$\mathbf{H}_\infty$ Reliable Control of Uncertain Linear State Delayed Systems *

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Abstract

This paper deals with the problem of robust and reliable H_∞ control design for linear uncertain time-delay systems with time-varying norm-bounded parameter uncertainty, and also with actuator failures among a prespecified subset of actuators. A state feedback control design is presented that stabilizes the plant and guarantees an H_∞-norm bound constraint on attenuation of the augmented disturbances, including failure signals, for all admissible uncertainties as well as actuator failures. It is shown that, the existence of the desired controllers is related to the positive definite solution of a parameter-dependent Riccati-like matrix equation, whose solving algorithm is also discussed in detail. Two illustrative examples are provided to demonstrate the applicability of the proposed method.

Key Words - Linear systems; uncertain systems; state delay; actuator failures; robust control; H_∞ control; reliable control

1 Introduction

In the past two decades, significant advances have been made in the H_∞ control since the original work in [1]. The standard H_∞ control problem was completely solved in [2] for linear systems by deriving simple state-space formulas for all controllers. Furthermore, in order to improve the performance robustness against parameter uncertainty, the so-called robust H_∞ control problem was extensively studied for both linear uncertain continuous- and discrete-time systems, see for example [3-9]. On the other hand, since the inherent time delays and parameter uncertainty contained in the dynamical behavior of many physical processes are unavoided [10], the H_∞ control problem as well as the robust stabilization problem for time-delay systems have recently received increasing attention, and various related work on these two issues has been reported, see, respectively, [11-18]. Also, the

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robust H_∞ state feedback control for linear systems with both state delay and parameter uncertainty was considered in [19].

However, although the robust and/or H_∞ controller design for time-delay systems have well been developed, these control designs may result in unsatisfactory control system performance, or even instability, in the event of control component failures (i.e., actuator or sensor outages), since failures of control components often occur in real world. In practical applications, it is always necessary to design controllers that achieve desired performance requirements, not only when the system is operating properly, but also in the presence of certain system measurement or control input failures. This motivated the study of the so-called reliable control, see for example [20-22].

Generally speaking, in addition to the basic stability, a good engineering control system should possess multiple expected performances, such as robustness to modeling error, reliability against sensor and actuator failures, disturbance attenuation property, etc. Recently, the problems of H_∞ reliable as well as robust reliable controller design have begun to attract much attention. [23] presented reliable centralized and decentralized control design methodologies to achieve both stability and H_∞ disturbance attenuation, but did not tackle the robustness issue. [24] proposed a reliable design approach for parameter uncertain systems, and gave the simulation results in a flight control system, but the system under study was assumed to be delay-free. [25] extended the results of [23] to the state delayed systems, but still did not take the system uncertainty into account. More recently, the robust reliable H_∞ control for linear systems with parameter uncertainty and actuator failure was investigated by [26], but the influence of time-delay was unfortunately not considered. In [27-28], the reliable control problems were investigated for a class of nonlinear deterministic and stochastic time-delay systems, respectively. However, the H_∞ norm restriction, which reflects the disturbance attenuation behavior, was not dealt with in [27-28]. As a summary, in the existing results concerning reliable control, either the robustness issue against parameter uncertainties, or the H_∞ constraint on the disturbance attenuation, or the state delay feature of the system under consideration, has not been addressed.

To the authors' best knowledge, so far, there have been very few papers focusing on the robust reliable H_∞ control design problem for uncertain state-delayed systems. That is, the problem for simultaneous realization of robustness, H_∞ performance, reliability for parameter uncertain time-delay control systems is still open, owing to its complexity. This situation motivates the investigation in the present paper on the multiobjective H_∞ reliable control for uncertain linear state delayed systems.

This paper focuses on the problem of robust H_∞ reliable control design for linear systems with state delay and parameter uncertainty. The goal of this problem is to design the state feedback controller such that, for all admissible uncertainties as well as actuator failures, the plant is robustly stabilized and the prescribed H_∞-norm bound constraint on disturbance attenuation is guaranteed, simultaneously. It is assumed that, the parameter uncertainties are norm-bounded and the actuator failures occur among a prespecified subset of actuators. A simple, effective, modified algebraic Riccati equation approach is developed to solve the addressed problem. The resulting time-delay control systems are reliable in that they provide guaranteed robust stability and H_∞ performance not only when all control components are operational, but also in case of actuator failures. two illustrative examples are presented to demonstrate the applicability of the proposed method.

The rest of this paper is organized as follows. In Section 2 the robust H_∞ reliable state feedback control problem is formulated for a class of linear uncertain time-delay systems. In Section 3, the analysis results are first given for an uncertain time-delay system to be robustly stable with prescribed H_∞ performance constraint, in the presence of possible actuator failures. Then, the corresponding synthesis results are established. It is shown that, the existence of the desired controllers, which guarantee both the reliability and the H_∞ disturbance attenuation level for the uncertain state delayed system, is closely related to the positive definite solution to a class of modified Riccati equations. Furthermore, the numerical algorithm on such class of modified Riccati equations are discussed. Two simulation examples are given in Section 4 to illustrate the usefulness of the proposed theory, and finally, some concluding remarks are drawn in Section 5.

Notation. The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. $L_2[0,\infty)$ is the space of square integrable vector over $[0,\infty)$. For a given transfer function $T(s)$, the H_∞ norm of $T(s)$ is defined as $\|T(s)\|_\infty = \sup_{\omega\in\mathbb{R}} \sigma_{\max}[T(j\omega)]$ where $\sigma_{\max}[\cdot]$ denotes the largest singular value of $[\cdot]$. The capital letters are used to represent matrices, while vector variables are described in bold faces. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2 Problem formulation and preliminaries

Consider a linear continuous-time uncertain state delayed system represented by

$$\begin{aligned}
\dot{\boldsymbol{x}}(t) &= (A+\Delta A)\boldsymbol{x}(t) + (A_d + \Delta A_d)\boldsymbol{x}(t-d) + B\boldsymbol{u}(t) + D\boldsymbol{w}(t), && (2.1)\\
\boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \;\; t \in [-d, 0], && (2.2)\\
\boldsymbol{y}(t) &= C\boldsymbol{x}(t), && (2.3)
\end{aligned}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state, $\boldsymbol{u}(t) \in \mathbb{R}^m$ is the control input, $\boldsymbol{w}(t) \in \mathbb{R}^p$ is the square-integrable exogenous disturbance, $\boldsymbol{y}(t) \in \mathbb{R}^l$ is the measured output. A, A_d, B, D, C are known constant matrices with appropriate dimensions, $d \geq 0$ denotes the unknown real state delay, $\boldsymbol{\phi}(t)$ is a continuous vector valued initial function. ΔA and ΔA_d are real valued matrix functions representing norm-bounded parameter uncertainties and satisfy

$$\begin{bmatrix} \Delta A & \Delta A_d \end{bmatrix} = M \begin{bmatrix} \Xi_1 N_1 & \Xi_2 N_2 \end{bmatrix}, \qquad (2.4)$$

where $\Xi_1 \in \mathbb{R}^{i \times j}$ and $\Xi_2 \in \mathbb{R}^{i \times j}$, which may be time-varying, are real uncertain matrices with Lebesgue measurable elements and meet

$$\Xi_1^T \Xi_1 \leq I, \quad \Xi_2 \Xi_2^T \leq I, \tag{2.5}$$

and M, N_1, N_2, which denote the structure of the uncertainties, are fixed matrices with appropriate dimensions.

The uncertainties $\Delta A, \Delta A_d$ are said to be admissible if both (2.4) and (2.5) are satisfied.

We now consider the reliability with respect to actuator failures. In general, the actuators of a control system can be classified into two selected subsets. That is, the actuators *susceptible* to failures and the actuators *robust* to failures. The first class of actuators is denoted as $\Omega \subseteq \{1, 2, \cdots, m\}$ and is possible to fail, while the second class of actuators, which is essential to stabilize a given system, is therefore denoted as $\bar{\Omega} \subseteq \{1, 2, \cdots, m\} - \Omega$, and is assumed never to fail.

Introduce the decomposition

$$B = B_\Omega + B_{\bar{\Omega}}, \tag{2.6}$$

where B_Ω means the control matrix associated with the set Ω, and $B_{\bar{\Omega}}$ denotes the control matrix associated with the complementary subset of control inputs, i.e, B_Ω and $B_{\bar{\Omega}}$ are generated by zeroing out the columns corresponding to $\bar{\Omega}$ and Ω respectively. Furthermore, let $\omega \subseteq \Omega$ correspond to a particular subset of the susceptible actuators that actually fail, we adopt the following notation that will be used in the derivation of the main result

$$B = B_\omega + B_{\bar{\omega}}, \tag{2.7}$$

where B_ω and $B_{\bar{\omega}}$ have meanings analogous to those of B_Ω and $B_{\bar{\Omega}}$.

To this end, the purpose of this paper can be stated as designing the fixed state feedback controller

$$\boldsymbol{u}(t) = F\boldsymbol{x}(t) \tag{2.8}$$

that stabilizes the linear time-delay system (2.1)-(2.3) with a given H_∞-norm constraint on disturbance attenuation, for all admissible uncertainties and all actuator failures occurred within the prespecified subset Ω.

3 Main results and proofs

Applying the state feedback control law (2.8) to the system (2.1)-(2.3), we obtain the closed-loop system

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= (A + \Delta A + BF)\boldsymbol{x}(t) + (A_d + \Delta A_d)\boldsymbol{x}(t-d) + D\boldsymbol{w}(t), & (3.1)\\ \boldsymbol{y}(t) &= C\boldsymbol{x}(t). & (3.2) \end{aligned}$$

The following lemmas will be essentially used in the proof of the main results.

Lemma 3.1 (see [29]) *For an arbitrary positive scalar $\varepsilon_1 > 0$ and a positive definite matrix $P > 0$, we have*

$$(\Delta A)^T P + P(\Delta A) \leq \varepsilon_1 PMM^T P + \varepsilon_1^{-1} N_1^T N_1. \tag{3.3}$$

Lemma 3.2 (see [29]) *Let a positive scalar $\varepsilon_2 > 0$ and a positive definite matrix $Q > 0$ be such that $\varepsilon_2 N_2 Q^{-1} N_2^T < I$. Then the following inequality holds*

$$\begin{aligned}(A_d + \Delta A_d)Q^{-1}(A_d + \Delta A_d)^T &\leq A_d Q^{-1} A_d^T \\ +A_d Q^{-1} N_2^T (\varepsilon_2^{-1} I - N_2 Q^{-1} N_2^T)^{-1} N_2 Q^{-1} A_d^T &+ \varepsilon_2^{-1} MM^T.\end{aligned} \tag{3.4}$$

Lemma 3.3 *The closed-loop system (3.1)-(3.2) is asymptotically stable for all time-delay $d \geq 0$ if there exist positive definite matrices $P > 0$ and $Q > 0$ which satisfy the following inequality:*

$$\begin{aligned}&(A + \Delta A + BF)^T P + P(A + \Delta A + BF) \\ &+P(A_d + \Delta A_d)Q^{-1}(A_d + \Delta A_d)^T P + Q < 0\end{aligned} \tag{3.5}$$

Proof: Choose the Lyapunov function candidate

$$Y(\boldsymbol{x}(t)) := \boldsymbol{x}^T(t) P \boldsymbol{x}(t) + \int_{t-d}^{t} \boldsymbol{x}^T(s) Q \boldsymbol{x}(s) ds \tag{3.6}$$

where P is the positive definite solution to the inequality (3.5).

For notation convenience, we define

$$\bar{A} := A + \Delta A + BF, \quad \bar{A}_d := A + \Delta A_d, \quad \bar{C} = C, \quad \bar{D} = D, \quad \bar{\boldsymbol{w}} = \boldsymbol{w}. \tag{3.7}$$

The corresponding Lyapunov derivative along a given trajectory is

$$\frac{dY(\boldsymbol{x}(t))}{dt} = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-d) \end{bmatrix}^T \begin{bmatrix} \bar{A}^T P + P\bar{A} + P\bar{A}_d + Q & P\bar{A}_d \\ \bar{A}_d^T P & -Q \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-d) \end{bmatrix}. \tag{3.8}$$

Note that the matrix in (3.8) is negative definite if the inequality

$$\bar{A}^T P + P\bar{A} + P\bar{A}_d Q^{-1} \bar{A}_d^T P + Q < 0 \tag{3.9}$$

is satisfied. This completes the proof of this lemma. □

Remark 3.1 Lemma 3.3 provides a delay independent stability criteria which may be suitable for the case when the time-delay is unknown.

Next, sufficient conditions under which the controller (2.8) guarantees the H_∞ norm constraint and simultaneously stabilizes the closed-loop system are given in the following theorem.

Theorem 3.1 *Given the H_∞-norm constraint $\gamma > 0$ and a positive definite matrix $Q > 0$. If there exists a positive definite matrix P such that the inequality*

$$\begin{aligned}(A + \Delta A + BF)^T P + P(A + \Delta A + BF) \\ +P(A_d + \Delta A_d)Q^{-1}(A_d + \Delta A_d)^T P \\ +Q + \bar{C}^T\bar{C} + \gamma^{-2}P\bar{D}\bar{D}^T P < 0\end{aligned} \tag{3.10}$$

is satisfied, then the closed-loop system (3.1)-(3.2), i.e.,

$$\dot{\boldsymbol{x}}(t) = \bar{A}\boldsymbol{x}(t) + \bar{A}_d\boldsymbol{x}(t-d) + \bar{D}\bar{\boldsymbol{w}}(t), \tag{3.11}$$

$$\boldsymbol{y}(t) = \bar{C}\boldsymbol{x}(t), \tag{3.12}$$

is asymptotically stable, and the transfer function from the disturbance input to the measured output

$$T_{\boldsymbol{y}\bar{\boldsymbol{w}}}(s) = \bar{C}(sI - \bar{A} - \bar{A}_d e^{-sd})^{-1}\bar{D}$$

satisfies the constraint $\|T_{\boldsymbol{y}\bar{\boldsymbol{w}}}(s)\|_\infty \le \gamma$ for all $d \ge 0$.

Proof: See the Appendix. □

Since the actuators play a role in transmitting the controller output to the plant, the significance of possible actuator failures is considered now. Without loss of generality, the transfer function of an actuator is assumed to be 1. Generally, the output signals of faulty actuators may greatly affect the system behavior. As these signals act on the system in unexpected manner, they are considered as disturbances too. Like [26], in this paper, the output of faulty actuators is assumed to be any arbitrary energy-bound signal (that is, the output of a failed actuator belongs to $L_2[0,\infty)$ and acts on the system as disturbance input). We denote $\boldsymbol{w}_c = [\boldsymbol{w}^T(t)\ \boldsymbol{v}_\omega^T(t)]^T$ where $\boldsymbol{v}_\omega(t)$ is the disturbance contribution from the actuators in the subset ω ($\omega \subseteq \Omega$) which actually fail and hence cause extra/unexpected disturbance signals. Note that the transfer function of the resulting closed-loop system from the disturbance signal $\boldsymbol{w}_c(t)$ to the measured output $\boldsymbol{y}(t)$ is :

$$T_{\boldsymbol{y}\boldsymbol{w}_c} = C[sI - (A + B_{\bar{\omega}}F + \Delta A) - (A_1 + \Delta A_1)e^{-sd}]^{-1}[D \ \ B_\omega].$$

The above discussion motivates the major goal of designing a feedback controller that guarantees satisfactory closed-loop behavior *even when* there are actuator failures in the prespecified subset of susceptible actuators, and also simultaneously meets the H_∞-norm constraint below a given level while maintaining the stability of the closed-loop system.

We now define the following modified Riccati equation:

$$A^T P + PA + P\Theta P + \varepsilon_1^{-1}N_1N_1^T + C^T C + Q + Q_1 = 0, \tag{3.13}$$

where

$$\begin{aligned}\Theta :=& -\varepsilon_3^{-1}B_{\bar{\Omega}}B_{\bar{\Omega}}^T + \gamma^{-2}B_\Omega B_\Omega^T + A_dQ^{-1}A_d^T \\ &+ A_dQ^{-1}N_2^T(\varepsilon_2^{-1}I - N_2Q^{-1}N_2^T)^{-1}N_2Q^{-1}A_d^T + (\varepsilon_1 + \varepsilon_2^{-1})MM^T + \gamma^{-2}DD^T.\end{aligned}$$

We are ready to give our main results as follows.

Theorem 3.2 *Given the required H_∞-norm constraint $\gamma > 0$. If there exist appropriate positive scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ and positive definite matrices $Q > 0$, $Q_1 > 0$ such that $I - \varepsilon_2 N_2 Q^{-1} N_2^T > 0$ and the modified Riccati equation (3.13) has a positive definite solution $P > 0$, then the state feedback law*

$$\boldsymbol{u}(t) = F\boldsymbol{x}(t), \quad F = -0.5\varepsilon_3^{-1} B^T P \tag{3.14}$$

stabilizes the uncertain time-delay system (2.1)-(2.3), independent of the delay d, and simultaneously satisfies the H_∞-norm constraint below the given level γ for all admissible uncertainties and possible actuator failures corresponding to $\omega \subseteq \Omega$.

Proof: It follows from (2.7) and (3.14) that

$$\begin{aligned} BF &= (B_{\bar\omega} + B_\omega)[-0.5\varepsilon_3^{-1}(B_{\bar\omega} + B_\omega)^T P] \\ &= -(0.5\varepsilon_3^{-1} B_{\bar\omega} B_{\bar\omega}^T P + 0.5\varepsilon_3^{-1} B_\omega B_\omega^T P). \end{aligned}$$

Define

$$\nu_{\bar\omega} := -0.5\varepsilon_3^{-1} B_{\bar\omega} B_{\bar\omega}^T P, \quad \nu_\omega := -0.5\varepsilon_3^{-1} B_\omega B_\omega^T P.$$

Applying the control law (3.14) to the system (2.1)-(2.2) yields the closed-loop system

$$\dot{\boldsymbol{x}}(t) = A_c \boldsymbol{x}(t) + A_1 \boldsymbol{x}(t-d) + D_c \boldsymbol{w}_c(t) \tag{3.15}$$

$$\boldsymbol{y}(t) = C_c \boldsymbol{x}(t) \tag{3.16}$$

where

$$\begin{gathered} A_c = A + \nu_{\bar\omega} + \Delta A = A - 0.5\varepsilon_3^{-1} B_{\bar\omega} B_{\bar\omega}^T P + \Delta A, \\ A_1 = A_d + \Delta A_d, \quad C_c = C, \quad D_c = [D \ \ B_\omega], \\ \boldsymbol{w}_c = \begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{v}_\omega \boldsymbol{x}(t) \end{bmatrix}, \quad \boldsymbol{v}_\omega(t) = \nu_\omega P \boldsymbol{x}(t). \end{gathered}$$

From Lemma 3.1 and Lemma 3.2, we obtain the following two inequalities:

$$\begin{gathered} A_c^T P + P A_c = A^T P + PA - \varepsilon_3^{-1} P B_{\bar\omega} B_{\bar\omega}^T P + (\Delta A)^T P + P(\Delta A) \\ \leq A^T P + PA + P(-\varepsilon_3^{-1} B_{\bar\omega} B_{\bar\omega}^T + \varepsilon_1 M M^T) P + \varepsilon_1^{-1} N_1 N_1^T \end{gathered} \tag{3.17}$$

$$\begin{gathered} P A_1 Q^{-1} A_1^T P = P(A_d + \Delta A_d) Q^{-1} (A_d + \Delta A_d) P \\ \leq P[A_d Q^{-1} A_d^T + A_d Q^{-1} N_2^T (\varepsilon_2^{-1} I - N_2 Q^{-1} N_2^T)^{-1} N_2 Q^{-1} A_d^T + \varepsilon_2^{-1} M M^T] P. \end{gathered} \tag{3.18}$$

Also, it is easy to see that

$$C_c^T C_c = C^T C, \quad \gamma^{-2} P D_c D_c^T P = \gamma^{-2} P(D D^T + B_\omega B_\omega^T) P. \tag{3.19}$$

From the definitions of B_ω, $B_{\bar\omega}$, B_Ω and $B_{\bar\Omega}$, it is clear that the relationships

$$B_\Omega B_\Omega^T = B_\omega B_\omega^T + B_{\Omega-\omega} B_{\Omega-\omega}^T, \quad B_{\bar\Omega} B_{\bar\Omega}^T = B_{\bar\omega} B_{\bar\omega}^T - B_{\Omega-\omega} B_{\Omega-\omega}^T$$

are true. This leads to

$$B_\omega B_\omega^T \leq B_\Omega B_\Omega^T, \quad B_{\bar{\omega}} B_{\bar{\omega}}^T \geq B_{\bar{\Omega}} B_{\bar{\Omega}}^T. \tag{3.20}$$

From relations (3.17)-(3.20) and the modified Riccati equation (3.13), we have

$$\begin{aligned}
&A_c^T P + PA_c + PA_1 Q^{-1} A_1^T P + Q + C_c^T C_c + \gamma^{-2} P D_c D_c^T P \\
\leq &A^T P + PA + P(-\varepsilon_3^{-1} B_{\bar{\omega}} B_{\bar{\omega}}^T + \varepsilon_1 M M^T) P + \varepsilon_1^{-1} N_1 N_1^T \\
&+ P[A_d Q^{-1} A_d^T + A_d Q^{-1} N_2^T (\varepsilon_2^{-1} I - N_2 Q^{-1} N_2^T)^{-1} N_2 Q^{-1} A_d^T \\
&+ \varepsilon_2^{-1} M M^T] P + Q + C^T C + \gamma^{-2} P (D D^T + B_\omega B_\omega^T) P \\
\leq &A^T P + PA + P(-\varepsilon_3^{-1} B_{\bar{\Omega}} B_{\bar{\Omega}}^T + \varepsilon_1 M M^T) P + \varepsilon_1^{-1} N_1 N_1^T \\
&+ P[A_d Q^{-1} A_d^T + A_d Q^{-1} N_2^T (\varepsilon_2^{-1} I - N_2 Q^{-1} N_2^T)^{-1} N_2 Q^{-1} A_d^T \\
&+ \varepsilon_2^{-1} M M^T] P + Q + C^T C + \gamma^{-2} P (D D^T + B_\Omega B_\Omega^T) P \\
= &A^T P + PA + P\Theta P + \varepsilon_1^{-1} N_1 N_1^T + C^T C + Q = -Q < 0.
\end{aligned} \tag{3.21}$$

Thus, the proof of this theorem is immediately completed from Theorem 3.1. □

Remark 3.2 Theorem 3.2 shows that the mixed robustness, H_∞-norm upper bound of closed-loop transfer function and reliability can be guaranteed when a positive definite solution P to the algebraic Riccati equation (3.13) is known to exist. When there is no actuator failure, i.e, $\Omega = \emptyset$, Theorem 3.2 generalizes the result of [19], and when there is no time-delay in the system state equation, Theorem 3.2 includes the result of [26] as a special case.

Remark 3.3 Clearly, it is important to check the existence of a positive definite solution to (3.13). When the symmetric matrix Θ is nonnegative definite, (3.13) is a generalized algebraic Riccati equation, and the discussion on numerical solution to such a parameter-dependent Riccati equation can be found in many papers, see for example [30-31]. The investigation of the case for $\Theta > 0$ is performed in the following theorem which gives the existence condition of an expected positive definite solution to (3.13). It should be pointed out that $\Theta > 0$ means that the system matrix A must be stable.

Theorem 3.3 *Consider the algebraic matrix equation (3.13) with $\Theta > 0$. There exists a positive definite solution $P > 0$ to (3.13) if and only if*

$$\Pi := A^T \Theta^{-1} A - (\varepsilon_1^{-1} N_1 N_1^T + C^T C + Q + Q_1) \geq 0. \tag{3.22}$$

Furthermore, in this case, the desired solutions can be expressed as

$$P = TV\Theta^{-1/2} - A^T \Theta^{-1} \tag{3.23}$$

where $T \in \mathbb{R}^{n\times n}$ is the square root of $\Pi = A^T \Theta^{-1} A - (\varepsilon_1^{-1} N_1 N_1^T + C^T C + Q + Q_1)$, $V \in \mathbb{R}^{n\times n}$ is an arbitrary orthogonal matrix.

Proof: We can rewrite (3.13) as

$$\begin{aligned}&(P\Theta^{1/2} + A^T\Theta^{-1/2})(P\Theta^{1/2} + A^T\Theta^{-1/2})^T \\ &= A^T\Theta^{-1}A - (\varepsilon_1^{-1}N_1N_1^T + C^TC + Q + Q_1) \geq 0\end{aligned} \tag{3.24}$$

and the first conclusion of this theorem follows immediately. Moreover, take the square root of Π, i.e.

$$TT^T = \Pi = A^T\Theta^{-1}A - (\varepsilon_1^{-1}N_1N_1^T + C^TC + Q + Q_1), \quad T \in \mathbb{R}^{n\times n}.$$

Equation (3.13) is then equivalent to

$$(P\Theta^{1/2} + A^T\Theta^{-1/2})(P\Theta^{1/2} + A^T\Theta^{-1/2})^T = TT^T, \tag{3.25}$$

or

$$P\Theta^{1/2} + A^T\Theta^{-1/2} = TV \tag{3.26}$$

where $V \in \mathbb{R}^{n\times n}$ is an arbitrary orthogonal matrix. The expression (3.23) can be directly derived from (3.26). This finishes the proof. □

Remark 3.4 It is apparent from the above results that there exists much freedom contained in the design steps, such as the choices of appropriate ε_1, ε_2, ε_3, Q, Q_1. This design freedom may be exploited to achieve other desired closed-loop properties, such as low-energy control input requirement and good transient behavior. This will be the subject of further studies.

4 Numerical examples

In this section two numerical examples together with the corresponding simulation results are given to illustrate the applicability of the proposed design approach.

Example 1: Consider the uncertain delay linear system (2.1)-(2.3) with the following data

$$A = \begin{bmatrix} 4 & 0.1 & -0.5 \\ -0.2 & 2 & 0.2 \\ 1 & -0.2 & -6 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.02 & 0 & 0.01 \\ 0.01 & -0.02 & 0 \\ -0.01 & -0.02 & -0.5 \end{bmatrix},$$

$$B = \begin{bmatrix} 2 & 0.1 & 1 \\ 0 & 4 & 0.1 \\ 0.2 & 0 & 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0.2 \\ 2 & 0.5 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.06 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.03 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.04 \end{bmatrix},$$

$$\Xi_1 = \mathrm{diag}\{0.1, 0.1, 0.2\}, \quad \Xi_2 = \mathrm{diag}\{0.2, 0.1, 0.4\}.$$

Since the set of the eigenvalues of A is $\{3.9395, -5.9447, 2.0052\}$, A is unstable. We now give the following system parameters: the time-delays for the first, second and third states are 1.5 second, 2.5 second and 5 second, respectively; the initial (state) conditions are $\boldsymbol{x}(t_0) = [20.5 \quad -10 \quad -25]^T$; the H_∞-norm constraint is $\gamma = 0.59$; $\Omega = \{3\}$; and the disturbance inputs are sine waves with unit intensities.

For $\Omega = \{3\}$, we have

$$B_{\bar{\Omega}} = \begin{bmatrix} 2 & 0.1 & 0 \\ 0 & 4 & 0 \\ 0.2 & 0 & 0 \end{bmatrix}, \quad B_{\Omega} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0.1 \\ 0 & 0 & 0.1 \end{bmatrix},$$

Set $\varepsilon_1 = 0.01$, $\varepsilon_2 = 1$, $\varepsilon_3 = 1.3333$ and

$$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

such that $I - \varepsilon_2 N_2 Q^{-1} N_2^T > 0$ is met. The symmetric matrix Θ and the positive definite solution P to the modified Riccati equation (3.13) are obtained, respectively, as follows

$$\Theta = \begin{bmatrix} -0.1290 & -0.0148 & -0.0615 \\ -0.0148 & -11.8512 & 0.0375 \\ -0.0615 & 0.0375 & 2.7972 \end{bmatrix}, \quad P = \begin{bmatrix} 67.1897 & 0.9567 & -3.3591 \\ 0.9567 & 0.4593 & 0.0169 \\ -3.3591 & 0.0169 & 0.3015 \end{bmatrix}.$$

From (3.14), the required feedback control law is obtained by

$$\boldsymbol{u}(t) = F\boldsymbol{x}(t), \quad F = \begin{bmatrix} -50.1403 & -0.7188 & 2.4967 \\ -3.9546 & -0.7248 & 0.1006 \\ -25.1060 & -0.3766 & 1.2477 \end{bmatrix}.$$

We now discuss the following two cases.

- Case 1: There are no actuator failures (i.e., all actuators are normal).
- Case 2: There is a failure of the third actuator (i.e., an actuator failure corresponding to $\omega \subseteq \Omega = \{3\}$ occurs).

The simulation (state response) results for the closed-loop system in case 1 and case 2 are shown in Fig. 1 and Fig. 2.

Next, we set $\gamma = 0.75$. In this case, the following new results are obtained:

$$P = \begin{bmatrix} 7.1181 & 0.1525 & -0.1466 \\ 0.1525 & 0.4453 & 0.0567 \\ -0.1466 & 0.0567 & 0.1238 \end{bmatrix}, \quad F = \begin{bmatrix} -5.3276 & -0.1186 & 0.1007 \\ -0.4957 & -0.6737 & -0.0795 \\ -2.6695 & -0.0760 & 0.0482 \end{bmatrix}.$$

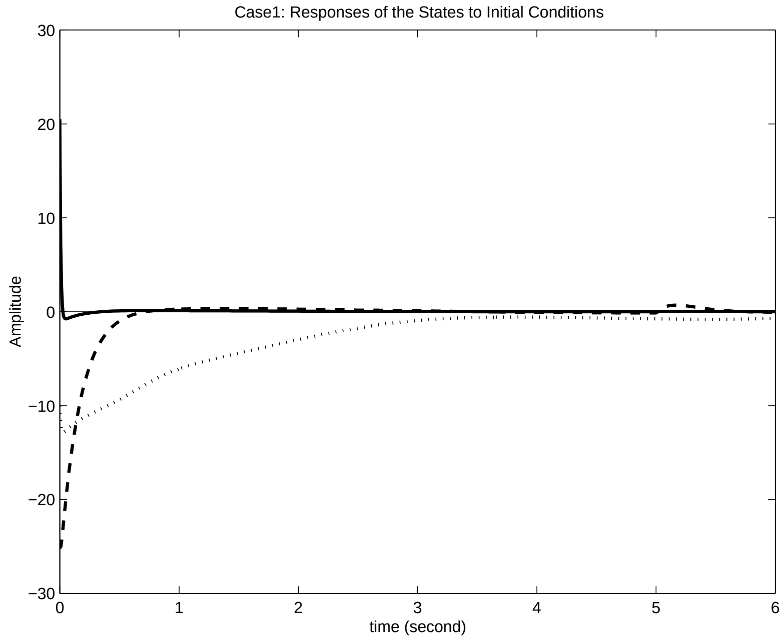

Figure 1: x_1 (solid), x_2 (point), x_3 (dashed).

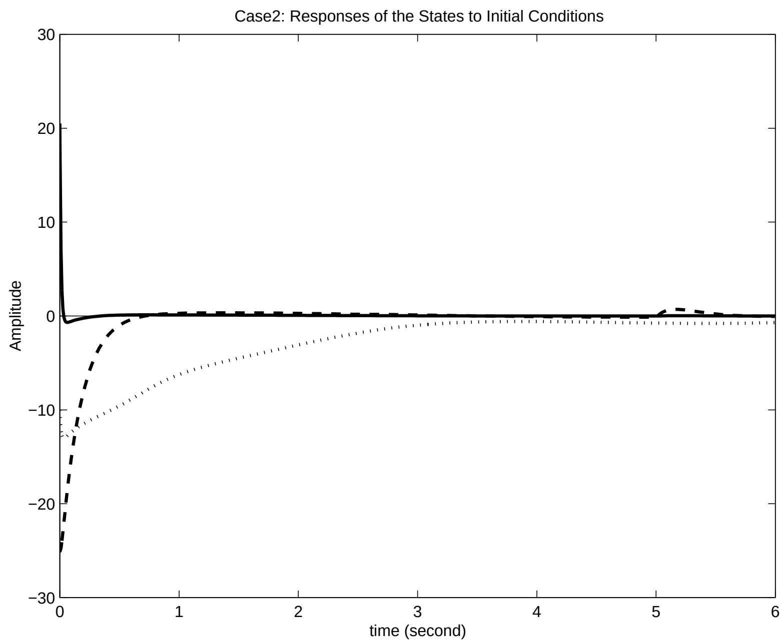

Figure 2: x_1 (solid), x_2 (point), x_3 (dashed).

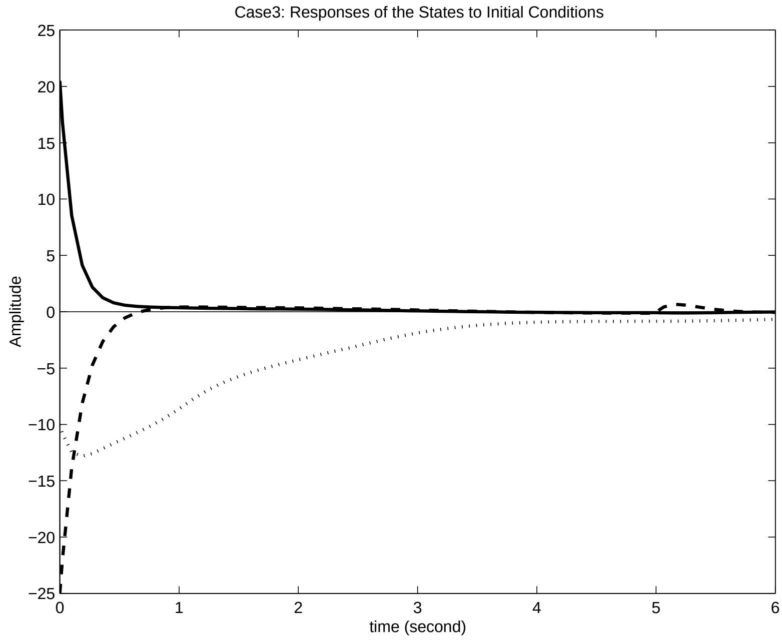

Figure 3: x_1 (solid), x_2 (point), x_3 (dashed).

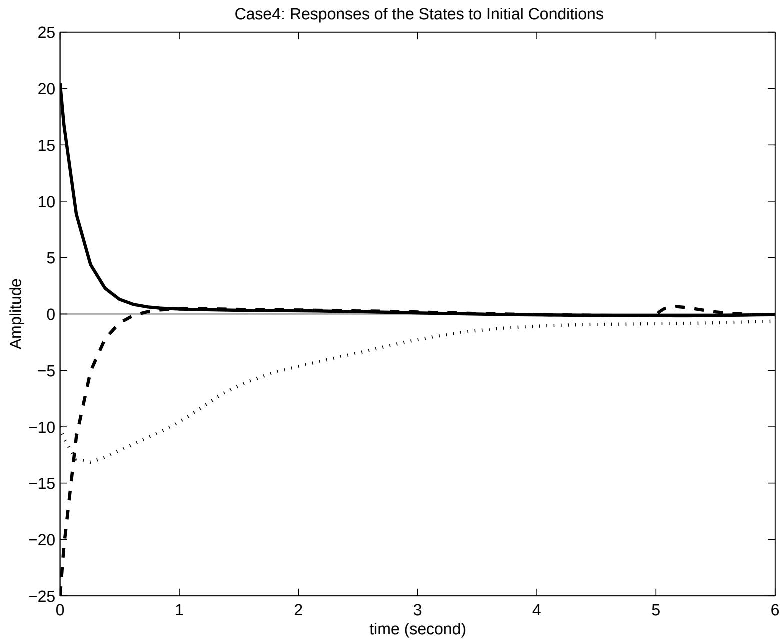

Figure 4: x_1 (solid), x_2 (point), x_3 (dashed).

Also, case 3 is associated with the the normal situation, and case 4 corresponds to the failure of the third actuator. Simulation results for these two cases are displayed in Fig. 3 and Fig. 4.

It can be seen from the numerical simulation results that all our goals are well achieves.

Example 2: Consider the uncertain delay linear system (2.1)-(2.3)with system parameters being given as follows:

$$A = \begin{bmatrix} -5 & -0.1 & 0.5 \\ 0.2 & -2 & -0.2 \\ -1 & 0.2 & -0.7 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.02 & 0 & 0.01 \\ 0.01 & -0.02 & 0 \\ -0.01 & -0.02 & -0.05 \end{bmatrix},$$

$$B = \begin{bmatrix} 6 & 0.1 & 1 \\ 0 & 4 & 0.1 \\ 0.2 & 10 & 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0.2 \\ 2 & 0.5 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 2 \\ .3 & 0.1 \end{bmatrix}, \quad M = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.06 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.03 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.04 \end{bmatrix},$$

$$\Xi_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad \Xi_2 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}.$$

The time delays for the first, second and third states are set as 1 second, 2 second and 3 second, respectively; the initial (state) conditions are $\boldsymbol{x}(t_0) = [50 \quad -25 \quad 10]^T$; the H_∞ norm constraint is $\gamma = 0.7$; and the set of actuators susceptible to failures is $\Omega = \{3\}$. Therefore, we have

$$B_{\bar{\Omega}} = \begin{bmatrix} 6 & 0.1 & 0 \\ 0 & 4 & 0 \\ 0.2 & 10 & 0 \end{bmatrix}, \quad B_\Omega = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0.1 \\ 0 & 0 & 0.1 \end{bmatrix},$$

Setting $\varepsilon_1 = .01$, $\varepsilon_2 = 1$, $\varepsilon_3 = 1.3333$, $Q = Q_1 = 0.1 I_3$, we obtain the symmetric matrix Θ, and the positive definite solution P to the modified Riccati equation (3.13) as follows:

$$\Theta = \begin{bmatrix} -24.9408 & -0.0980 & -1.3877 \\ -0.0980 & -3.8111 & -29.5684 \\ -1.3877 & -29.5684 & -74.5225 \end{bmatrix}, \quad P = \begin{bmatrix} 0.2637 & 0.0498 & 0.0942 \\ 0.0498 & 0.1270 & -0.0047 \\ 0.0942 & -0.0047 & 0.1182 \end{bmatrix}.$$

Hence, the desired state feedback gain matrix is calculated by:

$$u(t) = Fx(t), \quad F = \begin{bmatrix} -0.6004 & -0.1117 & -0.2209 \\ -0.4380 & -0.1747 & -0.4397 \\ -0.1043 & -0.0233 & -0.0396 \end{bmatrix}.$$

Again, we discuss the following two cases, that is, the case when all actuators are normal and the case when there is a failure for the third actuator. The simulation (state response) results of the closed-loop system for these two cases are depicted, respectively, in Fig. 5 and Fig. 6, which verify our theoretical results.

Remark 4.1 In this paper, we actually consider a multiobjective control (sub-optimal) problem. That is, for a class of linear uncertain time-delay systems, we want the controlled systems to have expected reliability and disturbance rejection attenuation level, for all admissible parameter uncertainties and all possible actuator failures. As can be seen in the numerical examples, the desired solution set, if not empty, must be very large. In other words, as long as a state feedback gain could make the corresponding closed-loop system satisfy prespecified multiple objectives, it will belong to the desired solution set.

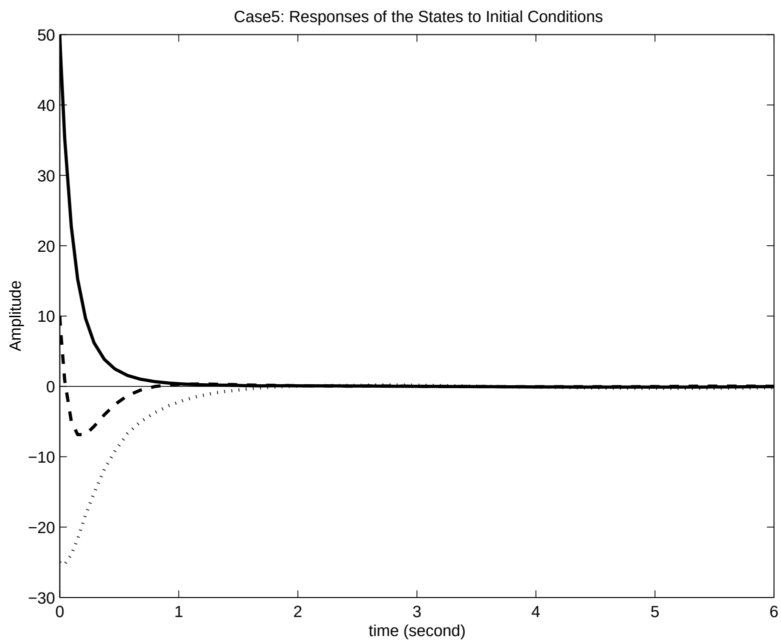

Figure 5: x_1 (solid), x_2 (point), x_3 (dashed).

5 Conclusion

A design approach for robust reliable H_∞ state feedback control of linear systems with state delay and parameter uncertainty has been presented. Based on a positive definite solution to a modified parameter-dependent Riccati equation, the proposed robust reliable H_∞ state feedback controller guarantees robust stability and robust H_∞ performance for uncertain time-delay systems, independent of the time delay, not only when all control components are operational, but also in case of actuator failures occurred within the prescribed set of susceptible actuators. The contribution of this paper, which can be summarized as follows, is twofold:

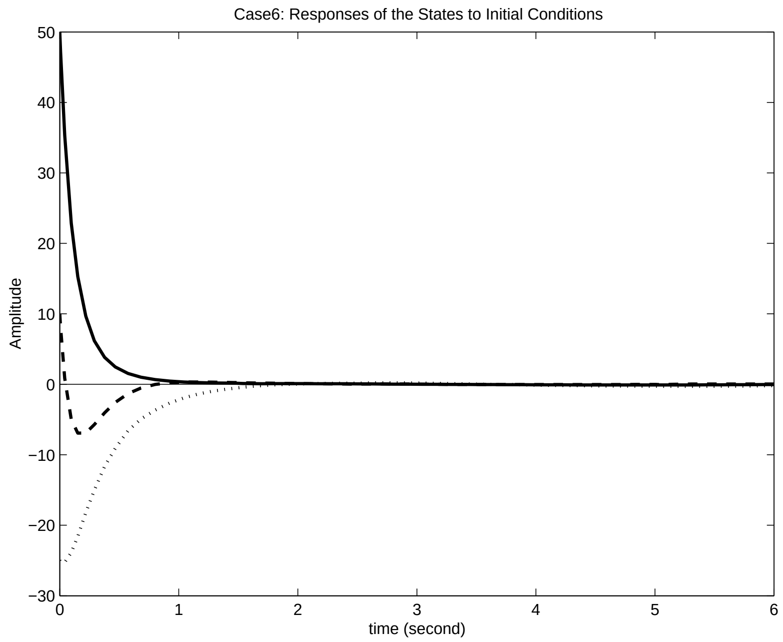

Figure 6: x_1 (solid), x_2 (point), x_3 (dashed).

- Previous results on robust control and robust H_∞ control for time-delay systems have been extended to the case in which an actuator failure is allowed.
- A modified algebraic Riccati equation approach has been developed to deal with the addressed design problem. Numerical algorithm on this kind of algebraic Riccati equations has also been discussed.

Two simulation examples have shown that the present design approach is both simple and effective.

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7 Appendix (Proof of Theorem 3.1)

Proof: From Lemma 3.3, a controller which satisfies the inequality (3.10) stabilizes the state delayed system (2.1)-(2.3) for all $d \geq 0$.

Define the matrix Υ as follows

$$\Upsilon = -(\bar{A}^T P + P\bar{A} + P\bar{A}_d Q^{-1}\bar{A}_d^T P + Q + \bar{C}^T\bar{C} + \frac{1}{\gamma^2}P\bar{D}\bar{D}^T P).$$

Then, we have

$$\bar{A}^T P + P\bar{A} + P\bar{A}_d Q^{-1}\bar{A}_d^T P + Q + \bar{C}^T\bar{C} + \frac{1}{\gamma^2}P\bar{D}\bar{D}^T P + \Upsilon = 0$$

and

$$\begin{aligned}
&(-j\alpha I - \bar{A}^T - e^{j\alpha d}\bar{A}_d^T)P + P(j\alpha I - \bar{A} - e^{-j\alpha d}\bar{A}_d) \\
&\quad -P\bar{A}_d Q^{-1}\bar{A}_d^T P - Q - \Upsilon - \bar{C}^T\bar{C} - \frac{1}{\gamma^2}P\bar{D}\bar{D}^T P \\
&\qquad = -e^{j\alpha d}\bar{A}_d^T P - e^{-j\alpha d}P\bar{A}_d
\end{aligned} \tag{7.1}$$

for all $\alpha \in \mathbb{R}$.

Next, we define the following matrices:

$$\begin{aligned}
U(j\alpha) :&= Q + P\bar{A}_d Q^{-1}\bar{A}_d^T P - \bar{A}_d^T P e^{j\alpha d} - P\bar{A}_d e^{-j\alpha d} \\
&= [P\bar{A}_d e^{-j\alpha d} - Q]Q^{-1}[\bar{A}_d^T P e^{j\alpha d} - Q] \\
V(j\alpha) :&= (j\alpha I - \bar{A} - \bar{A}_d e^{-j\alpha d})^{-1}
\end{aligned}$$

for all $\alpha \in \mathbb{R}$.

Note that $U(j\alpha)$ is nonnegative definite from the definition. Using the matrices $U(j\alpha)$ and $V(j\alpha)$, we can rewrite (7.1) as

$$(V^T(-j\alpha))^{-1}P + PV^{-1}(j\alpha) - U(j\alpha) - \Upsilon - \bar{C}^T\bar{C} - \frac{1}{\gamma^2}P\bar{D}\bar{D}^T P = 0 \tag{7.2}$$

or

$$\begin{aligned}
&PV(j\alpha) + V^T(-j\alpha)P - \frac{1}{\gamma^2}V^T(-j\alpha)P\bar{D}\bar{D}^T PV(j\alpha) \\
&\qquad = V^T(-j\alpha)[U(j\alpha) + \Upsilon + \bar{C}^T\bar{C}]V(j\alpha)
\end{aligned} \tag{7.3}$$

This implies that

$$\begin{aligned}
&\bar{D}^T PV(j\alpha)\bar{D} + \bar{D}^T V^T(-j\alpha)P\bar{D} - \frac{1}{\gamma^2}\bar{D}^T V^T(-j\alpha)P\bar{D}\bar{D}^T PV^T(j\alpha)\bar{D} - \gamma^2 I \\
&\qquad = \gamma I + \bar{D}^T V^T(-j\alpha)[U(j\alpha) + \Upsilon + \bar{C}^T\bar{C}]V(j\alpha)\bar{D}
\end{aligned} \tag{7.4}$$

and

$$\begin{aligned}
&-\frac{1}{\gamma^2}(\gamma^2 I - \bar{D}^T PV(-j\alpha)\bar{D})^T(\gamma^2 I - \bar{D}^T PV(j\alpha)\bar{D}) \\
= \;& -\gamma^2 I + \bar{D}^T V^T(-j\alpha)[U(j\alpha) + \Upsilon]V(j\alpha)\bar{D} \\
&+ \frac{1}{\gamma^2}\bar{D}^T V^T(-j\alpha)\bar{C}^T\bar{C}V(j\alpha)\bar{D}.
\end{aligned} \tag{7.5}$$

for all $\alpha \in \mathbb{R}$.

The left hand side of equation (7.5) is non-positive definite and

$$\bar{C}V(j\alpha)\bar{D} = T_{\boldsymbol{y}\bar{\boldsymbol{w}}}.$$

Hence

$$-\gamma^2 I + \bar{D}^T V(-j\alpha)[U(j\alpha) + \Upsilon]V(j\alpha)\bar{D} + \frac{1}{\gamma^2}T^T_{\boldsymbol{y}\bar{\boldsymbol{w}}}(-j\alpha)T_{\boldsymbol{y}\bar{\boldsymbol{w}}}(j\alpha) \leq 0 \tag{7.6}$$

and

$$T^T_{\boldsymbol{y}\bar{\boldsymbol{w}}}(-j\alpha)T_{\boldsymbol{y}\bar{\boldsymbol{w}}}(j\alpha) \leq \gamma^2 - \gamma\bar{D}^T V^T(-j\alpha)[U(j\alpha) + \Upsilon]V(j\alpha)\bar{D} \leq \gamma^2 I \tag{7.7}$$

for all $\alpha \in \mathbb{R}$, that is, $\|T_{\boldsymbol{y}\bar{\boldsymbol{w}}}\|_\infty \leq \gamma$. □

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