HIGH FREQUENCY INVERTER-TRANSFORMER-CYCLOCONVERTER

SYSTEM FOR DC TO AC(3-PHASE) POWER CONVERSION

A thesis submitted for the degree of Doctor of philosophy

by

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Dedicated to My:

1- Mother and late Father with respect

2- wife Nahid and Daughter Shima

3- Sister and Brothers
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ABSTRACT:

This thesis is concerned with a 3-phase multistage high frequency link DC to AC power conversion with a novel inverter-cycloconverter circuitry. The conversion system is composed of a high frequency PWM inverter, step-up high frequency transformer and cycloconverter with bidirectional switching devices.

In first stage the DC voltage of the power source, say a submarine battery, is inverted to a system of 3-phase sinusoidally modulated 1 kHz alternative wave forms. For this purpose a suggested optimized PWM technique for 3-phase inverter operation is adopted, in which harmonic components up to 17th (17 kHz) are eliminated from the inverter output voltages.

In the second stage, for DC input isolation from AC output and also for a voltage transfomation (here stepping-up) a high frequency (size reduced) transformer is employed. Generalized high frequency operation, influence and side effects of the transformer on overall system design & performance is investigated.

In the final stage the 1 kHz-to-50 Hz conversion process is accomplished by a 3-phase cycloconverter. The proposed "nonlinear modulation strategy" for cycloconverter output voltage and associated harmonic analysis is demonstrated, in which the harmonic components up to 38th (1.9 kHz) are eliminated from the conversion system output voltage.

To assess the suggested functioning principles for the inverter & cycloconverter, the prototype conversion system was developed. Some design criteria and switching device selection are presented, together with different voltage & current wave forms of the prototype system under resistive & inductive load (induction motor) and their respective spectra.
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<th>Description</th>
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</thead>
<tbody>
<tr>
<td>AC</td>
<td>Alternative Current</td>
</tr>
<tr>
<td>CRD</td>
<td>Capacitor Resistor Diode (snubber)</td>
</tr>
<tr>
<td>D</td>
<td>Drain (MOSFET)</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>EMI</td>
<td>Electro-Magnetic Interference</td>
</tr>
<tr>
<td>G</td>
<td>Gate (MOSFET)</td>
</tr>
<tr>
<td>GOSS</td>
<td>Grain Oriented Silicon Steel</td>
</tr>
<tr>
<td>GTO</td>
<td>Gate Turn Off thyristor</td>
</tr>
<tr>
<td>H.D</td>
<td>Harmonic Distortion</td>
</tr>
<tr>
<td>h.f</td>
<td>high frequency</td>
</tr>
<tr>
<td>HVDC</td>
<td>High Voltage Direct Current</td>
</tr>
<tr>
<td>k Hz</td>
<td>kilo Hertz</td>
</tr>
<tr>
<td>M Hz</td>
<td>Mega Hertz</td>
</tr>
<tr>
<td>MOSFET</td>
<td>Metal Oxide Semiconductor Field Effect Transistor</td>
</tr>
<tr>
<td>PRI</td>
<td>Parallel Resonant Inverter</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>S</td>
<td>Source (MOSFET)</td>
</tr>
<tr>
<td>SIT</td>
<td>Static Induction Transistor</td>
</tr>
<tr>
<td>SITH</td>
<td>Static Induction Thyristor</td>
</tr>
<tr>
<td>SMES</td>
<td>Super-Conducting Magnetic Energy Storage</td>
</tr>
<tr>
<td>SMPC</td>
<td>Switched mode Power Conversion</td>
</tr>
<tr>
<td>SPICE</td>
<td>Simulation Program with Integrated Circuit Emphasis</td>
</tr>
<tr>
<td>SRI</td>
<td>Series Resonant Inverter</td>
</tr>
<tr>
<td>T.H.D</td>
<td>Total Harmonic Distortion</td>
</tr>
<tr>
<td>UPS</td>
<td>Uninterruptible Power Supply</td>
</tr>
<tr>
<td>vol.</td>
<td>volume</td>
</tr>
<tr>
<td>ZVS</td>
<td>Zero Voltage Switching</td>
</tr>
<tr>
<td>ZCS</td>
<td>Zero Current Switching</td>
</tr>
</tbody>
</table>
List of Principal Symbols:

A  ampere
A  area( m² )
\( A_n \)  transfer matrix coefficients
\( A_n \)  cycloconverter switching pattern amplitude
\( A_{1 \& A_2} \)  load line trajectory
\( a_n \)  Fourier Series coefficients in general
A  area( m² )
B  magnetic flux density( Tesla )
\( B_r \)  remanent density
\( B_{max} \)  maximum density
\( B_s \)  saturation density
\( b_n \)  Fourier Series coefficients in general
C  capacitance( Farad )
\( C_f \)  filter capacitor
\( C_r \)  resonant capacitor
\( C_s \)  snubber capacitor
\( C_{gs} \)  MOSFET gate to source capacitor
\( C_{gd} \)  MOSFET gate to drain capacitor
\( C_{ds} \)  MOSFET drain to source capacitor
\( C_p \)  primary interwinding capacitance
\( C_s \)  secondary interwinding capacitance
\( C_t \)  total interwinding capacitance
\( C^* \)  temperature
\( D_{1,2,3,...} \)  antiparallel diodes
\( D_{11,22,...} \)  snubber circuit diodes
\( \frac{di}{dt} \)  rate of change of instantaneous current
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>E</td>
<td>source voltage (volts)</td>
</tr>
<tr>
<td>E</td>
<td>energy (J)</td>
</tr>
<tr>
<td>$f_{11}$-$f_{l1}$-$f_{mn}$</td>
<td>cycloconverter switching function</td>
</tr>
<tr>
<td>$f_c$</td>
<td>carrier wave frequency</td>
</tr>
<tr>
<td>$f_i$</td>
<td>cycloconverter input frequency</td>
</tr>
<tr>
<td>$f_m$</td>
<td>modulating wave frequency</td>
</tr>
<tr>
<td>$f_o$</td>
<td>cycloconverter output frequency</td>
</tr>
<tr>
<td>$f_r$</td>
<td>resonance frequency</td>
</tr>
<tr>
<td>$f_{rotor}$</td>
<td>rotor frequency</td>
</tr>
<tr>
<td>$f_{stator}$</td>
<td>stator frequency or supply frequency</td>
</tr>
<tr>
<td>$F_1,F_2,F_3$</td>
<td>switching functions in direct 3-phase conversion</td>
</tr>
<tr>
<td>$h$</td>
<td>heat transfer coefficient</td>
</tr>
<tr>
<td>$H$</td>
<td>magnetizing force or magnetic field intensity</td>
</tr>
<tr>
<td>$i$</td>
<td>instantaneous current (A)</td>
</tr>
<tr>
<td>$I_a,I_b,I_c$</td>
<td>cycloconverter input current</td>
</tr>
<tr>
<td>$I_A,I_B,I_C$</td>
<td>cycloconverter output current</td>
</tr>
<tr>
<td>$I_{ah}$</td>
<td>input current harmonics</td>
</tr>
<tr>
<td>$I_C$</td>
<td>collector current</td>
</tr>
<tr>
<td>$I_d$</td>
<td>dual converter output current</td>
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<td>$I_D$</td>
<td>drain current</td>
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<td>$I_\phi$</td>
<td>primary magnetizing current</td>
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<tr>
<td>$k$</td>
<td>constant</td>
</tr>
<tr>
<td>$L$</td>
<td>inductance (H)</td>
</tr>
<tr>
<td>$L_c$</td>
<td>winding length</td>
</tr>
<tr>
<td>$L_e$</td>
<td>magnetic path length</td>
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<tr>
<td>$L_f$</td>
<td>filter inductor</td>
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<tr>
<td>$L_m$</td>
<td>primary magnetizing inductance</td>
</tr>
<tr>
<td>$L_{mt}$</td>
<td>mean turn length</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$L_p$</td>
<td>primary leakage inductance</td>
</tr>
<tr>
<td>$L_r$</td>
<td>resonant inductor</td>
</tr>
<tr>
<td>$L_s$</td>
<td>secondary magnetizing inductance</td>
</tr>
<tr>
<td>$L_{Ss}$</td>
<td>snubber inductance (turn-on)</td>
</tr>
<tr>
<td>$m$</td>
<td>number of input phases</td>
</tr>
<tr>
<td>$M$</td>
<td>modulation index</td>
</tr>
<tr>
<td>$n$</td>
<td>number of output phases</td>
</tr>
<tr>
<td>$n_1, n_2$</td>
<td>transformer turns numbers</td>
</tr>
<tr>
<td>$N$</td>
<td>number of pulses over a quarter cycle</td>
</tr>
<tr>
<td>$N_p$</td>
<td>primary turns</td>
</tr>
<tr>
<td>$N_r$</td>
<td>rotor speed (round per minute)</td>
</tr>
<tr>
<td>$N_s$</td>
<td>synchronous speed (round per minute)</td>
</tr>
<tr>
<td>$N_S$</td>
<td>secondary turns</td>
</tr>
<tr>
<td>$p$</td>
<td>number of pole pairs</td>
</tr>
<tr>
<td>$p_d$</td>
<td>dielectric loss</td>
</tr>
<tr>
<td>$p_e$</td>
<td>eddy current loss</td>
</tr>
<tr>
<td>$p_h$</td>
<td>hysteresis loss</td>
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<tr>
<td>$p_i$</td>
<td>input power</td>
</tr>
<tr>
<td>$p_o$</td>
<td>output power</td>
</tr>
<tr>
<td>$p_r$</td>
<td>prototype power</td>
</tr>
<tr>
<td>$Q_o$</td>
<td>quality factor</td>
</tr>
<tr>
<td>$R$</td>
<td>ratio of input to output frequency</td>
</tr>
<tr>
<td>$R$</td>
<td>resistance</td>
</tr>
<tr>
<td>$R$</td>
<td>ratio of carrier wave to modulating wave frequency</td>
</tr>
<tr>
<td>$Re$</td>
<td>transformer core loss shunt resistance</td>
</tr>
<tr>
<td>$R_g$</td>
<td>gate resistance</td>
</tr>
<tr>
<td>$R_o$</td>
<td>load resistance</td>
</tr>
<tr>
<td>$R_p$</td>
<td>primary resistance</td>
</tr>
</tbody>
</table>
Rs secondary resistance
RS snubber circuit resistance
S slip( per unit )
S1,S2,.....,S18 semiconductor switches
t transformer sheet thickness
T1,T2 semiconductor switches
Ta ambient temperature
td(off) delay time on switch turn-OFF
td(on) delay time on switch turn-ON
tf switching fall time
Ton switch on( conduction ) time
tr switching rise time
Ts switching period( on + off )
To operating temperature
uAN,uBN average voltage during device switching
dv/dt rate of change of the instantaneous voltage
vab,vbc,vca 3-phase inverter line voltages
van,vbn,vcn 3-phase inverter phase voltages
va'n,vb'n,vc'n 3-phase rectifier phase voltages
vAN,vBN cycloconverter input voltages
vANh output voltage harmonics
VC voltage across the capacitor
VCE transistor Collector to Emitter voltage
Vd dual converter output voltage
VDC phase controlled converter output voltage
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{DS}$</td>
<td>MOSFET drain to source voltage</td>
</tr>
<tr>
<td>$V_{GS}$</td>
<td>MOSFET gate to source voltage</td>
</tr>
<tr>
<td>$v_{in}$</td>
<td>input voltage</td>
</tr>
<tr>
<td>$v_{(}\text{harm.}\text{)}$</td>
<td>harmonic voltage amplitude</td>
</tr>
<tr>
<td>$v_{(}\text{harm.}\text{)}$</td>
<td>harmonic voltage amplitude</td>
</tr>
<tr>
<td>$V_m$</td>
<td>maximum source voltage</td>
</tr>
<tr>
<td>$v_{no}$</td>
<td>inverter third harmonic voltage</td>
</tr>
<tr>
<td>$v_o$</td>
<td>output voltage</td>
</tr>
<tr>
<td>$v_{o-p}$</td>
<td>positive group output voltage</td>
</tr>
<tr>
<td>$v_R$</td>
<td>voltage across the resistor</td>
</tr>
<tr>
<td>$v_{spike}$</td>
<td>spike voltage</td>
</tr>
<tr>
<td>$v_{2}, v_{3}, \ldots, v_n$</td>
<td>harmonic voltage amplitude</td>
</tr>
<tr>
<td>$y$</td>
<td>lamination sheet thickness (meter)</td>
</tr>
<tr>
<td>$Z_{AB}, Z_{BC}, Z_{CA}$</td>
<td>3-phase load impedances</td>
</tr>
<tr>
<td>Greek letters</td>
<td>Description</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------</td>
</tr>
<tr>
<td>α</td>
<td>delay firing angle (inverter-phase R)</td>
</tr>
<tr>
<td>α₁, α₂,..., αₖ</td>
<td>pulse width in proposed modulation strategy</td>
</tr>
<tr>
<td>αₚ</td>
<td>delay firing angles for the positive converter</td>
</tr>
<tr>
<td>αₙ</td>
<td>delay firing angles for the negative converter</td>
</tr>
<tr>
<td>β</td>
<td>delay firing angle (inverter-phase Y)</td>
</tr>
<tr>
<td>γ</td>
<td>delay firing angle (inverter-phase B)</td>
</tr>
<tr>
<td>δ</td>
<td>magnetic skin dept</td>
</tr>
<tr>
<td>δ</td>
<td>pulse width</td>
</tr>
<tr>
<td>Δ</td>
<td>proportional to</td>
</tr>
<tr>
<td>ε</td>
<td>dielectric constant</td>
</tr>
<tr>
<td>η</td>
<td>efficiency</td>
</tr>
<tr>
<td>θ</td>
<td>phase shift between waveforms (in phase modulation)</td>
</tr>
<tr>
<td>θ</td>
<td>permissible temperature (°C)</td>
</tr>
<tr>
<td>μ</td>
<td>absolute permeability (H/m)</td>
</tr>
<tr>
<td>μ₀</td>
<td>magnetic space constant = 4π.10⁻⁷ (H/m)</td>
</tr>
<tr>
<td>μᵣ</td>
<td>relative permeability (per unit)</td>
</tr>
<tr>
<td>ρ</td>
<td>resistivity (Ω.m)</td>
</tr>
<tr>
<td>Φ</td>
<td>core flux (Wb)</td>
</tr>
<tr>
<td>Φₘ</td>
<td>maximum flux (Wb)</td>
</tr>
<tr>
<td>dΦ/dt</td>
<td>rate of change of the magnetic flux (Wb/sec.=volt)</td>
</tr>
<tr>
<td>ω</td>
<td>source angular frequency (rad/sec.)</td>
</tr>
<tr>
<td>ωₖ</td>
<td>carrier wave angular frequency (rad./sec.)</td>
</tr>
<tr>
<td>ωₒ</td>
<td>output voltage angular frequency (rad./sec.)</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction
1. **Switched Mode Power Conversion (SMPC)**

1.1 **General**

In a general sense, power conversion can be defined as a process (series of action, changes, manipulations) that converts one form of energy (Electrical, Thermal, Radiant, ...) to another on a continuous basis. There are many types of devices which can provide such a function with varying degrees of cost, reliability, complexity and efficiency.

Here, only power converters which perform energy translation electrically and in a dynamic fashion, are the area of interest. Generally the linear (dissipative) and switched mode (nondissipative) are two alternatives for delivery of electric power from a DC source in a controllable manner.

Linear power conversion is based on the presence of a series linear element, either a resistor (mechanical control), or a transistor used in linear mode (electronic control) such that the total load current is passed through the series linear element and more power is lost in the controlling device Fig. 1-1(a). Thus, linear power conversion even in its ideal form is dissipative and inefficient (according to the maximum power transfer theorem $\eta(\text{max}) = 50\%$).

In switched mode power conversion, the controlling device is an ideal switch either closed or open Fig. 1-1(b). By controlling the ratio of the time intervals spent in the closed and open position ($T_{\text{on}}/T_{\text{s}}$) the power flow to the load can be controlled in a very efficient way, ideally 100%, because when the device is fully on it has only a small saturation voltage drop across it (0.3 to 1.0).
In the off condition the reverse leakage current is usually negligible so that the power loss is nearly zero. A detailed comparison between linear and switched mode power conversion may be found in [1].

Fig. 1.1 Power conversion circuits:

(a) - linear (dissipative)  
(b) - switch mode (nondissipative)

A general switched mode power conversion (SMPC) system has components including, Inductors, Capacitors, Transformers, Semiconductor Switches, and associative Controllers (microcomputers, microprocessors) as shown in Fig. 1-2.
In SMPC systems, the semiconductor switches; Bipolar Junction Transistor (BJT), Metal Oxide Semiconductor Field Effect Transistor (MOSFET),...and Gate Turn-Off Thyristors (GTO), that controlling the dynamic transfer of power from input to output, are either fully ON or fully OFF, with very short transition times from one of these states to the other.

![Diagram of general switched mode power conversion (SMPC) system]

**Switch ON, OFF controller (microprocessor)**

Fig. 1-2 General switched mode power conversion (SMPC) system

### 1.2 SMPC with high frequency link

The basic objective of high frequency power conversion is size reduction, that is, having compact and lighter weight power converters for following applications:
Mainframe computers, airborne and mobile equipments, artificial satellite, artificial organs for biological replacement, space stations, space vehicles, robotics, ships and submarines (in link with the work presented in this thesis).

Reducing the physical size with result of less surface area for system cooling, necessitates higher efficiencies for achieving higher power densities. On the other hand higher efficiency dictates lower conversion loss in all devices and components.

In SMPC circuits, magnetic elements and capacitors play the major roles of energy storage/transfer and ripple filtering. Since the required values and volumes of magnetic elements (Transformers and Inductors) and Capacitors decrease as the operating frequency is increased, it is imperative to design converter circuits capable of operating at high power and high frequencies to achieve high power density and element size reduction.

It should be emphasized that, the operation of any converter at very high frequencies (k Hzs and M Hz) is strongly influenced by the effects of parasitic elements such as interconnected and leakage inductances, interwinding capacitances, skin effect in conductors, and proximity effect in inductors.

Some parasitics such as the leakage inductance of the transformers can be constructively used as a part of circuit [1], [2], [3]. When the resonant elements are placed on the primary side of the transformer, the leakage inductance and interwinding capacitances act as an independent parasitic elements between the resonant elements ($L_r$ & $C_r$) and output filter. If the resonant capacitor be pushed appropriately through the transformer to the secondary then the leakage inductance and interwinding capacitances values can be added to resonant elements. The result is a welcomed absence of the voltage spikes on all the windings. Other parasitics may have adverse effects on the circuit performance and should be
minimized. The effects of high frequency operation in transformers will be studied in chapter 3.

During the last decade, different topologies which use a high frequency AC link for DC-to-DC and DC-to-AC power conversion with different control requirement, have been presented in [4], [5], ..., [12].

Reference [13] illustrates a DC-to-AC power conversion technique which employs two high frequency inverters operating with frequency modulation. The link frequency is typically of the order of a few kilohertz and low frequency (50 Hz) output power is obtained from the high frequency inverters through a suitable Transformer-Cycloconverter circuitry with following block diagram.

Fig. 1.3 DC-to-AC power conversion based on frequency modulation

A brief mathematical system description is:

\[ v_1 = V_m \sin(\omega_c + \omega_o)t, \quad v_2 = V_m \sin(\omega_c - \omega_o)t, \quad v_3 = v_1 - v_2, \quad v_4 = k \cdot v_3 \]  \hspace{1cm} (1-1)

\[ v_4 = kV_m \left[ \sin(\omega_c + \omega_o)t - \sin(\omega_c - \omega_o)t \right] = \left[ 2kV_m \sin(\omega_o)t \right] \cos(\omega_c t) \]  \hspace{1cm} (1-2)
\( v_5 \) can be obtained by multiplying \( v_4 \) and a square wave \( v_s \) with unit amplitude and in phase with \( \cos(\omega_c t) \). After some arithmetic manipulation:

\[
v_5 = v_o + v_{(\text{harmon.})}
\]

\[
v_o = \frac{4kV_m}{\pi}\sin(\omega_o t)
\]

\[
v_{(\text{harmon.})} = -\sum_{m=1}^{\infty} \left[ \sin(2m\omega_c + \omega_o) t - \sin(2m\omega_c - \omega_o) t \right] (-1)^m \left[ \frac{2kV_m}{\pi} \right] \left[ \frac{1}{2k-1} - \frac{1}{2k+1} \right]
\]

If \( f_1 = f_c + f_o = 20.1 \text{ kHz} \), \( f_2 = f_c - f_o = 20 \text{ kHz} \), then in (1-4) the \( v_o \) is required output with \( f_o = (f_1 - f_2)/2 = 50 \text{ Hz} \) and (1-5) shows harmonic components starting with \( 2f_c + f_o = 40.15 \text{ kHz} \) and \( 2f_c - f_o = 40.05 \text{ kHz} \), so unwanted components can be easily filtered.

Reference [14] shows a controllable DC-to-DC and DC-to-AC power conversion process employing two high frequency Inverters operating with phase-modulation strategy, Fig. 1-4, with following mathematical description

\[
v1(\omega t) = \frac{4V_m}{\pi} \sum_{n=1,3,5}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n} \cos\left[n(\omega_c t - \theta)\right]
\]

\[
v1(\omega t) = \frac{4V_m}{\pi} \sum_{n=1,3,5}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n} \cos\left[n(\omega_c t + \theta)\right]
\]

\[
v3(\omega t) = v1(\omega t) - v2(\omega t)
\]

\[
v3(\omega t) = \frac{8V_m}{\pi} \sum_{n=1,3,5}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n} \sin(n\theta) \sin(n\omega_c t)
\]

Suppressing of harmonics by tank circuit:

\[
v4(\omega t) = \frac{8V_m}{\pi} \sin(\theta) \sin(\omega_c t)
\]

\( v_4(\omega t) \) after passing through full bridge rectifier is processed to \( v_5(\omega t) \) which can be obtained by multiplying \( v_4(\omega t) \) with a unit square wave \( v_s(\omega t) \) having the same phase relationship:
Fig. 1.4 DC-to-(DC & AC) power conversion based on phase modulation
\[ v_5(\omega t) = \frac{16V_m}{\pi^2} \sin(\theta) - v_{(\text{harmonics})} \]  
\[ v_{(\text{harmonics})} = \frac{32V_m}{\pi^2} \sin(\theta) \left\{ \sum_{n=3,5,7}^{\infty} \frac{1}{(n+1)(n-1)} \cos(n\omega_c t) \right\} \]  
\[ V_{\text{out}(DC)} = \frac{16V_m}{\pi^2} \sin(\theta) \]  

after suppressing terms containing \( \omega_c \) yields:

The output of the output Inverter is yielded from (1-12) with frequency of \( f_0 \) and modulation index of \( M \). Phase modulation [14] has some advantages over frequency modulation [13]:

1- Control range is extended to include zero output (\( \theta = 0^\circ \))
2- Power factor of the system is unity when \( \theta = 90^\circ \) and zero when \( \theta = 0^\circ \).

**1-3 SMPC with Resonant Topologies**

Generally, when the conversion frequency of conventional pulse width-modulation (PWM) power converters increase and approaches to 100 kHz, the switching losses and switch stresses become excessive due to the forced commutation and consequently, simultaneous presence of a high current and high voltage during turn-on and turn-off, Fig. 1-5.

On the other hand, industrial applications strongly demand high frequency operation in off-line power conversion, because they offer higher power density, faster transient response, and size reduction in energy storage/transfer components.
The fundamental departure from the conventional forced commutation approach was the "zero-current switching" technique reported in [15], then "resonant switches" was proposed by Liu and Lee in [16]. By adding a high-frequency resonant circuit around the switch it is possible to shape the switch current and voltage wave forms so that a high current and a high voltage are not present at the same time.
Depending on how the high frequency resonant circuit is connected to the switch, the resonant or quasi-resonant technique eliminates either turn-on or turn-off switching losses.

### 1.3.1 Zero Current & Zero Voltage Switching: (ZCS) & (ZVS)

In the zero-current switched (ZCS) resonant converters [17] the current of the transistor is shaped by a resonant network, so that it is reduced to zero prior to turn-off to eliminate turn-off losses. In the zero-voltage switched (ZVS) resonant converters [17] the voltage waveform of the transistor is shaped by a resonant network, so that the voltage reduces to zero prior to turn-on to eliminate turn-on losses.

For clear understanding Fig. 1-6 shows the Buck converter with the conventional, ZCS and ZVS versions and their equivalent [18]. In Fig. 1-6 (b), when switch is conducting, Inductor L and Capacitor C constitute a series resonant circuit with its oscillation initiated by the turn-on of S. Current through S is sinusoidal and reduced to zero before S is turned-off.

Similar events take place in Fig. 1-6 (d). Fig. 1-6 (c) & (e) are equivalent circuit of ZC & ZV resonant switches. Each of these topologies represent a high frequency sub-circuit obtaining from a resonant converter by replacing voltage sources and filter capacitors with short circuits, and filter inductors with open circuits [19].
The load line trajectory for an inductive load with conventional and resonant switching is shown in Fig. 1-7. In the conventional switching (path A1 & A2) the path goes through a high stress region where the switch...
is subject to high voltage and high current simultaneously, but in the resonant switching the path B moves along either the voltage axis or the current axis, consequently, the switching stresses and losses are eliminated.

Fig. 1-7 Load line trajectory for inductive load:

A, A2 for conventional switching & B for resonant switching

1.4 **Uninterruptible Power Supply (UPS)**

An UPS system is a device which is placed between the user's equipment input and the power-source output, to act as a reserve source of energy in the event of a complete power failure or mains disturbances (missing cycles, sags, spikes and transients).
Most of the equipment served by the utility can tolerate short term voltage variations or disturbances without affecting the operation or life of the equipment. There is however growing use of computer and other critical equipments that are more sensitive to input power quality than almost all others.

Once a computer system stops for very short duration due to a power disturbance or failure; the result may be malfunction, production of "bad data", and total system shut down. Nevertheless, in general, voltage can fall to zero as long as it recovers within 0.5 cycles or 8.33 milliseconds, and it can drop to 30% below normal for up-to 0.5 seconds. This is due to stored energy built in to the computer power supply filters and motors [20].

The need for a UPS will always be determined by comparing the danger, inconvenience and expense caused by unpredicted power problems against the cost of installing and maintaining UPS system itself [20-chapter 2],[21]

1.4.1 General application of UPS systems

In general the following are the most common areas of applications:

1- where power interruptions endanger lives and property; hospital diagnostic, intensive-care systems, industrial safety monitors and process control systems where quality and continuity of supply are of vital importance, stock-market quotation, bank transfers, air-line reservations, and alarms and surveillance equipment.

2- where power interruptions cost time and money; computer complexes, remote terminals, and instrumentation centres.
3- where power interruptions disrupt critical data networks; military installations, automatic production lines, satellite and radar communications, pipe-line and electrical distribution monitoring systems.

1.4.2 UPS technology

There are basically two types of power conditioning devices available to provide the reserve energy source:
1-static UPS system
2-rotary UPS system

In addition there are 3 types of operating principle; "on-line", "off-line" and "hybrid" [22]. Each type has its own advantages and disadvantages.

1.4.2.1 static UPS system

Fig. 1-8 shows typical static UPS, OFF-line and ON-line [23]. In Fig.1-8 (a) OFF-line UPS provides power for less sensitive loads such as lighting, heating, systems and motors that only require protection against complete mains failure. Under normal conditions the load is powered directly by the input AC supply and is only driven by the inverter on failure of the mains supply. The transfer to the inverter is automatic and is normally performed by a mechanical device such as contactor.

ON-line UPS, Fig. 1-8 (b) eliminates any switching of load between the failed mains and inverter output. This system protect the sensitive loads against of whole range of daily occurrence hazards to include; line disturbances, over voltage spikes, voltage sags and drop-outs.
(a) OFF-line (stand-by) UPS

(b) ON-line UPS

Fig. 1-8 Static UPS systems (a) OFF-line  (b) ON-line
1.4.2.2 Rotary UPS system

Although the majority of UPS in use today are of the static variety, a number of very reliable and useful systems using rotating components. Fig. 1-9 shows typical rotary UPS (a) [20] and hybridized static-rotary (b) [24].

Fig. 1-9 (a) - Rotary & (b) - Hybridized (static-rotary) UPS system
The rotary system fulfils the same basic requirements as the static UPS. The DC/AC converter of the rotary system consists of a DC motor with shunt field control driving a separately exited AC alternator mounted on a common shaft with or without a flywheel. Stable frequency operation is achieved by regulating the speed of the DC motor and the output voltage of the AC alternator is regulated by the use of an automatic voltage regulator.

Further development of rotary UPS includes a synchronous machine in which the motor and the generator windings are located on a single common frame. The rotor is DC exited and has a damper winding which provides a low impedance to high frequency harmonic currents. This construction of machine offers the advantages of a reduction in size and weight, improvement of efficiency, reduction of source impedance and increase of system reliability. Detailed study about rotary and hybrid version of UPS systems may be found in [20].

A full optimum voltage and harmonic control PWM technique for 3-phase UPS systems has been presented in [25].

1.4.3 Other type of UPS system (Triport UPS system)

This system employs the versatility of reversible charger/inverter and combine some of the benefits of both OFF-line and ON-line systems. Basically it consists of a ferro-resonant Transformer [26] and a single-phase square wave inverter. The Transformer has two primary windings, one of them is connected to the mains through a static transfer switch and the other connected to the output of inverter and the secondary winding connected to the load as Fig. 1-10 shows.
under normal condition, the load and ferro-resonant transformer and consequently the battery charger are powered from the mains. During mains disturbance transfer switch disconnect the mains, allowing inverter to rebuilt AC power from the battery without any interruption [27], [28] and [29].

Fig. 1-10 Triport UPS system
1.5 Superconducting Magnetic Energy Storage (SMES)

To satisfy the need for a power conditioner (future large scale UPS system) and peak power, several technologies including compressed air, underground pumped hydro-storage, batteries, and superconducting magnetic energy storage [30, 31] are available. In each of these technologies, except SMES, the Electrical energy is converted to another form, Mechanical or Chemical, through several transformations and then back to Electrical. The conversion processes are inherently inefficient (65% to 75%). SMES may be as high as 95% efficient because there is no conversion of energy from one form to other [32].

Research on SMES started in 1970 at the university of Wisconsin (U.S.A) and industrial participation in the development of commercially feasible SMES began in 1980 by Bechtel company [33]. The general schematic of a SMES and its components [32] is shown in Fig. 1-11.

The current in the superconducting coil will be on the order of several kilo-ampers. Since power systems normally operate at low current levels so a transformer is needed to convert the high voltage and low current of the AC system to the low voltage and high current required by the coil. The individual semiconductor elements can only carry a few thousand amperes so many must be used in parallel.

The converter is an AC-to-DC rectifier and DC-to-AC inverter that changes the alternating current from the utility into the direct current that must flow continuously in the coil. Here extremely reliable protection measures are required because, unlike other energy sources, an open coil circuit could result in huge economic loss. Since the AC-to-DC and DC-to-AC conversion processes are very efficient, losses attributable to these conversion will be between 3% and 8% of the energy charge.
During normal operation the coil will be charged to maximum Amps, storage the maximum energy:

\[ E = \frac{1}{2}LI^2 \]  

(1-14)

where: \( E = \) stored energy, \( L = \) coil inductance, \( I = \) coil current
and when energy be required by the power line the coil will be discharged. The converter delay angle of $\alpha$ (0° to 180° degrees ideally) is the key to the control of rate of charge and discharge. The SMES, in addition to its use in the load leveling of a system, may also be used as a power supply for large scale UPS systems or as a system stabilizer that damps power oscillation by mean of the rapid charge and discharge of energy.

1.6 Future developments

Power conditioning technology, particularly in applications, has progressed rapidly over the past 15 to 20 years due mainly to the growing importance and needs for computer related and other critical loads. The UPS market is expected to grow [34] at a rate of about 30% per year in units over the next several years, tilted in favour of the smaller units [20] and compact UPS capable of powering more load units.

At present, the volume of a typical UPS is divided into four main sections:

1- switching devices (rectifier, inverter)
2- control and filter circuit
3- battery
4- transformer

The proportion of the total volume occupied by each of the four sections is shown in Fig. 1-12.

Since the transformer occupy about one third of the volume of a UPS system [27] the reduction of battery and the transformer size are the main
research targets. It may become economical to store sufficient electromagnetic energy using superconductivity to eliminate the need for storage batteries or fly wheels for momentary power loss protection \[20\] & \[35\].

![Component distribution in a conventional 50 Hz UPS](image)

Fig. 1-12 Component distribution in a conventional 50 Hz UPS

### 1.7 Multi-stage power conversion

The remarkable progress in high frequency switching power-Electronics, mainly concerned with new power semiconductor switches; MOSFET, IGBT, GTO thyristor, Static Induction Transistor & Thyristor, and control devices has made possible the development of high frequency switched mode power conversion with high frequency AC link and DC link.
Recently, high performance and high power density DC (solar array)-to-AC power conversion systems with a high frequency transformer & AC link have become of major interest in telecommunication, space/satellite stations and decentralized residential solar photovoltaic generation systems [36] & [37].

The multi-stage sine-wave modulated AC power conditioning system with high frequency link, which is composed of a PWM or series/parallel resonant h.f. inverter, h.f. transformer for isolation & voltage transformation (up/down) and a synchronous switching cycloconverter with bidirectional switching devices becomes more attractive and efficient.

Fig. 1-13 (a) shows the conventional UPS and DC-to-AC conversion which has simple structure, but it has following disadvantages:

1- The size of the isolation transformer is large.
2- the size of the output filter for removing low order harmonics is high.
3- the transient response to fluctuation of the load or input voltage is not good.
4- acoustic noise occurs from the transformer & filter reactor.

In Fig. 1-13 (b), the h.f. inverter output is a pulse-width modulated h. f. voltage. The h.f. transformer provides isolation between DC and AC parts, and probably steps of the voltage, the cycloconverter output's is a modulated voltage whose frequency is the same as the commercial AC line frequency by converting the polarity of rectangular pulses output by the inverter.

Since the system is operated with h.f. link, the transformer and filter component size can be reduced. Moreover by employing a high frequency link in
(a) Block diagram of conventional UPS

(b) Block diagram of multi-stage h.f. link power conversion

Fig. 1-13 Multistage power conversion (a)-conventional (b)-with h.f. link
excess of 20 k Hz, acoustic noise is reduced and transient is improved.

Generally there are 4 different topologies are practically acceptable for multi-stage DC-to-AC power conversion either with DC link or h. f. AC link. Fig.1-14 schematically represents these topologies.

The systems shown in Fig. 1-14, can be divided into two unidirectional and bidirectional groups. High frequency inverter can be chosen either the PWM or series/parallel resonant configuration.

The topology of Fig. 14-(a) will be fully discussed in chapter 4 and the cases of 14-(b) & 14-(c) have been discussed in [36] & [13] respectively. The topology of 14-(d), with cycloconverter supplied by sinusoidal wave form, is subject of the research reported in this thesis.
Fig. 1-14 Possible topologies for multi-stage DC/AC power conversion
1.8 Research background & proposed system

On the basis of section 1-7 discussion, for proposed system with 3-phase version, the possible configurations are proposed through Fig. 1-15 (a, b, c, d) in coming pages.

The principles of (3-phase)-to-(3-phase) in Fig. 1-15 (a), (1-phase)-to-(3-phase) in Fig. 1-15 (b), indirect (1-phase)-to-(3-phase) in Fig. 1-15 (c) and direct (3-phase)-to-(3-phase) in Fig. 1-15 (d) will be fully discussed in chapter 4. The case of Fig. 1-15 (a) is the subject of study in this thesis.
Fig. 1-15 Multi-stage DC-to-AC conversion (a)

Load
3-phase
1000/50 Hz

Cycloconverter & h. f. filter

Transformer
Step-up & I kHz

DC supply & I kHz inverter

Load
3-phase

S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15, S16, S17, S18

S7, S8, S9, S10, S11, S12, S13, S14, S15, S16, S17, S18 = Bidirectional switches
Fig. 1-15 Multi-phase DC-to-AC conversion (p)

Diagram of a multi-phase DC-to-AC conversion circuit with labeled components and switches.

Switches: S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12, S13.
Fig. 1-15 Multi-stage DC-to-AC conversion (continued)
Fig. 1-15 Multi-stage DC-to-AC conversion (d)

Load

DC Supply & 1 kHz Inverter

Step-up & 1 kHz

Transformer

Cycle converter

1000/50 Hz

3-phase Load

Phase

Phase

S1, S2, ..., S15 = Bidirectional switches

S7, S8, .......... S15 = Bidirectional switches
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Chapter 2

Inverters & The Prototype System Inverter

(proposed modulation strategy)
2.1 Inverters, General

Since "energy storage" can only be obtained in the form of DC (battery, fuel cell [1], solar array, other DC sources), inverters accomplish the DC-to-AC conversion process.

Application of inverters include the following areas:
1- variable speed AC motor drives.
2- induction heating.
3- stand by power supplies (off-line).
4- uninterruptible power supplies (on-line).
5- air craft and space craft power supplies.
6- output of HVDC transmission lines.
7- photovoltaic application for utility interface.

2.2 Type of Inverters

Inverters can be broadly classified into two types:
1- Single phase inverters
2- 3-phase inverters

Each type can be subdivided into mainly two categories depending on the type of power semiconductor switch commutation:

1- pulse width modulation (PWM) inverters with forced commutated operation.
2- resonant inverters with naturally commuted operation.

For the proposed work the pulse width modulation inverter will be studied with 3-phase and half-bridge mode of connection. Moreover, in order to achieve any selected transformation ratio of voltage or current of inverter, and provide complete isolation between it's DC input and AC output a transformer
is employed on output side of the inverter. The high frequency orientated operation of this transformer (emergence of significant parasitic elements) and drastic side effects on the operation of the prototype system will be taken into consideration in chapters 3 & 5.

2.3 Half bridge and Full bridge Inverters

Inverters provide single phase or three phase AC balance system either by half bridge or full bridge mode of connections. Theoretically any poly-phase AC system may be derived [2],[3]. Fig. 2-1 shows the basic structure of a single phase inverter.

Fig. 2-1 Single phase inverter (a)- half bridge (b)- full bridge

With resistive loads, the antiparallel diodes (D1, D2,..., D4 in Fig. 2-1) across the MOSFETs have no functions. If the load is non-resistive (mainly inductive) the current in each arm of the inverter would be delayed on its voltage since the conduction time of the MOSFETs and diodes depends on the load power factor, details in [23]. When the diodes conduct, the trapped electric energy is fed back to the DC source, thus preventing voltage instability. For this reason these diodes are called as a feedback diodes. For the Inverter-Transformer system, the effect of different power factors; resistive with
Cos(Φ) = 1, inductive and capacitive with Cos(Φ) = 0.0 lagging & leading respectively, is shown in Fig. 2-2. (further discussion in Appendix C)

Fig. 2-2 Voltage and current waveforms of an inverter for pure:
(a) Resistive  (b) Inductive  (c) Capacitive loads
In the half bridge circuit, S1 and S2 conduct alternately, i.e., when S1 is on, S2 is off and vice versa. The voltage appearing across any open switch is equal to the E and the peak current through any conducting switch is the half of average source current \( i = \frac{E}{2R} \).

In the full bridge circuit, switches S1 and S2 are closed simultaneously during the first half-cycle of the conversion process, and then S3 and S4 are closed simultaneously during the second half cycle. The voltage appearing across any open switch is equal to the E and the peak current through any conducting switch is \( 2i = \frac{E}{R} \), so for full bridge inverter with resistive load, the output power is 4 times higher and the fundamental current component is twice than that of half bridge inverters.

2.4 Three Phase Inverters

Three phase inverters are normally used for high power application. Three single phase half bridge (6 MOSFETs) or three single phase full bridge (12 MOSFETs) inverters can be connected in parallel to form a 3-phase inverter. Fig. 2-3 shows the most common connection of the 3-phase half bridge inverter.

The gating signals of single phase inverters should be advanced or delayed by 120° with respect to each other in order to obtain three phase balanced (fundamental) voltages. The three outputs may then be brought to a three phase transformer. The transformer primary windings must be isolated from one another, the secondary windings may be connected in star or delta to supply the load circuit.
In order to eliminate triplen harmonics ( \( n=3, 6, 9, \ldots \) ) the transformer secondary is normally in delta. When S1 is switched on, terminal a is connected to the positive of the DC input voltage. When S4 is switched on, terminal a is brought to the negative terminal of DC source and the same happens to terminals b and c.

![Diagram of 3-phase half bridge inverter with isolating & stepping transformer](image)

Fig. 2-3 General 3-phase half bridge inverter with isolating & stepping transformer.

There are 6R modes of conduction for creation a cycle of inverter output voltage, where R is the number of pulses per full cycle of output voltage. The MOSFETs are numbered in the sequence of gating them (i.e., 123 - 234 - 345 - 456 - 561 - 612 ) when each MOSFET conducts for 180° ( \( R=1 \) ). For proposed system inverter R=9, thus for creation of one cycle there will be 54 modes of conduction, table 2-10.
2.5 Voltage Control of Single Phase Inverters

In many Industrial applications, it is often required to control the output voltage of inverters. The techniques available differ in the harmonic content that they produce in the inverter output voltage, thus the acceptable harmonic content is the factor that determines the choice of technique. There are number of strategies employing pulse width modulation (PWM) and may be listed as:

1- single pulse width modulation.
2- multi-pulse width modulation (uniform pulse width modulation-UPWM [4])
3- sinusoidal pulse width modulation: natural sampled PWM, regular sampled PWM (symmetrical & asymmetrical) and optimized PWM [5], [6].
4- modified sinusoidal pulse width modulation [7].
5- staircase pulse width modulation [8], [9].

The optimized PWM will be employed for proposed system inverter.

2.5.1 Single pulse width modulation

Fig. 2-4 shows the output voltage of single pulse width modulation on full bridge inverter.

![Fig. 2-4 Single pulse width modulation](image-url)

Fig. 2-4 Single pulse width modulation
This wave form may be analysed according to Fourier Series.

$$V_0(t) = \sum_{n=1,3,5}^{\infty} \left[ a_n \sin(n\omega t) + b_n \cos(n\omega t) \right] \tag{2-1}$$

$$a_n = \frac{2}{\pi} \int_0^\pi E \sin(n\omega t) d(\omega t) = \frac{2E}{n\pi} \int_{\pi/2}^{n\delta/2} \sin(n\omega t) d(\omega t) = \frac{4E}{n\pi} \sin\left(\frac{n\delta}{2}\right) \tag{2-2}$$

$$b_n = \frac{2}{\pi} \int_0^\pi E \cos(n\omega t) d(\omega t) = \frac{2E}{\pi} \int_{n\delta/2}^{(n+1)\delta/2} \cos(n\omega t) d(\omega t) = 0 \tag{2-3}$$

$$V_0(t) = \sum_{n=1,3,5}^{\infty} a_n \sin(n\omega t) = \sum_{n=1,3,5}^{\infty} \left[ \frac{4E}{n\pi} \sin\left(\frac{n\delta}{2}\right) \right] \sin(n\omega t) \tag{2-4}$$

In (2-4), the output voltage, $V_0(t)$, is controlled by varying $\delta$ from $0^\circ$ to $180^\circ$.

### 2.5.2 Multiple pulse width modulation

This method is also called uniform pulse width modulation (UPWM), since the width and space between pulses are equal, Fig. 2-5.

The harmonic content can be significantly reduced by increasing the number of pulses in each half cycle of output voltage. The $V_0(t)$ wave form Fourier Series coefficients are determined by deriving an expression for a general pair of pulses situated at $\omega t = \alpha_m$ and $\omega t = \pi + \alpha_m$, and then combining the effects of all such pairs of pulses in the cycle.
The Fourier coefficients for a pair of pulses are:

\[ a_n = \frac{2}{\pi} \int_{0}^{\alpha_m+\delta} E \sin(n\omega t) d(\omega t) = \frac{2E}{n\pi} \left[ \cos(n\alpha_m) - \cos(n(\alpha_m+\delta)) \right] = \frac{4E}{n\pi} \sin(n_{2}^{\delta}) \sin\left(\frac{n(\alpha_m+\delta)}{2}\right) \] (2-6)

\[ b_n = \frac{2}{\pi} \int_{0}^{\alpha_m+\delta} E \cos(n\omega t) d(\omega t) = \frac{2E}{n\pi} \left[ -\sin(n\alpha_m) + \sin(n(\alpha_m+\delta)) \right] = \frac{4E}{n\pi} \sin(n_{2}^{\delta}) \cos\left(\frac{n(\alpha_m+\delta)}{2}\right) \] (2-7)

Then:

\[ A_n = \sum_{m=1,2}^{R} \left[ \frac{4E}{n\pi} \sin\left(\frac{\delta}{2}\right) \sin\left(\frac{n(\alpha_m+\delta)}{2}\right) \right] \] (2-8)

\[ B_n = \sum_{m=1,2}^{R} \left[ \frac{4E}{n\pi} \sin\left(\frac{\delta}{2}\right) \cos\left(\frac{n(\alpha_m+\delta)}{2}\right) \right] \] (2-9)
Where:

\[ R = \frac{f_c}{f_m} \text{ or } R = \text{number of pulses per one cycle} \]

\[ f_c = \text{carrier frequency} \]

\[ f_m = \text{output voltage frequency} \]

By substituting (2-8) & (2-9) into (2-5) the output voltage results:

\[
V_o(t) = \sum_{n=1,3,5}^{\infty} \left[ \sum_{m=1,2}^{R} \frac{4E}{n\pi} \sin\left(\frac{\delta}{2}\right) \sin\left(n\left(\frac{\alpha_m + \frac{\delta}{2}}{2}\right)\right) \right] \sin(n\omega t) + \\
\sum_{n=1,3,5}^{\infty} \left[ \sum_{m=1,2}^{R} \frac{4E}{n\pi} \sin\left(\frac{\delta}{2}\right) \cos\left(n\left(\frac{\alpha_m + \frac{\delta}{2}}{2}\right)\right) \right] \cos(n\omega t)
\]  

(2-10)

By changing \( R \) and varying \( \delta \) the \( V_o(t) \) and harmonic contents are controlled.

2.5.3 Sinusoidal pulse-width modulation (Naturally sampled)

Unlike the multiple pulse width modulation that the width of pulses was the same, here the width of each pulse is varied in proportion to the amplitude of a sine wave evaluated at the centre of the same pulse [10], Fig. 2-6.

Fig. 2-6 Sinusoidal PWM principle

\[
\frac{\alpha_1}{y_1} = \frac{\alpha_2}{y_2} = \frac{\alpha_3}{y_3} = \text{cte.} 
\]  

(2-11)
In this strategy the output voltage is controlled efficiently, moreover distortion factor and low order harmonics are reduced significantly [7].

In sinusoidal PWM, a sinusoidal modulating wave is compared directly with a triangular carrier wave (Sampling signal) to determine the switching instants, and therefore the resultant pulse widths, Fig. 2-7.

![Fig. 2-7 Modulating & carrier wave](image)

Each switching point is determined via a non-linear equation expressing the equality at the switching point of the carrier wave and modulating sine wave as follow (with reference to Fig. 2-7)

Modulating wave:

$$y = M\sin(\alpha) \quad \text{(2-12)}$$

Carrier wave:

$$y_1 = 4(k-1) - \frac{2R}{\pi}\alpha \quad \text{Negative going transitions}$$

$$y_2 = 2 - 4k + \frac{2R}{\pi}\alpha \quad \text{positive going transitions}$$

$$k = 1, 2, 3, \ldots, R \quad \text{(2-13)}$$
Where:

\[ \alpha = \omega_m t \]
\[ \omega_m = 2\pi f_m \]
\[ R = \frac{f_c}{f_m} \]

\[ f_m = \text{is frequency of modulating wave} \]
\[ f_c = \text{is frequency of carrier wave} \]
\[ M = \text{is modulating index} \]
\[ R = \text{is frequency ratio or pulse numbers forming one output cycle} \]

Thus the \( 2R \) switching points are given by the solution of following set:

\[ M \sin \left( \alpha \right) = 4(k-1) - \frac{2R}{\pi} \alpha \]
\[ M \sin \left( \alpha \right) = 2-4k + \frac{2R}{\pi} \alpha \]

\[ k = 1, 2, 3, \ldots, R \]  \hspace{1cm} (2-15)

### 2.6 Discussion on \( M,R \)

For modulation index less than unity, the number of pulses in the resulting PWM wave is always equal to the frequency ratio \( R \). For \( M > 1 \), the number of pulses may be less than \( R \). As modulation index increases beyond unity, pulses are successively dropped and pulse numbers decreases. For significantly large \( M \), the pulse number becomes 1 and, finally, a square wave results.

To utilize fully the harmonic minimization capabilities of PWM control, it is desirable the inverter to operate at the maximum permissible switching frequency over the widest possible inverter output frequency range.
For low values of R the following consideration should be taken into account:

1- R should be an integer, this means that the two intersecting wave forms are synchronous, and therefore discontinuities and fluctuations are avoided. Subharmonics or "beat frequency" effects can be produced if the modulating and carrier frequencies are not synchronized [11].

2- R should be an odd number, ensuring that the positive and negative half-cycle have the same wave form, which means that the output voltage contains no even harmonics.

3- The harmonic of the order R will be dominant, thus in a 3-phase system it can be eliminated if R is chosen as a multiple of 3.

4- With a high value of the frequency ratio it is possible to cancel or reduce a number of low order harmonics, but at the same time a high frequency ratio results in relatively high switching losses in MOSFETs.

5- As has been shown in [12] by a 3-dimension model, the harmonic distortion (H.D.) decreases with increasing M till unity, and increases thereafter. Harmonic distortion also decreases with increasing of R.

Fig. 2.8 shows the 3-dimensional relationship (variation) between R, M, Total Harmonic Distortion (T.H.D).

\[ (*) \quad H.D = \sqrt{\sum_{n=1}^{\infty} \left( \frac{v_n}{v_1} \right)^2} \]
\[ (***) \quad T.H.D = \sqrt{\sum_{n=1}^{\infty} \left( \frac{v_n}{v_1} \right)^2} \]
Fig. 2-8 3-Dimensional relationship between R, M, T.H.D. in sinusoidal PWM
2.7 PWM inverter for proposed system

A sinusoidal PWM method is applied to the three phase half bridge prototype system inverter (Fig. 2-3) for obtaining switching instants. The set of equations (2-15) is used here by the following specifications:

\[ M \sin(\alpha) = 4(k-1) - \frac{2R}{\pi} \alpha \]
\[ M \sin(\alpha) = 2-4k + \frac{2R}{\pi} \alpha \quad (2-16) \]
\[ k = 1, 2, 3, \ldots \]

\[ M = 0.9 \]
\[ f_m = 1000 \text{ hz inverter output voltage frequency} \quad (2-17) \]
\[ R = 9 \]

By substituting (2-17) into (2-16):

\[ 0.9 \sin(\alpha) = 4(k-1) - \frac{18}{\pi} \alpha \]
\[ 0.9 \sin(\alpha) = 2-4k + \frac{18}{\pi} \alpha \quad (2-18) \]
\[ k = 1, 2, \ldots \]

The 2R = 18 switching angles; \( \alpha_0, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{17}, \alpha_{18} \) for \( \gamma \) an formation are obtained from solving of (2-18) by computer. Then by referring to Fig. 2-9 and considering 3-phase symmetry & quarter cycle symmetry property the switching angles for \( \gamma_{bn} \); \( \beta_0', \beta_1, \beta_2, \beta_3, \ldots, \beta_{17}, \beta_{18} \) and \( \gamma_{cn} \); \( \gamma_0, \gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_{17}, \gamma_{18} \) are arranged in following tables:
Fig. 2-9 3-phase inverter (Fig. 2-3) phase voltages
Table 2-1 Switching angles for \( v_{an} \) formation

<table>
<thead>
<tr>
<th>Switching angles</th>
<th>Relationship with ( \alpha_1, \alpha_2, \alpha_3, \alpha_4 )</th>
<th>Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>0</td>
<td>0.00000</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>( \alpha_1 )</td>
<td>23.6001</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>( \alpha_2 )</td>
<td>34.8000</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>( \alpha_3 )</td>
<td>68.3995</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>( \alpha_4 )</td>
<td>71.4995</td>
</tr>
<tr>
<td>( \alpha_5 )</td>
<td>180–( \alpha_4 )</td>
<td>108.500</td>
</tr>
<tr>
<td>( \alpha_6 )</td>
<td>180–( \alpha_3 )</td>
<td>111.600</td>
</tr>
<tr>
<td>( \alpha_7 )</td>
<td>180–( \alpha_2 )</td>
<td>145.200</td>
</tr>
<tr>
<td>( \alpha_8 )</td>
<td>180–( \alpha_1 )</td>
<td>159.399</td>
</tr>
<tr>
<td>( \alpha_9 )</td>
<td>180</td>
<td>180.000</td>
</tr>
<tr>
<td>( \alpha_{10} )</td>
<td>180+( \alpha_1 )</td>
<td>203.600</td>
</tr>
<tr>
<td>( \alpha_{11} )</td>
<td>180+( \alpha_2 )</td>
<td>214.800</td>
</tr>
<tr>
<td>( \alpha_{12} )</td>
<td>180+( \alpha_3 )</td>
<td>248.399</td>
</tr>
<tr>
<td>( \alpha_{13} )</td>
<td>180+( \alpha_4 )</td>
<td>251.499</td>
</tr>
<tr>
<td>( \alpha_{14} )</td>
<td>360–( \alpha_4 )</td>
<td>288.590</td>
</tr>
<tr>
<td>( \alpha_{15} )</td>
<td>360–( \alpha_3 )</td>
<td>291.699</td>
</tr>
<tr>
<td>( \alpha_{16} )</td>
<td>360–( \alpha_2 )</td>
<td>325.200</td>
</tr>
<tr>
<td>( \alpha_{17} )</td>
<td>360–( \alpha_1 )</td>
<td>336.400</td>
</tr>
<tr>
<td>( \alpha_{18} )</td>
<td>360</td>
<td>360.000</td>
</tr>
<tr>
<td>Switching angles</td>
<td>Relationship with $\alpha_1, \alpha_2, \alpha_3, \alpha_4$</td>
<td>Degrees</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>$\alpha_3 - 60$</td>
<td>8.4000</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>$\alpha_4 - 60$</td>
<td>11.500</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>$120 - \alpha_4$</td>
<td>48.500</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>$120 - \alpha_3$</td>
<td>51.600</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>$120 - \alpha_2$</td>
<td>85.500</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>$120 - \alpha_1$</td>
<td>96.500</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>$120$</td>
<td>120.00</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>$120 + \alpha_1$</td>
<td>143.60</td>
</tr>
<tr>
<td>$\beta_8$</td>
<td>$120 + \alpha_2$</td>
<td>154.80</td>
</tr>
<tr>
<td>$\beta_9$</td>
<td>$120 + \alpha_3$</td>
<td>188.40</td>
</tr>
<tr>
<td>$\beta_{10}$</td>
<td>$120 + \alpha_4$</td>
<td>191.50</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>$300 - \alpha_4$</td>
<td>228.50</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>$300 - \alpha_3$</td>
<td>231.60</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>$300 - \alpha_2$</td>
<td>265.20</td>
</tr>
<tr>
<td>$\beta_{14}$</td>
<td>$300 - \alpha_1$</td>
<td>276.40</td>
</tr>
<tr>
<td>$\beta_{15}$</td>
<td>$300$</td>
<td>300.00</td>
</tr>
<tr>
<td>$\beta_{16}$</td>
<td>$300 + \alpha_1$</td>
<td>323.71</td>
</tr>
<tr>
<td>$\beta_{17}$</td>
<td>$300 + \alpha_2$</td>
<td>334.80</td>
</tr>
<tr>
<td>$\beta_{18}$</td>
<td>$360$</td>
<td>360.00</td>
</tr>
</tbody>
</table>
Fig. 2-11

\[ f(x) = v_{bn}, M=0.9, R=9, f_m=1 \text{ KHz} \]
Table 2-3 Switching angles for $v_{cn}$ formation

<table>
<thead>
<tr>
<th>Switching angles</th>
<th>Relationship with $\alpha_1, \alpha_2, \alpha_3, \alpha_4$</th>
<th>Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>$60-\alpha_2$</td>
<td>25.200</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>$60-\alpha_1$</td>
<td>34.400</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>60</td>
<td>60.000</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>$60+\alpha_1$</td>
<td>83.600</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>$60+\alpha_2$</td>
<td>94.800</td>
</tr>
<tr>
<td>$\gamma_5$</td>
<td>$60+\alpha_3$</td>
<td>128.40</td>
</tr>
<tr>
<td>$\gamma_6$</td>
<td>$60+\alpha_4$</td>
<td>131.50</td>
</tr>
<tr>
<td>$\gamma_7$</td>
<td>$240-\alpha_4$</td>
<td>168.50</td>
</tr>
<tr>
<td>$\gamma_8$</td>
<td>$240-\alpha_3$</td>
<td>171.60</td>
</tr>
<tr>
<td>$\gamma_9$</td>
<td>$240-\alpha_2$</td>
<td>205.20</td>
</tr>
<tr>
<td>$\gamma_{10}$</td>
<td>$240-\alpha_1$</td>
<td>216.40</td>
</tr>
<tr>
<td>$\gamma_{11}$</td>
<td>240</td>
<td>240.00</td>
</tr>
<tr>
<td>$\gamma_{12}$</td>
<td>$240+\alpha_1$</td>
<td>263.60</td>
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<td>$\gamma_{13}$</td>
<td>$240+\alpha_2$</td>
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<td>$240+\alpha_3$</td>
<td>308.40</td>
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<td>311.50</td>
</tr>
<tr>
<td>$\gamma_{16}$</td>
<td>$420-\alpha_4$</td>
<td>348.50</td>
</tr>
<tr>
<td>$\gamma_{17}$</td>
<td>$420-\alpha_3$</td>
<td>351.60</td>
</tr>
<tr>
<td>$\gamma_{18}$</td>
<td>360</td>
<td>360.00</td>
</tr>
</tbody>
</table>
2.8 Other waveforms of inverter

Based on Fig. 2-3 the following relationship can be derived by assuming.

i- balanced resistive load.
ii- the n is zero voltage reference point.

\[ v_{ab} = v_{an} - v_{bn} \]
\[ v_{bc} = v_{bn} - v_{cn} \] \hspace{1cm} (2–19)
\[ v_{ca} = v_{cn} - v_{an} \]

\[
\begin{bmatrix}
  v_{ao} \\
  v_{bo} \\
  v_{co}
\end{bmatrix} = \frac{1}{3}
\begin{bmatrix}
  2 & -1 & -1 \\
  1 & 2 & -1 \\
  -1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
  v_{an} \\
  v_{bn} \\
  v_{cn}
\end{bmatrix}
\] \hspace{1cm} (2–20)

The third harmonic (when in Fig. 2-3 the K is open) is:

\[ v_{no} = \frac{1}{3} (v_{ao} + v_{bo} + v_{co}) - (v_{an} + v_{bn} + v_{cn}) \] \hspace{1cm} (2–21)

The respective waveforms are shown in next pages.
Fig. 2-13 3-phase inverter line voltages

$$f(x) = V_{ab}, \quad R = 9, \quad M = 0.9, \quad f_m = 1 \text{ KHz}$$

$$f(x) = V_{bc}, \quad R = 9, \quad M = 0.9, \quad f_m = 1 \text{ KHz}$$

$$f(x) = V_{ca}, \quad R = 9, \quad M = 0.9, \quad f_m = 1 \text{ KHz}$$
Fig. 2-14

$f(x) = V_{ao}, \ R = 9, \ M = 0.9, \ fm = 1 \text{ KHz}$

$f(x) = V_{bo}, \ R = 9, \ M = 0.9, \ fm = 1 \text{ KHz}$

$f(x) = V_{co}, \ R = 9, \ M = 0.9, \ fm = 1 \text{ KHz}$

Fig. 2-14
Fig. 2-15 third harmonic \( (v_{no}) \) waveform
2.9 Harmonic spectrum of output voltage ($v_{an}$)

The harmonic spectrum of the $v_{an}$ waveform (analysed by Fourier Series) is shown in Fig. 2-16. In Table 2-4 the amplitude of low order and predominant components is presented.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Harmonic Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.901369</td>
</tr>
<tr>
<td>3</td>
<td>0.002669</td>
</tr>
<tr>
<td>5</td>
<td>0.011683</td>
</tr>
<tr>
<td>7</td>
<td>0.266405</td>
</tr>
<tr>
<td>9</td>
<td>0.710470</td>
</tr>
<tr>
<td>11</td>
<td>0.267798</td>
</tr>
<tr>
<td>13</td>
<td>0.005674</td>
</tr>
<tr>
<td>15</td>
<td>0.176882</td>
</tr>
<tr>
<td>17</td>
<td>0.256467</td>
</tr>
<tr>
<td>19</td>
<td>0.255778</td>
</tr>
<tr>
<td>21</td>
<td>0.159450</td>
</tr>
<tr>
<td>23</td>
<td>0.127635</td>
</tr>
<tr>
<td>25</td>
<td>0.163148</td>
</tr>
</tbody>
</table>
Fig. 2-16 inverter phase voltage (v an) harmonic spectrum
2.10 Harmonic elimination importance

The output voltage waveform of ideal inverters should be sinusoidal. Nevertheless, the waveforms of practical inverters are nonsinusoidal and contain certain harmonics, for example: the output of conventional PWM inverters is square wave (2 level on half bridge, 3 level on full bridge) and for resonant inverters the output voltage is approximately sinusoidal.

The total harmonic distortion (T.H.D.) factor is defined as a measure of closeness in shape between a waveform and its fundamental component:

\[
T\text{.H.D.} = \frac{\sqrt{v_2^2 + v_3^2 + v_4^2 + \ldots + v_n^2}}{v_1} = \frac{1}{v_1} \left( \sum_{n=2, 3, \ldots}^{\infty} v_n^2 \right)^{1/2}
\]

Where \( v_1 \) is the rms value of the fundamental component of the output voltage, and \( v_n \) (\( n \neq 1 \)) is the rms value of the \( n \)th harmonic. In some applications (fast response power supplies) T.H.D. may be varied from 0.05, and this requires a low pass filter at inverter output to eliminate the higher harmonics.

As it is clear from resonant L-C filtering circuits (\( f_r = \frac{1}{2\pi \sqrt{L}} \)), the low order harmonics is accompanied weight and bulky filter elements. Thus if low order harmonics in the output voltage can be reduced or eliminated by other means (unfiltered inverter), the size of the filter can be reduced. This also improves the speed of response to load change of the entire inverter filter system.
2.11 Harmonic elimination and voltage control of PWM inverters

For the harmonic elimination and control process of the output voltage of the 3-phase inverters, the Fig. 2-17 is used as a simplified reference circuit.

![Fig. 2-17 The simplified 3-phase half bridge inverter](Image)

A generalized quarter-wave symmetric PWM switching pattern can be defined as shown in Fig. 2-18, where the odd switching angles $\alpha_1, \alpha_3, \ldots$ define the negative going transitions and $\alpha_2, \alpha_4, \ldots$ define the positive going transitions.
The $v_{an}(\omega t)$, $v_{bn}(\omega t)$, and $v_{cn}(\omega t)$ will be referred as the inverter phase voltages, where the following relationships are valid between them:

$$v_{an}(\omega t) = v_{an}(\omega t + 2\pi)$$

$$v_{an}(\omega t) = v_{an}(\pi - \omega t)$$

$$v_{an}(\omega t) = -v_{an}(\pi + \omega t)$$

$$v_{bn}(\omega t) = v_{an}(\omega t - \frac{2}{3}\pi)$$

$$v_{cn}(\omega t) = v_{an}(\omega t + \frac{2}{3}\pi)$$

Fig. 2-18 Generalized output voltage waveform of single phase half bridge inverter
The first three are the character of the wave: periodic, with quarter-cycle symmetry, and half-cycle antisymmetry. The last two requirements show the symmetric three-phase character of the inverter voltages. The $v_{ab}(\omega t)$, $v_{bc}(\omega t)$ and $v_{ca}(\omega t)$ define the line voltages of inverter.

When the inverter is commutated $N$ (number of switching angles) times over a quarter cycle, the expansion of $v_{an}(\omega t)$ in a Fourier series yields only odd-sinusoidal components [13] & [14].

$$v_{an}(\omega t) = \sum_{n=1,3,5}^{\infty} \left[ a_n \sin(n\omega t) + b_n \cos(n\omega t) \right] \quad (2-24)$$

where:

$$b_n = 0.0$$

$$a_n = \frac{4E}{n\pi} \left[ 1 - 2 \sum_{k=1}^{N} (-1)^{k} \cos(n\alpha_k) \right] \quad (2-25)$$

$$n = 1, 3, 5, \ldots, N = \text{number of pulses over a quarter cycle}$$

The term $\alpha_k$ denotes the commutation angles ($k = 1, 2, \ldots, N$), such that:

$$0 < \alpha_1 < \alpha_2 < \ldots < \alpha_k < \frac{\pi}{2} \quad (2-26)$$

The (2-25) may be shown in an expanded form:

$$a_n = \frac{4E}{n\pi} \left[ 1 - 2 \cos(n\alpha_1) + 2 \cos(n\alpha_2) - 2 \cos(n\alpha_3) + 2 \cos(n\alpha_4) - \ldots \right] \quad (2-27)$$
It is clear from (2-25) that, there are N degrees of freedom in controlling the amplitudes of the component harmonics, making it possible to eliminate N low order harmonics (put related \( a_n = 0.0 \)) or eliminate (N-1) low order and to control the amplitude of the fundamental wave between the limits:

\[
0 \leq V_{\text{max}}(1) < \frac{4}{\pi} \quad \text{or} \quad 0 \leq V_{\text{max}}(1) < 1.2732E
\]

(2-28)

For elimination N low order harmonics following set can be arranged:

\[
f_N(\alpha) = 1 + 2\sum_{k=1}^{N} (-1)^k \cos(n_k \alpha) = 0
\]

\[
f_N(\alpha) = 1 + 2\sum_{k=1}^{N} (-1)^k \cos(n_k \alpha) = 0
\]

(2-29)

The (2-29) represent a system of N nonlinear equations. In solving that numerically [15] & [16] the main concern is the convergence of iteration results (for example in using Newton-Raphson method assigning an initial guess for the N angles, which is the most difficult step of solution). So there is a trial and error process, rather than a guarantee for convergence to a solution.

After determining the optimized switching angles, they can subsequently be programmed into the microprocessor memory and used to generate the PWM waveform in real-time. A number of solutions have been reported with different computer facilities [13], [17] and [18].
In ref. [13], the system of equations has been solved for \( N = 2 \) & \( N = 5 \) for half bridge and full bridge inverters. Ref. [17] is more interesting for programmed PWM techniques, and [19] gives the trajectories of switching angles with \( N = 9 \) for high performance induction motor type loads.

Here the control of fundamental wave amplitude between the limits given in (2-28) and eliminating of 5th, 7th, 11th components by varying of \( \alpha_1 \), \( \alpha_2 \), \( \alpha_3 \) and \( \alpha_4 \) [18], is shown in Fig. 2-19.

![Fig. 2-19 Trajectories of switching angles for 3-phase inverter](image)

Not 5th, 7th, 11th harmonics of 3-phase Inverter

\( \alpha_1 \), \( \alpha_2 \), \( \alpha_3 \), \( \alpha_4 \)
2.12 Harmonic elimination for prototype 3-phase half bridge system with \( N=4 \) and \( V_{\text{max}}(1) = 1.17 \) p.u.

In determining the N angles over a quarter-cycle, with a view to eliminating N harmonics, a set of nonlinear equations, based on (2-25) should be solved. Since the prototype system is 3-phase & half bridge, the triplen harmonics \( v_{3k}, k=1, 2, 3, \ldots \) produce no current in it and need not be eliminated. Putting \( N=4 \) in (2-25) yields:

\[
\begin{align*}
V_{\text{max}}(5) &= 0 \\
V_{\text{max}}(7) &= 0 \\
V_{\text{max}}(11) &= 0 \\
V_{\text{max}}(13) &= 0
\end{align*}
\]

or in a expanded form:

\[
\begin{align*}
0.5 - \cos(5\alpha_1) + \cos(5\alpha_2) - \cos(5\alpha_3) + \cos(5\alpha_4) &= 0.0 \\
0.5 - \cos(7\alpha_1) + \cos(7\alpha_2) - \cos(7\alpha_3) + \cos(7\alpha_4) &= 0.0 \\
0.5 - \cos(11\alpha_1) + \cos(11\alpha_2) - \cos(11\alpha_3) + \cos(11\alpha_4) &= 0.0 \\
0.5 - \cos(13\alpha_1) + \cos(13\alpha_2) - \cos(13\alpha_3) + \cos(13\alpha_4) &= 0.0
\end{align*}
\]

This set, (2-31) was solved by using NAG (Numerical Algorithm Groups) routines in University computer center (Computer program in Appendix A) with following results:

\[
\begin{align*}
\alpha_1 &= 10.548153^\circ \\
\alpha_2 &= 16.094384^\circ \\
\alpha_3 &= 30.905343^\circ \\
\alpha_4 &= 32.864859^\circ \\
V_{\text{max}}(1) &= 1.17 \text{ p.u.}
\end{align*}
\]

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2.13 Waveforms construction for prototype system inverter

With reference to Fig. 2-17, related relationships of (2-19), (2-20), (2-21), (2-23) and based on obtained angles of (2-32) the inverter phase & line voltages and associated tables of switching angles are arranged as follows:

Table 2-5 Switching angles for \( v_{an} \) (harmonic eliminated) formation

<table>
<thead>
<tr>
<th>Switching angles</th>
<th>Relationship with ( \alpha_1, \alpha_2, \alpha_3, \alpha_4 )</th>
<th>Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>0</td>
<td>0.00000</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>( \alpha_1 )</td>
<td>10.5482</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>( \alpha_2 )</td>
<td>16.0944</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>( \alpha_3 )</td>
<td>30.9053</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>( \alpha_4 )</td>
<td>32.8649</td>
</tr>
<tr>
<td>( \alpha_5 )</td>
<td>( 180-\alpha_4 )</td>
<td>147.135</td>
</tr>
<tr>
<td>( \alpha_6 )</td>
<td>( 180-\alpha_3 )</td>
<td>149.095</td>
</tr>
<tr>
<td>( \alpha_7 )</td>
<td>( 180-\alpha_2 )</td>
<td>163.906</td>
</tr>
<tr>
<td>( \alpha_8 )</td>
<td>( 180-\alpha_1 )</td>
<td>169.452</td>
</tr>
<tr>
<td>( \alpha_9 )</td>
<td>( 180 )</td>
<td>180.000</td>
</tr>
<tr>
<td>( \alpha_{10} )</td>
<td>( 180+\alpha_1 )</td>
<td>190.548</td>
</tr>
<tr>
<td>( \alpha_{11} )</td>
<td>( 180+\alpha_2 )</td>
<td>196.094</td>
</tr>
<tr>
<td>( \alpha_{12} )</td>
<td>( 180+\alpha_3 )</td>
<td>210.905</td>
</tr>
<tr>
<td>( \alpha_{13} )</td>
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<td>327.135</td>
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<td>( 360-\alpha_3 )</td>
<td>329.095</td>
</tr>
<tr>
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<td>( 360-\alpha_2 )</td>
<td>343.906</td>
</tr>
<tr>
<td>( \alpha_{17} )</td>
<td>( 360-\alpha_1 )</td>
<td>349.452</td>
</tr>
<tr>
<td>( \alpha_{18} )</td>
<td>( 360 )</td>
<td>360.000</td>
</tr>
<tr>
<td>Switching angles</td>
<td>Relationship with $\alpha_1, \alpha_2, \alpha_3, \alpha_4$</td>
<td>Degrees</td>
</tr>
<tr>
<td>------------------</td>
<td>------------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>$120-\alpha_4$</td>
<td>87.1351</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>$120-\alpha_3$</td>
<td>89.0947</td>
</tr>
<tr>
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<td>$120-\alpha_2$</td>
<td>103.906</td>
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<tr>
<td>$\beta_3$</td>
<td>$120-\alpha_1$</td>
<td>109.452</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>$120$</td>
<td>120.000</td>
</tr>
<tr>
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<td>$120+\alpha_1$</td>
<td>130.548</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>$120+\alpha_2$</td>
<td>136.094</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>$120+\alpha_3$</td>
<td>150.905</td>
</tr>
<tr>
<td>$\beta_8$</td>
<td>$120+\alpha_4$</td>
<td>152.865</td>
</tr>
<tr>
<td>$\beta_9$</td>
<td>$300-\alpha_4$</td>
<td>267.135</td>
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<tr>
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</tr>
<tr>
<td>$\beta_{12}$</td>
<td>$300-\alpha_1$</td>
<td>289.452</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>$300$</td>
<td>300.000</td>
</tr>
<tr>
<td>$\beta_{14}$</td>
<td>$300+\alpha_1$</td>
<td>310.548</td>
</tr>
<tr>
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<td>$300+\alpha_2$</td>
<td>316.094</td>
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<tr>
<td>$\beta_{16}$</td>
<td>$300+\alpha_3$</td>
<td>330.905</td>
</tr>
<tr>
<td>$\beta_{17}$</td>
<td>$300+\alpha_4$</td>
<td>332.865</td>
</tr>
<tr>
<td>$\beta_{18}$</td>
<td>$360$</td>
<td>360.000</td>
</tr>
<tr>
<td>Switching angles</td>
<td>Relationship with $\alpha_1, \alpha_2, \alpha_3, \alpha_4$</td>
<td>Degrees</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>$60-\alpha_4$</td>
<td>27.1351</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>$60-\alpha_3$</td>
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</tr>
<tr>
<td>$\gamma_2$</td>
<td>$60-\alpha_2$</td>
<td>43.9056</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>$60-\alpha_1$</td>
<td>49.4518</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>$60$</td>
<td>60.0000</td>
</tr>
<tr>
<td>$\gamma_5$</td>
<td>$60+\alpha_1$</td>
<td>70.5480</td>
</tr>
<tr>
<td>$\gamma_6$</td>
<td>$60+\alpha_2$</td>
<td>76.0944</td>
</tr>
<tr>
<td>$\gamma_7$</td>
<td>$60+\alpha_3$</td>
<td>90.9053</td>
</tr>
<tr>
<td>$\gamma_8$</td>
<td>$60+\alpha_4$</td>
<td>92.8649</td>
</tr>
<tr>
<td>$\gamma_9$</td>
<td>$240-\alpha_4$</td>
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<tr>
<td>$\gamma_{10}$</td>
<td>$240-\alpha_3$</td>
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<td>$\gamma_{11}$</td>
<td>$240-\alpha_2$</td>
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<td>229.452</td>
</tr>
<tr>
<td>$\gamma_{13}$</td>
<td>$240$</td>
<td>240.000</td>
</tr>
<tr>
<td>$\gamma_{14}$</td>
<td>$240+\alpha_1$</td>
<td>250.548</td>
</tr>
<tr>
<td>$\gamma_{15}$</td>
<td>$240+\alpha_2$</td>
<td>256.094</td>
</tr>
<tr>
<td>$\gamma_{16}$</td>
<td>$240+\alpha_3$</td>
<td>270.905</td>
</tr>
<tr>
<td>$\gamma_{17}$</td>
<td>$240+\alpha_4$</td>
<td>272.865</td>
</tr>
<tr>
<td>$\gamma_{18}$</td>
<td>360</td>
<td>360.000</td>
</tr>
</tbody>
</table>
Fig. 2-22
Fig. 2-23 3-phase inverter (Fig 2-17) line voltages

\[ f(x) = V_{ab}, \text{ no } 5\text{th}, 7\text{th}, 11\text{th}, 13\text{th }, \text{ fm}=1 \text{ KHz} \]

\[ f(x) = V_{bc}, \text{ no } 5\text{th}, 7\text{th}, 11\text{th}, 13\text{th }, \text{ fm}=1 \text{ KHz} \]

\[ f(x) = V_{ca}, \text{ no } 5\text{th}, 7\text{th}, 11\text{th}, 13\text{th }, \text{ fm}=1 \text{ KHz} \]
Fig. 2-24
Fig. 2-25 Third harmonic ($v_{no}$) waveform
2.14 Harmonic spectrum of output voltages ($v_{an}$ & $v_{ab}$)

The harmonic spectrum of $v_{an}$ waveform (analysed by Fourier Series) is shown in Fig. 2-26(a). The harmonic spectrum of $v_{ab}$ (line voltage) is represented in Fig. 2-26(b) where triplen components are eliminated and amplitudes should be multiplied by $\sqrt{3}$. In Table 2-8 the amplitude of low order and predominant components is presented.

Table 2-8 The $v_{an}$ harmonic component amplitudes

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Harmonic Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.170470</td>
</tr>
<tr>
<td>3</td>
<td>0.180014</td>
</tr>
<tr>
<td>5</td>
<td>0.000040</td>
</tr>
<tr>
<td>7</td>
<td>0.000060</td>
</tr>
<tr>
<td>9</td>
<td>0.017429</td>
</tr>
<tr>
<td>11</td>
<td>0.000080</td>
</tr>
<tr>
<td>13</td>
<td>0.000224</td>
</tr>
<tr>
<td>15</td>
<td>0.085448</td>
</tr>
<tr>
<td>17</td>
<td>0.237163</td>
</tr>
<tr>
<td>19</td>
<td>0.349063</td>
</tr>
<tr>
<td>21</td>
<td>0.329534</td>
</tr>
<tr>
<td>23</td>
<td>0.195669</td>
</tr>
<tr>
<td>25</td>
<td>0.055028</td>
</tr>
</tbody>
</table>
Fig. 2-26(a) inverter phase voltage $v_{an}$ harmonic spectrum

$\nu_{an}$ HARM. SPEC., $M=0.9$, $R=9$
Fig. 2-26(b) inverter line voltage \( v_{ab} \) harmonic spectrum.
2.15 Spectrum comparison before and after harmonic elimination

The comparison between $v_{an}$ harmonic spectrums before and after low order component elimination is presented in Table 2-9.

Table 2-9 The $v_{an}$ harmonic spectrum comparison before & after elimination

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Harmonic Amplitude</th>
<th>Harmonic Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Elimination</td>
<td>5, 7, 11, 13th Eliminated</td>
</tr>
<tr>
<td>1</td>
<td>0.901369</td>
<td>1.170470</td>
</tr>
<tr>
<td>3</td>
<td>0.002669</td>
<td>0.180014</td>
</tr>
<tr>
<td>5</td>
<td>0.011683</td>
<td>0.000000</td>
</tr>
<tr>
<td>7</td>
<td>0.266405</td>
<td>0.000000</td>
</tr>
<tr>
<td>9</td>
<td>0.710470</td>
<td>0.017429</td>
</tr>
<tr>
<td>11</td>
<td>0.267798</td>
<td>0.000000</td>
</tr>
<tr>
<td>13</td>
<td>0.005674</td>
<td>0.000000</td>
</tr>
<tr>
<td>15</td>
<td>0.176882</td>
<td>0.085448</td>
</tr>
<tr>
<td>17</td>
<td>0.256467</td>
<td>0.237163</td>
</tr>
<tr>
<td>19</td>
<td>0.255778</td>
<td>0.349063</td>
</tr>
<tr>
<td>21</td>
<td>0.159450</td>
<td>0.329534</td>
</tr>
<tr>
<td>23</td>
<td>0.127635</td>
<td>0.195669</td>
</tr>
<tr>
<td>25</td>
<td>0.163148</td>
<td>0.055028</td>
</tr>
</tbody>
</table>

In prototype 3-phase half bridge inverter there are no:

1- DC components (because of area symmetry).
2- Even harmonics (because of half-wave symmetry).
3- Odd cosine terms (because of quarter-wave symmetry).
4- Triplen components (because of 3-phase symmetry).
5- 5th, 7th, 11th, 13th components (because of elimination process).
2.16 Microprocessor implementation of prototype system inverter

By considering of $\alpha$, $\beta$ and $\gamma$ switching angles for modulation of $v_{an}$ (Fig. 2-20), $v_{bn}$ (Fig. 2-21) and $v_{cn}$ (Fig. 2-22) waveforms, Table 2-10 is made up which illustrates that there are 54 conduction stages ($6R & R=9$) for creation of one cycle of 3-phase, symmetric and 1 kHz AC waveforms from DC source. The prototype system inverter is shown in Fig. 2-27 for convenience.

![Fig. 2-27 3-phase half bridge inverter with isolating & stepping transformer](image-url)
Table 2-10 one cycle switching sequences for 3-phase 1 Khz half bridge inverter

<table>
<thead>
<tr>
<th>Stage</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>Conduction Duration (degrees)</th>
<th>Step</th>
<th>Conduction Duration (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>α1-α0=10.5482</td>
<td>α1</td>
<td>29.30000</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>α2-α1=5.54620</td>
<td>α2</td>
<td>15.40555</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>γ1-α2=11.0407</td>
<td>γ1</td>
<td>30.66944</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>γ2-γ1=1.95960</td>
<td>γ2</td>
<td>5.444444</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>α3-γ2=1.810600</td>
<td>α3</td>
<td>5.027777</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>α4-α3=1.959600</td>
<td>α4</td>
<td>5.444444</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>γ3-α4=11.04070</td>
<td>γ3</td>
<td>30.66944</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>γ4-γ3=5.54620</td>
<td>γ4</td>
<td>15.40555</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>γ5-γ4=10.54820</td>
<td>γ5</td>
<td>29.30000</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>γ6-γ5=10.54820</td>
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<td>29.30000</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>1</td>
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<td>15.40555</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>30.66944</td>
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<tr>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>γ8-β2=1.81060</td>
<td>γ8</td>
<td>5.027777</td>
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<tr>
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<td>0</td>
<td>0</td>
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<td>5.444444</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>β3-γ9=11.0411</td>
<td>β3</td>
<td>30.66944</td>
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<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>β4-β3=5.54600</td>
<td>β4</td>
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The different practical waveforms about the prototype system inverter will be presented in chapter 5.
2.17 Resonant inverters

There is increasing demand for smaller and lighter power processing equipment for DC-to-DC or DC-to-AC conversion. The natural approach is to raise the converter's operating frequency. PWM inverters fail to accommodate such a requirement owing to high frequency harmonic components associated with the square wave current or voltage wave forms. Moreover the power dissipation in the switching device of PWM inverters increases significantly at high frequency because of the their forced commutation operation.

As a result, resonant power conversion employing a high frequency link emerges as a leading technique and resonant inverters being superior to conventional PWM inverters due to possessing several attractive features such as:

1- light wight.
2- fast response.
3- high reliability.
4- naturally sinusoidal voltage and current wave forms.
5- reduced EMI (electromagnetic interference).
6- simple circuit implementation.

Features 1, 2 result from link operation at high frequency levels. While features 3, 4, 5, 6 are common to all circuits in which the commutation of the current in the MOSFETs occurs naturally in the course of circuit operation.

The resonant inverters are based on resonant current oscillation on a L-C-R circuit with under damped \( R < 2(L/C)^{1/2} \) mode of operation [20], [21] & [22], which is exited from a fixed DC supply through a half bridge or full bridge inverter, Fig. 2-28.
This type of inverter produces an approximately sinusoidal wave form at high output frequency, up to several hundred kHz and is commonly used in relatively fixed output load application, for example; induction heating, ultrasonic generator, fluorescent lighting.

2.18 Series resonant & Parallel resonant inverters

In general, resonant converters are classified into two types, depending on the manner by which energy is extracted from the resonant tank. These two types are:

1- The series resonant converters (SRCs)
2- The parallel resonant converters (PRCs)

In the SRCs, Fig. 2-29 (a), the load (output converter) is connected to the h.f. link circuit in series with link capacitor in which power transfer from the source to the load is accomplished through series resonance. The steady-state and the dynamic properties of SRCs have been analysed in many literature [23], [24] & [25].
Fig. 2-29 (a) Series resonant inverter  (b) Parallel resonant inverter
In the PRCs, Fig. 2-29(b), the load (output converter) is connected to the h.f. link in parallel with the link capacitor in which the capacitor voltage is used to transfer energy from the source to the load. The steady-state analysis and performance of PRCs have been reported by [26], [27] & [28].

In both cases the h.f. feature of the link capacitor allows the use of a h.f. transformer to provide voltage transformation and isolation between the DC source and the load.

In Fig. 2-29, the load on the inverter is the h.f. transformer and output converter consisting of the four bidirectional switches S3, ...... & S6 which can be turned ON and OFF as desired. In a DC-to-DC conversion scheme the output converter will be a rectifier (controlled or uncontrolled) whereas for DC-to-AC conversion process, it will be a cycloconverter and firing angles is modulated.

In DC-to-AC conversion, the link inverter can encounter regenerative loads (power flow from the load to the link and thus from the link to the DC supply. But this condition cannot be analysed by conventional methods.

The parallel resonant inverters can be operate either below resonance \( f_0 < f_r \), leading power factor mode) or above resonance \( f_0 > f_r \), lagging power factor mode). Operation and analysis of PRC operating below resonance are presented in [27], [29]. The operation, analysis and performance of operating above resonance are reported in [30], [31].
2.19 References:


8- K. Thorborg "Power Electronics" 1988, prentice Hall.


21- L. P. Huelsman "Basic circuit theory with digital computer" 1972, Prentice Hall.


26- V. T. Ranganathan, P. D. Ziogas, V. R. Stefanovic "Performance characteristic of high frequency link under forward and regenerative power flow conditions" IEEE IAS Annual meeting 1983, pp. 831-839.


Chapter 3

The transformer in

high frequency & switched mode operation
3.1 General

The conversion process in power Electronics (DC-to-DC, DC-to-AC, AC-to-DC, and AC-to-AC) requires the use of Transformers and Inductors. With the advent of Integrated Magnetics [1] and the increased use of Transformer leakage inductances as the resonant inductors in resonant circuits [2 & 3], a crucial role in modern Power Electronics circuits is taken by these components.

An ideal transformer:
1. would pass all signal frequencies with no power loss.
2. provide any selected transformation ratio of voltage or current.
3. provide complete isolation between its input and its output.
4. process power in either direction with equal facility.
5. provide for additional inputs or outputs if are required.

Obviously, such a perfect transformation device does not exist physically.

Since the dramatic miniaturization of integrated logic circuits and power semi-conductor devices; MOSFET, GTO & SIT (static induction transistor), SITH (static induction thyristor) has not yet been matched by similar reduction in the size of transformers, so frequently they are the heaviest and bulkiest item in the conversion circuits. The use of high frequency switching on inverter-transformer circuitry, allows the design to reduce the size and weight of the transformer significantly.

The design of high frequency magnetic components is complicated as several considerations and constraints come in to effect, that are less important at low frequencies. Skin and proximity effects dominate copper losses, domain wall resonance phenomena raise hysteresis losses in the core magnetic materials. Parasitic inductance become as a proportion of total circuit inductance, and winding capacitance becomes considerable.
Transformers also have a significant effect upon the overall performance and efficiency of the power conversion system. Accordingly, the design of such transformers has an important influence on overall system weight, conversion efficiency and cost. Because of the interdependence and interaction of parameters, reasonable trade-offs are necessary to achieve design optimization, particularly if high frequency operation for conversion is wanted.

3.2 The design problems generally

The design process is faced with a set of constraints which must be observed in the design of any transformer, which are outlined as follows:

1- Efficiency in link with copper and core losses
2- Size and weight in connection with operating frequency
3- Maximum permissible temperature rise in link with core material
4- Cost effectiveness

Depending upon application, certain of these constraints will be dominant. It is not possible to optimize all parameters in a single design because of the interaction and interdependence of parameters. For example, if volume and weight are of great significance, reduction in both can be effected by operating the transformer at high frequency but at a penalty in efficiency and regulation. When the frequency can not be raised, reduction in weight and volume may still be possible by selecting a more efficient core material, but at a penalty of increased cost. Again necessary trade-offs should be made to achieve desirable design.

A detailed design procedure for different type of transformers may be found in [4, 5 & 6]. Here those factors and topics which are in some way associated with the transformer design and operation at high frequency and switching circuitries (inverters and cycloconverters) are principally reviewed.
3.3 The core

In square-wave application (chopping by inverter) the core is rapidly switched from full magnetization in one direction to full magnetization in the opposite direction. Characteristic of such application is the large class of inverters and converters in which the power transformer is not operated in a saturated mode but is driven nearly to saturation by a transistor (MOSFET) or the thyristor (GTO) switching circuit having an independent drive-signal source.

Generally, the core material for the Inverter-transformer is selected for maximum flux density and maximum squareness. Fig. 3.1

![Fig. 3-1 square hysteresis loop](image)

Cores with higher flux capabilities enable smaller transformers to be built. Maximum squareness is the more important characteristic, however, this must be tempered by the operating frequency, allowable temperature rise, maximum power output, as well as cost considerations.
A lack of a good B-H squareness results in spikes being generated on the output of the square wave which is potentially dangerous for the turned-off switch. Theoretically, when the core is saturated there is no flux change, however, in practice due to imperfect saturation when the squareness ratio ($Br/Br_s$) is less than one, there is an unwanted flux change ($d\Phi/dt$). This flux change, in addition to other causes (stray inductance and capacitance), generate a spike which, even though of short duration, causes heating of the semiconductor switch junctions, so the ideal inverter transformer should have a high squareness ratio to minimize the spike effect. Finally it should be added that square loop material has the steepest slope and thus the highest primary inductance.

3.4 Core materials and development

The first development from iron wire cores was around 1900 when non-oriented silicon-alloyed steel sheets were introduced for transformer use [7]. This new material was made by alloying 3% silicon to offer electrical steel with improved loss qualities and omnidirectional properties.

The next significant development was that in the late 1930s GOSS (Grain Oriented Silicon Steel) was invented. This was produced by adjusting the chemistry of previous steels, cold rolling the strip to the required thickness, followed by high temperature annealing at 1200 C° to induce secondary recrystallisation. This process produced large grains that were well oriented along the rolling direction of the steel sheet.

Then Hi-B steel was introduced, differing from GOSS in that improved grain orientation resulted in an improvement in magnetizing characteristics and a decrease in hysteresis loss. The loss in Hi-B steel sheets consists of hysteresis
loss which varies linearly with frequency:

\[ p_h = k \cdot B_{\text{max}}^{1.7} \cdot f \frac{\text{watt}}{\text{m}^3} , \quad k = \text{cons.} \quad (3-1) \]

and eddy current loss:

\[ p_e = \frac{\pi^2 \cdot B_{\text{max}}^2 \cdot f^2 \cdot t^2}{6 \cdot \rho} \frac{\text{watt}}{\text{m}^3} \quad (3-2) \]

Where \( \rho \) is the resistivity and \( t \) is sheet thickness.

The total loss has traditionally consisted of hysteresis and eddy current loss. But, using improved measuring techniques it has been shown that the sum of hysteresis and classical eddy loss does not equate with the total loss in the steel. The term anomalous loss is used to describe the discrepancy. The anomalous loss is thought to be caused by domain wall movement and has been shown to be proportional to the distance between domain walls, hence any reduction in this distance will result in loss reduction[7].

This fact is the basis of the second major development that of laser scribed, or surface scribed steels[7]. The laser surface treatment can reduce Hi-B type steel losses by a further 8% - 10%.

**3.4.1 High frequency core material**

For high frequency purposes, the four materials generally available for use in high frequency transformers are:

1. powdered iron.
2. powdered amorphous (not having crystalline structure) metal.
3. Manganese-Zinc Ferrite (Mn + Zn + Fe₂O₄).
4. Nikel-Zinc Ferrite (Ni + Zn + Fe₂O₄).
The Ferromagnetic material like iron family generally have high electrical conductivity and must be laminated even at 50 Hz to prevent excessive eddy current loss. Since:

\[
\delta = 0.5642 \sqrt{\frac{\rho}{\mu \cdot f}} \quad (3-3)
\]

Where:
- \( \delta \) is skin depth,
- \( \rho \) is resistivity,
- \( \mu = \mu_0 \mu_r \) is permeability (assumed constant) of the material,
- \( f \) is the frequency of the alternating field.

To avoid high losses due to eddy currents, the iron would have to be laminated thinner than the skin depth. Lamination of much less than 2 \( \mu \)m are quite impractical \[8\]. So, the amorphous metals which are prepared in the glassy state to lower their conductivity are employed. This process typically lowers the conductivity by a factor 3 at best, so the potential increase in skin depth is limited to a factor of 1.732.

Ferrites are ferromagnetic iron oxides, prepared commercially as ceramics and distinguished by their high electrical resistivity compared to metals. (Mn + Zn) Ferrite typically displays a \( \mu = 200 \mu_0 \) but a much lower conductivity \( 10^{-5} (\Omega \cdot m)^{-1} \). So (Ni + Zn) Ferrite is an appropriate choice for transformers operating at MHz ranges for extremely low conductivity and its ignorable eddy current loss. A transformer computer design aid has been presented in ref. [9] on the basis of the total power loss minimization at \( f > 100 \text{kHz} \).

### 3.5 Effect of high frequency on dielectric losses

Generally most of power losses constitutes Iron loss and Copper loss in commercial (50, 60 Hz) transformers. By elevating operation frequency, some stray loss occurs which is not accounted for by Iron loss or Copper loss. The mechanism of generating such stray loss is not clear so far. The experimental
results have been demonstrated that the stray loss in high voltage/high frequency transformer [60 v/1000 v, 30 kHz] is originated from the dielectric loss in the insulating materials of transformer as follow [10]

\[ P_d = 2\pi f \varepsilon E^2 \tan(\delta) \]  \hspace{1cm} (3-4)

Where:
- \( P_d \) is dielectric loss per unit volume
- \( f \) is frequency
- \( \varepsilon \) is dielectric constant
- \( \tan(\delta) \) is dielectric dissipation factor
- \( E \) is electric field

The dielectric loss according to [10] arises from various portions of insulation:
1. between high voltage and low voltage windings
2. between high voltage winding and core
3. between insulating layer of high voltage winding

Moreover in some materials \( \tan(\delta) \) changes with temperature. The thermal stability of transformer in operation depends on the balance between the heat generation and heat dissipation to avoid pushing transformer operation toward the curie point. Curie temperature is the temperature at which a material changes its ferromagnetic properties and become paramagnetic. For example, Ferrites have a curie temperature about 200°C, which limits their operation core temperature to about 100°C [11].

### 3.6 Effect of high frequency on size and output rating

Mathematical analysis indicate that the higher the power frequency, the smaller the size and weight of the transformer, motor or alternator. For
example a convectively cooled 4 kHz transformer has weight 28% less than its 400 Hz counter part [5].

The basic relationship among the volt-amper rating \( (p_o) \), volume \( (\text{vol}) \), magnetic flux density \( (B) \), operating frequency \( (f) \), permissible temperature \( (\theta) \), where \( \theta = T_o - T_a \), \( T_o \) is the operating temperature of the winding hottest point and \( T_a \) is ambient temperature is as follow [5]:

\[
p_o = k \left( \frac{3}{\text{vol}} \right)^4 \cdot f \cdot B \cdot (h \cdot \theta)^2 \tag{3-5}
\]

or

\[
\text{vol} = k_1 \left( \frac{p_o}{f \cdot B} \right)^\frac{3}{4} \cdot \frac{1}{(h \cdot \theta)^\frac{3}{8}} \tag{3-6}
\]

Where \( k, k_1 \) constants and \( h \) is coefficient of heat transfer.

From (3-5), (3-6), it is easy to say that the size and rating of a transformer are more responsive to a change in frequency and magnetic flux density than to a change in the temperature rise.

An increase in frequency more effectively reduces size\( (\text{vol}) \). To evaluate the extent to which size may be reduced by increasing the frequency, we must first consider the relation among flux density, frequency, and core losses.

When the frequency of the input to a transformer is increased the flux density decreases, because \( B \) varies inversely with \( f \):

\[
E = 4.44 f \cdot \Phi_{mr} N \quad \text{or} \quad E = 4.44 f \cdot B_{mr} A \cdot N \tag{3-7}
\]
In (3-7) if \( B_m \) be kept constant, any increasing of \( f \), reduces \( A \) (core cross section) or \( N \) (number of turns) which in turn, cause more reduction in size and weight.

From (3-6) it can be inferred that by having fixed \( p_0 \), \( B \), \( h_0 \) if frequency increases \( m \) times, the vol. of transformer decreases with following multiple:

\[
\frac{1}{(m)^{3/4}} = \frac{1}{m^{0.75}}
\]

As has been stated in [12] for a fixed transformer configuration, and for a constant loss power, the volume is determined by the expression:

\[
\text{Vol.} = \left[ \frac{1}{B_{\text{max}} f} \right]^6,
\]

where \( B_{\text{max}} \) is the maximum flux density and \( f \) is the operating frequency.

For a wide range considering the dependency of total transformer weight from increased frequency may be referred to [13]. For a prototype rating \( p_r \) for 60 Hz transformer with a 40° rise any increasing in \( f \) or \( \theta \) and \( \theta \) will result increase \( p_r \), which follows the formula [5]:

\[
p_o = p_r \left( \frac{f}{60} \right)^{0.76} \left( \frac{\theta}{40} \right)^{0.63}
\]  

(3-8)

3.7 Effect of high frequency on permeability (\( \mu_r \))

At high frequencies, eddy currents in the laminations may be great enough that the magnetic flux density will fall from a value \( B_0 \) at the surface of core sheet to some lower value in the interior. The effect depends on the
properties of the material, the frequency of the alternating field and the
dimensions of the sheet.

It is possible the magnetic flux density falls very rapidly in the interior
of the sheet, so that most of the flux is confined to a thin layer or skin near
the surface. This phenomenon is called eddy-current shielding or magnetic skin
effect and is analogous to the more familiar Copper skin effect.

The variation of flux density with distance is given by [14].

\[ B = B_0 \left( e^{\frac{-x}{\delta}} \right) \]  \hspace{1cm} (3-9)

Where, \( x \) is the distance in to the material from the surface. The \( \delta \) is given by

\[ \delta = \sqrt{\frac{2\rho}{\mu_0 \mu_r \omega}} \]  \hspace{1cm} (3-10)

Where, \( \rho \) is the resistivity of the material, \( \omega \) is the angular frequency of the
alternating field. At a depth \( x = \delta \) the \( B = B_0 e^{-1} = 0.3678 (B_0) \). So \( \delta \) is
known as the depth of penetration or magnetic skin depth.

The reduction in permeability from \( \mu_r \) to \( \mu'_r \) because of reduced flux
penetration is described by [5].

\[ \frac{\mu_r}{\mu'_r} = \frac{1}{y} \left[ \frac{\text{Sinh}(y) + \text{Sin}(y)}{\text{Cosh}(y) + \text{Cos}(y)} \right] \]  \hspace{1cm} (3-11)

Where \( y = a / \delta \) and \( a \) is lamination sheet thickness (meters), \( \delta \) is from (3-10).
Harmonic currents or voltages in transformer, result from the non linear
relationship between flux and magnetizing current which is exemplified in the
shape of the hysteresis loop, Fig. 3-2, and the shape of the hysteresis curve
depend on the properties of the core material.
Fig. 3-2 dependency of magnetizing current (accordingly, voltage & current harmonics) in the transformer from the hysteresis loop:

(a) Sine flux and Cosine induced voltage
(b) Sine current with linear B-H curve
(c) Peak current with round B-H loop
(d) Square-wave current with square B-H loop
From equation (3-11) dependency of \( \mu_r \) on frequency variation is realized, and since \( \mu = \mu_0 \cdot \mu_r = \frac{dB}{dH} \), therefore any reformation of voltage or current harmonics due to variation on operation frequency are expected. Fig. 3-3 shows the hysteresis loop/frequency variation for a typical transformer [4].

![Hysteresis loop with operating frequency variation](image)

Fig. 3-3 variation of hysteresis loop with operating frequency
There is a critical frequency $f_c$ at which $y=2$ and penetration is half the thickness of the sheet [5].

$$f_c = 101.5 \frac{\rho}{\mu r \cdot a^2}$$

(3 - 12)

Where $\rho$ is in microhm-centimeters and $a$ is in centimeters. For example for 4% Silicon, 50% Nickel and 80% Nickel sheet, with 14 mils thick is 12000, 1300, and 200 Hz respectively. It should be noted that these are audio frequencies, much lower than the radio frequencies associated with copper skin effect.

### 3.8 Effect of high frequency on magnetizing current

The equivalent circuit of a transformer with parameters as lumped constants is shown in Fig. 3-4.

![Transformer Equivalent Circuit](image)

**Fig. 3-4 transformer equivalent circuit**

where:

- $L_m$: the primary magnetizing inductance
\( L_p \): the leakage inductance of primary
\( L_s \): the leakage inductance of secondary

\( N_1, N_2 \) are primary and secondary number of turns

\( R_e \): the equivalent core loss shunt resistance

\( R_o \): the load resistance

\( R_p \): the equivalent D.C. primary resistance

\( R_s \): the equivalent D.C. secondary resistance

\( V_p \) and \( V_o \) are the input and output voltages

Referring to Fig. 3-4:

\[
I_m = \frac{1}{L_m} \int_0^{T_{on}} V_p \, dt \tag{3-13}
\]

\[
L_m = K N_1^2 \quad , \quad K = \text{core constant} \tag{3-14}
\]

\( T_{on} \) = on time of Inverter switch, inversely proportional to frequency

\( I_m \) = primary magnetizing current.

From Fig. 3.5, maximum magnetizing current is limited by the maximum usable flux density. The flux density also depends on the number of turns and core parameters:

\[
B = \frac{\mu_o \mu_r N_1 I_m}{l_e} \tag{3-15}
\]

where, \( l_e \) is the equivalent magnetic path length

By substituting (3-13), (3-14) in to (3-15)

\[
B = \frac{\mu_o \mu_r}{k l_e N_1} \int_0^{T_{on}} V_p \, dt \tag{3-16}
\]

putting \( N_1 \) from (3-15) in to (3-14) yields:
Fig. 3-5 magnetizing current waveform construction
(a) without hysteresis  (b) with hysteresis
by equating $L_m$ from (3-17) and (3-13) we have:

$$\frac{1}{I_m} \int_0^{T_{on}} v_p \, dt = k \left[ \frac{1_C \cdot B}{\mu_0 \cdot \mu_r \cdot I_m} \right]^2$$

or:

$$I_m = \frac{k \left[ \frac{1_C \cdot B}{\mu_0 \cdot \mu_r} \right]^2}{\int_0^{T_{on}} v_p \, dt} \quad (3-18)$$

In (3-18) by increasing $f$ (decreasing $T_{on}$), $I_m$ will be increased provided that all transformers parameters be kept constant. So very clearly, the magnetizing current $I_m$ will generally increase with frequency if the high-frequency core material is fully utilized up to $B_{max}$. This is confirmed through Fig. 3-5.

3.9 Transformer saturation on high-frequency switched mode power conversion

In practical magnetic component design, one of most important principles is that the magnetic flux is proportional to the volt-seconds applied to the winding:

$$v = N \frac{d\Phi}{dt}, \quad d\Phi = \frac{1}{N} v \cdot dt, \quad \Phi(t_1) - \Phi(t_0) = \frac{1}{N} \int_{t_0}^{t_1} v \cdot dt \quad (3-19)$$
The (3-19) shows that the change in flux during a time interval $t_1 - t_0$ is proportional to the integral of the voltage over the interval, or volt-seconds applied to the winding.

In push-pull type inverters (half-bridge, full-bridge), it is clear that the volt-second unbalance can cause transformer saturation [15], which can have harmful consequences:

1. increased conduction and switching losses
2. increased EMI (Electro Magnetic Interference)
3. decreased reliability and failure of the power transistors

So for these reasons, transformer saturation should be prevented. In a transformer, volt-second unbalance can originate from:

1. differences in the duty ratios of the "push" and "pull" switches.
2. differences in the positive and negative voltages of a center tap supply for half bridge converter.
3. differences in the voltage drops of the "push" and "pull" switches.

The voltage drop differences result from component tolerances, and duty ratio differences can result from differences in the time delays in the "push" and "pull" sections from load current pulsation. In association with PWM inverter operation, case 1 (duty ratio differences) is the usual one.

Fig. 3-6 shows a full bridge inverter with duty ratio unbalance. If there is a duty ratio unbalance in a full-bridge inverter (Fig. 3-6(a)), which does not have capacitive coupling, the flux excursion in the core, equivalent magnetizing current and primary current become asymmetrical, Fig. 3-6(c). As mentioned before this problem may cause saturation.
Fig. 3-6 Full bridge inverter (a), switch status (b), primary current without capacitor (c), primary current with capacitor
One method to eliminate transformer saturation is to add a coupling capacitor in series with the transformer as shown in Fig. 3-6(a), for achieving $A \neq B$ becoming $A = B$, Fig. 3-6(d). In connection with Fig. 3-6(a), it has been discovered recently [16] that a low frequency oscillation can develop in the primary side of transformer, resulting from the change of the transistor Bipolar storage time in response to change in the collector current. This oscillation is a periodic exchange of energy between the coupling capacitor $C_c$ and primary magnetizing inductance $L_m$. The oscillation can produce a large enough magnetizing current to drive the transformer into severe saturation and can also cause excessive low-frequency EMI.

The above mentioned oscillation takes place only in the primary side of power circuit, therefore it is not detectable by observing only the inverter output voltage. It has been shown in [16] that a resistor shunting the coupling capacitor Fig. 3-6(a) provides a simple but effective method for preventing both system oscillation caused by the storage time modulation [17] and transformer saturation caused by duty-ratio unbalance. For more detailed discussion about oscillation frequency and cure for that oscillation refer to [16].

### 3.10 Leakage Inductance

The existence of leakage inductance lies in the fact that part of the primary magnetic flux is not coupled to the secondary winding and vice versa, in result, gives rise to leakage inductance in each winding without contributing to the mutual flux. Leakage inductance is actually distributed throughout the winding in a transformer. However, for simplicity, this inductance is shown as a lumped constant $L_p$ in Fig. 3-4.
The leakage inductance depends very little on the core material, and strongly depends on number of turns. This inductance can be reduced drastically by bifilar winding. A pair of insulated wires are wound simultaneously and contiguously (close enough to touch each other). Each wire constitutes a winding; this very close proximity to its neighbour reduces leakage inductance by several orders of magnitude more than does ordinary interleaving. Therefore, by this method, toroid transformers provide negligible leakage inductance.

By the following means leakage inductance can be reduced:
1- minimizing turns (will be discussed later on)
2- increasing winding width (windings cover the magnetic path as far as possible)
3- minimizing isolation between primary and secondary
4- using bifilar windings
5- inter weaving the primary and secondary windings, more detailed in [4]

It should be emphasized that reduction of leakage inductance usually increases transformer capacitances [5].

### 3.11 Transformer capacitances

When the transformer is energized, different voltage gradients arise almost everywhere and consequently there are a large variety of capacitance which correspond to each of these gradients.
1- turn-to-turn
2- layer-to-layer
3- winding-to-winding
4- winding-to-core
5- terminal-to-terminal
The turn-to-turn capacitance is so small because they are in series when referred the whole winding. The effect of these capacities on transformer varies with the application, particularly they emerge in high frequency operation.

They are considered as a combined lumped capacitance of the primary and the secondary windings Fig. 3-7.

Fig. 3-7 transformer capacitances
By following means transformer capacitances can be reduced:

1- increased dielectric thickness between layer, turns, and windings
2- reduce winding width
3- increase number of layers
4- avoid bifilar winding

Since the corrective measures work in opposite directions for minimizing transformer capacitance and leakage inductance, a trade off has to be made between them.

3.12 Effect of frequency on leakage inductance

It is difficult to scale transformer design over a wide frequency range, since materials characteristics and design criteria changes. It is important to know how the leakage inductance changes with frequency to obtain an idea how overlap might affect output waveform if the system was designed to operate at much higher frequencies.

Here investigation is based on two mode of operation

\[ I - f \cdot \Phi = \text{const.}, \quad (\Phi = A.B_{\text{max}}) \quad (3-20) \]

Where \( f \) is frequency and \( \Phi \) is the core flux. This means that the benefits of increasing the transformer frequency are absorbed by reducing the core size which is accompany with weight reduction of windings, but number of turns is unchanged. The leakage inductance of primary and secondary windings for concentric cylindrical coils are described in [18]:

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\[ L_p = K_p N_1^2 \frac{L_{mt}}{L_c} \quad (3 - 21) \]

\[ L_s = K_s N_2^2 \frac{L_{mt}}{L_c} \quad (3 - 22) \]

\[ L_t = L_p + \left( \frac{N_1}{N_2} \right)^2 L_s \quad (3 - 23) \]

Where:
- \( L_t \): total transformer leakage inductance in primary terms
- \( K_p, K_s \): constants (irrelevant to this study)
- \( N_1, N_2 \): number of primary and secondary turns
- \( L_{mt} \): the mean turn length
- \( L_c \): winding length

Since:

\[ E_1 = 4.44f \Phi_m N_1 \]
\[ E_2 = 4.44f \Phi_m N_2 \quad (3 - 24) \]
\[ f \Phi_m = \text{cte} \]

Hence for constant terminal voltages, \( N \) does not change with frequency as the core area is inversely proportional to frequency, then the dependency of \( L_{mt} \) from frequency is:

\[ L_{mt} \Delta \frac{1}{f} \quad (3 - 25) \]

\( L_c \) can be assumed to be independent of frequency (since \( N = \text{const.} \)). The above conditions lead to the overall result that:

\[ L_p \Delta \frac{1}{f} \quad (3 - 26) \]
II - $\Phi = \text{const.}$ \hspace{1cm} (3-27)

From equations (3 - 24), with constant $E_1$, $\Phi_m$, the following proportions can be derived:

$$N_1 \Delta \frac{1}{f} \quad \text{or} \quad N_1^2 \Delta \frac{1}{f^2} \quad (3-28)$$

$$L_c \Delta N \quad \text{or} \quad L_c \Delta \frac{1}{f} \quad (3-29)$$

Therefore:

$$L_p \Delta \frac{1}{f^3} \quad (3 - 30)$$

### 3.13 Effect of leakage inductance on overlap and converter operation

In phase controlled converter and cycloconverters, the transfer or commutation of the current from one switch (MOSFET) to the next does not take place instantaneously. Due to existence of resistance and inductance in the supply source (transformer here) it takes a finite time (called overlap period) for the current to decay to zero in the outgoing switch, whilst the current will rise at the same rate in incoming switch. Consequently, on the output of cycloconverter, which is theoretically designed for maximum fundamental output voltage and clean harmonic spectra, there will be loss of voltage and a modification of the harmonic distortion. This deterioration will be higher as long as the system operation frequency is increasing.

In transformer feeding the cycloconverter each phase may be represented by its Thevenin equivalent circuit consist of a voltage source in series with its inductance. The major contributor to the supply impedance here, is the transformer leakage reactance.
In ref. [19] it can be seen that the overlap period (angle):

1- increases with load current increasing
2- increases with supply frequency increasing
3- decreases with supply voltage wave amplitude increasing
4- increases with supply reactance (leakage inductance) increasing

A further effect of the transformer leakage inductance is to produce commutation notches in the waveforms of the voltages appearing at the cycloconverter output terminals Fig. 3-8.

![Fig. 3-8](image)

(a) no-load converter output  (b) on-load converter output  
(c) voltage loss due to overlap due to leakage inductance
If the proper precautions are not taken, the presence of these notches can lead to some harmful effects [20]:

1- to influence the firing angles limit.
2- due to overlap period the fundamental component of cycloconverter input current will have a slight lagging phase shift with respect to input source voltage and consequently affects the input displacement factor and cycloconverter input power factor [19].

Of course this lagging phase shift and mean voltage reduction is consistent with presence of transformer leakage inductance.

For a more detailed discussion on leakage inductance and overlap problems, such as mean voltage reduction calculation for different configurations see refs [19], [20] & [21].
3.14 Study of interaction between transformer inductances and snubber circuits

As it was outlined in chapter 1, in the proposed DC-to-3 phase AC multistage conversion system, the transformer operates between inverter and cycloconverter on basis of storage/transfer of energy with two purposes:

1- making isolating AC output from DC input.
2- any desired voltage/current level transformation.

Since the transformer inductances have key role in power conversion circuits, hence it is important to know the behaviour of these inductances with regard to "over lap" problem and "snubber circuit design" for the inverter on the primary side, and the cycloconverter on the secondary side.

It is quite difficult and far beyond the scope of this chapter to show analytically the details of interaction between the inverter and its snubber circuits, the transformer and its inductances (magnetizing & leakage), the cycloconverter and its snubbers, output filter elements and the load.

For overall prototype system operation and interactions investigation, the computer base circuit simulation packages, for example, SPICE (Simulation Program with Integrated Circuit Emphasis) can be employed[22].

In single capacitor snubber arrangement (no $R_1, D_{11}, R_2, D_{22}$ in Fig. 3-9) the energy stored in the capacitors is returned to the load via transformer leakage inductance and causes ringing effects. In other words there is a natural frequency, $f_r = \frac{1}{2\pi \sqrt{2LC}}$, in the system which is not in connection with inverter operation frequency (inverter with single capacitor snubbers can be analyzed). This is the interaction between the total inductance (primary leakage and magnetizing inductance) of transformer and the snubber capacitors.
The employing of CRD snubber arrangements, Fig. 3-9, overcome this problem. Here the capacitor acts in the same way to control the rate of rise of voltage across its associated switch at turn-OFF. At switch turn-ON, the capacitor discharges through the resistor (R₁ & R₂) in which it's stored energy is dissipated.

As Fig. 3-10 shows there are 3 distinct transient mechanisms during T₁-OFF & T₂-ON period.

1- At t = t' MOSFET T₁ turn-OFF; the associated equivalent circuit is shown in Fig. 3-10 (a). It has been assumed that the MOSFETs and Diodes are ideal, so the switching-ON and switching-OFF times of transistors, also recovery time for diodes are neglected. In consequence i₁ immediately is transferred to snubber circuit and goes through C₁, D₁₁. The capacitor C₁ starts charging, so vₜ₁ increases, consequently i₁ and vₐ decrease. At the same time i₂ starts increasing through C₂ and R₂. C₂ starts discharging and vₜ₂ falls from 2E.

During this transient period changes on iₐ are insignificant \( \left( \frac{d}{dt} i_L = 0 \right) \), due to this fact that the rate of change of i₁ and i₂ approximately equal \( \left( \frac{d}{dt} i_1 = \frac{d}{dt} i_2 \right) \). This stage would last until vₜ₂ approaches to zero, so that D₂ becomes forward biased and start to conduct i₂ current. The duration t"-t', \( \frac{d}{dt} i_1 \) and \( \frac{d}{dt} i_2 \) are affected by the size of snubber's capacitor and resistor (shown in Fig. 3-10).
Fig. 3-9 single phase, half bridge inverter with C-R-D snubbers and transformer with open circuit secondary winding

\( L = L_m + L_1 \)

(\( L_m \) is the primary magnetizing inductance & \( L_1 \) is the primary leakage inductance)
Fig. 3-10 3-occurrence circuit diagram between T1 turn-OFF and T2 turn-ON in inverter circuit of Fig. 3-9
2- at \( t = t'' \), when \( v_{T_2} \) reaches to zero and \( D_2 \) starts to conduct \( i_2 \), Fig. 3-10 (b). Since \( i_2 \) is no longer limited by \( R_2 \), it increases suddenly up to the load current, whereas \( i_1 \) decreases accordingly. Due to this rapid change in \( i_1 \) and \( i_2 \), presence of any inductance in the way of snubber to transformer, will produce a voltage spike equal to \( 2E + \) induced voltage across the \( T_1 \) which may be harmful for switch \( T_1 \).

The voltage spike can be reduced by increasing snubber circuit capacitors:

\[
\frac{1}{2} CV_{\text{spike}}^2 = \frac{1}{2} L_{\text{leakage + stray}} I^2
\]

\( (3-31) \)

\[ V_{\text{spike}} = I \sqrt{ \frac{L_{\text{leakage + stray}}}{C} } \]

This interval ends when \( i_1 = 0 \), \( v_{T_1} = v_{C_1} = 2E \), and \( i_2 = i_L \).

3- At \( t = t''' \), when \( i_1 = 0 \), \( v_{C_1} = 2E \) the related circuit becomes as shown in Fig. 3-10 (c). Now any current because of voltage spikes, can flow through \( R_1, C_1 \) to the supply.

At this moment \( v_{T_2} = 0 \) (\( D_2 \) conducts), therefore \( v_L \) approaches to \(-E\) consequently \( i_2 \) decreases and when it reaches zero MOSFET \( T_2 \) can be triggered. Therefore the under investigation period between \( T_1\)-OFF & \( T_2\)-ON is ended.
3.15 References:


6- M. G. Say" The performance and design of alternating current machines" 1949, Pitman, London.

7- Brain Richardson "Transformer core losses" IEE Electronics and power, May 1986, pp. 365-368


Chapter 4

Cycloconverters & the Prototype System Cycloconverter
(Proposed Modulation Strategy)
4.1 Two quadrant operation

In phase controlled converters the output DC voltage of either polarity can be controlled continuously by varying \( \alpha \), delaying firing angles \([1, 2 \& 3]\). For example in Fig. 4-1 (a) for:

\[
\begin{align*}
\alpha &= 0^\circ & V_{DC} &= \text{Maximum} \\
\alpha &= 90^\circ & V_{DC} &= 0 \\
\alpha &= 180^\circ & V_{DC} &= \text{Minimum}
\end{align*}
\]

And because of the unidirectional current carrying property of the semiconductor switches within the converter, the current at the DC terminals can flow in only one direction (2-quadrant operation) as shown in Fig. 4-1 (b).

![2-quadrant operation](a)

Fig. 4-1 2-quadrant operation

4.2 Four quadrant operation and dual converters

By connecting the DC terminals of two phase controlled converters in parallel and oppositely poled with one another, the resultant system is called dual converter Fig. 4-2 (a), which can operate with both polarities of voltage.

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and current at its DC terminals (4-quadrant operation) as shown in Fig. 4-2 (b).

Fig. 4-2 4-quadrant operation

If \( \alpha_p \) is the firing angle for positive converter and \( \alpha_N \) is the firing angle of the Negative converter, in order to producing the exactly equal voltages and the same circuit polarity at the DC terminal of dual converter the following relationship should be satisfied.

\[
\bar{\alpha}_p + \bar{\alpha}_N = 180^\circ \quad (4-1)
\]

It means when one converter operates as a rectifier (\( 0^\circ \leq \alpha \leq 90^\circ \)), the other operates as an inverter (\( 90^\circ \leq \alpha \leq 180^\circ \)) [1].

If (4-1) is not be satisfied, due to inequality between DC voltages of two groups, there will be a circulating current, flowing through converters which in many cases is harmful and unwanted.

It is possible that by a process of continuous phase modulation of the converter firing angles such that the output voltage in Fig. 4-2(a) is varied continuously with a sinusoidal mode and with frequency different from input
source frequency. Now this dual converter is called "cycloconverter" or "frequency changer" [2].

4.3 Power frequency changers (cycloconverters)

The "power frequency changer" term, in general can apply to any electrical system that converts AC power of a given frequency to AC power with another chosen frequency. Such a system can employ static systems using solid-state and controllable switching devices or rotating electrical machinery:

\[ f_{\text{rotor}} = S \cdot f_{\text{stator}} \]

\[ S = \frac{N_s - N_r}{N_s} \quad (4-2) \]

\[ N_s = 120 \frac{f_{\text{stator}}}{p} \quad \text{(synchronous speed)} \]

where; \( f_{\text{stator}} \) is supply frequency and \( p \) is number of stator pole pairs. It is clear that any change in \( N_r \) will be reflected on \( f_{\text{rotor}} \).

4.4 Static power frequency changers

There are two categories of static system. In the first type, the power is converted in two stages with an intermediate DC link (AC-to-DC-to-AC). This type is called indirect "AC-to-AC" power conversion or "Rectifier-inverter type" frequency conversion. In the second type, the power is converted in one stage (AC-to-AC), which is termed "direct AC-to-AC power converters" or "cycloconverters", devised in the early 1930s. Recently many circuit configurations for static frequency changers have been designed.
The frequency changers are closely related to conventional rectifiers or AC- to -DC converter circuits. In fact, for any given AC- to -DC converter circuit, there is a corresponding AC- to -AC frequency changer circuit.

As has been outlined in chapter one, for a multistage DC- to -3 phase AC power conversion, different inverter-cycloconverter circuit configurations have been proposed (Fig. 1-15). From cycloconverter operation points of view and on the basis of direct and indirect conversion modes, these configurations can be classified:

i - Direct conversion

1- Single phase (1000 Hz)- to -single phase (50 Hz), Fig. 1-15 (a)
2- Single phase (50 Hz)- to -3 phase (50 Hz), Fig. 1-15 (b)
3- 3 phase (1000 Hz)- to -3 phase (50 Hz), Fig. 1-15 (d)

ii - Indirect conversion

1- Single phase (1000 Hz)- to -DC- to 3 phase (50 Hz), Fig. 1-15 (c)
2- 3 phase (1000 Hz)- to -DC- to -3 phase (50 Hz), expansion of Fig.1-15 (c)

Here, first the general theoretical principle of operation for different modes are discussed and at the end, the proposed technique for direct single phase (1000 Hz)- to -single phase (50 Hz) in comparison with the conventional method will be studied.

4.5 Direct AC- to -AC conversion (general discussion)

The direct AC- to -AC power frequency changer, consists of arrays of solid-state and bidirectional controllable switches, connected directly between the
input and output terminals. Operationally, it is a wave synthesizer; output voltage waves of the desired amplitude and frequency are produced by sequentially switching chosen segments of the voltage waves of the AC input source to the output terminals. The resulting waveforms of current flowing at the input terminals are built up of segments of the current waves flowing at the output terminals.

The basic principle of operation of the cycloconverter is based on the fact that, the multiplication of a set of \( n \cdot 1 \) sinusoidal quantities (input voltage matrix) by a set of \( m \cdot n \) balanced sinusoidal quantities (cycloconverter switching functions) yields a third set of \( m \cdot 1 \) sinusoidal quantities (output voltage matrix), whose frequency can be varied by varying the frequency of the second set. The graphical representation of the direct AC-to-AC power conversion is shown in Fig. 4-3 [4].

We can write:

\[
\text{Matrix (} m \cdot 1 \text{)} = \text{Matrix (} m \cdot n \text{)} \cdot \text{Matrix (} n \cdot 1 \text{)} \quad (4-3)
\]

or:

\[
[v_0(\omega_o t)] = [s_f(\omega_s t)] [v_1(\omega_1 t)]
\quad (4-4)
\]

Referring to Fig. 4-3, the equation (4-4) can be expanded:

\[
\begin{bmatrix}
  v_{o1} \\
  \vdots \\
  v_{om}
\end{bmatrix}
= A
\begin{bmatrix}
  f_{11} & \cdots & f_{1k} & \cdots & f_{1n} \\
  \vdots & \cdots & \vdots & \cdots & \vdots \\
  f_{m1} & \cdots & f_{mk} & \cdots & f_{mn}
\end{bmatrix}
\cdot
\begin{bmatrix}
  v_{i1} \\
  \vdots \\
  v_{ik} \\
  \vdots \\
  v_{in}
\end{bmatrix}
\quad (4-5)
\]
Fig. 4-3 Generalized cycloconverter topology for:

(n-phase input with $f_i$ Hz) - to -(m-phase output with $f_o$ Hz)
For input current the following matrix can be derived.

\[
\text{Matrix } [n \times 1] = \text{Matrix } [n \times m] \cdot \text{Matrix } [m \times 1] \quad (4\text{-}6)
\]

Comparing (4-6) with (4-3), it results:

\[
[I_i(\omega_s t)] = [S_f(\omega_s t)]^T[I_o(\omega_s t)] \quad (4\text{-}7)
\]

With reference to Fig. 4-3, the equation (4-7) can be expanded:

\[
\begin{bmatrix}
I_{i1} \\
\vdots \\
I_{ik} \\
\vdots \\
I_{in}
\end{bmatrix} = A
\begin{bmatrix}
f_{11} \ldots f_{i1} \ldots f_{m1} \\
\vdots \\
f_{1k} \ldots f_{ik} \ldots f_{mk} \\
\vdots \\
f_{1n} \ldots f_{in} \ldots f_{mn}
\end{bmatrix}
\begin{bmatrix}
I_{o1} \\
\vdots \\
I_{ol} \\
\vdots \\
I_{om}
\end{bmatrix} 
\quad (4\text{-}8)
\]

It should be noted again, all elements in (4-5) and (4-8) are sinusoidal time functions. In Fig. 4-3, the input is a system of n balanced and symmetric phases, in which each phase is related by \(\frac{1}{n}360^\circ\) with respect to the one ahead and similarly the output is a system of m balanced and symmetric phases, in which each phase is related by \(\frac{1}{m}360^\circ\) with respect to the one ahead. Therefore the general term of the transform matrix of \(S_f(\omega_s t)\) can be written:

\[
f_{i,k}(t) = A \cos\left[\omega_s t - \frac{k-1}{n}360^\circ + \frac{l-1}{m}360^\circ\right] \quad (4\text{-}9)
\]

\(k = 1, 2, \ldots, n\)
\(l = 1, 2, \ldots, m\)

More details may be found in [5].
4.5.1 Direct 3 phase- to -3 phase conversion

For \( n = m = 3 \) the common and practical case, (3-phase)- to -(3-phase) is yielded [6], Fig. 4-4 and part of Fig. 1-14 (d).

![Diagram of 3-phase to 3-phase conversion](image)

**Fig. 4-4 direct 3-phase -to -3-phase conversion**

With reference to Fig. 4-4 and following assumptions:

\[
\begin{bmatrix}
  v_{an} \\
v_{bn} \\
v_{cn}
\end{bmatrix} = V_i (\text{max}) \begin{bmatrix}
  \cos (\omega_i t) \\
  \cos (\omega_i t - 120^\circ) \\
  \cos (\omega_i t + 120^\circ)
\end{bmatrix}
\]

(4-10)

\[
\begin{bmatrix}
  I_A \\
  I_B \\
  I_C
\end{bmatrix} = I_o (\text{max}) \begin{bmatrix}
  \cos (\omega_o t) \\
  \cos (\omega_o t - 120^\circ) \\
  \cos \omega_o t + 120^\circ
\end{bmatrix}
\]

(4-11)
\[
\begin{bmatrix}
F_1 \\
F_2 \\
F_3
\end{bmatrix}
= A
\begin{bmatrix}
\cos (\omega_s t) \\
\cos (\omega_s t - 120^\circ) \\
\cos (\omega_s t + 120^\circ)
\end{bmatrix}
\]

The equations (4-5), (4-8) become:

\[
\begin{bmatrix}
V_{AN} \\
V_{BN} \\
V_{CN}
\end{bmatrix}
= A
\begin{bmatrix}
F_1 & F_2 & F_3 \\
F_3 & F_1 & F_2 \\
F_2 & F_3 & F_1
\end{bmatrix}
\begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix}
\]

(4-13)

\[
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
= A
\begin{bmatrix}
F_1 & F_3 & F_2 \\
F_2 & F_1 & F_3 \\
F_3 & F_2 & F_1
\end{bmatrix}
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix}
\]

(4-14)

Putting (4-10), (4-11), (4-12) in the (4-13) and (4-14) yields:

\[
\begin{bmatrix}
V_{AN} \\
V_{BN} \\
V_{CN}
\end{bmatrix}
= A
\begin{bmatrix}
\cos (\omega_s t) & \cos (\omega_s t - 120^\circ) & \cos (\omega_s t + 120^\circ) \\
\cos (\omega_s t + 120^\circ) & \cos (\omega_s t) & \cos (\omega_s t - 120^\circ) \\
\cos (\omega_s t - 120^\circ) & \cos (\omega_s t + 120^\circ) & \cos (\omega_s t)
\end{bmatrix}
\begin{bmatrix}
V_{i} (\text{max}) \\
\cos (\omega_i t - 120^\circ) \\
\cos (\omega_i t + 120^\circ)
\end{bmatrix}
\]

(4-15)

\[
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
= A
\begin{bmatrix}
\cos (\omega_s t) & \cos (\omega_s t + 120^\circ) & \cos (\omega_s t - 120^\circ) \\
\cos (\omega_s t - 120^\circ) & \cos (\omega_s t) & \cos (\omega_s t + 120^\circ) \\
\cos (\omega_s t + 120^\circ) & \cos (\omega_s t - 120^\circ) & \cos (\omega_s t)
\end{bmatrix}
\begin{bmatrix}
I_{o} (\text{max}) \\
\cos (\omega_o t - 120^\circ) \\
\cos (\omega_o t + 120^\circ)
\end{bmatrix}
\]

(4-16)

Noting that:

\[
\cos (P) \cdot \cos (Q) = \frac{1}{2} \left[ \cos (P + Q) + \cos (P - Q) \right]
\]

And:

141
\[ \omega_o = \omega_s - \omega_i \]

After trigonometric manipulation, the (4-15) and (4-16) become:

\[
\begin{bmatrix}
V_{AN} \\
V_{BN} \\
V_{CN}
\end{bmatrix} = \frac{3}{2} AV_i (\text{max}) \begin{bmatrix}
\cos \left( (\omega_s - \omega_i)t \right) \\
\cos \left( (\omega_s - \omega_i)t - 120^\circ \right) \\
\cos \left( (\omega_s - \omega_i)t + 120^\circ \right)
\end{bmatrix} = \frac{3}{2} AV_i (\text{max}) \begin{bmatrix}
\cos(\omega_o t) \\
\cos(\omega_o t - 120^\circ) \\
\cos(\omega_o t + 120^\circ)
\end{bmatrix}
\]

(4-17)

\[
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix} = \frac{3}{2} AI_r (\text{max}) \begin{bmatrix}
\cos \left( (\omega_s - \omega_d)t \right) \\
\cos \left( (\omega_s - \omega_d)t - 120^\circ \right) \\
\cos \left( (\omega_s - \omega_d)t + 120^\circ \right)
\end{bmatrix} = \frac{3}{2} AI_r (\text{max}) \begin{bmatrix}
\cos(\omega_f t) \\
\cos(\omega_f t - 120^\circ) \\
\cos(\omega_f t + 120^\circ)
\end{bmatrix}
\]

(4-18)

4.5.2 Harmonic studies in 3 phase- to -3 phase conversion

The coefficient of the input voltages in (4-5) and (4-13) are termed cycloconverter "transfer matrix" or in terms of switch operation they are called cycloconverter "existence function" or "switching pattern".

In practice the switching patterns, \( f_{i,k} \), represent the ON/OFF states of the power switches, resulting in a train of rectangular pulses with unity amplitude.

They can be expressed mathematically as follow:

\[
f_{i,k}(t) = A_0 + A_1 \cos \left[ \omega_s t - \frac{k-1}{n} 360^\circ + \frac{1-1}{m} 360^\circ \right] + \sum_{n=2,3,4} A_n \cos \left( n \left[ \omega_s t - \frac{k-1}{n} 360^\circ + \frac{1-1}{m} 360^\circ \right] \right)
\]

\[ k = 1, 2, \ldots, n \]
\[ 1 = 1, 2, \ldots, m \]

(4-19)
For \( n = m = 3 \) case:

\[
\begin{align*}
\mathbf{F}_1 = (A_0 + A_1 \cos \omega_s t) + \sum_{n=2,3,4} A_n \cos(n \omega_s t) \\
\mathbf{F}_2 = A_0 + A_1 \cos(\omega_s t - 120^\circ) + \sum_{n=2,3,4} A_n \cos\left[n(\omega_s t - 120^\circ)\right] \\
\mathbf{F}_3 = (A_0 + A_1 \cos(\omega_s t + 120^\circ)) + \sum_{n=2,3,4} A_n \cos\left[n(\omega_s t + 120^\circ)\right]
\end{align*}
\]

Comparing (4 - 12) and (4 - 20) reveals the harmonic components in the switching patterns as follow:

\[
F_h = \begin{bmatrix}
\sum_{n=2,3,4} A_n \cos(n \omega_s t) \\
\sum_{n=2,3,4} A_n \cos\left[n(\omega_s t - 120^\circ)\right] \\
\sum_{n=2,3,4} A_n \cos\left[n(\omega_s t + 120^\circ)\right]
\end{bmatrix}
\]

The infinite series of harmonics present in the cycloconverter output voltage and input current yields from:

\[
\begin{bmatrix}
V_{ANh} \\
V_{BNh} \\
V_{CNh}
\end{bmatrix} = \begin{bmatrix}
F_{1h} & F_{2h} & F_{3h} \\
F_{3h} & F_{1h} & F_{2h} \\
F_{2h} & F_{3h} & F_{1h}
\end{bmatrix} \begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix}
\]

(4 - 22)
and:

$$
\begin{align*}
[I_{ah}] &= \begin{bmatrix} F_{1h} & F_{3h} & F_{2h} \\
[I_{bh}] &= \begin{bmatrix} F_{2h} & F_{1h} & F_{3h} \\
[I_{ch}] &= \begin{bmatrix} F_{3h} & F_{2h} & F_{1h} \\
[I_A] \\
[I_B] \\
[I_C] \\
\end{bmatrix}
\end{align*}
(4-23)
$$

By substituting (4-10), (4-11), (4-21) into (4-22) and (4-23), after some trigonometric manipulation the first term of output voltage and input current can be expressed:

$$
V_{ANh}(\omega_o t) = \sum_{k=1}^{\infty} \frac{3(A_{3k+1}) \cdot V_i(\text{max})}{2} \cos \left[ 3k\omega_i + (3k \pm 1)\omega_o \right] t 
(4-24)
$$

$$
I_{nh}(\omega_i t) = \sum_{k=1}^{\infty} \frac{3(A_{3k+1}) \cdot I_o(\text{max})}{2} \cos \left[ 3k\omega_o + (3k \pm 1)\omega_i \right] t 
(4-25)
$$

From (4-24) the harmonic frequencies of the output voltage are:

$$
3k\omega_i + (3k \pm 1)\omega_o
$$

$$
(2\omega_i + 3\omega_i), (4\omega_i + 3\omega_i), (5\omega_o + 6\omega_i), (7\omega_o + 6\omega_i), \ldots \quad (4-26)
$$

k = 1, 2, 3, \ldots

and from the (4-25) the harmonic frequencies of the input current are:

$$
3k\omega_o + (3k \pm 1)\omega_i
$$

$$
(2\omega_i + 3\omega_o), (4\omega_i + 3\omega_o), (5\omega_i + 6\omega_o), (7\omega_i + 6\omega_o), \ldots
$$

k = 1, 2, 3, \ldots

by assuming:

$$
R = \frac{\omega_i}{\omega_o} > 1 \text{ and integer}.
$$
the output voltage harmonic frequencies = \( \omega_0 \left[ 3k(R + 1) \pm 1 \right] \) \( k=1,2,3,\ldots \)

the input current harmonic frequencies = \( \omega_0 \left[ 3k (R + 1) \pm R \right] \) \( k=1,2,3,\ldots \)

(4-28)

From (4-28) it is obvious that harmonic spectrum will be free from low order harmonics if R be chosen higher regardless of its side effects.

4.6 More discussion about cycloconverter switching pattern

From (4-5) and (4-20) it can be realized that each element of transform matrix, \( f_{l,k}(t) \), describes the operation of one switch in the power circuit such that \( f_{l,k}(t) = 0 \) when switch is open and \( f_{l,k}(t) = 1 \) when switch is conducting and connecting output terminal l to input terminal k (Fig. 4-3).

Generally each element is a function of \( f_i, f_o, \frac{v_o}{v_i}, \) and \( t \) [8]

\[
f_{l,k}(t) = f_{l,k} \left( f_i, f_o, \frac{v_o}{v_i}, t \right) \quad (4-29)
\]

\( l = 1, 2, \ldots m \)

\( k = 1, 2, \ldots n \)

From (4-24) and (4-25) it can be understood that the quality of the input current waveform and output voltage waveform is greatly dependent upon the character of the switching pattern. The various character of \( f_{l,k}(t) \) (pulse position, width, and repetition rate), represent different modulation techniques, which may produce wave forms with identical wanted components but widely differing spectral characteristics. So it is important to find the optimal cycloconverter switching pattern for specific application requirements (minimum total harmonic distortion and maximum fundamental component). This problem will be considered in indirect AC-to-AC power conversion.
4.7 Some important conclusion about direct 3 phase-to-3 phase conversion

1- In Fig. 4-4, it is assumed that the 3 phase-to-3 phase cycloconverter feeds a 3 phase inductive load with \( \cos\Phi \) power factor, therefore (4-11) becomes:

\[
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} = I_o \max \begin{bmatrix}
\cos (\omega_o t - \Phi) \\
\cos (\omega_o t - \Phi - 120^\circ) \\
\cos (\omega_o t - \Phi + 120^\circ)
\end{bmatrix}
\]  
(4-30)

by substituting (4-30) in to (4-23) and doing trigonometric manipulation, the first term of the input current matrix is expressed. [7]

\[
I_{ah}(\omega_i t) = \sum_{k=1}^{\infty} \frac{3(A_{3k} \pm 1)I_{o-max}}{2} \cos \left\{ \left[ 3k\omega_o + (3k\pm 1)\omega_i \right] t + \Phi \right\}
\]  
(4-31)

considering the sign of \( \Phi \) in (4-30) and (4-31) shows the input power factor appears as the inverse of the output power factor, that is an inductive load at the output of cycloconverter is seen by the source as a capacitive load, and vice versa. This is a welcome side benefit for a direct AC-to-AC frequency changer which it may operate as a VAR compensator [8].

2- As (4-28) shows, there is no subharmonic below the fundamental components in the output voltage.

3- The 3 phase-to-3 phase cycloconverter can be viewed as a generalized solid-state transformer with turn ratio of:
\[
\frac{N_s}{N_p} = \frac{V_{out}}{V_{in}} = \frac{3}{2} \frac{A_1 V_{in\, (max)}}{V_{in\, (max)}} = \frac{3}{2} A_1 \tag{4-32}
\]

As has been shown in [6] and [8-Chap. 3]

\[
A_{1\, max} = \frac{\sqrt{3}}{\pi} \tag{4-33}
\]

therefore:

\[
\frac{N_s}{N_p\, (max)} = \frac{3}{2} \cdot \frac{\sqrt{3}}{\pi} = 0.827 \, \text{(step down)} \tag{4-34}
\]

equation (4-34) demonstrates that 3 phase-to-3 phase cycloconverter of Fig. 4-4 operates as a step down transformer.

4.8 Indirect AC-to-AC conversion (AC-to-DC-to AC)

Generally this mode of operation is based on that:

\[
\text{Matrix (m.1). } \left[ \text{Matrix (1.n). Matrix (n.1)} \right] = \text{Matrix (m.1)} \tag{4-35}
\]

The multiplication of a set of (1.n) balanced sinusoidal quantities (rectifier switching matrix, \(f_r(\omega_1 t)\)) by a set of (n.1) sinusoidal quantities (input voltage matrix, \(V_i(\omega_i t)\)) results in a DC quantity free of harmonics, then multiplication of DC quantity (rectifier output voltage) by a set of (m.1) balanced sinusoidal quantities (inverter switching matrix, \(f_i(\omega_0 t)\)) results in a set of (m.1) balanced sinusoidal quantities (output voltage matrix, \(V_0(\omega_0 t)\)).

This process is shown in Fig. 4-5. Referring to Fig. 4-5 and (4-35):
Fig 4-5 Generalized indirect:

\( (n\text{-phase input with } f_1 \text{ Hz}) \rightarrow (m\text{-phase output with } f_0 \text{ Hz}) \)
\[
\begin{bmatrix}
\text{inverter} \\
F_i(\omega_i t)
\end{bmatrix}
\begin{bmatrix}
\text{rectifier} \\
F_r(\omega_i t)
\end{bmatrix}
\begin{bmatrix}
\text{input} \\
V_i(\omega_i t)
\end{bmatrix} = \begin{bmatrix}
\text{output} \\
V_o(\omega_i t)
\end{bmatrix}
\tag{4-36}
\]

or:

\[
\begin{bmatrix}
f_{i1} \\
\vdots \\
f_{im}
\end{bmatrix}
A
B\begin{bmatrix}
f_{r1} \\
\vdots \\
f_{rm}
\end{bmatrix}V_i
= \begin{bmatrix}
V_{i1} \\
\vdots \\
V_{in}
\end{bmatrix}
= \frac{\text{mABV}_i}{2}
\begin{bmatrix}
V_{o1} \\
\vdots \\
V_{om}
\end{bmatrix}
\tag{4-37}
\]

The all matrix elements in (4-37) are sinusoidal and function of time.

The general term of the different matrixes are:

1- Input voltage matrix:

\[V_{ik} = V_i \cos \left(\omega_i t - \frac{k-1}{n} 360^\circ\right) \quad k = 1, 2, 3, \ldots, n\]

2- Input rectification matrix:

\[f_k = B \cos \left(\omega_i t - \frac{k-1}{n} 360^\circ\right) \quad k = 1, 2, 3, \ldots, n\]

3- Inversion matrix:

\[f_{i1} = A \cos \left(\omega_o t - \frac{1-1}{m} 360^\circ\right) \quad l = 1, 2, 3, \ldots, m\]

4- Output voltage matrix:

\[f_{i1} = A \cos \left(\omega_o t - \frac{1-1}{m} 360^\circ\right) \quad l = 1, 2, 3, \ldots, m\]  \tag{4-38}
Where:

- \( n \) is the input phase number
- \( m \) is the output phase number
- \( \omega_i \) is the cycloconverter input angular frequency rad/sec
- \( \omega_o \) is the cycloconverter output angular frequency rad/sec

similar to (4-6) the input current matrix can be derived as follows:

\[
\begin{align*}
\begin{bmatrix}
F_r(\omega_i t) \\
F_i(\omega_o t) \\
I_o(\omega_o t)
\end{bmatrix}
= \begin{bmatrix}
I_i(\omega_i t)
\end{bmatrix}
\end{align*}
\]  

(4-39)

(4-39) can be expanded in terms of matrix elements:

\[
B \begin{bmatrix}
f_{r1} \\
\vdots \\
f_{rm}
\end{bmatrix}
A \begin{bmatrix}
f_{i1} & \cdots & f_{il} & \cdots & f_{im}
\end{bmatrix}
I_o = \frac{mABI_o}{2} \begin{bmatrix}
i_{i1} \\
\vdots \\
i_{im}
\end{bmatrix}
\]

(4-40)

the general terms in (4-40) for the input and output currents are:

1- input current:

\[
I_{ik} = \frac{mABI_o}{2} \cos \left[ \omega_i t - \frac{k-1}{n} 360^\circ \right], \quad k = 1, 2, \ldots, n
\]

2- output current:

\[
I_{ol} = I_o \cos \left[ \omega_o t - \frac{l-1}{m} 360^\circ \right], \quad l = 1, 2, \ldots, m
\]
4.8.1 *Indirect 3 phase- to - 3 phase conversion*

For $m = n = 3$, the known and practical configuration (3 phase- to -DC- to -3 phase ) is achieved (Fig. 4-6). For $n = 1, m = 3$ part of Fig. 1-15 (b) is resulted.

Referring to Fig. 4-6, (4 - 37) and (4 - 40) the output voltage and input current matrixes can be derived as follows.

![Diagram of indirect 3-phase to 3-phase power conversion](image)

**Fig. 4-6** Indirect 3-phase($f_i$) - to - 3-phase ($f_o$) power conversion
output voltages:

\[
\begin{bmatrix}
\text{inversion of DC link voltage} \\
\cos (\omega_0 t) \\
\cos (\omega_0 t - 120^\circ) \\
\cos (\omega_0 t + 120^\circ)
\end{bmatrix}
\cdot
\begin{bmatrix}
\text{input rectification} \\
\cos (\omega_i t) \\
\cos (\omega_i t - 120^\circ) \\
\cos (\omega_i t + 120^\circ)
\end{bmatrix}
\begin{bmatrix}
3 \text{ phase input } (f_i) \\
\cos (\omega_i t) \\
\cos (\omega_i t - 120^\circ) \\
\cos (\omega_i t + 120^\circ)
\end{bmatrix}
= \frac{3}{2} B.A.V_i
\begin{bmatrix}
\text{output voltage } (f_o) \\
\cos (\omega_o t) \\
\cos (\omega_o t - 120^\circ) \\
\cos (\omega_o t + 120^\circ)
\end{bmatrix}
\quad (4 - 42)
\]

Input current:

\[
\begin{bmatrix}
\text{inversion} \\
\cos (\omega_i t) \\
\cos (\omega_i t - 120^\circ) \\
\cos (\omega_i t + 120^\circ)
\end{bmatrix}
\cdot
\begin{bmatrix}
\text{output current rectification} \\
\cos (\omega_o t) \\
\cos (\omega_o t - 120^\circ) \\
\cos (\omega_o t + 120^\circ)
\end{bmatrix}
\begin{bmatrix}
3 \text{ phase output } (f_o) \\
\cos (\omega_o t) \\
\cos (\omega_o t - 120^\circ) \\
\cos (\omega_o t + 120^\circ)
\end{bmatrix}
= \frac{3}{2} B.A.I_o
\begin{bmatrix}
\text{Input current } (f_i) \\
\cos (\omega_i t) \\
\cos (\omega_i t - 120^\circ) \\
\cos (\omega_i t + 120^\circ)
\end{bmatrix}
\quad (4 - 43)
\]

As (4 - 42) and (4 - 43) shows, the output of rectifier is pure DC voltage \((3/2 B V_i)\), that is, a filter is not needed. Also, since the semiconductor switches operate in the ON/OFF mode and switching patterns for inversion, rectification stages are train of pulses with unity amplitude and different ON/OFF durations. As a result these switching patterns can be expressed mathematically similar to (4 - 20) and in order to feeding the inverter without any ripple, the filtering components \(C_f\) and \(L_f\) in Fig. 4-6 are employed. Here the output waveforms are obtained with selective PWM technique and associated harmonic elimination, voltage control strategy.
4.8.2 **Indirect 3 phase- to - 3 phase conversion with removed DC link components**

In Fig. 4-6 the filter components fulfil the following duties: Link inductor $L_f$ is used to smooth the DC link current and suppress link current discontinuities. The capacitor $C_f$ is mainly employed to store the energy flowing back the inverter (due to inductance of the load). In other words capacitor $C_f$, plays the role of regenerative and flow back energy sink. In addition these components have the following disadvantages:

1- are heavy and bulky
2- slow down system response
3- limit future load development

As it has been explained in [9], it is possible to remove the DC link components provided that an approach for the release of trapped and regenerative energy should be implemented. To satisfy that in Fig. 4-6, the regenerative PWM rectifier, comprised of bidirectional switches is selected. The power flow in new structure is controlled by varying the modulation index of the corresponding inverter, therefore an optimized switching pattern may be applied to the rectifier.

4.8.3 **Harmonic studies in indirect 3 phase- to -3 phase conversion with suppressed DC link**

Since filter components are not employed, the harmonic components of the rectifier output voltage have a direct effect on the inverter operation. Similarly, the harmonic components of the inverter input current have a direct effect on the input line current of the rectifier. This design of the switching pattern for both inverter and rectifier has significant importance.
The output of rectifier of Fig. 4-6 with removed DC link can be expressed by [9]:

\[ v_i(\omega t) = E + \sum_{k=6,12}^{\infty} v_{r,k} \sin(k\omega t + \Phi_{r,k}) \]  

(4-44)

where:

- \( E \) is the DC component of rectifier output voltage
- \( v_{r,k} \) is the amplitude of \( k \)-th unwanted component of \( v_r \)
- \( \Phi_{r,k} \) is the phase displacement of \( k \)-th unwanted component of \( v_r \)
- \( \omega_i = 2\pi f_i \), \( f_i \) is the frequency changer input frequency

If the switching pattern of the inverter be assumed as follows:

\[
S_I(\omega_o t) = \begin{bmatrix}
  f_{i1} \\
  f_{i2} \\
  f_{i3}
\end{bmatrix} = \begin{bmatrix}
  A_1 \sin \omega_o t + \sum_{n=5,7}^{\infty} A_n \sin n \omega_o t \\
  A_1 \sin (\omega_o t - 120^\circ) + \sum_{n=5,7}^{\infty} A_n \sin (\omega_o t - 120^\circ) \\
  A_1 \sin (\omega_o t + 120^\circ) + \sum_{n=5,7}^{\infty} A_n \sin (\omega_o t + 120^\circ)
\end{bmatrix}
\]

(4-45)

Then the output voltage of inverter \( v_o(\omega_o t) \) or across the load can be expressed by:

\[
v_o(\omega_o t) = \begin{bmatrix}
  v_{ab}(\omega_o t) \\
  v_{bc}(\omega_o t) \\
  v_{ca}(\omega_o t)
\end{bmatrix} = v_r(\omega_i t) \cdot S_I(\omega_o t) \]  

(4-46)
The (4-46) can be expanded by substituting (4-44) and (4-45)

\[
\begin{bmatrix}
    v_d(\omega t) \\
v_b(\omega t) \\
v_q(\omega t)
\end{bmatrix}
= E + \sum_{k=612}^{\infty} \sum_{n=57}^{\infty} v_{nk} A_n \sin(k \omega t + \Phi_{nk})
\]

\[
A \sin(\omega_1 t) + \sum_{n=57}^{\infty} A_n \sin(n \omega_1 t)
\]

\[
A \sin(\omega_1 t - 120^\circ) + \sum_{n=57}^{\infty} A_n \sin(n \omega_1 t - 120^\circ)
\]

\[
A \sin(\omega_1 t + 120^\circ) + \sum_{n=57}^{\infty} A_n \sin(n \omega_1 t + 120^\circ)
\]

\[
\begin{bmatrix}
    \text{output} \\
    \sin(\omega_1 t) \\
    \sin(\omega_1 t - 120^\circ) \\
    \sin(\omega_1 t + 120^\circ)
\end{bmatrix}
= E A_1 + \sum_{n=57}^{\infty} A_n \sin(n \omega_1 t)
\]

\[
\sum_{n=57}^{\infty} A_n \sin(n (\omega_1 t - 120^\circ))
\]

\[
\sum_{n=57}^{\infty} A_n \sin(n (\omega_1 t + 120^\circ))
\]

For one phase:

\[
v_{ab}(\omega_o t) = E A_1 \sin(\omega_o t) + E \sum_{n=57}^{\infty} A_n \sin(n (\omega_o t)) +
\]

\[
\sum_{k=612}^{\infty} \sum_{n=157}^{\infty} v_{nk} A_n \sin(k \omega_o t + \Phi_{nk}) \sin(n (\omega_o t))
\]

(4-47)
Since:
\[ \sin(P) \cdot \sin(Q) = \frac{1}{2} \left[ \cos(P - Q) - \cos(P + Q) \right] \]
the (4-48) can be rewritten:

\[
v_{ab}(\omega_o t) = EA_1 \sin(\omega_o t) + E \sum_{n=5,7}^\infty A_n \sin(\omega_o t) + \sum_{k=6,12}^\infty \sum_{n=1,5,7}^\infty \frac{1}{2} v_{r,k} A_n \cos[(k\omega_i - m\omega_o) t + \Phi_{r,k}] \\
- \sum_{k=6,12}^\infty \sum_{1,5,7}^\infty \frac{1}{2} v_{r,k} A_n \cos[(k\omega_i + m\omega_o) + \Phi_{r,k}]
\]

(4 - 49)

In (4-49) there are 4 group of frequencies:

1- \( f_o \) fundamental
2- \( n.f_o \) \( n = 5, 7, \ldots \) 
3- \( k.f_i - n.f_o \) \( k = 6, 12, \ldots \) & \( n = 1, 3, 5, \ldots \) 
4- \( k.f_i + n.f_o \) \( k = 6, 12, \ldots \) & \( n = 1, 5, 7, \ldots \)

4.8.4 Subharmonics and their effects

As (4-50) shows, in group 3, there are harmonic components which have frequency lower than the fundamental frequency (\( f_o \)). They are called subharmonics. In induction motor applications these subharmonics can cause a number of problems such as torque pulsations, energy losses, light flickering and magnetic saturation (the amplitude of flux wave produced in the AC machine by a harmonic component is inversely proportional to its frequency). The presence of subharmonics in the inverter output voltage, is the main disadvantage for indirect AC-to-AC conversion with removed DC link components. In (4-49) the subharmonics will appear when:
\[ kf_i - mf_o < f_o \quad \text{or} \quad kf_i < (m+1)f_o \quad (4-51) \]

to avoid the creation of subharmonics always:

\[ kf_i - mf_o > f_o \quad \text{or} \quad kf_i > (m+1)f_o \quad k=6,12,\ldots \quad m=1,5,7,\ldots \quad (4-52) \]

in (4-52) the simultaneous values for \( k \) and \( m \) should be considered.

In reference [9] & [10] the method in which the generation of low frequency components in the inverter output voltage can be avoided has been discussed.

4.9 phase conversion

In order to reduce the size and maintenance problems in conventional rotary phase conversion systems [11], several types of static phase converter circuits have been developed with varying performance and circuit complexities. Three-phase electrical equipments such as three-phase induction motors are significantly more efficient and economical than their single-phase counterparts. In addition in many cases the extension of three phase power lines to rural and light industrial areas is not economical, and also other applications such as, operation of the 3-phase auxiliary pumps and fans (from single-phase traction supply) in electric locomotives, necessitate single-phase to three-phase power conversion process.
4.9.1 General approach for single phase-to-three phase power conversion

This approach is based on the principle of direct cycloconversion which was discussed in section (4-5). In equations (4-5), (4-8) by setting \( n = 1 \) and \( m = 3 \), the single phase-to-three phase configuration will be resulted as shown in Fig. 4-7 and also Fig. 1-15(b).

According to (4-5), (4-8) and \( n = 1 \), \( m = 3 \) the general relationship for cycloconverter input/output voltage and currents can be derived.

\[
\begin{bmatrix}
V_{\alpha}\nl
V_{\beta}
\end{bmatrix} = A \begin{bmatrix}
f_1
f_2
f_3
\end{bmatrix} \cdot \begin{bmatrix}
V_{i\alpha}\nl
V_{i\beta}
\end{bmatrix} \quad (4-53)
\]

\[
[I] = A \begin{bmatrix}
f_1 & f_2 & f_3
\end{bmatrix} \begin{bmatrix}
I_{\alpha}\nl
I_{\beta}
\end{bmatrix} \quad (4-54)
\]
Referring to relationships (4-10), (4-12) the (4-53) and (4-54) becomes:

\[
\begin{bmatrix}
V_{AB} \\
V_{BC} \\
V_{CA}
\end{bmatrix} = A \begin{bmatrix}
\cos(\omega_s t) \\
\cos(\omega_s t - 120^\circ) \\
\cos(\omega_s t + 120^\circ)
\end{bmatrix} \cdot V_i \begin{bmatrix}
\cos(\omega_1 t)
\end{bmatrix} = \frac{A V_i}{2} \begin{bmatrix}
\cos(\omega_o t) + \cos(\omega_o + \omega_1) t \\
\cos(\omega_o t - 120^\circ) + \cos(\omega_o + \omega_1) t - 120^\circ \\
\cos(\omega_o t + 120^\circ) + \cos(\omega_o + \omega_1) t + 120^\circ
\end{bmatrix}
\]

(4-55)

\[
[I_o] = A \begin{bmatrix}
\cos(\omega_s t) & \cos(\omega_s t - 120^\circ) & \cos(\omega_s t + 120^\circ)
\end{bmatrix}.
\]

\[
\begin{bmatrix}
\cos(\omega_o t) \\
\cos(\omega_o t - 120^\circ) \\
\cos(\omega_o t + 120^\circ)
\end{bmatrix}
\]

\[
= \frac{3 A I_o}{2} \cos(\omega_o - \omega_s) t = \frac{3 A I_o}{2} \cos(\omega_1 t)
\]

(4-56)

The (4-55) and (4-56) are based on the ideal phase converter. For the actual converter the switching pattern is comprised of the ON/OFF states of switches S1, S2, ..., S6 in Fig. 4-7, consequently a train of rectangular pulses with unity amplitude and different ON/OFF durations, which their Fourier Series expansions can be written as follows [12].

\[
\begin{bmatrix}
F_1 \\
F_2 \\
F_3
\end{bmatrix} = \begin{bmatrix}
A_1 \cos(\omega_s t) + \sum_{n \neq 3, 5, 7} A_n \cos(n \omega_s t) \\
A_1 \cos(\omega_s t - 120^\circ) + \sum_{n \neq 3, 5, 7} A_n \cos(n (\omega_s t - 120^\circ)) \\
A_1 \cos(\omega_s t + 120^\circ) + \sum_{n \neq 3, 5, 7} A_n \cos(n (\omega_s t + 120^\circ))
\end{bmatrix}
\]

(4-57)
Therefore (4 - 53) becomes:

\[
\begin{bmatrix}
V_{AB} \\
V_{BC} \\
V_{CA}
\end{bmatrix} =
\begin{bmatrix}
A_1 \cos (\omega_s t) + \sum_{n=3,5,7} A_n \cos(n \omega_s t) \\
A_1 \cos (\omega_s t - 120^\circ) + \sum_{n=3,5,7} A_n \cos[(n \omega_s t - 120^\circ)] \\
A_1 \cos (\omega_s t + 120^\circ) + \sum_{n=3,5,7} A_n \cos[(n \omega_s t + 120^\circ)]
\end{bmatrix} \cdot V_i \left[ \cos (\omega_o t) \right]
\]

(4 - 58)

Since:
\[
\cos(P) \cdot \cos(Q) = \frac{1}{2} [\cos(P + Q) + \cos(P - Q)]
\]

\[
\omega_s = \omega_o + \omega_i \text{ and for single phase (50 Hz) - to - three phase (50 Hz) purpose}
\]

\[
\omega_o = \omega_i , \text{ so } \omega_s = 2\omega_o
\]

and:

\[
\omega_s + \omega_o = 3\omega_o
\]

Thus:

\[
\begin{bmatrix}
V_{AB} \\
V_{BC} \\
V_{CA}
\end{bmatrix} = \frac{A_1 V_i}{2} \begin{bmatrix}
\cos (\omega_o t) \\
\cos (\omega_o t - 120^\circ) \\
\cos (\omega_o t + 120^\circ)
\end{bmatrix} + \frac{A_i V_i}{2} \begin{bmatrix}
\cos (\omega_o t) \\
\cos (\omega_o t - 120^\circ) \\
\cos (\omega_o t + 120^\circ)
\end{bmatrix}
\]

\[
+ \sum_{n=3,5,7} \frac{A_n V_i}{2} \begin{bmatrix}
\cos [(n \omega_s - \omega_i) t + n 120^\circ] + \cos [(n \omega_s + \omega_i) t + n 120^\circ] \\
\cos [(n \omega_s - \omega_i) t + (n - 1) 120^\circ] + \cos [(n \omega_s + \omega_i) t + (n - 1) 120^\circ]
\end{bmatrix}
\]

(4 - 59)
in the same way:

\[
[I_d] = [F_1 \ F_2 \ F_3].I_o \begin{bmatrix}
\cos(\omega_o t) \\
\cos(\omega_o t - 120^\circ) \\
\cos(\omega_o t + 120^\circ)
\end{bmatrix} = \frac{3A_1 I_o}{2}\cos(\omega_t t) + \sum_{n=5,11}^{\infty} \frac{3A_n I_o}{2}\cos(n \omega_s + \omega_o) t
\]

\[+ \sum_{n=7,13}^{\infty} \frac{3A_n I_o}{2}\cos(n \omega_s - \omega_o) t\]

(4 - 60)

As (4 - 59) shows the output line voltage contain third harmonics, whose amplitude is equal to the amplitude of the respective fundamental component, these component are inherent with the proposed general approach and independent of the PWM strategy employed. More details and practical example may be found in [12].

4.10 Single phase- to -single phase cycloconverters

The main power circuits of single phase -to- single phase cycloconverters are shown in Fig. 4-8.
Fig. 4-8 single-phase cycloconverter circuit configuration:

(a) for conventional modulation technique
(b) for proposed modulation strategy
In fig. 4-8, the (single-phase)-to-(single-phase) cycloconverter configurations are:

(a) with centre tapped transformer and unidirectional switching devices (with conventional modulation technique).

(b) bridge connection with bidirectional switches and full voltage utilization (with proposed modulation strategy).

The principle of operation of the single phase-to-single phase cycloconverter[13] can be explained with the help of Fig. 4-8 (a) & (b). The two phase controlled converter (positive and negative groups) are operated as bridge rectifiers. Their delay angles are such that the output voltage of positive group is equal and opposite to the negative group. For satisfying this requirement, as indicated in section 4-2, if $\alpha_p$ is the delay angle of positive group, then the delay angle of negative group should be:

$$\alpha_n = 180^\circ - \alpha_p \quad (4 - 61)$$

consequently the average output voltage of the positive converter is equal and opposite of the negative converter (output is alternative with any frequency of $f_o < f_i$).

In cycloconverter operation with continuously variable output frequency and increasing capability of cycloconverter to produce sinusoidal output voltage (to reduce torque pulsation in AC drives) any effort should be taken in to effect. Since in all cases the output voltage waveform is necessarily made up of segments of the source voltage waveforms (Fig. 4-9(d)), so it is possible to arrange that these segments such as to contain the minimum output voltage (proposed strategy), or input current harmonics. Here, two different strategies for cycloconverter control are presented.
4.10.1 Conventional modulation strategy for sinusoidal cycloconverter

This modulation strategy which is based on cosine wave crossing method \[2, 13\] is applied for determining the timing of the firing pulses for cycloconverter switches. Referring to Fig. 4-9 (a) and assuming:

\[v_{AN} = V_m \sin(\omega t)\] \hspace{1cm} (4-62)
\[u_{AN} = \frac{2}{\pi} V_m \cos(\omega t)\] \hspace{1cm} (4-63)
\[v_{BN} = V_m \sin(\omega t - \pi) = -V_m \sin(\omega t)\] \hspace{1cm} (4-64)
\[u_{BN} = \frac{2}{\pi} \cos(\omega t)\] \hspace{1cm} (4-65)
\[v_o = M V_m \sin(\omega t) = MV_m \sin \left( \frac{1}{R} \omega t \right)\] \hspace{1cm} (4-66)

Where:

- \(v_o\) is the cycloconverter output voltage
- \(M\) is the depth of modulation for output voltage and \(0.0 \leq M < 2/\pi\)
- \(\omega_o = 2\pi f_o\) \hspace{1cm} (4-67)
- \(f_o\) is the cycloconverter output frequency
- \(R = \frac{f_i}{f_o}\) is the input/output frequency ratio

A few cycles of the anti-phase sinusoidal source voltages \(v_{AN}\) and \(v_{BN}\) and also \(u_{AN}\), \(u_{BN}\), and \(v_o\) are shown in Fig. 4-9.

The firing angles are determined so that the average output voltage, produced across the load by the conducting switch, is equal to the instantaneous value of the required output voltage at the firing instant.
Fig. 4-9
From Fig. 4-10 (a), for firing angle $\alpha$, the average value of the output is

\[ v_{\text{ave}} = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t) \, d(\omega t) = \frac{2}{\pi} V_m \cos(\alpha) \quad , \quad \alpha = \omega t = 2\pi f t \quad (4-68) \]

Therefore:

\[ \frac{2}{\pi} V_m \cos(\alpha) = M V_m \sin\left(\frac{\alpha}{R}\right) \quad (4-69) \]

(4-69) implies that the intersection points of a sinusoidal reference voltage $M \sin\left(\frac{1}{R} \omega t\right)$, with a series of cosine timing waves are the required switching points of the MOSFETs. The cosine timing waves are derived from, and synchronized to the AC input voltage (4-63), (4-65), and it's phase is such that it's peak occurs at the earliest possible commutation angle ($\alpha = 0$) of the associated MOSFET.
In Fig. 4-9 the switching points for a few cycles have been identified. Switch S1 of positive group is triggered at α1, α3, α5, .. when $v_{AN}$ is positive and switch S2 of positive group is triggered at α2, α4, α6, .. when $v_{BN}$ is positive. The resulting output modulated voltage wave form of the positive group $v_{o-p}$ is also shown in Fig. 4-9 (d).

In a similar way, the next half of output modulated voltage wave form of the negative group can be sketched. More details are given in [2 & 13]. It should be noted that in Fig. 4-9 the current is positive and continuous.

The theoretical results of switching angles (table 4-1) and modulated output voltage and its harmonic spectrum are presented in Fig.s 11, 12, 13 for this modulation strategy (4.10.1) with $f_i = 1$ kHz, $f_o = 50$ Hz, $R = 20$, and $M=0.63$. 
<table>
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<th>ON (deg.)</th>
<th>ON + OFF</th>
</tr>
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</table>
Conventional Modulation Strategy

Fig. 4-11

Cyclo. out-put, 1000 hz - to - 50 hz, m = 0.63

x-deg

vo(t) - p.u.

0.00

1.00

0.75

0.50

0.25
Conventional Modulation Strategy

Fig. 4-12

Cyclo. out-put, 1000 Hz to 50 Hz, m=0.63

\( V_0(t) \) - P.U.
Conventional Modulation Strategy

Fig. 4-13
4.11 proposed modulation strategy for sinusoidal cycloconverter

This strategy is based on the "equal area criterion". Referring to Fig. 4-8(b), in Fig. 4-14 the two dotted and hatched areas are equated:

4.11.1- equivalent hatched area is built symmetrically on the both sides of 90° line.

![Diagram showing modulation strategy](image)

Fig. 4-14 equivalent hatched area is built symmetrically around the 90° line

\[
\begin{align*}
t &= \frac{T_o}{2R\pi} = \frac{1}{f_o} = \frac{R}{2R\pi f_i} = \frac{R}{2R\pi f_i} = \frac{1}{\omega_i} \text{ rad (time unit for X axis)} \quad (4-70)
\end{align*}
\]

\[
A_{\phi o i} = \int_{\frac{(k-1)\pi}{\omega_i}}^{\frac{k\pi}{\omega_i}} M\sin(\omega_o t) dt = \frac{M}{\omega_o} \left( - \cos \left[ \omega_o \left( \frac{k\pi}{\omega_i} \right) \right] + \cos \left[ \omega_o \left( \frac{(k-1)\pi}{\omega_i} \right) \right] \right) \quad (4-71)
\]

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or:

\[ A_{\text{dot.}} = \frac{M}{\omega_o} \left[ \cos(\pi - \frac{\pi}{R} \alpha) - \cos(\frac{\alpha}{R}) \right] \]  \hspace{1cm} (4-72)

and:

\[ A_{\text{hatch.}} = \int \frac{\cos(\frac{(k-1)\pi}{2} + \frac{\alpha}{2})}{\omega_i} \sin(\omega o t) dt = \] 
\[ \int \frac{\cos(\frac{(k-1)\pi}{2} - \frac{\alpha}{2})}{\omega_i} \] 
\hspace{1cm} (4-73)

\[ = \frac{1}{R \omega_o} \left\{ - \cos(\frac{(k-1)\pi}{2} + \frac{\alpha}{2}) + \cos(\frac{(k-1)\pi}{2} - \frac{\alpha}{2}) \right\} \]

or:

\[ A_{\text{hatch.}} = \frac{1}{R \omega_o} \left\{ \cos(\frac{(k-0.5)\pi}{2} - \frac{\alpha}{2}) - \cos(\frac{(k-0.5)\pi}{2} + \frac{\alpha}{2}) \right\} \]  \hspace{1cm} (4-74)

Since:

\[ \cos(Q) - \cos(P) = 2 \sin\left(\frac{P+Q}{2}\right) \sin\left(\frac{P-Q}{2}\right) \]  \hspace{1cm} (4-75)

then:

\[ A_{\text{hatch.}} = \frac{2}{R \omega_o} \sin(k-0.5)\pi \cdot \sin\left(\frac{\alpha}{2}\right) \]  \hspace{1cm} (4-76)
by equating (4-72) with (4-76) yields:

$$A_{\dot{\dot{}}} = A_{\text{hatch}} \quad (4-77)$$

$$\frac{M}{\omega_o} \left[ \cos (k-1) \frac{\pi}{R} - \cos k \frac{\pi}{R} \right] = \frac{2}{R \omega_o} \sin \left( k - \frac{1}{2} \right) \pi \sin \frac{\alpha}{2} \quad (4-78)$$

The amount of $\alpha(k)$ can be determined:

$$\alpha(k) = 2 \sin^{-1} \left[ \frac{RM \left[ \cos (k-1) \frac{\pi}{2} - \cos k \frac{\pi}{2} \right]}{2 (-1)^{k+1} \sin \left( k - \frac{1}{2} \right) \pi} \right] \cdot \frac{180^\circ}{\pi} \quad (4-79)$$

$$k = 1, 2, 3, \ldots, 2R$$

$$M = 0.0, \ldots, 0.636$$

For $f_1 = 1 \text{ kHz}$, $f_0 = 50 \text{ Hz}$, $R = 20$ and $M = 0.63$ the OFF and ON = $\alpha(k)$ durations for this mode are given in table 4-2. The simulation of cycloconverter modulated output voltage is presented in Fig. 4-15 and modulated output voltage harmonic spectrum is shown in Fig. 4-16.
## Table 4-2

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4.11.2 The equivalent areas are built equally around the $45^\circ$ and $135^\circ$ lines as shown in Fig.4-17.

By equating the dotted area with the sum of the two hatched areas the following expression for ON durations can be derived:

$$\alpha (k) = 2 \sin^{-1} \left[ \frac{RM \left( \cos \frac{(k - 1)\pi R}{R} - \cos \frac{k\pi R}{R} \right)}{2\sqrt{2} \sin \frac{(k - 1/2)\pi}{2}} \right] \cdot \frac{180^\circ}{\pi} \quad (4 - 80)$$

$$k = 1, 2, 3, \ldots, 2R$$

$$M = 0.0, \ldots, 0.63$$

Table 4-3 presents the $ON = \alpha(k)$ and $OFF$ durations and Fig. 4-18 shows the modulated output voltage harmonic spectrum.
Table 4-3

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Fig. 4-18
Proposed Modulation Strategy: Line 45-135
Cyclic Harmonic Spec. 1000 to 50 Hz, M = 0.63
4.11.3 The equivalent areas are built equally around the 45°, 90° and 135° lines as shown in Fig.4-19.

\[
\varphi = \sin(n\alpha)
\]

\[
\psi = -\sin(n\alpha)
\]

\[
\varphi = M\sin(n\alpha) = M\sin(n\alpha/R)
\]

\[
\text{Fig. 4-19 dotted area} = \Sigma \text{hatched areas}
\]

By equating the dotted area with the sum of the three hatched areas the ON durations, \( \alpha(k) \), is determined from:

\[
\alpha(k) = 2\sin^{-1} \left[ \frac{RM \left( \cos(k-1)\frac{\pi}{R} - \cos k\frac{\pi}{R} \right) \left( \sqrt{2} + 1 \right) \sin \left( k - \frac{1}{2} \right) \pi}{2} \right] \cdot \frac{180^\circ}{\pi} \quad (4-81)
\]

\[k = 1, 2, 3, \ldots, 2R\]
\[R = \frac{f_i}{f_o}\]
\[M = 0.0, \ldots, 0.63\]

Table 4-4 gives the ON durations and Fig. 4-20 shows the harmonic component spectrum for this mode.
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Fig. 4-20

Harmonic Orders

Amplitude

Proposed Modulation Strategy, Line 45-90-135

Cyclic Harm. Spec., 1000 to 50 Hz, m = 0.63
4.11.4 The equivalent area is built as shown in Fig. 4-21.

![Fig. 4-21 equivalent hatched area is built on the right side of input half cycles](image)

From the equality of two areas following expression for this mode can be derived:

\[ \alpha (k) = \left\{ \left[ 1 - (-1)^{k+1} \right] \frac{\pi}{2} - (-1)^{k} \right\} \cos^{-1} \left[ \frac{R M \left[ \cos \left( k - \frac{\pi}{R} \right) - \cos \left( \frac{\pi}{R} \right) \right] - (-1)^{k} \cos k \pi}{(-1)^{k+1}} \right] \]

\( k = 1, 2, 3, \ldots, 2R \)  
\( R = \frac{f_i}{f_o} \)  
\( M = 0.0, \ldots, 0.63 \)

The related results and harmonic spectrum are shown in table 4-5 and Fig. 4-22 respectively.
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<th>ON (deg.)</th>
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Fig. 4.22

Harmonic Orders

Amplitude

Proposed Modulation Strategy

Cyclic Harmonic Spectrum, 1000-70-50 Hz, M=0.63
4.12 Harmonic Analysis for proposed modulation strategy
"General mathematical expression"

Due to absence of low order harmonics and decreased device switching among the four "equal area criterion" modes, those were discussed in section 4-11. Mode number 1 is selected for microprocessor implementation.

Although the harmonic spectra of the modes are to some extent close to each other, mode number 1 has a clearer spectrum and is easier to implement than the others.

Referring to Fig. 4-15, the general expression of a Fourier Series for the instantaneous output voltage in Fig. 4-8(b) is:

\[ V_o(\omega t) = \sum_{n=1,3,5,...}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t) \]  

(4-83)

The coefficients \( A_n \) and \( B_n \) are determined by considering a pair of pulses such that the positive pulse of duration \( \alpha'k \) starts at \( \omega t = \Phi_k \) and the negative one of the same width starts at \( \omega t' = 3600^* + \Phi_k \) (Fig. 4-15). The effects of all pulses can be combined together to obtain the effective output voltage. If the positive pulse of \( k \)th pair starts at \( \omega t = \Phi_k \) and ends at:

\[ \omega t' = 3600^* + \Phi_k \]

or:

\[ \omega t' = R \pi + \Phi_k \text{ (rad.)} \]

the Fourier coefficients for this pair of pulses are:
\[ a_n = \frac{2}{R\pi} \int_{\Phi_k}^{\Phi_k+\alpha_k} V_m \sin(R\omega t) \cdot \cos(n\omega t) \, d(\omega t) \quad (4 - 84) \]

Since:

\[ \sin \left( \frac{P + Q}{2} \right) \cos \left( \frac{P - Q}{2} \right) = \frac{1}{2} \left[ \sin P + \sin Q \right] \]

Then:

\[ \sin (R\omega o t) \cdot \cos (n\omega o t) = \frac{1}{2} \left[ \sin (R + n) \omega o t + \sin (R - n) \omega o t \right] \]

and (4 - 84) becomes:

\[ a_n = \frac{2}{R\pi} \cdot \frac{v_m}{2} \left\{ \int_{\Phi_k}^{\Phi_k+\alpha_k} \sin (R + n) \omega o t \, d(\omega o t) + \int_{\Phi_k}^{\Phi_k+\alpha_k} \sin (R - n) \omega o t \, d(\omega o t) \right\} \quad (4 - 85) \]

\[ a_n = \frac{v_m}{R\pi} \left\{ \frac{1}{R+n} \left[ \cos(R+n)\Phi_k - \cos \left( (R+n)(\Phi_k+\alpha_k) \right) \right] + \frac{1}{(R-n)} \left[ \cos(R-n)\Phi_k - \cos \left( (R-n)(\Phi_k+\alpha_k) \right) \right] \right\} \quad (4 - 86) \]

and:

\[ b_n = \frac{2}{R\pi} \int_{\Phi_k}^{\Phi_k+\alpha_k} V_m \sin(R\omega o t) \cdot \sin(n\omega o t) \, d(\omega o t) \quad (4 - 87) \]
since:

\[
\sin \left( \frac{P + Q}{2} \right) \sin \left( \frac{P - Q}{2} \right) = \frac{1}{2} \left[ \cos(P) - \cos(Q) \right]
\]

Then:

\[
\sin (R\omega_c t) \cdot \sin (n\omega_c t) = \frac{1}{2} \left[ \cos (R + n) \omega_c t - \cos (R - n) \omega_c t \right]
\]

and (4-87) becomes:

\[
b_n = \frac{2}{R\pi \cdot \frac{V_m}{2}} \left\{ \int_{\phi_k}^{\phi_{k+\alpha_k}} \cos (R + n)\omega_c t \, d(\omega_c t) - \int_{\phi_k}^{\phi_{k+\alpha_k}} \cos (R - n)\omega_c t \, d(\omega_c t) \right\}
\]

(4-88)

\[
b_n = \frac{V_m}{R\pi} \left\{ \frac{1}{R+n} \left[ \sin (R+n)(\Phi_k + \alpha_k) \right] - \sin (R+n)\Phi_k \right\} - \frac{1}{R-n} \left[ \sin (R-n)(\Phi_k + \alpha_k) - \sin (R-n)\Phi_k \right]
\]

(4-89)

Where:

\[
R = \frac{f_i}{f_o}
\]

\[n = 1, 3, 5, \ldots\]

\[\alpha_k = \left( k - \frac{1}{2} \right) \pi - \frac{1}{2} \alpha_k \quad \text{(in Fig. 4-15)}\]

\[k = 1, 2, 3, \ldots\]

\[R \quad \text{(number of pulses per cycloconverter output voltage half cycle)}\]

\[V_m \quad \text{amplitude of cycloconverter input voltage in Fig. 4-8(b)}\]
Therefore the $A_n$ and $B_n$ can be expressed as follows:

$$A_n = \sum_{k=1}^{R} \frac{V_m}{R\pi} \left\{ \frac{1}{(R+n)} \left[ \cos(R+n)\Phi_k - \cos[(R+n)(\Phi_k + \alpha_k)] \right] + \frac{1}{(R-n)} \left[ \cos(R-n)\Phi_k - \cos[(R-n)(\Phi_k + \alpha_k)] \right] \right\}$$

$$B_n = \sum_{k=1}^{R} \frac{V_m}{R\pi} \left\{ \frac{1}{(R+n)} \left[ \sin(R+n)\Phi_k + \alpha_k \right] - \sin(R+n)\Phi_k \right\} - \frac{1}{(R-n)} \left[ \sin(R-n)\Phi_k + \alpha_k \right] - \sin(R-n)\Phi_k \right\}$$

(4-90)

(4-91)

By substituting (4-90) and (4-91) in to following equation:

$$v_o(\omega_d) = \sum_{n=1,3,5,...}^{\infty} \left[ A_n \cos n\omega_d + B_n \sin n\omega_d \right]$$

(4-92)

The general expression of the cycloconverter out-put voltage will be resulted.

It should be emphasized that if the $R = \frac{f_i}{f_o}$ is not integral, the positive and negative half-cycle waveforms may have different voltage profiles. As a result the output voltage may have even harmonics and subharmonics in addition to the ordinary odd harmonics[14]. The effects of the subharmonics on the induction motor operation was discussed in this chapter.
4.13 References:


5- S. I. Khan, P. D. Ziogas, & M. H. Rashid "Forced commutated cycloconverter for High Frequency link application" 1986, IEEE, pp. 476-487.


Chapter 5

Design & performance of the prototype system
5.1 Introduction

This chapter deals with the design and performance of the prototype DC-to-AC(3-phase & 1 k Hz)-to-AC(3-phase & 50 Hz) power conversion through the Inverter-Transformer-Cycloconverter system. The block diagram for this system is shown in Fig. 5-1. The chapter contains six main sections:

1- the 3-phase inverter
2- the high frequency transformer
3- the 3-phase cycloconverter
4- the control circuits
5- the phase shifters
6- the experimental results of the prototype system

5.2 The 3-phase inverter

There are three inverter circuit configurations:

5.2.1 Push-pull inverter( with center tapped transformer )

The push-pull type inverter shown in Fig. 5-2 has two disadvantages. First, in this type the peak value of the Drain-to-source voltage of each MOSFET is equal to twice the supply voltage plus the amplitude of the voltage spike generated by transformer leakage inductance and stray inductances in series with the MOSFET [1]. Their associated energy is suppressed by the snubber capacitors. This can be consider as a limitation for the high power inverters energized from high DC source voltages, but less likely for battery fed inverters.

Secondly, in PWM application due to a small difference in the switching times of the transistors, or in other words, as a result of unsymmetrical switching of the transistors a net DC voltage component is applied.
The block diagram of DC-10-AC(1kHz)-10-AC(50Hz) power conversion system.

- Zero-crossing detector
- Microprocessor
- Supply
- DC
- Vol
- 50 Hz mains
- 3-phase shifter
- 3-phase circuits
- 12 Base-drive circuits
- Filter capacitor
- Cycloconverter 6
- 1:1 transformer
- PWM inverter
- 3-phase & 1 kHz
- 1 kHz, step-up
- 1 kHz
- 50 Hz
- 120° 240° 0
to the transformer and consequently the core enters saturation in one direction as a result of the magnetizing current spike. Therefore in core materials like Ferrite the saturation flux is lower than that of laminated core materials and this problem will be more apparent [1] & [2].

![Fig. 5-2 push-pull inverter](image)

For handling the same load power, this problem relatively can be solved by inserting an air gap in the transformer magnetic circuit but at the cost of higher magnetizing current, greater losses & increased leakage flux which increases the voltage spike across the MOSFET at switch off. In reference [3] symmetry correction circuit on basis of feedback from the two primary current difference has been suggested but at the cost of circuit complexity. As an advantage for this type of inverter, the MOSFET base drive circuits may be left non-isolated from each other.
5.2.2 **Half bridge inverter (with center tapped power supply)**

This type is shown in Fig. 5-3. Here the peak value of Drain-to Source voltage of each MOSFET is equal to the supply voltage \( E \) plus the amplitude of the voltage spike, and therefore suitable for higher supply voltages (advantage). This type offers two disadvantages.

![Half bridge inverter diagram](image)

Fig. 5-3 half-bridge inverter (with center tapped power supply)

Firstly for handling the same load power the current will be doubled which causes limitations for high power circuit configurations and secondly for avoiding any supply short circuit isolated gate drive circuits for each MOSFET are required.
5.2.3 The full bridge inverter

As Fig. 5-4 shows, in this inverter, the push-pull and half bridge advantages are combined, that is the MOSFET voltage rating is $E$ and the current through it is half of the half bridge inverter for the same load power, but in contrast four MOSFET have to be used and isolated base drive circuits are required.

![Diagram of full-bridge inverter](image)

Fig. 5-4 full-bridge inverter

5.2.4 prototype system inverter

By review of the merits and shortcomings of three explained types, the 3-phase version of half bridge inverter shown in Fig. 5-5 is chosen for the prototype system to generate 3-phase symmetric and 1000 Hz sinusoidal waveforms (all details have been presented in chapter 2) for applying to the cycloconverter through the step-up transformer.
5.2.5 Selection of power semiconductor switches for the inverter

The selection of a switch for use in a high-speed inverter is dictated by the following conditions:

1- the peak value of the Drain-to-Source voltage of each MOSFET during turn off should be equal to or greater than the total applied Electric pressure across it due to supply voltage plus voltage spikes caused by transformer leakage inductance and stray inductances.
2- the frequency cut-off characteristics of the switch must be high compared to the actual switching frequency. If the transistor can not switch rapidly between the states of saturation and cut-off, excessive junction heating results. When the transistor frequency cut-off characteristic is several times that of the switching device the output waveform would be more nearly a square wave (free of pulse corner oscillation).

3- the transistor must be able to handle the currents that are necessary to produce the required output power at the given source voltage, moreover in order to achieve the wanted efficiency the switch saturation voltage at rated currents should be low enough.

4- as was mentioned in chapter 2, when the inverter is running an inductive load, for keeping voltage stability and releasing the trapped energy (feed back from load to supply source), antiparallel diodes (feed back diodes) are necessary (Fig. 5-5).

In modern power MOSFET transistors a parasitic anti-parallel diode is inherently built in the process of device fabricating. The voltage and current ratings are equal to the MOSFET values, but there is in general, a slow reverse recovery [4].

For the prototype system inverter, by considering the above mentioned conditions and the fact that the maximum DC input voltage used at the primary side of the transformer will be 110 V and give application flexibility for MOSFETS to be employed in different inverter configurations, it was decided to work at maximum primary current 10A. So the SILICONIX power MOSFET "SMM 20N 50, N-channel, Enhancement Mode transistor" is selected. More information and data sheets for this MOSFET and associated antiparallel diode are given in Appendix B.
5.2.6 The base drive circuit

The base drive circuit shown in Fig. 5-6 was developed successfully. This drive circuit mainly includes the Optocoupler 6N137 and newly developed power MOSFET drive ICL7667 CPA by Harris semiconductor.

![Base drive circuit diagram](image)

Fig. 5-6 The base drive circuit for (SMM20N50) MOSFET

5.2.6.1 Optocouplers and isolation of gatecontrolling signals from power circuits (MOSFETs)

The power circuits (inverters) are generally multiple transistors and each transistor must be gated individually. The gate signals should be applied between gate and source terminals. This gate signals can be isolated from the
power circuit (MOSFETS) by employing pulse transformers or optocouplers. The pulse transformer is simple. It has one primary and one or more secondary windings for simultaneous gating signals to series or parallel MOSFETs. But they have the leakage inductance, rise time of the output pulse and saturation problems [5]. Therefore optocouplers are preferable in spite of the requirement for a separate power supply.

The optocoupler (optical isolator) is comprised of an infrared light-emitting diode (ILED) and a silicon phototransistor, Fig. 5-7.

Fig. 5-7 The basic optocoupler structure

The input signal (from microprocessor) is applied to the ILED and the output is taken from phototransistor. This phototransistor has rapid rise and fall times, for example 25 nsec. for optocoupler 6N137 which was employed for gate drive circuits of prototype system inverter MOSFETs. The data sheets are given in Appendix B.
5.2.6.2 The ICL7667 CPA Chip

The ICL7667CPA is a high speed driver designed to convert T.T.L level signal (from microprocessor) into high current outputs at voltages up to 15v. This chip is ideal for driving power MOSFETs in high frequency switched mode power converters. It has rise and fall time about 30 nsec quite compatible with optocoupler 6N137 and MOSFET of SMM20N50. The respective data sheets are included in Appendix B.

5.2.7 The snubber circuits design

The protection of the power semiconductor devices, here, MOSFETs are accomplished by turn-ON and turn-OFF snubbers against excessive $di/dt$ and $dv/dt$ respectively, shown in Fig. 5-8.

![Turn-ON (di/dt) snubber](a)

![Turn-OFF (dv/dt) snubber](a)

![Fig. 5-8 Turn-ON & Turn-OFF snubbers:](b)

(a) schematic  
(b) with circuit elements
5.2.7.1 The MOSFET equivalent circuit

The switching equivalent circuit of a power MOSFET is shown in Fig. 5-9 [6].

![The MOSFET equivalent circuit diagram](image)

**Fig. 5-9** The MOSFET equivalent

Where:

- $R_g$: gate resistance
- $R_D$: epitaxial layer resistance
- $R_C$: channel resistance
- $R_{DS\ (on)} = R_D + R_C$: Drain-Source resistance at turn-on
- $D$: parasitic anti-parallel diode (MOSFET body diode)
- $C_{input} = C_{gd} + C_{gs}$ (with the source ac shorted to the drain)
- $C_{output} = C_{gd} + C_{ds}$ (with the gate ac shorted to the source)

These quantities of the MOSFET-SMM 20N50 may be found in Appendix B.
5.2.7.2 MOSFET switching waveforms and times

General switching waveforms and times are shown in Fig. 5-10 [7].

![Diagram of MOSFET switching waveforms]

\[ t_{on} = t_{d(on)} + t_r \]
\[ t_{off} = t_{d(off)} + t_f \]

Fig. 5-10 General MOSFET switching waveforms

The switching times[ \( t_{d(on)} \), \( t_r \), \( t_{d(off)} \) & \( t_f \)] for SMM20N50 MOSFET can be found in related data sheets (Appendix B).

i- Turn-on period

In Fig. 5-10 the timing relationship between the gate-source voltage and the drain-source voltage for a complete (ON and OFF) drive pulse applied to the gate is presented. During the turn-on period a delay occurs (turn-on delay). This delay, \( t_{d(on)} \) is the time that is required to charge the input capacitance (\( C_{input} \)) to gate threshold voltage level. This duration, \( t_{d(on)} \)
is determined by the gate-to-source ($c_{gs}$) and drain-to-source ($c_{ds}$) capacitances. After the delay, the drain current rises as the drain-to-source voltage falls. The rise time, $t_r$, is the gate charging time from the threshold level to the full-gate voltage, which is required to drive the MOSFET into the linear region. This time ($t_r$) is measured using the 10% and 90% points on the rising waveform. The sum of turn-on delay time and rise time is known as turn-on time:

$$t_{on} = t_{d(on)} + t_r \quad (5-1)$$

ii- **Turn-off period**

During the turn-off period, another delay occurs (turn-off delay, $t_{d-off}$). This delay, which is followed by a fall in drain current as the drain-source voltage rises, is the time required for the input capacitance ($C_{input}$) to discharge from the full gate voltage to the 90% falling waveform point. The $V_{gs}$ must decrease significantly before $V_{ds}$ begins to rise. The fall time, $t_f$, is the time that is required for the input capacitance to discharge from the 90% falling waveform point to threshold voltage and if $V_{gs} < V_{threshold}$ the transistor turns off. The sum of turn-off delay time $t_{d(off)}$ and fall time is known as turn-off time.

$$t_{off} = t_{d(off)} + t_f \quad (5-2)$$

Switching-time calculations, and further information about switching characteristics, may be found on section 2.2.1 and 3.2 in reference [4].

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5.2.7.3 $\text{di/dt protection or turn-on snubber design}$

The typical voltage and current waveforms of a MOSFET switch during turn-on and turn-off are shown in Fig. 5-11.

Fig. 5-11 The typical MOSFET voltage & current waveforms

During turn-on, the drain current ($I_D$) rises and from Fig. 5-11 $\text{di/dt}$ is:

$$\frac{\text{di}}{\text{dt}} = \frac{I_L}{t_{\text{on}}} = \frac{I_D}{t_{\text{on}}} \quad (5 - 3)$$

If the rate of rise of drain current ($\text{di/dt}$) is greater than the allowable limit, due to high current density switch failure may be inevitable. The $\text{di/dt}$
may be held down to an acceptable value by including a small amount of inductance (turn-on snubber) in the drain circuit. The amount of this inductance is calculated such that after elapsed specified \( t_{on} \) time, the switch will turn-to full conduction. In Fig. 5-8(b) immediately after voltage \( E \) being applied:

\[
E = R.i + (L_s + L) \frac{di}{dt} \quad (5-4)
\]

If (5-4) be solved for \( i(t) \):

\[
i(t) = \frac{E}{R} \left( 1 - \left[ e \left( \frac{-R}{L + L_s} \right) t \right] \right) \quad (5-5)
\]

\[
\frac{di(t)}{dt} = -\frac{E}{R} \cdot \frac{-R}{L + L_s} e^{\frac{-R}{L + L_s} t} = \frac{E}{L + L_s} e^{\frac{R}{L + L_s} t} \quad (5-6)
\]

\[
\frac{[di(t)]}{dt}_{\text{max}(t=0)} = \frac{E}{L + L_s} \quad (5-7)
\]

and:

By equating (5-3) with (5-7):

\[
\frac{I_D}{t_{on}} = \frac{E}{L + L_s} \quad (5-8)
\]

From (5-8):

\[
L + L_s = \frac{E \cdot t_{on}}{I_D} \quad (5-9)
\]
Since the $t_{on}$ for power MOSFETs (like SMM20N50) is very short (120nsec), by referring to (5-9), the presence of any load inductance and even any stray inductance will be sufficient for $di/dt$ protection.

5.2.7.4 $dv/dt$ protection or turn-off snubber design

During turn-off, the drain-to-source voltage rise in relation to the fall of drain current. From Fig. 5-11:

$$\frac{dv}{dt} = \frac{v_{DS}}{\tau_{off}} = \frac{E}{\tau_{off}} \quad (5-10)$$

Referring to Fig. 5-8 (b) at the instant that the MOSFET carrying $I_D$ (amp.), the switch is turned-off the current will divert and pass through the turn-off snubber circuit. Here the size of the capacitor can be determined such that:

$$I_D = C_S \frac{dv}{dt} \quad \text{or} \quad \frac{dv}{dt} = \frac{I_D}{C_S} \quad (5-11)$$

Where $I_D$ is maximum drain current and $dv/dt$ is the allowable rate of drain-to-source voltage. By equating (5-10) with (5-11) yields:

$$E \tau_{off} = I_D C_S \quad (5-12)$$

$$C_S = \frac{I_D \tau_{off}}{E}$$

During turn-on the capacitor $C_S$ has to discharge through the transistor and this increases the peak current rating of the transistor. The discharge through the transistor can be avoided by placing $R_S$ across $C_S$ instead of placing $R_S$ across $D_S$.
5.2.7.5 Discussion about $R_S$ determination

The resistor $R_S$ is presented to prevent the immediate discharge of the capacitor-stored energy into the switch at turn-on. This resistor must be capable of dissipating a power of:

$$P = \frac{1}{2} C_S E^2 f_{\text{switching}}$$  \hspace{1cm} (5-13)

In choosing the value of $R_S$, taking the time constant $R_S C_S$ as one-fifth of the switching period (before the switch turn-off, the capacitor has completely been discharged):

$$5 R_S C_S = T_{\text{on}} \quad \text{(the shortest one in PWM operation)} \hspace{1cm} (5-14)$$

$$R_S = \frac{1}{5} \frac{T_{\text{on}}}{C_S} \hspace{1cm} (5-15)$$

From other hand, as has been discussed in Chap. 2, resonant inverter section, due to the energy stored in $L_S$, there will be a formation of damped resonant circuit between turn-on and turn-off snubber elements. The $R_S - L_S - C_S$ circuit is normally designed critically damped to avoid oscillations.

Thus:

$$R_S > 2 \sqrt{\frac{L + L_S}{C_S}}$$  \hspace{1cm} (5-16)

Therefore $R_S$ is chosen such that the (5-13), (5-15) and (5-16) should be satisfied.
The diode $D_S$ in Fig. 5-8(b) is presented to eliminate the delay that the resistor introduces in protecting against rapid $dv/dt$ (rapid charging). It should be emphasized that the snubber circuits must be located physically as near as possible to the device in order to minimize stray inductance effects.

For the prototype system the snubber circuit shown in Fig. 5-8(b) with $C_S = 0.1 \mu F$ and $R_S = 15 \Omega$ was employed with satisfactory operation.

5.3 High-Frequency Transformer

In DC-to-AC (1 kHz)-to-AC (50 Hz) power conversion system the high-frequency transformer plays a crucial role. The power conversion with high-frequency link and the transformer behavior in high-frequency & switched mode operation have been discussed in Chapter-1 and Chapter-3 respectively, where the inter-effect of leakage inductance on system (in connection with the snubber capacitance & generation of the harmful voltage spikes) was discussed.

For the prototype system, 3 units of the single-phase transformer were ordered & manufactured externally with following specifications:

**High-frequency transformer**

1- operating frequency, 1 kHz
2- power rating, 800 w
3- winding configurations:

The $P1$, $P2$ and $P3$ windings also $S1$ and $S2$ windings are bifilar wound to minimize leakage inductance. The secondary parts should be sandwiched between the parts of primary sections as shown in Fig. 5-12. A very tiny air gap is left between $E$ & $I$ parts of the core.
Fig. 5-12 1 kHz transformer winding configuration (operating in prototype system)

4 - voltage rating:

As can be seen from Fig. 5-3 & Fig. 5.5 the each primary winding will be supplied by two MOSFETs and centre-tapped DC source of 60 v \( \left( \frac{E}{2} = 60\text{ v} \right) \). The turn ratio of this transformer is 1:3, that is the secondary output voltage is 360 v.
5 - current rating:

The primary winding is to be designed for maximum 7 amp. and secondary winding is to be designed for maximum 2.5 amp. The 3-phase connection of this transformer will be feeding an induction motor. A very tiny air-gap (0.08 mm) was left because of reasons explained in section 5.2.1.

This transformer was supplied by width modulated pulses with 1 kHz fundamental [according to Figs. 2-(20, 21 & 22)] by connecting the primary windings to the DC source through the inverter. To avoid any saturation associated problem, the conversion system was operated at 60-80 v of DC supply.

5.4 3-Phase Cycloconverter For Prototype System

The comprehensive discussions on power frequency changers (cycloconverters) have been presented in chapter 4, including:

1- Direct frequency conversion:
   i - generalized m-phase input with $f_i$ frequency -to- n-phase output with $f_o$ frequency
   ii - 3-phase with $f_i$ frequency -to- 3-phase with $f_o$ frequency
   iii - single-phase with $f_i$ frequency -to- single-phase with $f_o$ frequency
   iv - single-phase with $f$ frequency -to- 3-phase with $f$ frequency (phase-conversion)

2 - Indirect frequency conversion:
   i - generalized m-phase with $f_i$ frequency -to- DC -to- n-phase with $f_o$ frequency
   ii - 3-phase with $f_i$ frequency -to- DC -to- 3-phase with $f_o$ frequency
The suggested technique (demonstrated in chapter 4) for cycloconverter operation was implemented on prototype system. The same switching devices (used in inverter) are employed for the prototype system force commutated cycloconverter, shown in Fig. 5-13.

S7, S8, ..., S17, S18 = bidirectional switches

Fig. 5-13 3-phase force commutated cycloconverter for prototype system
5.5 The control circuits

This section deals with the those components that take part in control of the prototype system operation.

5.5.1 Microprocessor for prototype system control

The existing INTEL-8085 microprocessor system was used to generate:

I- gate signals for 3-phase, half bridge inverter MOSFETs for formation of 3-phase symmetric and 1000 Hz sinusoidal waveforms from DC supply on basis of optimized PWM strategy which have been fully discussed in chapter 2. The prospective switching pattern are shown in Fig. 5-14 (referring to Fig. 5-5).

II- gate signals for three signal phase-to-signal phase cycloconverter MOSFETs for converting 1000 Hz-to-50 Hz loading power.

The switching pattern for one phase of the cycloconverter MOSFETs are presented in Fig. 5-15 (referring to Fig. 5-13)
Fig. 5-14 Switching patterns for one cycle of 3-phase & 1 kHz prototype system inverter in Fig. 5-5
Fig. 5-15 Switching patterns for one cycle of the 1000-to-50 Hz prototype system cycloconverter (in Fig. 5-13)
The basic clock frequency of the INTEL-8085 is 3.072 MHz (internally divided by 2 from the 6.144 MHz quartz crystal input) or state duration = 325.6 nsec [8] & [9].

The gate triggering angles for both inverter (table 2-10) and cycloconverter (table 4-2) are stored in suitable addresses in the microprocessor EPROM. The program starts to execute when zero crossing pulse arrives through the interrupt. Then the required control pulses are transmitted to the drive circuits through the output port.

5.5.2 The zero crossing detector

The prospective application of minicomputers and microprocessors to the control of static converters can only materialize with development of suitable transducers to extract the required informations from the operating converter circuit. For example the zero point detection of current or voltage waveforms.

Since, the cycloconverter in the prototype system, converts the 1000 Hz to 50 Hz and by reconsidering the proposed strategy for cycloconverter switching pattern structure it will be clear that the generation of 1000 Hz by 3 phase inverter should be synchronized by generation of the modulated and 50 Hz cycloconverter output. In other words all the timing circuitry for both inverter and cycloconverter will be referenced to the same point (the mains supply). A convenient way of this implementation is to use a zero crossing detector which gives a pulse every time the A.C. voltage passed through the zero point. The circuit diagram of the zero-crossing detector is shown in Fig. 5-16.
Fig. 5-16 zero crossing detector circuit configuration
A miniature transformer provides isolated low voltage to the circuit. One end of the secondary is grounded to the reset of the system while the other is connected to the inverting input of a differential voltage comparator (SN 52-741 C). The non-inverting input is held at 0.0V. The back-to-back diodes across the inputs of the comparator and in series with a resistor ensure that both the current and the voltage ratings of it are never exceeded.
5.5.3 The phase shifter

Referring to Fig. 5-13, for 3-phase cycloconverter MOSFETs, 12 gating signals are required. Although the microprocessor is capable of generating 12 switching pulses, but it is more efficient (from the software design points of view and avoiding excessive device switching) if the microprocessor generates only gating signals for No. 1 single phase cycloconverter and then by using appropriate logic circuit converters (phase-shifters) for generating 120° and 240° delayed pulses, the No. 2 and No. 3 single-phase cycloconverters will efficiently be gated and finally the 3-phase cycloconverter will be ready to convert the 1000 Hz to 50 Hz according to suggested modulation strategy.

Fig. 5-17 shows a $0^\circ$, $120^\circ$ and $240^\circ$ phase shifter circuit diagram. Where two 4-bit down counter (741 S193), monostable (74121), D-Flip-Flop (7474) and 25 8-bit Registers (741 S164), and NAND gate (74000) are taken part to generate $120^\circ$ and $240^\circ$ delayed pulses. The 4 units of this phase shifters are needed for prototype system.

5-6 Experimental results taken from the prototype system

The prototype system, shown in Fig. 1-18, was designed and performed based on the circuit diagram of Fig. 5-1. The different practical results of the normal system operation is presented in coming pages.
Fig. 5-17 The phase shifter circuit diagram, employed for gating of the cycloconverter MOSFETs (S11, S12, ..., S18)
FIG. 5-18 Prototype system circuit configuration for DC-10-AC (3-phase, 1 KHz) to AC (3-phase, 50 Hz) conversion

Load

Cycloconverter & Filter

3-phase

Transformer

Step-up & I KHz

Inverter

DC Supply & I KHz

1000/50 Hz

Phase

Process

3

Directional switches

S7, S8, ..., S17, S18
Fig. 5-19 Typical switching waveforms (turn-off) of resistive load for Siliconix SMM20N50 power MOSFET employed in prototype system inverter & cycloconverter
Fig. 5-20 (a) Control signals generated by microprocessor for the base drive circuits of $S_1$, $S_3$ and $S_5$ MOSFETs in inverter of Fig. 5.18
(b) Expansion of (a) in time domain
Fig. 5-21 (a) Triggering signals measured at the Gate-Source of S1, S3 and S5 MOSFETs in inverter of Fig. 5-18

(b) Expansion of (a) in time domain for S1 & S3
Fig. 5-22 (a) Inverter output (transformer primary) voltage waveforms with secondary open circuit in Fig. 5-18.

(b) Harmonic spectrum of $V_{an}$ (K is closed)
Fig. 5-23  (a) Inverter output (transformer primary) voltage waveforms with secondary connected to the set of Cycloconverter+Filter+Induction motor  
(b) Primary current waveform
Fig. 5-24 (a) Inverter (primary) line voltage waveforms with secondary open circuit.

(b) Harmonic spectrum of the $V_{ab}$
Fig. 5-25 Secondary open circuit waveforms (in Fig. 5-18)

(a) n to o is closed

(b) n to o is open
Fig. 5-26 (a) Secondary open circuit waveforms with L-C series filter connected between inverter & primary, tuned for 1 kHz (may be compared with Fig 5-25).

(b) Harmonic spectrum of $v_{RR'}$ in Fig. 5-25 (b)
Fig. 5-27 (a) Secondary waveforms during load (induction motor) operation
(b) Harmonic spectrum of $V_{RR'}$
\[ \Phi(v_{RR}', v_{YY}) = \sin^{-1}\left(\frac{a}{b}\right) = \sin^{-1}\left(\frac{2.2}{2.6}\right) = 117.85^\circ \]

\[ \Phi(v_{YY}, v_{BB}) = \sin^{-1}\left(\frac{a}{b}\right) = \sin^{-1}\left(\frac{2.2}{2.6}\right) = 117.85^\circ \]

Fig. 5-28  Secondary voltage waveforms combination for phase angle determination

(a) \( v_{RR}' \) with \( v_{YY}' \)          (b) \( v_{YY}' \) with \( v_{BB}' \)

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Fig. 5-29 Prototype system transformer characteristics at 1 kHz (in Fig. 5-18)

(a) Flux and magnetizing current
(b) Hysteresis loop
Fig. 5-30 (a) Control signals generated by microprocessor for the base drive circuits of S7, S8, S9 and S10 MOSFETs in fig. 5-18
(b) Expansion of (a) in the time domain
Fig. 5-31  (a) Triggering signals measured at the Gate-Source of $S_7$, $S_8$, $S_9$ and $S_{10}$ MOSFETs in Cycloconverter of Fig 5-18
(b) Cycloconverter output voltage ($v_{u1-v1}$) in Fig. 5-18 (no filter, no load)
Fig. 5-32 (a) & (b) Expansion of Fig. 5-31(b) to show intereffect of transformer leakage reactance with snubber circuit capacitance (which was not established in this thesis)
Fig. 5-33 (a) Cycloconverter crude output voltage $v_{u1-vl}$ in Fig. 5-18 (no filter, no load) (b) Harmonic spectrum of (a)
Fig. 5-34 (a) Cycloconverter output voltage waveform, the cycloconverter input was supplied by 1 kHz sinusoidal waveforms, generated by power amplifier rather than by inverter & transformer for spectrum comparison.

(b) Expansion of (a) in time domain.

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Fig. 5-35 (a) Voltage waveforms across S7 or S8 MOSFET in Fig. 5-18.

(b) Effect of 4 µF filtering capacitor on (a)
Fig. 5-36 (a) Harmonic spectrum of Fig. 5-34 (a) which may be compared and verified with Fig. 4-16

(b) Harmonic spectrum of the waveform shown in Fig. 5-33 (a)
Fig. 5-37 (a) Cycloconverter output voltage (Fig. 5-33(a)) after filtering with $C = 0.1 \mu F$

(b) Filtering with $C = 1 \mu F$
Fig. 5-38 (a) Cycloconverter output voltage (Fig. 5-33 (a)) after filtering with $C = 4 \ \mu F$

(b) Voltage waveform across inductive load ($\cos \Phi = 0.716$)
Fig. 5-39 (a) Voltagewaveform across resistive load (10 Ω, Cosφ =1)
(b) Voltage & current waveforms of resistive load (15 Ω, Cos φ =1)
Fig. 5-40 (a) Voltage waveforms across 3-phase induction motor load with 

\[ \cos \Phi = 0.716 \]

(b) waveforms in (a) after filtering with 4 \( \mu \)F connected across the output
Fig. 5-41 (a) Cycloconverter input voltage \( v_{RR'} \), 1000 Hz and output \( v_{u2-v2} \), 50 Hz voltage (inductive load) waveforms 

(b) waveforms of (a) for reverification of proposed modulation strategy.
5-7 References:


7- "Siliconix MOSPOWER application handbook" Siliconix incorporated 1986

9- "8085 Assembly language programming manual" Intel corporation.

10- "SDK-85 system design kit user's manual" Intel corporation - 1978.
Chapter 6

Results, Discussions, Conclusion & Future Work
6.1 General

The work which has been presented here is an investigation of a high frequency link multistage power conditioning system for DC-to-3 phase (1 kHz)-to-3 phase (50 Hz) power processing.

System operation is based on the high-frequency inverter, high-frequency transformer and cycloconverter circuitry with the following objectives:

1- conversion of DC Power to AC form with higher voltage level than the DC source, which necessitate that a power transformer should be employed for both voltage transformation and system input from output isolation purposes.
2- transfer of power by high-frequency link for size reduction of transformer and energy store/transfer elements (inductors and capacitors).

A detailed investigation on such a system (high-frequency and multistage) presents a formidable array of problems that could be tackled. For example the cause and side effects of input current & output voltage harmonics (also in some applications subharmonics) and crucial role of the transformer on overall conversion system design and operation may be future research targets.

6.2 Results and discussions

6.2.1 Inverter connected to the primary and secondary is open circuit

The inverter phase voltages ($V_{an}, V_{bn}, V_{cn}$), inverter line voltages ($V_{ab}, V_{bc}, V_{ca}$), $V_{no}$ and $V_{an}$ harmonic spectrum, those presented in chapter 5, were in very good agreement with those have shown in chapter 2 with computer simulation. Any slight discrepancy on the waveforms is due to the tiny distortion of microprocessor output signals caused by the MOSFET base drive circuits,
Fig. 5-20(b) and Fig. 5-21(b) may be compared.

The inverter fundamental frequency is 1 kHz and each cycle is modulated by $R=9$ pulses with different widths (according to Table 2-10), and from other hand there is oscillatory tendency between transformer inductances and snubber capacitors during switch turn-off (Chapter 3), so attribution of any discrepancy in practical waveforms to these problems is quite recognizable.

From Chapter 3, resonance frequency between snubber capacitors ($C$) and transformer inductances ($L = $ magnetizing + leakage) is:

$$f_r = \frac{1}{2\pi \sqrt{2LC}}$$

and developed spike voltage across switch during turn-off is:

$$v_{spike} = I(\text{primary current}) \sqrt{\frac{L}{C}}$$

Therefore any attempt to increase $C$ to lower $v_{spike}$ will decrease $f_r$ which is undesirable for inverter output. By significantly reduced leakage inductance and using high quality core material the effect on both $f_r$ and $v_{spike}$ will be more desirable. It should be reminded this conclusion is concerned with the case with open circuit secondary.

The primary and secondary voltages were measured 1 kHz sinewave (Fig. 5-25(b)) which is expected, because up to order of 17th (17 kHz), the harmonic components theoretically and practically are eliminated and the rest have insignificant effect on the fundamental.
6.2.2 Inverter connected to the primary through filtering elements and secondary is open circuit

By connecting L-C series filter between primary and inverter and tuned for 1 kHz the secondary voltages will be a pure sinewave (Fig. 5-26(a)). But adding parallel L-C across primary did not make any change which means that effects of high order harmonics is negligible.

6.2.3 Unloaded cycloconverter is connected to the secondary

With rated voltage transformer operation, the cycloconverter sinusoidally modulated output voltage (Fig. 5-33(a)) and it's harmonic spectrum (Fig. 5-33(b)) are satisfactory, but different slightly from those shown in Fig.s (4-15) & (4-16). The possible unwanted distortion factors are:

1- distortion on conducted input half-cycle segments (pulse durations from 4-79) is mainly due to transformer leakage inductance acting as a source impedance causing overlap between switches. By considering both conventional and proposed modulation strategies, it is clear that in proposed technique any possibility of source short circuit via in-coming and out-going MOSFETs is eliminated.

2- interaction between snubber capacitors of the cycloconverter and transformer inductances (Fig. 5-32).

3- effect of filtering capacitor on output voltage(Fig.5-35(b)).

Since a study of these interactions on overall conversion system using classical analysis is very complex, they were not discussed in this thesis in detail (simulation packages, for example SPICE, may be used).
6.2.4 Cycloconverter output filtering

Since system operating link frequency is 1 kHz, the transformer offers some leakage reactance. In addition some stray inductances are present, so by connecting filtering capacitor of 2-4 \( \mu \)F across the cycloconverter output, the overall system output is a 50 Hz sinusoidal wave, Figures 5(37-41).

6.2.5 System loading

A three-phase induction motor was connected to the cycloconverter output. Its starting and unloaded performance was normal, but further motor load caused excessive voltage drop across secondary. The cycloconverter measured output voltage verifies the theoretical value, 64% of secondary voltage, Figures 4-16 and 5-41(b) may be compared. But in load condition, as long as load current increases the transformer regulation gets worse. The same problem occurred with the resistive load.

By considering all facts and explanations about overall system operation the following points are made:

1- the 3-phase inverter operation under the suggested optimized PWM strategy was well satisfied. The low order harmonics up to 17th(17 kHz) are eliminated and output voltage fundamental is achieved by 1.17 p.u. of DC source voltage (table 2-9).

2- the cycloconverter operation (1 kHz-to-50 Hz) under implementation of proposed modulation strategy results 50 Hz sinusoidal output with 64% of input or secondary voltage (Fig.5-41(b)), which is satisfactory in compare with conventional and other methods demonstrated in chapter 4.

3- no need for bulky filtering elements except a small capacitor (2-4 \( \mu \)F) across cycloconverter output.
Therefore the prototype system operation for **DC-to-3 phase (1kHz)**-to-3 phase (50 Hz) power conversion and 3-phase induction motor application purposes is well approved provided that the high-frequency transformer should meet all appropriate requirements dictated by conversion system such as minimized and optimized leakage inductance, interwinding capacitance and design recommendation core material which in turn require another research period itself.

### 6.3 Future Work

There are different aspects of the system raised as a result of this research work which could be given more attention. These aspects can be summarized as follows:

1. It is possible to provide closed loop control such that the constant output voltage is maintained as the load varies. The existing microprocessor based waveform generating system could be incorporated in the loop.

2. Referring to Fig. 2-19, by controlling of the inverter fundamental output voltage (in terms of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ variation which is accomplished with the elimination of 5th, 7th and 11th components) accordingly the output of cycloconverter can be controlled either by open loop or close loop. The aspect seems very attractive especially in induction motor application with constant maximum torque and variable speed by high frequency inverter-transformer-cycloconverter configuration. Because in this mode, in the inverter side components up-to 13th ($13 \text{ kHz}$) and in cycloconverter output the components up-to 39th ($1.8 \text{ kHz}$) will be eliminated.

3. Microprocessor implementation for variable cycloconverter output voltage and frequency based on relationships of (4-79) & (4-82) which produce predictable no load output voltage harmonics.
4- if system load is a slip ring induction motor, the rotor energy recovery and injection of this energy to the system as a close loop.

5- for the prototype system the transformer was designed and manufactured externally but it was not sought after one. The exact transformer design to predict the leakage inductance and inter-winding capacitances reliably would help in the overall system performance. For precise system efficiency assessment, the calculation of transformer losses under non-sinusoidal operation conditions is necessary. Therefore for switched mode and high-frequency operation, the transformer is worth much further study, though this is beyond of the scope of this thesis and itself is a major project in its own right, especially for following targets:

1- high-frequency and high-power handling capability
2- low cost & high efficiency design
Appendix A (Computer programs)
**Program for 3-Dimension Plotting of T.H.D=f(R,M)**

- **R=fc/fm**
- **fc** is carrier wave frequency
- **fm** is modulating wave frequency
- **M** is depth of modulation
- **T.H.D** is total harmonic distortion factor on inverter output

```
INTEGER NUMX, NUMY, NP, NW, NCONT, ISM, I, IVIEW
REAL XLOW, XHIGH, YLOW, YHIGH, PI, XAV, YAV, SCALE, ZANGLE,
x1, y1, x2, y2, x3, y3, x4, y4
REAL X(25), Y(25), Z(25), GRIDZ(60,60), W(15000)
NUMX=60
NUMY=60
NW=15000
XLOW=3.0
XHIGH=11.0
YLOW=0.2
YHIGH=1.0
read(*,*)id,un,iv
ISM=0
READ (70,*) NP
DO 10 I=1,NP
  READ (70,*) X(I), Y(I), Z(I)
10 CONTINUE
CALL GINO
if(id.eq.1) call t4107
if(id.eq.2) call laserw
CALL UNITS(un)
CALL RANGRD(NP, X, Y, Z, NUMX, XLOW, XHIGH, NUMY, YLOW, YHIGH, GRIDZ, NW, W)
call isoprj(numx, xlow, xhigh, numy, ylow, yhigh, gridz, iv, nw, w)
call ginend
CLOSE (UNIT=5)
STOP
END
```
C ***********************************************************************
C SET OF NONLINEAR EQUATION SOLUTION, NAG-(c05nbf) ROUTINE
C nag4.f, harmonic elimination (5, 7, 11, 13th) and \( v_{1}(\text{max})=1.17 \)
C alfa1=10.548153, alfa2=16.094384, alfa3=30.905343, alfa4=32.864859
C 20/SEP/1989, LIB.-TERMINAL, 4 PM
C ***********************************************************************
C DOUBLE PRECISION FNORM, TOL
C IMPLICIT DOUBLE PRECISION (A-H, O-Z)
C INTEGER IFAIL, J, NOUT
C DOUBLE PRECISION FVEC(4), WA(54), X(4)
C DOUBLE PRECISION F05ABF, DSQRT, X02AAF
C EXTERNAL FCN
C DATA NOUT /6/
C WRITE (NOUT, 99999)
C DO 20 J=1,4
C X(J)=49813000. E-08
C 20 CONTINUE
C TOL = DSQRT(X02AAF(0. 0))
C IFAIL = 1
C N1=1
C CALL C05NBF(FCN, 4, X, FVEC, TOL, WA, 54, IFAIL)
C N2=N1+1
C FNORM = F05ABF(FVEC, 4)
C PI=3.1415926
C Y=(PI/2)
C IF (X(2)-X(1))8,8,9
C IF (X(3)-X(2))8,8,7
C IF (X(4)-X(3))8,8,6
C IF (X(4)-Y)5,8,8
C WRITE (NOUT, 99998) FNORM, IFAIL, (X(J), J=1,4)
C STOP
C 99999 FORMAT (4(1X/), 31H C05NBF EXAMPLE PROGRAM RESULTS/1X)
C 99998 FORMAT (5X, 31H FINAL L2 NORM OF THE RESIDUALS, E12.4//5X,
C * 15H EXIT PARAMETER, I10//5X, 27H FINAL APPROXIMATE SOLUTION//
C * (5X, 3E12.4))
C END
C SUBROUTINE FCN(N, X, FVEC, IFLAG)
C INTEGER IFLAG, N
C DOUBLE PRECISION FVEC(N), X(N)
C INTEGER K
C I=1
C DO 30 M=5,13,2
C K=1
C IF (M-9)11,30,11
C WRITE (6,*M)
C TEMP=COS (M*X(K)) -COS (M*X(K+1)) +COS (M*X(K+2)) -COS (M*X(K+3)) -0.5
C K=I
C FVEC(K) =TEMP
C I=1+K
C 30 CONTINUE
C RETURN
C END
proposed modulation strategy, equal area, 90 deg. line

\[
\pi = 3.1415926
\]
\[
z = 0.6360
\]
\[
r = 20.
\]
do 14 \( k = 1 \), 40
al = \( \pi / r \)
a2 = \( k \times a1 \)
a3 = \( \cos(a2) \)
j = \( k - 1 \)
a4 = \( j \times a1 \)
a5 = \( \cos(a4) \)
a6 = \( a5 - a3 \)
a7 = \( a6 \times r \times z \)
a8 = \( k - 0.5 \)
a9 = \( a8 \times a1 \)
a10 = \( \sin(a9) \)
l = \( k + 1 \)
al1 = \( (-1) \times \pi \)
al2 = \( 2 \times a1 \times a10 \)
al3 = \( a7 / a12 \)
if (k = 20) 1, 1, 2
a14 = \( \sin(a13) \)
3  beta = \( a14 \times (180. / \pi) \)
alfal = \( 2 \times \beta \)
n = \( k - 1 \)
x1 = 90. - beta
x2 = \( x1 + a1f \)
x3 = \( x1 + x2 \)
landa = \( 180 \times n \)
x4 = \( x1 + \text{landa} \)
x5 = \( x2 + \text{landa} \)
x6 = \( x3 + \text{landa} \)
write (50, \*) x4, x5, x6
alfaa = 90. - beta
alfa2 = 90. - beta
alfa3 = 90. + beta
alfoff = 90. - beta
toff = alfoff * (20000. / 7200)
alfo = 2 * beta
ton = alfo * (20000. / 7200)
write (71, \*) aloff, alfo, alfo
	total = toff + ton + toff
write (72, \*) toff, ton, toff, toff
write (46, \*) k, alfa2, alfa3, alfo
write (47, \*) k, toff, ton, toff
write (48, \*) k, alfoff, alfoa, alfoff
write (49, \*) alfoff, alfoa, alfoff
write (6, \*) k

go to 14
2 al1 = \( (-1) \times \sin(a13) \)
go to 3
14 continue
stop
end
Appendix B ( SMM20N50 Power MOSFET data sheets )
PRODUCT SUMMARY

<table>
<thead>
<tr>
<th>PART NUMBER</th>
<th>V(BR)DSS (VOLTS)</th>
<th>rDS(on) (OHMS)</th>
<th>ID (AMPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMM20N50</td>
<td>500</td>
<td>0.30</td>
<td>20</td>
</tr>
</tbody>
</table>

ABSOLUTE MAXIMUM RATINGS (TC = 25°C unless otherwise noted)

<table>
<thead>
<tr>
<th>PARAMETERS/TEST CONDITIONS</th>
<th>Symbol</th>
<th>SMM20N50</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drain-Source Voltage</td>
<td>VDS</td>
<td>500</td>
<td>V</td>
</tr>
<tr>
<td>Gate-Source Voltage</td>
<td>VGS</td>
<td>40</td>
<td>V</td>
</tr>
<tr>
<td>Continuous Drain Current, TC = 25°C</td>
<td>ID</td>
<td>20</td>
<td>A</td>
</tr>
<tr>
<td>TC = 100°C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pulsed Drain Current¹</td>
<td>IDM</td>
<td>80</td>
<td>A</td>
</tr>
<tr>
<td>Avalanche Current (see figure 9)</td>
<td>IA</td>
<td>20</td>
<td>A</td>
</tr>
<tr>
<td>Power Dissipation</td>
<td>PD</td>
<td>250</td>
<td>W</td>
</tr>
<tr>
<td>TC = 25°C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC = 100°C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operating Junction &amp; Storage Temp</td>
<td>TJ, TSTG</td>
<td>-55 to 150</td>
<td>°C</td>
</tr>
<tr>
<td>Lead Temperature</td>
<td>TL</td>
<td>55</td>
<td>°C</td>
</tr>
</tbody>
</table>

THERMAL RESISTANCE RATINGS

<table>
<thead>
<tr>
<th>THERMAL RESISTANCE</th>
<th>Symbol</th>
<th>Typ.</th>
<th>Max.</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Junction-to-Case</td>
<td>RJC</td>
<td>-</td>
<td>0.5°C</td>
<td>K/W</td>
</tr>
<tr>
<td>Junction-to-Ambient</td>
<td>RJA</td>
<td>-</td>
<td>20</td>
<td>K/W</td>
</tr>
<tr>
<td>Case-to-Sink</td>
<td>RCS</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

¹Pulse width limited by maximum junction temperature (refer to transient thermal impedance data, figure 11.)
### ELECTRICAL CHARACTERISTICS (T<sub>J</sub> = 25°C unless otherwise noted)

<table>
<thead>
<tr>
<th>PARAMETERS/TEST CONDITIONS</th>
<th>Symbol</th>
<th>Min.</th>
<th>Typ.</th>
<th>Max.</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drain-Source Breakdown Voltage</td>
<td>V&lt;sub&gt;(BR)DSS&lt;/sub&gt;</td>
<td>500</td>
<td>-</td>
<td>-</td>
<td>V</td>
</tr>
<tr>
<td>Gate Threshold Voltage</td>
<td>V&lt;sub&gt;GS(th)&lt;/sub&gt;</td>
<td>2.0</td>
<td>2.6</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>Gate-Body Leakage</td>
<td>I&lt;sub&gt;GSS&lt;/sub&gt;</td>
<td>-</td>
<td>-</td>
<td>100</td>
<td>µA</td>
</tr>
<tr>
<td>Zero Gate Voltage Drain Current</td>
<td>I&lt;sub&gt;DSS&lt;/sub&gt;</td>
<td>-</td>
<td>-</td>
<td>250</td>
<td>µA</td>
</tr>
<tr>
<td>Zero Gate Voltage Drain Current</td>
<td>I&lt;sub&gt;DSS&lt;/sub&gt;</td>
<td>-</td>
<td>-</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>On-State Drain Current&lt;sup&gt;2&lt;/sup&gt;</td>
<td>I&lt;sub&gt;D(on)&lt;/sub&gt;</td>
<td>20</td>
<td>-</td>
<td>-</td>
<td>A</td>
</tr>
<tr>
<td>Drain-Source On-State Resistance&lt;sup&gt;2&lt;/sup&gt;</td>
<td>R&lt;sub&gt;DS(on)&lt;/sub&gt;</td>
<td>0.26</td>
<td>0.30</td>
<td></td>
<td>Ω</td>
</tr>
<tr>
<td>Forward Transconductance&lt;sup&gt;2&lt;/sup&gt;</td>
<td>g&lt;sub&gt;fs&lt;/sub&gt;</td>
<td>6.0</td>
<td>11</td>
<td>-</td>
<td>S(Ω&lt;sup&gt;-1&lt;/sup&gt;)</td>
</tr>
<tr>
<td>Input Capacitance</td>
<td>C&lt;sub&gt;iss&lt;/sub&gt;</td>
<td>-</td>
<td>3800</td>
<td>4500</td>
<td>pF</td>
</tr>
<tr>
<td>Output Capacitance</td>
<td>C&lt;sub&gt;Coss&lt;/sub&gt;</td>
<td>-</td>
<td>750</td>
<td>1000</td>
<td>pF</td>
</tr>
<tr>
<td>Reverse Transfer Capacitance</td>
<td>C&lt;sub&gt;rss&lt;/sub&gt;</td>
<td>-</td>
<td>350</td>
<td>500</td>
<td>pF</td>
</tr>
<tr>
<td>Total Gate Charge</td>
<td>Q&lt;sub&gt;g&lt;/sub&gt;</td>
<td>-</td>
<td>70</td>
<td>100</td>
<td>nC</td>
</tr>
<tr>
<td>Gate-Source Charge</td>
<td>Q&lt;sub&gt;gs&lt;/sub&gt;</td>
<td>-</td>
<td>15</td>
<td>-</td>
<td>nC</td>
</tr>
<tr>
<td>Gate-Drain Charge</td>
<td>Q&lt;sub&gt;gd&lt;/sub&gt;</td>
<td>-</td>
<td>34</td>
<td>-</td>
<td>nC</td>
</tr>
<tr>
<td>Turn-On Delay Time</td>
<td>t&lt;sub&gt;d(on)&lt;/sub&gt;</td>
<td>-</td>
<td>34</td>
<td>45</td>
<td>ns</td>
</tr>
<tr>
<td>Rise Time</td>
<td>t&lt;sub&gt;r&lt;/sub&gt;</td>
<td>-</td>
<td>97</td>
<td>70</td>
<td>ns</td>
</tr>
<tr>
<td>Turn-Off Delay Time</td>
<td>t&lt;sub&gt;d(off)&lt;/sub&gt;</td>
<td>-</td>
<td>120</td>
<td>150</td>
<td>ns</td>
</tr>
<tr>
<td>Fall Time</td>
<td>t&lt;sub&gt;f&lt;/sub&gt;</td>
<td>-</td>
<td>62</td>
<td>75</td>
<td>ns</td>
</tr>
</tbody>
</table>

### SOURCE-DRAIN DIODE RATINGS & CHARACTERISTICS (T<sub>J</sub> = 25°C unless otherwise noted)

<table>
<thead>
<tr>
<th>PARAMETERS/TEST CONDITIONS</th>
<th>Symbol</th>
<th>Min.</th>
<th>Typ.</th>
<th>Max.</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous Current</td>
<td>I&lt;sub&gt;S&lt;/sub&gt;</td>
<td>-</td>
<td>-</td>
<td>20</td>
<td>A</td>
</tr>
<tr>
<td>Pulsed Current&lt;sup&gt;1&lt;/sup&gt;</td>
<td>I&lt;sub&gt;SM&lt;/sub&gt;</td>
<td>-</td>
<td>-</td>
<td>110</td>
<td>A</td>
</tr>
<tr>
<td>Forward Voltage&lt;sup&gt;2&lt;/sup&gt;</td>
<td>V&lt;sub&gt;SD&lt;/sub&gt;</td>
<td>-</td>
<td>-</td>
<td>1.6</td>
<td>V</td>
</tr>
<tr>
<td>Reverse Recovery Time&lt;sup&gt;2&lt;/sup&gt;</td>
<td>t&lt;sub&gt;rr&lt;/sub&gt;</td>
<td>-</td>
<td>300</td>
<td>650</td>
<td>ns</td>
</tr>
<tr>
<td>Reverse Recovered Charge</td>
<td>Q&lt;sub&gt;rr&lt;/sub&gt;</td>
<td>-</td>
<td>2.0</td>
<td>-</td>
<td>µC</td>
</tr>
</tbody>
</table>

<sup>1</sup>Pulse width limited by maximum junction temperature (refer to transient thermal impedance data, figure 11)

<sup>2</sup>Pulse test: Pulse width ≤ 300 µsec, Duty Cycle ≤ 2%
FIGURE 1: Typical Output Characteristics

FIGURE 2: Typical Transfer Characteristics

FIGURE 3: Typical Transconductance

FIGURE 4: Typical On-Resistance

FIGURE 5: Typical Capacitance

FIGURE 6: Typical Gate Charge
PERFORMANCE CURVES (25°C Unless otherwise noted)

FIGURE 7: On-Resistance vs. Junction Temperature

FIGURE 8: Typical Source-Drain Diode Forward Voltage

FIGURE 9: Maximum Avalanche and Drain Current vs. Case Temperature

FIGURE 10: Safe Operating Area

FIGURE 11: Normalized Effective Transient Thermal Impedance, Junction-to-Case
Appendix C

1 - Semi-Graphic & Semi-Numerical Analysis
2 - Effect of the load power factor on the Inverter-Transformer operation
Semi-Graphical & Semi-Numerical Analysis

Any periodic and non-sinusoidal function may be expressed by a Fourier Series:

\[ y = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \sin(nx) + b_n \cos(nx) \right] \]  \hspace{1cm} (1)

Where

\[ a_0 = \frac{1}{\pi} \int_{0}^{\pi} y \, dx \quad , \quad a_n = \frac{1}{\pi} \int_{0}^{\pi} y \sin(nx) \, dx \quad , \quad b_n = \frac{1}{\pi} \int_{0}^{\pi} y \cos(nx) \, dx \]  \hspace{1cm} (2)

The following cases may occur:

1- if the areas of the positive and negative half cycles are equal, then \( a_0 = 0 \)

2- if \( f(x + \pi) = -f(x) \), then odd harmonics only are present

3- if \( f(-x) = -f(x) \), then \( b_n = 0 \)

4- if \( f(-x) = f(x) \), then \( a_n = 0 \)

If it is difficult to find a manageable mathematical expression to \( f(x) \), the "semi-Graphical, semi-Numerical" method can be applied\([1, 2]\). Here, one cycle of the curve is divided into \( m \) vertical strips of equal width (the \( m \) an even number). If \( y_k \) is the ordinate of \( k\)-th strip, then the coefficients of the \( n\)-th harmonic are

\[ a_n = \frac{2}{m} \sum_{k=1}^{m} y_k \sin(nk) \]  \hspace{1cm} (3)

\[ b_n = \frac{2}{m} \sum_{k=1}^{m} y_k \cos(nk) \]  \hspace{1cm} (4)
or

\[
a_n = \frac{2}{m} \sum_{k=1}^{m} y_k \sin \left( \frac{2\pi k}{m} n \right)
\]

(5)

\[
b_n = \frac{2}{m} \sum_{k=1}^{m} y_k \cos \left( \frac{2\pi k}{m} n \right)
\]

(6)

For the greater accuracy, \( m \) (the number of ordinate) >> \( n \) (the harmonic order) should be considered.

1- Bela E.F. Karsa "Electrical measuring instruments and measurements"
Budapest, 1967

company (UK) limited, 1987

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Effect of the load power factor on the Inverter-Transformer operation

Referring to the Figs. 2-2 & 2-3 of the chapter 2:

1- when the load is a pure resistive ($\cos \Phi = 1$), the current waveform will be identical to the voltage waveform. So the power delivered to the load via the switches is proportional to the dotted areas in the current waveforms, shown in Fig.2-2( a ).

2- when the load is completely inductive ($\cos \Phi = 0$ lagging) the current waveform is similar to the Fig. 2-2( b ), $i = \frac{1}{L} \int_{n}^{t} Edt$. When the power (trapped in the inductive load) is feed backed in to the source (hatched areas in the current waveform), at the same time power is pushed into the load by the source. Because of the unidirectional current carrying property of the semiconductor switches the reverse current is blocked by the switch, in result causes a reverse voltage spike at the primary. By connecting the anti-parallel diodes across the MOSFETs this problem can be avoided. when the load is a pure inductive, the diode conduction duration is 90°.

3- when the load is completely capacitive ($\cos \Phi = 0$ leading), in the beginning of any MOSFET conduction the power is supplied to the reverse charged capacitor through a very low impedance (transformer impedance + switch saturation resistance) and the current rises to extremely high levels until the capacitor begins to represent a rising impedance (charging to source voltage). A general current waveform for this mode of operation is shown in Fig. 2-2( c ).
Published papers

These papers have been accepted for presentation at the:


A NOVEL STRATEGY FOR CONTROL OF CYCLOCONVERTERS
Asghar Karamat  Tom Thomson  Pratap Mehta
Brunel University  UK

ABSTRACT

This paper presents a novel technique for controlling cycloconverters. The main advantages claimed are (i) much improved harmonic spectrum in the output voltage, (ii) the proposed strategy allows much easier implementation using microprocessors and (iii) it reduces the possibility of short-circuit between the positive and the negative groups of converters.

INTRODUCTION

Multi-stage power conversion from dc to ac with high frequency-links and inverter-transformer-phase controlled cycloconverter topologies have been of growing interest for uninterruptible power supplies and solar converters. During the last decade, there have been a number of suggestions for dc to ac systems (1,2,3). For a required voltage transformation and/or electrical isolation between the dc input and ac output conventionally a transformer is employed. In the new generation of lighter power converters these demands are met by small transformers operating at high frequency. The relative merits and shortcomings of these new systems, however, have not all been fully explored, but eventually are dependent to some extent on the application and power and voltage levels for which these systems are designed.

CONVENTIONAL MODULATION STRATEGY

Conventional control strategy is based on the cosine-wave crossing method (4,5) for determining the timing of the triggering pulses for cycloconverter switches. Referring to Figure 1 the voltage equations for this configuration are:

\[ v_{AN} = V_m \sin(\omega_1 t) \]  
\[ u_{AN} = \frac{2}{\pi} V_m \cos(\omega_1 t) \]  
\[ v_{BN} = V_m \sin(\omega_1 t - \pi) = -V_m \sin(\omega_1 t) \]  
\[ u_{BN} = \frac{2}{\pi} V_m \cos(\omega_1 t) \]  
\[ v_o = M V_m \sin(\omega_o t) \]

\( v_o \) is the cycloconverter output voltage,  
\( M \) is the depth of modulation for output voltage and \( 0.1 < M < 2/\pi \),  
\( \omega_o = 2\pi f_c \)  
\( f_c \) is the cycloconverter output frequency.
\( R = \frac{f_i}{f_o} \) is the input/output frequency ratio. Referring to equations (1) to (5) it is clear that the firing angles are determined so that the average output voltage, produced across the load by a conducting switch, is equal to the instantaneous value of the required output voltage at that firing instant.

From equation (1), for triggering angle \( \alpha \), the average value of the output voltage is

\[
\overline{V_{ave}} = \frac{1}{\pi} \int_{0}^{\alpha} V_m \sin(\omega_1 t) d(\omega_1 t) = \frac{2}{\pi} V_m \cos(\alpha)
\]

(6)

\( \alpha = \omega_1 t = 2\pi f_1 t \)

Therefore the (5) and (6) values should be equal at any instant of conduction, so

\[
\frac{2}{\pi} V_m \cos(\alpha) = M V_m \sin(\frac{\alpha}{R})
\]

(7)

Equation (7) implies that the intersecting points of a sinusoidal reference voltage with a series of cosine timing waves[(2) and (4)], which are derived from and synchronized to the ac input voltage, are the required switching points of the switches.

Depending on \( M, R \) and \( f_i \), equation (7) may be applied by appropriate implementation. For \( M = 0.63 \), \( R = 20 \), \( f_i = 1 \text{ kHz} \) and resistive load the full output modulated voltage and associated harmonic spectrum are presented in Figure 2.

**Proposed modulation strategy**

Figure 3 shows the configuration suitable for implementing the new strategy. S1 to S4 are bi-trival switches as shown. S1 and S2 conduct when point A is positive with respect to B and similarly S3 and S4 conduct when point B is positive with respect to A, thereby producing the positive half-cycle of the cycloconverter.
output. The negative half-cycle of the output is produced in a similar fashion. Figure 4 illustrates the strategy for deriving the triggering angles of the switches which uses the “equal area criterion” i.e. the hatched area being equal to the dotted area. The two areas are given by the following expressions.

Since:

$$\cos(Q) - \cos(P) = 2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

(11)

Then:

$$A_{hatch} = 2\sin(k-0.5)n.\sin(\alpha)$$

(12)

By equating (8) with (12):

$$A_{det} = A_{hatch}$$

or

$$\frac{M}{\omega o}\left[\cos(k-1)\frac{\pi}{R} - \cos(k)\frac{\pi}{R}\right] = \frac{2}{R\omega_o}.\sin(k-0.5)n.\sin(\alpha)$$

(13)

From equation (13) the value of $\alpha(k)$ can be determined from:

$$\alpha(k) = \sin^{-1}\left[\frac{X}{Y}\right].\frac{360}{\pi}$$

(14)

Where

$$X = RM\left[\cos(k-1)\frac{\pi}{R} - \cos(k)\frac{\pi}{R}\right]$$

(15)

$$Y = 2(-1)^{k+1}\sin(k-0.5)n\pi$$

Where

$$k = 1, 2, 3, \ldots, 2R$$

$$M = 0, \ldots, 2/\pi$$
For $M = 0.63$, $R = 20$ and $f_i = 1 \text{kHz}$, the modulated output voltage and associated harmonic spectrum are presented in Figure 5.

![Figure 5: Cycloconverter modulated output voltage and associated harmonic spectrum with proposed strategy](image)

**EXPERIMENTAL RESULTS**

The proposed control strategy was verified on a three-phase cycloconverter supplied at $1 \text{kHz}$ from a dc source via a $1 \text{kHz}$ inverter-transformer system.

Figure 6 shows the output voltage harmonic spectrum of the $50 \text{Hz}$ output cycloconverter. As can be seen there are no significant harmonics below $2 \text{kHz}$ for the input frequency of $1 \text{kHz}$. Also this results compare very favourably with the predicted spectrum shown in Figure 5.

![Figure 6: Output voltage harmonic spectrum](image)

Figure 7 shows the three-phase voltage waveforms. For measurement purposes these oscillograms were obtained with a 2 microfarad capacitor connected across the output to filter out harmonics in excess of $1.8 \text{kHz}$.

![Figure 7: Three-phase output voltage waveforms](image)

**CONCLUSIONS**

The proposed control strategy for a non-circulating current cycloconverter offers the following advantages:

1. Typically for a $1 \text{kHz}$ input system the control strategy assures that there are no significant harmonics below $2 \text{kHz}$.

2. The strategy is amenable to implementation using on-line microprocessor control i.e. equation (14) can be programmed into the system.

3. Unlike conventional cycloconverter the proposed configuration does not require any centre-tapped windings.
APPENDIX

Harmonic analysis for the proposed modulation strategy

Refering to Fig. (8), the general expression of a Fourier Series for the instantaneous output voltage is

\[
v_o(\omega_o t) = \sum_{1,3,5} (A_n \cos n\omega_o t + B_n \sin n\omega_o t)
\]

(16)

The coefficients An and Bn are determined by considering a pair of pulses such that the positive pulse of duration \(\alpha^*_k\) starts at \(t = \Phi^*_k\) and the negative one of the same width starts at \(t' = 3600^\circ + \Phi^*_k\) (Fig. 8). The effects of all pulses can be combined together to obtain the effective output voltage. If the positive pulse of \(k\)th pair starts at \(t = \Phi^*_k\) and ends at:

\[
\omega_t = 3600^\circ + \Phi^*_k
\]

or:

\[
\omega_t = R \, \pi + \Phi^*_k \text{ (rad.)}
\]

The Fourier coefficients for this pair of pulses are:

\[
a_n = \frac{2}{R \pi} \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} V_m \sin(R\omega_o t) \cdot \cos(n\omega_o t) \, d(\omega_o t)
\]

(17)

Since

\[
\sin(R\omega_o t) \cdot \cos(n\omega_o t) = \frac{1}{2} \left[ \sin((R + n)\omega_o t) + \sin((R - n)\omega_o t) \right]
\]

then (17) becomes

\[
a_n = \frac{2}{R \pi} \cdot \frac{V_m}{2} \left\{ \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \sin((R + n)\omega_o t) \, d(\omega_o t) + \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \sin((R - n)\omega_o t) \, d(\omega_o t) \right\}
\]

(18)

or

\[
a_n = \frac{V_m}{\pi} \left\{ \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \sin((R + n)\omega_o t) \, d(\omega_o t) + \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \sin((R - n)\omega_o t) \, d(\omega_o t) \right\}
\]

(19)

\[
b_n = \frac{2}{R \pi} \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} V_m \sin(R\omega_o t) \cdot \sin(n\omega_o t) \, d(\omega_o t)
\]

since

\[
\sin(R\omega_o t) \cdot \sin(n\omega_o t) = \frac{1}{2} \left[ \cos((R + n)\omega_o t) - \cos((R - n)\omega_o t) \right]
\]

then (21) becomes

\[
b_n = \frac{2}{R \pi} \cdot \frac{V_m}{2} \left\{ \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \cos((R + n)\omega_o t) \, d(\omega_o t) - \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \cos((R - n)\omega_o t) \, d(\omega_o t) \right\}
\]

(22)

or

\[
b_n = \frac{V_m}{\pi} \left\{ \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \cos((R + n)\omega_o t) \, d(\omega_o t) - \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \cos((R - n)\omega_o t) \, d(\omega_o t) \right\}
\]

(23)

\[
a_n = \frac{V_m}{\pi} \left\{ \int_{\Phi^*_k}^{\Phi^*_k+\alpha_k} \sin((R + n)(\Phi_k^{(k)}) \cdot \sin((R + n)\Phi_k^{(k)}) \right\}
\]

(24)

Where

\[
R = \frac{f_i}{f_o}
\]

\[
V_m = \text{cycloconverter input voltage amplitude}
\]

as shown in Figure 3

\[
n = 1,2,3,5,...
\]

\[
\alpha_k^{(k)} = \text{ON durations for kth pulse pair (Fig. 8), resulting from (14)}
\]

\[
k = 1,2,3,...., R \text{ (number of pulses per cycloconverter output voltage half cycle)}
\]

\[
\Phi_k = \left( k - \frac{1}{2} \right) \pi - \frac{1}{2} \alpha_k^{(k)}
\]
Therefore $A_n$ and $B_n$ can be expressed as follows:

$$A_n = \sum_{k=1}^{R\pi} \frac{V_m}{R+n} \left( \frac{1}{R+n} \cos((R+n)\Phi_k \cos((R+n)(\Phi_k + \alpha_k)) \right)$$

(25)

$$B_n = \sum_{k=1}^{R\pi} \frac{V_m}{R+n} \left( \frac{1}{R+n} \sin((R+n)(\Phi_k + \alpha_k)) \right)$$

(26)

By substituting (25) and (26) in to following expression

$$v_0(\omega t) = \sum_{n=1,3,5} \left[ A_n \cos(n\omega t) + B_n \sin(n\omega t) \right]$$

(27)

The general expression of the cycloconverter output voltage will can be obtained.

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A NOVEL STRATEGY FOR CONTROL OF CYCLOCONVERTERS

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ABSTRACT

This paper presents a novel technique for controlling cycloconverters. The main advantages claimed are (i) much improved harmonic spectrum in the output voltage, (ii) the proposed strategy allows much easier implementation using microprocessors and (iii) it reduces the possibility of short-circuit between the positive and the negative groups of converters.

INTRODUCTION

Multi-stage power conversion from dc to ac with high frequency-links and inverter-transformer-phase controlled cycloconverter topologies have been of growing interest for uninterruptible power supplies and solar converters. During the last decade, there have been a number of suggestions for dc to ac systems (1,2,3). For a required voltage transformation and/or electrical isolation between the dc input and ac output conventionally a transformer is employed. In the new generation of lighter power converters these demands are met by small transformers operating at high frequency. The relative merits and shortcomings of these new systems, however, have not all been fully explored, but eventually are dependent to some extent on the application and power and voltage levels for which these systems are designed.

CONVENTIONAL MODULATION STRATEGY

Conventional control strategy is based on the cosine-wave crossing method (4,5) for determining the timing of the triggering pulses for cycloconverter switches. Referring to Figure 1 the voltage equations for this configuration are:

\[ v_{AN} = V_m \sin(\omega_1 t) \]  
\[ u_{AN} = \frac{2}{\pi} V_m \cos(\omega_1 t) \]  
\[ v_{BN} = V_m \sin(\omega_1 t - \pi) = -V_m \sin(\omega_1 t) \]  
\[ u_{BN} = \frac{2}{\pi} V_m \cos(\omega_1 t) \]  
\[ v_o = M \cdot V_m \sin(\omega_0 t) \]

\( v_o \) is the cycloconverter output voltage
\( M \) is the depth of modulation for output voltage and \( 0.1 < M < 2/\pi \)
\( \omega_o = 2\pi f_c \)
\( f_c \) is the cycloconverter output frequency
Refering to equations (1) to (5) it is clear that the firing angles are determined so that the average output voltage, produced across the load by a conducting switch, is equal to the instantaneous value of the required output voltage at that firing instant.

From equation (1), for triggering angle α, the average value of the output voltage is

\[ V_{av} = \frac{1}{\pi} \int_{-\alpha}^{\pi+\alpha} V_m \sin(\omega t) \, d(\omega t) = \frac{2}{\pi} V_m \cos(\alpha) \]

(6)

\[ \alpha = \omega t = 2\pi f_t \]

Therefore the (5) and (6) values should be equal at any instant of conduction, so

\[ \frac{2}{\pi} V_m \cos(\alpha) = MV_m \sin\left(\frac{\alpha}{R}\right) \]

(7)

Equation (7) implies that the intersecting points of a sinusoidal reference voltage with a series of cosine timing waves[2] and [4], which are derived from and synchronized to the ac input voltage, are the required switching points of the switches.

Depending on M, R and f, equation (7) may be applied by appropriate implementation. For M=0.63, R=20, f=1 kHz and resistive load the full output modulated voltage and associated harmonic spectrum are presented in Figure 2.

**Proposed modulation strategy**

Figure 3 shows the configuration suitable for implementing the new strategy. S1 to S4 are biletal switches as shown. S1 and S2 conduct when point A is positive with respect to B and similarly S3 and S4 conduct when point B is positive with respect to A, thereby producing the positive half-cycle of the cycloconverter.
output. The negative half-cycle of the output is produced in a similar fashion. Figure 4 illustrates the strategy for deriving the triggering angles of the switches which uses the “equal area criterion” i.e. the hatched area being equal to the dotted area. The two areas are given by the following expressions.

\[
\begin{align*}
\text{Since:} & \\
\cos(Q) - \cos(P) &= 2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right) \\
\text{Then:} & \\
A_{\text{hatch}} &= 2\sin(k-0.5)\pi \sin\left(\frac{\alpha}{2}\right) \\
\text{By equating (8) with (12):} & \nonumber \\
A_{\text{area}} &= A_{\text{hatch}} \\
\text{or} & \\
\frac{M}{\omega_o} \left[ \cos(k-1)\frac{\pi}{R} - \cos(k+1)\frac{\pi}{R} \right] &= \frac{2}{R\omega_o} \sin(k-0.5)\pi \sin\left(\frac{\alpha}{2}\right) \\
\text{From equation (13) the value of } \alpha(k) \text{ can be determined from:} & \\
\alpha(k) &= \sin^{-1}\left[ \frac{X}{Y} \right] \cdot \frac{360}{\pi} \\
\end{align*}
\]

Figure 4 Equal area criterion for proposed modulation strategy
For \( M = 0.63, R = 20 \) and \( f_i = 1 \text{kHz} \), the modulated output voltage and associated harmonic spectrum are presented in Figure 5.

![Modulated output voltage](image)

Harmonic orders

![Harmonic spectrum](image)

Figure 5 cycloconverter modulated output voltage and associated harmonic spectrum with proposed strategy

**EXPERIMENTAL RESULTS**

The proposed control strategy was verified on:

(i) a single-phase cycloconverter supplied from a 1 kHz source. Figure 6 (a) & (b) show the output voltage waveform (50 Hz fundamental) and harmonic content respectively.

(ii) a three-phase cycloconverter supplied at 1kHz from a dc source via a 1kHz inverter-transformer system.

Figure 6 shows the output voltage waveform of the 50Hz output cycloconverter. As can be seen there are no significant harmonics below 2kHz for the input frequency of 1kHz. Also this results compare very favourably with the predicted spectrum shown in Figure 5.

Figure 7 shows the output voltage harmonic spectrum of the 50Hz output cycloconverter. For measurement purposes these oscillograms were obtained with a 2 microfarad capacitor connected across the output to filter out harmonics in excess of 1.8 kHz.
3 Unlike conventional cycloconverter the proposed configuration does not require any centre-tapped windings.

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The proposed control strategy for a non-circulating current cycloconverter offers the following advantages:

1 Typically for a 1kHz input system the control strategy assures that there are no significant harmonics below 2kHz.

2 The strategy is amenable to implementation using on-line microprocessor control i.e. equation (14) can be programmed into the system.