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LONG MEMORY AND VOLATILITY DYNAMICS IN THE US DOLLAR EXCHANGE RATE

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Abstract

This paper focuses on nominal exchange rates, specifically the US dollar rate vis-à-vis the Euro and the Japanese Yen at a daily frequency. We model both absolute values of returns and squared returns using long-memory techniques, being particularly interested in volatility modelling and forecasting given their importance for FOREX dealers. Compared with previous studies using a standard fractional integration framework such as Granger and Ding (1996), we estimate a more general model which allows for dependence not only at the zero but also at other frequencies. The results show differences in the behaviour of the two series: a long-memory cyclical model and a standard I(d) model seem to be the most appropriate for the US dollar rate vis-à-vis the Euro and the Japanese Yen respectively.

JEL Classification: C22, O40
Keywords: Fractional integration, Long memory, Exchange rates, Volatility

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1. Introduction

The empirical literature analysing the statistical properties of exchange rates is vast. Most studies focus on the behaviour of *real* exchange rates in order to establish whether it is consistent with the theory of Purchasing Power Parity (PPP), which is one of the central tenets of the theory of exchange rate determination. In particular, they test the null hypothesis that the real exchange rate follows a random walk, the alternative being that PPP holds in the long run. However, such unit root tests are now well known to have very low power, and to be unable to distinguish between random-walk behaviour and very slow mean-reversion in the PPP-consistent level of the real exchange rate (see, e.g., Frankel, 1986, and Lothian and Taylor, 1997), unless very long spans of data are used (see, e.g., Lothian and Taylor, 1996, and Cheung and Lai, 1994). Moreover, whilst in a flexible-price monetary model PPP is assumed to hold continuously, in a sticky-price model it holds only in the long run. Therefore the relevant issue to investigate is whether deviations from PPP are transitory or permanent.

As a result of the increasing awareness of the limitations of standard unit root tests as well as of possible frictions in foreign exchange markets, long-memory and fractional integration methods have been used much more frequently. For instance, applying R/S techniques to daily rates for the British pound, French franc and Deutsche mark, Booth, Kaen and Koveos (1982) found positive memory during the flexible exchange rate period (1973-1979) but negative one (i.e., anti-persistence) during the fixed exchange rate period (1965-1971). Cheung (1993) also found evidence of long-memory behaviour in foreign exchange markets during the managed floating regime. On the other hand, the results obtained by Baum, Barkoulas and Caglayan (1999) estimating an ARFIMA model for real exchange rates in the post-Bretton Woods era do not support long-run PPP.

Other studies focus on the behaviour of *nominal* exchange rates. In this case, the main motivation is often building a model with better forecasting properties, rather than test theories of exchange rate determination, and in particular the financial modelling and forecasting of exchange rate volatility. This is because, from a dealer's perspective, what is of interest is not so much the ability to predict fluctuations in the exchange rate level, but rather in its volatility. The reason is that generally dealers (and also fund managers) when managing FOREX books (or diversified portfolios) can easily hedge their cash positions by using the derivatives market.[1] However, because of the hedging positions, they may incur substantial losses if volatility remains flat. Forecasting FOREX volatility adequately has acquired additional importance for market participants as a result of Basel II, which has introduced a Revised Framework for International Convergence of Capital Measurement and Capital Standards. The reason is that the new framework relies to a greater extent on the assessment of market risk provided by banks and market participants themselves for capital calculations, and gives them the option to choose between various approaches to determining the (minimum) capital requirements. Within the new regulatory framework the more accurately a company can identify and measure its risk exposure in the market, the lower the cost of raising funds it faces.

Some examples of recent studies analysing nominal exchange rate dynamics using fractional integration (looking at futures in particular) are those by Fang, Lai and

[1] With the current amount of liquidity in most FOREX markets, the cost of hedging a cash position has become relatively low.

Lai (1994), Crato and Ray (2000) and Wang (2004). Volatility dynamics in foreign exchange rates (mainly the Deutsche mark vis-à-vis US dollar rate) have also been examined with the FIGARCH-model, introduced by Baillie, Bollerslev and Mikkelsen (1996), and subsequent papers using this approach are Andersen and Bollerslev (1997, 1998), Tse (1998 – examining the Japanese Yen-US dollar rate), Baillie, Cecen and Han (2000), Kihc (2004) and Morana and Beltratti (2004 – analysing volatility).

The present study also focuses on nominal exchange rates, specifically the US dollar rate vis-à-vis the Euro and the Japanese Yen at a daily frequency. We model both absolute values of returns and squared returns using long-memory techniques, being particularly interested in volatility modelling and forecasting given their importance for FOREX dealers. Compared with previous studies using a standard fractional integration framework such as Granger and Ding (1996), we estimate a more general model which allows for dependence not only at the zero but also at other frequencies.

The layout of the paper is the following. Section 2 describes the methodology. Section 3 presents the empirical results. Section 4 examines the stability of the relationships over time. Section 5 examines the forecasting properties of the estimated models, while Section 6 offers some concluding remarks.

2. Methodology

Given a covariance stationary process $\{x_t, t = 0, \pm 1, \ldots \}$, with autocovariance function $E[(x_t - Ex_t)(x_{t-j} - Ex_t)] = \gamma_j$, according to McLeod and Hipel (1978), x_t displays the property of long memory if

$$\lim_{T \to \infty} \sum_{j=-T}^{T} \left| \gamma_j \right|$$

is infinite. An alternative definition, based on the frequency domain is as follows. Suppose that x_t has an absolutely continuous spectral distribution, and therefore a spectral density function, denoted by f(λ), and defined as

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \cos \lambda j, \qquad -\pi < \lambda \leq \pi.$$

Then, x_t displays long memory if this function has a pole at some frequency λ in the interval [0, π]. Most of the empirical literature has focused on the case when the singularity or pole in the spectrum occurs at the zero frequency. This is true of standard fractionally integrated or I(d) models of the form:

$$(1 - L)^d x_t = u_t, \qquad t = 0, \pm 1, \ldots, \qquad (1)$$

with $x_t = 0$, $t \leq 0$, and where L is the lag operator ($Lx_t = x_{t-1}$), d is a positive real value, and u_t is an I(0) process defined as a covariance stationary process with a spectral density function that is positive and bounded at all frequencies.[2] As previously mentioned these processes are characterised by a spectral density function which is unbounded at the zero frequency. They were first analysed in the 1960s when Granger (1966) and Adelman (1965) pointed out that most aggregate economic time series have

[2] The I(0) class of models includes the classical white noise process but also other structures allowing a weak dependence structure, such as the stationary autoregressive moving average (ARMA) models.

a typical shape where the spectral density increases dramatically as the frequency approaches zero. However, differencing the data frequently leads to over-differencing at the zero frequency.

However, a process may also display a pole or singularity in the spectrum at a frequency away from zero. In this case, the process may still display the property of long memory but the autocorrelations exhibit a cyclical structure that is decaying very slowly. This is the case of the Gegenbauer processes defined as:

$$(1 - 2\cos w_r L + L^2)^d x_t = u_t, \qquad t = 1, 2, ..., \tag{2}$$

where w_r and d are real values, and u_t is I(0). For practical purposes we define $w_r = 2\pi r/T$, with r = T/s, and thus s will indicate the number of time periods per cycle, while r refers to the frequency that has a pole or singularity in the spectrum of x_t. Note that if r = 0 (or s = 1), the fractional polynomial in (2) becomes $(1 - L)^{2d}$, which is the polynomial associated with the common case of fractional integration at the long-run or zero frequency. This type of process was introduced by Andel (1986) and subsequently analysed by Gray, Zhang and Woodward (1989, 1994), Chung (1996a,b) and Dalla and Hidalgo (2005) among many others.

Gray et al. (1989, 1994) showed that the polynomial in (2) can be expressed in terms of the Gegenbauer polynomial, such that, denoting $\mu = \cos w_r$, for all $d \neq 0$,

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j, \tag{3}$$

where $C_{j,d}(\mu)$ are orthogonal Gegenbauer polynomial coefficients recursively defined as:

$$C_{0,d}(\mu) \;=\; 1, \qquad\qquad C_{1,d}(\mu) \;=\; 2\,\mu\, d,$$

$$C_{j,d}(\mu) \;=\; 2\,\mu\left(\frac{d-1}{j}+1\right)C_{j-1,d}(\mu) \;-\; \left(2\frac{d-1}{j}+1\right)C_{j-2,d}(\mu), \quad j \;=\; 2, 3, \ldots.,$$

(see, for instance, Magnus et al., 1966, Rainville, 1960, etc. for further details on Gegenbauer polynomials). Gray et al. (1989) showed that x_t in (2) is (covariance) stationary if $d < 0.5$ for $|\mu = \cos w_r| < 1$ and if $d < 0.25$ for $|\mu| = 1$.[3] The model just presented can be generalised to the case of more than one cyclical structure to consider processes of the form:

$$\prod_{j=1}^{k}(1 - 2\cos w_r^{(j)} L + L^2)^{d_j}\, x_t \;=\; u_t, \qquad t \;=\; 1, 2, \ldots, \qquad (4)$$

where k is a finite integer indicating the maximum number of cyclical structures, and $w_r^{(j)} = 2\pi / s_{(j)}$ where $s_{(j)}$ indicates the number of time periods per cycle corresponding to the j^{th} cyclical structure. Empirical studies based on multiple cyclical structures of this type (also named k-factor Gegenbauer processes) are Ferrara and Guegan (2001), Sadek and Khotanzad (2004) and Gil-Alana (2007).

[3] Note that if $|\mu| < 1$ and d in (2) increases beyond 0.5, the process becomes "more nonstationary" in the sense, for example, that the variance of the partial sums increases in magnitude.

In this paper we also adopt a flexible specification that allows us to analyse long-memory models of the form (1) and (2) in a single framework. Specifically, we consider processes of the form:

$$(1 - L)^{d_1}(1 - 2\cos w_r L + L^2)^{d_2} x_t = u_t, \quad t = 1,2,\ldots, \qquad (5)$$

where u_t is again I(0), d_1 indicates the order of integration at the long-run or zero frequency, and d_2 refers to the cyclical long-run dependence component.

We employ a parametric approach developed by Robinson (1994) that is very general in the sense that it allows to consider all the above specifications in a single framework. This method, based on the Whittle function in the frequency domain, is briefly described in the Appendix. One advantage of Robinson's (1994) approach is that it is valid for any real value d (or d_1 and d_2 in (5)), thus encompassing stationary ($d < 0.5$) and nonstationary ($d \geq 0.5$) hypotheses. Moreover, the limiting distribution is standard (normal, in the cases of equations (1) and (2)) and chi-square in the case of (5)), and this limit behaviour holds independently of the inclusion or exclusion of deterministic terms in the model and the modelling approach for the I(0) disturbances. Moreover, Gaussianity is not a requirement, a moment condition of only 2 being necessary.

3. Empirical results

The time series data we examine are the US foreign exchange rates with respect to the Euro and the Japanese Yen, daily, for the time period January 4th, 1999 – October

2nd, 2009. These data were obtained from the Federal Reserve Bank of St. Louis database (DEXUSEU and DEXJPUS for the US-Euro and US-Yen rates respectively).

[Insert Figure 1 about here]

Plots of the two series are displayed in the upper half of Figure 1, while their corresponding returns, obtained as the first differences of the logged values, are shown in the bottom half. First, the order of integration of the log-series is estimated to determine if they contain unit roots. For this purpose, initially we carried out standard unit root tests (Dickey and Fuller, ADF, 1979; Phillips and Perron, PP, 1988; Elliot et al., 1996; and Ng and Perron, NP, 2001), finding evidence of unit roots (the results are not reported for brevity's sake) in the two series. However, it is well known that these procedures may have very low power if the true data generating processes are fractionally integrated. Therefore, we also performed tests that, unlike the above, are not based on autoregressive alternatives but on fractional ones. In particular, we considered a regression model of the form:

$$y_t = \alpha + \beta t + x_t; \qquad t = 1, 2, \ldots, \qquad (6)$$

where x_t is assumed to be fractionally integrated as in equation (1). Thus, if d = 1, the series displays a unit root process.

[Insert Table 1 about here]

Table 1 reports the estimates of d in (6) and (1) for the three standard cases of no regressors (i.e., $\alpha = \beta = 0$ a priori in equation (6)), an intercept (α unknown, and $\beta = 0$ a priori), and an intercept with a linear time trend (α and β unknown), under the assumption that the error term (u_t in (1)) follows a white noise process, an AR(1), and the exponential spectral model of Bloomfield (1973) in turn. The latter is a non-parametric specification that produces autocorrelations decaying exponentially as in the AR case and allows to approximate ARMA structures with a small number of parameters.[4]

We display in Table 1 the 95% confidence intervals formed by the non-rejection values of d, using Robinson's (1994) parametric approach in the frequency domain. We also present (in parentheses inside the square brackets) the Whittle estimates of d (Dahlhaus, 1989) in each case. It can be seen that the intervals almost always include the unit root, the only exceptions being the US dollar-Yen rate with an intercept and with a linear trend, where the estimated value of d is slightly below 1. In all other cases, the estimated d is around 1, hence supporting the unit root model and justifying the use of returns in the remainder of the paper.

[Insert Figure 2 about here]

In what follows we focus on the variance of the return series and examine the squared and absolute returns, which are used as proxies for volatility. These two measures have been widely employed in the financial literature to measure volatility.[5] Plots of these series are displayed in Figure 2.

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[4] See Gil-Alana (2004) for the use of fractional integration with Bloomfield disturbances in the context of Robinson's (1994) tests.

[5] Absolute returns were employed among others by Ding et al. (1993), Granger and Ding (1996), Bollerslev and Wright (2000), Gil-Alana (2005), Cavalcante and Assaf (2004), Sibbertsen (2004) and Cotter (2005), whereas squared returns were used in Lobato and Savin (1998), Gil-Alana (2003), Cavalcante and Assaf (2004) and Cotter (2005).

[Insert Figures 3 and 4 about here]

Figure 3 shows the first 1,000 sample autocorrelation values for the absolute and squared returns of the two series. It can be seen that the four series display some degree of dependence with these values decaying very slowly, which may be consistent with fractionally integrated processes of the form given by equation (1). Moreover, there is some type of cyclical structure (especially for the Euro returns) which may imply that models of the form given by (2) or even (5) may also be plausible for these series. The periodograms, displayed in Figure 4, have the highest values at the smallest frequencies, which is again an indication of possible I(d) behaviour with $d > 0$, though this may be obscuring other peaks at non-zero frequencies.

We start by presenting the results based on model 1, which is the one that displays long memory exclusively at the long-run or zero frequency, that is,

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, \ldots, \qquad \text{(M1)}$$

and, similarly to the results presented in Table 1, we consider the three cases of no regressors, an intercept, and an intercept with a linear trend, assuming that the disturbances follow a white noise, an AR(1) and a Bloomfield-type process in turn.[6] The results are displayed in Tables 2 and 3 for the absolute and squared returns respectively.

[Insert Tables 2 and 3 about here]

[6] When using higher AR orders very similar results were obtained.

Starting with the absolute returns (in Table 2), the estimated values of d are in all cases strictly positive and smaller than 0.5, i.e. inside the stationary region, though with some degree of long-memory behaviour. When not allowing for autocorrelation the estimated value of d is around 0.10 for the US dollar-Euro rate, and is slightly higher for the US dollar-Yen one. If autocorrelation is allowed, in the form of either an AR process or of the Bloomfield model, the values of d are higher and close to 0.2 in the two series. Very similar results are obtained in Table 3 for the squared returns, with values close to 0.1 with uncorrelated errors and close to 0.2 with weak autocorrelation. Finally, regarding the deterministic terms (not reported), the time trend coefficients were found to be insignificant in all cases, while the intercept was statistically significant at the 5% level, implying that the model including an intercept is the one that should be selected in all cases.

Because of the differences in the results depending on how we specify the error term, we also applied a semi-parametric method (Robinson, 1995) where the disturbances u_t are simply assumed to be I(0) with no functional form required for them. This method is based on a "local" Whittle estimate in the frequency domain; it considers a band of frequencies that degenerates to zero, and the estimate of d is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2\, d \, \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right),$$

$$for \;\; d \in (-1/2\,,\, 1/2)\,; \quad \overline{C(d)} = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j)\, \lambda_j^{2d}, \quad \lambda_j = \frac{2\,\pi\, j}{T}, \;\; \frac{m}{T} \to 0,$$

where m is a bandwidth parameter.[7]

[Insert Figure 5 about here]

Figure 5 displays for each series the estimates of d based on the above procedure using the whole range of parameters for the bandwidth (displayed on the horizontal axis)[8], including the 95% confidence interval corresponding to the I(0) case. It is clear that the four series exhibit long-memory (d > 0) behaviour, consistently with the results based on the parametric approach outlined above and with other studies such as Granger and Ding (1996).

[Insert Figure 6 about here]

Next we consider a cyclical long-memory model of the form given by equation (2). This is motivated by the periodograms of the series.[9] Figure 6 displays the first 100 values of the periodogram for the Fourier frequencies $\lambda_r = 2\pi r/T$, ($r = T/s$), for $r = 1, \ldots, 100$. It is noteworthy that for the US Dollar-Euro case the highest value of the periodogram does not occur at the smallest frequency ($r = 1$) but instead at $r = 4$, which should correspond to cycles with a periodicity of $T/4 \approx 677$ periods (days). By contrast, for the US Dollar-Yen case the highest value is found at the smallest frequency $r = 1$, followed by $r = 19$ ($T/19 \approx 142$ periods). Therefore, <u>model 2</u> is specified as:

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7 Further refinements of this approach can be found in Velasco (1999), Phillips and Shimotsu (2004, 2005); etc. Applying some of these methods we obtain almost identical results to those reported here.

8 The choice of the bandwidth is crucial since it affects the trade-off between bias and variance; specifically, the asymptotic variance and the bias of this estimator are decreasing and increasing with m respectively.

9 Note that the periodogram is an asymptotic unbiased (though not consistent) estimate of the spectral density function.

$$y_t = \alpha + x_t; \quad (1 - 2\cos w_r L + L^2)^d x_t = u_t, \quad t = 1, 2, ..., \qquad \text{(M2)}$$

with $w_r = 2\pi/677$ in case of the US Dollar-Euro absolute and squared return series and $w_r = 2\pi/142$ for the US Dollar-Yen values. The results using the above model are displayed in Tables 4 and 5.

[Insert Tables 4 and 5 about here]

Two models, without regressors ($\alpha = 0$ in (M2)) and with an intercept, are considered. We employ here another version of Robinson's (1994) parametric tests, testing the null hypothesis H_o: $d = d_o$, in (M2) for a range of values of d_o from 0 to 1 with 0.001 increments, and s in w_r equal to 600, …, 700 for the US Dollar-Euro case, and s = 100, …, 200 for the US Dollar-Yen one. We select the model that produces the lowest statistic in Robinson (1994) for different values for s and d. It is noteworthy that the estimated values of s are equal to 677 and 142 respectively for the two series, which correspond to some of the highest peaks in the periodograms displayed in Figure 6 (specifically, the highest peak for the US dollar-Euro rate, and the second highest for the US dollar- Yen rate).

Starting with the absolute values of the returns (see Table 4), we find that the differencing parameter is strictly positive and significant, though very close to 0 in all cases: the estimated values of d are 0.035 (US Dollar-Euro) and 0.049 (US Dollar-Yen) for the cases of white noise and Bloomfield disturbances, and 0.075 (US Dollar-Euro) and 0.080 (US Dollar-Yen) with AR(1) errors. For the squared returns (Table 5) the values are again significant though slightly higher: 0.042 (US Dollar-Euro) and 0.050

(US Dollar-Yen) with uncorrelated and Bloomfield errors, and 0.092 (US Dollar-Euro) and 0.083 (US Dollar-Yen) with AR(1) disturbances. Once more, the intercepts are statistically significant in all cases.

Finally we examine the case of a long-memory model that simultaneously takes into account the long-run and the cyclical structures. Therefore, model 3 is specified as:

$$y_t = \alpha + x_t; \quad (1 - L)^{d_1} (1 - 2\cos w_r L + L^2)^{d_2} x_t = u_t, \quad t = 1, 2, ..., \qquad \text{(M3)}$$

once more focusing on the cases of no regressors (α = 0 in (M3)) and an intercept, for uncorrelated and correlated (AR and Bloomfield) errors.

[Insert Tables 6 and 7 about here]

The results based on (M3) are displayed in Tables 6 and 7. Interestingly, the selected models are once more those for the frequency r that corresponds to s = 677 for the US Dollar-Euro series case and to s = 142 for the US Dollar-Yen one. Concerning the estimates of the fractional differencing parameters, for the US Dollar-Euro d_1 is not significantly different from zero, and the same holds for d_2 in the case of the US Dollar-Yen. Therefore, model 2 and model 1 appear to be the most adequate ones for the US Dollar-Euro and the US Dollar-Yen cases respectively. We also perform LR tests to choose between models 1 and 3 for the US Dollar-Yen, and between models 2 and 3 for the US Dollar-Euro; these provide further evidence that model 2 (long-run cyclical dependence) is more appropriate for the US Dollar-Euro (absolute and squared) returns, and model 1 (standard I(d)) for the US Dollar-Yen values.

On the basis of this evidence as well as the t-values for the deterministic terms we choose the models below. For the US dollar-Euro series:

$$y_t = 0.00479 + x_t; \quad (1 - 2\cos w_4 L + L^2)^{0.036} x_t = u_t, \quad t = 1, 2, ..., \tag{7}$$

in the case of the absolute returns, and

$$y_t = 0.000042 + x_t; \quad (1 - 2\cos w_4 L + L^2)^{0.042} x_t = u_t, \quad t = 1, 2, ..., \tag{8}$$

for the squared returns.

However, for the US dollar-Yen values, a model with long memory only at the zero frequency seems to be more adequate, namely

$$y_t = 0.00526 + x_t; \;\; (1 - L)^{0.185} x_t = u_t; \;\; u_t = -0.144 u_{t-1} + \varepsilon_t, \;\; t = 1, 2, ..., \tag{9}$$

for the absolute returns, and

$$y_t = 0.000051 + x_t; \;\; (1 - L)^{0.186} x_t = u_t; \;\; u_t = -0.139 u_{t-1} + \varepsilon_t, \;\; t = 1, 2, ..., \tag{10}$$

for the squared values.

Clearly, both volatility series are characterised by long memory, but in case of the US dollar – Yen rate this affects the long run structure of the process, while in case of the US dollar – Euro there is an underlying cyclical structure.

4. Stability tests and structural breaks

In this section we examine whether the results reported in Section 3 are stable over the sample period or instead subject to structural change. For this purpose we performed once more the versions of Robinson's (1994) tests employed in Section 3, using the specifications described above, starting with a sample of 1,500 observations and then adding recursively five observations each time till the end of the sample (with 2,710 observations). We report in Figure 7 the estimated values of d for the absolute return series, for the Euro case (the upper plot) and for the Japanese Yen (in the lower part of the figure) respectively. In the former case, we employ a model of a similar form to the one given by equation (8), i.e., using cyclical fractional integration,

$$y_t = \mu + x_t; \quad (1 - 2\cos w_r L + L^2)^d x_t = u_t, \quad t = 1, 2, \ldots,$$

while in the latter case (Japanese yen absolute returns) we use a model similar to equation (10), i.e., based on a standard I(d) model,

$$y_t = \mu + x_t; \quad (1 - L)^d x_t = u_t; \qquad u_t = \rho\, u_{t-1} + \varepsilon_t, \qquad t = 1, 2, \ldots .$$

[Insert Figure 7 about here]

As can be seen, for the US dollar-Euro absolute returns the estimated value of the (cyclical) fractional differencing parameter remains close to 0 (and statistically insignificant) for each subsample until the one ending at the observation 2035, which corresponds to February 5, 2007. If observations after that date are included, the estimate is above 0.010, becoming significantly different from 0 for each subsample till the end of the sample period, with another increase at observation 2460 (October 6, 2008). Focusing now on the US dollar-Yen case, the estimate of the fractional differencing parameter, d, is relatively stable till observation 2320 (March 20, 2008), with values around 0.12; there is then an increase (with values close to 0.15) till observation 2460 (October 6, 2008), and another one (d about 0.20) until the end of the sample. Similar results (not reported for reasons of space) were obtained for the squared returns.

Because of the instability in the estimated fractional differencing parameter (see Figure 7) in what follows we consider three different subsamples for each series. These are: for the US dollar-Euro, [January 4, 1999 – February 5, 2007]; [February 6, 2007 – October 6, 2008] and [October 7, 2008 – October 2, 2009], and for the US dollar-Yen [January 4, 1999 – March 20, 2008], [March 21, 2008 – October 6, 2008] and [October 7, 2008 – October 2, 2009].

[Insert Table 8 about here]

Table 8 displays the estimates of the long-run and the cyclical fractional differencing parameters using model 1 and model 2 for each subsample and each series.

The upper and lower half of the table concern the absolute and squared returns respectively. Considering the subsamples separately it can be seen that some of the estimates are statistically significant, especially in the case of model 1 (with long memory at the long-run or zero frequency). Also, the estimated value of d for model 1 is higher in the second subsample and lower in the third subsample for the US dollar-Euro rate (for both absolute and squared returns), whilst for the US dollar-Yen rate there is a decrease in the second subsample and an increase in the third one (see Table 8).

5. Forecasting performance

In this section we examine the forecasting accuracy of the models presented in previous sections. For this purpose, we consider for each of the four series (i.e. the absolute and squared returns of the US dollar exchange rates against the Euro and the Japanese Yen) the three models that have been presented in Section 3, i.e., model 1 (M1): fractional integration at the zero frequency; model 2 (M2): fractional cyclical integration; and model 3 (M3): fractional integration at both the zero and the cyclical frequencies.

We perform an in-sample forecasting experiment to establish which of the three models (M1, M2 or M3) performs best for each series. First, we computed the root mean squared errors for the last 100 observations in the sample. Then, we computed the modified Diebold and Mariano (M-DM, 1995) statistic as suggested by Harvey, Leybourne and Newbold (1997).[10]

[10] Harvey et al. (1997) and Clark and McCracken (2001) show that this modified test statistic performs better than the DM test statistic in finite samples, and also that the power of the test is improved when p-values are computed with a Student t-distribution.

Using the M-DM test statistic, we evaluate the relative forecast performance of the different models by making pairwise comparisons. We use the root mean squared errors in the computations. The results are displayed in Tables 9, 10 and 11 respectively for 50, 75 and 100-period ahead predictions.

[Tables 9 - 11 near here]

For each prediction-horizon we indicate in the tables in bold the rejections of the null hypothesis that the forecast performance of model (Mi) and model (Mj) is equal in favour of the one-sided alternative that model (Mi)'s performance is superior at the 5% significance level. The results for the three time horizons are consistent with the conclusions based on the estimation results of Section 3: model 2 (M2), i.e., the cyclical fractionally integrated one, seems to be the most adequate specification for the US dollar- Euro absolute and squared returns, while model 1 (M1), the standard I(d) model, is the preferred one for the two US dollar/ Yen returns series.

6. Conclusions

This paper has applied long-memory methods to analyse the US dollar rate vis-à-vis the Euro and the Japanese Yen at a daily frequency, with particular attention being paid to volatility modelling and forecasting given its importance for FOREX dealers. Specifically, we have estimated a more general fractional integration model compared with previous studies, allowing for dependence not only at the zero but also at other frequencies. The results show differences in the behaviour of the two series: a long–memory (Gegenbauer) process capturing the underlying cyclical structure and a standard I(d) model seem to be the most appropriate for the US dollar rate vis-à-vis the Euro and the Japanese Yen respectively. Consequently, mean reversion with hyperbolical decay occurs in both cases in response to exogenous shocks to the volatility process, but in the former cyclicality is present. The in-sample forecasting analysis also indicates that the cyclical fractional model outperforms other models in case of the Euro return series, while a standard I(d) model outperforms other long memory models in the case of the Yen returns.

The analysis carried out in this paper can be extended to allow for non-linear structures and possible structural breaks (whose presence is suggested by some of the evidence presented here); for detecting the latter the method suggested by Gil-Alana (2008) for breaks in fractionally integrated models or the Markov-Switching approach proposed by Tsay and Hardle (2009) could be applied.

References

Adelman, I (1965). “Long cycles: Fact or artifacts” American Economic Review, 55: 444-463.

Andel, J., 1986, Long memory time series models, Kybernetika 22, 105-123.

Andersen, T.G. and T. Bollerslev (1997) Heterogeneous information arrivals and return volatility dynamics: uncovering the long run in high frequency returns. The Journal of Finance 52, 975-1005.

Andersen, T.G. and T. Bollerslev (1998) Deutsche Mark-Dollar volatility: intraday activity patterns, macroeconomic announcements, and longer run dependencies. The Journal of Finance 53, 219-265.

Baillie, R.T., T. Bollerslev and H.O. Mikkelsen (1996) Fractionally integrated generalized autoregressive conditional heteroscedasticity. Journal of Econometrics 74, 3-30.

Baillie, R.T., A.A. Cecen and Y.W. Han (2000) High frequency Deutsche mark – US dollar return. FIGARCH representations and non-linearities. Multinational Finance Journal 4, 247-267.

Baum, C.F., J. Barkoulas and M. Caglayan (1999) Persistence in the international inflation rates. Southern Economic Journal 65, 900-913.

Bloomfield, P., 1973, An exponential model in the spectrum of a scalar time series, Biometrika 60, 217-226.

Bollerslev, T., Wright, J.H., 2000, High frecuency data, frequecy domain inference and volatility forecasting, Review of Economics and Statistics 83, 596-602.

Booth, G.G., F.R. Kaen and P.E. Koveos (1982) R/S analysis of foreign exchange markets under two international monetary regimes. Journal of Monetary Economics 10, 407-415.

Cavalcante, J., Assaf, A., 2004, Long range dependence in the returns and volatility of the Brazilian stock market, European Review of Economics and Finance 3, 5-22.

Cheung, Y.W. (1993) Long memory in foreign exchange rates. Journal of Business and Economic Statistics 11, 93-101.

Cheung, Y.-W. and K.S. Lai (1994), “Mean reversion in real exchange rates”, Economics Letters, 46, 3, 251-256.

Chung, C.-F., 1996a, A generalized fractionally integrated autoregressive moving-average process, Journal of Time Series Analysis 17, 111-140.

Chung, C.-F., 1996b, Estimating a generalized long memory process, Journal of Econometrics 73, 237-259.

Cotter, J., 2005, Uncovering long memory in high frequency UK futures, European Journal of Finance 11, 325-337.

Crato, N. and B.K. Ray (2000) Memory in returns and volatilities of future's contracts. Journal of Futures Markets 20, 525-543.

Dalla, V. and J. Hidalgo, 2005, A parametric bootstrap test for cycles, Journal of Econometrics 129, 219-261.

Dickey, D. A., and W. A. Fuller (1979). "Distribution of the estimators for autoregressive time series with a unit root". Journal of the American Statistical Association, 74: 427-431.

Ding, Z., Granger, C.W.J., Engle, R.F., 1993, A long memory property of stock markets and a new model, Journal of Empirical Finance 1, 83-106.

Elliot, G., T. Rothenberg and J. Stock, 1996, Efficient tests for an autoregressive unit root, Econometrica 64, 813-836.

Fang, H., K.S. Lai and M. Lai (1994) Fractal structure in currency futures price dynamics. Journal of Futures Markets 14, 169-181.

Ferrara, L. and D. Guegan (2001) Forecasting with k-factor Gegenbauer processes. Theory and Applications. Journal of Forecasting 20, 581-601.

Frankel, J. (1986), "International capital mobility and crowding out in the U.S. economy: imperfect integration of financial markets or goods markets?" in R. Hafer (ed.), How Open is the U.S. Economy?, Lexington Books, Lexington.

Gil-Alana, L.A., 2003, Fractional integration in the volatility of asset returns, European Review of Economics and Finance 2, 41-52.

Gil-Alana, L.A., 2004, The use of the Bloomfield (1973) model as an approximation to ARMA processes in the context of fractional integration, Mathematical and Computer Modelling 39, 429-436.

Gil-Alana, L.A., 2005, Long memory in daily absolute and squared returns in the Spanish stock market, Advances in Investment Analysis and Portfolio Management 1, 198-217.

Gil-Alana, L.A. (2007) Testing the existence of multiple cycles in financial and economic time series. Annals of Economics and Finance 1, 1-20.

Gil-Alana, L.A. (2008) Fractional integration and structural breaks at unknown periods of time, Journal of Time Series Analysis 29, 163-185.

Granger, C.W.J (1966). “The typical spectral shape of an economic variable”. Econometrica, 37: 150-161.

Granger, C.W.J., Ding, Z., 1996, Varieties of long memory models, Journal of Econometrics 73, 61-78.

Gray, H.L., Yhang, N., Woodward, W.A. (1989) On generalized fractional processes, Journal of Time Series Analysis, 10, pp. 233-257.

Gray, H.L., Yhang, N., Woodward, W.A. (1994) On generalized fractional processes. A correction, Journal of Time Series Analysis, 15, pp. 561-562.

Kihc, R. (2004) On the long memory properties of emerging capital markets. Evidence from Istambul exchange. Applied Financial Economics 14, 915-922.

Lobato, I.N., Savin, N.E., 1998, Real and spurious long memory properties of stock market data, Journal of Business and Economic Statistics 16, 261-268.

Lothian, J.R. and M. P. Taylor (1996), “Real exchange rate behavior: the recent float from the perspective of the past two centuries”, Journal of Political Economy, 104, 3, 488-509.

Lothian, J.R. and M. P. Taylor (1997), “Real exchange rate behavior”, Journal of International Money and Finance, 16, 6, 945-954.

Magnus, W., Oberhettinger, F. and R.P. Soni, 1966, Formulas and theorema for the special functions of mathematical physics, Springer, Berlin.

McLeod, A.I. and K.W. Hipel, 1978 Preservation of the rescaled adjusted range. A reassessment of the Jurst phenomenon, Water Resources Research 14, 491-507.

Morana, C. and A. Beltratti (2004) Structural change and long range dependence in volatility of exchange rates, either, neither or both?. Journal of Empirical Finance 11, 629-658.

Ng, S. and P. Perron (2001): “Lag selection and the construction of unit root tests with good size and power”, Econometrica, vol. 69, pp. 1519-1554.

Phillips, P. C. B., and P. Perron (1988). “Testing for a unit root in a time series regression”. Biometrika, vol. 75, pp. 335-346.

Phillips, P.C.B. and K. Shimotsu, 2004. Local Whittle estimation in nonstationary and unit root cases. Annals of Statistics 32, 656-692.

Phillips, P.C.B. and K. Shimotsu, 2005. Exact local Whittle estimation of fractional integration. Annals of Statistics 33, 1890-1933.

Rainville, E.D., 1960, Special functions, MacMillan, New York.

Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association 89, 1420-1437.

Robinson, P.M., 1995, Gaussian semi-parametric estimation of long range dependence, Annals of Statistics 23, 1630-1661.

Sadek, N. and A. Khotanzad (2004) K-factor Gegenbauer ARMA process for network traffic simulation. Computers and Communications 2, 963-968.

Sibbertsen, P., 2004, Long memory in volatilities of German stock returns, Empirical Economics 29, 477-488.

Tsay, W. and W. Hardle, 2009, A generalized ARFIMA process with Markov-Switching fractional differencing parameter, Journal of Statistical Computation and Simulation, forthcoming.

Tse, Y.K. (1998) The conditional heteroskedasticity of the Yen-Dollar exchange rate. Journal of Applied Econometrics 13, 49-55.

Velasco, C., 1999. Gaussian semiparametric estimation of nonstationary time series, Journal of Time Series Analysis 20, 87-127.

Wang, C. (2004) Futures trading activity and predictable foreign exchange movements. Journal of Banking and Finance 28, 1023-1041.

Table 1: Estimates of d in the log exchange rates series

i) White noise disturbances			
	No regressors	An intercept	A time trend
Log of US-Euro	**[0.973 (0.996) 1.022]**	**[0.985 (1.007) 1.032]**	**[0.985 (1.007) 1.032]**
Log of US-Yen	**[0.974 (0.997) 1.023]**	[0.946 (0.969) 0.994]	[0.946 (0.969) 0.994]
ii) AR(1) disturbances			
	No regressors	An intercept	A time trend
Log of US-Euro	**[0.927 (0.970) 1.017]**	**[0.971 (1.005) 1.043]**	**[0.971 (1.005) 1.043]**
Log of US-Yen	**[0.980 (0.999) 1.018]**	**[0.940 (0.976) 1.018]**	**[0.940 (0.976) 1.018]**
iii) Bloomfield disturbances			
	No regressors	An intercept	A time trend
Log of US-Euro	**[0.949 (0.992) 1.032]**	**[0.971 (1.008) 1.041]**	**[0.971 (1.008) 1.041]**
Log of US-Yen	**[0.953 (0.991) 1.032]**	**[0.943 (0.980) 1.022]**	**[0.943 (0.980) 1.022]**

In brackets the 95% confidence interval for the values of d. In parentheses, the Whittle estimates. We report in bold the cases where the unit root hypothesis cannot be rejected.

Table 2: Estimates of d in model (M1) using the absolute returns

i) White noise disturbances			
	No regressors	An intercept	A time trend
US-Euro	**[0.090 (0.103) 0.118]**	**[0.086 (0.098) 0.112]**	**[0.086 (0.098) 0.112]**
US-Yen	**[0.113 (0.130) 0.148]**	**[0.101 (0.116) 0.133]**	**[0.100 (0.115) 0.132]**
ii) AR(1) disturbances			
	No regressors	An intercept	A time trend
US-Euro	**[0181 (0.201) 0.224]**	**[0.170 (0.188) 0.209]**	**[0.170 (0.188) 0.209]**
US-Yen	**[0.186 (0.212) 0.241]**	**[0.162 (0.185) 0.212]**	**[0.160 (0.184) 0.212]**
iii) Bloomfield disturbances			
	No regressors	An intercept	A time trend
Log of US-Euro	**[0.204 (0.230) 0.259]**	**[0.189 (0.209) 0.236]**	**[0.189 (0.209) 0.236]**
Log of US-Yen	**[0.198 (0.228) 0.259]**	**[0.169 (0.196) 0.226]**	**[0.168 (0.196) 0.226]**

Table 3: Estimates of d in model (M1) using the squared returns

i) White noise disturbances			
	No regressors	An intercept	A time trend
US-Euro	**[0.092 (0.105) 0.118]**	**[0.093 (0.106) 0.120]**	**[0.091 (0.104) 0.120]**
US-Yen	**[0.010 (0.116) 0.135]**	**[0.097 (0.114) 0.132]**	**[0.095 (0.112) 0.131]**
ii) AR(1) disturbances			
	No regressors	An intercept	A time trend
US-Euro	**[0.190 (0.210) 0.236]**	**[0.190 (0.212) 0.237]**	**[0.188 (0.211) 0.235]**
US-Yen	**[0.163 (0.191) 0.224]**	**[0.158 (0.186) 0.218]**	**[0.156 (0.184) 0.217]**
iii) Bloomfield disturbances			
	No regressors	An intercept	A time trend
US-Euro	**[0.201 (0.228) 0.351]**	**[0.201 (0.228) 0.352]**	**[0.200 (0.227) 0.356]**
US-Yen	**[0.163 (0.193) 0.235]**	**[0.160 (0.189) 0.221]**	**[0.159 (0.188) 0.221]**

Table 4: Estimates of model (M2) using the absolute returns

	i) White noise disturbances				
Series	No regressors		With an intercept		
	j	d	j	d	μ
US-Euro	**677**	**[0.029 (0.035) 0.043]**	**677**	**[0.029 (0.036) 0.043]**	**0.00479 (41.358)**
US-Yen	**142**	**[0.040 (0.049) 0.059]**	**142**	**[0.040 (0.049) 0.059]**	**0.00504 (42.230)**
	ii) AR(1) disturbances				
Series	No regressors		With an intercept		
	j	d	j	d	μ
US-Euro	**677**	**[0.064 (0.075) 0.087]**	**677**	**[0.065 (0.076) 0.088]**	**0.00478 (28.773)**
US-Yen	**142**	**[0.064 (0.080) 0.097]**	**142**	**[0.067 (0.082) 0.099]**	**0.00503 (34.515)**
	iii) Bloomfield disturbances				
Series	No regressors		With an intercept		
	j	d	j	d	μ
US-Euro	**677**	**[0.032 (0.035) 0.040]**	**677**	**[0.032 (0.036) 0.040]**	**0.00479 (41.358)**
US-Yen	**142**	**[0.043 (0.049) 0.054]**	**142**	**[0.044 (0.049) 0.055]**	**0.00503 (42.230)**

Table 5: Estimates of model (M2) using the squared returns

	i) White noise disturbances				
Series	No regressors		With an intercept		
	j	d	j	d	μ
US-Euro	**677**	**[0.035 (0.042) 0.049]**	**677**	**[0.036 (0.042) 0.049]**	**0.000042 (16.661)**
US-Yen	**142**	**[0.040 (0.050) 0.060]**	**142**	**[0.040 (0.050) 0.060]**	**0.000047 (15.897)**
	ii) AR(1) disturbances				
Series	No regressors		With an intercept		
	j	d	j	d	μ
US-Euro	**677**	**[0.080 (0.092) 0.104]**	**677**	**[0.080 (0.092) 0.105]**	**0.000041 (10.528)**
US-Yen	**142**	**[0.066 (0.083) 0.103]**	**142**	**[0.066 (0.084) 0.103]**	**0.000047 (12.908)**
	iii) Bloomfield disturbances				
Series	No regressors		With an intercept		
	j	d	J	d	μ
US-Euro	**677**	**[0.038 (0.042) 0.046]**	**677**	**[0.038 (0.042) 0.046]**	**0.000042 (16.661)**
US-Yen	**142**	**[0.044 (0.050) 0.056]**	**142**	**[0.044 (0.050) 0.056]**	**0.000047 (15.897)**

Table 6: : Estimates of model (M3) using the absolute returns

	i) White noise disturbances			
Series	No regressors		With an intercept	
	d_1 (long run)	d_2 (cyclical)	d_1 (long run)	d_2 (cyclical)
US-Euro (j = 677)	**0.011** **[-0.066, 0.098]**	**0.087** **[0.048, 0.101]**	**0.007** **[-0.059, 0.093]**	**0.078** **[0.051, 0.093]**
US-Yen (j = 142)	**0.136** **[0.108, 0.153]**	**0.009** **[-0.017, 0.039]**	**0.131** **[0.101, 0.148]**	**0.012** **[-0.013, 0.027]**
	ii) AR(1) disturbances			
Series	No regressors		With an intercept	
	d_1 (long run)	d_2 (cyclical)	d_1 (long run)	d_2 (cyclical)
US-Euro (j = 677)	**0.004** **[-0.032, 0.066]**	**0.093** **[0.050, 0.107]**	**0.006** **[-0.036, 0.077]**	**0.088** **[0.045, 0.099]**
US-Yen (j = 142)	**0.127** **[0.101, 0.151]**	**0.004** **[-0.032, 0.051]**	**0.127** **[0.103, 0.155]**	**0.003** **[-0.033, 0.057]**
	iii) Bloomfield disturbances			
Series	No regressors		With an intercept	
	d_1 (long run)	d_2 (cyclical)	d_1 (long run)	d_2 (cyclical)
US-Euro (j = 677)	**0.014** **[-0.061, 0.111]**	**0.092** **[0.032, 0.104]**	**0.006** **[-0.064, 0.097]**	**0.081** **[0.049, 0.096]**
US-Yen (j = 142)	**0.142** **[0.109, 0.166]**	**0.007** **[-0.022, 0.051]**	**0.139** **[0.097, 0.159]**	**0.014** **[-0.016, 0.030]**

Table 7: : Estimates of model (M3) using the squared returns

	i) White noise disturbances			
Series	No regressors		With an intercept	
	d_1 (long run)	d_2 (cyclical)	d_1 (long run)	d_2 (cyclical)
US-Euro (j = 677)	**0.007** **[-0.036, 0.088]**	**0.071** **[0.033, 0.094]**	**0.008** **[-0.045, 0.043]**	**0.073** **[0.035, 0.098]**
US-Yen (j = 142)	**0.117** **[0.098, 0.136]**	**0.004** **[-0.011, 0.024]**	**0.116** **[0.095, 0.138]**	**0.003** **[-0.017, 0.027]**
	ii) AR(1) disturbances			
Series	No regressors		With an intercept	
	d_1 (long run)	d_2 (cyclical)	d_1 (long run)	d_2 (cyclical)
US-Euro (j = 677)	**0.010** **[-0.054, 0.103]**	**0.091** **[0.038, 0.116]**	**0.009** **[-0.055, 0.089]**	**0.086** **[0.046, 0.111]**
US-Yen (j = 142)	**0.129** **[0.114, 0.161]**	**0.005** **[-0.028, 0.044]**	**0.129** **[0.113, 0.155]**	**0.004** **[-0.041, 0.036]**
	iii) Bloomfield disturbances			
Series	No regressors		With an intercept	
	d_1 (long run)	d_2 (cyclical)	d_1 (long run)	d_2 (cyclical)
US-Euro (j = 677)	**0.011** **[-0.049, 0.091]**	**0.074** **[0.031, 0.110]**	**0.008** **[-0.045, 0.043]**	**0.075** **[0.033, 0.096]**
US-Yen (j = 142)	**0.121** **[0.100, 0.144]**	**0.006** **[-0.015, 0.025]**	**0.120** **[0.099, 0.142]**	**0.005** **[-0.014, 0.025]**

Table 8: : Estimates of models(M1) and (M2) for each subsample

	i) Absolute returns			
Series	US – Euro		US – Yen	
	Model 1	Model 2	Model 1	Model 2
1st sub-sample	**d = 0.109 (0.084, 0.137)**	d= -0.001 (j=1017) (-0.007, 0.009)	**d = 0.155 (0.127, 0.186)**	**d= 0.034 (j=1160) (0.027, 0.042)**
2nd sub-sample	**d = 0.189 (0.135, 0.258)**	d= -0.057 (j = 17) (-0.088, -0.024)	d = 0.097 (-0.087, 0.341)	d= -0.017 (j = 4) (-0.097, 0.078)
3rd sub-sample	d = -0.174 (-.281, -.035)	d = -0.064 (j = 7) (-0.103, 0.011)	**d = 0.129 (0.035, 0.247)**	**d= 0.043 (j=19) (0.004, 0.086)**
	ii) Squared returns			
Series	US – Euro		US – Yen	
	Model 1	Model 2	Model 1	Model 2
1st sub-sample	**d = 0.090 (0.065, 0.119)**	d= -0.001 (j=1017) (-0.013, 0.009)	**d = 0.159 (0.127, 0.194)**	**d= 0.042 (j=1160) (0.034, 0.053)**
2nd sub-sample	**d = 0.239 (0.179, 0.311)**	d= -0.045 (j = 17) (-0.073, -0.014)	d = -0.116 (-0.284, 0.351)	d= -0.001 (j = 4) (-0.094, 0.108)
3rd sub-sample	**d = 0.198 (0.112, 0.325)**	d = -0.040 (j = 7) (-0.083, 0.007)	d = 0.106 (-0.022, 0.269)	d= 0.022 (j=124) (-0.004, 0.053)

Table 9. Pairwise comparison using the modified DM statistic (RMSE, h = 50)

Absolute returns								
$/Euro	(M1)	(M2)	(M3)		$ / Yen	(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX		(M1)	XXXX	XXXX	XXXX
(M2)	2.45 **(M2)**	XXXX	XXXX		(M2)	-2.9 **(M1)**	XXXX	XXXX
(M3)	1.156	-2.0 **(M2)**	XXXX		(M3)	-2.3 **(M1)**	0.133	XXXX
Series 3					**Series 4**			
$/Euro	(M1)	(M2)	(M3)		$ / Yen	(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX		(M1)	XXXX	XXXX	XXXX
(M2)	2.71 **(M2)**	XXXX	XXXX		(M2)	-2.8 **(M1)**	XXXX	XXXX
(M3)	1.177	-2.2 **(M2)**	XXXX		(M3)	-2.1 **(M1)**	0.987	XXXX

Table 10. Pairwise comparison using the modified DM statistic (RMSE, h = 75)

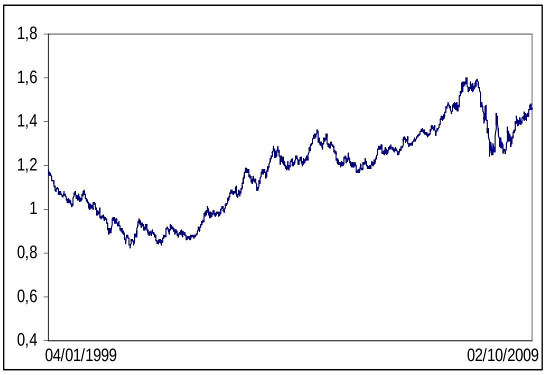
Absolute returns								
$/Euro	(M1)	(M2)	(M3)		$ / Yen	(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX		(M1)	XXXX	XXXX	XXXX
(M2)	3.55 **(M2)**	XXXX	XXXX		(M2)	-3.1 **(M1)**	XXXX	XXXX
(M3)	1.77	-2.4 **(M2)**	XXXX		(M3)	-2.7 **(M1)**	1.437	XXXX
Series 3					**Series 4**			
$/Euro	(M1)	(M2)	(M3)		$ / Yen	(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX		(M1)	XXXX	XXXX	XXXX
(M2)	3.69 **(M2)**	XXXX	XXXX		(M2)	-3.1 **(M1)**	XXXX	XXXX
(M3)	1.68	-2.8 **(M2)**	XXXX		(M3)	-2.9 **(M1)**	1.606	XXXX

Table 11. Pairwise comparison using the modified DM statistic (h = 100)

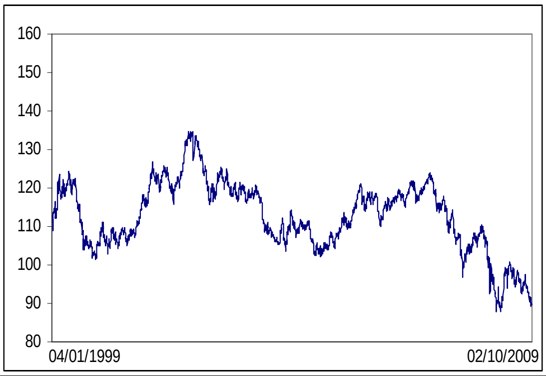
Absolute returns								
$/Euro	(M1)	(M2)	(M3)		$ / Yen	(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX		(M1)	XXXX	XXXX	XXXX
(M2)	3.87 **(M2)**	XXXX	XXXX		(M2)	-3.5 **(M1)**	XXXX	XXXX
(M3)	2.08 **(M3)**	-3.4 **(M2)**	XXXX		(M3)	-3.2 **(M1)**	1.454	XXXX
Series 3					**Series 4**			
$/Euro	(M1)	(M2)	(M3)		$ / Yen	(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX		(M1)	XXXX	XXXX	XXXX
(M2)	3.94 **(M2)**	XXXX	XXXX		(M2)	-3.4 **(M1)**	XXXX	XXXX
(M3)	2.11 **(M3)**	-3.6 **(M2)**	XXXX		(M3)	-3.0 **(M1)**	1.239	XXXX

Figure 1: Foreign exchange rate time series and their corresponding returns

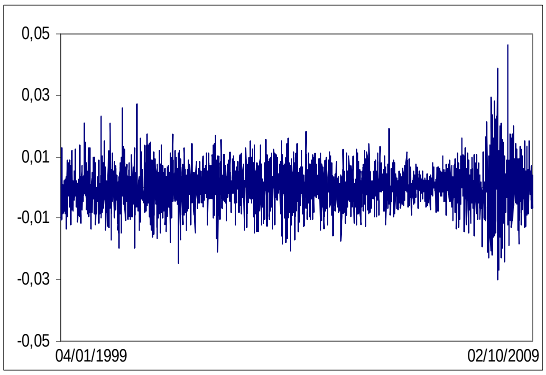
US – Euro



US – Yen



US – Euro returns



US – Yen returns

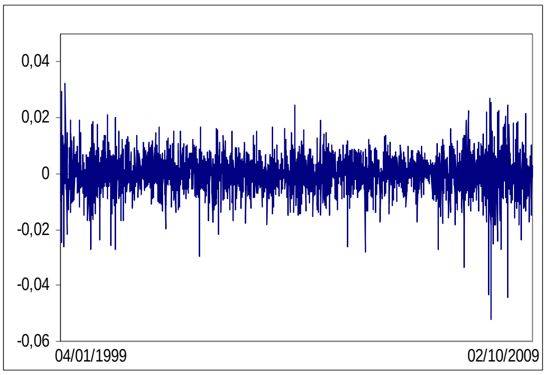


Figure 2: Absolute and squared returns series

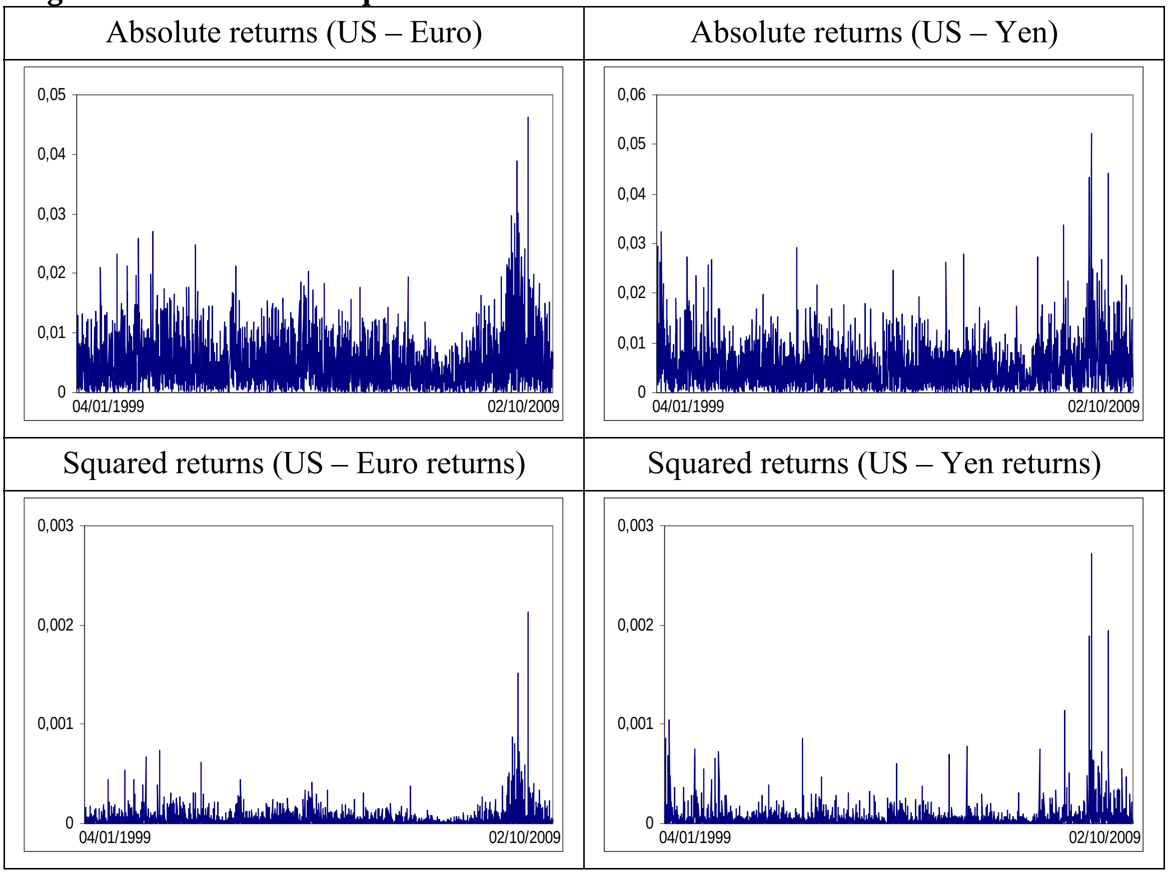


Figure 3: Correlogram of the absolute and squared returns series

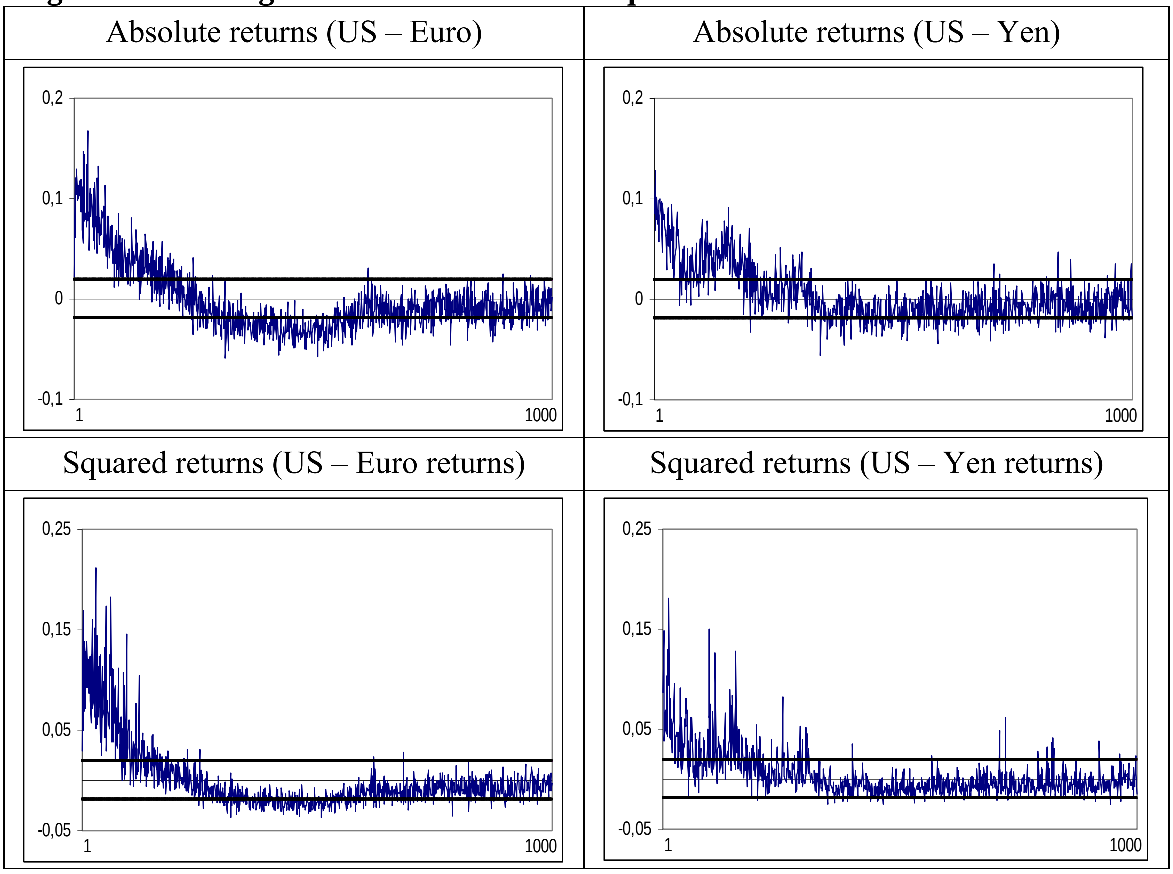
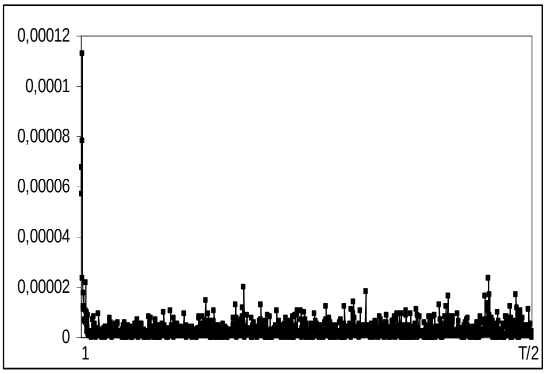
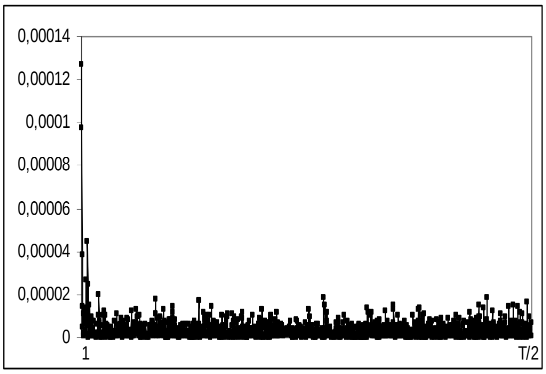


Figure 4: Periodograms of the absolute and squared returns series

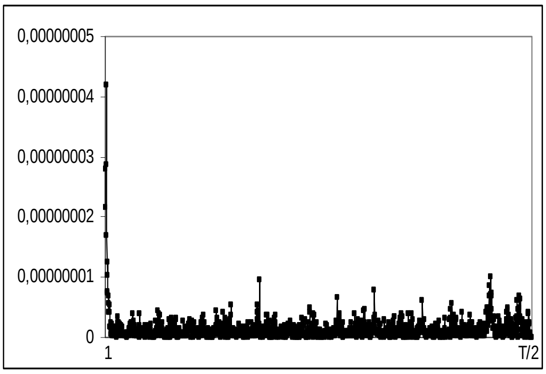
Absolute returns (US – Euro)



Absolute returns (US – Yen)



Squared returns (US – Euro returns)



Squared returns (US – Yen returns)

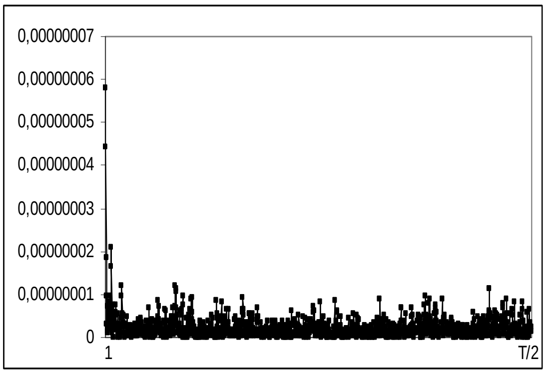
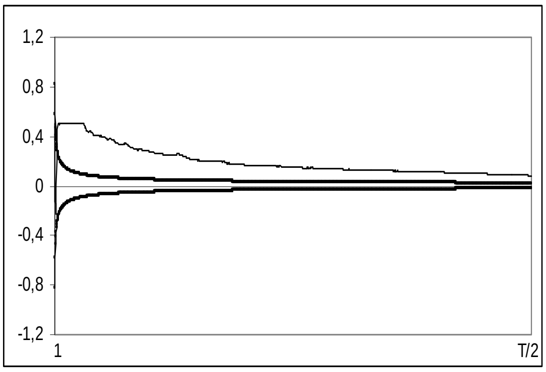
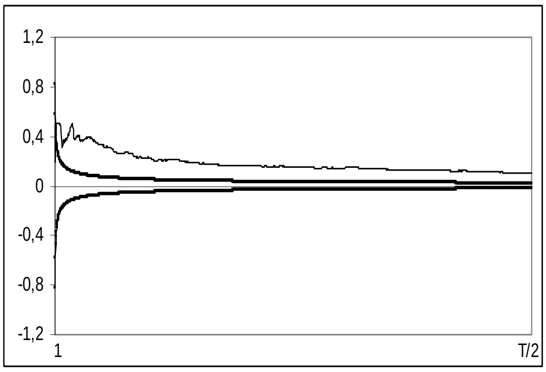


Figure 5: Estimates of d based on the Whittle semiparametric method (Robinson, 1995)

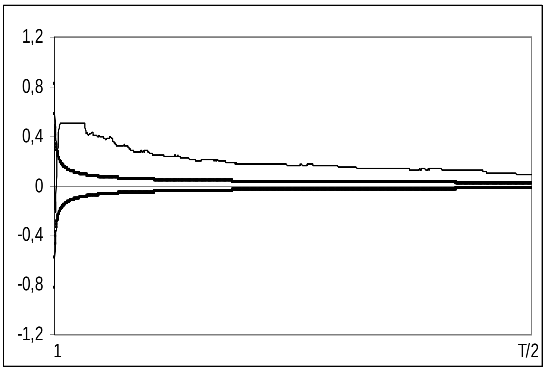
Absolute returns (US – Euro)



Absolute returns (US – Yen)



Squred returns (US – Euro returns)



Squared returns (US – Yen returns)

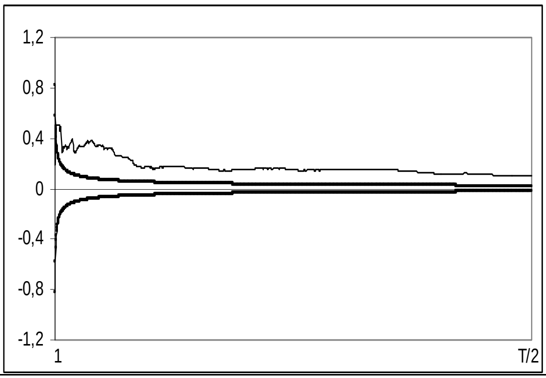


Figure 6: First 100 values in the periodograms of the absolute and squared returns

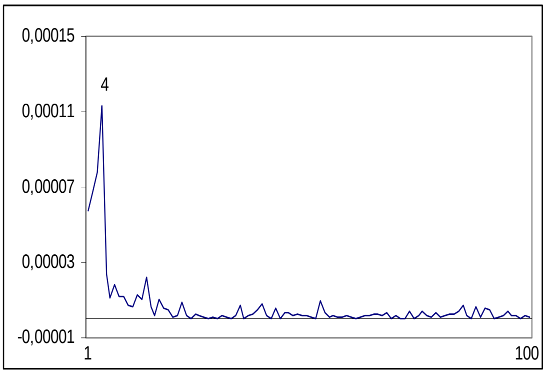
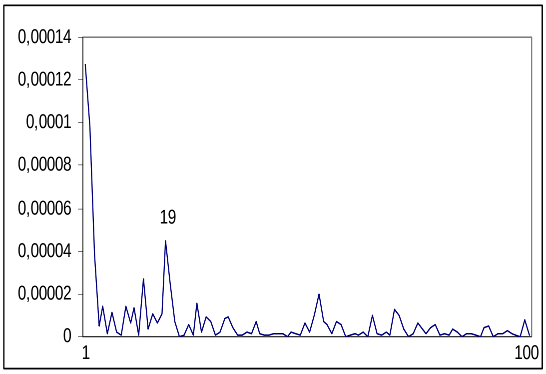
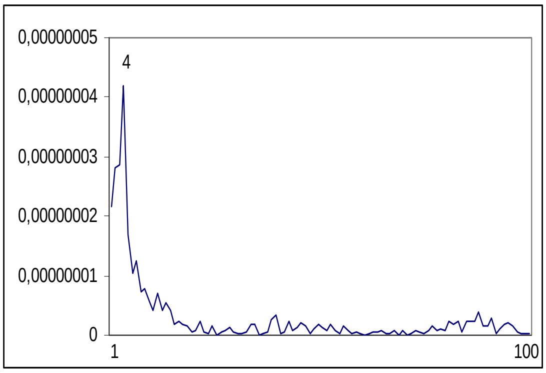
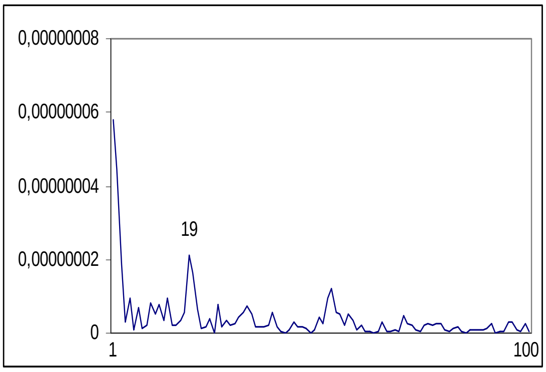
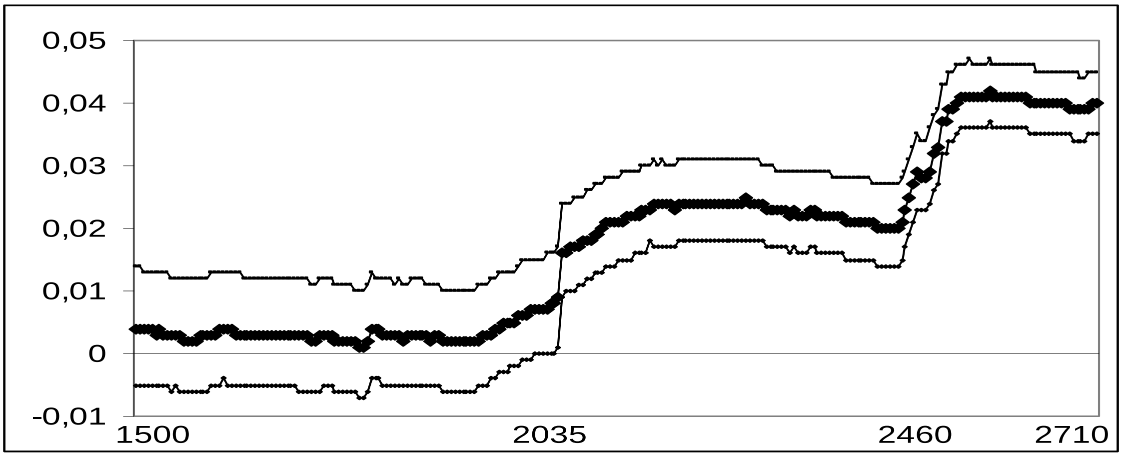
Absolute returns (US – Euro)	Absolute returns (US – Yen)
	
Squred returns (US – Euro returns)	Squared returns (US – Yen returns)
	

Figure 7: Recursive estimates of the fractional differencing parameter

Absolute returns for the US – Euro rate



Absolute returns for the US – Yen rate