Robust H_∞ Filtering for a Class of Nonlinear Networked Systems With Multiple Stochastic Communication Delays and Packet Dropouts

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***Abstract*—In this paper, the robust H_∞ filtering problem is studied for a class of uncertain nonlinear networked systems with both multiple stochastic time-varying communication delays and multiple packet dropouts. A sequence of random variables, all of which are mutually independent but obey Bernoulli distribution, are introduced to account for the randomly occurred communication delays. The packet dropout phenomenon occurs in a random way and the occurrence probability for each sensor is governed by an individual random variable satisfying a certain probabilistic distribution in the interval [0 1]. The discrete-time system under consideration is also subject to parameter uncertainties, state-dependent stochastic disturbances and sector-bounded nonlinearities. We aim to design a linear full-order filter such that the estimation error converges to zero exponentially in the mean square while the disturbance rejection attenuation is constrained to a give level by means of the H_∞ performance index. Intensive stochastic analysis is carried out to obtain sufficient conditions for ensuring the exponential stability as well as prescribed H_∞ performance for the overall filtering error dynamics, in the presence of random delays, random dropouts, nonlinearities, and the parameter uncertainties. These conditions are characterized in terms of the feasibility of a set of linear matrix inequalities (LMIs), and then the explicit expression is given for the desired filter parameters. Simulation results are employed to demonstrate the effectiveness of the proposed filter design technique in this paper.**

***Index Terms*—Networked systems, nonlinear systems, packet dropout, robust H_∞ filtering, stochastic systems, stochastic time-varying communication delays.**

I. Introduction

THE objective of H_∞ filtering is to design an estimator for a given system such that the L_2 gain from the exogenous disturbance to the estimation error is less than a given level γ. H_∞ filtering is closely related to many robustness problems such as stabilization and sensitivity minimization of uncertain systems, and has therefore gained persistent attention from the early 1980s; see [1], [2], [6], [8], [22], [27], [29], [33], and the references therein.

To address the robustness issue, in recent years, the robust H_∞ filtering problems have been extensively investigated for a variety of complex dynamical systems, such as Markovian jumping systems [28], fuzzy systems [5], [26], time-varying systems [12], [18], [30], stochastic systems [32], and nonlinear systems [7], [21]. Furthermore, recognizing that both nonlinearity and stochasticity are commonly encountered in engineering practice, the robust H_∞ filtering problems for nonlinear stochastic systems have stirred a great deal of research interests. For example, the stochastic H_∞ filtering problem for time-delay systems subject to sensor nonlinearities have been dealt with in [23] and [24]. In [32], the robust H_∞ filtering problem for affine nonlinear stochastic systems with state and external disturbance-dependent noise has been studied, where the filter can be designed by solving second-order nonlinear Hamilton–Jacobi inequalities. So far, in comparison with the fruitful literature available for continuous-time systems, the corresponding H_∞ filtering results for discrete-time systems have been relatively few.

On another research frontier, in the past decade, networked control systems (NCSs) have attracted much attention owe to their successful applications in a wide range of areas for the advantage of decreasing the hardwiring, the installation cost and implementation difficulties. Nevertheless, the NCS-related challenging problems arise inevitably that would degrade the system performances. Such network-induced problems include, but are not limited to, the communication delays (also called network-induced time-delays) [3], [7], [10], [22], [23], [30], [35], and packet dropouts (probabilistic information missing, missing measurement) [4], [13], [17], [19], [20], [25]. In most relevant literature, the network-induced time-delays have been commonly assumed to be deterministic, which is fairly unrealistic as, by nature, delays resulting from network transmissions are typically time-varying and random. Very recently, researchers have started to model the communication delays in various probabilistic ways and, accordingly, some initial results have appeared [31], [34], among which the binary random communication delay has received much research attention due to its practicality and simplicity in describing network-induced delays.

The packet dropout (often named as missing measurement) serves as one of the most frequently occurred phenomenon with

Manuscript received July 21, 2009; accepted November 19, 2009. First published December 18, 2009; current version published March 10, 2010. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. James Lam. This work was supported in part by the Engineering and Physical Sciences Research Council (EPSRC) of the U.K. under Grant GR/S27658/01, the Royal Society of the U.K., the Alexander von Humboldt Foundation of Germany, National Natural Science Foundation of China under Grant 60825303, 60834003, 973 Project under Grant 2009CB320600, Fok Ying Tung Education Foundation under Grant 111064, and the Youth Science Fund of Heilongjiang Province under Grant QC2009C63.

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Digital Object Identifier 10.1109/TSP.2009.2038965

the networked control systems that has attracted considerable research attention during the past few years. In most results reported until now, however, it has been implicitly assumed that the measurement signal is either completely available (denoted by 1) or completely missing (denoted by 0), and all the sensors have the same missing probability [8], [22]. Unfortunately, such an assumption cannot cover some practical cases where partial/multiple missing measurements take place for an array of sensors, such as the case when the individual sensor has different missing probability and the case when only partial information is missing [19], [20], [25]. Note that the nonlinear filtering problem has been intensively investigated from researchers for several decades. However, to the best of the authors' knowledge, the filtering problem has not yet been addressed for uncertain stochastic nonlinear systems with multiple randomly occurred communication delays and partial missing measurements from individual sensors. It is, therefore, the main purpose of this paper to shorten such a gap by investigating the robust H_∞ filtering for discrete nonlinear networked systems with multiple stochastic communication delays and multiple missing measurements.

Motivated by the above analysis, in this paper, we aim to investigate the robust H_∞ filtering problem for discrete uncertain nonlinear networked systems with multiple stochastic time-varying communication delays and multiple missing measurements.

The main contributions of this paper are summarized as follows: 1) a new model is proposed to describe the multiple network communication delays, each of which satisfies an individual Bernoulli distribution; 2) a combination of important factors contributing to the complexity of NCSs are investigated in a unified framework which comprises partial measurement missing, sector nonlinearities and parameter uncertainties; and 3) stochastic analysis is conducted to enforce the H_∞ performance for the addressed "complex" systems in addition to the usual stability requirement. By means of LMIs, a sufficient condition for the robustly exponential stability of the filtering error dynamics is obtained and a prescribed H_∞ disturbance rejection attenuation level is guaranteed, and the explicit expression of the desired filter parameters is also derived. A numerical simulation example is used to demonstrate the effectiveness of the presented filtering scheme in this paper.

The rest of this paper is organized as follows. Section II formulates the problem under consideration. The exponentially stability condition and robust H_∞ performance of the filter error system are given in Section III. The filter design problem is solved in Section IV. An illustrative example is given in Section V and we conclude the paper in Section VI.

Notation: The notation used in the paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m\times n$; I and 0 represent the identity matrix and zero matrix, respectively. The notation $P>0$ means that P is real symmetric and positive definite; the notation $\|A\|$ refers to the norm of a matrix A defined by $\|A\|=\sqrt{\mathrm{tr}(A^TA)}$ and $\|\cdot\|_2$ stands for the usual l_2 norm. In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry, and $\mathrm{diag}\{\ldots\}$ stands for a block-diagonal matrix. In addition, $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ will, respectively, mean expectation of x and expectation of x conditional on y. The set of all nonnegative integers is denoted by $\mathbb{I}^+$ and the set of all nonnegative real numbers is represented by $\mathbb{R}^+$. If A is a matrix, $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) means the largest (respectively, smallest) eigenvalue of A. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. Problem Formulation

To start with, let us denote the following for presentation clarity:

$$\tilde{x}(k):=\sum_{i=1}^{q}\alpha_i(k)x\left(k-\tau_i(k)\right) \tag{1}$$

where $\tau_i(k)$ $(i=1,2,\ldots,q)$ are the random communication delays to be discussed in detail.

Consider the following discrete-time uncertain nonlinear networked system with multiple stochastic communication delays:

$$\begin{cases} x(k+1)=(A+\Delta A)x(k)+A_d\tilde{x}(k)+Ff\left(x(k)\right) \\ \qquad\qquad +g\left(x(k),\tilde{x}(k),k\right)w(k)+D_1v(k) \\ \tilde{y}(k)=Cx(k)+D_2v(k) \\ z(k)=Lx(k) \\ x(j)=\varphi(j), j=-d_M,-d_M+1,\cdots,0 \end{cases} \tag{2}$$

where $x(k)\in\mathbb{R}^n$ represents the state vector; $\tilde{x}(k)\in\mathbb{R}^n$ is defined in (1); $\tilde{y}(k)\in\mathbb{R}^r$ is the process output; $z(k)\in\mathbb{R}^m$ is the signal to be estimated; $v(k)\in\mathbb{R}^q$ is the exogenous disturbance signal belonging to $l_2[0,\infty)$. $\varphi(j)$ $(j=-d_M,-d_M+1,\cdots,0)$ are the initial conditions. $w(k)$ is a scalar Wiener process (Brownian motion) satisfying

$$\mathbb{E}\left\{w(k)\right\}=0,\ \mathbb{E}\left\{w^2(k)\right\}=1,\ \mathbb{E}\left\{w(k)w(j)\right\}=0\ (k\neq j),$$

and $g:\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{N}\to\mathbb{R}^n$ is the continuous function quantifying the noise intensity which satisfies

$$g^T\left(x(k),\tilde{x}(k),k\right)g(x(k),\tilde{x}(k),k) \le \rho_1x^T(k)x(k)+\rho_2\tilde{x}^T(k)\tilde{x}(k)$$

where $\rho_1>0$ and $\rho_2>0$ are known constant scalars. The parameter uncertainties ΔA is a real-valued matrix of the form

$$\Delta A=HF(k)E \tag{3}$$

where H and E are known real constant matrices with appropriate dimensions, and $F(k)$ is the unknown time-varying matrix function satisfying $F^T(k)F(k)\le I$.

The vector-valued nonlinear functions f is assumed to satisfy the following sector-bounded conditions with $f(0)=0$:

$$\left[f(x)-f(y)-R_1(x-y)\right]^T\left[f(x)-f(y)-R_2(x-y)\right]\le 0 \tag{4}$$

where $R_1,R_2\in\mathbb{R}^{n\times n}$ and R_1-R_2 is a positive definite matrix.

Remark 1: It is customary that the nonlinear function f is said to belong to $[R_1\ R_2]$ (see [14]). The nonlinear description in (4) is quite general that include the usual Lipschitz conditions as a special case. Note that both the control analysis and model reduction problems for systems with sector nonlinearities have been intensively studied; see, e.g., [9] and [15].

The random variables $\alpha_i(k) \in \mathbb{R}$ $(i = 1, 2, \cdots, q)$ in (2) are mutually uncorrelated Bernoulli distributed white sequences obeying the following probability distribution law:

$$\begin{aligned} \text{Prob}\{\alpha_i(k) = 1\} &= \mathbb{E}\{\alpha_i(k)\} = \bar{\alpha}_i, \\ \text{Prob}\{\alpha_i(k) = 0\} &= 1 - \bar{\alpha}_i. \end{aligned}$$

The following assumption is needed on the random communication time-delays considered.

Assumption 1: The communication delays $\tau_i(k)$ $(i = 1, 2, \cdots, q)$ are time-varying and satisfy $d_m \leq \tau_i(k) \leq d_M$, where d_m and d_M are constant positive scalars representing the lower and upper bounds on the communication delays, respectively.

Remark 2: The way the communication delays are described in (1) is believed to be new because, different from most existing literature, such a description exhibits the following two features: 1) the communication delays are allowed to occur in any fashion, either discretely, successively, or even distributely; and 2) each possible delay could occur independently and randomly according to an individual probability distribution which can be specified *a prior* through statistical test.

In this paper, the packet dropout (missing measurement) phenomenon constitutes another focus of our present research. The multiple packet dropouts are described by

$$\begin{aligned} y(k) &= \Xi C x(k) + D_2 v(k) \\ &= \sum_{j=1}^{r} \beta_j C_j x(k) + D_2 v(k) \end{aligned} \tag{5}$$

where $y(k) \in \mathbb{R}^r$ is the *actual* measurement signal of (2), $\Xi := \text{diag}\{\beta_1, \ldots, \beta_r\}$ with β_j $(j = 1, \ldots, r)$ being r unrelated random variables which are also unrelated with $\alpha_i(k)$ and $w(k)$. It is assumed that β_j has the probabilistic density function $q_j(s)$ $(j = 1, \ldots, r)$ on the interval [0 1] with mathematical expectation μ_j and variance σ_j^2. C_j is defined by

$$C_j := \text{diag}\{\underbrace{0, \cdots, 0}_{j-1}, 1, \underbrace{0, \cdots, 0}_{r-j}\}C.$$

Note that β_j could satisfy any discrete probabilistic distributions on the interval [0 1], which includes the widely used Bernoulli distribution as a special case. In the sequel, we denote $\bar{\Xi} = \mathbb{E}\{\Xi\}$.

Remark 3: In real systems, the measurement data may be transferred through multiple sensors. On one hand, for different sensor, the data missing probability may be different. On the other hand, due to various reasons such as sensor aging and sensor temporal failure, the data missing at one moment might be partial, and therefore the missing probability cannot be simply described by 0 or 1. In (5), the diagonal matrix Ξ accounts for the probabilistic missing status of the array of sensors where the random variable β_j corresponds to the jth sensor. Also, β_j can take value on the interval [0 1] and the probability for β_j to take different values may differ from each other. It is easy to see that the widely used Bernoulli distribution is included here as a special case.

According to the above analysis, we have the following system to be investigated:

$$\begin{cases} x(k+1) = (A + \Delta A)x(k) + A_d \tilde{x}(k) + Ff(x(k)) \\ \qquad\qquad + g(x(k), \tilde{x}(k), k) w(k) + D_1 v(k) \\ y(k) = \Xi C x(k) + D_2 v(k) \\ \qquad = \sum_{j=1}^{r} \beta_j C_j x(k) + D_2 v(k) \\ z(k) = Lx(k) \\ x(j) = \varphi(j), j = -d_M, -d_M + 1, \cdots, 0. \end{cases} \tag{6}$$

In this paper, we are interested in obtaining $\hat{z}(k)$, the estimate of the signal $z(k)$, from the *actual* measured output $y(k)$. The full-order filter to be considered is given as follows:

$$\begin{cases} \hat{x}(k+1) = A_f \hat{x}(k) + B_f y(k) \\ \hat{z}(k) = C_f \hat{x}(k) \end{cases} \tag{7}$$

where $\hat{x}(k) \in \mathbb{R}^n$ represents the state estimate, $\hat{z}(k) \in \mathbb{R}^m$is the estimated output, and A_f, B_f, C_f are appropriately dimensioned filter matrices to be determined.

Augmenting the model of (6) to include the states of the filter (7), the filtering error system is given by

$$\begin{cases} \bar{x}(k+1) = (\bar{A} + \tilde{A})\bar{x}(k) + \sum_{i=1}^{q} (\bar{A}_{di} + \tilde{A}_{di})\bar{x}(k - \tau_i(k)) \\ \qquad\qquad + \bar{D}w(k) + \bar{F}f(x(k)) + \bar{D}_1 v(k) \\ \bar{z}(k) = \bar{L}\bar{x}(k) \end{cases} \tag{8}$$

where

$$\begin{aligned} &\bar{x}(k) = \begin{bmatrix} x^T(k) & \hat{x}^T(k) \end{bmatrix}^T, \ \bar{z}(k) = z(k) - \hat{z}(k), \\ &\bar{L} = [L \quad -C_f], \ \bar{F} = [F^T \quad 0]^T, \\ &\bar{A} = \begin{bmatrix} A + \Delta A & 0 \\ B_f \bar{\Xi} C & A_f \end{bmatrix}, \ \tilde{A} = \begin{bmatrix} 0 & 0 \\ B_f(\Xi - \bar{\Xi})C & 0 \end{bmatrix}, \\ &\bar{A}_{di} = \begin{bmatrix} \bar{\alpha}_i A_d & 0 \\ 0 & 0 \end{bmatrix}, \ \tilde{A}_{di} = \begin{bmatrix} \tilde{\alpha}_i(k) A_d & 0 \\ 0 & 0 \end{bmatrix}, \\ &\bar{D} = \begin{bmatrix} g(x(k), \tilde{x}(k), k) \\ 0 \end{bmatrix}, \ \bar{D}_1 = \begin{bmatrix} D_1 \\ B_f D_2 \end{bmatrix}. \end{aligned}$$

With $\tilde{\alpha}_i(k) = \alpha_i(k) - \bar{\alpha}_i$. It is clear that $\mathbb{E}\{\tilde{\alpha}_i(k)\} = 0$ and $\mathbb{E}\{\tilde{\alpha}_i^2(k)\} = \bar{\alpha}_i(1 - \bar{\alpha}_i)$.

Due to the existence of the stochastic variable $\alpha_i(k)$ and β_j, the definition for the exponential stability in the mean square is needed for the forthcoming issue of stochastic analysis.

Definition 1: [22] The filtering error system (8) is said to be exponentially stable in the mean square if, in case of $v(k) = 0$, for any initial conditions, there exist constants $\delta > 0$ and $0 < \kappa < 1$ such that

$$\mathbb{E}\left\{\|\bar{x}(k)\|^2\right\} \leq \delta \kappa^k \sup_{-d_M \leq i \leq 0} \mathbb{E}\left\{\|\varphi(i)\|^2\right\}, \quad \forall k \geq 0.$$

Our aim in this paper is to develop techniques to deal with the robust $\mathcal{H}_\infty$ filtering problem for uncertain discrete nonlinear systems with multiple communication delays and packet dropouts. More specifically, given a disturbance attenuation level $\gamma > 0$, we like to design the filter of the form (7) for the system (6) such that, for all admissible parameter uncertainties, nonlinearities, multiple communication delays and packet dropouts, the following two requirements are satisfied simultaneously:

R1) the filter error system (8) is exponentially stable in the mean square;

R2) under zero initial condition, the filtering error $\bar{z}(k)$ satisfies

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\bar{z}(k)\|^2\right\} \leqslant \gamma^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\|v(k)\|^2\right\} \tag{9}$$

for all nonzero $v(k)$, where $\gamma > 0$ is a prescribed scalar.

III. Robust $\mathcal{H}_\infty$ Filtering Performances Analysis

Before proceeding further, we introduce the following lemmas which will be needed for the derivation of our main results.

Lemma 1: (Schur Complement) Given constant matrices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ where $\mathcal{S}_1 = \mathcal{S}_1^{\mathrm{T}}$ and $0 < \mathcal{S}_2 = \mathcal{S}_2^{\mathrm{T}}$, then $\mathcal{S}_1 + \mathcal{S}_3^{\mathrm{T}}\mathcal{S}_2^{-1}\mathcal{S}_3 < 0$ if and only if

$$\begin{bmatrix}\mathcal{S}_1 & \mathcal{S}_3^{\mathrm{T}}\\ \mathcal{S}_3 & -\mathcal{S}_2\end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix}-\mathcal{S}_2 & \mathcal{S}_3\\ \mathcal{S}_3^{\mathrm{T}} & \mathcal{S}_1\end{bmatrix} < 0. \tag{10}$$

Lemma 2: (S-procedure) Let $L = L^{\mathrm{T}}$ and H and E be real matrices of appropriate dimensions with F satisfying $FF^T \le I$, then $L + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$, if and only if there exists a positive scalar $\varepsilon > 0$ such that $L + \varepsilon^{-1}HH^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E < 0$ or equivalently,

$$\begin{bmatrix} L & H & \varepsilon E^{\mathrm{T}}\\ H^{\mathrm{T}} & -\varepsilon I & 0\\ \varepsilon E & 0 & -\varepsilon I\end{bmatrix} < 0. \tag{11}$$

Let us first consider the robust exponential stability analysis problem for the filter error system (8) with $v(k) = 0$.

Theorem 1: Let the filter parameters A_f, B_f and C_f be given and the admissible conditions hold. Then, the filtering error system (8) with $v(k) = 0$ is robustly exponentially stable in the mean square if there exist matrices $P > 0$, $Q_j > 0$ $(j = 1, 2, \cdots q)$ and positive constant scalars λ_1, λ_2 satisfying

$$\Omega = \begin{bmatrix}\Omega_{11} & * & *\\ \hat{Z}^T P\bar{A} & \Omega_{22} & *\\ \Omega_{31} & \bar{F}^T P\hat{Z} & \Omega_{33}\end{bmatrix} < 0, \tag{12}$$

$$P \le \lambda_1 I, \tag{13}$$

where

$$\Omega_{11} = \lambda_1 A_\rho - P + \sum_{j=1}^{q}(d_M - d_m + 1)Q_j + \bar{A}^T P\bar{A} + \sum_{j=1}^{r}\sigma_j^2\bar{C}_j^T P\bar{C}_j - \lambda_2 G^T\tilde{R}_1 G,$$

$$\Omega_{22} = \mathrm{diag}\left\{-Q_1 + \tilde{A}_1, -Q_2 + \tilde{A}_2, \cdots, -Q_q + \tilde{A}_q\right\} + \hat{Z}^T P\hat{Z} + \lambda_1\rho_2\hat{Z}_a^T\hat{Z}_a$$

$$\Omega_{31} = \bar{F}^T P\bar{A} - \lambda_2\tilde{R}_2^T G,\ \Omega_{33} = \bar{F}^T P\bar{F} - \lambda_2 I,$$

$$\tilde{A}_i = \bar{\alpha}_i(1-\bar{\alpha}_i)\hat{A}_d^T P\hat{A}_d + \lambda_1\rho_2 W_i,\ i = 1, 2, \cdots, q.$$

$$\tilde{R}_1 = \left(R_1^T R_2 + R_2^T R_1\right)/2,\ \tilde{R}_2 = -\left(R_1^T + R_2^T\right)/2,$$

$$G = [I \quad 0],\ \hat{Z} = [\bar{A}_{d1} \quad \bar{A}_{d2} \quad \cdots \quad \bar{A}_{dq}],$$

$$\hat{Z}_a = [T_1 \quad T_2 \quad \cdots \quad T_q],\ \bar{C}_j = \begin{bmatrix}0 & 0\\ B_f C_j & 0\end{bmatrix},$$

$$\hat{A}_d = \begin{bmatrix}A_d & 0\\ 0 & 0\end{bmatrix},\ A_\rho = \begin{bmatrix}\rho_1 I & 0\\ 0 & 0\end{bmatrix},$$

$$W_i = \begin{bmatrix}\bar{\alpha}_i(1-\bar{\alpha}_i)I & 0\\ 0 & 0\end{bmatrix},\ T_i = \begin{bmatrix}\bar{\alpha}_i I & 0\\ 0 & 0\end{bmatrix}.$$

Proof: Let

$$\Theta_j(k) = \{x(k-\tau_j(k)), x(k-\tau_j(k)+1), \cdots, x(k)\} \quad (j = 1, 2, \cdots, q)$$

$$\chi(k) = \{\Theta_1(k)\cup\Theta_2(k)\cup\cdots\cup\Theta_q(k)\} = \bigcup_{j=1}^{q}\Theta_j(k).$$

Choose the following Lyapunov functional for system (8):

$$V(\chi(k)) = \sum_{i=1}^{3}V_i(k)$$

where

$$V_1(k) = \bar{x}^T(k)P\bar{x}(k),\ V_2(k) = \sum_{j=1}^{q}\sum_{i=k-\tau_j(k)}^{k-1}\bar{x}^T(i)Q_j\bar{x}(i),$$

$$V_3(k) = \sum_{j=1}^{q}\sum_{m=-d_M+1}^{-d_m}\sum_{i=k+m}^{k-1}\bar{x}^T(i)Q_j\bar{x}(i)$$

with $P > 0$, $Q_j > 0$ $(j = 1, 2, \cdots, q)$ being matrices to be determined. Then, along the trajectory of system (8) with $v(k) = 0$, we have

$$\begin{aligned}\mathbb{E}\{\Delta V|x(k)\} &= \mathbb{E}\{V(\chi(k+1))|\chi(k)\} - V(\chi(k))\\ &= \mathbb{E}\{(V(\chi(k+1)) - V(\chi(k))|\chi(k)\}\\ &= \sum_{i=1}^{3}\mathbb{E}\{\Delta V_i|\chi(k)\}.\end{aligned} \tag{14}$$

From (8), we can obtain that

$$\begin{aligned}&\mathbb{E}\{\Delta V_1|\chi(k)\}\\ &= \mathbb{E}\left\{\left(\bar{x}^T(k+1)P\bar{x}(k+1) - \bar{x}^T(k)P\bar{x}(k)\right)|\chi(k)\right\}\\ &= \mathbb{E}\Big\{\Big(\bar{x}^T(k)(\bar{A}^T P\bar{A} + \tilde{A}^T P\tilde{A} - P)\bar{x}(k)\\ &\quad + 2\bar{x}^T(k)\bar{A}^T P\left(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\right)\\ &\quad + 2\bar{x}^T(k)\bar{A}^T P\bar{F}f(x(k))\\ &\quad + \sum_{i=1}^{q}\bar{x}^T(k-\tau_i(k))\tilde{A}_{di}^T P\tilde{A}_{di}\bar{x}(k-\tau_i(k))\\ &\quad + \left(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\right)^T\\ &\quad \times P\left(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\right)\\ &\quad + 2\left(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\right)^T P\bar{F}f(x(k))\\ &\quad + \bar{D}^T P\bar{D} + f^T(x(k))\bar{F}^T P\bar{F}f(x(k))\Big)|\chi(k)\Big\}\end{aligned}$$

$$
\begin{aligned}
&\mathbb{E}\left\{\tilde{A}_{di}^T P \tilde{A}_{di}\right\} \\
&\quad = \mathbb{E}\left\{\begin{bmatrix}\tilde{\alpha}_i(k)A_d & 0\\ 0 & 0\end{bmatrix}^T P \begin{bmatrix}\tilde{\alpha}_i(k)A_d & 0\\ 0 & 0\end{bmatrix}\right\} \\
&\quad = \bar{\alpha}_i(1-\bar{\alpha}_i)\begin{bmatrix}A_d & 0\\ 0 & 0\end{bmatrix}^T P \begin{bmatrix}A_d & 0\\ 0 & 0\end{bmatrix} \\
&\quad = \bar{\alpha}_i(1-\bar{\alpha}_i)\hat{A}_d^T P \hat{A}_d \\
&\mathbb{E}\{\bar{D}^T P \bar{D}\} \\
&\quad \le \lambda_1\rho_2\left(\sum_{i=1}^q T_i\bar{x}(k-\tau_i(k))\right)^T\left(\sum_{i=1}^q T_i\bar{x}(k-\tau_i(k))\right) \\
&\qquad + \lambda_1\rho_2\sum_{i=1}^q \bar{x}^T(k-\tau_i(k))W_i\bar{x}(k-\tau_i(k)) \\
&\qquad + \lambda_1\bar{x}^T(k)A_\rho\bar{x}(k). \qquad (15)
\end{aligned}
$$

Taking (14), (15) into consideration, we have

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_1|\chi(k)\} \\
&\quad \le \bar{x}^T(k)\left(\bar{A}^T P\bar{A} + \sum_{j=1}^r \sigma_j^2\bar{C}_j^T P\bar{C}_j + \lambda_1 A_\rho - P\right)\bar{x}(k) \\
&\qquad + 2\bar{x}^T(k)\bar{A}^T P\left(\sum_{i=1}^q \bar{A}_{di}\bar{x}(k-\tau_i(k))\right) \\
&\qquad + 2\bar{x}^T(k)\bar{A}^T P\bar{F}f(x(k)) + \sum_{i=1}^q\sum_{j=1}^q \bar{x}^T(k-\tau_i(k)) \\
&\qquad \times \left(\bar{A}_{di}^T P\bar{A}_{dj} + \lambda_1\rho_2 T_i^T T_j\right)\bar{x}(k-\tau_j(k)) \\
&\qquad + \sum_{i=1}^q \bar{x}^T(k-\tau_i(k))\left(\bar{\alpha}_i(1-\bar{\alpha}_i)\hat{A}_d^T P\hat{A}_d + \lambda_1\rho_2 W_i\right) \\
&\qquad \times \bar{x}(k-\tau_i(k)) + 2\left(\sum_{i=1}^q \bar{A}_{di}\bar{x}(k-\tau_i(k))\right)^T \\
&\qquad \times P\bar{F}f(x(k)) + f^T(x(k))\bar{F}^T P\bar{F}f(x(k)). \qquad (16)
\end{aligned}
$$

Next, it can be derived that

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_2|\chi(k)\} \\
&\quad \le \mathbb{E}\left\{\sum_{j=1}^q\left(\bar{x}^T(k)Q_j\bar{x}(k) - \bar{x}^T(k-\tau_j(k))Q_j\right.\right. \\
&\qquad\quad \left.\left. \times (k-\tau_j(k)) + \sum_{i=k-d_M+1}^{k-d_m}\bar{x}^T(i)Q_j\bar{x}(i)\right)|\chi(k)\right\} \\
&\mathbb{E}\{\Delta V_3|\chi(k)\} \\
&\quad = \mathbb{E}\left\{\sum_{j=1}^q\left((d_M-d_m)\bar{x}^T(k)Q_j\bar{x}(k)\right.\right. \\
&\qquad\quad \left.\left. - \sum_{i=k-d_M+1}^{k-d_m}\bar{x}^T(i)Q_j\bar{x}(i)\right)|\chi(k)\right\}. \qquad (17)
\end{aligned}
$$

Letting

$$
\xi(k) = \left[\bar{x}^T(k)\bar{x}^T(k-\tau_1(k))\cdots\bar{x}^T(k-\tau_q(k))\,f^T(x(k))\right]^T
$$

the combination of (16) and (17) results in

$$
\mathbb{E}\{\Delta V|x(k)\} \le \xi^T(k)\Omega_1\xi(k) \qquad (18)
$$

where

$$
\Omega_1 = \begin{bmatrix}\Omega_{11}+\lambda_2 G^T\tilde{R}_1 G & * & *\\ \hat{Z}^T P\bar{A} & \Omega_{22} & *\\ \bar{F}^T P\bar{A} & \bar{F}^T P\hat{Z} & \bar{F}^T P\bar{F}\end{bmatrix}.
$$

Notice that (4) implies

$$
\begin{bmatrix}\bar{x}(k)\\ f(x(k))\end{bmatrix}^T\begin{bmatrix}G^T\tilde{R}_1 G & G^T\tilde{R}_2\\ \tilde{R}_2^T G & I\end{bmatrix}\begin{bmatrix}\bar{x}(k)\\ f(x(k))\end{bmatrix} \le 0. \qquad (19)
$$

From (18) and (19), it follows that

$$
\mathbb{E}\{\Delta V|x(k)\} \le \xi^T(k)\Omega\xi(k).
$$

According to Theorem 1, we have $\Omega < 0$. Hence, for all $\xi(k) \ne 0$, $\mathbb{E}\{\Delta V|x(k)\} \le \xi^T(k)\Omega\xi(k) < 0$. Furthermore, from Theorem 1 in [22], the robustly exponential stability of system (8) can be confirmed in the mean square sense. The proof is complete. ■

Next, we will analyze the $\mathcal{H}_\infty$ performance of the filtering error system (8).

Theorem 2: Let the filter parameters A_f, B_f and C_f be given and γ be a prespecified positive constant. Then the filtering error system (8) is robustly exponentially stable in the mean square for $v(k) = 0$ and satisfies $\|\bar{z}(k)\|_2 \le \gamma\|v(k)\|_2$ under the zero initial condition for any nonzero $v(k) \in l_2[0,+\infty)$, if there exist matrices $P > 0$, $Q_j > 0$ $(j = 1, 2, \cdots q)$ and positive constant scalars λ_1, λ_2 satisfying

$$
\Phi < 0 \qquad (20)
$$

$$
P \le \lambda_1 I \qquad (21)
$$

where

$$
\Phi = \begin{bmatrix}\Phi_{11} & * & * & *\\ \hat{Z}^T P\bar{A} & \Omega_{22} & * & *\\ \Omega_{31} & \bar{F}^T P\hat{Z} & \Omega_{33} & *\\ \bar{D}_1^T P\bar{A} & \bar{D}_1^T P\hat{Z} & \bar{D}_1^T P\bar{F} & \bar{D}_1^T P\bar{D}_1 - \gamma^2 I\end{bmatrix}
$$

$$
\Phi_{11} = \Omega_{11} + \bar{L}^T\bar{L},
$$

with Ω_{11}, Ω_{22}, Ω_{31}, Ω_{33}, $\bar{C}_j$, $\hat{A}_d$, A_ρ, W_i, T_i, $\hat{Z}$, $\hat{Z}_a$, $\hat{R}_1$, $\hat{R}_2$, and $\tilde{A}_i$ being defined as in Theorem 1.

Proof: It is clear that $\Phi < 0$ implies $\Omega < 0$. According to Theorem 1, the filtering error system (8) is robustly exponentially stable in the mean square.

Let us now deal with the $\mathcal{H}_\infty$ performance of the system (8). Construct the same Lyapunov-Krasovskii functional as in Theorem 1. A similar calculation as in the proof of Theorem 1 leads to

$$
\mathbb{E}\{\Delta V|\chi(k)\} \le \xi_0^T(k)\Omega_2\xi_0(k) \qquad (22)
$$

where

$$\xi_0(k)=\left[\bar{x}^T(k)\bar{x}^T(k-\tau_1(k)\cdots \bar{x}^T(k-\tau_q(k))\ f^T(x(k))\ v^T(k)\right]^T$$

$$\Omega_2=\begin{bmatrix}\Omega_{11}+\lambda_2 G^T\tilde{R}_1 G & * & * & * \\ \hat{Z}^T P\bar{A} & \Omega_{22} & * & * \\ \bar{F}^T P\bar{A} & \bar{F}^T P\hat{Z} & \bar{F}^T P\bar{F} & * \\ \bar{D}_1^T P\bar{A} & \bar{D}_1^T P\hat{Z} & \bar{D}_1^T P\bar{F} & \bar{D}_1^T P\bar{D}_1\end{bmatrix}.$$

In order to deal with the $\mathcal{H}_\infty$ performance of the filtering system (8), we introduce the following index:

$$J(n)=\mathbb{E}\sum_{k=0}^{\infty}\left[\bar{z}^T(k)\bar{z}(k)-\gamma^2 v^T(k)v(k)\right] \tag{23}$$

where n is non-negative integer. Obviously, our aim is to show $J(n)<0$ under the zero initial condition. From (19), (22), and (23), one has

$$\begin{aligned}J(n)=&\mathbb{E}\sum_{k=0}^{n}\left[\bar{z}^T(k)\bar{z}(k)-\gamma^2 v^T(k)v(k)+\Delta V(\chi(k))\right]\\ &-\mathbb{E}V(\chi(n+1))\\ \le&\mathbb{E}\sum_{k=0}^{n}\Big\{\bar{x}^T(k)\bar{L}^T\bar{L}\bar{x}(k)-\gamma^2 v^T(k)v(k)\\ &+\xi_0^T(k)\Omega_2\xi_0(k)-\lambda_2\left[\bar{x}(k)\ \ f(x(k))\right]\\ &\times\begin{bmatrix}G^T\tilde{R}_1 G & G^T\tilde{R}_2\\ \tilde{R}_2^T G & I\end{bmatrix}\begin{bmatrix}\bar{x}(k)\\ f(x(k))\end{bmatrix}\Big\}\\ =&\xi_0^T(k)\Phi\xi_0(k).\end{aligned}$$

According to Theorem 2, we have $J(n)\le 0$. Letting $n\to\infty$, we obtain

$$\|\bar{z}(k)\|_2\le\gamma\|v(k)\|_2$$

which completes the proof of Theorem 2. ■

IV. Robust $\mathcal{H}_\infty$ Filter Design

In this section, we aim at solving the $\mathcal{H}_\infty$ filter design problem for the system (6), that is, we are interested in determining the filter matrices in (7) such that the filtering error system in (8) is exponentially stable with a guaranteed $\mathcal{H}_\infty$ performance. The following theorem provides sufficient conditions for the existence of such $\mathcal{H}_\infty$ filters for system (8).

Theorem 3: Let $\gamma>0$ be a given positive constant and the admissible conditions hold. Then, for the nonlinear system (6) with multiple communication delays and packet dropouts, there exists an admissible $\mathcal{H}_\infty$ filter of the form (7) such that the filtering error system (8) is robustly exponentially stable in the mean square for $v(k)=0$ and also satisfies $\|\bar{z}(k)\|_2\le\gamma\|v(k)\|_2$ under the zero initial condition for any nonzero $v(k)\in l_2[0,+\infty)$, if there exist positive definite matrices $P, Q_j>0$ $(j=1,2,\cdots q)$, positive constant scalars $\varepsilon,\lambda_1,\lambda_2$ and matrices X, C_f satisfying

$$\Lambda<0, \tag{24}$$

$$P\le\lambda_1 I \tag{25}$$

where

$$\Lambda=\begin{bmatrix}\Lambda_1 & * & * & * \\ 0 & -\gamma^2 I & * & * \\ \Lambda_2 & \Lambda_3 & \Lambda_4 & * \\ 0 & 0 & \Lambda_5 & -\varepsilon I\end{bmatrix},$$

$$\Lambda_1=\begin{bmatrix}\Lambda_{11} & * & * \\ 0 & \Lambda_{22} & * \\ -\lambda_2\tilde{R}_2^T G & 0 & -\lambda_2 I\end{bmatrix},$$

$$\Lambda_2=\begin{bmatrix}\bar{X} & 0 & 0\\ \hat{L}_0+C_f\hat{R}_3 & 0 & 0\\ P\hat{A}_0+X\hat{R}_1 & P\hat{Z} & P\bar{F}\end{bmatrix},$$

$$\Lambda_3=\left[0\quad 0\quad (P\hat{D}_0+X\hat{D}_1)^T\right]^T,$$

$$\Lambda_4=\mathrm{diag}\{-\bar{P},-I,-P\},\ \Lambda_5=\left[0\quad 0\quad H_0^T P\right],$$

$$\begin{aligned}\Lambda_{11}=&\lambda_1 A_\rho-P+\sum_{j=1}^{q}(d_M-d_m+1)Q_j-\lambda_2 G^T\tilde{R}_1 G\\ &+\varepsilon E_0^T E_0,\\ \Lambda_{22}=&\mathrm{diag}\{-Q_1+\tilde{A}_1,-Q_2+\tilde{A}_2,\cdots,-Q_q+\tilde{A}_q\}\\ &+\lambda_1\rho_2\hat{Z}_a^T\hat{Z}_a,\\ \bar{P}=&\mathrm{diag}\{\underbrace{P,\cdots,P}_{r}\},\ E_0=[E\quad 0],\end{aligned}$$

$$\bar{X}=\left[\sigma_1\hat{R}_{21}^T X^T\quad\cdots\quad\sigma_r\hat{R}_{2r}^T X^T\right]^T,\ \hat{L}_0=[L\quad 0],$$

$$\hat{R}_{2j}=\begin{bmatrix}0 & 0\\ C_j & 0\end{bmatrix},\ \hat{D}_1=\begin{bmatrix}0\\ D_2\end{bmatrix},\ \hat{A}_0=\begin{bmatrix}A & 0\\ 0 & 0\end{bmatrix},$$

$$\hat{R}_3=[0\quad -I],\ \hat{E}=[0\quad I]^T,\ \hat{D}_0=\left[D_1^T\quad 0\right]^T,$$

$$\hat{R}_1=\begin{bmatrix}0 & I\\ \bar{\Xi}C & 0\end{bmatrix},\ H_0=[H^T\quad 0]^T.$$

Furthermore, if $(P,Q_j,X,C_f,\varepsilon,\lambda_1,\lambda_2)$ is a feasible solution of (24) and (25), then the system matrices of the admissible $\mathcal{H}_\infty$ filter in the form of (7) can be obtained by means of the matrices X and C_f, where

$$[A_f\quad B_f]=[\hat{E}^T P\hat{E}]^{-1}\hat{E}^T X. \tag{26}$$

Proof: From Theorem 2, we know that there exists an admissible filter in the form of (7) such that the filtering error system (8) is robustly exponentially stable with a guaranteed $\mathcal{H}_\infty$ performance γ if there exist matrices $P>0$, $Q_j>0$ $(j=1,2,\cdots q)$, and positive constant scalars λ_1,λ_2 satisfying (20), (21). By the Schur complement, (20) is equivalent to

$$\begin{bmatrix}\lambda_1 A_\rho+\check{\Omega}_{11} & * & * & * & * & * & * \\ 0 & \Lambda_{22} & * & * & * & * & * \\ -\lambda_2\tilde{R}_2^T G & 0 & -\lambda_2 I & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ \bar{P}\hat{C} & 0 & 0 & 0 & -\bar{P} & * & * \\ \bar{L} & 0 & 0 & 0 & 0 & -I & * \\ P\bar{A} & P\hat{Z} & P\bar{F} & P\bar{D}_1 & 0 & 0 & -P\end{bmatrix}<0 \tag{27}$$

where

$$\check{\Omega}_{11} = -P + \sum_{j=1}^{q}(d_M - d_m + 1)Q_j - \lambda_2 G^T \tilde{R}_1 G,$$
$$\hat{C} = \begin{bmatrix}\sigma_1 \bar{C}_1^T & \sigma_2 \bar{C}_2^T & \cdots \sigma_r \bar{C}_r^T\end{bmatrix}^T.$$

In order to avoid partitioning the positive definite matrices P and Q_j, we rewrite the parameters in Theorem 2 in the following form:

$$\bar{A} = \hat{A}_0 + H_0 F(k) E_0 + \hat{E} K \hat{R}_1, \bar{C}_j = \hat{E} K \hat{R}_{2j},$$
$$\bar{L} = \hat{L}_0 + C_f \hat{R}_3,\ \bar{D}_1 = \hat{D}_0 + \hat{E} K \hat{D}_1,$$
$$K = [A_f \quad B_f],\ X = P\hat{E}K,$$

and therefore we can get (26). Then, from Lemma 2, we can obtain (24). This completes the proof of this theorem. ■

Remark 4: In Theorem 3, the robust H_∞ filtering problem is solved for a class of discrete-time nonlinear networked systems with multiple stochastic communication delays and multiple packet dropouts by using an LMI approach. Obviously, our main results can be easily specialized to many special cases, for example, the cases when there are no nonlinearities, or no stochastic disturbances, or no parameter uncertainties, etc. These specialized results are not listed here to keep the exposition concise. It is also worth pointing out that the main results in this paper can be easily extended to the delayed jumping systems with sensor nonlinearities [24] and other more complicated systems. Note that we mainly focus on the effects brought by multiple stochastic communication delays and packet dropouts, which are two of the most important network-induced characteristics.

Remark 5: Lemma 2 is used to tackle the norm-bounded parameter uncertainties in the proof of Theorem 3. Comparing to existing literature, the system we consider is more comprehensive since the random delays, partial measurement missing, sector nonlinearities, parameter uncertainties and stochastic disturbances are simultaneously taken into account. For deterministic time-delay system, a lot of research attention has been paid on the selection of Lyapunov functionals to reduce the conservatism; see, e.g., [11]. Similarly, for the discrete-time stochastic system considered in this paper, we could further reduce the conservatism of the main results by paying an effort towards the construction of more general Lyapunov functionals (e.g., the one used in [16]), which leaves a relatively minor research issue for further investigation.

Remark 6: Our main results are based on the LMI conditions. The LMI Control Toolbox implements state-of-the-art interior-point LMI solvers. While these solvers are significantly faster than classical convex optimization algorithms, it should be kept in mind that the complexity of LMI computations remains higher than that of solving, say, a Riccati equation. For instance, problems with a thousand design variables typically take over an hour on today's workstations. However, research on LMI optimization is a very active area in the applied math, optimization and the operations research community, and substantial speed-ups can be expected in the future.

V. An Illustrative Example

In this section, we present an illustrative example to demonstrate the effectiveness of the developed method. The system data of (2) are given as follows:

$$A = \begin{bmatrix}0.2 & 0 & 0.1\\ 0.1 & -0.3 & 0.1\\ 0.1 & 0 & -0.2\end{bmatrix},\ H = \begin{bmatrix}0.1\\ 0.2\\ 0.1\end{bmatrix},$$
$$F(k) = \sin(0.6k),\ E = [0.1 \quad 0.1 \quad 0.1],$$
$$A_d = \begin{bmatrix}0.2 & 0 & 0.1\\ 0.1 & -0.3 & 0.1\\ 0.1 & 0 & -0.2\end{bmatrix},\ F = \begin{bmatrix}0.2 & 0 & 0.1\\ 0.1 & 0.3 & 0.1\\ 0.1 & 0 & 0.2\end{bmatrix},$$
$$D_1 = \begin{bmatrix}-0.2 & 0 & 0.1\\ -0.1 & 0.1 & 0.1\\ 0 & 0.2 & 0.1\end{bmatrix},\ C = \begin{bmatrix}1 & 0.8 & 0.7\\ -0.6 & 0.9 & 0.6\\ 0.2 & 0.1 & 0.1\end{bmatrix},$$
$$D_2 = \begin{bmatrix}0.9 & -0.6 & 0.1\\ 0.5 & 0.8 & 0.1\\ 0.2 & 0.3 & 0.1\end{bmatrix},\quad L = [-0.1 \quad 0 \quad 0.1].$$

Let $\gamma = 0.9$, $f(x(k)) = 0.4\sin(x(k))$ and $g(x(k), \tilde{x}(k), k) = 0.5x(k) + 0.5\tilde{x}(k)$. Assume that the time-varying communication delays satisfy $2 \le \tau_i(k) \le 6$ $(i = 1, 2)$ and

$$\bar{\alpha}_1 = \mathbb{E}\{\alpha_1(k)\} = 0.8, \quad \bar{\alpha}_2 = \mathbb{E}\{\alpha_2(k)\} = 0.6.$$

Suppose that the probabilistic density functions of β_1, β_2 and β_3 in [0 1] are described by

$$q_1(s_1) = \begin{cases}0 s_1 = 0\\ 0.1 s_1 = 0.5\\ 0.9 s_1 = 1\end{cases}, \quad q_2(s_2) = \begin{cases}0.1 s_2 = 0\\ 0.1 s_2 = 0.5\\ 0.8 s_2 = 1\end{cases},$$
$$q_3(s_3) = \begin{cases}0 s_3 = 0\\ 0.2 s_3 = 0.5\\ 0.8 s_3 = 1\end{cases}.$$

from which the expectations and variances can be easily calculated as $\mu_1 = 0.95, \mu_2 = 0.85, \mu_3 = 0.9, \sigma_1 = 0.15, \sigma_2 = 0.32$ and $\sigma_3 = 0.2$. The initial condition is set to be $x_0 = [1\ 0\ -1]^T$, $\hat{x}_0 = [0\ 0\ 0]^T$ and the external disturbance v_k is described by

$$v_k = \begin{cases}0.1, & 20 \le k \le 50\\ -0.1, & 70 \le k \le 100\\ 0, & \text{else.}\end{cases}$$

We would like to design a filter in the form of (7) so that the filtering error system in (8) is exponentially stable with a guaranteed H_∞ norm bound γ. By applying Theorem 3 with help from Matlab LMI toolbox, we can obtain the desired H_∞ filter parameters as follows (other matrices are omitted for space saving):

$$A_f = \begin{bmatrix}0.3170 & 0.2021 & 0.1123\\ 0.3169 & 0.3169 & 0.3169\\ 0.3170 & 0.1106 & 0.3170\end{bmatrix}$$
$$B_f = \begin{bmatrix}-0.0079 & -0.0407 & 0.0944\\ -0.0080 & -0.0408 & 0.0948\\ -0.0079 & -0.0406 & 0.0942\end{bmatrix}$$
$$C_f = [0.3280 \quad 0.1220 \quad 0.4231].$$

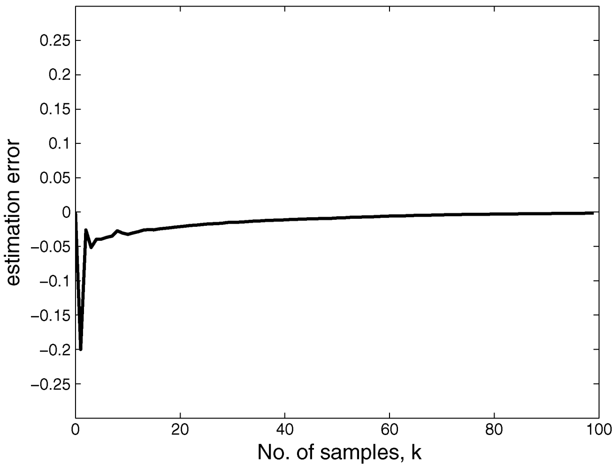

Fig. 1. Estimation error $z(k) - \hat{z}(k)$.

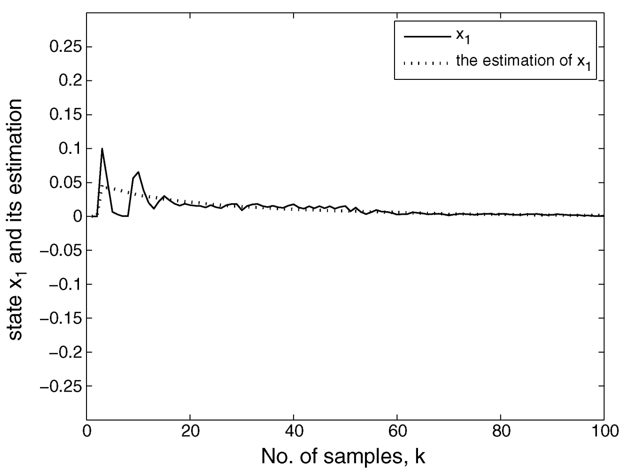

Fig. 2. The state $x_1(k)$ and its estimate $\hat{x}_1(k)$.

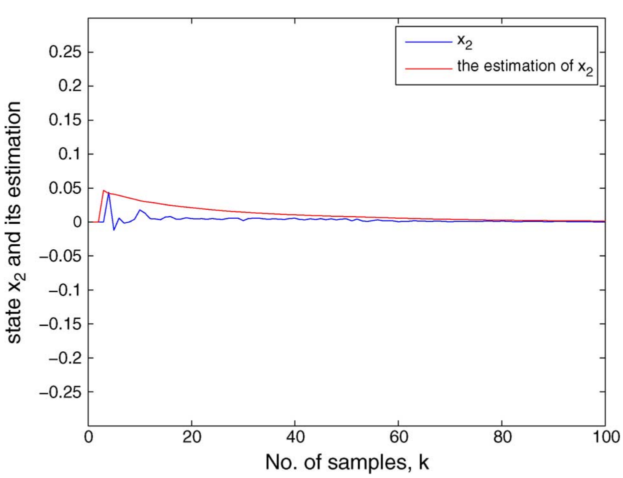

Fig. 3. The state $x_2(k)$ and its estimate $\hat{x}_2(k)$.

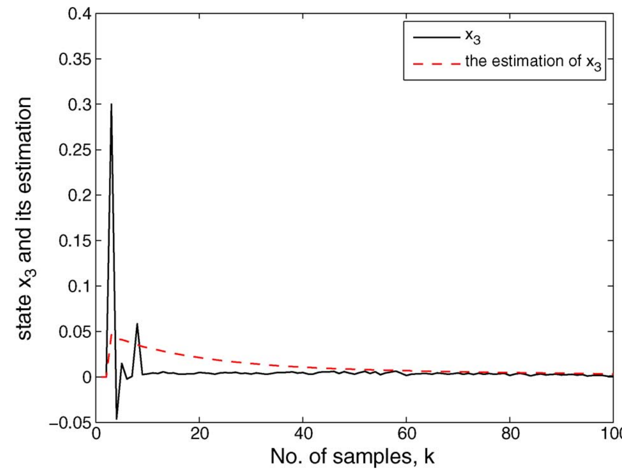

Fig. 4. The state $x_3(k)$ and its estimate $\hat{x}_3(k)$.

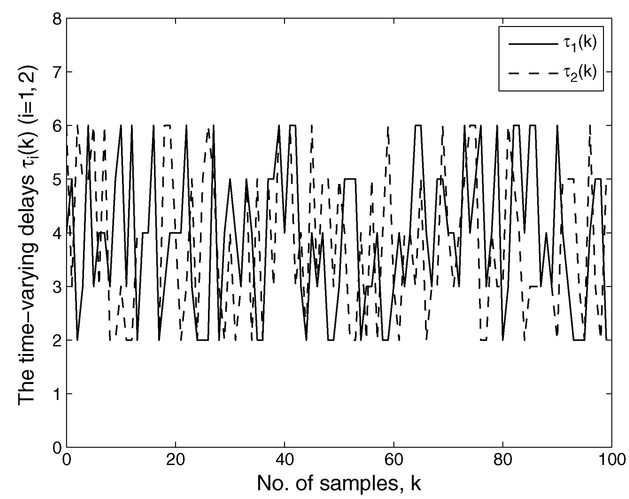

Fig. 5. The time-varying delays $\tau_i(k)$ $(i = 1, 2)$.

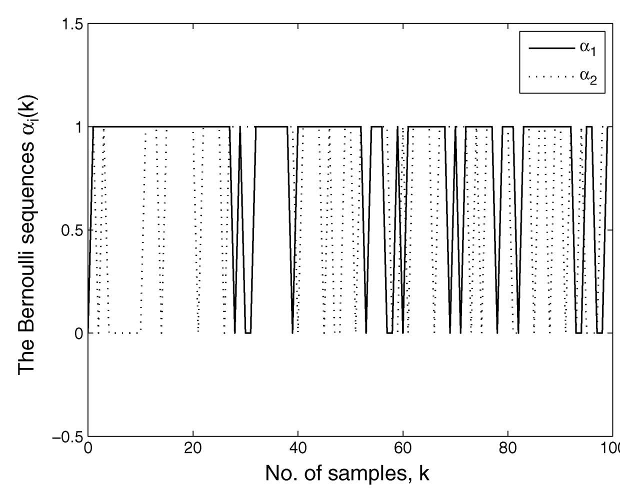

Fig. 6. The Bernoulli sequences $\alpha_i(k)$ $(i = 1, 2)$.

The simulation results are shown in Figs. 1–6. Fig. 1 plots the estimation error $\bar{z}(k)$. The actual state response $x_i(k)$ and the estimate $\hat{x}_i(k)$ $(i = 1, 2, 3)$ are depicted in Figs. 2–4, respectively. Fig. 5 shows the time-varying delays $\tau_i(k)$ $(i = 1, 2)$. The Bernoulli sequences $\alpha_i(k)$ $(i = 1, 2)$ are drawn in Fig. 6.

All the simulation have confirmed our theoretical analysis for the problems of robust H_∞ filtering for discrete nonlinear networked systems with multiple time-varying random communication delays and multiple packet dropouts.

VI. Conclusion

In this paper, we have studied the robust H_∞ filtering problem for nonlinear networked systems with multiple time-varying random communication delays and multiple packet dropouts. The discrete-time system under study involves parameter uncertainties, state-dependent stochastic disturbances (multiplicative noises or Itô-type noises), multiple stochastic time-varying delays, sector-bounded nonlinearities and multiple packet dropouts. By means of LMIs, sufficient conditions for the robustly exponential stability of the filtering error dynamics have been obtained and, at the same time, the prescribed H_∞ disturbance rejection attenuation level has been guaranteed. Then, the explicit expression of the desired filter parameters has been derived. A numerical example has been provided to show the usefulness and effectiveness of the proposed design method.

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