Correlations for non-Hermitian Dirac operators: chemical potential in three-dimensional QCD

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In the presence of a non-vanishing chemical potential the eigenvalues of the Dirac operator become complex. We use a Random Matrix Model (RMM) approach to calculate analytically all correlation functions at weak and strong non-Hermiticity for three-dimensional QCD with broken flavor symmetry and four-dimensional QCD in the bulk.

1. The Model

In [1] a RMM has been introduced to study the influence of a chemical potential on the support of Dirac operator eigenvalues. Here, we aim to derive microscopic correlations of complex eigenvalues at zero virtuality which are important for spontaneous symmetry breaking. For simplicity we consider a non-chiral RMM relevant for three-dimensional QCD with broken flavor symmetry [2] as well as in four dimension away from zero. In the spirit of [2] we replace the QCD Dirac operator with a constant complex matrix of size $N \times N$ as the Dirac eigenvalues become complex at finite density:

$$\mathcal{Z}_{QCD3}^{(2N_f)}(\{m_f\}) \;=\; \int dJdJ^\dagger \prod_{f=1}^{N_f} |\det[J - im_f]|^2 \exp\left[-\frac{N}{1-\tau^2}\mathrm{Tr}\left(JJ^\dagger - \tau\Re\mathrm{e}J^2\right)\right] \; .(1)$$

The measure follows from taking the same Gaussian distribution for both the Hermitian and anti-Hermitian parts of J, H and $i[(1-\tau)/(1+\tau)]^{1/2}A$, respectively. We present results in the limit of weak non-Hermiticity where $\lim_{N\to\infty} 2N(1-\tau) \equiv \alpha$ is kept fixed as well as in the limit of strong non-Hermiticity with $\tau \in [0,1)$. In the weak limit the parameter α mimics the influence of the chemical potential. The model (1) has been solved for $N_f = 0$ both in the strong [3] and weak limit [4], where in the latter the existence of orthogonal (Hermite) polynomials in the complex plane was exploited. Here, we extend both results to include $2N_f$ fermions flavors, massive or massless, using the same method as in [5]. For that reason we have to take the absolute value of the fermion determinant.

2. Results

We present the two-flavor case as an example and refer to [6] for the general case. In the weak limit the eigenvalues z of the matrix J and the quark masses m_f are rescaled as

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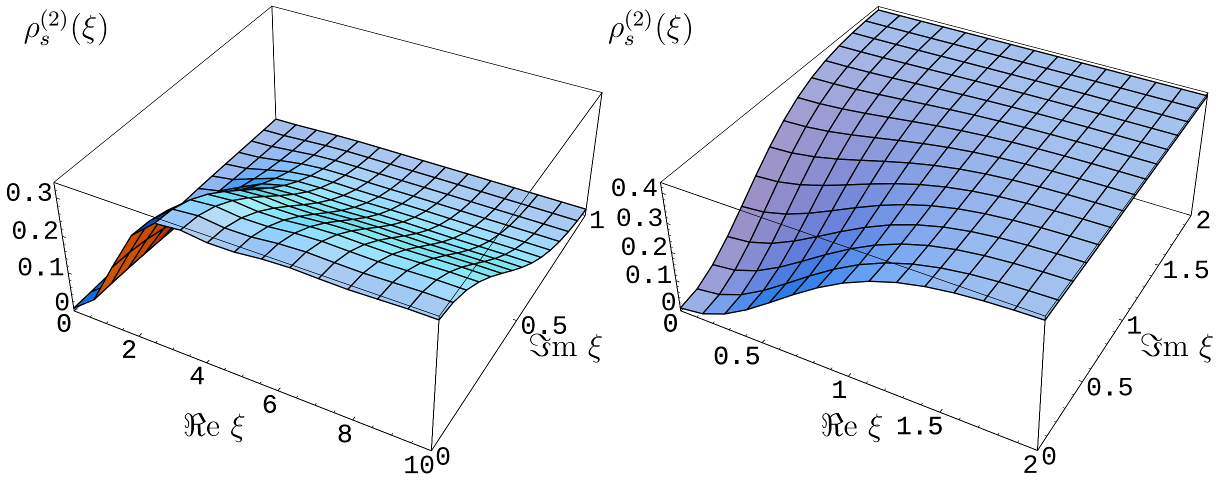

Figure 1. The 2-flavor massless density at weak (left) and strong non-Hermiticity (right).

$Nz = \xi$ and $Nm_f = \mu_f$, respectively. The microscopic density shown in Fig. 1 reads [6]

$$\rho_S^{(2)}(\xi) \;=\; \frac{1}{\pi\alpha}\exp\left[-\frac{2}{\alpha^2}\Im\mathrm{m}^2\xi\right]\left\{g_\alpha(2i\Im\mathrm{m}\ \xi) - \frac{g_\alpha(\xi+i\mu)g_\alpha(i\mu-\xi^*)}{g_\alpha(2i\mu)}\right\}\ , \tag{2}$$

with $g_\alpha(\xi) \equiv \int_{-\Sigma}^{\Sigma} du \exp\left[-\alpha^2u^2/2 + i\xi u\right]/\sqrt{2\pi}$ and Σ being the condensate. It displays the exponential suppression in the complex plane as for $N_f = 0$ [4] and the oscillations and level repulsion at the origin of the density of real eigenvalues [7]. Analytically this is seen as eq. (2) matches in the Hermitian limit $\alpha \to 0$ with the corresponding density [7].

In the strong non-Hermitian limit we rescale $\sqrt{N}z = \xi$ due to the change of mean level spacing. Formally it can also be obtained from eq. (2) by taking $\alpha \to \infty$. The density

$$\rho_S^{(2)}(\xi) \;=\; \frac{1}{\pi(1-\tau^2)}\left\{1 \;-\; \exp\left[-\frac{1}{1-\tau^2}|\xi - i\mu|^2\right]\right\} \tag{3}$$

displayed in Fig. 1 (right) shows an additional level repulsion at zero compared to the entirely flat density of Ginibre [3] at $N_f = 0$. The transition from real ($\alpha = 0$) to strong non-Hermitian ($\alpha = \infty$) correlations resembles the results [8] from lattice simulations of QCD in four dimensions with chemical potential.

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