Robust Passivity and Passification of Stochastic Fuzzy Time-Delay Systems

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Abstract

In this paper, the passivity and passification problems are investigated for a class of uncertain stochastic fuzzy systems with time-varying delays. The fuzzy system is based on the Takagi-Sugeno (T-S) model that is often used to represent the complex nonlinear systems in terms of fuzzy sets and fuzzy reasoning. To reflect more realistic dynamical behaviors of the system, both the parameter uncertainties and the stochastic disturbances are considered, where the parameter uncertainties enter into all the system matrices and the stochastic disturbances are given in the form of a Brownian motion. We first propose the definition of robust passivity in the sense of expectation. Then, by utilizing the Lyapunov functional method, the Itô differential rule and the matrix analysis techniques, we establish several sufficient criteria such that, for all admissible parameter uncertainties and stochastic disturbances, the closed-loop stochastic fuzzy time-delay system is robustly passive in the sense of expectation. The derived criteria, which are either delay-independent or delay-dependent, are expressed in terms of linear matrix inequalities (LMIs) that can be easily checked by using the standard numerical software. Illustrative examples are presented to demonstrate the effectiveness and usefulness of the proposed results.

Keywords

Fuzzy system; passivity; passification; stochastic disturbance; parameter uncertainty; linear matrix inequality (LMI).

I. Introduction

In the past decades, the fuzzy logic theory [40] has been shown to be an effective approach to dealing with the analysis and synthesis problems of nonlinear systems. Among various models available for fuzzy systems, the Takagi-Sugeno (T-S) fuzzy model in [28] was a notable one which was a linear system constructed to approximate a nonlinear plant. Roughly speaking, the T-S fuzzy model is a system described by fuzzy IF-THEN rules which can give local linear representation of the nonlinear system by decomposing the whole input space into several partial fuzzy spaces and representing each output space with a linear equation. Such a model is capable of approximating a wide class of nonlinear systems. Since the model in the consequent part is linear, conventional linear system theory can be conveniently applied for the system analysis and synthesis, and therefore the past decade has seen a rich body of literature utilizing T-S fuzzy models for analysis and control of complex dynamic systems [17, 23, 26, 29, 36, 39].

In reality, the time-delay, modeling errors and stochastic disturbances commonly exist in various engineering, biological, and economical systems due to the finite speed of information processing. They are arguably three of the main sources that may cause the instability of the system [31–33, 37, 38]. To cope with the modeling errors, the robust stability analysis and synthesis problems have first been addressed for T-S fuzzy models with parameter uncertainties, see [4, 14] for some earlier publications. Subsequently, time-delay was introduced in [5] for the fuzzy feed-back systems and sufficient stabilization conditions were established. Recently, there have been a large amount of results on how to check the stability of time-delay T-S systems by using various delay-dependent or delay-independent approaches, see e.g. [6, 11, 19]. For a comprehensive survey of the research

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for time-delay fuzzy systems, we refer the readers to [20]. Moreover, in the past few decades, a great deal of attention has been devoted to the stochastic systems governed by Itô stochastic differential equations since this kind of stochastic systems has many practical applications. Subsequently, stochastic fuzzy T-S systems with or without time-delays have recently become a research focus. For example, in [13, 30], both parameter uncertainties and stochastic disturbances were considered for the T-S model and the exponential mean square stability was discussed. In [12], the sliding-mode control (SMC) problem for nonlinear stochastic time-delay systems was considered by means of a fuzzy approach.

On another research front, the passivity and passification problems for a variety of practical systems have been attracting renewing attention for many years. The passivity theory was firstly proposed in the circuit analysis [1] and since then has found successful applications in diverse areas such as stability, signal processing, complexity, fuzzy control, chaos control and synchronization [3,7,34,35]. Since the well-known T-S fuzzy model has been proven to be a popular and convenient tool in functional approximations of nonlinear systems, it is not surprising that the passivity theory has been generalized to T-S fuzzy models with or without time-delays. For instance, the passivity problem of the T-S fuzzy system with constant delays was considered in [15]. In [25], the passivity was analyzed for neural networks as well as linear time-delay systems [22, 24]. Very recently, the passivity and passification problems were dealt with in [9] for the networked control systems. As discussed already, stochastic systems have been successfully applied in modeling practical systems in many areas such as biology, economics, and engineering, mainly due to the fact that stochastic disturbances exist universally in reality. Therefore, the robust stability, stabilization, control and filtering problems for stochastic systems have been intensively investigated by many researchers, and a great number of results on these topics have been reported in the literature. Unfortunately, to the best of the authors' knowledge, there have been few results on *the passivity and passification problems of stochastic T-S fuzzy time-delay systems with or without parameter uncertainties*. Therefore, the purpose of this paper is to shorten such a gap.

Motivated by the above discussions, in this paper, we aim to investigate the passivity and passification problems of a class of generalized time-delay T-S model with both parameter uncertainties and stochastic disturbances. The main contributions of this paper can be summarized as follows: 1) the fuzzy system under consideration is comprehensive that comprises stochasticity, uncertainties and time-delays, and can therefore reflect more realistic dynamical behaviors; 2) the definition of passivity is first extended to the stochastic setting (i.e., in the sense of expectation); and 3) a novel Lyapunov functional method combined with the matrix analysis techniques is developed to obtain sufficient conditions under which the closed-loop system is globally robustly passive in the sense of expectation. These sufficient conditions are expressed in terms of linear matrix inequalities (LMIs) that can be solved numerically. The rest of the paper is organized as follows. In Section II, the system studied in this paper is proposed and some preliminaries are given. In Section III, by utilizing the Lyapunov functional method, passivity conditions and state feedback passification of the stochastic uncertain T-S model are presented. In Section IV, illustrative examples are constructed to demonstrate the effectiveness and usefulness of the acquired results and, finally, conclusions are drawn in Section V.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m \times n$ real matrices, respectively. $\|\cdot\|$ refers to the Euclidean vector norm and I_n is the n-dimensional identity matrix. $P > 0$ means that P is a real, symmetric and positive definite matrix. Q^T represents the transpose of matrix Q and the asterisk "$*$" in a matrix is used to represent the term which is induced by symmetry. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ containing all $\mathcal{P}$-null sets and being right continuous. $L^p_{\mathcal{F}_0}$ denotes the family of all $\mathcal{F}_0$-measurable $C([-\tau_M, 0], \mathbb{R}^n)$-valued random variables $\varphi = \{\varphi(s)| - \tau_M \leq s \leq 0\}$ such that $\sup_{-\tau_M\leq s\leq 0} \mathbb{E}\{|\varphi(s)|^p\} < \infty$, where $\mathbb{E}\{\cdot\}$ is the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. Sometimes, when no confusion would arise, the dimensions of a function or a matrix will be omitted for convenience.

II. Model description and preliminaries

In this section, we consider a generalization of the traditional Takagi-Sugeno fuzzy system that includes both the parameter uncertainties and stochastic disturbances. The corresponding ith rule is formulated in the following form:

Plant rule i:

IF $\theta_1(t)$ is η_{i1} and $\ldots$ $\theta_p(t)$ is η_{ip}, THEN

$$\begin{cases} dx(t) &= \big[(A_i+\triangle A_i)x(t)+(B_i+\triangle B_i)x(t-\tau(t))+(G_i+\triangle G_i)J(t)\big]dt \\ & \quad +\sigma_i(t,x(t),x(t-\tau(t)))d\omega_i(t), \\ y(t) &= (C_i+\Delta C_i)x(t)+(D_i+\Delta D_i)x(t-\tau(t))+(W_i+\Delta W_i)J(t); \quad i=1,2,\ldots,r \end{cases} \tag{1}$$

where η_{ij} $(j=1,2,\ldots,p)$ are fuzzy sets; $x(t)\in\mathbb{R}^n$ is the state vector and $y(t)\in\mathbb{R}^m$ is the output vector; $J(t)\in\mathbb{R}^m$ is the square-integrable exogenous input and r is the number of IF-THEN rules; $\omega_i(t)\in\mathbb{R}^q$ $(i=1,2,\ldots,r)$ are uncorrelated zero mean Gaussian white noise process with covariances I_q; $\theta(t)=(\theta_1(t),\theta_2(t),\ldots,\theta_p(t))$ is the premise variable vector and it is assumed that the premise variables do not depend on the noise-input variables $\omega_i(t)$ explicitly; $0\le\tau_m\le\tau(t)\le\tau_M$ denotes the time-varying differentiable delay with $\dot{\tau}(t)\le d$. Note that the assumption of $\dot{\tau}(t)\le d$ implies that the increasing rate of the time-delay is limited which is indeed the case in most engineering practice. A_i, B_i, G_i, C_i, D_i and W_i are known constant matrices with appropriate dimensions and ΔA_i, ΔB_i, ΔG_i, ΔC_i, ΔD_i and ΔW_i are real matrices representing norm-bounded parameter uncertainties that satisfy:

$$[\Delta A_i \ \Delta B_i \ \Delta G_i]=H_{1i}F_{1i}[E_{1i} \ E_{2i} \ E_{3i}], \quad [\Delta C_i \ \Delta D_i \ \Delta W_i]=H_{2i}F_{2i}[E_{4i} \ E_{5i} \ E_{6i}] \tag{2}$$

where H_{1i}, H_{2i}, E_{ji} $(j=1,2,\ldots,6)$ are known real constant matrices with appropriate dimensions and F_{1i}, F_{2i} are unknown matrices satisfying

$$F_{1i}^T F_{1i}\le I, \qquad F_{2i}^T F_{2i}\le I. \tag{3}$$

$\sigma_i(\cdot,\cdot,\cdot):\mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^{n\times q}$ is the noise intensity function satisfying the Lipschitz condition, i.e., there exist constant matrices R_{1i} and R_{2i} of appropriate dimensions such that the following inequality

$$\text{trace}\big(\sigma_i^T(t,u,v)\sigma_i(t,u,v)\big)\le\|R_{1i}u\|^2+\|R_{2i}v\|^2 \tag{4}$$

holds for all $i=1,2,\ldots,r$ and $(t,u,v)\in\mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n$.

Remark 1: In the fuzzy system (1), both the parameter uncertainties and the stochastic disturbances are introduced. The reason is twofold: 1) the system parameters are usually obtained by way of statistical estimation which definitely results in some estimation errors; and 2) stochastic disturbances are nearly inevitable due to a variety of causes such as the thermal noise in the electronic devices or the noisy environment. Note that parameter uncertainties were firstly proposed in [16] for the passivity and passification problems of uncertain fuzzy systems without taking into account the stochastic influences and the delay effects.

Let

$$\nu_i(\theta(t))=\prod_{j=1}^{p}\eta_{ij}(\theta_j(t)) \quad \text{and} \quad \mu_i(\theta(t))=\frac{\nu_i(\theta(t))}{\sum\limits_{j=1}^{r}\nu_j(\theta(t))} \tag{5}$$

where $\eta_{ij}(\theta_j(t))$ is the grade of membership of $\theta_j(t)$ in η_{ij}. The basic property of $\nu_i(\theta(t))$ is that $\nu_i(\theta(t))\ge 0$ and $\sum\limits_{j=1}^{r}\nu_j(\theta(t))>0$ hold uniformly for all $t\ge 0$. Obviously, one has

$$\mu_i(\theta(t))\ge 0 \ (i=1,2,\ldots,r), \quad \sum_{i=1}^{r}\mu_i(\theta(t))=1; \qquad \forall t\in\mathbb{R}^+. \tag{6}$$

The initial condition associated with (1) is given by

$$x(s) = \varphi(s), \qquad -\tau_M \le s \le 0 \tag{7}$$

where $\varphi(\cdot) \in L^2_{\mathcal{F}_0}([-\tau_M, 0], \mathbb{R}^n)$ and the corresponding state trajectory is denoted as $x(t, \varphi)$.

We are now ready to introduce the notion of robust passivity for system (1) with stochastic disturbances. In the literature, there are several different definitions of passivity. In terms of the stochastic nature of the T-S fuzzy systems under consideration, we define the notion of *passivity in the sense of expectation* by extending the concept of passivity proposed in [21].

Definition 1: The fuzzy system (1) is called globally robustly passive in the sense of expectation if there exists a scalar $\beta \ge 0$ such that

$$2\mathbb{E}\left\{\int_0^t J^T(s)y(s)ds\right\} \ge -\beta\mathbb{E}\left\{\int_0^t J^T(s)J(s)ds\right\}, \ \ \forall t \ge 0$$

for all admissible uncertainties (2)-(3) and solution $x(t, \mathbf{0})$ of (1).

By denoting $\bar{A}_i = A_i + \Delta A_i$, $\bar{B}_i = B_i + \Delta B_i$, $\bar{C}_i = C_i + \Delta C_i$, $\bar{D}_i = D_i + \Delta D_i$, $\bar{G}_i = G_i + \Delta G_i$ and $\bar{W}_i = W_i + \Delta W_i$, the defuzzified system of model (1) can be represented as follows:

$$\begin{cases} dx(t) &= \sum_{i=1}^{r} \mu_i(\theta(t))[\bar{A}_i x(t) + \bar{B}_i x(t-\tau(t)) + \bar{G}_i J(t)]dt + \sum_{i=1}^{r} \mu_i(\theta(t))\sigma_i(t, x(t), x(t-\tau(t)))d\omega_i(t), \\ y(t) &= \sum_{i=1}^{r} \mu_i(\theta(t))[\bar{C}_i x(t) + \bar{D}_i x(t-\tau(t)) + \bar{W}_i J(t)]. \end{cases} \tag{8}$$

Before starting the main results, we need to introduce two more notations and some lemmas which will be used in the next section.

Let $C^{1,2}(\mathbb{R}\times\mathbb{R}^n, \mathbb{R}^+)$ denote the family of all nonnegative function $V(t,x)$ on $\mathbb{R}\times\mathbb{R}^n$ which are continuously twice differentiable in x and once differentiable in t. For each $V \in C^{1,2}(\mathbb{R}\times\mathbb{R}^n, \mathbb{R}^+)$, by Itô's differential formula [8, 18], the stochastic derivative of $V(t, x(t))$ along (8) can be obtained as:

$$dV(t, x(t)) = \mathcal{L}V(t, x(t))dt + V_x(t, x(t))\sum_{i=1}^{r} \mu_i(\theta(t))\sigma_i(t, x(t), x(t-\tau(t)))d\omega_i(t), \tag{9}$$

where $\mathcal{L}$ is the weak infinitesimal operator $\mathcal{L}$ of the stochastic process $\{x_t = x(t+s)|t \ge 0, -\tau_M \le s \le 0\}$, and the mathematical expectation of $\mathcal{L}V(t, x(t))$ is given by

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V(t, x(t))\} &= \mathbb{E}\Bigg\{V_t(t, x(t)) + V_x(t, x(t))\left[\sum_{i=1}^{r} \mu_i(\theta(t))(\bar{A}_i x(t) + \bar{B}_i x(t-\tau(t)) + \bar{G}_i J(t))\right] \\ &\quad + \frac{1}{2}\text{trace}\left[\sum_{i=1}^{r} \mu_i^2(\theta(t))\sigma_i^T(t, x(t), x(t-\tau(t)))V_{xx}(t, x(t))\sigma_i(t, x(t), x(t-\tau(t)))\right]\Bigg\} \end{aligned} \tag{10}$$

with

$$V_t(t, x(t)) = \frac{\partial V(t, x(t))}{\partial t}; \quad V_x(t, x(t)) = (\frac{\partial V(t, x(t))}{\partial x_1}, \ldots, \frac{\partial V(t, x(t))}{\partial x_n}); \quad V_{xx}(t, x(t)) = (\frac{\partial^2 V(t, x(t))}{\partial x_i \partial x_j})_{n\times n}.$$

Lemma 1: [2] Let $Q(x) = Q^T(x)$, $R(x) = R^T(x)$, and $S(x)$ depend affinely on x. Then the following linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

holds if and only if one of the following conditions holds:

(1) $R(x) > 0$, $Q(x) - S(x)R^{-1}(x)S^T(x) > 0$;
(2) $Q(x) > 0$, $R(x) - S^T(x)Q^{-1}(x)S(x) > 0$.

Lemma 2: [10] For any constant matrix $P \in \mathbb{R}^{n\times n}$, $P^T = P > 0$, scalar $r > 0$, and vector function $u : [0, r] \to \mathbb{R}^n$, one has

$$r\int_0^r u^T(s)Pu(s)ds \geq (\int_0^r u(s)ds)^T P(\int_0^r u(s)ds),$$

provided that the integrals are well defined.

Lemma 3: Let X, Y, Ξ be matrices with Ξ satisfying $\Xi^T\Xi \leq I$ and scalar $\varepsilon > 0$. Then the following inequality holds:

$$X\Xi Y + (X\Xi Y)^T \leq \varepsilon^{-1}XX^T + \varepsilon Y^TY.$$

III. Main Results

In this section, the passivity conditions are first derived for the T-S fuzzy system (1) with both parameter uncertainties and stochastic disturbances, and the sufficient passive criteria are then obtained for the model (1) with parameter uncertainties only but using a different Lyapunov functional and, finally, the passification problem of the state feedback closed-loop fuzzy model is considered.

A. Passivity analysis

Theorem 1: The delayed fuzzy system (1) is globally robustly passive in the sense of expectation if there exist two positive definite matrices P_1, P_2 and $2(r+1)$ scalars $\beta \geq 0$, $\lambda > 0$, $\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$ such that the following LMIs hold for all $i = 1, 2, \ldots, r$:

$$P_1 < \lambda I, \tag{11}$$

$$\Xi^{(i)} = \begin{bmatrix} \Xi_{11}^{(i)} & P_1B_i & -C_i^T + P_1G_i & P_1H_{1i} & 0 & \varepsilon_{1i}E_{1i}^T & \varepsilon_{2i}E_{4i}^T \\ * & -(1-d)P_2 + \lambda R_{2i}^TR_{2i} & -D_i^T & 0 & 0 & \varepsilon_{1i}E_{2i}^T & \varepsilon_{2i}E_{5i}^T \\ * & * & -(\beta I + W_i^T + W_i) & 0 & H_{2i} & \varepsilon_{1i}E_{3i}^T & \varepsilon_{2i}E_{6i}^T \\ * & * & * & -\varepsilon_{1i}I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{2i}I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_{1i}I & 0 \\ * & * & * & * & * & * & -\varepsilon_{2i}I \end{bmatrix} < 0, \tag{12}$$

where $\Xi_{11}^{(i)} = P_1A_i + A_i^TP_1 + P_2 + \lambda R_{1i}^TR_{1i}$.

Proof: By Lemma 1, we know that condition (12) is equivalent to

$$\Xi_1^{(i)} + \varepsilon_{1i}^{-1}\hat{H}_{1i}\hat{H}_{1i}^T + \varepsilon_{1i}\hat{E}_{1i}\hat{E}_{1i}^T + \varepsilon_{2i}^{-1}\hat{H}_{2i}\hat{H}_{2i}^T + \varepsilon_{2i}\hat{E}_{4i}\hat{E}_{4i}^T < 0, \tag{13}$$

where

$$\Xi_1^{(i)} = \begin{bmatrix} P_1A_i + A_i^TP_1 + P_2 + \lambda R_{1i}^TR_{1i} & P_1B_i & -C_i^T + P_1G_i \\ * & -(1-d)P_2 + \lambda R_{2i}^TR_{2i} & -D_i^T \\ * & * & -(\beta I + W_i^T + W_i) \end{bmatrix},$$

and

$$\hat{H}_{1i} = \begin{bmatrix} P_1H_{1i} \\ 0 \\ 0 \end{bmatrix}, \quad \hat{E}_{1i} = \begin{bmatrix} E_{1i}^T \\ E_{2i}^T \\ E_{3i}^T \end{bmatrix}, \quad \hat{H}_{2i} = \begin{bmatrix} 0 \\ 0 \\ H_{2i} \end{bmatrix}, \quad \hat{E}_{4i} = \begin{bmatrix} E_{4i}^T \\ E_{5i}^T \\ E_{6i}^T \end{bmatrix}.$$

Choose a Lyapunov functional candidate $V_1(t, x(t)) \in C^{1,2}(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^+)$ as

$$V_1(t, x(t)) = x^T(t)P_1x(t) + \int_{t-\tau(t)}^t x^T(s)P_2x(s)ds. \tag{14}$$

By Itô's differential rule, the mathematical expectation of the stochastic derivative of $V_1(t,x(t))$ along the trajectory of fuzzy system (8) can be obtained as

$$\begin{aligned}
\mathbb{E}\{dV_1(t,x(t))\} &= \mathbb{E}\Bigg\{\Bigg\{\sum_{i=1}^{r}\mu_i(\theta(t))\big[2x^T(t)P_1(\bar{A}_ix(t)+\bar{B}_ix(t-\tau(t))+\bar{G}_iJ(t)) \\
&\quad +x^T(t)P_2x(t)-(1-\dot{\tau}(t))x^T(t-\tau(t))P_2x(t-\tau(t))\big] \\
&\quad +\mathrm{trace}[\sum_{i=1}^{r}\mu_i^2(\theta(t))\sigma_i^T(t,x(t),x(t-\tau(t)))P_1\sigma_i(t,x(t),x(t-\tau(t)))]\Bigg\}\,dt\Bigg\} \\
&\leq \mathbb{E}\Bigg\{\sum_{i=1}^{r}\mu_i(\theta(t))\,\big[x^T(t)(P_1\bar{A}_i+\bar{A}_i^TP_1+P_2)x(t)+2x^T(t)P_1\bar{B}_ix(t-\tau(t)) \\
&\quad +2x^T(t)P_1\bar{G}_iJ(t)-(1-d)x^T(t-\tau(t))P_2x(t-\tau(t)) \\
&\quad +\mathrm{trace}(\sigma_i^T(t,x(t),x(t-\tau(t)))P_1\sigma_i(t,x(t),x(t-\tau(t))))\big]\,dt\Bigg\}; \qquad (15)
\end{aligned}$$

here, to achieve (15), conditions $\dot{\tau}(t)\leq d$ and $0\leq\mu_i(\theta(t))\leq 1$ $(i=1,2,\ldots,r)$ have been exploited and the relationship of $\mathbb{E}\left\{2\sum_{i=1}^{r}\mu_i(\theta(t))x^T(t)P_1\sigma_i(t,x(t),x(t-\tau(t)))d\omega_i(t)\right\}=0$ has been utilized.

On the other hand, conditions (4) and (11) ensure that

$$\begin{aligned}
&\mathbb{E}\left\{\mathrm{trace}\big(\sigma_i^T(t,x(t),x(t-\tau(t)))P_1\sigma_i(t,x(t),x(t-\tau(t)))\big)\right\} \\
\leq\ &\lambda\mathbb{E}\left\{\big(x^T(t)R_{1i}^TR_{1i}x(t)+x^T(t-\tau(t))R_{2i}^TR_{2i}x(t-\tau(t))\big)\right\}, \quad i=1,2,\ldots,r. \qquad (16)
\end{aligned}$$

Substituting (16) into (15) gives

$$\begin{aligned}
&\mathbb{E}\{dV_1(t,x(t))-2J^T(t)y(t)dt-\beta J^T(t)J(t)dt\} \\
\leq\ &\mathbb{E}\Bigg\{\sum_{i=1}^{r}\mu_i(\theta(t))\big[x^T(t)(P_1\bar{A}_i+\bar{A}_i^TP_1+P_2+\lambda R_{1i}^TR_{1i})x(t)+2x^T(t)P_1\bar{B}_ix(t-\tau(t)) \\
&+2x^T(t)(P_1\bar{G}_i-\bar{C}_i^T)J(t))+x^T(t-\tau(t))(\lambda R_{2i}^TR_{2i}-(1-d)P_2)x(t-\tau(t)) \\
&-2x^T(t-\tau(t))\bar{D}_i^TJ(t)-J^T(t)(\bar{W}_i^T+\bar{W}_i+\beta I)J(t)\big]\,dt\Bigg\} \\
=\ &\mathbb{E}\left\{\sum_{i=1}^{r}\mu_i(\theta(t))\zeta^T(t)\Xi_2^{(i)}\zeta(t)dt\right\}; \qquad (17)
\end{aligned}$$

where $\zeta^T(t)=(x^T(t)\ x^T(t-\tau(t))\ J^T(t))$ and

$$\Xi_2^{(i)}=\begin{bmatrix} P_1\bar{A}_i+\bar{A}_i^TP_1+P_2+\lambda R_{1i}^TR_{1i} & P_1\bar{B}_i & -\bar{C}_i^T+P_1\bar{G}_i \\ * & -(1-d)P_2+\lambda R_{2i}^TR_{2i} & -\bar{D}_i^T \\ * & * & -(\beta I+\bar{W}_i^T+\bar{W}_i)\end{bmatrix}.$$

Obviously, $\Xi_2^{(i)}=\Xi_1^{(i)}+\Delta\Xi_1^{(i)}$, in which

$$\begin{aligned}
\Delta\Xi_1^{(i)} &= \begin{bmatrix} P_1(\Delta A_i)+(\Delta A_i)^TP_1 & P_1\Delta B_i & -(\Delta C_i)^T+P_1\Delta G_i \\ * & 0 & -(\Delta D_i)^T \\ * & * & -(\Delta W_i)^T-\Delta W_i\end{bmatrix} \\
&= \hat{H}_{1i}F_{1i}\hat{E}_{1i}^T+\hat{E}_{1i}F_{1i}^T\hat{H}_{1i}^T-\hat{H}_{2i}F_{2i}\hat{E}_{4i}^T-\hat{E}_{4i}F_{2i}^T\hat{H}_{2i}^T.
\end{aligned}$$

It follows from Lemma 3 that

$$\Delta\Xi_1^{(i)} \le \varepsilon_{1i}^{-1}\hat{H}_{1i}\hat{H}_{1i}^T + \varepsilon_{1i}\hat{E}_{1i}\hat{E}_{1i}^T + \varepsilon_{2i}^{-1}\hat{H}_{2i}\hat{H}_{2i}^T + \varepsilon_{2i}\hat{E}_{4i}\hat{E}_{4i}^T. \tag{18}$$

Considering (13), it can be derived that

$$\frac{\mathbb{E}\{dV_1(t,x(t))\}}{dt} - \mathbb{E}\{2J^T(t)y(t) + \beta J^T(t)J(t)\} \le \sum_{i=1}^{r}\mu_i(\theta(t))\mathbb{E}\{\zeta^T(t)\Xi_2^{(i)}\zeta(t)\} \le 0, \tag{19}$$

which means

$$\begin{aligned} 2\mathbb{E}\left\{\int_0^t J^T(s)y(s)ds\right\} &\ge \mathbb{E}\left\{V_1(t,x(t)) - V_1(t,\mathbf{0}) - \beta\int_0^t J^T(s)J(s)ds\right\} \\ &= \mathbb{E}\left\{V_1(t,x(t)) - \beta\int_0^t J^T(s)J(s)ds\right\} \\ &\ge -\beta\mathbb{E}\left\{\int_0^t J^T(s)J(s)ds\right\}. \end{aligned} \tag{20}$$

From Definition 1, we know this indicates that the stochastic uncertain fuzzy system (1) is globally robustly passive in the sense of expectation, and the proof of Theorem 1 is then completed. ■

From the proof of Theorem 1, it is easy to know that if there are no uncertainties in the fuzzy model (1), i.e., ΔA_i, ΔB_i, ΔG_i, ΔC_i, ΔD_i and $\Delta W_i \equiv 0$, the following corollary can be obtained easily.

Corollary 1: The delayed fuzzy system (1) without parameter uncertainties is globally passive in the sense of expectation if there exist two positive definite matrices P_1, P_2 and two scalars $\beta \ge 0$, $\lambda > 0$ such that the following LMIs hold for all $i = 1, 2, \ldots, r$:

$$P_1 < \lambda I \quad \text{and} \quad \Xi_1^{(i)} < 0,$$

where $\Xi_1^{(i)}$ is defined as that in Theorem 1.

Remark 2: In Corollary 1, if there are no stochastic disturbances either, i.e., R_{1i}, $R_{2i} \equiv 0$ $(i = 1, 2, \ldots, r)$; and the time delay becomes constant, i.e., $\tau(t) \equiv \tau$; then Corollary 1 turns out to be the same result as Theorem 1 in [15].

In the following, let us consider the fuzzy model (1) without stochastic disturbances in (8), where the system is simplified to

$$\left\{ \begin{aligned} dx(t) &= \sum_{i=1}^{r}\mu_i(\theta(t))[\bar{A}_i x(t) + \bar{B}_i x(t-\tau(t)) + \bar{G}_i J(t)]dt, \\ y(t) &= \sum_{i=1}^{r}\mu_i(\theta(t))[\bar{C}_i x(t) + \bar{D}_i x(t-\tau(t)) + \bar{W}_i J(t)]. \end{aligned} \right. \tag{21}$$

Since the stochastic differential is not involved in (21), we are able to employ a different Lyapunov functional with the hope to reduce possible conservatism. In this case, the definition of the passivity is the same as that proposed in [21] for deterministic systems.

Theorem 2: The delayed fuzzy system (21) is globally robustly passive if there exist positive definite matrices Q_j, matrices X_j $(j = 1, 2, \ldots, 5)$ and $3r+1$ scalars $\beta \ge 0$, $\delta_{1i} > 0$, $\delta_{2i} > 0$, $\delta_{3i} > 0$ such that the following LMIs hold for all $i = 1, 2, \ldots, r$:

$$\Gamma^{(i)} = \begin{bmatrix} \Gamma_1^{(i)} & \Gamma_2^{(i)} \\ * & \Gamma_3^{(i)} \end{bmatrix} < 0, \tag{22}$$

where $\Gamma_3^{(i)} = \mathrm{diag}(-\delta_{1i}I, -\delta_{1i}I, -\delta_{2i}I, -\delta_{2i}I, -\delta_{3i}I, -\delta_{3i}I)$,

$$\Gamma_1^{(i)} = \begin{bmatrix} \Gamma_{11}^{(i)} & \frac{1}{\tau_m}Q_5 + A_i^T X_2^T & A_i^T X_3^T + X_1 B_i & Q_1 - X_1 + A_i^T X_4^T & A_i^T X_5^T + X_1 G_i - C_i^T \\ * & \Gamma_{22} & \frac{1}{\tau_M - \tau_m}Q_4 + X_2 B_i & -X_2 & X_2 G_i \\ * & * & \Gamma_{33}^{(i)} & B_i^T X_4^T - X_3 & B_i^T X_5^T + X_3 G_i - D_i^T \\ * & * & * & \Gamma_{44} & X_4 G_i - X_5^T \\ * & * & * & * & \Gamma_{55}^{(i)} \end{bmatrix},$$

$$\Gamma_2^{(i)} = \begin{bmatrix} X_1 H_{1i} & \delta_{1i} E_{1i}^T & 0 & \delta_{2i} E_{4i}^T & 0 & \delta_{3i} E_{1i}^T \\ 0 & 0 & 0 & 0 & X_2 H_{1i} & 0 \\ 0 & \delta_{1i} E_{2i}^T & 0 & \delta_{2i} E_{5i}^T & X_3 H_{1i} & \delta_{3i} E_{2i}^T \\ 0 & 0 & 0 & 0 & X_4 H_{1i} & 0 \\ 0 & \delta_{1i} E_{3i}^T & H_{2i} & \delta_{2i} E_{6i}^T & X_5 H_{1i} & \delta_{3i} E_{3i}^T \end{bmatrix},$$

and $\Gamma_{11}^{(i)} = Q_3 - \frac{1}{\tau_m}Q_5 + X_1 A_i + A_i^T X_1^T$, $\Gamma_{22} = Q_2 - Q_3 - \frac{1}{\tau_m}Q_5 - \frac{1}{\tau_M - \tau_m}Q_4$, $\Gamma_{33}^{(i)} = -(1-d)Q_2 + X_3 B_i + B_i^T X_3^T - \frac{1}{\tau_M - \tau_m}Q_4$, $\Gamma_{44} = (\tau_M - \tau_m)Q_4 + \tau_m Q_5 - X_4 - X_4^T$, $\Gamma_{55}^{(i)} = X_5 G_i + G_i^T X_5^T - (W_i + W_i^T + \beta I)$.

Proof: It follows from Lemma 1 that the inequality (22) is equivalent to

$$\Gamma_1^{(i)} + \delta_{1i}^{-1} \bar{H}_{1i} \bar{H}_{1i}^T + \delta_{2i}^{-1} \bar{H}_{2i} \bar{H}_{2i}^T + \delta_{3i}^{-1} \tilde{H}_{1i} \tilde{H}_{1i}^T + (\delta_{1i} + \delta_{3i}) \bar{E}_{1i} \bar{E}_{1i}^T + \delta_{2i} \bar{E}_{4i} \bar{E}_{4i}^T < 0, \tag{23}$$

where

$$\bar{H}_{1i} = \begin{bmatrix} X_1 H_{1i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tilde{H}_{1i} = \begin{bmatrix} 0 \\ X_2 H_{1i} \\ X_3 H_{1i} \\ X_4 H_{1i} \\ X_5 H_{1i} \end{bmatrix}, \bar{H}_{2i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ H_{2i} \end{bmatrix}, \bar{E}_{1i} = \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \end{bmatrix}, \bar{E}_{4i} = \begin{bmatrix} E_{4i}^T \\ 0 \\ E_{5i}^T \\ 0 \\ E_{6i}^T \end{bmatrix}.$$

Consider the following Lyapunov functional candidate for model (21) as

$$V_2(t, x(t)) = V_{21}(t, x(t)) + V_{22}(t, x(t)) + V_{23}(t, x(t)), \tag{24}$$

where

$$\begin{aligned} V_{21}(t, x(t)) &= x^T(t) Q_1 x(t), \\ V_{22}(t, x(t)) &= \int_{t-\tau(t)}^{t-\tau_m} x^T(s) Q_2 x(s) ds + \int_{t-\tau_m}^{t} x^T(s) Q_3 x(s) ds, \\ V_{23}(t, x(t)) &= \int_{-\tau_M}^{-\tau_m} \int_{\alpha}^{0} \dot{x}^T(t+s) Q_4 \dot{x}(t+s) ds d\alpha + \int_{-\tau_m}^{0} \int_{\alpha}^{0} \dot{x}^T(t+s) Q_5 \dot{x}(t+s) ds d\alpha. \end{aligned}$$

Note that the Lyapunov functional (24) is significantly different from that in (14) and, with the derivative terms involved in $V_{23}(t, x(t))$, a less conservative result could be established. Nevertheless, such a Lyapunov functional (24) does not seem to work for the stochastic system (8) and this is the reason why (14) is selected.

Calculating the time derivative of $V_{2i}(t,x(t))$ $(i=1,2,3)$ along the solutions of model (21) and we have

$$
\begin{aligned}
\frac{d}{dt}V_{21}(t,x(t)) &= 2x^T(t)Q_1\dot{x}(t); \qquad (25)\\
\frac{d}{dt}V_{22}(t,x(t)) &= x^T(t)Q_3x(t)+x^T(t-\tau_m)(Q_2-Q_3)x(t-\tau_m)-(1-\dot{\tau}(t))x^T(t-\tau(t))Q_2x(t-\tau(t))\\
&\le x^T(t)Q_3x(t)+x^T(t-\tau_m)(Q_2-Q_3)x(t-\tau_m)-(1-d)x^T(t-\tau(t))Q_2x(t-\tau(t)); \qquad (26)\\
\frac{d}{dt}V_{23}(t,x(t)) &= \dot{x}^T(t)[(\tau_M-\tau_m)Q_4+\tau_mQ_5]\dot{x}(t)-\int_{t-\tau_M}^{t-\tau_m}\dot{x}^T(s)Q_4\dot{x}(s)ds-\int_{t-\tau_m}^{t}\dot{x}^T(s)Q_5\dot{x}(s)ds\\
&\le \dot{x}^T(t)[(\tau_M-\tau_m)Q_4+\tau_mQ_5]\dot{x}(t)-\frac{1}{\tau_M-\tau_m}\left(\int_{t-\tau_M}^{t-\tau_m}\dot{x}(s)ds\right)^TQ_4\left(\int_{t-\tau_M}^{t-\tau_m}\dot{x}(s)ds\right)\\
&\quad -\frac{1}{\tau_m}\left(\int_{t-\tau_m}^{t}\dot{x}(s)ds\right)^TQ_5\left(\int_{t-\tau_m}^{t}\dot{x}(s)ds\right)
\end{aligned}
$$

$$
\begin{aligned}
&\le \dot{x}^T(t)[(\tau_M-\tau_m)Q_4+\tau_mQ_5]\dot{x}(t)-\frac{1}{\tau_M-\tau_m}\left(\int_{t-\tau(t)}^{t-\tau_m}\dot{x}(s)ds)^TQ_4(\int_{t-\tau(t)}^{t-\tau_m}\dot{x}(s)ds\right)\\
&\quad -\frac{1}{\tau_m}(x(t)-x(t-\tau_m))^TQ_5(x(t)-x(t-\tau_m))\\
&= \dot{x}^T(t)[(\tau_M-\tau_m)Q_4+\tau_mQ_5]\dot{x}(t)-\frac{1}{\tau_m}(x(t)-x(t-\tau_m))^TQ_5(x(t)-x(t-\tau_m))\\
&\quad -\frac{1}{\tau_M-\tau_m}(x(t-\tau_m)-x(t-\tau(t)))^TQ_4(x(t-\tau_m)-x(t-\tau(t))). \qquad (27)
\end{aligned}
$$

Note that we have used the condition $\dot{\tau}(t)\le d$ and Lemma 2 to obtain (26) and (27).

From the equation (21), it follows that for a matrix $X=(X_1^T\ X_2^T\ X_3^T\ X_4^T\ X_5^T)^T$, the relation

$$
2\xi^T(t)X\left[\sum_{i=1}^{r}\mu_i(\theta(t))(\bar{A}_ix(t)+\bar{B}_ix(t-\tau(t))+\bar{G}_iJ(t))-\dot{x}(t)\right]=0 \qquad (28)
$$

is true, where $\xi^T(t)=(x^T(t)\ x^T(t-\tau_m)\ x^T(t-\tau(t))\ \dot{x}^T(t)\ J^T(t))$.

Combining (21), (25)-(28) together with (2)-(3), we have

$$
\frac{dV_2(t,x(t))}{dt}-2J^T(t)y(t)-\beta J^T(t)J(t)\le\sum_{i=1}^{r}\mu_i(\theta(t))\xi^T(t)(\Gamma_1^{(i)}+\Delta\Gamma_1^{(i)})\xi(t), \qquad (29)
$$

where

$$
\begin{aligned}
\Delta\Gamma_1^{(i)} &= \begin{bmatrix}
X_1\Delta A_i+(\Delta A_i)^TX_1^T & (\Delta A_i)^TX_2^T & (\Delta A_i)^TX_3^T+X_1\Delta B_i & (\Delta A_i)^TX_4^T & \Delta\Gamma_{15}^{(i)}\\
* & 0 & X_2\Delta B_i & 0 & X_2\Delta G_i\\
* & * & X_3\Delta B_i+(\Delta B_i)^TX_3^T & (\Delta B_i)^TX_4^T & \Delta\Gamma_{35}^{(i)}\\
* & * & * & 0 & X_4\Delta G_i\\
* & * & * & * & \Delta\Gamma_{55}^{(i)}
\end{bmatrix}\\
&= \bar{H}_{1i}F_{1i}\bar{E}_{1i}^T+\bar{E}_{1i}F_{1i}^T\bar{H}_{1i}^T-\bar{H}_{2i}F_{2i}\bar{E}_{4i}^T-\bar{E}_{4i}F_{2i}^T\bar{H}_{2i}^T+\tilde{H}_{1i}F_{1i}\bar{E}_{1i}^T+\bar{E}_{1i}F_{1i}^T\tilde{H}_{1i}^T
\end{aligned}
$$

and $\Delta\Gamma_{15}^{(i)}=(\Delta A_i)^TX_5^T+X_1\Delta G_i-(\Delta C_i)^T$, $\Delta\Gamma_{35}^{(i)}=(\Delta B_i)^TX_5^T+X_3\Delta G_i-(\Delta D_i)^T$, $\Delta\Gamma_{55}^{(i)}=X_5\Delta G_i+(\Delta G_i)^TX_5^T-(\Delta W_i+(\Delta W_i)^T)$.

It follows from Lemma 3 that

$$
\Delta\Gamma_1^{(i)}\le\delta_{1i}^{-1}\bar{H}_{1i}\bar{H}_{1i}^T+\delta_{2i}^{-1}\bar{H}_{2i}\bar{H}_{2i}^T+\delta_{3i}^{-1}\tilde{H}_{1i}\tilde{H}_{1i}^T+\delta_{1i}\bar{E}_{1i}\bar{E}_{1i}^T+\delta_{3i}\bar{E}_{1i}\bar{E}_{1i}^T+\delta_{2i}\bar{E}_{4i}\bar{E}_{4i}^T. \qquad (30)
$$

Also, it can be seen (23) ensures that $\Gamma_1^{(i)} + \Delta\Gamma_1^{(i)} < 0$. Thus, from (29), one has

$$\begin{aligned} 2\int_0^t J^T(s)y(s)ds &\geq V_1(t,x(t)) - V_1(t,\mathbf{0}) - \beta\int_0^t J^T(s)J(s)ds \\ &= V_1(t,x(t)) - \beta\int_0^t J^T(s)J(s)ds \\ &\geq -\beta\int_0^t J^T(s)J(s)ds \end{aligned} \tag{31}$$

for all $t \geq 0$. The proof is then completed. ■

The following corollary is readily accessible.

Corollary 2: The delayed fuzzy model (21) without parameter uncertainties is globally passive, if there exist definite matrices Q_j, matrices X_j $(j = 1, 2, \ldots, 5)$ and a scalar $\beta \geq 0$ such that $\Gamma_1^{(i)} < 0$ for all $i = 1, 2, \ldots, r$; where $\Gamma_1^{(i)}$ is defined as that in Theorem 2.

Remark 3: In Theorem 1, a delay-independent passive condition is obtained for the addressed uncertain stochastic fuzzy systems with time-delays. From LMI (12), it infers that the time-varying delay needs to be differentiable and $\dot{\tau}(t) \leq d < 1$. Such conservatism is mainly due to the consideration of the stochastic disturbances that influence the construction of the Lyapunov functional. On the other hand, Theorem 2 is concerned with the deterministic fuzzy systems and hence a delay-dependent passive criterion could be derived which are dependent on not only the upper bound but also the lower bound of the time-varying delay, where the time derivative of $\tau(t)$ is no longer required to be less than 1.

B. Passification problem

Now, we are ready to consider the passification problem, i.e., design of a state feedback controller that makes the closed-loop fuzzy system passive.

We first consider the following general stochastic uncertain T-S fuzzy model with control input:

Plant rule i:

IF $\theta_1(t)$ is η_{i1} and ... $\theta_p(t)$ is η_{ip}, THEN

$$\begin{cases} dx(t) = \big[(A_i + \triangle A_i)x(t) + (B_i + \triangle B_i)x(t-\tau(t)) + (G_i + \triangle G_i)J(t)\big]dt \\ \qquad\qquad +\sigma_i(t, x(t), x(t-\tau(t)))d\omega_i(t) + S_i u(t), \\ y(t) = (C_i + \Delta C_i)x(t) + (D_i + \Delta D_i)x(t-\tau(t)) + (W_i + \Delta W_i)J(t); \quad t \geq 0 \end{cases} \tag{32}$$

where $i = 1, 2, \ldots, r$; $u(t) \in \mathbb{R}^l$ is the control input and S_i is a constant matrix with appropriate dimensions.

In this paper, the state feedback controller is taken to be as follows:

$$u(t) = \sum_{j=1}^{r} \mu_j(\theta(t))K_j x(t). \tag{33}$$

Then, the closed-loop fuzzy system can be represented as

$$\begin{cases} dx(t) = \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r} \mu_i(\theta(t))\mu_j(\theta(t))\big[(\bar{A}_i + S_iK_j)x(t) + \bar{B}_i x(t-\tau(t)) + \bar{G}_i J(t)\big]dt \\ \qquad\qquad + \sum\limits_{i=1}^{r} \mu_i(\theta(t))\sigma_i(t, x(t), x(t-\tau(t)))d\omega_i(t), \\ y(t) = \sum\limits_{i=1}^{r} \mu_i(\theta(t))[\bar{C}_i x(t) + \bar{D}_i x(t-\tau(t)) + \bar{W}_i J(t)]. \end{cases} \tag{34}$$

Theorem 3: The delayed feedback closed-loop fuzzy model (34) is globally robustly passive in the sense of expectation if there exist two positive definite matrices U_1, U_2, matrices T_j and $2(r+1)$ scalars $\beta \geq 0$, $\gamma > 0$,

$\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$ such that the following LMIs hold for all $i, j = 1, 2, \ldots, r$:

$$U_1 > \gamma I, \tag{35}$$

$$\Pi^{(i,j)} = \begin{bmatrix} \Pi_{11}^{(i,j)} & B_i U_1 & -U_1 C_i^T + G_i & \varepsilon_{1i} H_{1i} & 0 & U_1 E_{1i}^T & U_1 E_{4i}^T & U_1 R_{1i}^T & 0 \\ * & -(1-d)U_2 & -U_1 D_i^T & 0 & 0 & U_1 E_{2i}^T & U_1 E_{5i}^T & 0 & U_1 R_{2i}^T \\ * & * & \Pi_{33}^{(i)} & 0 & \varepsilon_{2i} H_{2i} & E_{3i}^T & E_{6i}^T & 0 & 0 \\ * & * & * & -\varepsilon_{1i} I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{2i} I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_{1i} I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_{2i} I & 0 & 0 \\ * & * & * & * & * & * & * & -\gamma I & 0 \\ * & * & * & * & * & * & * & * & -\gamma I \end{bmatrix} < 0; \tag{36}$$

where $\Pi_{11}^{(i,j)} = A_i U_1 + U_1 A_i^T + U_2 + S_i T_j + T_j^T S_i^T$, $\Pi_{33}^{(i)} = -(\beta I + W_i^T + W_i)$. Moreover, the state feedback gain can be constructed as

$$K_j = T_j U_1^{-1}. \tag{37}$$

Proof: The detailed proof follows a similar line of that of Theorem 1 and is therefore omitted to save space. ■

Corollary 3: The delayed feedback closed-loop fuzzy system (34) with no parameter uncertainties is globally passive in the sense of expectation if there exist two positive definite matrices U_1, U_2, matrices T_j and two scalars $\beta \geq 0$, $\gamma > 0$ such that the LMIs hold for all $i, j = 1, 2, \ldots, r$:

$$U_1 > \gamma I \qquad \text{and} \qquad \Pi_1^{(i,j)} < 0;$$

where $\Pi_1^{(i,j)}$ is defined as that in Theorem 3.

Remark 4: In Corollary 3, if R_{1i}, $R_{2i} \equiv 0$ $(i = 1, 2, \ldots, r)$ and the time delay becomes constant, i.e., $\tau(t) \equiv \tau$, then Corollary 3 reduces to be Theorem 2 in [15].

When there are no stochastic disturbances in (34), the system specializes to

$$\begin{cases} dx(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\theta(t))\mu_j(\theta(t)) \big[(\bar{A}_i + S_i K_j)x(t) + \bar{B}_i x(t-\tau(t)) + \bar{G}_i J(t)\big] dt, \\ y(t) = \sum_{i=1}^{r} \mu_i(\theta(t))[\bar{C}_i x(t) + \bar{D}_i x(t-\tau(t)) + \bar{W}_i J(t)]; \end{cases} \tag{38}$$

and we can obtain the following result.

Theorem 4: The delayed feedback closed-loop fuzzy model (38) is globally robustly passive if there exist five positive definite matrices Z_j $(j = 1, 2, \ldots, 5)$, matrices Y and T_j, $3r+1$ scalars $\beta \geq 0$, $\delta_{1i} > 0$, $\delta_{2i} > 0$, $\delta_{3i} > 0$ such that the following LMIs hold for all $i, j = 1, 2, \ldots, r$:

$$\Sigma^{(i,j)} = \begin{bmatrix} \Sigma_1^{(i,j)} & \Sigma_2^{(i)} \\ * & \Sigma_3^{(i)} \end{bmatrix} < 0, \tag{39}$$

where $\Sigma_3^{(i)} = \mathrm{diag}(-\delta_{1i} I, -\delta_{1i} I, -\delta_{2i} I, -\delta_{2i} I, -\delta_{3i} I, -\delta_{3i} I)$,

$$\Sigma_1^{(i,j)} = \begin{bmatrix} \Sigma_{11}^{(i,j)} & \frac{1}{\tau_m} Z_5 + Y A_i^T + T_j^T S_i^T & B_i Y^T & Z_1 + T_j^T S_i^T + Y A_i^T - Y^T & G_i - Y C_i^T \\ * & \Sigma_{22} & \frac{1}{\tau_M - \tau_m} Z_4 + B_i Y^T & -Y^T & G_i \\ * & * & \Sigma_{33} & Y B_i^T & -Y D_i^T \\ * & * & * & \Sigma_{44} & G_i \\ * & * & * & * & -(\beta I + W_i + W_i^T) \end{bmatrix},$$

$$\Sigma_2^{(i)} = \begin{bmatrix} \delta_{1i}H_{1i} & YE_{1i}^T & 0 & YE_{4i}^T & 0 & YE_{1i}^T \\ 0 & 0 & 0 & 0 & \delta_{3i}H_{1i} & 0 \\ 0 & YE_{2i}^T & 0 & YE_{5i}^T & 0 & YE_{2i}^T \\ 0 & 0 & 0 & 0 & \delta_{3i}H_{1i} & 0 \\ 0 & E_{3i}^T & \delta_{2i}H_{2i} & E_{6i}^T & 0 & E_{3i}^T \end{bmatrix};$$

and $\Sigma_{11}^{(i,j)} = Z_3 - \frac{1}{\tau_m}Z_5 + A_iY^T + YA_i^T + S_iT_j + T_j^TS_i$, $\Sigma_{22} = Z_2 - Z_3 - \frac{1}{\tau_m}Z_5 - \frac{1}{\tau_M-\tau_m}Z_4$, $\Sigma_{33} = -(1-d)Z_2 - \frac{1}{\tau_M-\tau_m}Z_4$, $\Sigma_{44} = (\tau_M - \tau_m)Z_4 + \tau_mZ_5 - Y - Y^T$. Moreover, the state feedback gain can be constructed as

$$K_j = T_j(Y^T)^{-1}. \tag{40}$$

Proof: The detailed proof follows a similar line of that of Theorem 2 and is therefore omitted to save space. ■

IV. Numerical Examples

Consider an embedded linear module described by passive state-space equations for RLC interconnect circuits [27]. Without loss of generality, consider the input $J(t)$ to be the port current vector and the output $y(t)$ to be the port voltage vector and the state-space equations. The time delay could be made up of a simple adjustable timer circuit which controls the actual relay. The circuit with uncertain component parameters is embedded in a feedback structure, which is designed to achieve the system specifications. Furthermore, the electronic noise is a random signal characteristic of all electronic circuits. Depending on the circuit, the noise generated by electronic devices can vary greatly. For example, thermal noise and shot noise are inherent to all devices. The other types depend mostly on manufacturing quality and semiconductor defects. Therefore, RLC interconnect circuits could exhibit time-delay, parameter uncertainties and stochastic noises. On the other hand, passivity implies that a network/circuit cannot generate more energy than it absorbs, and no passive termination of the network will cause the system to go unstable. The loss of passivity can be a serious problem because transient simulations of reduced networks may encounter artificial oscillations when connected to the rest of the circuitry. To this end, it is of great importance to investigate the robust passivity and passification of stochastic fuzzy time-delay systems.

In this section, two examples are illustrated to show the effectiveness of our results, one dealing with the stochastic uncertain fuzzy delay system (8), and the other corresponding to the deterministic fuzzy model (21).

Example 1: Consider the uncertain fuzzy system (8) with stochastic disturbances and time-varying delay $\tau(t) = 0.1 + 0.05\sin(10t)$, that is, $d = 0.5$. Take the number of IF-THEN rules $r = 2$ and the other parameters are as follows:

$$A_1 = \begin{bmatrix} -0.8 & 0 & 0.1 \\ 0.1 & -1.7 & 0.1 \\ 0.1 & 0 & -0.6 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & -0.1 & 0 \\ 0.1 & -0.7 & 0 \\ 0 & -0.2 & -0.9 \end{bmatrix}; \quad B_1 = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.1 & 0.2 & 0 \\ 0.1 & 0 & 0.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.1 & 0.1 & 0 \\ 0 & 0.2 & 0.1 \\ -0.1 & 0 & 0.1 \end{bmatrix}; \quad G_1 = \begin{bmatrix} 1 & -0.6 \\ 0.8 & 0.9 \\ 0.7 & 0.6 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.9 & 0.5 \\ -0.6 & 0.8 \\ 0.8 & 0.7 \end{bmatrix};$$

$$C_1 = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0 & 0.2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ -0.1 & 0 & 0.1 \end{bmatrix}; \quad D_1 = \begin{bmatrix} -0.1 & 0.2 & 0.1 \\ 0 & -0.1 & 0 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -0.1 & 0.1 & 0 \\ 0 & 0.2 & 0.1 \end{bmatrix}; \quad W_1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.2 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

The parameter uncertainties ΔA_i, ΔB_i, ΔG_i, ΔC_i, ΔD_i and ΔW_i satisfy the conditions (2)-(3) with

$$H_{11} = H_{12} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, E_{11}^T = E_{12}^T = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, E_{21}^T = E_{22}^T = \begin{bmatrix} 0.1 \\ -0.1 \\ 0 \end{bmatrix}, E_{31}^T = E_{32}^T = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix};$$

$$H_{21}^T = H_{22}^T = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, E_{41}^T = E_{42}^T = \begin{bmatrix} 0 \\ -0.2 \\ -0.1 \end{bmatrix}, E_{51}^T = E_{52}^T = \begin{bmatrix} -0.2 \\ 0.15 \\ 0 \end{bmatrix}, E_{61}^T = E_{62}^T = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

The noise intensity functions $\sigma_i(\cdot,\cdot,\cdot)$ $(i = 1, 2)$ meet the inequality (4) with

$$R_{11} = R_{12} = \begin{bmatrix} -0.1 & 0 & 0.1 \\ -0.1 & -0.1 & 0 \end{bmatrix}, \qquad R_{21} = R_{22} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ -0.2 & 0 & 0.1 \\ 0.1 & -0.1 & 0.1 \end{bmatrix}.$$

LMIs (11)-(12) can be solved with the feasible solutions as follows:

$$P_1 = \begin{bmatrix} 7.8397 & -0.9420 & -1.8243 \\ -0.9420 & 7.5320 & -0.7854 \\ -1.8243 & -0.7854 & 6.6648 \end{bmatrix}, P_2 = \begin{bmatrix} 4.6018 & -0.4591 & -1.3436 \\ -0.4591 & 4.5679 & -0.9538 \\ -1.3436 & -0.9538 & 3.7231 \end{bmatrix};$$

$$\varepsilon_{11} = 6.4899, \varepsilon_{12} = 8.5367; \varepsilon_{21} = 5.2175, \varepsilon_{22} = 5.9586; \beta = 29.0493, \lambda = 12.0170.$$

According to Theorem 1, the fuzzy model (8) with parameters as above is globally robustly passive in the sense of expectation.

In the following, we consider the passification problem. Take

$$S_1 = \begin{bmatrix} -0.3 & 0 & -0.1 \\ 0.1 & -0.3 & 0 \\ 0 & 0.1 & -0.4 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 0 & -0.45 & 0.2 \\ -0.2 & -0.5 & 0 \\ 0.1 & -0.1 & -0.56 \end{bmatrix}.$$

LMIs (35)-(36) can be solved with a feasible solution given as follows:

$$U_1 = \begin{bmatrix} 3.3738 & -0.3984 & 0.2495 \\ -0.3984 & 2.5440 & 0.0113 \\ 0.2495 & 0.0113 & 3.4647 \end{bmatrix}, U_2 = \begin{bmatrix} 3.9343 & 1.1431 & -0.2911 \\ 1.1431 & 4.6714 & 0.0110 \\ -0.2911 & 0.0110 & 4.4277 \end{bmatrix};$$

$$T_1 = T_2 = \begin{bmatrix} 1.7804 & 5.3470 & -1.8584 \\ 4.5403 & 1.5965 & -4.9487 \\ 2.9129 & 5.1539 & 2.1634 \end{bmatrix};$$

$$\varepsilon_{11} = 2.7530, \varepsilon_{12} = 2.7482, \varepsilon_{21} = 2.8610, \varepsilon_{22} = 2.8529; \beta = 3.0061, \gamma = 2.2577.$$

According to Theorem 3, we can construct a state feedback controller to make the closed-loop fuzzy system (34) passive with the feedback gain as follows:

$$K_1 = K_2 = \begin{bmatrix} 0.8363 & 2.2355 & -0.6039 \\ 1.5638 & 0.8793 & -1.5438 \\ 1.0825 & 2.1930 & 0.5393 \end{bmatrix}.$$

Example 2: Consider the uncertain fuzzy system (21) with $\tau(t) = 0.1+0.05\sin(30t)$, i.e., $d = 1.5$, $\tau_M = 0.15$ and $\tau_m = 0.05$. Take the number of IF-THEN rules $r = 2$ and the other parameters as follows:

$$A_1 = \begin{bmatrix} -0.6 & 0.4 \\ -0.1 & -0.8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.9 & -0.3 \\ 0 & -0.7 \end{bmatrix}; \quad B_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.16 & 0 \\ 0.1 & 0.2 \end{bmatrix};$$
$$G_1 = \begin{bmatrix} 0.3 & -0.3 \\ 0.1 & -0.2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0 \end{bmatrix}; \quad C_1 = \begin{bmatrix} -0.2 & 0.05 \\ -0.1 & -0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.1 & -0.2 \\ -0.3 & -0.1 \end{bmatrix};$$
$$D_1 = \begin{bmatrix} 0.2 & 0.3 \\ -0.1 & 0.2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.3 & -0.1 \\ 0.4 & -0.1 \end{bmatrix}; \quad W_1 = \begin{bmatrix} 0.15 & 0.02 \\ 0.1 & 0.16 \end{bmatrix}, \quad W_2 = \begin{bmatrix} -0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}.$$

The parameter uncertainties ΔA_i, ΔB_i, ΔG_i, ΔC_i, ΔD_i and ΔW_i satisfy the conditions (2)-(3) with

$$H_{11} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.05 \end{bmatrix}, \quad H_{12} = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & -0.1 \end{bmatrix}; \quad E_{11} = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & -0.2 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & 0 \end{bmatrix};$$
$$E_{21} = \begin{bmatrix} -0.05 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} 0 & 0.01 \\ 0.2 & -0.1 \end{bmatrix}; \quad E_{31} = \begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad E_{32} = \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & -0.2 \end{bmatrix};$$
$$H_{21} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & -0.1 \end{bmatrix}, \quad H_{22} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix}; \quad E_{41} = \begin{bmatrix} 0.1 & -0.1 \\ 0 & 0.2 \end{bmatrix}, \quad E_{42} = \begin{bmatrix} -0.1 & 0.1 \\ 0.2 & 0 \end{bmatrix};$$
$$E_{51} = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad E_{52} = \begin{bmatrix} 0.2 & -0.2 \\ -0.1 & -0.1 \end{bmatrix}; \quad E_{61} = \begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad E_{62} = \begin{bmatrix} 0.2 & -0.2 \\ -0.1 & 0.1 \end{bmatrix}.$$

Solving LMIs (22) gives

$$Q_1 = \begin{bmatrix} 2.7468 & 0.2005 \\ 0.2005 & 3.8103 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.4288 & -0.0104 \\ -0.0104 & 0.4660 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 1.6951 & 0.0846 \\ 0.0846 & 2.3420 \end{bmatrix},$$
$$Q4 = \begin{bmatrix} 0.2042 & -0.0067 \\ -0.0067 & 0.2077 \end{bmatrix}, \quad Q5 = \begin{bmatrix} 0.0789 & -0.0130 \\ -0.0130 & 0.0404 \end{bmatrix}; \quad X_1 = \begin{bmatrix} 1.8594 & 0.4280 \\ -0.0451 & 2.9488 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 0.7867 & -0.1943 \\ -0.2833 & 0.3816 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 0.0383 & 0.1979 \\ 0.1155 & 0.0871 \end{bmatrix}, \quad X_4 = \begin{bmatrix} 1.3120 & 0.3578 \\ -0.3466 & 1.3247 \end{bmatrix},$$
$$X_5 = \begin{bmatrix} 0.0924 & 0.0708 \\ -0.0007 & -0.2059 \end{bmatrix};$$
$$\delta_{11} = 2.6552, \delta_{12} = 2.4950, \delta_{21} = 2.5973, \delta_{22} = 2.3611, \delta_{31} = 2.6347, \delta_{32} = 2.4730, \beta = 3.1354.$$

According to Theorem 2, the fuzzy model (21) with parameters as above is globally robustly passive.

In order to consider the passification problem, we take

$$S_1 = \begin{bmatrix} 0.1 & 0 & 0.26 \\ 0.1 & 0.25 & -0.3 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.15 & -0.27 & 0 \\ 0 & -0.23 & 0.1 \end{bmatrix};$$

and then solve LMIs (39) to obtain

$$Z_1 = \begin{bmatrix} 99.3108 & 1.1459 \\ 1.1459 & 95.3975 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 17.0189 & 0.8299 \\ 0.8299 & 8.5669 \end{bmatrix}, \quad Z_3 = \begin{bmatrix} 80.6531 & 5.3812 \\ 5.3812 & 53.7594 \end{bmatrix},$$

$$Z_4 = \begin{bmatrix} 6.7463 & -0.0715 \\ -0.0715 & 6.3229 \end{bmatrix}, \quad Z_5 = \begin{bmatrix} 3.4591 & -0.1209 \\ -0.1209 & 3.4525 \end{bmatrix};$$

$$Y = \begin{bmatrix} 35.9069 & -2.0818 \\ -6.5350 & 46.2361 \end{bmatrix}, \qquad T_1 = T_2 = \begin{bmatrix} -239.4619 & -69.4844 \\ -16.4446 & -1.9395 \\ -58.1799 & -24.5773 \end{bmatrix};$$

$$\delta_{11} = 105.8715, \delta_{12} = 104.4078, \delta_{21} = 105.0035, \delta_{22} = 106.1356, \delta_{31} = 105.3239, \delta_{32} = 103.3992, \beta = 106.4114.$$

According to Theorem 4, we can construct a state feedback controller to make the closed-loop fuzzy system (38) passive with the feedback gain as follows:

$$K_1 = K_2 = \begin{bmatrix} -6.8119 & -2.4656 \\ -0.4642 & -0.1076 \\ -1.6648 & -0.7669 \end{bmatrix}.$$

V. Conclusions

In this paper, the passivity and passification problems have been investigated for a class of uncertain stochastic fuzzy systems with time-varying delay. To reflect more realistic dynamical behaviors of the system, both the parameter uncertainties and stochastic disturbances have been considered. We have proposed the definition of robust passivity in the sense of expectation. Then, by utilizing the Lyapunov functional method and the Itô differential rule combined with the matrix analysis techniques, we have established several sufficient criteria such that, for all admissible parameter uncertainties and stochastic disturbances, the closed-loop stochastic fuzzy time-delay system is robustly passive in the sense of expectation. The derived criteria, which are either delay-independent or delay-dependent, have been expressed in terms of LMIs. Illustrative examples have been presented to demonstrate the effectiveness and usefulness of the proposed results.

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