The Optimal Passive Filters to Minimize Voltage Harmonic Distortion at a Load Bus

Ahmed Faheem Zobaa, *Senior Member, IEEE*

***Abstract*—A method is presented for finding the optimum fixed inductance–capacitance combination to minimize voltage harmonic distortion at a load bus where it is desired to maintain the displacement power factor at a desired level constraining the compensator values, which would create resonance conditions, and the manufacturers' standard values for power shunt capacitors according to IEEE Standard 18-2002. A comparison study of using the constraint, holding either the displacement power factor or the power factor at a desired value, is done. Finally, the contribution of the newly developed method is demonstrated in six examples taken from existing publications.**

***Index Terms*—Harmonics, power factor, reactive power optimization.**

NOMENCLATURE

R_{LK}	Load resistance and reactance at harmonic number.
X_{LK}	K (Ω).
G_{LK}	Load conductance and susceptance at harmonic.
B_{LK}	Number K (Ω).
R_{TK}	Transmission system resistance and reactance at.
X_{TK}	Harmonic number K (Ω).
X_L	Fundamental inductive and capacitive reactance of.
X_C	Compensator (Ω).
R	Resistance of the compensator reactor (Ω).
I_{SK}	Supply current at harmonic number K (A).
I_S	Rms value of supply current (A).
I_{lK}	Load current at harmonic number K (A).
I_{LK}	Load harmonic current (A).
I_{CK}	Capacitor current at harmonic number K (A).
P_L	load power (W)
P_S	supply power (W)
V_{LK}	Load voltage at harmonic number K (V).
V_{SK}	Supply voltage at harmonic number K (V).
V_L	Rms value of load voltage (V).
I_N	Rms value of neutral current (A).

I. INTRODUCTION

HARMONIC distortion complicates the computation of power and power factor (PF) because many of the simplification power engineers' use for power frequency analysis do not apply. The fundamental frequency current component related to the reactive power is useful for helping engineers' size capacitors for PF correction. The term displacement power factor (dPF) is used to describe the PF using the fundamental frequency components only. Power quality monitoring instruments now commonly report this quantity, as well as the truePF.

There are several measures commonly used for indicating the harmonic content of a waveform with a single number. One of the most common is total harmonic distortion (THD), which can be calculated for either voltage THD (VTHD) or current THD (ITHD). THD is a measure of the effective value of the harmonic components of a distorted waveform, that is, the potential heating of the harmonics relative to the fundamental.

In case of nonsinusoidal sources, maximum transmission efficiency (η), minimum transmission loss (TL), and maximum PF do not lead to the same shunt LC compensator values [1]. When the three criteria are combined into one model and solved by the Penalty Function method [2], the results do satisfy the three criteria by one value of LC compensator [3]. In other attempts at optimizing the LC compensators [4], the main objective has been to maximize the load PF with minimum TL. This may also reduce the total harmonic distortion of voltage and current, but it may not minimize them. In other words, maximizing PF does not solve the problem of minimizing voltage harmonic distortion.

This paper presents a method for minimizing the VTHD at the load bus where it is desired to maintain the dPF at a desired level by using Penalty Function method as a tool of optimization. An optimum fixed LC compensator will be selected that will minimize the expected value of VTHD for a specified range of source harmonic and impedance values, while constraining the compensator values which would create resonant conditions. One problem to be addressed is whether the values obtained from theoretical optimization solution can be obtained from standard manufactured values. Depending on the voltage, manufacturers have discrete capacitors available. In the presented method, manufacturers standard values for shunt capacitor are taken into consideration. The standard values are considered as constraint in the sense that chosen capacitor should be one of these values. It is assumed that the load harmonics are not sufficiently serious to suggest tuned filters, but when combined with source harmonics, the use of a pure capacitive compensator would degrade PF [5] and overload the equipment. The major attribute of this method is that it, unlike conventional approaches, guarantees convergence to the optimal solution.

II. DESCRIPTION OF THE SYSTEM UNDER STUDY

At the fundamental frequency, power systems are primarily inductive, and the equivalent impedance is sometimes called

Manuscript received July 28, 2003; revised January 21, 2004. Paper no. TPWRD-00393-2003.

The author is with Electrical Power and Machines Department, Faculty of Engineering, Cairo University, Giza 12613, Egypt (e-mail: a.zobaa@eng.cu.edu.eg).

Digital Object Identifier 10.1109/TPWRD.2004.838458

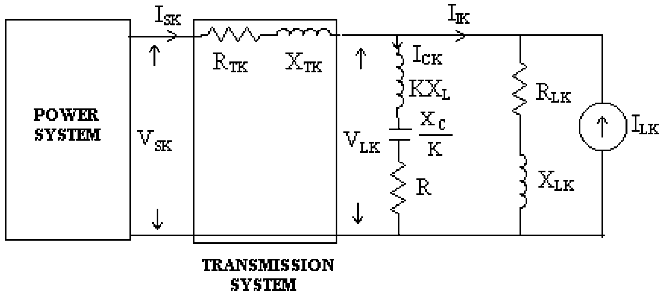


Fig. 1. Single-phase equivalent circuit for Kth harmonic with shunt LC compensator.

simply the short-circuit impedance. Capacitive effects are frequently neglected on utility distribution systems and industrial power systems. One of most used frequently quantities in the analysis of harmonics on power systems is the short-circuit impedance to the point on a network at which a compensator is located [6]. If not directly available, it can be computed from short-circuit study results that gives either the short-circuit MVA or the short-circuit current.

Fig. 1 is a single-phase equivalent circuit of a bus with LC compensator, experiencing VTHD at harmonic order K because of a voltage source V_{SK} and harmonic current sources within the load itself, I_{LK}.

Thevenin voltage source representing the utility supply and the harmonic current source representing the nonlinear load are

$$v_S(t) = \sum_K v_{SK}(t) \tag{1}$$

and

$$i_L(t) = \sum_K i_{LK}(t) \tag{2}$$

where K is the order of harmonic present. The Kth harmonic Thevenin source and load impedances are

$$Z_{TK} = R_{TK} + jX_{TK} \tag{3}$$

and

$$Z_{LK} = R_{LK} + jX_{LK}. \tag{4}$$

Finally, this model (Fig. 1) is adequate where VTHD is less than 10% [6].

III. Formulation of the Problem

Formulating an optimization problem requires the knowledge of the objective function to be optimized, the constraints imposed on the variables and/or the objective function, and the solution method from which the solution is determined.

A. Formulation of the Objective Function

The VTHD at the compensated load terminals is defined as

$$VTHD = \frac{\sqrt{\sum_{K>1} V_{LK}^2}}{V_{L1}} \tag{5}$$

and

$$V_{LK} = \frac{V_{SK}(CR) - I_{LK}(DR * ER)}{A_{IK} + jA_{JK}} \tag{6}$$

where

$$\begin{aligned}
AR &= R + R_{LK} \\
BR &= \left(X_{LK} + KX_L - \frac{X_C}{K}\right) \\
CR &= R_{CLK} + jX_{CLK} \\
DR &= R + j\left(KX_L - \frac{X_C}{K}\right) \\
ER &= R_{TLK} + jX_{TLK} \\
A_{IK} &= R_{TLK} + R(R_{LK} + R_{TK}) \\
&\quad - (X_{LK} + X_{TK})\left(KX_L - \frac{X_C}{K}\right) \\
A_{JK} &= X_{TLK} + R(R_{LK} + X_{TK}) \\
&\quad + (R_{LK} + R_{TK})\left(KX_L - \frac{X_C}{K}\right).
\end{aligned}$$

$$Z_{CLK} = \frac{(R_{CLK} + jX_{CLK})}{(Z_{LK} + Z_{CK})} \tag{7}$$

where

$$\begin{aligned}
R_{CLK} &= RR_{LK} - X_{LK}\left(KX_L - \frac{X_C}{K}\right) \\
X_{CLK} &= RX_{LK} + R_{LK}\left(KX_L - \frac{X_C}{K}\right).
\end{aligned}$$

$$Z_{TLK} = \frac{(R_{TLK} + jX_{TLK})}{(Z_{TK} + Z_{LK})} \tag{8}$$

where

$$\begin{aligned}
R_{TLK} &= R_{TK}R_{LK} - X_{TK}X_{LK} \\
X_{TLK} &= R_{TK}X_{LK} + R_{LK}X_{TK}.
\end{aligned}$$

B. Formulation of the Main Constraint

The compensated dPF at the load is given as

$$dPF = \frac{P_{L1}}{V_{L1}I_{S1}} = \frac{G_{L1}V_{L1}}{I_{S1}} \tag{9}$$

The compensated PF at the load is given as

$$PF = \frac{P_L}{V_L I_S} = \frac{\sum_K G_{LK}V_{LK}^2}{\sqrt{\sum_K I_{SK}^2 \sum_K V_{LK}^2}} \tag{10}$$

where

$$I_{SK} = \frac{V_{SK}(AR + jBR) + I_{LK}CR}{A_{IK} + jA_{JK}} \tag{11}$$

$$Y_{LK} = G_{LK} - jB_{LK}. \tag{12}$$

C. Performance Indications

The TL is given as

$$\mathrm{TL} = \sum_{\mathrm{K}} \mathrm{I}_{\mathrm{SK}}^2 \mathrm{R}_{\mathrm{TK}}. \tag{13}$$

The η is given as

$$\eta = \frac{\mathrm{P_L}}{\mathrm{P_S}} = \frac{\sum_{\mathrm{K}} \mathrm{G_{LK}} \mathrm{V}_{\mathrm{LK}}^2}{\sum_{\mathrm{K}} \mathrm{I}_{\mathrm{SK}}^2 \mathrm{R_{TK}} + \sum_{\mathrm{K}} \mathrm{G_{LK}} \mathrm{V}_{\mathrm{LK}}^2}. \tag{14}$$

In some countries, utilities impose a penalty in the form of higher charges for a customer whose PF is less 90% [7]. For dPF improvement calculation, the higher limit of 95% is used [8]. After formulating the objective function and the constraints, the problem becomes

$$\begin{aligned} \text{Minimize} \quad & \mathrm{VTHD}(\mathrm{X_C}, \mathrm{X_L}) \\ \text{Subject to:} \quad & \mathrm{dPF}(\mathrm{X_C}, \mathrm{X_L}) \geq 95\% \end{aligned} \tag{15}$$

or

$$\begin{aligned} \text{Minimize} \quad & \mathrm{VTHD}(\mathrm{X_C}, \mathrm{X_L}) \\ \text{Subject to:} \quad & \mathrm{PF}(\mathrm{X_C}, \mathrm{X_L}) \geq 90\% \end{aligned}$$

The additional constraints involved are the effect of supply frequency on the ac resistance, the compensator values, which would create resonant conditions, and the manufacturer's standard values for power shunt capacitors.

IV. Additional Constraints Involved

A. Effect of Supply Frequency on the AC Resistance

In most power systems, one can generally assume that the resistance does not change significantly when studying the effects of harmonics less than the ninth [6]. In this study, it is assumed that

$$\begin{aligned} \mathrm{R_{TK}} &= \mathrm{R_T} \\ \mathrm{R_{LK}} &= \mathrm{R_L} \end{aligned} \tag{16}$$

where

- $\mathrm{R_T}$ resistance of the transmission system at the fundamental frequency;
- $\mathrm{R_L}$ resistance of the load at the fundamental frequency.

B. Resonance Constraint

The expected impedance seen from the Thevenin source is given by

$$\mathrm{Z} = \mathrm{Z_{TK}} + \mathrm{Z_{CLK}}. \tag{17}$$

The resonance peaks can be obtained by setting the imaginary part of (17) to zero, resulting in a quadratic equation in X_C and X_L for any given harmonic order K

$$\mathrm{K_1}\left(\mathrm{KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right)^2 + \mathrm{K_2}\left(\mathrm{KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right) + \mathrm{K_3} = 0 \tag{18}$$

where

$$\begin{aligned} \mathrm{K_1} &= \mathrm{X_{TK}} + \mathrm{X_{LK}} \\ \mathrm{K_2} &= \mathrm{R}_{\mathrm{LK}}^2 + \mathrm{X}_{\mathrm{LK}}^2 + 2\mathrm{X_{LK}}\mathrm{X_{TK}} \\ \mathrm{K_3} &= \mathrm{R}^2\mathrm{X_{LK}} + \mathrm{X_{TK}}\left[(\mathrm{R} + \mathrm{R_{LK}})^2 + \mathrm{X}_{\mathrm{LK}}^2\right]. \end{aligned}$$

The pre-calculated compensator values for series resonance, by taking the solution of (18) where the square root of the discriminant is positive, are used to subdivide the entire search region into smaller regions. In each region, total minimums are identified, leading to the eventual identification of the global minimum of any of the functions. This makes the optimal *LC* compensator value is not a part of (18).

C. Standard Ratings of the Capacitors

IEEE Standard 18-2002 [9] shows the voltage and reactive power ratings of the capacitors. Each value of the reactive power ratings of the particular voltage is used to calculate the corresponding value of X_{Ci}. This value is then substituted into (16) to become one variable equation in X_L.

Then, the problem becomes

$$\begin{aligned} \text{Minimize} \quad & \mathrm{VTHD}(\mathrm{X_{Ci}}, \mathrm{X_L}) \\ \text{Subject to:} \quad & \mathrm{dPF}(\mathrm{X_{Ci}}, \mathrm{X_L}) \geq 95\% \\ & \mathrm{X_{Ci}} \text{ and } \mathrm{X_L} \text{ are not a part of (18)} \end{aligned} \tag{19}$$

or

$$\begin{aligned} \text{Minimize} \quad & \mathrm{VTHD}(\mathrm{X_{Ci}}, \mathrm{X_L}) \\ \text{Subject to:} \quad & \mathrm{PF}(\mathrm{X_{Ci}}, \mathrm{X_L}) \geq 90\% \\ & \mathrm{X_{Ci}} \text{ and } \mathrm{X_L} \text{ are not a part of (18).} \end{aligned}$$

The objective function and constraints are complicated to be solved directly. Hence, an iterative method is needed to generate the solution. From experience, the Penalty Function method is chosen due to that it requires fewer steps and function evaluations [10], [11].

V. Basic Approach of the Penalty Function Method

Penalty Function methods transform the basic optimization problem into alternative formulation such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. The search algorithm [2] is described in Appendix. In the optimization process, the resistance of the compensator reactor has been neglected due to its small value with respect to its fundamental reactance [12].

VI. Simulated Results and Its Discussion

Four cases of an industrial plant (Table I) were simulated using the optimization method. The numerical data were primarily taken from an example in [8], where the inductive three-phase load is 5100 kW with a dPF of 71.65%. The 60-cycle supply bus voltage is 4.16 kV.

Also, we will study another two cases, 5 and 6, by changing the $\% V_{S5}$ and $\% V_{S7}$ of cases 2 and 4 to 7% and 4%.

Harmonic contents of the supply voltage, and the load current are arbitrary selected in all cases. The advantages of the presented method are explicitly demonstrated in the following tables and figures.

TABLE I
FOUR CASES OF AN INDUSTRIAL PLANT UNDER STUDY

PARAMETERS & HARMONICS	CASE 1	CASE 2	CASE 3	CASE 4
Short Circuit MVA	150	150	80	80
$R_{T1}(\Omega)$	0.01154	0.01154	0.02163	0.02163
$X_{T1}(\Omega)$	0.1154	0.1154	0.2163	0.2163
$R_{L1}(\Omega)$	1.742	1.742	1.742	1.742
$X_{L1}(\Omega)$	1.696	1.696	1.696	1.696
V_{S1}(kV)	2.4	2.4	2.4	2.4
$V_{S5}(\%V_{S1})$	5	1	5	1
$V_{S7}(\%V_{S1})$	3	7	3	7
$V_{S11}(\%V_{S1})$	2	2	2	2
$V_{S13}(\%V_{S1})$	1	1	1	1
$I_{L5}(\% I_{S1})$	5	5	5	5
$I_{L7}(\% I_{S1})$	3	3	3	3
$I_{L11}(\% I_{S1})$	2	2	2	2
$I_{L13}(\% I_{S1})$	1	1	1	1

TABLE II
SIMULATED RESULTS WHERE PF IS TAKEN AS A CONSTRAINT

CASE	X_C^* (Ω)	X_L^* (Ω)	PF (%)	I_S(A)	η(%)	TL (kW)
1	6.78	0.3085	89.12	777.58	99.57	6.98
2	6.78	0.3085	90.38	766.28	99.59	6.78
3	6.78	0.2948	90.26	755.45	99.22	12.34
4	6.78	0.2948	90.71	751.42	99.23	12.21
5	6.40	0.3038	88.67	782.02	99.57	7.06
6	6.78	0.2948	89.45	762.19	99.21	12.57

TABLE III
SIMULATED RESULTS WHERE dPF IS TAKEN AS A CONSTRAINT

CASE	X_C^* (Ω)	X_L^* (Ω)	PF (%)	I_S (A)	η (%)	TL (kW)
1	5.49	0.2925	93.53	743.81	99.61	6.38
2	5.49	0.2925	94.34	737.24	99.62	6.27
3	5.49	0.2788	94.35	728.12	99.29	11.47
4	5.49	0.2788	94.67	725.45	99.29	11.38
5	5.49	0.2925	92.09	755.37	99.60	6.58
6	5.49	0.2788	93.64	733.60	99.28	11.64

Comparison of the results, shown Tables II and III, show that a lower short-circuit capacity corresponds to a higher PF at the same conditions. This has to be expected since, with higher transmission impedance, less harmonic current will flow into the compensated load. Also, it shows that an additional harmonic content results in lower PF. This is caused by the increase in compensated line current due to the additional harmonics.

Comparison of the results, shown in Figs. 2 and 3 and Tables II–IV, shows that the general performance of the proposed method where dPF is taken as a constraint is satisfactory, providing improvement of the overall performance compared with the method where PF is taken as a constraint.

Comparison of the results, shown in Tables V and VI, show that the dPF-constrained method is satisfactory, providing higher dPF and lower ITHD compared with the PF-constrained method. Also, the resultant values of VTHD all come out well below standard limits [8].

From the above-simulated results, LC compensators can achieve a higher best performance than pure capacitive compensation [5], [13] for nonlinear loads when source harmonics are present.

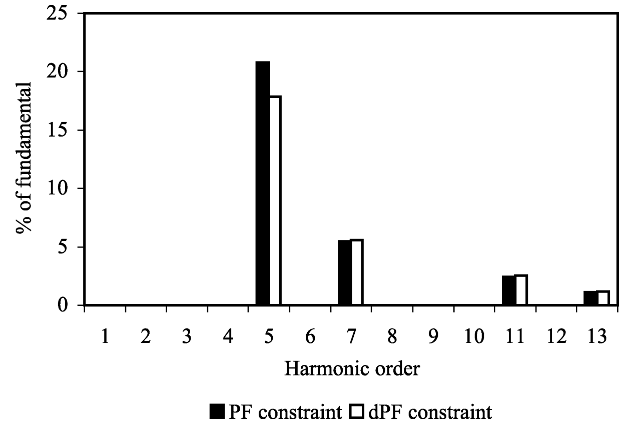


Fig. 2. Harmonic contents of the supply current after compensation for case 1.

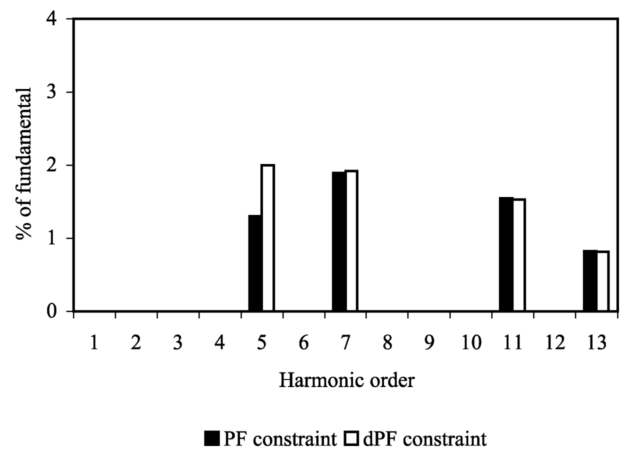


Fig. 3. Harmonic contents of the load voltage after compensation for case 1.

TABLE IV
FUNDAMENTAL COMPONENTS OF THE LOAD VOLTAGE AND THE SUPPLY CURRENT AFTER COMPENSATION FOR CASE 1

CASE	*PF* constraint	*dPF* constraint
Fundamental load voltage (V)	2354.82	2364.95
Fundamental supply current (A)	760.70	731.81

TABLE V
HARMONIC DISTORTIONS AND DISPLACEMENT FACTOR AFTER COMPENSATION WHERE PF IS TAKEN AS A CONSTRAINT

CASE	ITHD (%)	VTHD (%)	dPF (%)
1	21.69	2.83	91.23
2	12.99	4.46	91.23
3	14.18	2.37	91.19
4	9.71	3.46	91.19
5	28.65	3.62	92.30
6	19.61	2.68	91.19

TABLE VI
HARMONIC DISTORTIONS AND DISPLACEMENT FACTOR AFTER COMPENSATION WHEREdPF IS TAKEN AS A CONSTRAINT

CASE	ITHD (%)	VTHD (%)	dPF (%)
1	18.94	3.22	95.24
2	13.10	4.56	95.24
3	13.12	2.62	95.19
4	9.86	3.56	95.19
5	26.05	4.04	95.24
6	18.04	3.07	95.19

TABLE VII
THE NOTCH FREQUENCY OF THE OPTIMAL COMPENSATORS

CASE	1	2	3	4	5	6
PF constraint	4.69	4.69	4.69	4.69	4.69	4.69
dPF constraint	4.33	4.33	4.33	4.33	4.33	4.33

TABLE VIII
CAPACITOR CAPABILITIES FOR CASE 1

Harmonic No.	Frequency (Hz)	*PF* constraint		*dPF* constraint	
		Voltage (%)	Current (%)	Voltage (%)	Current (%)
1	60	102.72	102.87	104.01	104.17
5	300	8.78	43.95	5.71	28.58
7	420	1.26	8.87	0.99	6.92
11	660	0.24	2.65	0.20	2.18
13	780	0.08	1.01	0.06	0.83

TABLE IX
CAPACITOR BANK LIMITS FOR CASE 1

Item	*PF* constraint	*dPF* constraint	Limit [9]
% Peak voltage	113.08	110.97	120.00
% rms voltage	103.10	104.17	110.00
% rms current	112.25	108.26	135.00
% kvar	115.74	112.78	135.00

One important side effect of adding a tuning inductor is that it creates a sharp parallel resonance point at a frequency below the notch frequency [6]

$$\mathrm{h}_{\mathrm{notch}} = \sqrt{\frac{\mathrm{X_C}}{\mathrm{X_L}}}. \tag{20}$$

This resonant frequency must be safely away from any significant harmonic because of changes in either capacitance or inductance with temperature, or failure might shift the parallel resonance higher into the harmonic. This could present a worse situation because the resonance is very sharp. Also, filters are added to the system starting with the lowest problem harmonic [6].

Table VII shows that notch frequencies are safely away from any significant harmonic, and near the fifth.

Table VIII summarizes capacitor evaluation for case 1 that is designed to help evaluate the various capacitor duties against the standards.

Table IX shows that the resultant values all come out well below standard limits, and the method where dPF is taken as constraint is satisfactory providing lower limitations compared with the method where PF is taken as a constraint.

Finally, any current flowing within an impedance generates voltage at the impedance terminal. This fact also applies to harmonic currents flowing through the electrical system. Therefore, the higher the harmonic current levels, the greater the resulting harmonic voltages, thus creating distortion in the electrical system voltage. As transformers also have impedance, voltage distortion appears at the transformer's secondary terminals when harmonic currents flow through them. Thus, to reduce voltage distortion, two factors can be modified: the level of harmonic currents and transformer impedance. Using phase-shifting techniques may reduce the level of harmonic currents, and low impedance plays a crucial role in reducing voltage distortion. Now, low-impedance phase-shifting transformers [14] have thus been designed. They allow the treatment of harmonic currents while providing a path of low impedance. The quality and reliability of the electrical system can thus be considerably improved through the use of a single piece of equipment.

VII. CONCLUSIONS

A novel method is developed to minimize the voltage harmonic distortion at a load bus while holding displacement power factor constant using an LC compensator. Such compensators have dual purposes. The first is that it acts as a compensator to improve the power factor of the nonlinear loads. The second is that it acts as a filter of the harmonic load currents, thus preventing the proliferation of the network with these currents. The problem is formulated as an optimization problem and solved with a Penalty Function method.

Whether or not the solution generated by the presented method is indeed optimal depends on the knowledge of the system configuration, operating condition and harmonic voltage distortion. Nevertheless, the method represents a useful tool for providing an optimal solution under a given situation.

Finally, the presented method includes the improvement in the accuracy of the solution and in the ability of it to guarantee convergence to the optimal solutions as illustrated by the examples taken from existing publications. Six cases are tested, and the general performance of the proposed method is satisfactory, providing improvement of distortion levels, and power factor correction, compared with a method used to minimize the voltage harmonic distortion at a load bus while holding power factor constant. Using this method, the global optimal solutions, as well as the local optimums, are determined. These additional information can be useful for performing a cost-benefit decision analysis, paper under preparation, before implementing the optimal LC compensator.

Ongoing research efforts consist of the modification and application of this method to take into account load profiles, other constraints (for example, the one based on the LC compensator cost and/or the allowable overloads for capacitor imposed by standard or recommendations), and the probability density function of several indices that are well known in recent international standards; for example, IEC 1000-3-6.

APPENDIX
PENALTY FUNCTION METHOD ALGORITHM

Let the basic optimization problem, with inequality constraints, be of the form (19). This problem is converted into an unconstrained minimization problem by constructing a function of the form

$$\mathrm{f(X_L)} = \mathrm{VTHD(X_C, X_L)} + \sum_{\mathrm{m}} \mu_{\mathrm{m}} \left(max\left[0, \mathrm{g_m(X_C, X_L)}\right]\right) \tag{A1}$$

where g_m is some function of the constraint (i.e. dPF or PF), μ_m is a positive constant known as the penalty parameter, and $max[0, \mathrm{g_m}(X_C, X_L)]$ is commonly used form of the penalty parameter, which is the second part of (A1).

Step 1) Start with an initial feasible point X_C^0, X_L^0 satisfying all the constraints with strict inequality sign. Start with an initial value of $\mu_1 > 0$. Set $\mathrm{J} = 1$.

Step 2) Minimize $\mathrm{f}(X_L)$ by using any of the unconstrained minimization methods and obtain the solution X_C^*, X_L^*. The Golden Section search method [2] can be applied for obtaining the optimal X_L^*.

Step 3) Test whether the solution X_C^*, X_L^* is the optimum solution of the original problem. The algorithm will stop when a feasible point will be reached or when the relative change in the objective function is small

$$\varepsilon < 10^{-6}. \tag{A2}$$

If X_C^*, X_L^* is found to be optimum, terminate the process. Otherwise, go to the next step.

Step 4) Find the value of the next penalty parameter as

$$\mu_{\mathrm{m}}^{(\mathrm{J}+1)} = \beta \mu_{\mathrm{m}}^{(\mathrm{J})} \tag{A3}$$

where $\beta < 1$.

Step 5) Set the new value of $\mathrm{J} = \mathrm{J}+1$, take the new starting point as X_C^*, X_L^*, and go to step 2.

REFERENCES

[1] M. M. Abdel-Aziz, E. E. Abou El-Zahab, A. M. Ibrahim, and A. F. Zobaa, "LC compensators for power factor correction of nonlinear loads," *IEEE Trans. Power Delivery*, vol. 19, no. 1, pp. 331–336, Jan. 2004.

[2] S. S. Rao, *Engineering Optimization Theory and Practice*, 3rd ed. New York: Wiley-Interscience, 1996.

[3] A. F. Zobaa and E. M. Tag El-Din, "Power factor optimization based on manufacturers standards of the capacitors for nonlinear loads," in *Proc. 2003 Large Engineering Systems Conf. Power Engineering (LESCOPE'03)*, Montreal, QC, Canada, May 2003, pp. 138–142.

[4] M. M. Abdel Aziz, E. E. Abou El-Zahab, and A. F. Zobaa, "*LC* compensators based on cost minimization for nonlinear loads," in *Proc. 2003 Large Engineering Systems Conf. Power Engineering (LESCOPE'03)*, Montreal, QC, Canada, May 2003, pp. 143–147.

[5] M. M. Abdel-Aziz, E. E. Abou El-Zahab, A. M. Ibrahim, and A. F. Zobaa, "Comparing capacitive and *LC* compensators for power factor correction," in *Proc. 10th Int. Conf. Harmonics and Quality of Power (ICHQP'02)*, vol. 1, Rio de Janeiro, Brazil, Oct. 2002, pp. 42–45.

[6] R. C. Dugan, M. F. McGranaghan, and H. W. Beaty, *Electrical Power Systems Quality*. New York: McGraw-Hill, 1996.

[7] M. M. Abdel Aziz, E. E. Abou El-Zahab, and A. F. Zobaa, "Power factor and your electrical utility bill in Egypt," *IEEE Trans. Power Delivery*, vol. 18, no. 4, pp. 1567–1568, Oct. 2003.

[8] *IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems*, IEEE Std. 519-1992, 1992.

[9] *IEEE Standard for Shunt Power Capacitors*, IEEE Std. 18-2002, 2002.

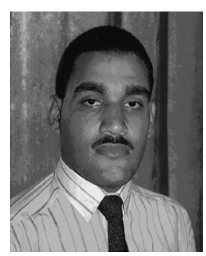
[10] D. A. Pierre, *Optimization Theory With Applications*. New York: Wiley, 1969.

[11] G. H. Hadley, *Nonlinear Programming and Dynamic Programming*. Reading, MA.: Adison-Wesley, 1964.

[12] G. G. Richards, O. T. Tan, P. Klinkhachorn, and N. I. Santoso, "Cost-constrained power factor optimization with source harmonics using *LC* compensators," *IEEE Trans. Ind. Electron.*, vol. 34, no. 2, pp. 266–270, May 1987.

[13] I. M. El-Amin, S. O. Duffuaa, and A. U. Bawah, "Optimal shunt compensators at nonsinusoidal busbars," *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 716–723, May 1995.

[14] *IEEE Guide for the Application, Specification, and Testing of Phase-Shifting Transformers*, IEEE Std. C57.135-2001, 2002.



Ahmed Faheem Zobaa (M'01–SM'04} received the B.Sc. (Hons.), M.Sc., and Ph.D. degrees in electrical power and machines from the Faculty of Engineering at Cairo University, Giza, Egypt, in 1992, 1997, and 2002, respectively.

Currently, he is an Assistant Professor in the Department of Electrical Power and Machines with the Faculty of Engineering, Cairo University. He was an Instructor in the Department of Electrical Power and Machines, with the Faculty of Engineering at Cairo University from 1992 to 1997 and Teaching Assistant from 1997 to 2002. He regularly reviews papers for eight IEEE TRANSACTIONS especially all IEEE/PES TRANSACTIONS and seven journals in his areas of interest. He is author or co-author of many refereed Journal and Conference papers. His areas of research include harmonics, compensation of reactive power, power quality, photovoltaics, wind energy, education and distance learning. He is an Editorial Board member for *Electrical Power Quality and Utilization Journal*, *Electric Power Components & Systems Journal* and *International Journal of Computational Intelligence*. He is an Editor for *IEEE Power Engineering Letters* and IEEE TRANSACTIONS ON ENERGY CONVERSION. Also, he is an Associate Editor for IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, *International Journal of Power and Energy Systems*, *International Journal on Modeling and Simulation*, and *Neurocomputing Journal*.

Dr. Zobaa is a member of the IEEE Power Engineering/Industry Applications/Industrial Electronics/Power Electronics Societies, the Institution of Electrical Engineers, International Association of Science and Technology for Development and the International Solar Energy Society.