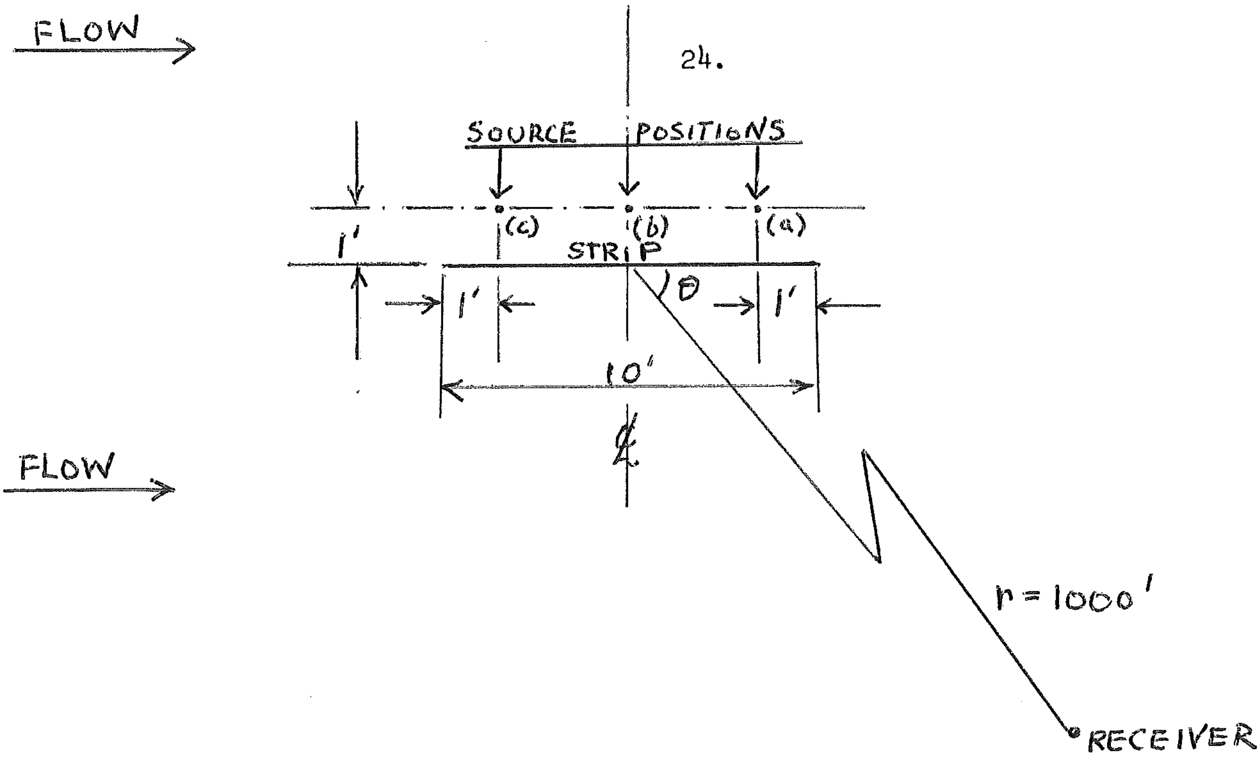


Fig. 2.

The graphs show:

(i) As the Mach number increases the sound level decreases in the shadow of the strip for all source positions.

(ii) For a given fixed M and source position the sound level in the shadow of the leading edge decreases more slowly (as we go into the shadow of the strip) than it does at the trailing edge.

(iii) For M = 0.5 and 0.9 the sound level at the leading (trailing) edge shadow region is least for source position (a) ((c)). For M = 0.1 the sound level at both the leading and trailing edge is least for source position (b).

(iv) The source position (b) gives the largest shadow region and its least attenuation is greater than all the other source positions at approximately $\theta = -90^{o}$. It also appears that the average least attenuation is greater for this position than for source positions (a) and (c).

(v) In graphs 4 to 6 it can be seen that the best source position, as far as attenuation is concerned, is (b). These graphs show that the wake does have a shielding effect which becomes more prominent as the Mach number gets larger.

7. CONCLUSIONS

For a choice of source position the best would seem to be that which gives the largest angular shadow region below the wing. Thus if the source was fixed at a specific height above a wing a central position would yield the best attenuation in the shadow of the wing. The effect of the wake is to produce a shield at the trailing edge of the wing. This shield is more effective at Mach numbers which are rather higher than are achieved by most aircraft at take off and approach conditions.

For the more practical situation where the engine consists of a jet then the jet can be modelled by a line of sources. The present work can, in fact, be extended to cope with this situation by the principle of superposition. Thus if the jet was modelled by apparent sources at positions (a_1), (a_2), (a_3), ... (a_n) then by the principle of superposition the attenuation is given be $20 \log_{10}|(\chi a_1 + \chi a_2 + \chi a_3 \ldots)/(\phi_{oa_1} + \phi_{oa_2} + \phi_{oa_3} \ldots)|$ where χa_i represents the field produced by a source ϕ_{oa_i} at position (a_i). Without going through the exact detailed solution for such a situation the analysis for one source would seem to indicate that the jet should be placed on the centre line of the wing so that the apparent sources down stream of the jet will be shielded by the wake. This assumes the apparent sources are all equally noisy. If this is not the case it might be more expedient to arrange the jet so that the noisiest apparent source is at the centre line of the wing.

Finally, we note that the present technique together with the work Rawlins [4], [8] , can be extended to consider the situation where an absorbing strip replaces the rigid strip. It would be expected that such an arrangement would give even greater attenuation in the shadow of the strip.

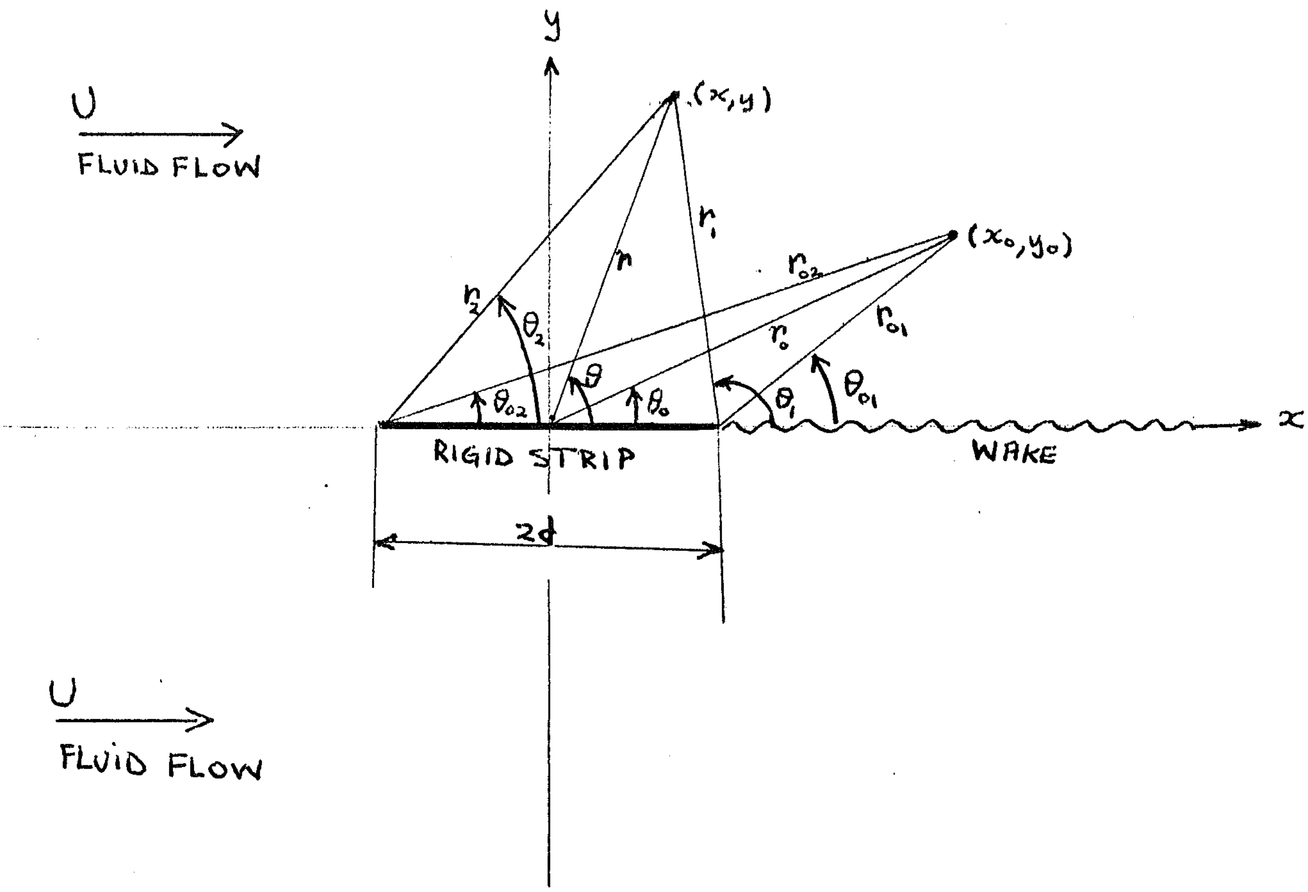
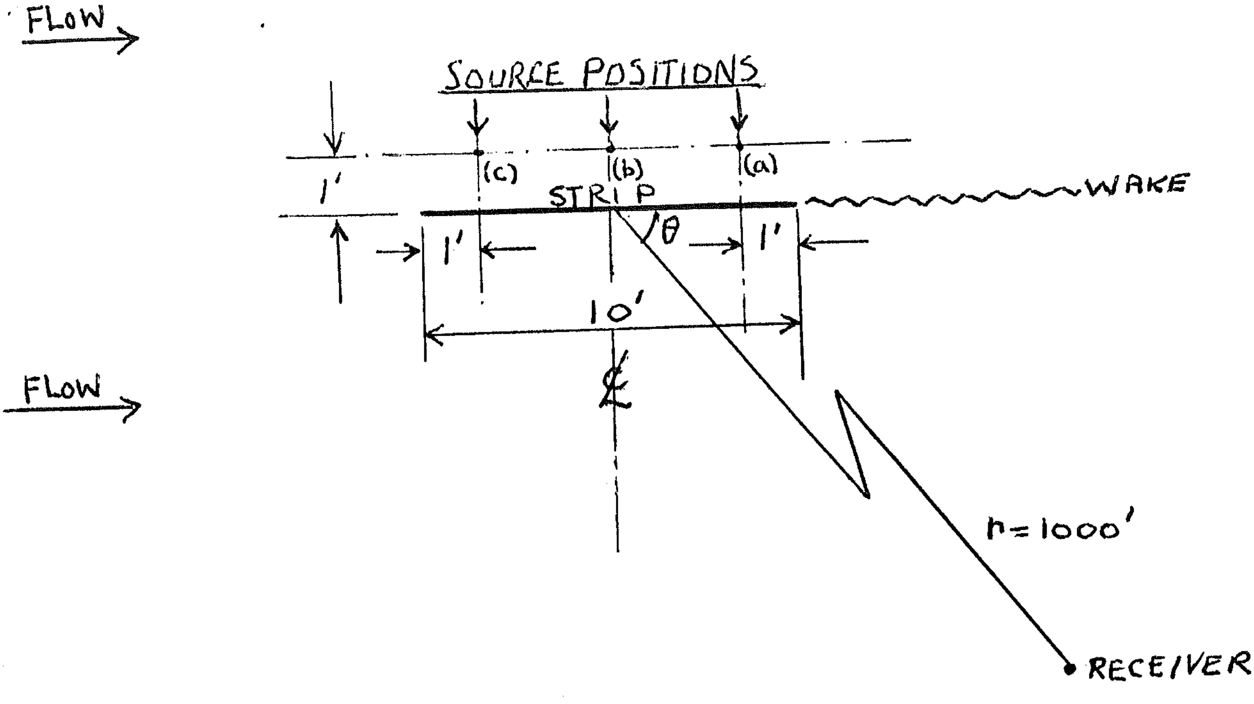


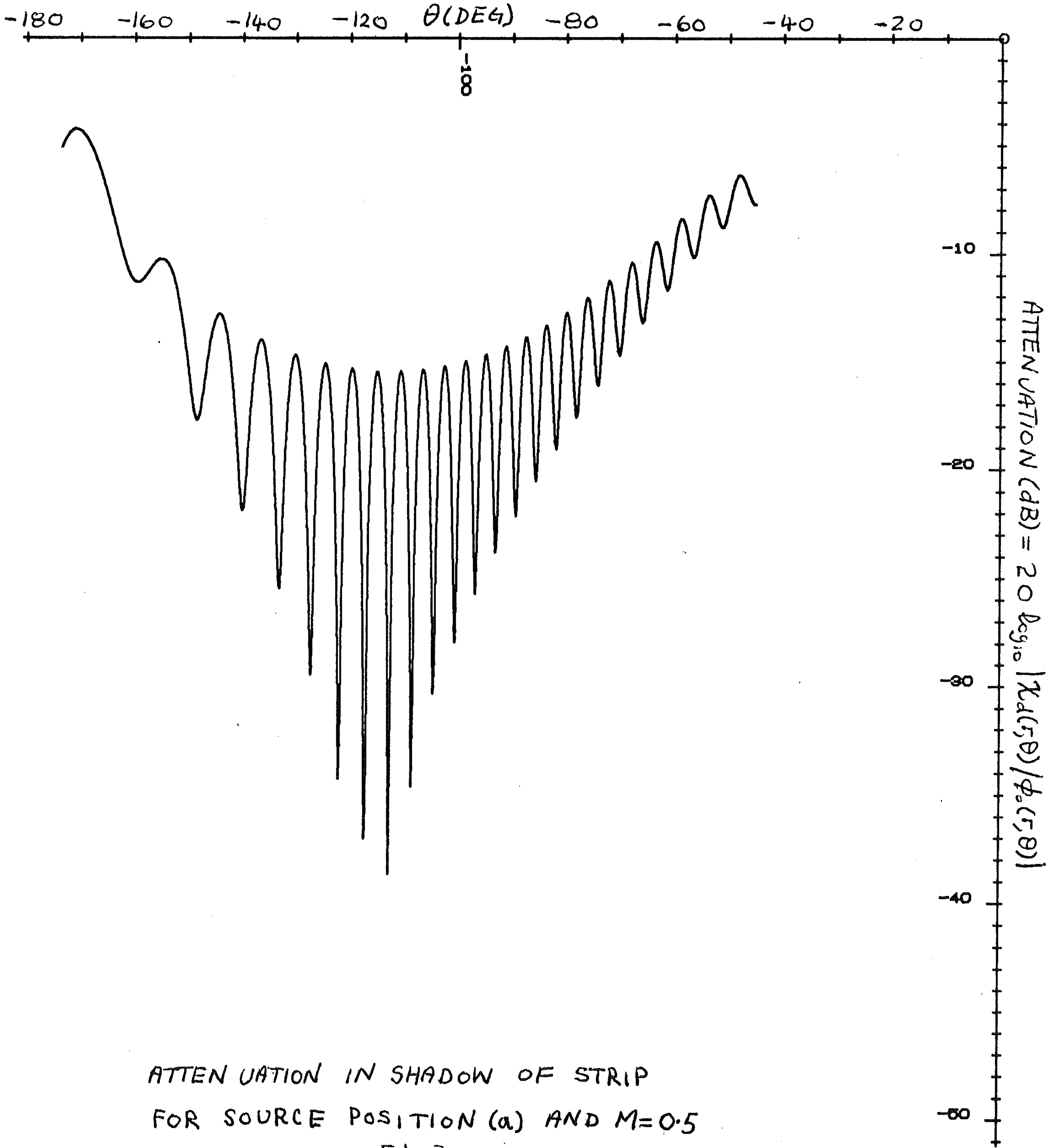
Fig 1.



FLOW
SOURCE POSITIONS
(c)
(b)
(a)
1'
STRIP
WAKE
θ
1'
1'
10'
℄
FLOW
r = 1000'
RECEIVER

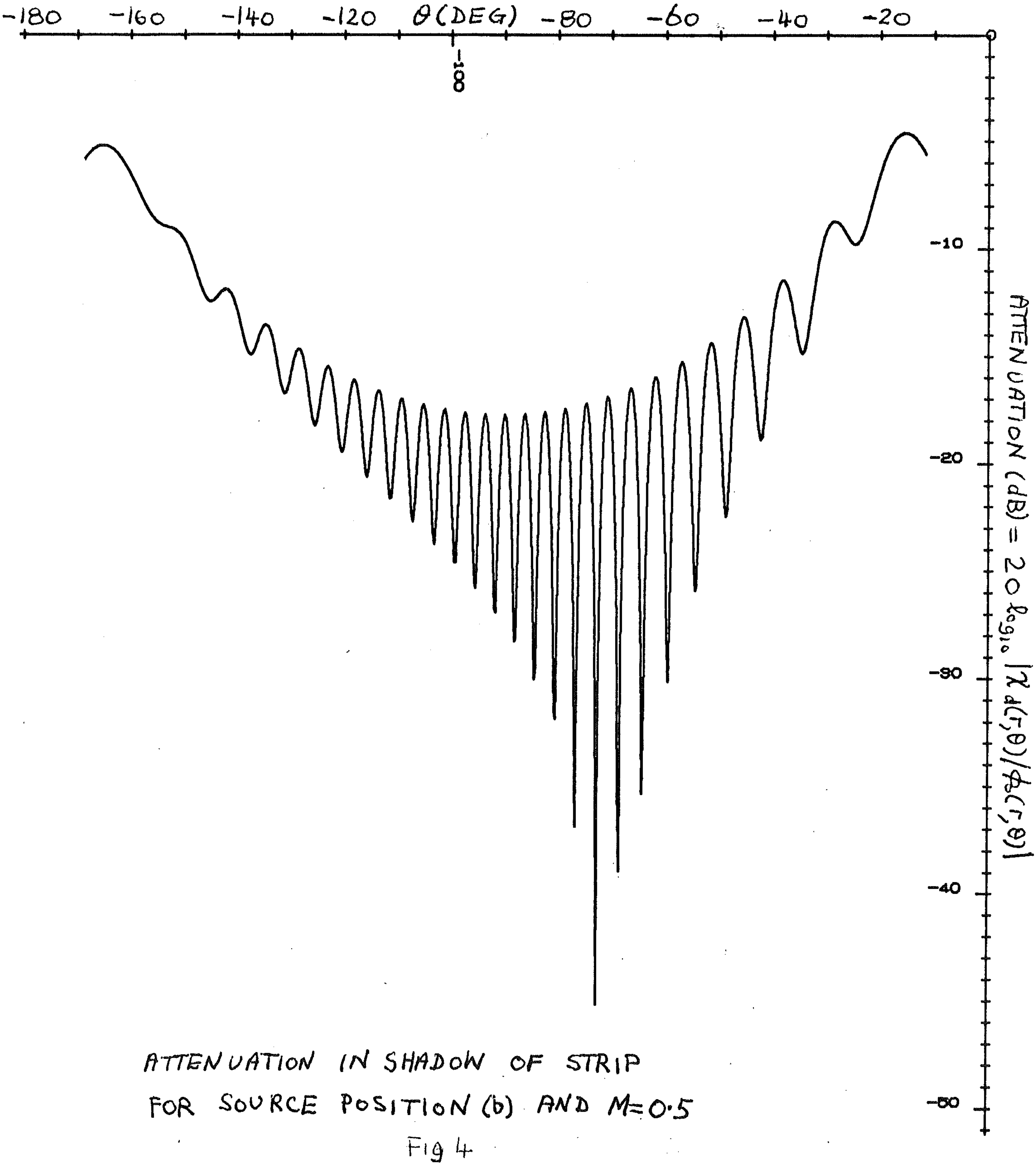
fig 2.

KRO= 25.806237 KR= 6283.1853 KD= 31.415927 M= .50000000 THO= 14.036243



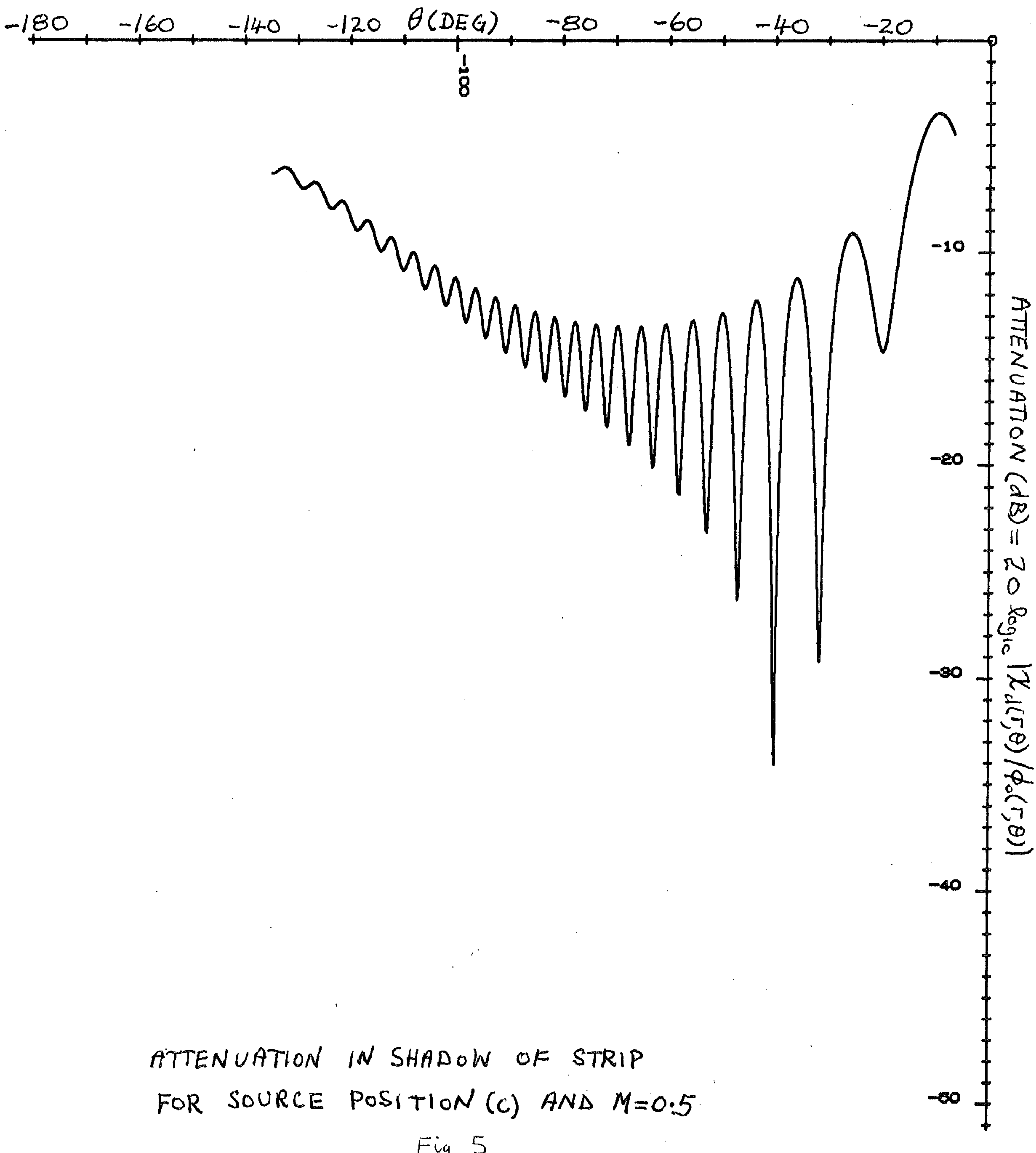
ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITION (a) AND M=0·5
Fig. 3.

KRO= 6.2831853 KR= 6283.1853 KD= 91.415927 M= .50000000 THO= 90.000000



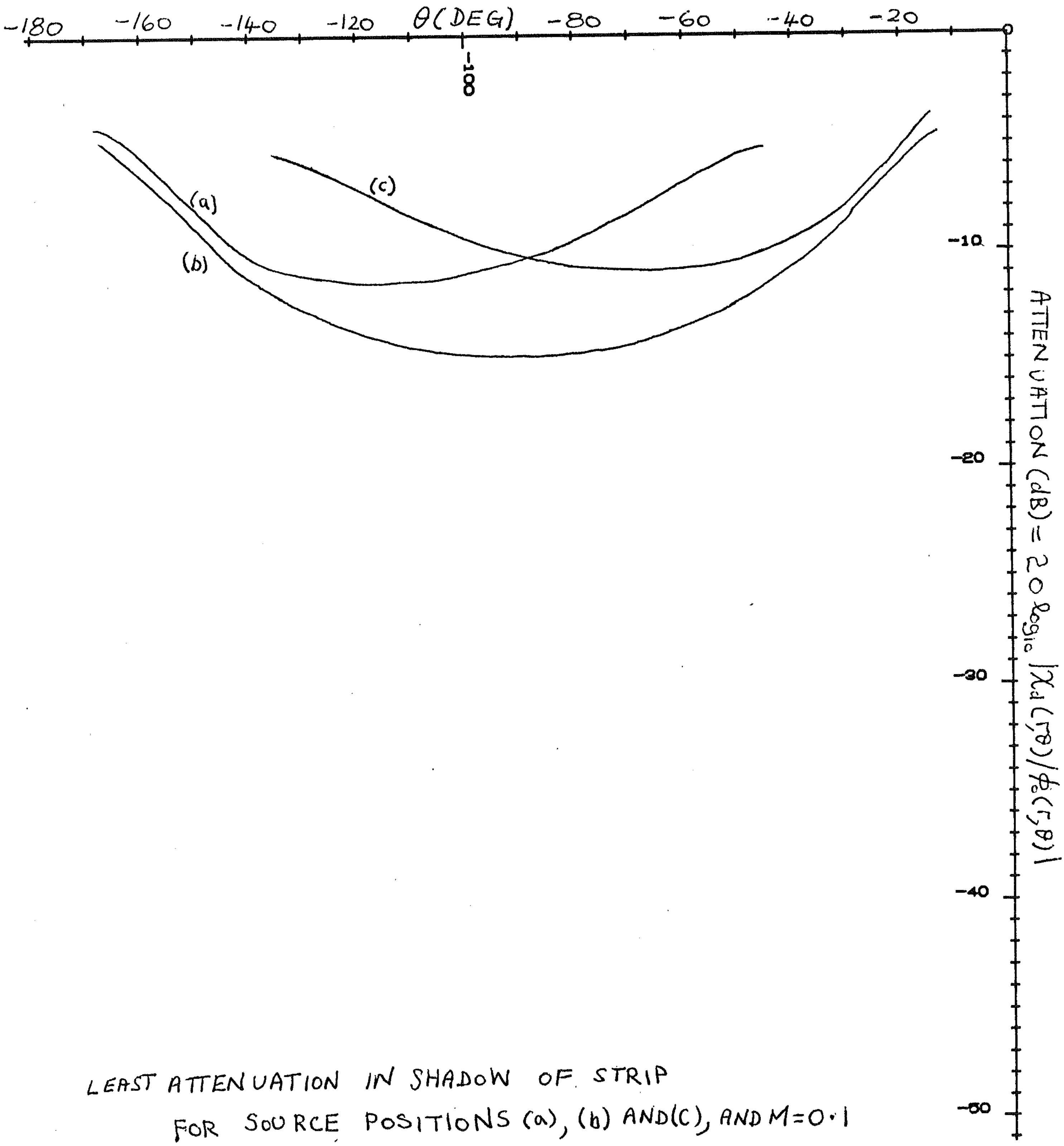
ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITION (b) AND M=0·5
Fig 4

KRO= 25.806237 KR= 6283.1853 KD= 31.415927 M= .50000000 THO= 165.86376



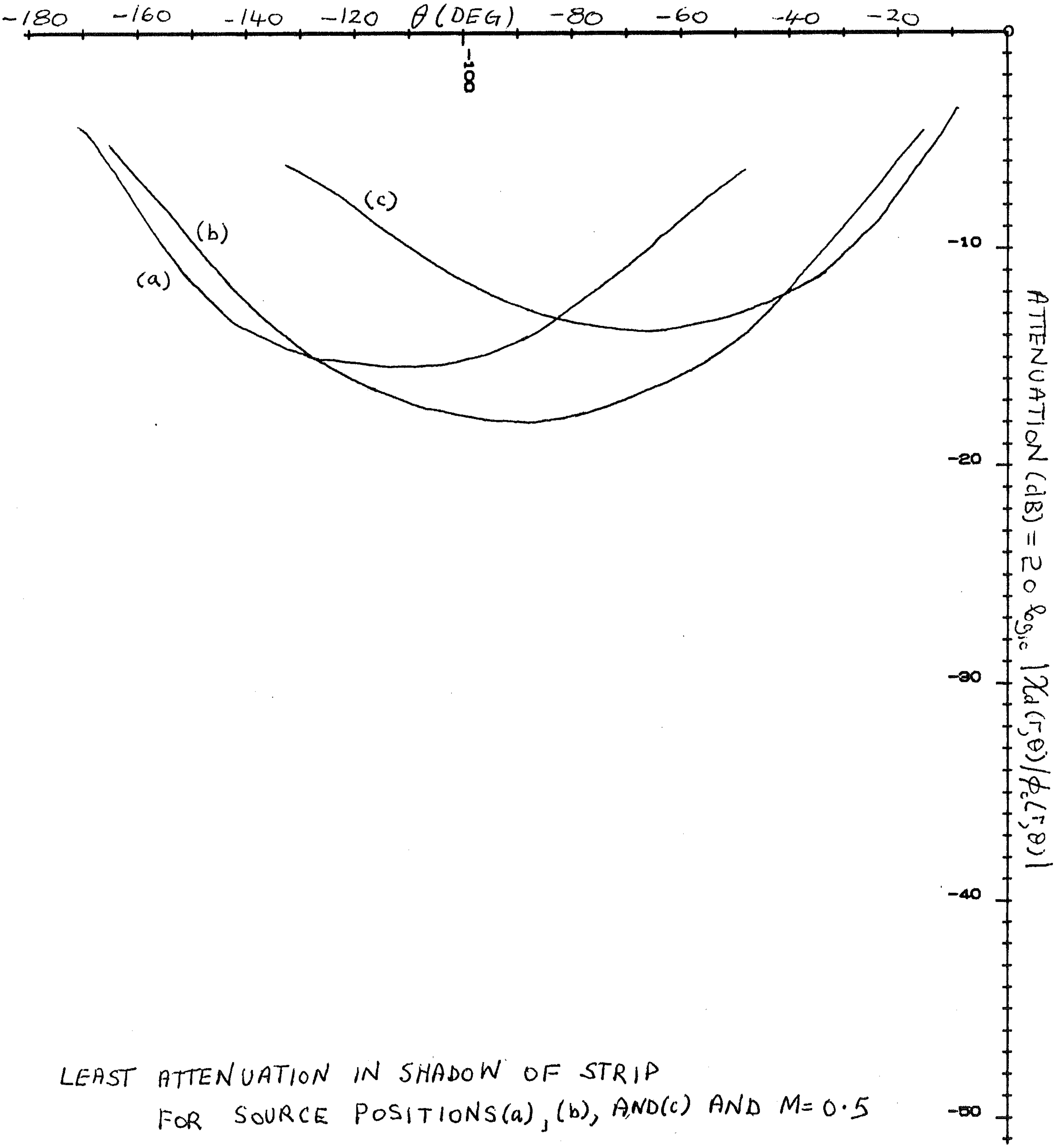
ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITION (c) AND M=0·5

Fig. 5



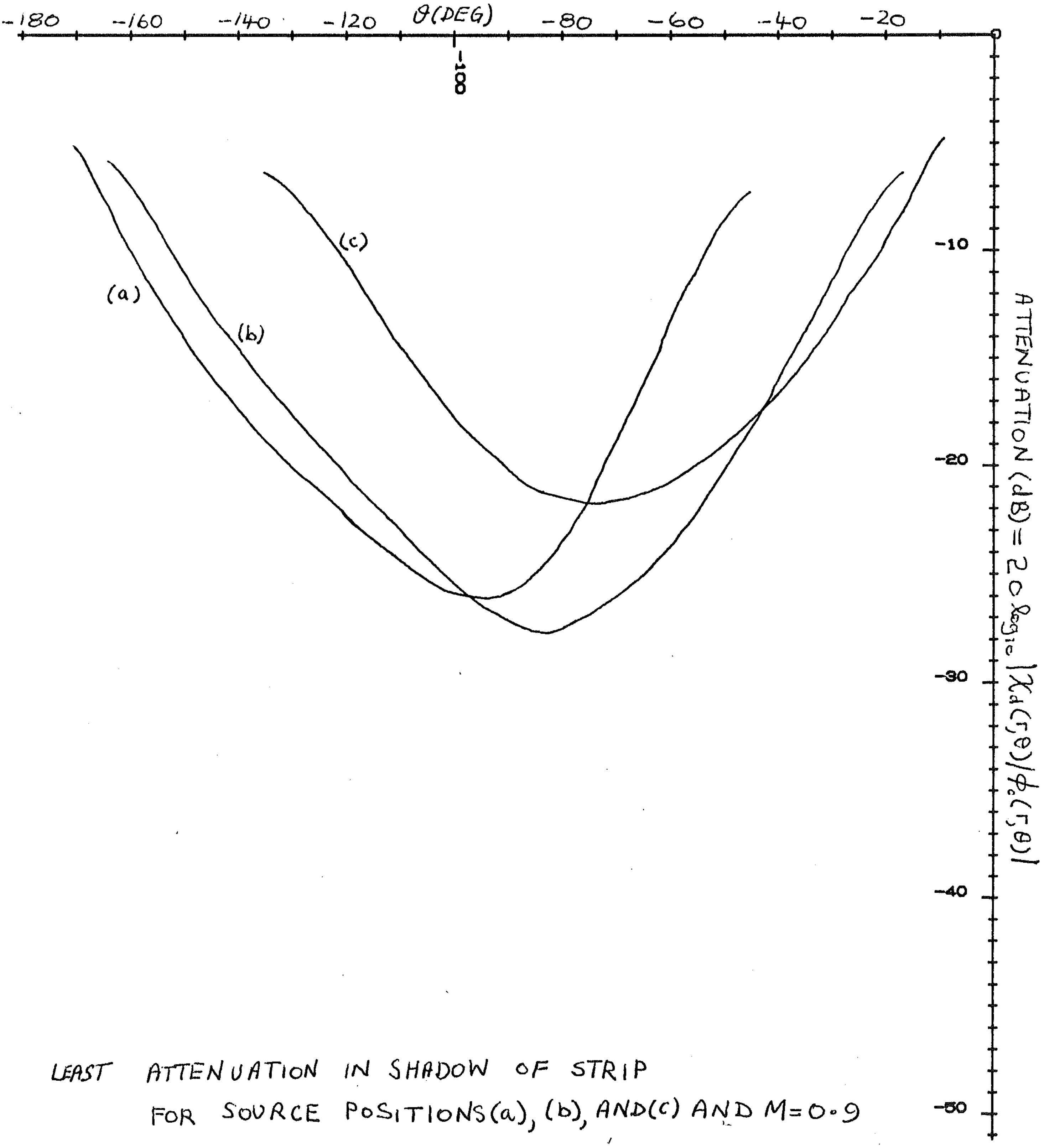
LEAST ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITIONS (a), (b) AND (C), AND $M = 0{\cdot}1$

Fig 6



LEAST ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITIONS (a), (b), AND (c) AND $M = 0\cdot 5$

Fig 7



LEAST ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITIONS (a), (b), AND (c) AND M=0·9

Fig 8

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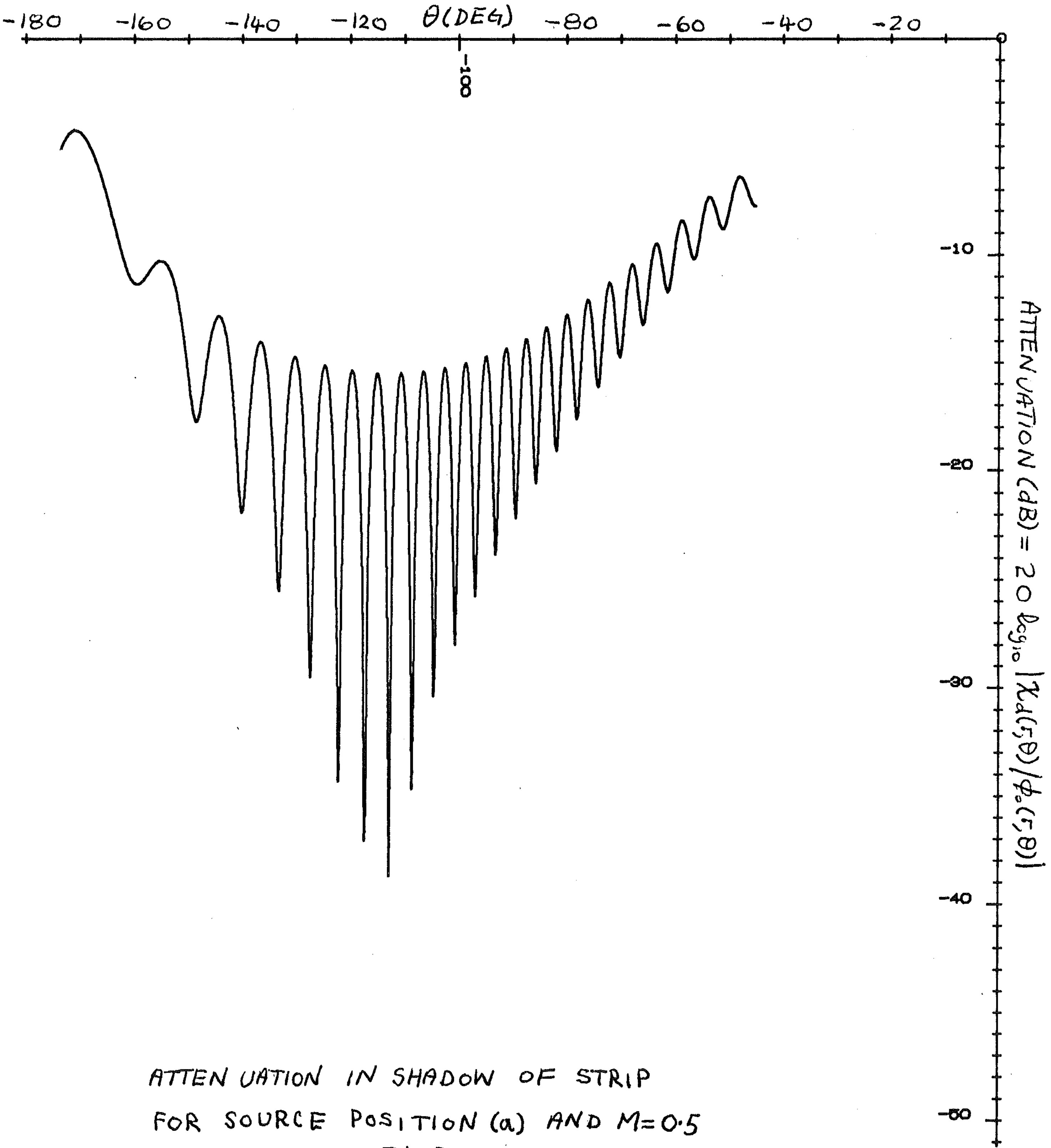
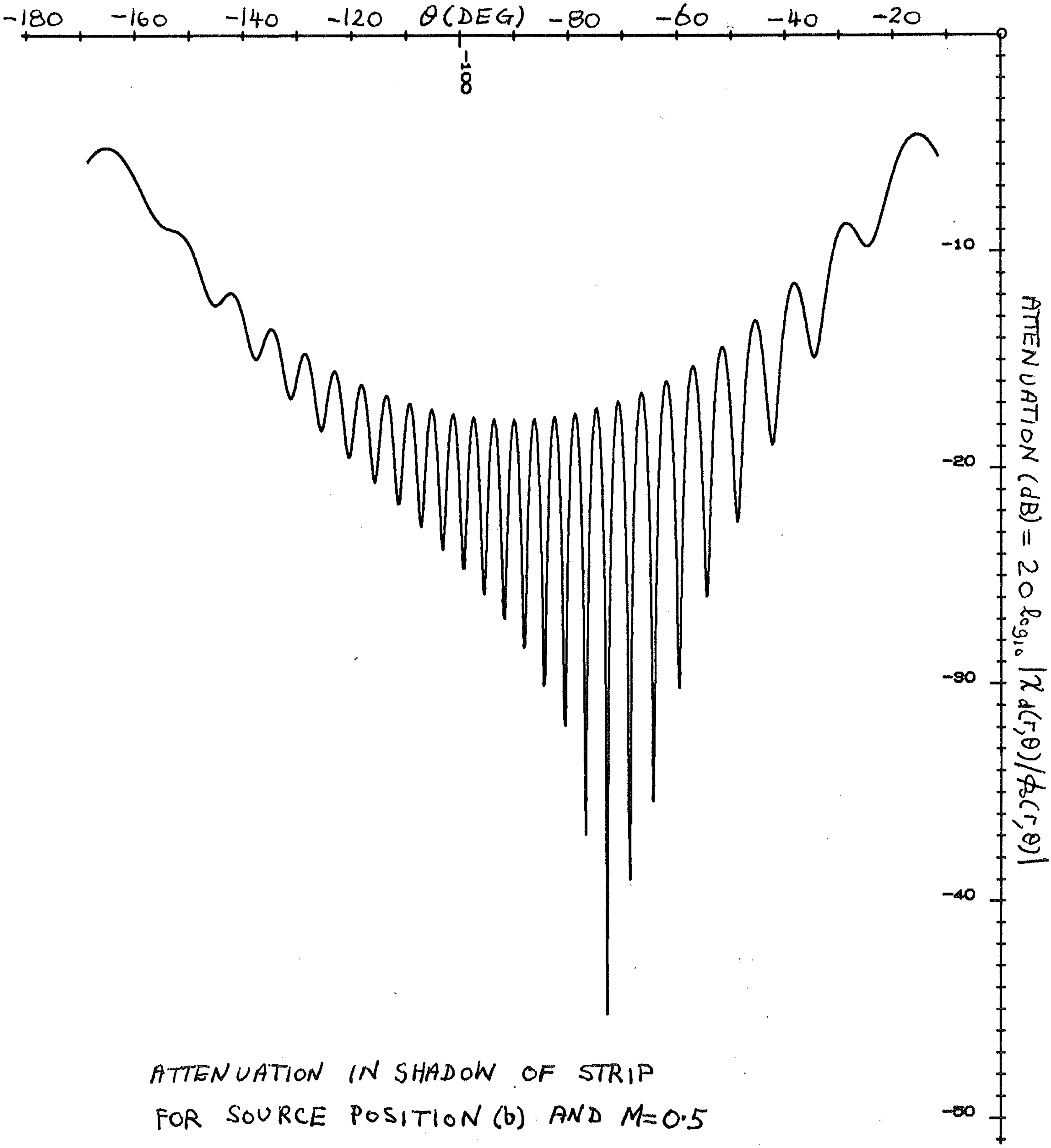
KRO= 25.806237 KR= 6283.1853 KD= 31.415927 M= .50000000 THO= 14.036243



ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITION (a) AND M=0·5

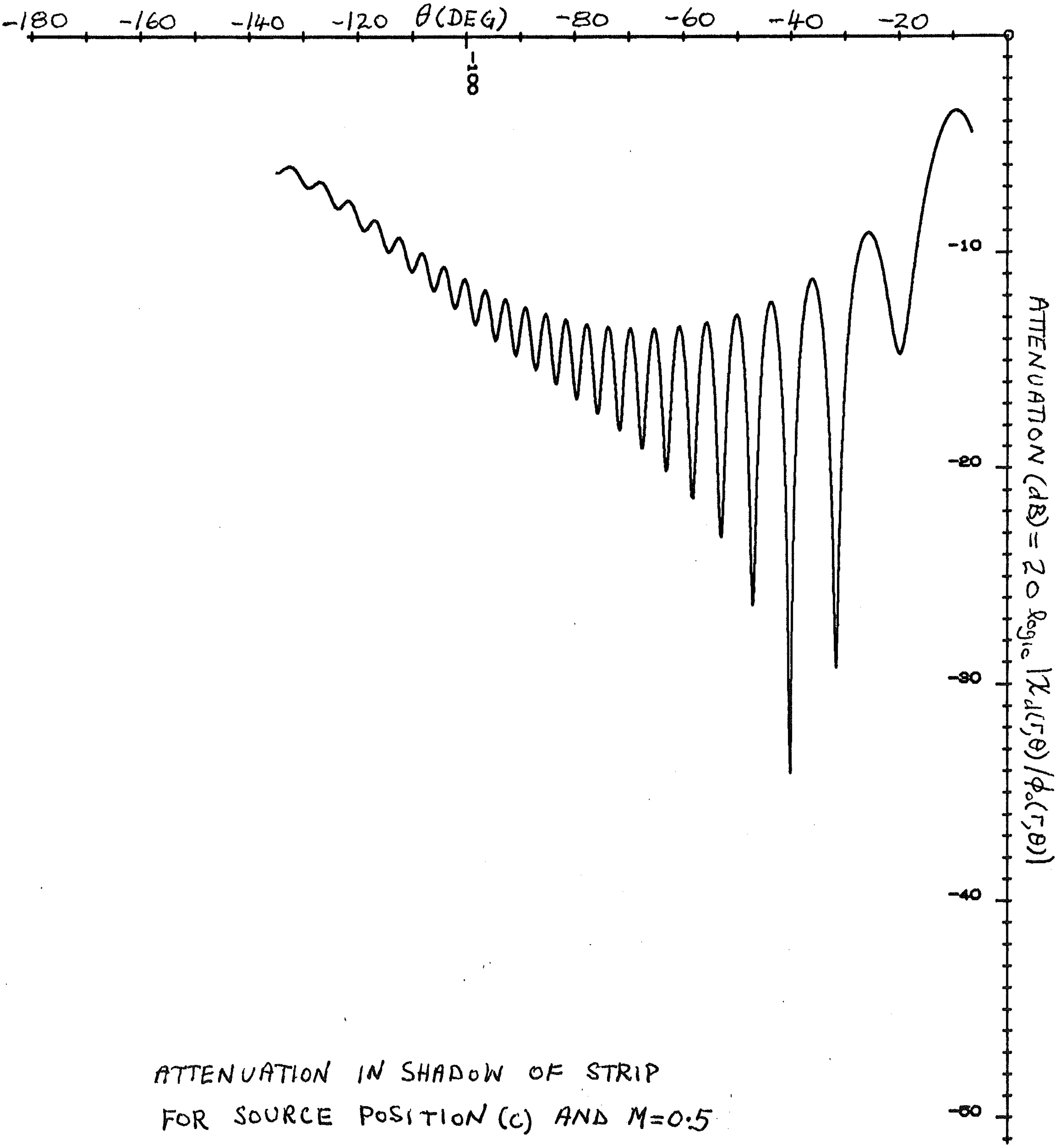
Fig. 3

KRO= 6.2831853 KR= 6283.1853 KD= 31.415927 M= .50000000 THO= 90.000000



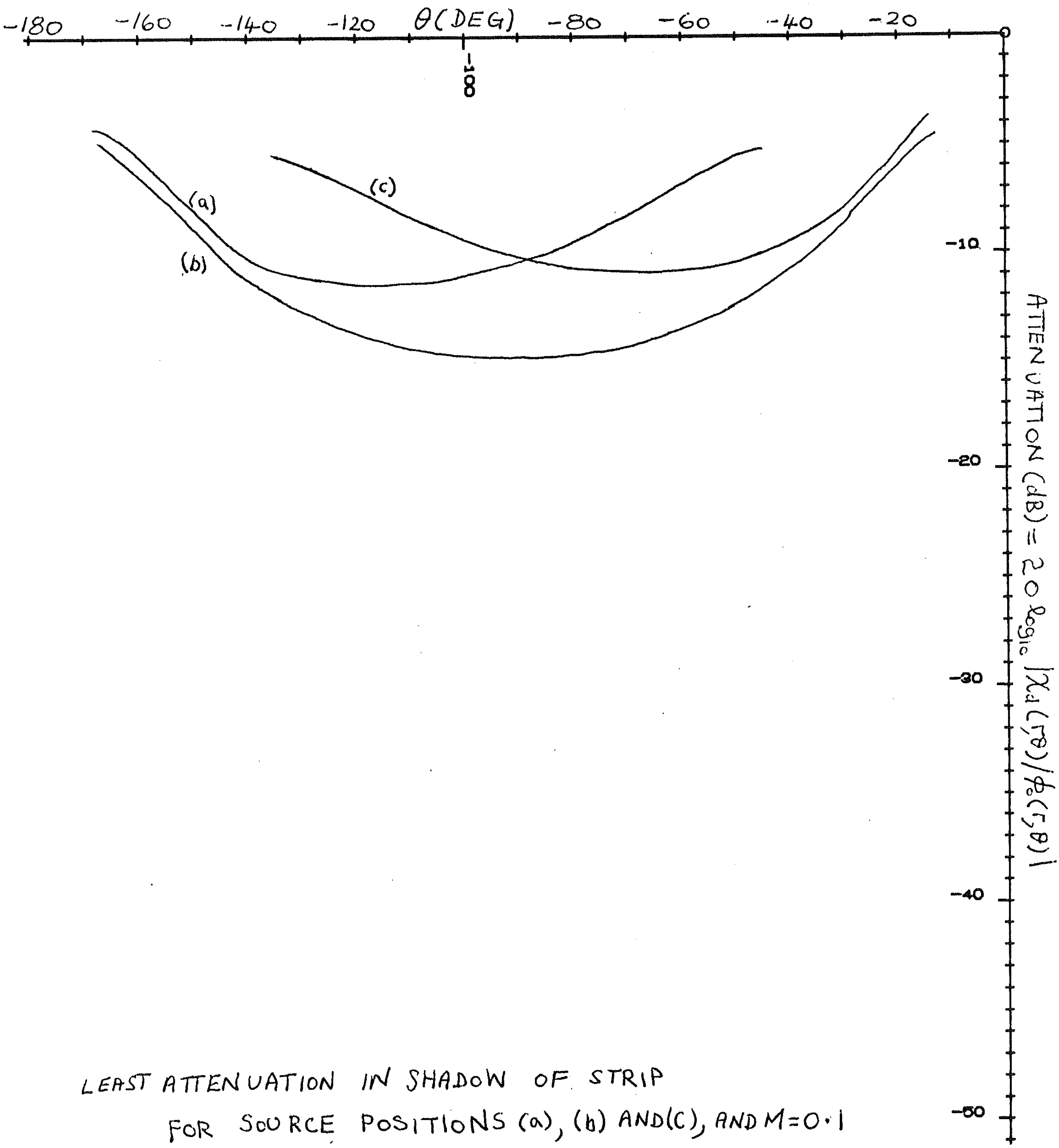
ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITION (b) AND M=0·5
Fig 4

KRO= 25.806237 KR= 6283.1853 KD= 31.415927 M= .50000000 THO= 165.86376



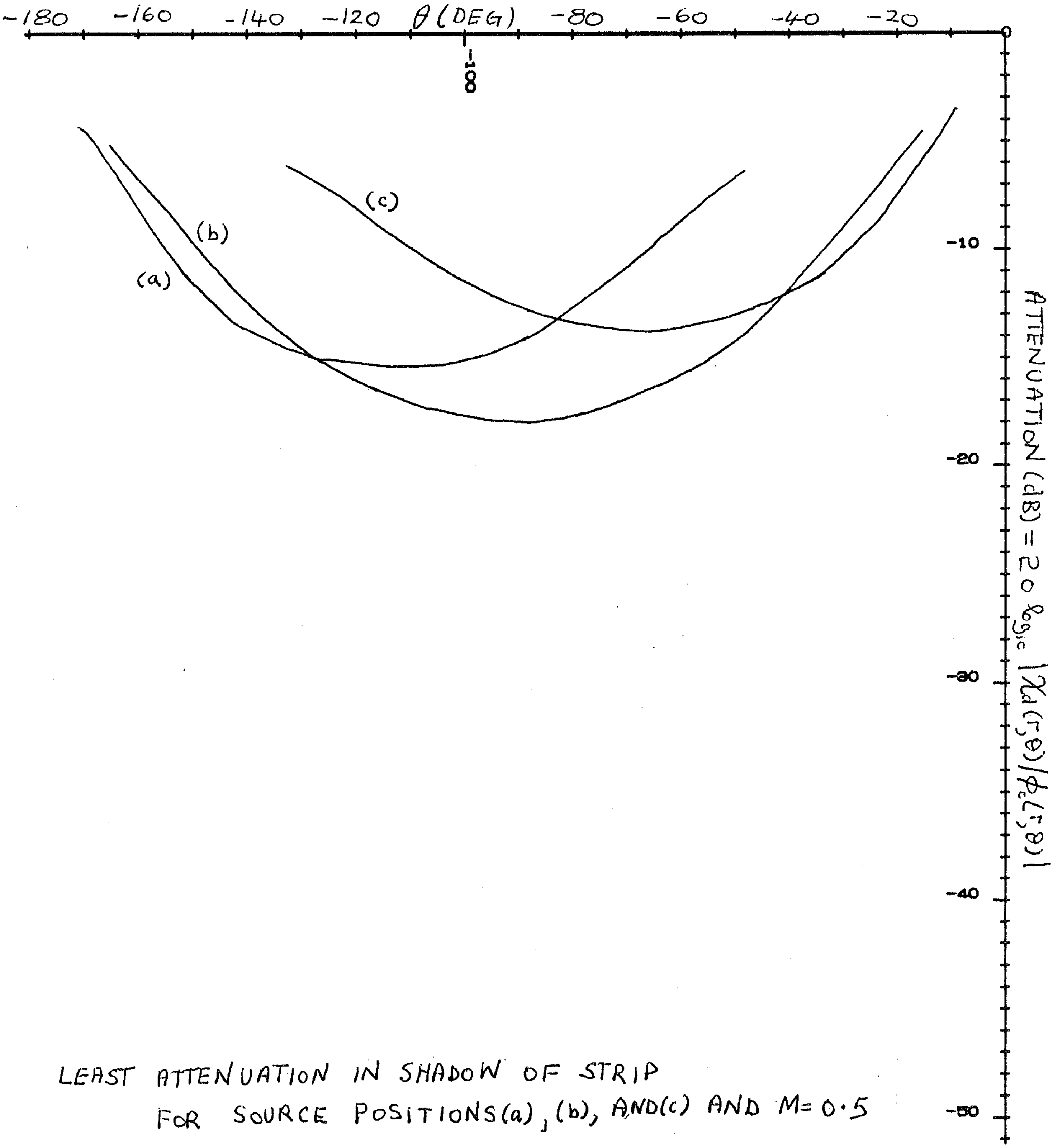
ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITION (c) AND M=0.5

Fig. 5



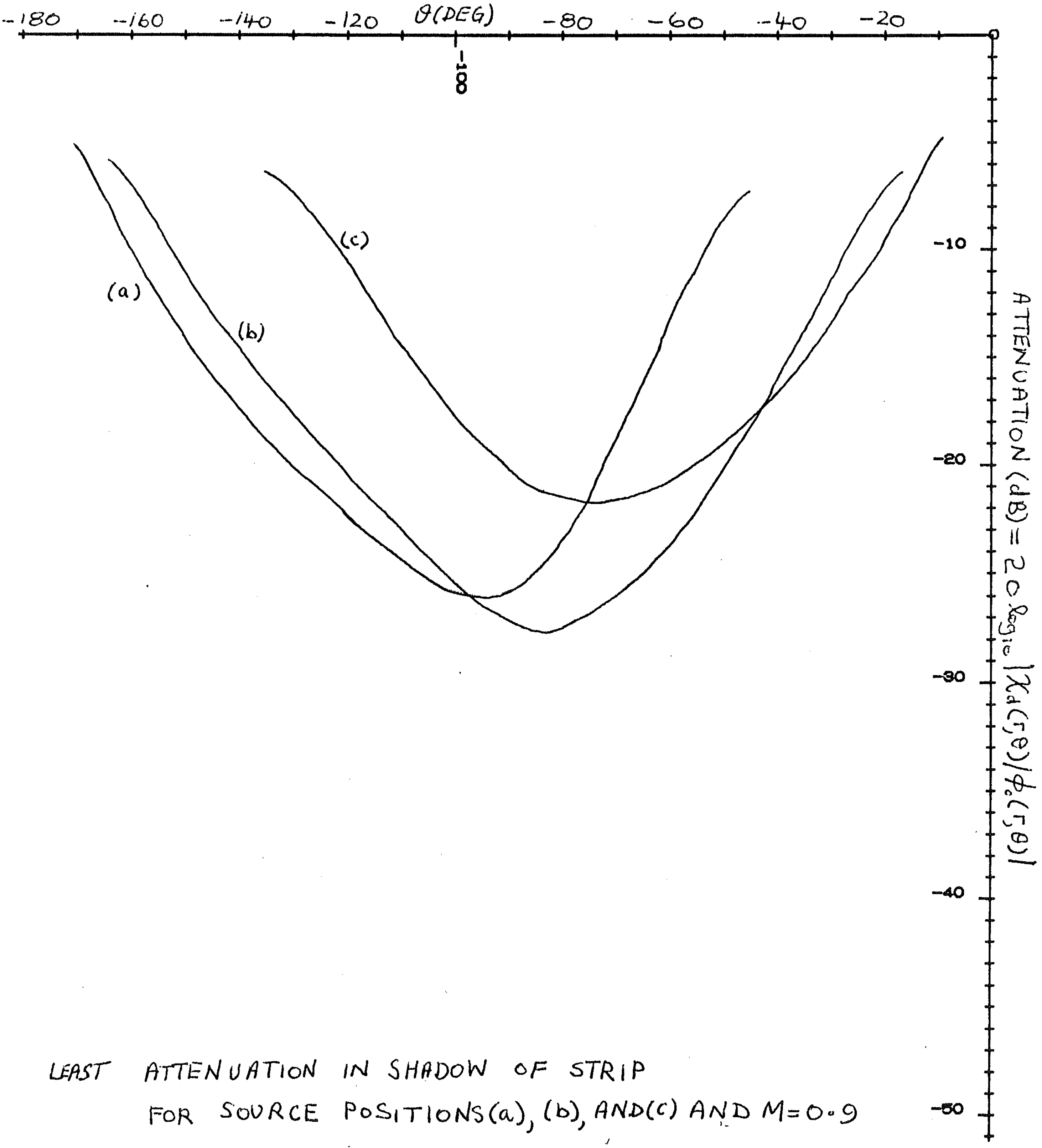
LEAST ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITIONS (a), (b) AND (C), AND M=0·1

Fig 6



LEAST ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITIONS (a), (b), AND (c) AND M = 0·5

Fig 7



LEAST ATTENUATION IN SHADOW OF STRIP
FOR SOURCE POSITIONS (a), (b), AND (c) AND M=0·9

Fig 8