

Long-run Equilibrium Price Targeting

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Abstract

This article describes a characterisation of competitive market behaviour using the concepts of cointegration analysis. It requires all (n) firms to set prices to follow a single stochastic trend (equivalently the vector of n prices should relate to cointegrating rank $n - 1$). This implies that, in the long run, prices are driven by the shocks that impact on all companies, ruling out the possibility that the price set by any one firm is weakly exogenous for the set of cointegrating vectors.

Key Words: Cointegration, Common trend, Competition, Equilibrium Price Adjustment, Stochastic Trend, Weak Exogeneity.

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1 Introduction

In this article we define statistical criteria for determining competitive behaviour from the long-run decomposition of prices. If a market is efficient then arbitrage should eliminate misspricing. Regulatory authorities and firms have exploited tests of stationarity and cointegration to attempt to determine non-competitive behaviour (Forni, 2004, London Economics, 2002). Here, tests for stationary relative prices are seen as a special case of cointegration. When market prices are sufficiently inter-related in the long-run via cointegration, then the market is viewed as having a broad definition or being more competitive. It follows from arbitrage in the long-run that prices should reflect misspricing or the demand and supply shocks across a market.

Here, we generalize the approach outlined by Hendry and Juselius (2001) to the case of multi-product price comparisons for a competitive market with n commodities. It is assumed that all prices are integrated of the same order. In the bivariate case competitive behaviour can often be seen as being consistent with parallel pricing (Buccirossi, 2006 and Forni, 2004) and this proposition might be appropriately tested by determining whether in their natural logarithm (log) price proportions are stationary. Here, n price responses are consistent with competitive behaviour when all prices are $I(1)$, there are $n-1$ cointegrating relationships or a single common trend, and the common trend is driven by a combination of shocks to all n prices. The test of cointegration is a primary test of the proposition that all series are driven by a single common trend and thus a weighted average of the price shocks of all firms, but in the multiproduct case this does not imply parallel pricing (Buccirossi, 2006).

Pure parallel pricing only arises when $n-1$ prices respond to a single price and this price is then weakly exogenous for the vector of cointegrating relationships (Johansen, 1992). In the latter case the price set by one firm defines a stochastic trend and all firms respond to the prices set by that firm. The price that is weakly exogenous responds only to past values of that price and more particularly to the shocks that apply to that firm's price. In this article, the common stochastic trend is not restricted to being generated in the above manner.

in the next section Long-run Equilibrium Price Targeting (LEPT) is defined, followed by a discussion of generic and empirical identification of the cointegrating vectors, then the effect of weak exogeneity is considered and finally conclusions are offered.

2 The Stochastic Trend, Long-run Equilibrium Price Targeting (LEPT) and Cointegration.

Consider a market consisting of n firms. These firms are viewed as being competitive when they all respond to a single common stochastic trend, itself consisting of a linear combination of the vector of shocks to individual firms (ϵ_t). This common trend we refer to as an Equilibrium Price Target (EPT) when

each of the firms responds to it in the same way and the relationship between each firm's price and this trend defines a set of restrictions on the $n - 1$ cointegrating relations (β), sufficient to exactly identify the $n \times (n - 1)$ matrix of cointegrating vectors, β .¹

Competitive firms are viewed as correcting their price behaviour in response to some equilibrium price target. The underlying target to which the competitive firm responds is a weighted average of the vector of all firms prices $x'_t = [p_{1t} \dots p_{nt}]$ and for series that are all $I(1)$ is defined by the non-stationary component of a single common trend.

Let us consider the case where prices have a k^{th} order Vector Error Correction form:

$$\Gamma(L)\Delta x_t = \alpha\beta'x_{t-1} + \epsilon_t$$

where $\Gamma(L) = I - \Gamma_1 L - \Gamma_2 L^2 \dots - \Gamma_{k-1} L^{k-1}$ and we define $\Gamma = -(I - \Gamma_1 - \Gamma_2 \dots - \Gamma_{k-1})$. The following common trends definition of the equilibrium price target derives from Theorem 4.2 in Johansen (1995) that gives rise to a cointegrating rank of $n - 1$.

Definition Let p_t^* be:

$$\begin{aligned} p_t^* = w'x_t = w'Cx_0 + w'C\left(\sum_{i=1}^t \epsilon_i + \mu\right) \\ + w'\alpha(\beta'\alpha)^{-1}\sum_{i=0}^{\infty} (I + \beta'\alpha)^i \beta'(\epsilon_i + \mu). \end{aligned} \quad (1)$$

Where the price weights are $w' = [w_1 \dots w_n]$, $C = \beta_{\perp}(\alpha'_{\perp}\Gamma\beta_{\perp})^{-1}\alpha'_{\perp}$, $\alpha\alpha'_{\perp} = 0$, $\beta'_{\perp}\beta = 0$, x_0 are initial values and μ is the drift. Then for $p_{it} \sim I(1)$, $\forall i = 1, \dots, n$, Long-run Equilibrium Price Targeting (LEPT) implies that:

$$p_{it} - p_t^* \sim I(0).$$

A case of special interest is where the price weights sum to one ($w'\iota = 1$, $\iota' = [1, \dots, 1]$) or prices are homogenous of degree zero. Then:

$$\begin{aligned} p_{it} - p_t^* &= p_{it} - w'x_t \\ &= (w'\iota j_i - w')x_t = w'(\iota j_i - I_n)x_t. \end{aligned}$$

Where j_i is the transpose of the i^{th} unit vector. When $(\iota j_i - I_n) = R_i$ then there are n cointegrating vectors of the form $\beta_{\cdot i} = w'(\iota j_i - I_n)$ that are dependent, when all prices have the same order of integration.

¹We use information that derives from the long-run inter-action of prices, because: we believe that arbitrage is likely to require firms to respond to the forces of competition, and this defines an informationally efficient starting point from which to detect anomalous pricing behaviour. There are alternative measures of competitive behaviour (for example, Froeb and Werden, 1998), but they are informationally burdensome and sensitive to the nature of the uncertainty (Hunter, Ioannidis, Iossa and Skerratt, 2001). This was first proposed by Hunter and Burke (2007).

Here we consider a trivariate system² with $w' = [w_1 \quad w_2 \quad w_3]$ and:

$$\begin{aligned} \beta'_n &= \begin{bmatrix} \beta_{.1} \\ \beta_{.2} \\ \beta_{.3} \end{bmatrix} = \begin{bmatrix} w'R_1 \\ w'R_2 \\ w'R_3 \end{bmatrix} \\ &= \begin{bmatrix} w_2 + w_3 & -w_2 & -w_3 \\ -w_1 & w_1 + w_3 & -w_3 \\ -w_1 & -w_2 & w_1 + w_2 \end{bmatrix}, \end{aligned}$$

where

$$R_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{and} \quad R_3 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

As $\text{rank}(\beta'_n) < n$ we consider $n - 1$ cointegrating vectors:

$$\beta' = \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \end{bmatrix} = \begin{bmatrix} w_2 + w_3 & -w_2 & -w_3 \\ -w_1 & w_1 + w_3 & -w_3 \end{bmatrix}_3.$$

The system requires a normalization to aid identification and the finding of parallel pricing may be sensitive to the normalization. Next identification is considered.

3 Empirical and Generic Identification

In general the unrestricted cointegrating relationships are not identified. First generic identification is considered (Johansen, 1995) and this is followed by empirical identification (Boswijk, 1996). This is often something ignored by practitioners, but an order condition can be determined by comparison of the restricted and unrestricted forms of β , and in the case of LEPT this gives rise to:

$$r^2 = (n - 1)^2 = (n - 1)(n - 2) + n - 1, \quad (2)$$

restrictions that are necessary and sufficient to identify β . Firstly, economic theory suggests $n - 1 = 2$ price homogeneity restrictions⁴ that fix the first column of β' :

$$\beta_{11} + \beta_{21} + \beta_{31} = 0 \quad \text{and} \quad \beta_{12} + \beta_{22} + \beta_{32} = 0.$$

²The n variable case can be easily imputed from the trivariate case.

³If one considers the inverse of $\begin{bmatrix} w_2 + w_3 & -w_2 \\ -w_1 & w_1 + w_3 \end{bmatrix}$ and applies it to each of the columns in β' , then this gives rise to $\beta' = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ subject to an appropriate normalization.

⁴Notice, that price homogeneity is a long-run property of LEPT. This means that in the short-run agents may mistake relative and absolute price movements. However, long-run pricing that does not satisfy this property would not appear to be consistent with competitive behaviour.

Secondly there are $(n-1)(n-2) = 2$ restrictions that fix $n-2 = 1$ elements in the remaining $n-1 = 2$ rows:

$$\beta_{31} - \beta_{32} = 0 \text{ and } \beta_{22} - \beta_{21} - 1 = 0.$$

Generic identification (see Burke and Hunter, 2005, Chapter 5) follows, because LEPT imposes just enough restrictions to satisfy the order condition (2). Now the formulae above can be used to solve $r^2 = 4$ equations in terms of $n-1 = 2$ identified parameters:

$$\begin{aligned} \beta_{32} &= \beta_{31} = -w_3 \\ \beta_{22} &= \beta_{21} + 1 = 1 - w_2 \\ \beta_{11} &= -\beta_{21} - \beta_{31} = w_2 + w_3 \\ \beta_{12} &= -\beta_{22} - \beta_{32} = -\beta_{21} - 1 - \beta_{31} = w_2 + w_3 - 1 = -w_1. \end{aligned}$$

Although, the above criterion are necessary and sufficient for generic identification, for empirical identification we require $\beta_{21} \neq 0$ and $\beta_{31} \neq 0$.

There are a number of different ways by which both α and β can be identified, Burke and Hunter (2005) present a sufficient condition for the generic identification that is implicit in being able to solve for the structural parameters from a long-run reduced form:

$$\beta' = \begin{bmatrix} 1 & 0 & \beta_{31} \\ 0 & 1 & \beta_{32} \end{bmatrix}.$$

This parameterization of β' is termed a Normalization Rule by Boswijk (1996) and it also implies the imposition of r^2 exactly identifying restrictions. Consider, an orientation that operates on the first two columns of β' :

$$B_{1,2} = \begin{bmatrix} w_2 + w_3 & -w_2 \\ -w_1 & w_1 + w_3 \end{bmatrix}.$$

A necessary condition for the long-run reduced form to exist is:

$$\det(B_{1,2}) = \det \left(\begin{bmatrix} w_2 + w_3 & -w_2 \\ -w_1 & w_1 + w_3 \end{bmatrix} \right) = (w_1 + w_2 + w_3)w_3 \neq 0.$$

However, empirical identification according to Theorem 3 in Boswijk (1996) implies that identification is not sensitive to the columns selected to generically identify β . This implies for the normalization associated with columns i and j :

$$\beta'_{ij} = \left[I_{n-1} \quad B_{i,j}^{-1} b_{\neq i,j} \right],$$

where I_{n-1} is an $n-1 \times n-1$ identity matrix and $b_{\neq i,j}$ the unrestricted vector of parameters related to the remaining price. If $b_{\neq i,j} = 0$, then one of the prices is long-run excluded and for the case considered here this would contradict the notion that all the series are $I(1)$ as for $r = n-1$, $\beta'_{ij} = \left[I_{n-1} \quad 0 \right]$ and therefore the remaining $n-1$ series are $I(0)$.

Hence for generic and empirical identification of β' based on the normalization rule of Boswijk, then in the trivariate case with $i = 1$ and $j = 2$, we require an ordering that gives rise to:

$$\det(B_{1,2}) = \det \left(\begin{bmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{bmatrix} \right) = \beta_{11}\beta_{22} - \beta_{12}\beta_{21} \neq 0$$

and

$$b_{\neq 1,2} = \begin{bmatrix} \beta_{31} \\ \beta_{32} \end{bmatrix} \neq 0.$$

In our case empirical identification follows when $\beta_{31} \neq 0$ and $\beta_{12} \neq 0$ and this is consistent with Theorem 2 and 3 in Boswijk (1996). Firstly, when $\beta_{31} \neq 0$, theorem 2 must hold as:

$$b'_{\neq 12} = [\beta_{31} \quad \beta_{32}] = [\beta_{31} \quad \beta_{31}] \neq 0.$$

Secondly, Theorem 3 is satisfied when $\det(B_{1,1}) = (w_1 + w_2 + w_3)w_3 \neq 0$ and this is obtained from LEPT as $w_3 \neq 0$ when $\beta_{31} \neq 0$ and $(w_1 + w_2 + w_3) = -\beta_{21} - \beta_{31} + \beta_{21} + 1 + \beta_{31} \neq 0$. It is important to point out that identification may be sensitive to the ordering of the system and this may occur, because the loadings on the common trend depend on the impact that shocks to that company price have on the market. Further, LEPT can be linked back to a number of normalized long-run reduced forms, but the restrictions that give rise to LEPT do not apriori fix the long-run to be:

$$\beta'_{ij} = [I_{n-1} \quad -\iota].$$

More specifically, when $\beta = \beta_{ij}$ this implies two further over-identifying restrictions not required for LEPT to hold, though LEPT might imply them.

4 Common Trends, Competition and Exogeneity

Here, the notion of weak exogeneity is used to distinguish between market inefficiencies, and a dominant market share and the pure arbitrage case. The notion that weak exogeneity in a single price gives rise to a particular structure for the stochastic trend is expressed for the VAR(1) case (see Hunter and Burke, 2007). In general, where one of the prices is weakly exogenous, then this drives the stochastic trend (Kurita, 2008).

To draw out the key aspects of the concept, consider the special case of the first order VECM ($k = 1$) and $w = \alpha_{\perp}$, so that $C = \beta_{\perp}(\alpha'_{\perp}\beta_{\perp})^{-1}\alpha'_{\perp}$, $\alpha'_{\perp}\alpha = 0$.⁵

⁵In the first order VECM case when the initial conditions are removed empirically using a procedure, such as that described by Taylor(1999), then the common trend is a weighted average of the prices. More generally, this does not hold though the non-stationarity in the price series is still driven by $\alpha'_{\perp}(\sum_{i=1}^t \epsilon_i)$.

Now we can isolate the trend component by multiplying (1) by α'_\perp . Therefore:

$$\alpha'_\perp x_t = \alpha'_\perp x_0 + \alpha'_\perp \left(\sum_{i=1}^t \epsilon_i + \mu \right) = \alpha'_\perp \sum_{i=1}^t \epsilon_i,$$

when the initial condition is set to zero.

From the definition of LEPT, for a broad market⁶ all series must follow the same order of integration otherwise different market segments may respond to different trends as $\text{rank}(\Pi) \leq n - 2$. However, this type of relation is only consistent with competitive behaviour when the cointegrating relations depend on the shocks that impact on all the prices or we preclude the case where, by any simple re-ordering, $\alpha'_\perp = [0 \ 0 \ \alpha_{3\perp}]$. More specifically the identifying cointegrating combination negates the possibility that $n - 1$ prices depend exactly on a single price; this is the case where one of the prices is long-run weakly exogenous and $n - 1$ prices react to the n^{th} price. If there are $n - 1$ cointegrating vectors and α is an $n \times r$ matrix of loadings, then it follows from Johansen (1992) that for WE of a variable for the parameters of interest (β), a row of α is set to zero. With $\text{rank}(\alpha) = n - 1$, then only one price can be weakly exogenous as otherwise $\text{rank}(\alpha) = r < n - 1$ and there is more than one common trend.

Now consider the case where there is a single common trend and a single weakly exogenous variable and $w = \alpha_\perp$. As a result the following Theorem applies.

Theorem $\text{rank}(\beta) = \text{rank}(\Pi) = n - 1$ and there exists $\alpha' = [\alpha'_{n-1} \ 0]$ for some ordering of the p_i , $i = 1, \dots, n$, that implies a broad market as all prices interact, but there is non-competitive behaviour as p_i for $i = 1, \dots, n - 1$ follow p_n .

Proof. In general, $\alpha'_\perp = [\alpha_{1\perp} \ \alpha_{2\perp} \ \dots \ \alpha_{n\perp}]$ and the common trend drives all prices:

$$p_t^* = w' x_t = \alpha'_\perp x_t = \alpha'_\perp x_0 + \alpha'_\perp \left(\sum_{i=1}^t \epsilon_i + \mu \right).$$

For WE $\alpha' = [\alpha'_{n-1} \ 0]$, α_{n-1} is an $n - 1 \times n - 1$ sub-matrix of loadings and:

$$\alpha'_\perp = [0 \ 0 \ \dots \ \alpha_{n\perp}].$$

Therefore:

$$[0 \ \dots \ \alpha_{n\perp}] \begin{bmatrix} p_{1t} \\ \vdots \\ p_{nt} \end{bmatrix} = \alpha_{n\perp} p_{nt} = \alpha_{n\perp} p_{n0} + \alpha_{n\perp} \left(\sum_{i=1}^t \epsilon_{ni} + \mu_n \right).^7$$

⁶The term broad market is used by Forni(2004) to consider cases where all prices in a market or market segment interact.

⁷In the case of the k^{th} order VECM:

$$p_t^* = w' x_t = \alpha'_\perp x_t = \alpha'_\perp x_0 + \tilde{\beta} \alpha'_\perp \left(\sum_{i=1}^t \epsilon_i + \mu \right) \text{ and } \tilde{\beta} = \alpha'_\perp \beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1}.$$

In the trivariate case under the restrictions implied by LEPT:

$$\beta' = \begin{bmatrix} \alpha_{3\perp} & 0 & -\alpha_{3\perp} \\ 0 & \alpha_{3\perp} & -\alpha_{3\perp} \end{bmatrix}$$

and:

$$\beta' x_t = \begin{bmatrix} \alpha_{3\perp} & 0 & -\alpha_{3\perp} \\ 0 & \alpha_{3\perp} & -\alpha_{3\perp} \end{bmatrix} \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} = \begin{bmatrix} \alpha_{3\perp}(p_{1t} - p_{3t}) \\ \alpha_{3\perp}(p_{2t} - p_{3t}) \end{bmatrix}.$$

With price homogeneity, $\alpha'_{\perp} \iota = 1$ and so $\alpha_{3\perp} = 1$. By induction for the n variable case, $p_t^* = p_{nt}$ and all prices are driven by the stochastic behaviour that underlies p_{nt} . ■

When all firms prices are conditioned on p_{nt} , then firm n is the long-run price leader and LEPT implies:

$$\beta = [I_{n-1} \quad -\iota_{n-1}] \text{ and } \iota'_{n-1} = [1 \quad \dots \quad 1].$$

Therefore, we have a broad market in the sense that firms follow the common trend, but when the common trend is driven by a single firm without reference to other firms or more pertinently without reference to the direct shocks associated with miss-pricing by these other firms, then the firm must hold a dominant position in the market place or that firm must define the barometer to which all other firms respond. However, the notion of a barometric price implies that reference must be made to the behaviour of all the other firms prices and so the barometric case should not be distinguishable from the case of long-run competitiveness

It follows, when one firms price (here firm n) is weakly exogenous for the parameters of interest, that the n^{th} firms price can be viewed as driving all the other firms prices. This, we would argue is a form of price leadership as the long-run is conditioned only on the behaviour of the n^{th} firm price. In this case, under the restrictions associated with LEPT all firms respond to those of the n^{th} firm, but in the long-run the n^{th} firm does not respond to any of the other firms prices. Hence, although there are $n - 1$ long-run price relations and β satisfies the restrictions associated with LEPT, this is not a competitive case. Hence, for competitive behaviour, we have a further requirement that the common trend is not defined by a single firms price or that none of the firms prices are weakly exogenous for β .

A number of side issues arise from $rank(\Pi) < n - 1$, there being at least two common trends. Firstly, individual prices may follow linear combinations of the common trends that happen to be different. In this case, one trend may eventually come to dominate. Secondly, the market may be partitioned, so a block of firms follow one price and another group responds to one or both prices. A case of some interest arises when Π is block triangular and this occurs when we have cointegrating exogeneity (Hunter, 1990). This structure links nicely with an earlier literature on Granger causality (LECG, 1999). If firms follow

different linear combinations of the common trends, this may not be consistent with equilibrium in the very long-run as such a divergence of prices is likely in the end to lead to death or dominance.

5 Conclusion

In this article we considered the conditions required for competitive behaviour using cointegration analysis. We argue that pricing is consistent with competitive behaviour when: i) there are $n - 1$ cointegrating relationships, ii) the restrictions associated with LEPT are satisfied, iii) non of the price series are WE. Beyond the bivariate case the restrictions associated with LEPT are not in general simple price or log price differentials often applied in the literature. This has the implication that tests of stationarity applied to price differentials will not generally be appropriate when $n > 2$.

Here, the argument is driven by consideration of a single price, this may be pertinent when a number of producers supply a fairly homogenous single commodity, petrol or gas. When the multi-product nature of firm competition is important, then we believe that this analysis can be extended to this case via panel cointegration. We can also allow for shifting short-run dynamics in a similar manner to Kurita and Nielsen (2009) and long memory processes with fractional cointegration (Robinson, 2006).

6 References

- Boswijk, H.P. (1996). 'Cointegration, identification and exogeneity: inference in structural error correction models', *Journal of Business and Economic Statistics*, 14, 153-160.
- Buccirosi, P. (2006) 'Does parallel behavior provide some evidence of collusion?', *Review of Law and Economics*, 2, 85-101.
- Burke, S.P. and J. Hunter. (2005). 'Modelling Non-stationary Economic Time Series: A Multivariate Approach', Palgrave, Basingstoke.
- Forni, M., (2004). 'Using stationarity tests in antitrust market definition', *American Law and Economics Review*, 6, 441-464.
- Froeb, L.M. and G.J. Werden. (1998). A robust test for consumer welfare enhancing mergers among sellers of homogeneous products', *Economics Letters*, 58, 367-369.
- Hendry, D.F. and K. Juselius. (2001). 'Explaining cointegration analysis: Part II', *The Energy Journal*; 22, 75-120.
- Hunter, J. (1990). 'Cointegrating Exogeneity', *Economics Letters*, 34, 33-35.
- Hunter, J. (2003). 'A Critical Report on the London Economics Study of Sectoral Price Inter-dependence', prepared for KPN Mobile, the Netherlands and submitted to the Nederlandse Mededingingsautoriteit, August 2003.
- Hunter J and Burke, S.P (2007), Common trends, Cointegration and Competitive Price Behaviour, paper presented at the RES conference, Warwick, 2007

and the Econometrics Study Group Conference, Bristol, June, 2007.

Hunter, J., Ioannidis, C., Iossa, E., and L. Skerratt. (2001). 'Measuring, Consumer Detriment Under Conditions of Imperfect Information', Office of Fair Trading, Economic Research Paper 21.

Johansen, S. (1992). 'Testing weak exogeneity and the order of cointegration in UK money demand data', *Journal of Policy Modeling*, Special Issue: Cointegration, Exogeneity and Policy Analysis, 14, 313-334.

Kurita, T. (2008) Common stochastic trends and long-run price leadership in the US gasoline market. Working paper WP-2008-003. Faculty of Economics, Fukuoka University

Kurita, T. and B. Nielsen. (2009). 'Cointegrated Vector Autoregressive Models with Adjusted Short-Run Dynamics', *QASS*, Vol. 3 (3), 2009, 43-77.

LECg Ltd., (1999), Quantitative techniques in competition analysis, Research paper 17, OFT 266

London Economics. (2002). 'Study of Sectoral Price Inter-dependence', paper prepared for the Nederlandse Mededingingsautoriteit.

Robinson, P.M. (2006). 'Multiple Local Whittle Estimation in Stationary Systems', mimeo, London School of Economics.

Taylor, A.R.M. (1999). 'Recursive Mean Adjustment to Tests of the Seasonal Unit Root Hypothesis', Birmingham University Discussion paper, 99-11.