State Estimation for Coupled Uncertain Stochastic Networks With Missing Measurements and Time-Varying Delays: The Discrete-Time Case

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***Abstract*—This paper is concerned with the problem of state estimation for a class of discrete-time coupled uncertain stochastic complex networks with missing measurements and time-varying delay. The parameter uncertainties are assumed to be norm-bounded and enter into both the network state and the network output. The stochastic Brownian motions affect not only the coupling term of the network but also the overall network dynamics. The nonlinear terms that satisfy the usual Lipschitz conditions exist in both the state and measurement equations. Through available output measurements described by a binary switching sequence that obeys a conditional probability distribution, we aim to design a state estimator to estimate the network states such that, for all admissible parameter uncertainties and time-varying delays, the dynamics of the estimation error is guaranteed to be globally exponentially stable in the mean square. By employing the Lyapunov functional method combined with the stochastic analysis approach, several delay-dependent criteria are established that ensure the existence of the desired estimator gains, and then the explicit expression of such estimator gains is characterized in terms of the solution to certain linear matrix inequalities (LMIs). Two numerical examples are exploited to illustrate the effectiveness of the proposed estimator design schemes.**

***Index Terms*—Complex networks, discrete-time systems, Lyapunov functional, missing measurements, parameter uncertainties, state estimator, stochastic disturbances.**

I. INTRODUCTION

THE complexity of networks in the social, biological, engineering, and physical sciences gives rise to many challenges for scientists and engineers, which have been overlooked by the traditional disciplines. Recently, dynamical behaviors of complex networks have attracted recurrent research interests and a huge amount of results have been reported in the literature. In particular, the synchronization problem has been paid renewed attention for complex networks in various fields. Among many others, we mention here that the synchronization problem has been thoroughly investigated for the large-scale networks of chaotic oscillators [13], [31], the coupled systems exhibiting spatio–temporal chaos and autowaves [27], [40], and the array of coupled neural networks with or without delays [3], [15], [21], [23]–[26], [32], [37], [25]. Note that the delay effects in synchronization has been extensively studied; see, e.g., [4], [8], [14], [22], [38], and the references cited therein. The main reasons can be summarized as follows: 1) delays often occurs as a natural consequence of finite information transmission and processing speeds among the units of networks [8], [9], [29]; 2) delayed couplings arise frequently in biological neural networks, gene regulatory networks, communication networks, and electrical power grids [14], [8], [14], [16]; and 3) time delays can induce complex dynamics such as periodic or quasi-periodic motions, Hopf bifurcation, and higher dimensional chaos.

It is worth pointing out that most of the existing research concerning synchronization problems has been carried out for *continuous-time and deterministic* complex networks with or without delays. In reality, however, the existence of parameter uncertainties and stochastic disturbances are ubiquitous in a discrete-time fashion. First, the connection weights of the nodes of complex networks depend on certain resistance and capacitance values that include uncertainties (modeling errors). Second, signal transmission within a digital network is conducted in a *discrete-time* rather than a continuous-time way, and therefore, it is not surprising that the discrete-time networks have already been applied in a wide range of areas, such as image processing, time-series analysis, quadratic optimization problems, and system identification. Third, the signal transfer could be perturbed randomly from the release of probabilistic causes such as neurotransmitters [34] and packet dropouts [36]. Subsequently, the synchronization problem for *stochastic networks* has begun to receive initial research attention; see, e.g., [5], [18], and [39] for some recent publications. Unfortunately, the *discrete and random* nature of the network topology, though vitally important for understanding the interaction topology [30], has not received sufficient research attention due primarily to the difficulty in mathematical analysis. Note that, in [19], the global exponential stability problem has been studied for a class of discrete-time uncertain stochastic neural networks with time delays. Furthermore, in [16], one of the first few attempts has been made to address the synchronization problem for stochastic discrete-time complex networks with time delays, where the synchronization criteria are expressed in the form of linear matrix inequalities (LMIs).

Due to the complexity of large-scale networks, it is often the case that only partial information about the states of the key

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nodes is available in the network outputs. For the purpose of better understanding the complex networks, it becomes necessary to estimate the states of the key nodes through available measurements, and then use the estimated key node states to carry out specified tasks such as dynamics analysis and synchronization control for the complex networks. However, in most literature regarding complex networks, it has been implicitly assumed that the network states are fully accessible, which is not always the case in reality. Therefore, from a practical viewpoint, the state estimation problem for complex networks has become vitally important. It might be worth mentioning that, the state estimation problem for neural networks (a special class of complex networks) was addressed in [6], [7], [28], and [33], and has then drawn particular research interests; see, e.g., [11], [12], and [20], where the networks are assumed to be deterministic and continuous time. As pointed out in [33], the neuron state estimation problem is precursor for many applications such as system modeling, signal processing, and control engineering. In order to make use of relatively large-scale neural networks, one often needs to estimate the neuron state through measured network output, and then utilize the estimated neuron state to achieve certain practical performances, e.g., approximation of stochastic nonlinear systems based on a radial basis function neural network using [6], estimation of online immeasurable states based on a recurrent neural network [7], adaptive state estimation of a general neural network [28], and mathematical analysis of estimation error dynamics of various neural networks [11], [12], [20], [33].

It should be pointed out that, for neural networks as well as chaotic systems, numerous research results regarding synchronization problems based on the observer techniques have been developed. For the observer-based synchronization, a slave system (driven system) is designed such that its dynamics synchronizes that of the master system (drive system). From the viewpoint of control theory, the slave system is an observer of the master system, and the state of the master system can be estimated by the slave system. In this sense, the state estimation problem can be viewed partially as a synchronization one, for which a rich body of research outputs has been available. Nevertheless, a literature search reveals that the state estimation problem particularly addressed for coupled uncertain stochastic complex networks does not seem to be fully investigated. In order to reflect a more realistic situation, we further consider the state estimation problem with probabilistic missing measurements. Such a problem stems from the fact that the network measurements may not be consecutive but contain missing observations. The missing measurements for complex networks are caused for a variety of reasons, for example, a failure in the measurement, intermittent sensor failures, network congestion, accidental loss of some collected data, or some of the data may be jammed or coming from a very noisy environment, etc.; see [35] and the references therein. To the best of our knowledge, the state estimation problem for *discrete-time coupled uncertain stochastic complex networks with missing measurements and delays* has received very little research attention despite its significance in practice. It is, therefore, the main purpose of this paper to shorten such a gap by making one of the first few attempts.

In this paper, we focus on the state estimation problem for uncertain stochastic discrete-time complex networks with missing measurements and time-varying delay, where the stochastic disturbances are assumed to be Brownian motions that affect not only the network coupling but also the overall networks, and the parameter uncertainties are imbedded in both the network state and the network output. The subsystems in the network interact with each other through the coupled terms that are disturbed by Brownian motions. Through available actual output measurements, we aim to design a state estimator to estimate the network states such that, for all admissible parameter uncertainties and time-varying delays, the dynamics of the estimation error is guaranteed to be globally exponentially stable in the mean square. By employing the Lyapunov functional method combined with the stochastic analysis approach, several delay-dependent criteria are established that ensure the existence of the desired estimator gains, and then the explicit expression of such estimator gains is characterized in terms of the solution to certain LMIs. Note that the LMIs can be effectively solved and checked by the algorithms such as the interior-point method [1].

The rest of this paper is organized as follows. In Section II, the coupled discrete-time uncertain stochastic complex network with missing measurements is presented, and some definitions and lemmas are provided for designing the state estimators. In Section III, by employing the Lyapunov functional method combined with the matrix inequality techniques, sufficient conditions are established in the form of LMIs and the explicit expression of the estimation gains is given. Under the obtained conditions, the estimation error dynamics is globally exponentially stable for all admissible uncertainties and stochastic disturbances. In Section IV, two examples are given to demonstrate the effectiveness of the results obtained. Finally, conclusions are drawn in Section V.

Notations: Throughout this paper, I is the identity matrix with appropriate dimensions and $X > 0$ means that matrix X is real, symmetric, and positive definite. The superscript "T" stands for matrix transposition, and "$*$" in a matrix is used to represent the term which is induced by symmetry. $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix and $\|\cdot\|$ refers to the Euclidean vector norm. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions and generated by Brownian motion $\{\omega(s) : 0 \leq s \leq t\}$. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions.

II. Problem Formulation

In this paper, we consider the following coupled discrete-time uncertain stochastic complex network model with time-varying delay:

$$\begin{aligned} x_i(k+1) = &A_i(k)x_i(k) + D_i(k)x_i(k-\tau(k)) \\ &+ B_i f_i(x_i(k)) + W_i x_i(k)\omega_1(k) \\ &+ \sum_{j=1}^{2} G_{ij}\Gamma x_j(k)(l_i + \omega_2(k)) \end{aligned} \tag{1}$$

where $i = 1, 2$, $k = 1, 2, \ldots$, $x_i(k) = (x_{i1}(k), x_{i2}(k), \ldots, x_{in}(k))^T \in \mathbb{R}^n$ is the state vector of the ith subsystem at time k, and n denotes the number of nodes in each subsystem.

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B_i and W_i are known real matrices and $A_i(k)$ and $D_i(k)$ are time-varying matrices of the form

$$A_i(k) = A_i + \Delta A_i(k) \quad D_i(k) = D_i + \Delta D_i(k).$$

Here, the constant matrices A_i and D_i are known and $\Delta A_i(k)$ and $\Delta D_i(k)$ are unknown matrices representing the time-varying parameter uncertainties and are assumed to satisfy the following conditions:

$$\begin{aligned}[\Delta A_1(k), \Delta D_1(k)] &= E_1 F(k)[M_1, M_3]\\ [\Delta A_2(k), \Delta D_2(k)] &= E_2 F(k)[M_2, M_4]\end{aligned} \tag{2}$$

where E_1, E_2, M_1, M_2, M_3, and M_4 are known real constant matrices and $F(k)$ is the unknown time-varying matrix-valued function subject to the following condition:

$$F^T(k)F(k) \leq I, \qquad k = 1, 2, \ldots \tag{3}$$

The nonlinear function $f_i(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n_i}$ is known and the delay integer $\tau(k)$ is time-varying, which satisfies

$$\tau_m \leq \tau(k) \leq \tau_M, \qquad k = 1, 2, \ldots$$

with τ_m and τ_M being known positive integers representing the lower and upper bounds of the delay, respectively. Matrix $G = (G_{ij})_{2\times 2}$ is the coupling configuration matrix denoting the topological structure of the complex networks and Γ describes the inner coupling of the network. $L = \mathrm{diag}\{l_1, l_2\}$ and $l_i > 0 (i = 1, 2)$ stands for the coupling strength.

Remark 1: The addressed discrete-time delayed complex networks (1) are quite general that include many different kinds of networks (e.g., neural networks and social networks) as special cases. For example, we consider the following n-neuron discrete-time neural network with stochastic disturbances and time-varying delay of the form:

$$u(k+1) = Au(k) + Du(k-\tau(k)) + B\hat{F}(u(k)) + Wu(k)\omega(k) \tag{4}$$

where $u(k) = (u_1(k), u_2(k), \ldots, u_n(k))^T$ is the neural state vector, $A = \mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ with $|a_i| < 1$ is the state feedback coefficient matrix, and the $n \times n$ matrices $B = [b_{ij}]_{n\times n}$ and $D = [d_{ij}]_{n\times n}$ are, respectively, the connection weight matrix and the delayed connection weight matrix. The positive integer $\tau(k)$ is the same as in (1). $\omega(k)$ is the state-dependent white noise. In (4), $\hat{F}(u(k))$ denotes the neuron activation function. It is obvious that the neural network model (4) is just a subnetwork of an array of stochastically coupled neural networks described by (1).

Remark 2: For presentation simplification, we consider the case where there are two coupled subsystems (i $= 1, 2$), but our main results can be readily applied to the complex network of n subsystems. Also, the lower and upper bounds of the time-varying delay $\tau(k)$ are adjustable [9]. One typical example is the networked control system where the delay caused either from sensor to controller or from controller to actuator is actually time varying and bounded.

Remark 3: For notational convenience, in (1), it is temporarily assumed that the subsystems share the same time-varying delay. The reason why we adopt such an assumption is just to avoid unnecessarily complicated notations and make the presentation as concise as possible. We claim that, as will be shown later in Remark 8, our main results can be easily generalized to more general complex networks with different time delays in their subsystems.

As discussed in the introduction, it is usually the case in practice that: 1) the states of the complex networks are not completely accessible [20], [33]; and 2) the information one can have is just the actual measurement output of the network, which may contain probabilistic missing data encountered in practice such as networked system analysis (the data missing phenomenon is also called packet dropout [35]). It is, therefore, necessary to make use of the available information and design a state estimator to approximate the states of the complex networks (1) at an exponential rate, regardless of the parameter uncertainties, stochastic disturbances, and probabilistic data missing.

In this paper, the network measurements are of the following form:

$$y(k) = \gamma(k)\left[\sum_{i=1}^{2} C_i(k)x_i(k) + g(x_1(k), x_2(k))(q + \omega_3(k))\right] \tag{5}$$

where $y(k) \in \mathbb{R}^m$ is the actual measurement output; and the stochastic variable $\gamma(k) \in \mathbb{R}$ is a Bernoulli distributed white noise sequence specified by the following distribution law:

$$\begin{aligned}\mathrm{Prob}\{\gamma(k) = 1\} &= \mathbb{E}\{\gamma(k)\} = \bar{\gamma}\\ \mathrm{Prob}\{\gamma(k) = 0\} &= 1 - \mathbb{E}\{\gamma(k)\} = 1 - \bar{\gamma}\end{aligned} \tag{6}$$

with $\bar{\gamma} > 0$ being a known constant. Obviously, for the stochastic variable $\gamma(k)$, one has the variance $\sigma^2 = \bar{\gamma}(1-\bar{\gamma})$. $q \geq 0$ represents the nonlinearity strength. $C_i(k) = C_i + \Delta C_i(k)$, and $C_i \in \mathbb{R}^{m\times n}$ and $g(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^m$ are a known matrix and a nonlinear function, respectively. $\Delta C_i(k)$ $(i = 1, 2)$ is the time-varying parameter uncertainty and satisfies the following admissible conditions:

$$\begin{aligned}\Delta C_1(k) &= E_3 F(k) M_1\\ \Delta C_2(k) &= E_4 F(k) M_2, \qquad k = 1, 2, \ldots\end{aligned} \tag{7}$$

where E_3 and E_4 are known real constant matrices and $F(k)$ is defined in (3).

Remark 4: $\Delta A_i(k), \Delta D_i(k)$, and $\Delta C_i(k)$ $(i = 1, 2)$ are said to be admissible if conditions (2), (3), and (7) hold. This kind of parameter uncertainties has been widely used for studying the robustness of an uncertain systems, and can represent many practical situations (see, e.g., [36]).

In system (1) and its output measurement system (5), $\omega_1(k), \omega_2(k)$, and $\omega_3(k)$ are independent scalar Brownian motions on $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$\begin{aligned}\mathbb{E}\{\omega_i(k)\} = 0 \quad \mathbb{E}\left\{\omega_i^2(k)\right\} = 1\\ \mathbb{E}\{\omega_i(s)\omega_i(t)\} = 0\ (s \neq t), \qquad i = 1, 2, 3.\end{aligned} \tag{8}$$

It is further assumed that $\omega_i(k)$ $(i = 1, 2, 3)$ and $\gamma(k)$ are mutually independent.

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Remark 5: In the network model (1), stochastic disturbances are introduced on both the coupling term and the overall network because the network coupling could occur in both a deterministic and a stochastic way. Note that *the stochastic coupling term* was first proposed in [5] for investigating the complete synchronization problem of an array of linearly stochastically coupled neural networks with time delays, but the deterministic coupling as well as the overall stochastic disturbances have not been taken into account in [5].

In order to estimate the network states of (1), we construct the following Luenberger-like state estimator:

$$\begin{aligned}\hat{x}_i(k+1) &= A_i\hat{x}_i(k) + B_if_i(\hat{x}_i(k)) + l_i\sum_{j=1}^{2}G_{ij}\Gamma\hat{x}_j(k)\\ &\quad + K_i\left[y(k) - \bar{\gamma}\left(\sum_{i=1}^{2}C_i\hat{x}_i(k) + qg(\hat{x}_1(k),\hat{x}_2(k))\right)\right]\end{aligned} \tag{9}$$

where $i = 1,2, \hat{x}_i(k)$ is the estimation of the network state $x_i(k)$, and $K_i \in \mathbb{R}^{n\times m}$ is the estimator gain matrix to be designed.

Throughout this paper, the following assumptions are made on the nonlinear functions in (1) and (5).

Assumption 1: $f_1(0) = 0$ and $f_2(0) = 0$ and there exist matrices S_1 and S_2 such that

$$\|f_i(u) - f_i(v)\| \le \|S_i(u-v)\|, \qquad i = 1,2 \qquad \forall u,v \in \mathbb{R}^n.$$

Assumption 2: $g(0,0) = 0$ and there exists a matrix T such that the following inequality holds:

$$\|g(u_1,u_2) - g(v_1,v_2)\| \le \left\|T\begin{pmatrix}u_1 - v_1\\ u_2 - v_2\end{pmatrix}\right\|$$

for all $u_1,u_2,v_1,v_2 \in \mathbb{R}^n$.

By using the Kronecker product, complex networks (1) can be rewritten in a compact form as

$$\begin{aligned}x(k+1) &= (A(k) + (LG)\otimes\Gamma)x(k) + D(k)x(k-\tau(k))\\ &\quad + Bf(x(k)) + Wx(k)\omega_1(k) + (G\otimes\Gamma)x(k)\omega_2(k)\end{aligned} \tag{10}$$

where $x(k) = (x_1^T(k), x_2^T(k))^T, f(x(k)) = (f_1^T(x_1(k)), f_2^T(x_2(k)))^T$, $B = \mathrm{diag}\{B_1,B_2\}, W = \mathrm{diag}\{W_1,W_2\}$, $A(k) = A + \Delta A(k), D(k) = D + \Delta D(k), A = \mathrm{diag}\{A_1,A_2\}$, $D = \mathrm{diag}\{D_1,D_2\}, \Delta A(k) = \mathrm{diag}\{\Delta A_1(k),\Delta A_2(k)\}$, and $\Delta D(k) = \mathrm{diag}\{\Delta D_1(k),\Delta D_2(k)\}$.

Similarly, by denoting

$$\begin{aligned}C(k) &= C + \Delta C(k), C := [C_1, C_2]\\ \Delta C(k) &:= [\Delta C_1(k), \Delta C_2(k)]\\ g(x(k)) &:= g(x_1(k),x_2(k)), \hat{x}(k) := \left[\hat{x}_1^T(k), \hat{x}_2^T(k)\right]^T\\ K &:= \left[K_1^T, K_2^T\right]^T\end{aligned}$$

the network measurements (5) and the state estimator (9) turn out to be

$$y(k) = \gamma(k)[C(k)x(k) + g(x(k))(q + \omega_3(k))] \tag{11}$$

$$\begin{aligned}\hat{x}(k+1) &= (A + (LG)\otimes\Gamma)\hat{x}(k) + Bf(\hat{x}(k))\\ &\quad + K[y(k) - \bar{\gamma}(C\hat{x}(k) + qg(\hat{x}(k)))].\end{aligned} \tag{12}$$

In terms of Assumptions 1 and 2, one can easily obtain that

$$\begin{aligned}\|f(u) - f(v)\| &\le \|S_f(u-v)\|\\ \|g(u) - g(v)\| &\le \|T(u-v)\| \qquad \forall u,v \in \mathrm{R}^{2n}\end{aligned} \tag{13}$$

where $S_f = \mathrm{diag}\{S_1,S_2\}$.

Our main aim in this paper is to choose a suitable estimator gain $K \in \mathbb{R}^{2n\times m}$ such that the state estimation $\hat{x}(k)$ approaches the network state vector $x(k)$ exponentially fast. Letting the error state be

$$\tilde{x}(k) = x(k) - \hat{x}(k) \tag{14}$$

it follows from (10)–(12) that

$$\begin{aligned}\tilde{x}(k+1) &= (A+(LG)\otimes\Gamma)\tilde{x}(k) + \Delta A(k)x(k) + D(k)x(k-\tau(k))\\ &\quad + B(f(x(k)) - f(\hat{x}(k))) + Wx(k)\omega_1(k) - \bar{\gamma}\\ &\quad \times K[C\tilde{x}(k) + \Delta C(k)x(k) + q(g(x(k)) - g(\hat{x}(k)))]\\ &\quad + (G\otimes\Gamma)x(k)\omega_2(k) - \bar{\gamma}Kg(x(k))\omega_3(k)\\ &\quad - (\gamma(k) - \bar{\gamma})K[C(k)x(k) + g(x(k))(q + \omega_3(k))].\end{aligned} \tag{15}$$

The initial condition associated with (15) is given as

$$\tilde{x}(s) = \varphi(s), \qquad s = -\tau_M, -\tau_M + 1, \ldots, 0 \tag{16}$$

where $\varphi(\cdot) \in L^2_{\mathcal{F}_0}([-\tau_M,0],\mathbb{R}^{2n})$ and $L^2_{\mathcal{F}_0}([-\tau_M,0],\mathbb{R}^{2n})$ is the family of all $\mathcal{F}_0$-measurable $C([-\tau_M,0],\mathbb{R}^{2n})$-valued random variables satisfying $\sup_{-\tau_M\le s\le 0}\mathbb{E}\{\|\varphi(s)\|^2\} < \infty$.

Definition 1: The system (9) is said to be a globally robustly exponential state estimator of the complex networks (1) if the estimation error system (15) is globally robustly exponentially stable in the mean square, i.e., there exist two constants $\vartheta > 0$ and $\mu \in (0,1)$ such that

$$\mathbb{E}\{\|\tilde{x}(k)\|^2\} \le \vartheta\mu^k \sup_{s\in\mathbb{N}[-\tau_M,0]}\mathbb{E}\{\|\varphi(s)\|^2\}$$

holds for all $k \ge \kappa, \varphi(\cdot) \in L^2_{\mathcal{F}_0}([-\tau_M,0],\mathbb{R}^{2n})$ and parameter uncertainties satisfying (2), (3), and (7), where κ is a sufficiently large positive integer and $\mathbb{N}[-\tau_M,0]$ is the set defined as $\mathbb{N}[-\tau_M,0] = \{-\tau_M,-\tau_M+1,\ldots,0\}$.

III. Main Results

In this section, an LMI method is employed to solve the state estimation problem formulated in the previous section. Before deriving the main results, two useful lemmas are given as follows.

Lemma 1: Let X, Y and F be real matrices of appropriate dimensions with F satisfying $F^TF \le I$. Then, for any scalar $\varepsilon > 0$

$$XFY + (XFY)^T \le \varepsilon^{-1}XX^T + \varepsilon Y^TY.$$

Lemma 2 [1]: Let $Q(x) = Q^T(x), R(x) = R^T(x)$, and $S(x)$ depend affinely on x. Then, the following linear matrix inequality:

$$\begin{bmatrix}Q(x) & S(x)\\ S^T(x) & R(x)\end{bmatrix} > 0$$

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holds if and only if one of the following conditions holds:

1) $R(x) > 0, Q(x) - S(x)R^{-1}(x)S^T(x) > 0$;
2) $Q(x) > 0, R(x) - S^T(x)Q^{-1}(x)S(x) > 0$.

We are now in a position to give the main results in the following theorem.

Theorem 1: Under Assumptions 1 and 2, system (9) becomes a globally robustly exponential state estimator of the complex networks (1) if there exist four matrices $P_1 > 0, P_2 > 0, Q_{11} > 0$, and $Q_{22} > 0$, two matrices Q_{12} and X, and three scalars $\varepsilon > 0, \beta_1 > 0$, and $\beta_2 > 0$ such that the LMIs in (17) and (18), shown at the bottom of the page, hold. Moreover, the state estimator gain can be determined by

$$K = P_2^{-1}X. \tag{19}$$

Proof: Taking the augmented state vector to be

$$e(k) = \begin{bmatrix} x(k) \\ \tilde{x}(k) \end{bmatrix} \tag{20}$$

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} > 0 \tag{17}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & 0 & \Psi_{15} & 0 & \Psi_{17} & 0 & 0 & 0 \\ * & -P_1 & 0 & 0 & 0 & 0 & 0 & P_1\tilde{E}_1 & P_1\tilde{E}_1 & 0 \\ * & * & -P_2 & 0 & 0 & 0 & 0 & P_2\tilde{E}_1 - \bar{\gamma}X\tilde{E}_3 & P_2\tilde{E}_1 & 0 \\ * & * & * & -P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -P_2 & 0 & 0 & 0 & 0 & \sigma X\tilde{E}_3 \\ * & * & * & * & * & -\bar{\gamma}^{-1}P_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\bar{\gamma}^{-1}P_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\beta_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\beta_1 I & 0 \\ * & * & * & * & * & * & * & * & * & -\beta_2 I \end{bmatrix} < 0 \tag{18}$$

where

$$\Psi_{11} = \begin{bmatrix} \Lambda_1 & (\tau_M - \tau_m + 1)Q_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \Lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_{11} + \beta_1\tilde{M}_3^T\tilde{M}_3 & -Q_{12} & 0 & 0 & 0 & 0 \\ * & * & * & -Q_{22} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix}$$

$$\Psi_{12} = \begin{bmatrix} \Lambda_3 \\ 0 \\ D^TP_1 \\ 0 \\ B^TP_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \Psi_{13} = \begin{bmatrix} 0 \\ \Lambda_4 \\ D^TP_2 \\ 0 \\ 0 \\ 0 \\ B^TP_2 \\ -\bar{\gamma}qX^T \end{bmatrix} \quad \Psi_{15} = \begin{bmatrix} \sigma C^TX^T \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma qX^T \\ 0 \\ 0 \end{bmatrix} \quad \Psi_{17} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ X^T \\ 0 \\ 0 \end{bmatrix}$$

and

$$\begin{aligned}
\Lambda_1 &= -P_1 + W^T(P_1+P_2)W + (\beta_1+\beta_2)\tilde{M}_1^T\tilde{M}_1 + \varepsilon\left(S_f^TS_f + T^TT\right) + (\tau_M - \tau_m + 1)Q_{11} + (G^T \otimes \Gamma^T)(P_1+P_2)(G \otimes \Gamma) \\
\Lambda_2 &= -P_2 + (\tau_M - \tau_m + 1)Q_{22} + \varepsilon\left(S_f^TS_f + T^TT\right) \\
\Lambda_3 &= A^TP_1 + ((G^TL^T) \otimes \Gamma^T)P_1 \\
\Lambda_4 &= A^TP_2 - \bar{\gamma}C^TX^T + ((G^TL^T) \otimes \Gamma^T)P_2 \\
\tilde{E}_1 &= \mathrm{diag}\{E_1, E_2\}, \tilde{E}_3 = [E_3, E_4] \\
\tilde{M}_1 &= \mathrm{diag}\{M_1, M_2\} \\
\tilde{M}_3 &= \mathrm{diag}\{M_3, M_4\} \\
\sigma &= \sqrt{\bar{\gamma}(1-\bar{\gamma})}.
\end{aligned}$$

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it follows from (10) and (15) that

$$\begin{aligned} e(k+1) &= A_e(k)e(k) + D_e(k)e(k-\tau(k)) + B_e\xi(x(k),\hat{x}(k)) \\ &\quad + W_e e(k)\omega_1(k) + G_e e(k)\omega_2(k) - \bar{\gamma}K_e\xi(x(k),\hat{x}(k))\omega_3(k) \\ &\quad + (\bar{\gamma}-\gamma(k))[\mathcal{A}_e(k)e(k) + \mathcal{B}_e\xi(x(k),\hat{x}(k)) \\ &\qquad + K_e\xi(x(k),\hat{x}(k))\omega_3(k)] \end{aligned} \tag{21}$$

where $\xi(x(k),\hat{x}(k)) = [f^T(x(k)), g^T(x(k)), (f(x(k)) - f(\hat{x}(k)))^T, (g(x(k)) - g(\hat{x}(k)))^T]^T$ and

$$A_e(k) = \begin{bmatrix} A(k) + (LG)\otimes\Gamma & 0 \\ \Delta A(k) - \bar{\gamma}K\Delta C(k) & A + (LG)\otimes\Gamma - \bar{\gamma}KC \end{bmatrix}$$

$$B_e = \begin{bmatrix} B & 0 & 0 & 0 \\ 0 & 0 & B & -\bar{\gamma}qK \end{bmatrix}$$

$$D_e(k) = \begin{bmatrix} D(k) & 0 \\ D(k) & 0 \end{bmatrix} \quad W_e = \begin{bmatrix} W & 0 \\ W & 0 \end{bmatrix}$$

$$G_e = \begin{bmatrix} G\otimes\Gamma & 0 \\ G\otimes\Gamma & 0 \end{bmatrix} \quad \mathcal{A}_e(k) = \begin{bmatrix} 0 & 0 \\ KC(k) & 0 \end{bmatrix}$$

$$\mathcal{B}_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & qK & 0 & 0 \end{bmatrix} \quad K_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & K & 0 & 0 \end{bmatrix}.$$

From Assumptions 1 and 2, it is easy to show that

$$\|\xi(x(k),\hat{x}(k))\| \leq \|Se(k)\| \tag{22}$$

where $S = \mathrm{diag}\{\hat{S},\hat{S}\}$ and $\hat{S} = [S_f^T, T^T]^T$.

Choose a Lyapunov functional candidate as follows:

$$\begin{aligned} V(k) &= V_1(k) + V_2(k) + V_3(k) \\ &= e^T(k)Pe(k) + \sum_{j=k-\tau(k)}^{k-1} e^T(j)Qe(j) \\ &\quad + \sum_{i=k-\tau_M+1}^{k-\tau_m}\sum_{j=i}^{k-1} e^T(j)Qe(j) \end{aligned} \tag{23}$$

where $P = \mathrm{diag}\{P_1, P_2\}$.

Calculating the difference of $V_1(k)$ along the trajectories of (21) and taking the mathematical expectation, one has from (5) that

$$\begin{aligned} &\mathbb{E}\{\Delta V_1(k)\} \\ &= \mathbb{E}\{V_1(k+1) - V_1(k)\} \\ &= \mathbb{E}\{\zeta_1^T(k)P\zeta_1(k) + \bar{\gamma}(1-\bar{\gamma})\zeta_2^T(k)P\zeta_2(k) - e^T(k)Pe(k)\} \\ &= \mathbb{E}\Big\{ e^T(k)[A_e^T(k)PA_e(k) - P + \bar{\gamma}(1-\bar{\gamma})\mathcal{A}_e^T(k)P\mathcal{A}_e(k) \\ &\qquad + W_e^TPW_e + G_e^TPG_e]e(k) + e^T(k-\tau(k)) \\ &\qquad \times D_e^T(k)PD_e(k)e(k-\tau(k)) + 2e^T(k)A_e^T(k) \\ &\qquad \times PD_e(k)e(k-\tau(k)) + \xi^T(x(k),\hat{x}(k)) \\ &\qquad \times [B_e^TPB_e + \bar{\gamma}(1-\bar{\gamma})\mathcal{B}_e^TP\mathcal{B}_e + \bar{\gamma}K_e^TPK_e] \\ &\qquad \times \xi(x(k),\hat{x}(k)) + 2e^T(k) \\ &\qquad \times [A_e^T(k)PB_e + \bar{\gamma}(1-\bar{\gamma})\mathcal{A}_e^T(k)P\mathcal{B}_e]\xi(x(k),\hat{x}(k)) \\ &\qquad + 2e^T(k-\tau(k))D_e^T(k)PB_e\xi(x(k),\hat{x}(k)) \Big\} \end{aligned} \tag{24}$$

with $\zeta_1(k) = A_e(k)e(k) + D_e(k)e(k-\tau(k)) + B_e\xi(x(k),\hat{x}(k)) + W_e e(k)\omega_1(k) + G_e e(k)\omega_2(k) - \bar{\gamma}K_e\xi(x(k),\hat{x}(k))\omega_3(k)$, $\zeta_2(k) = \mathcal{A}_e(k)e(k) + \mathcal{B}_e\xi(x(k),\hat{x}(k)) + K_e\xi(x(k),\hat{x}(k))\omega_3(k)$ and $\bar{\gamma}$ defined in (6).

Similarly, we have

$$\begin{aligned} &\mathbb{E}\{\Delta V_2(k)\} \\ &= \mathbb{E}\left\{ \sum_{j=k+1-\tau(k+1)}^{k} e^T(j)Qe(j) - \sum_{j=k-\tau(k)}^{k-1} e^T(j)Qe(j) \right\} \\ &= \mathbb{E}\Bigg\{ e^T(k)Qe(k) - e^T(k-\tau(k))Qe(k-\tau(k)) \\ &\qquad + \sum_{j=k+1-\tau(k+1)}^{k-\tau_m} e^T(j)Qe(j) \\ &\qquad + \sum_{j=k-\tau_m+1}^{k-1} e^T(j)Qe(j) - \sum_{j=k-\tau(k)+1}^{k-1} e^T(j)Qe(j) \Bigg\} \\ &\leq \mathbb{E}\Bigg\{ e^T(k)Qe(k) - e^T(k-\tau(k))Qe(k-\tau(k)) \\ &\qquad + \sum_{j=k+1-\tau_M}^{k-\tau_m} e^T(j)Qe(j) \Bigg\} \end{aligned} \tag{25}$$

and

$$\begin{aligned} &\mathbb{E}\{\Delta V_3(k)\} \\ &= \mathbb{E}\left\{ \sum_{i=1-\tau_M}^{-\tau_m} \left[\sum_{j=k+1+i}^{k} e^T(j)Qe(j) - \sum_{j=k+i}^{k-1} e^T(j)Qe(j) \right] \right\} \\ &= \mathbb{E}\left\{ (\tau_M - \tau_m)e^T(k)Qe(k) - \sum_{j=k+1-\tau_M}^{k-\tau_m} e^T(j)Qe(j) \right\}. \end{aligned} \tag{26}$$

From (22), it follows that

$$\varepsilon\xi^T(x(k),\hat{x}(k))\xi(x(k),\hat{x}(k)) \leq \varepsilon e^T(k)S^TSe(k). \tag{27}$$

Using (23)–(27), we obtain

$$\mathbb{E}\{\Delta V(k)\} \leq \mathbb{E}\left\{\theta^T(k)\Phi_1(k)\theta(k)\right\} \tag{28}$$

where $\theta(k) = (e^T(k), e^T(k-\tau(k)), \xi^T(x(k),\hat{x}(k)))^T$ and $\Phi_1(k)$ is defined at the top of the next page, with $\Pi_1(k) = A_e^T(k)PA_e(k) - P + \bar{\gamma}(1-\bar{\gamma})\mathcal{A}_e^T(k)P\mathcal{A}_e(k) + W_e^TPW_e + G_e^TPG_e + (\tau_M - \tau_m + 1)Q + \varepsilon S^TS$. Also, it follows from Lemma 2 that $\Phi_1(k) < 0$ is equivalent to

$$\Phi_2(k) = \begin{bmatrix} \Pi_2 & 0 & 0 & A_e^T(k)P & \sigma\mathcal{A}_e^T(k)P & 0 \\ * & -Q & 0 & D_e^T(k)P & 0 & 0 \\ * & * & -\varepsilon I & B_e^TP & \sigma\mathcal{B}_e^TP & K_e^TP \\ * & * & * & -P & 0 & 0 \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -\bar{\gamma}^{-1}P \end{bmatrix} < 0 \tag{29}$$

with $\Pi_2 = -P + W_e^TPW_e + G_e^TPG_e + (\tau_M - \tau_m + 1)Q + \varepsilon S^TS$.

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$$\Phi_1(k) = \begin{bmatrix} \Pi_1(k) & A_e^T(k)PD_e(k) & A_e^T(k)PB_e + \bar{\gamma}(1-\bar{\gamma})\mathcal{A}_e^T(k)P\mathcal{B}_e \\ * & D_e^T(k)PD_e(k) - Q & D_e^T(k)PB_e \\ * & * & B_e^TPB_e - \varepsilon I + \bar{\gamma}(1-\bar{\gamma})\mathcal{B}_e^TP\mathcal{B}_e + \bar{\gamma}K_e^TPK_e \end{bmatrix}$$

Based on the derivation we have conducted so far, it follows that the uncertain nonlinear system (21) is globally robustly asymptotically stable in the mean square if $\Phi_2(k) = \Phi_3 + \Phi_3' < 0$, in which

$$\Phi_3 = \begin{bmatrix} \Pi_2 & 0 & 0 & A_e^TP & \sigma\bar{A}_e^TP & 0 \\ * & -Q & 0 & D_e^TP & 0 & 0 \\ * & * & -\varepsilon I & B_e^TP & \sigma\mathcal{B}_e^TP & K_e^TP \\ * & * & * & -P & 0 & 0 \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -\bar{\gamma}^{-1}P \end{bmatrix}$$

$$\begin{aligned} \Phi_3' &= \begin{bmatrix} 0 & 0 & 0 & \Delta A_e^T(k)P & \sigma\Delta\bar{A}_e^T(k)P & 0 \\ * & 0 & 0 & \Delta D_e^T(k)P & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \\ &= \hat{P}_1^T\hat{\aleph}_1 + \hat{\aleph}_1^T\hat{P}_1 + \hat{P}_2^T\hat{\aleph}_2 + \hat{\aleph}_2^T\hat{P}_2 \end{aligned} \tag{30}$$

with $\hat{P}_1 = [0,0,0,P,0,0], \hat{P}_2 = [0,0,0,0,\sigma P,0], \hat{\aleph}_1 = [\Delta A_e(k), \Delta D_e(k), 0,0,0,0], \hat{\aleph}_2 = [\Delta\bar{A}_e(k),0,0,0,0,0]$ and

$$\begin{aligned} A_e &= \begin{bmatrix} A + (LG)\otimes\Gamma & 0 \\ 0 & A + (LG)\otimes\Gamma - \bar{\gamma}KC \end{bmatrix} \\ \Delta A_e(k) &= \begin{bmatrix} \Delta A(k) & 0 \\ \Delta A(k) - \bar{\gamma}K\Delta C(k) & 0 \end{bmatrix} \\ D_e &= \begin{bmatrix} D & 0 \\ D & 0 \end{bmatrix} \quad \Delta D_e(k) = \begin{bmatrix} \Delta D(k) & 0 \\ \Delta D(k) & 0 \end{bmatrix} \\ \bar{A}_e &= \begin{bmatrix} 0 & 0 \\ KC & 0 \end{bmatrix} \quad \Delta\bar{A}_e(k) = \begin{bmatrix} 0 & 0 \\ K\Delta C(k) & 0 \end{bmatrix}. \end{aligned}$$

It is noticed from (2) and (7) that

$$\Delta A(k) = \begin{bmatrix} E_1F(k)M_1 & 0 \\ 0 & E_2F(k)M_2 \end{bmatrix} = \tilde{E}_1\tilde{F}(k)\tilde{M}_1 \tag{31}$$

$$\Delta D(k) = \begin{bmatrix} E_1F(k)M_3 & 0 \\ 0 & E_2F(k)M_4 \end{bmatrix} = \tilde{E}_1\tilde{F}(k)\tilde{M}_3 \tag{32}$$

$$\Delta C(k) = [E_3F(k)M_1, E_4F(k)M_2] = \tilde{E}_3\tilde{F}(k)\tilde{M}_1 \tag{33}$$

where $\tilde{F}(k) = \mathrm{diag}\{F(k), F(k)\}$.

From the definitions of $\Delta A_e(k), \Delta\bar{A}_e(k)$ and $\Delta D_e(k)$, one has

$$\Delta A_e(k) = \begin{bmatrix} \tilde{E}_1\tilde{F}(k)\tilde{M}_1 & 0 \\ \tilde{E}_1\tilde{F}(k)\tilde{M}_1 - \bar{\gamma}K\tilde{E}_3\tilde{F}(k)\tilde{M}_1 & 0 \end{bmatrix} = \hat{E}_1\tilde{F}(k)\hat{M}_1 \tag{34}$$

$$\Delta\bar{A}_e(k) = \begin{bmatrix} 0 & 0 \\ K\tilde{E}_3\tilde{F}(k)\tilde{M}_1 & 0 \end{bmatrix} = \hat{E}_3\tilde{F}(k)\hat{M}_1$$

$$\Delta D_e(k) = \begin{bmatrix} \tilde{E}_1\tilde{F}(k)\tilde{M}_3 & 0 \\ \tilde{E}_1\tilde{F}(k)\tilde{M}_3 & 0 \end{bmatrix} = \hat{E}_2\tilde{F}(k)\hat{M}_3 \tag{35}$$

where

$$\begin{aligned} \hat{E}_1 &= \begin{bmatrix} \tilde{E}_1 \\ \tilde{E}_1 - \bar{\gamma}K\tilde{E}_3 \end{bmatrix} \quad \hat{E}_2 = \begin{bmatrix} \tilde{E}_1 \\ \tilde{E}_1 \end{bmatrix} \\ \hat{E}_3 &= \begin{bmatrix} 0 \\ K\tilde{E}_3 \end{bmatrix} \\ \hat{M}_1 &= [\tilde{M}_1, 0] \quad \hat{M}_3 = [\tilde{M}_3, 0]. \end{aligned}$$

It can be shown from (34) and (35) that

$$\begin{aligned} \hat{\aleph}_1 &= [\hat{E}_1\tilde{F}(k)\hat{M}_1, \hat{E}_2\tilde{F}(k)\hat{M}_3, 0,0,0,0] = \bar{E}_1\bar{F}(k)\bar{M}_3 \\ \hat{\aleph}_2 &= [\hat{E}_3\tilde{F}(k)\hat{M}_1, 0,0,0,0,0] = \bar{E}_3\bar{F}(k)\bar{M}_1 \end{aligned} \tag{36}$$

where $\bar{E}_1 = [\hat{E}_1, \hat{E}_2, 0,0,0,0], \bar{E}_3 = [\hat{E}_3, 0,0,0,0,0]$ and $\bar{F}(k) = \mathrm{diag}\{\tilde{F}(k), \tilde{F}(k), \tilde{F}(k), \tilde{F}(k), \tilde{F}(k), \tilde{F}(k)\}, \bar{M}_1 = \mathrm{diag}\{\hat{M}_1, 0,0,0,0,0\}, \bar{M}_3 = \mathrm{diag}\{\hat{M}_1, \hat{M}_3, 0,0,0,0\}$.

Moreover, we have from Lemma 1 and condition (3) that

$$\begin{aligned} \Phi_3' &= \hat{P}_1^T\bar{E}_1\bar{F}(k)\bar{M}_3 + \bar{M}_3^T\bar{F}^T(k)\bar{E}_1^T\hat{P}_1 \\ &\quad + \hat{P}_2^T\bar{E}_3\bar{F}(k)\bar{M}_1 + \bar{M}_1^T\bar{F}^T(k)\bar{E}_3^T\hat{P}_2 \\ &\le \beta_1^{-1}\hat{P}_1^T\bar{E}_1\bar{E}_1^T\hat{P}_1 + \beta_1\bar{M}_3^T\bar{M}_3 \\ &\quad + \beta_2^{-1}\hat{P}_2^T\bar{E}_3\bar{E}_3^T\hat{P}_2 + \beta_2\bar{M}_1^T\bar{M}_1 \end{aligned} \tag{37}$$

with

$$\hat{P}_1^T\bar{E}_1\bar{E}_1^T\hat{P}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & P\hat{E}_1\hat{E}_1^TP + P\hat{E}_2\hat{E}_2^TP & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}$$

$$\bar{M}_3^T\bar{M}_3 = \begin{bmatrix} \hat{M}_1^T\hat{M}_1 & 0 & 0 & 0 & 0 & 0 \\ * & \hat{M}_3^T\hat{M}_3 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}$$

$$\hat{P}_2^T\bar{E}_3\bar{E}_3^T\hat{P}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & \sigma^2P\hat{E}_3\hat{E}_3^TP & 0 \\ * & * & * & * & * & 0 \end{bmatrix}$$

$$\bar{M}_1^T\bar{M}_1 = \begin{bmatrix} \hat{M}_1^T\hat{M}_1 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}$$

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and

$$\begin{aligned} P\hat{E}_1 &= \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} \tilde{E}_1 \\ \tilde{E}_1 - \bar{\gamma} K \tilde{E}_3 \end{bmatrix} \\ &= \begin{bmatrix} P_1 \tilde{E}_1 \\ P_2 \tilde{E}_1 - \bar{\gamma} P_2 K \tilde{E}_3 \end{bmatrix} \\ \hat{M}_1^T \hat{M}_1 &= \begin{bmatrix} \tilde{M}_1^T \tilde{M}_1 & 0 \\ 0 & 0 \end{bmatrix} \\ P\hat{E}_2 &= \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} \tilde{E}_1 \\ \tilde{E}_1 \end{bmatrix} = \begin{bmatrix} P_1 \tilde{E}_1 \\ P_2 \tilde{E}_1 \end{bmatrix} \\ P\hat{E}_3 &= \begin{bmatrix} 0 \\ P_2 K \tilde{E}_3 \end{bmatrix} \\ \hat{M}_3^T \hat{M}_3 &= \begin{bmatrix} \tilde{M}_3^T \tilde{M}_3 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

By Lemma 2 and noticing $K = P_2^{-1} X$, we know that (18) is equivalent to the following inequality:

$$\Phi_3 + \beta_1^{-1} \hat{P}_1^T \bar{E}_1 \bar{E}_1^T \hat{P}_1 + \beta_1 \bar{M}_3^T \bar{M}_3 + \beta_2^{-1} \hat{P}_2^T \bar{E}_3 \bar{E}_3^T \hat{P}_2 + \beta_2 \bar{M}_1^T \bar{M}_1 < 0 \quad (38)$$

which ensures $\Phi_2(k) < 0$. Therefore, it follows from the Lyapunov stability theory that the uncertain nonlinear system (21) is globally robustly asymptotically stable in the mean square. Furthermore, along the similar line of the proof of Theorem 1 in [16], we can prove that there exist scalars $\vartheta > 0$ and $\mu \in (0,1)$ such that

$$\mathbb{E}\{\|\tilde{x}(k)\|^2\} \le \vartheta \mu^k \sup_{s \in \mathbb{N}[-\tau_M, 0]} \mathbb{E}\{\|\varphi(s)\|^2\} \quad (39)$$

for all $k \ge \kappa, \varphi(\cdot) \in L^2_{\mathcal{F}_0}([-\tau_M, 0], \mathbb{R}^{2n})$ and parameter uncertainties satisfying (2), (3), (7), where κ is a sufficiently large positive integer. From Definition 1, we know that (39) means that system (9) is a globally robustly exponential state estimator of the complex network (1), and the proof is then completed. ■

When there are neither stochastic disturbances nor parameter uncertainties in the complex networks (1) and the corresponding output measurement system (5), the equations reduce to

$$x_i(k+1) = A_i x_i(k) + D_i x_i(k - \tau(k)) + B_i f_i(x_i(k)) + l_i \sum_{j=1}^{2} G_{ij} \Gamma x_j(k). \quad (40)$$

Also, assume further that there is no probabilistic missing data probability, in other words, the network measurements are the ideal outputs, i.e., $\gamma(k) \equiv \bar{\gamma} = 1$

$$y(k) = Cx(k) + g(x(k)). \quad (41)$$

By taking the same state estimator (9) and the same Lyapunov functional (23), along the similar proof lines of Theorem 1, one has the following result immediately.

Corollary 1: Under Assumptions 1 and 2, the system (9) becomes a globally exponential state estimator of the complex networks (40) with measurements (41) if there exist four matrices $P_1 > 0, P_2 > 0, Q_{11} > 0$, and $Q_{22} > 0$, two matrices Q_{12} and X, and one scalar $\varepsilon > 0$ such that the LMIs shown at the bottom of the page hold, where $\Upsilon_{11} = -P_1 + \varepsilon S_f^T S_f + (\tau_M - \tau_m + 1) Q_{11}$; the other symbols are the same as in Theorem 1. Moreover, the state estimator gain can be determined by $K = P_2^{-1} X$.

Consider the complex networks (40) with measurements (41). If we further implement the following state estimator:

$$\begin{aligned} \hat{x}_i(k+1) &= A_i \hat{x}_i(k) + D_i \hat{x}_i(k - \tau(k)) + B_i f_i(\hat{x}_i(k)) \\ &\quad + l_i \sum_{j=1}^{2} G_{ij} \Gamma \hat{x}_j(k) \\ &\quad + K_i \left[y(k) - \left(\sum_{i=1}^{2} C_i \hat{x}_i(k) + g(\hat{x}_1(k), \hat{x}_2(k)) \right) \right], \\ &\qquad i = 1, 2 \end{aligned} \quad (42)$$

then the error state $\tilde{x}(k) = x(k) - \hat{x}(k)$ satisfies the following equation:

$$\tilde{x}(k+1) = (A + (LG) \otimes \Gamma - KC)\tilde{x}(k) + D\tilde{x}(k - \tau(k)) + \tilde{B}\tilde{\xi}(x(k), \hat{x}(k)) \quad (43)$$

where $\tilde{B} = [B, -K]$ and $\tilde{\xi}(x(k), \hat{x}(k)) = [(f(x(k)) - f(\hat{x}(k)))^T, (g(x(k)) - g(\hat{x}(k)))^T]^T$.

Obviously, for any scalar $\varepsilon > 0$

$$\varepsilon \tilde{\xi}^T(x(k), \hat{x}(k)) \tilde{\xi}(x(k), \hat{x}(k)) \le \varepsilon \tilde{x}^T(k) \hat{S}^T \hat{S} \tilde{x}(k) \quad (44)$$

in which matrix $\hat{S}$ is defined as in (22).

$$\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} > 0 \quad \begin{bmatrix} \Upsilon_{11} & (\tau_M - \tau_m + 1)Q_{12} & 0 & 0 & 0 & 0 & 0 & \Lambda_3 & 0 \\ * & \Lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda_4 \\ * & * & -Q_{11} & -Q_{12} & 0 & 0 & 0 & D^T P_1 & D^T P_2 \\ * & * & * & -Q_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 & B^T P_1 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 & 0 & B^T P_2 \\ * & * & * & * & * & * & -\varepsilon I & 0 & -X^T \\ * & * & * & * & * & * & * & -P_1 & 0 \\ * & * & * & * & * & * & * & * & -P_2 \end{bmatrix} < 0$$

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Choose a Lyapunov functional candidate as follows:

$$\bar{V}(k) = \tilde{x}^T(k)P\tilde{x}(k) + \sum_{j=k-\tau(k)}^{k-1} \tilde{x}^T(j)Q\tilde{x}(j) + \sum_{i=k-\tau_M+1}^{k-\tau_m} \sum_{j=i}^{k-1} \tilde{x}^T(j)Q\tilde{x}(j). \quad (45)$$

Then, along the similar line of the proof of Theorem 1, we can obtain the following criterion.

Proposition 1: Under Assumptions 1 and 2, the system (42) becomes a globally exponential state estimator of the complex networks (40) with measurements (41) if there exist two matrices $P > 0$ and $Q > 0$, one matrix X, and one scalar $\varepsilon > 0$ such that the LMI (46) shown at the bottom of the page holds. Moreover, the state estimator gain can be determined by $K = P^{-1}X$.

Remark 6: In Theorem 1, Corollary 1, and Proposition 1, delay-dependent criteria are established that ensure the existence of the desired estimator gains, and the explicit expressions of such estimator gains are characterized in terms of the solutions to certain LMIs. Note that LMIs can be effectively solved and checked by the algorithms such as the interior-point method [1].

Remark 7: The superiority by employing the state estimator (42) lies in that it can estimate the states of a network that even has nonconvergent dynamics, and this can be illustrated in Example 2 of Section IV.

Remark 8: In this paper, for presentation simplicity, we consider the case where all time-varying delays in the complex network are equal. However, our main criteria can be easily applied to the system with different delays. For example, if we consider the following coupled discrete-time uncertain stochastic complex network model with time-varying delays:

$$\begin{aligned} x_i(k+1) = {} & A_i(k)x_i(k) + D_i(k)x_i(k-\tau_i(k)) \\ & + B_i f_i(x_i(k)) + W_i x_i(k)\omega_1(k) \\ & + \sum_{j=1}^{2} G_{ij}\Gamma x_j(k)(l_i + \omega_2(k)) \end{aligned}$$

where $\tau_{im} \le \tau_i(k) \le \tau_{iM}$ $(i = 1, 2)$. By utilizing the symbols defined in (10), this system can be rewritten in a compact form as follows:

$$\begin{aligned} x(k+1) = {} & (A(k) + (LG) \otimes \Gamma)x(k) \\ & + \sum_{i=1}^{2} \bar{D}_i(k)x(k-\tau_i(k)) + Bf(x(k)) \\ & + Wx(k)\omega_1(k) + (G \otimes \Gamma)x(k)\omega_2(k) \end{aligned}$$

where $\bar{D}_1(k) = \mathrm{diag}\{D_1(k), 0\}, \bar{D}_2(k) = \mathrm{diag}\{0, D_2(k)\}$. Take the same state estimator (9) and the similar Lyapunov functional as follows:

$$\begin{aligned} V(k) = {} & e^T(k)Pe(k) + \sum_{j=k-\tau_1(k)}^{k-1} e^T(j)Qe(j) \\ & + \sum_{i=k-\tau_{1M}+1}^{k-\tau_{1m}} \sum_{j=i}^{k-1} e^T(j)Qe(j) \\ & + \sum_{j=k-\tau_2(k)}^{k-1} e^T(j)\bar{Q}e(j) \\ & + \sum_{i=k-\tau_{2M}+1}^{k-\tau_{2m}} \sum_{j=i}^{k-1} e^T(j)\bar{Q}e(j). \end{aligned}$$

Along the similar line of the proof of Theorem 1, one can have similar corresponding results, which are omitted here for conciseness.

Remark 9: State estimation problem for general dynamical systems has long been a research topic attracting constant attention in the areas of control engineering, signal process, and communication. It should also be mentioned that the synchronization problem or regulation problem are, to some extent, similar to the state estimation problem for complex or neural networks [2], [10], [17], [41]. As indicated in Remark 1, the kind of complex networks investigated in this paper is general enough to include neural networks as a special case. Compared to the traditional systems in open literature, the addressed complex networks exhibit the following particular characteristics: 1) probabilistic missing measurement is taken into account on the network output; 2) the stochastic Brownian motions affect not only the overall network dynamics but also the coupling term of the network as well as the network output; and 3) the subsystems are explicitly coupled in terms of the coupling configuration matrix, the inner coupling, and the coupling strength. For the possibly large-scale coupled networks, in this paper, we introduce the Kronecker product to represent the complex networks in a compact form. Properties of Kronecker products are thoroughly exploited so as to simplify the derivation. Furthermore, in view of the three stochastic disturbances acting on the system and the stochastic variable describing the data missing phenomenon, stochastic analysis is intensively carried out throughout the paper, combined with the latest Lyapunov functional approach that leads to less conservative delay-dependent LMI-based results. It is believed that this paper represents one of the first few attempts to deal with the state estimation problem for the complex networks of the kind, and the results are useful in the area of complex networks in terms of both the problem addressed and the techniques developed.

$$\begin{bmatrix} (\tau_M - \tau_m + 1)Q - P + \varepsilon\left(S_f^T S_f + T^T T\right) & 0 & 0 & 0 & A^T P + ((LG)^T \otimes \Gamma^T)P - C^T X^T \\ * & -Q & 0 & 0 & D^T P \\ * & * & -\varepsilon I & 0 & B^T P \\ * & * & * & -\varepsilon I & -X^T \\ * & * & * & * & -P \end{bmatrix} < 0. \quad (46)$$

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IV. Numerical Examples

In this section, two examples are shown to justify the criteria obtained in the previous section.

Example 1: Consider a coupled complex network model (40) with parameters as follows:

$$A_1 = \begin{bmatrix} 0.14 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.24 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -0.2 & 0.3 & 0.1 \\ 0.2 & -0.1 & 0.2 \\ -0.1 & 0.2 & -0.3 \end{bmatrix}$$
$$\Gamma = \begin{bmatrix} 0.3 & -0.1 & 0.2 \\ 0 & -0.3 & 0.2 \\ -0.1 & -0.1 & -0.2 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} 0.2 & 0.2 & 0 \\ -0.1 & 0.1 & -0.3 \\ 0.2 & 0.1 & 0.1 \end{bmatrix}$$
$$D_2 = \begin{bmatrix} 0.3 & 0.1 & -0.2 \\ 0.1 & 0.1 & 0 \\ 0.05 & 0.2 & -0.1 \end{bmatrix}$$
$$L = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}$$
$$G = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}.$$

The time-varying delay $\tau(k)$ is assumed to have upper bound $\tau_M = 4$ and lower bound $\tau_m = 2$. The parameters of the relevant network measurement (41) are given as follows:

$$C_1 = \begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0 & 0.1 & 0.1 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} 0.1 & -0.2 & 0.1 \\ -0.1 & 0.2 & 0.1 \end{bmatrix}.$$

The nonlinearities $f_i(\cdot)$ and $g(\cdot)$ in system (40) and (41) are assumed to satisfy Assumptions 1 and 2 with

$$S_1 = \begin{bmatrix} -0.2 & 0.3 & 0.1 \\ 0.1 & -0.2 & 0.3 \end{bmatrix}$$
$$S_2 = \begin{bmatrix} 0.3 & 0.1 & -0.2 \\ 0.1 & -0.2 & 0.3 \end{bmatrix}$$
$$T = \begin{bmatrix} 0.2 & -0.1 & -0.15 & -0.1 & 0.1 & 0 \\ -0.5 & 0.25 & -0.2 & -0.4 & 0 & 0.1 \end{bmatrix}.$$

By using the Matlab LMI Control Toolbox, we solve the LMIs in Corollary 1 to obtain the equation shown at the bottom of the page (Q_{11}, Q_{12}, and Q_{22} are omitted for simplicity). Therefore, it follows from Corollary 1 that system (9) is an estimator of the coupled complex networks (40) with time-varying delay.

Example 2: Consider a coupled complex network system (40) with parameters as follows: $B_1 = B_2 = 0, \tau(k) = 4 + \sin(k\pi/2), L = \mathrm{diag}\{0.3, 0.12\}$, and

$$A_1 = \begin{bmatrix} -0.0277 & 0.0183 \\ 0.3287 & 0.0161 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0.1064 & 0.1271 \\ 0.5350 & 0.5467 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} 0.1223 & 0.0199 \\ 0.0636 & 0.0259 \end{bmatrix}$$
$$D_2 = \begin{bmatrix} -0.0138 & 0.3827 \\ 0.5910 & 0.0546 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 83.8679 & 20.6693 & -1.2878 & 1.5176 & 0.8494 & -0.2830 \\ 20.6693 & 70.6999 & -4.4931 & 1.9884 & 1.2210 & -0.3240 \\ -1.2878 & -4.4931 & 87.5894 & -0.1408 & 0.0413 & 0.4470 \\ 1.5176 & 1.9884 & -0.1408 & 73.5056 & -2.6079 & -24.7856 \\ 0.8494 & 1.2210 & 0.0413 & -2.6079 & 79.6573 & -4.2875 \\ -0.2830 & -0.3240 & 0.4470 & -24.7856 & -4.2875 & 75.9066 \end{bmatrix}$$
$$P_2 = \begin{bmatrix} 54.9881 & -7.0796 & -12.4294 & 3.7072 & 0.9411 & -0.9145 \\ -7.0796 & 41.8705 & 6.7480 & -1.9970 & 0.3910 & 0.0778 \\ -12.4294 & 6.7480 & 53.4407 & 1.4995 & -0.1316 & -0.5113 \\ 3.7072 & -1.9970 & 1.4995 & 49.8235 & -7.6956 & -10.3280 \\ 0.9411 & 0.3910 & -0.1316 & -7.6956 & 64.3763 & 3.3246 \\ -0.9145 & 0.0778 & -0.5113 & -10.3280 & 3.3246 & 55.6505 \end{bmatrix}$$
$$X = \begin{bmatrix} -0.0984 & -0.6751 \\ 3.6245 & 1.9025 \\ 4.5022 & 2.3026 \\ -2.5038 & 4.6604 \\ 4.8903 & -1.9889 \\ -4.2371 & -0.8033 \end{bmatrix} \quad \varepsilon = 32.5391 \quad K = \begin{bmatrix} 0.0277 & -0.0048 \\ 0.0746 & 0.0438 \\ 0.0822 & 0.0338 \\ -0.0593 & 0.0924 \\ 0.0729 & -0.0202 \\ -0.0904 & 0.0041 \end{bmatrix}.$$

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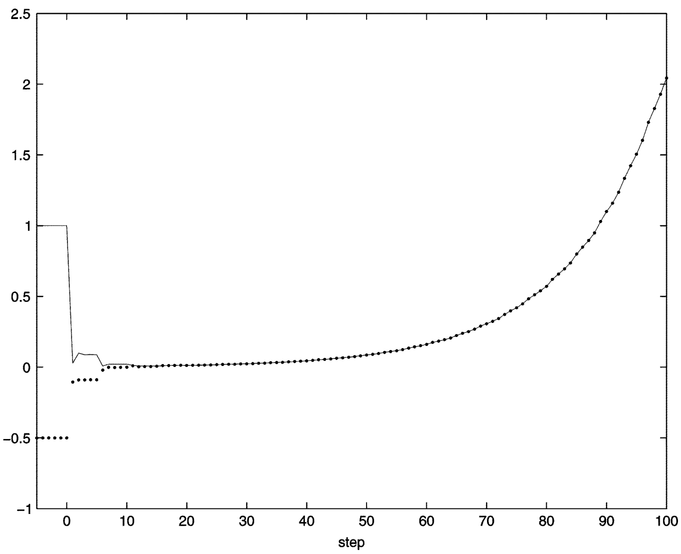

Fig. 1. State trajectories of x_{11} and $\hat{x}_{11}$.

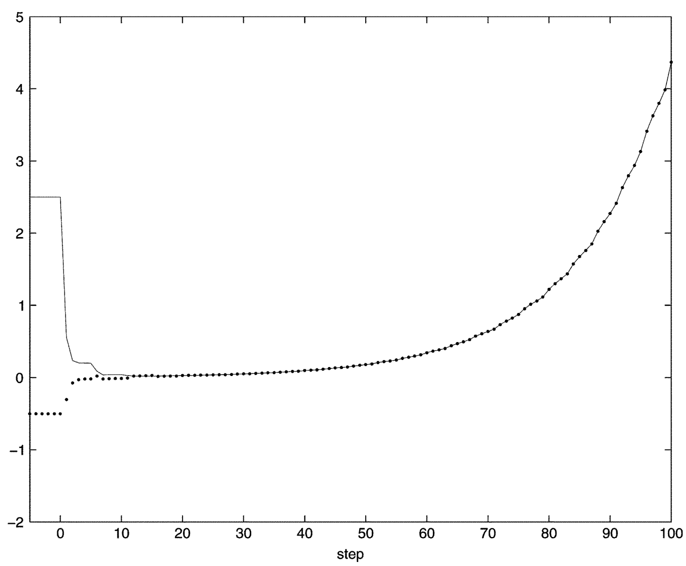

Fig. 2. State trajectories of x_{12} and $\hat{x}_{12}$.

$$G = \begin{bmatrix} 0.2 & 0.11 \\ -0.2 & 0.11 \end{bmatrix}$$
$$\Gamma = \begin{bmatrix} -0.13 & 0.24 \\ 0.23 & 0.25 \end{bmatrix}.$$

Here, we take the network measurement (41) with $g(\cdot) = 0$ and

$$C_1 = \begin{bmatrix} 0.1582 & 0.2825 \\ 0.4032 & 0.2092 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} -0.5257 & -0.2204 \\ -0.4068 & -0.3236 \end{bmatrix}.$$

By using the Matlab LMI Control Toolbox, the LMI in Proposition 1 has feasible solutions as follows:

$$P = \begin{bmatrix} 853.3893 & 181.5725 & -45.5398 & 33.3233 \\ 181.5725 & 544.8212 & -11.3038 & 6.1914 \\ -45.5398 & -11.3038 & 289.6969 & -204.1426 \\ 33.3233 & 6.1914 & -204.1426 & 162.7764 \end{bmatrix}$$

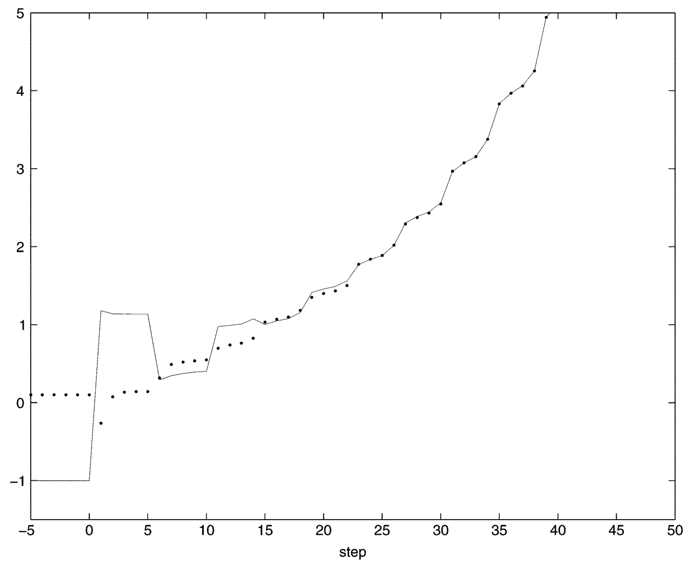

Fig. 3. State trajectories of x_{21} and $\hat{x}_{21}$.

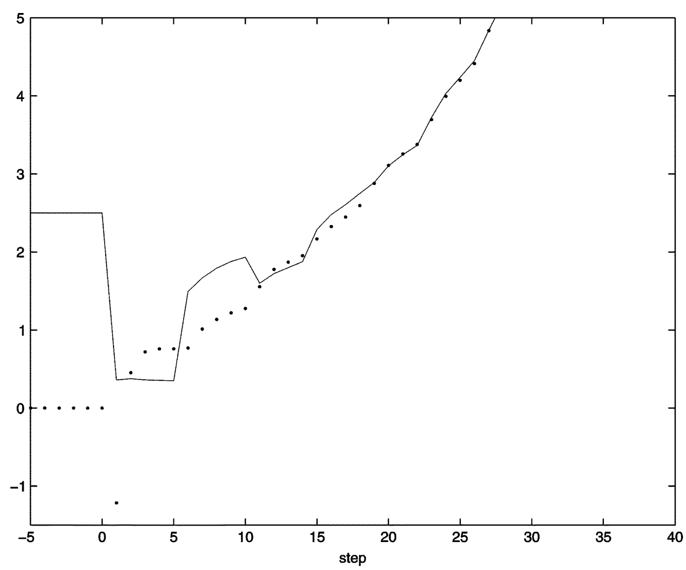

Fig. 4. State trajectories of x_{22} and $\hat{x}_{22}$.

$$Q = \begin{bmatrix} 79.2309 & -7.5621 & 3.8431 & -2.8271 \\ -7.5621 & 76.2042 & 4.0694 & -3.3450 \\ 3.8431 & 4.0694 & 67.4556 & -47.3623 \\ -2.8271 & -3.3450 & -47.3623 & 37.9673 \end{bmatrix}$$
$$X = \begin{bmatrix} 12.8328 & -58.7544 \\ -153.6470 & 130.2772 \\ 25.4106 & 189.5013 \\ -18.1685 & -161.9833 \end{bmatrix}.$$

Moreover, the state estimation gain is obtained as

$$K = \begin{bmatrix} 0.0856 & -0.0912 \\ -0.3088 & 0.2782 \\ 0.0550 & -0.3861 \\ -0.0484 & -1.4713 \end{bmatrix}.$$

Therefore, it follows from Proposition 1 that system (42) is an estimator of the coupled complex (40) with time-varying delay, which is further verified by the simulation results given

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by Figs. 1–4, where the solid line represents the network state, the dotted line stands for the state estimate, and the initial conditions are taken as $x(k) \equiv [1.0, 2.5, -1.0, 2.5]^T, \hat{x}(k) \equiv [-0.5, -0.5, 0.1, 0]^T$ $(k = -5, -4, \ldots, 0)$.

V. Conclusion

In this paper, we have investigated the state estimation problem for uncertain stochastic discrete-time complex networks with missing measurements and time-varying delay. The stochastic disturbances are assumed to affect not only the network coupling but also the overall networks, and the parameter uncertainties are imbedded in both the network state and the network output. The subsystems in the network interact with each other through the coupled terms that are disturbed by Brownian motions. We have designed a state estimator to estimate the network states such that, for all admissible parameter uncertainties and time-varying delays, the dynamics of the estimation error is guaranteed to be globally exponentially stable in the mean square. We have also shown that our method applies to unstable system as well when there are no parameter uncertainties. By employing the Lyapunov functional method combined with the stochastic analysis approach, several delay-dependent criteria have been established that ensure the existence of the desired estimator gains, and then the explicit expression of such estimator gains have been characterized in terms of the solution to certain LMIs. Two simulation examples have been presented to illustrate the usefulness and effectiveness of the main results obtained.

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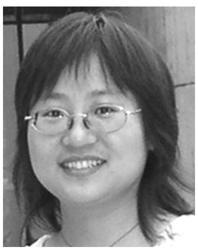

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