#### Thesis

## Supply Chain Network Design under Uncertainty and Risk

Submitted for the Degree of Doctor of Philosophy

by

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## **Abstract**

We consider the research problem of quantitative support for decision making in supply chain network design (SCND). We first identify the requirements for a comprehensive SCND as (i) a methodology to select uncertainties, (ii) a stochastic optimisation model, and (iii) an appropriate solution algorithm. We propose a process to select a manageable number of uncertainties to be included in a stochastic program for SCND. We develop a comprehensive two-stage stochastic program for SCND that includes uncertainty in demand, currency exchange rates, labour costs, productivity, supplier costs, and transport costs. Also, we consider conditional value at risk (CV@R) to explore the trade-off between risk and return. We use a scenario generator based on moment matching to represent the multivariate uncertainty. The resulting stochastic integer program is computationally challenging and we propose a novel iterative solution algorithm called adaptive scenario refinement (ASR) to process the problem. We describe the rationale underlying ASR, validate it for a set of benchmark problems, and discuss the benefits of the algorithm applied to our SCND problem. Finally, we demonstrate the benefits of the proposed model in a case study and show that multiple sources of uncertainty and risk are important to consider in the SCND. Whereas in the literature most research is on demand uncertainty, our study suggests that exchange rate uncertainty is more important for the choice of optimal supply chain strategies in international production networks. The SCND model and the use of the coherent downside risk measure in the stochastic program are innovative and novel; these and the ASR solution algorithm taken together make contributions to knowledge.

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## List of Abbreviations

**ASR** adaptive scenario refinement

**BoM** bill of material

BRL Brazilian Real

**CNY** Chinese Yuan

CV@R conditional value at risk

**DC** distribution centre

**DEM** deterministic equivalent model

**DEX** deterministic equivalent problem with explicit non-anticipativity constraints

**EDR** expected downside risk

**EP** expected profit

**EUR** Euro

**EV** expected value

**EVPI** expected value of perfect information

**F** facility

**HN** here-and-now

LSW stochastic programming test cases by Linderoth et al. (2001)

M market

MIP mixed integer program

**NPV** net present value

P product

**PF** probability of failure

**PT** power train

**s** supplier

**SAA** sample average approximation

**SCND** supply chain network design

**SCENGEN** scenario generation

**SIP** stochastic integer program

**SIPLIB** stochastic integer programming test problem library

**SLP** test-problem collection for stochastic linear programming

**SCENRED** scenario reduction

**SP** stochastic program

USD US Dollar

**V@R** value at risk

Var variance

**VMS** value of the multi-stage stochastic solution

**VSP** value of stochastic programming

ws wait-and-see

## 1. Introduction

#### 1.1. Problem Context and Motivation

Responding to changing environments of economy and commerce, as well as the impact of geopolitics and disruptive technologies, manufacturing companies have to embark upon strategic planning, which involves supply chain network design (SCND). In particular in the automotive industry, numerous factors require companies to analyse and improve their supply chain strategies, for example the rise of the Asian markets, new product introductions like hybrid and electric cars, mergers and acquisitions, fluctuating currency exchange rates, and increasingly volatile fuel costs. Further, modern manufacturing supply chains face customers with demand for increasing product variety, shorter product life cycles, lower cost, better quality, and faster response. To be successful in increasingly globalised and competitive markets, companies must constantly strive to reduce supply chain costs and improve customer service while planing for the unexpected. (Vonderembse et al., 2006; IBM, 2009).

Supply chain network design (SCND) is the strategic planning of supply chains, concerned with the number, location, and capacity of facilities and distribution centres, production technology to be employed at each facility, supplier selection, make-or-buy decisions, and the design of the transportation network (Shapiro, 2007). SCND determines the technology, process, and manufacturing assets for a company over the future years, in which it needs to fulfill the customer demands while remaining competitive. Further, the strategic decisions typically involve high investment costs and are not easily reversible. Therefore, SCND is crucial to the long term success of any manufacturing company. Four key aspects of the problem are (i) the topology of the supply chain, (ii) the timing of the decisions, (iii) consideration of uncertainty due to the stochasticity of input data, and (iv) the aim to maximise profit (Alonso-Ayuso et al., 2005b). To support managers at making decisions in this highly complex problem, quantitative, data-driven models are needed (Shapiro, 2007).

In this thesis, we study the scope of quantitative decision support in the SCND. We focus on SCND for manufacturing supply chains, especially in the automotive industry. In the remainder of this chapter, we pose research questions about SCND and set out the contents of this thesis.

#### 1.2. Research Questions

Set against the background presented in section 1.1, in this thesis we develop a methodology to support quantitative decision making in SCND and address the following research question:

Q1 How can we support decision making in SCND with a quantitative methodology?

To refine this question, we first need to specify the requirements for such a methodology:

Q2 What are the methodological requirements for quantitative decision support in SCND?

By answering this question, we identify three aspects of SCND that need further investigation: (i) uncertainty identification, (ii) SCND optimisation models, and (iii) solution algorithms. It is well accepted that uncertainties play a crucial role in the design of supply chains. However, there are numerous uncertainties that might have a possible effect on a production network, an issue addressed by the following question:

Q3 Which uncertainties need to be considered in SCND and how do we identify them?

Given the uncertainties, we need a model to support the decision making, which is addressed in the following research question:

Q4 What is an appropriate way to model the SCND problem?

We claim that the answer to this is a stochastic optimisation model, which – due to its complexity – requires an appropriate solution algorithm, investigated in the following question:

**Q5** How do we solve the SCND optimisation problem?

By providing answers in the form of models and solution algorithms to the series of these research questions Q2 - Q5 we develop a comprehensive methodology for decision support in SCND.

#### 1.3. Outline of the Thesis

The contents of this thesis is organised in the following way.

In chapter 2 we investigate the methodological requirements for quantitative decision support in SCND. To set the relevant background and refine the research questions, we first define SCND and introduce concepts of decision making under uncertainty. Then, we identify three key issues, based on the previous definitions and the related literature. (i) We find that uncertainty and risk play an important role and that we need

a methodology to select uncertainties that are included in the subsequent model. (ii) We suggest to employ a holistic optimisation model, including comprehensive strategic and tactical stages, detailed cost calculation in line with accountancy standards, and the consideration of multiple sources of uncertainty and risk. (iii) We set out the need for a solution approach that can deal with the resulting large scale optimisation problem but is still flexible enough to cope with evolving model structures in varying real world applications.

In chapter 3 we investigate how one can identify a set of relevant uncertainties for a SCND project, manageable in an optimisation model. First, we set the relevant background by introducing the concept of risk management in supply chains. Then, we review the related literature and deduce that while many concepts are related, no suitable methodology exists. Hence, we propose a new process for uncertainty selection based on the principle ideas of risk management. This is a structured, qualitative approach which aims to objectively capture expert opinion. For a case study we select six sources of uncertainty for an international production network in the automotive industry: uncertainty in demand, productivity, transport costs, labour costs, raw material costs, and exchange rates.

In chapter 4 we investigate the modeling of SCND under uncertainty. We give an extensive review of the literature and find that, although many of the features identified in chapter 2 appear in some research, no comprehensive study covering them altogether is reported. Therefore, we propose a two-stage stochastic program which includes the identified uncertainties, a holistic network structure, and detailed operations, especially in regards to exploiting flexibility. Since the decisions are made under uncertainty, the profit for a given network design is itself uncertain. To explore the resulting tradeoff between risk and return, we take into account conditional value at risk (CV@R) as a coherent downside risk measure and measure return as expected profit (EP). The two key features of the optimisation model are the consideration of multiple sources of uncertainty and the inclusion of the risk measure to explore the trade-off between risk and return. Further, we propose a scenario generator to represent the uncertainty. This is an extension of a moment matching scenario generator by Høyland et al. (2003) through a combination with sampling. Thereby we generate multi time period scenarios with the number of scenarios being constant over time, while capturing the correlation structure of the multivariate uncertainty.

In chapter 5, we investigate solution methodologies suitable for the proposed model, which is a stochastic integer program (SIP) with a large number of scenarios. We review existing algorithms and conclude that the problem can only be addressed by approximation heuristics. Among the reviewed algorithms, two are suitable: (i) a scenario reduction method by Heitsch and Römisch (2007) and (ii) using the scenario generator to produce a small scenario set for ex-ante decision making and a large set for

ex-post evaluation. Still, we propose a new solution heuristic called adaptive scenario refinement (ASR), that iteratively adds scenarios based on a maximum regret criterion. This is implemented in the FortSP stochastic programming solver system (see Ellison et al., 2010). We carry out an empirical study comparing the solution quality of the above approximation methods on publicly available linear and mixed integer test cases for stochastic programming, as well as on the SCND model.

Having developed a comprehensive methodology for SCND in the previous chapters, we apply it to a case study in chapter 6. The case study network is an international production network in the automotive industry with suppliers and facilities in Europe, Brazil, and China, and markets in Europe, Brazil, China, and the United States. The focus of the investigation is to analyse the effects and benefits of the key features of the optimisation model, that is, multiple sources of uncertainty and the CV@R risk measure. This is undertaken in three steps: (i) We explore the trade-off between risk and return and give insights into successful supply chain strategies. (ii) We use an exante decision making / ex-post evaluation approach to analyse the benefits from taking multiple uncertainties into account, in contrast to planning under single uncertainties. (iii) We investigate the effects of various uncertainties on given network structures.

Finally, we summarise the findings reported in this thesis and present our conclusions in chapter 7.

# 2. Methodological Requirements for Supply Chain Network Design (SCND)

In this chapter we investigate the relevant requirements for comprehensive SCND. This defines the structure of the thesis, as we expand on the identified topics in subsequent chapters. To set out the relevant background, we define 'supply chain network design' and related terms in section 2.1 and introduce the concepts of decision making under uncertainty in section 2.2. This enables us to identify the methodological requirements in section 2.3.

#### 2.1. SCND – Scope and Definition

#### 2.1.1. Supply Chain Management

Following the works of Mentzer et al. (2001) and Aitken (1998, in Christopher 2011), we define a *supply chain* as a network of connected and interdependent organisations, directly involved in the upstream and downstream flows of products, services, finances, and information from sources to customers. These organisations include manufacturers, suppliers, transporters, warehouses, and retailers (Chopra and Meindl, 2007).

On a more abstract level, a supply chain is a directed graph in which the set of nodes represents organisations or customers; these are connected by directed arcs, and products flow along these arcs. At the nodes, different products at different stages of the production process are bought, transformed, stored, or sold. The goal of the supply chain is to maximise the overall value added to the products as they pass through the network. The products in turn have to be supplied in required quantities, achieving a specified quality, at a competitive cost, and in a timely fashion. (Chopra and Meindl, 2007; Shapiro, 2007). In a typical supply chain, four types of nodes are identified: at the supplier nodes (S) raw materials or intermediate products are acquired; at facility nodes (F) manufacturing takes place, that is, physical product transformations; at distribution centre nodes (DC) intermediate operations such as sorting, storage, packaging, and dispatching take place, but no physical transformation (Shapiro, 2007); finally, at market nodes (M) products are sold to customers. A typical supply chain is set out in figure 2.1 (Shapiro, 2007).

Supply chain management is defined as the integrated planning within the supply

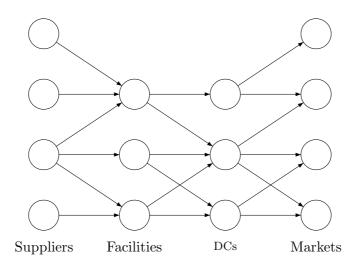


Figure 2.1. A typical supply chain (Shapiro, 2007)

chain across function, space, and time, with the purpose of improving the performance of the individual companies in the supply chain, as well as the entire network (Mentzer et al., 2001; Shapiro, 2007). In this context, functional integration refers to the business functions purchasing, manufacturing, transportation, warehousing, and inventory management. Spatial integration indicates the planning across geographically dispersed nodes in the supply chain, and, thereby, also across multiple businesses. Finally, intertemporal integration, also called *hierarchical planning*, is the integration across strategic, tactical, and operational planning and spanning across different time horizons; this is discussed in the next section.

#### 2.1.2. SCND in the Context of Supply Chain Management

The hierarchical levels of supply chain management are commonly classified as follows (Schmidt and Wilhelm, 2000; Mitra et al., 2006; Chopra and Meindl, 2007; Shapiro, 2007; Fleischmann et al., 2008; Simchi-Levi et al., 2008):

Given a product strategy as well as marketing and pricing plans, strategic supply chain management is concerned with resource acquisition and resource divestment decisions. This includes the number, location, and capacity of facilities and DCs, production technology to be employed at each facility, supplier selection, make-or-buy decisions, and the design of the transportation network. Hence, strategic supply chain decisions define the design of the supply chain and determine the framework in which tactical and operational supply chain management operate. This usually involves high investment costs and the decisions are difficult or expensive to alter on short notice. Strategic supply chain management has a relatively long planning horizon between two and twelve years, with the aim of determining the most effective long-term organisation

of the company's supply chain. This is achieved by maximising economic performance indicators such as return on investment or net revenues. In recent times, in addition to maximising economic performance, the consideration of risk has become an important decision criterion (Mitra, 1988; Chopra and Meindl, 2007; Shapiro, 2007). Strategic supply chain management is also called *supply chain network design (SCND)*.

Tactical supply chain management decides about resource adjustments and allocations on the basis of a fixed supply chain design. It manages how the supply chain is used and how market demand is met over the coming three to twelve months, which includes purchasing decisions, production levels, inventory policies, and transportation strategies. Tactical supply chain management aims to minimise manufacturing, transportation, and inventory holding costs while meeting customer demand.

Operational supply chain management coordinates the supply chain in response to customer demands. Therefore, it includes scheduling and sequencing decisions on a day-to-day basis for production, inventory, and distribution, while the planning policies set out in the tactical phase as well as the supply chain design are fixed. Operational supply chain management aims to minimise short-term production costs while assuring on-time delivery. These decisions are about the effectiveness of the resource utilisation within the supply chain.

To achieve intertemporal integration, a suite of hierarchical models is needed, that is consistent with the strategic, tactical, and operational planning problems faced by a company (Shapiro, 2007; Goetschalckx and Fleischmann, 2008). This is achieved through linking and overlapping the different model classes which address different supply chain activities (Dominguez-Ballesteros, 2001): strategic models about resource acquisition and divestments give input to and constrain aggregate tactical models that decide how to use these resources. Conversely, to evaluate a supply chain, operations carried out under a proposed design must at least approximately be anticipated in a strategic model (Shapiro, 2007). In the same way, detailed operational models are linked with more aggregate tactical models.

Within strategic supply chain management, the decision process covers four phases (Chopra and Meindl, 2007): (i) A supply chain strategy is defined, which includes a market strategy, the stages in the supply chain, and functions to be outsourced. (ii) Regions for the facilities are identified, with the potential role and approximate capacity of each facility. (iii) Desirable potential facility sites within each region are selected. (iv) The exact location and capacity of each facility is defined. To suit the different phases of the strategic decision process, there is a need for different models with varying detail and focus. As in the case of hierarchical integration, these need to be linked and overlapping, such that they can support progressive decision-making. A first step is to identify a set of potential new facility locations as well as the relevant existing facilities. Next, based on these facilities, first ideas for strategies are identified,

which might include the countries, in which to produce, a broad estimate of production capacities, and key suppliers. For the more promising strategies and facilities, a more detailed analysis is done which might include prices for machinery at different capacities, costs of building facilities and buying land, stock levels and warehouse capacities, as well as increased tactical detail. This follows the order of the planning process, starting from a management idea and leading to contracts with machinery suppliers, hiring new staff, and finally the implemented strategy.

Due to the long planning horizon and the difficulty to alter a network design on short notice, strategic supply chain decisions face a high exposure to uncertainty (Chopra and Meindl, 2007), while the extent and possible impact of uncertainties is lower for tactical planning, and still lower for operations (Mitra, 1988; Shapiro, 2007). Therefore, to optimise performance, companies need to build flexibility into the supply chain design in the strategic phase and exploit it in the tactical phase (Chopra and Meindl, 2007). As flexibility and uncertainty are closely connected in SCND, we next define flexibility in the context of manufacturing and subsequently summarise the principles of decision making under uncertainty.

#### 2.1.3. Manufacturing Flexibility

Flexibility is defined as 'the ability to change or react with little penalty in time, effort, cost, or performance' (Upton, 1994). In the context of manufacturing, this means 'being able to reconfigure manufacturing resources so as to efficiently produce different products of acceptable quality' (Sethi and Sethi, 1990). Besides environmental uncertainty, that is, the need to react to unexpected events, the need for flexibility arises due to variability in the products and processes (Toni and Tonchia, 1998). Manufacturing flexibility is further broken down into different dimensions, as set out in Sethi and Sethi (1990) and shown in table 2.1. Clearly, many of these dimensions are strongly influenced by the supply chain design, especially process, routing, product, and volume flexibility, as the network design defines the capacity and type of production facilities, as well as the assignment of products to facilities.

### 2.2. Decision Making under Uncertainty

In this section we introduce the basic principles of decision making under uncertainty. In section 2.2.1 we define the terms uncertainty, risk, and decision problem under uncertainty, with the related concept of decision trees. Risk preferences and risk measures are discussed in section 2.2.2 and stochastic programming is introduced in section 2.2.3.

Flexibility	Definition
Machine	The various types of operations that a machine can perform without requiring a prohibitive effort in switching from one operation to another.
Material Handling	The ability of a material handling system to move different product types efficiently for proper positioning and processing through the manufacturing facility it serves.
Operation	The ability of a product to be produced in different ways.
Process	The set of product types that manufacturing system can produce without major setups.
Routing	The ability of a manufacturing system to produce a product by alternate routes through the system.
Product	The ease with which new products can be added or substituted for existing products.
Volume	The ability of a manufacturing system to be operated profitably at different overall output levels.
Expansion	The ease with which the capacity and capability of a manufacturing system can be increased when needed.
Program	The ability of the system to run virtually untended for a long enough period.
Production	The universe of product types that the manufacturing system can produce without adding major capital equipment.
Market	The ease with which the manufacturing system can adapt to a changing market environment.

 $\textbf{Table 2.1.} \ \ \text{Dimensions of manufacturing flexibility (Sethi and Sethi, 1990)}$ 

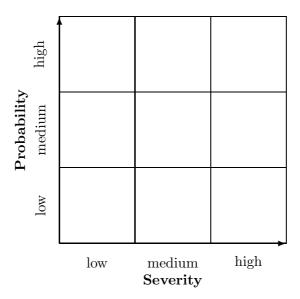


Figure 2.2. Risk matrix (adapted from Norrman and Jansson, 2004)

#### 2.2.1. Uncertainty and Risk

The terms uncertainty and risk have different meanings in a number of academic fields as well as the general public. While in economics these terms are sometimes used to distinguish between unknown outcomes with or without a known underlying probability distribution (see for example Knight, 2002), we take the more intuitive approach as found in dictionaries, finance, and classical risk management:

Decisions are made under *certainty* when perfect information is available and under *uncertainty* when one has only partial (or imperfect) information. (French, 1995; Zimmermann, 2000; Roy, 2005; Stewart, 2005). The term *uncertain* under this paradigm is value neutral, i.e. it includes the chance of gain and, conversely, the chance of damage or loss. As explained by Stewart (2005), uncertainty leads to *risk* which is the possibility that undesirable outcomes could occur. (Klibi et al., 2010).

Another perspective is given by Kaplan and Garrick (1981) who set out:

$$risk = uncertainty + damage$$

As we are concerned with quantitative decision making, we assume that we can identify uncertainty with a random variable with known probability distribution on an appropriate probability space, while risk is the possibility of loss or injury, together with the probability and severity of such loss (Kaplan and Garrick, 1981). Risk is often illustrated in a *risk matrix*, as shown in figure 2.2 (Norrman and Jansson, 2004). Note that risk is a subjective concept that depends on a decision maker's information, goals,

personal preferences, as well as the decision itself. For example, high market demand might be positive for a company if its supply chain has the ability to fulfill it, and negative otherwise.

A common way to describe decision problems under uncertainty are *decision trees*, represented by three types of nodes (Shapiro, 2007; Brandimarte, 2011):

**State nodes** correspond to the state of a system at points in time, just before decisions have to be made. At these nodes, the decision maker has to choose between mutually exclusive alternatives. State nodes are represented by squares.

Chance nodes correspond to points in time when random events occur. They are represented by circles and probabilities are denoted next to the arcs leaving the node. We assume that the realisation of the random event is independent from the decisions at the previous nodes, as needed in stochastic programming (see Kall and Wallace, 1994).

**Terminal nodes** are state nodes without decisions and mark the end of the decision process, whose gain is calculated at this stage. Terminal nodes are represented by bullets.

Usually, the root of a decision tree is a state node (otherwise the problem can be disaggregated into multiple smaller decision problems) and state nodes alternate with chance nodes (otherwise nodes can be aggregated). A typical decision tree is shown in figure 2.3.

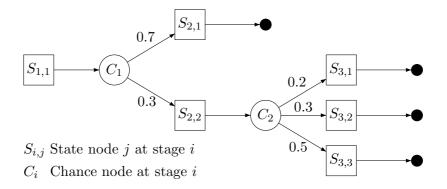


Figure 2.3. A sample decision tree

The decision problem under uncertainty is for the decision maker to choose one of the alternatives at each state node in accordance with his goals. The fundamental concept of decision problems under uncertainty is, that, at each state node the decision maker has to make a decision, before he knows what the realisation of uncertainty at the subsequent chance node is (if there is any). Possible criteria on which a decision maker might base these decisions are discussed in the following section. The maximum number of decision nodes in a path from the root node to a decision node is called the *stage* of the decision node and decisions on stage one are also called *here-and-now* (HN) decisions.

#### 2.2.2. Risk Aversion and Risk Measures

For our discussion of decision making under uncertainty, we take the economic point of view that our goal is to maximise profit, measured by some suitable financial performance indicator. In this situation, the profit or return following a given decision becomes a random variable itself (see Kall and Wallace, 1994) and we need a methodology by which we choose one random variable over another. An approach to this, widely used in economics is 'utility theory' (see for example Ingersoll, 1987), and related to this approach is 'stochastic dominance' (see for example Stewart, 2005; Gollmer et al., 2008, 2011). An approach, which is more practical in business applications (Eppen et al., 1989; Brandimarte, 2011), is the use of probability functionals; these are discussed next.

#### **Notations and Probability Functionals**

Let  $(\Omega, \mathcal{F}, P)$  be a probability space,  $\mathcal{L}$  a linear space of real valued, measurable functions defined on  $(\Omega, \mathcal{F}, P)$ , and  $\xi \in \mathcal{L}$  be a random variable with distribution function F and – in case  $\xi$  is absolutely continuous – probability density function f.  $\xi$  is interpreted as the profit distribution of a decision and its expected value is the expected profit (EP), calculated as

$$\mathrm{EP}[\xi] = \mathbb{E}[\xi] = \int_{\Omega} \xi dP(\omega) = \int_{\mathbb{R}} x \, dF(x) = \int_{\mathbb{R}} x \, f(x) \, dx.$$

A probability functional  $\mathcal{M}$  is a function  $\mathcal{M}: \mathcal{L} \to \mathbb{R} \cup \{\pm \infty\}$  (Pflug and Römisch, 2007).

#### The Trade-Off between Risk and Return

Modern portfolio theory (Markowitz, 1952) recognises a principal trade-off between risk and return which can be explored by using two probability functionals: a return measure, which is the expected profit, and a risk measure, which indicates the riskiness of the random variable, with large values representing high risk and low values low risk. Thereby, the decision problem under uncertainty becomes a bi-objective optimisation problem.

Given a bi-objective optimisation problem

$$\min_{x \in \mathcal{X}} f_i(x), \ i = 1, 2,$$

a solution  $x^* \in \mathcal{X}$  is called *efficient*, if there is no other solution  $\tilde{x} \in \mathcal{X}$ , such that  $f_i(x^*) \geq f_i(\tilde{x}), i = 1, 2$ , with strict inequality for at least one of the two objectives (Brandimarte, 2011). The set of all efficient solutions is called the *efficient frontier*. Among these, a decision maker has to choose according to his *risk preferences*, that is, how much risk he is willing to take for an additional expected return. (Figueira et al., 2005)

Assuming  $\mathcal{X}$ ,  $f_1$ , and  $f_2$  are convex, the efficient frontier can equivalently be calculated using the following optimisation problems by varying the parameters a and b, respectively (Krokhmal et al., 2002). Note that a mixed integer program (MIP) is not convex.

$$\min_{x \in \mathcal{X}} a f_1(x) + (1 - a) f_2(x), \qquad a \in [0, 1],$$
  
$$\min_{x \in \mathcal{X}} f_1(x), \text{ s.th. } f_2(x) \ge b, \qquad b \in \mathbb{R}.$$

Common risk measures used in the trade-off between risk and return are presented next.

#### Risk Measures

To simplify the notation for risk measures and to be in line with the usual interpretation that high values of a risk measure indicate high risk, we define a random variable  $\zeta = -\xi$  on  $(\Omega, \mathcal{F}, P)$ , which is the loss distribution associated with a decision. Also, let G and g denote the distribution function and (if it exists) probability density function for  $\zeta$ , respectively.

Traditionally, risk is measured by variance (Var) (Markowitz, 1952), defined as

$$\operatorname{\mathbb{V}ar}[\zeta] = \mathbb{E}[\zeta^2] - \mathbb{E}[\zeta]^2.$$

In some situations this gives a risk measure in line with our definition of risk. For example, in manufacturing the tolerance can be crucial and deviations from a target value in both directions are disadvantageous. Also, if  $\xi$  is symmetric around its mean, deviations below are equivalent to deviations above, and, therefore, variance is a meaningful risk measure. However, for general loss distributions it fails to capture the idea that high losses are negative and, therefore, pose a risk, while low losses (that is high profits) are not. Therefore, downside risk measures have been developed to account for the decision maker's different attitude towards downside losses as opposed to upside gains (Stulz, 1996; Ang et al., 2006). Further, variance is not a coherent risk measure in the sense of Artzner et al. (1999), that is, it fails to fulfill some properties desirable for risk measures. See appendix A for a discussion of the coherence property and note that the variance is obviously not transitive.

Besides variance, the most common downside risk measure in finance is the value at risk (V@R). For a continuous loss distribution and confidence level  $\beta \in (0,1)$ , the V@R is the  $\beta$ -quantile of the loss distribution (Pflug and Römisch, 2007)

$$V@R^{\beta}[\zeta] = G^{-1}(\beta).$$

For a formal definition that works for general distributions see Acerbi and Tasche (2002). For example, the 90 %-V@R is an upper estimate of losses, which is exceeded with at most 10 % probability. However, V@R is also not a coherent risk measure (see Roman and Mitra, 2009) and therefore, other risk measures have been proposed.

A coherent risk measure (see Acerbi and Tasche, 2002) closely related to the V@R is the conditional value at risk (CV@R). For continuous loss distributions and confidence level  $\beta \in (0,1)$ , the CV@R is defined as the expected losses exceeding the V@R, that is, the expected losses in the worst  $1 - \beta$  cases (Acerbi and Tasche, 2002):

$$CV@R^{\beta}[\zeta] = \frac{1}{1-\beta} \int_{V@R^{\beta}[\zeta]}^{\infty} x g(x) dx.$$

For general distributions a more careful definition is needed, which defines the CV@R as a weighted average of the V@R and profits falling short of V@R, (Acerbi and Tasche, 2002). The CV@R can easily be included in linear optimisation models using a formulation by Rockafellar and Uryasev (2000). Theoretical properties of the CV@R as well as its application in stochastic programs with mixed integer recourse are discussed in Schultz and Tiedemann (2006).

Another downside risk measure, similar to CV@R and often used in stochastic programming formulations, is the *expected downside risk (EDR)*. For a loss distribution and a maximum acceptable loss  $\tilde{x} \in \mathbb{R}$ , the expected downside risk is defined as (Fishburn, 1977)

$$EDR^{\tilde{x}}[\xi] = \mathbb{E}[\max(0, \zeta - \tilde{x})] = \int_{\tilde{x}}^{\infty} (x - \tilde{x}) g(x) dx.$$

Hence, the expected downside risk measures the expected value above the fixed target level  $\tilde{x}$ , while the CV@R measures the expected value above the value at risk. The expected downside risk is easy to include in linear optimisation models, but it is again not a coherent risk measure (see Roman and Mitra, 2009).

Based on a similar idea as expected downside risk is the *probability of failure (PF)* (also called excess probability), which simply measures the probability of exceeding the acceptable loss  $\tilde{x} \in \mathbb{R}$ :

$$PF^{\tilde{x}}[\zeta] = P(\zeta \ge \tilde{x}).$$

Theoretical properties of probability of failure as well as its application in stochastic programs with mixed integer recourse are discussed in Schultz and Tiedemann (2003),

but probability of failure is also not a coherent risk measure (see Roman and Mitra, 2009).

#### 2.2.3. Stochastic Programming

In very general terms, stochastic programming is defined as an optimisation model for decision making under uncertainty. The classical approach is to optimise the expected value. However, the use of expected value optimisation is not necessarily limiting, since all the previously discussed risk measures can be expressed in this setting. Thus, stochastic programming models are well equipped to explore the trade-off between risk and return in decision problems under uncertainty. We are only concerned with two-stage decision processes and linear or mixed integer optimisation problems; hence we restrict ourselves to two-stage stochastic linear programs with recourse in the definition and notations.

Let  $(\Omega, \mathcal{F}, P)$  be a probability space and  $\xi : (\Omega, \mathcal{F}, P) \to (\mathbb{R}^m, \mathcal{B})$  be a random vector with  $m \in \mathbb{N}$  and  $\mathcal{B}$  the Borel  $\sigma$ -algebra on  $\mathbb{R}^m$ . Consequently, the expected value of  $\xi$  is

$$\mathbb{E}_{P}[\xi] = \mathbb{E}_{P}[\xi(\omega)] = \int_{\omega \in \Omega} \xi(\omega) \, dP(\omega) = \sum_{\omega \in \Omega} \xi(\omega) P(\omega),$$

with the last equality being true if  $\Omega$  is finite. The two-stage stochastic linear program with recourse (SP) is defined as (Dantzig, 1955; Kall and Wallace, 1994; Birge and Louveaux, 1997; Ruszczynski and Shapiro, 2003a):

(SP) 
$$z_P^* = \min_{x \in \mathcal{X}} z_P(x) \text{ with } z_P(x) = c^T x + \mathbb{E}_P \left[ R(x, \omega) \right]$$
  
s.th.  $Ax = b$ ,  
 $x > 0$ .

where  $R(x,\omega)$  is the recourse problem

$$R(x,\omega) = R(x,\xi(\omega)) = \min_{y \in \mathcal{V}} \left\{ q(\omega)^T y | W(\omega) y = h(\omega) - T(\omega) x, \ y \ge 0 \right\}.$$

In this definition,  $\xi$  denotes the random components  $\xi = (q, W, h, T)$ , and, for some  $n_1, p_1, n_2, p_2 \in \mathbb{N}$ , the sets  $\mathcal{X} = \mathbb{R}^{n_1 - p_1} \times \mathbb{Z}^{p_1}$  and  $\mathcal{Y} = \mathbb{R}^{n_2 - p_2} \times \mathbb{Z}^{p_2}$  enforce integrality conditions.

(SP) is called relatively complete recourse if

$$|R(x,\omega)| < \infty, \ \forall \omega \in \Omega \text{ and } \forall x \in \mathcal{X} : Ax = b, x \geq 0.$$

That is, the second stage problem  $R(x,\omega)$  has a feasible solution and is bounded, for every random realisation  $\omega$  and for every feasible first stage solution x. If W is

constant, that is,  $W(\omega) = W$  for all  $\omega \in \Omega$ , then the corresponding problem is called fixed recourse.

(SP) can be restated as the so-called *deterministic equivalent model (DEM)*, which is – as the name suggests – an equivalent reformulation of (SP) as a deterministic optimisation problem. This is also the way we present the SCND model in chapter 4.

(DEM) 
$$\min_{x \in \mathcal{X}, y_{\omega} \in \mathcal{Y}, \omega \in \Omega} c^T x + \mathbb{E}_P \left[ q(\omega)^T y_{\omega} \right]$$
s.th.  $Ax = b$ ,
$$T(\omega)x + W(\omega)y_{\omega} = h(\omega), \quad \omega \in \Omega$$

$$x \ge 0, \ y_{\omega} \ge 0, \quad \omega \in \Omega.$$

For some algorithms, it is also useful to consider the deterministic equivalent problem with explicit non-anticipativity constraints (DEX). Here,  $\tilde{\omega} \in \Omega$  is an arbitrary but fixed scenario.

(DEX) 
$$\min_{x_{\omega} \in \mathcal{X}, y_{\omega} \in \mathcal{Y}, \omega \in \Omega} \mathbb{E}_{P} \left[ c^{T} x_{\omega} + q(\omega)^{T} y_{\omega} \right]$$
s.th.  $Ax_{\omega} = b, \quad \omega \in \Omega$ 

$$T(\omega) x_{\omega} + W(\omega) y_{\omega} = h(\omega), \quad \omega \in \Omega$$

$$x_{\omega} - x_{\tilde{\omega}} = 0, \quad \omega \in \Omega \setminus \{\tilde{\omega}\}$$

$$x_{\omega} \geq 0, \ y_{\omega} \geq 0, \quad \omega \in \Omega.$$

#### Minimising the conditional value at risk

Rockafellar and Uryasev (2000) show that the CV@R for a feasible solution  $x \in \mathcal{X}$  of (SP) can be calculated as

$$CV@R(x) = \min_{v \in \mathbb{R}} v + \frac{1}{1 - \beta} \mathbb{E}_{\omega \in \Omega} \left( \left[ c^T x + R(x, \omega) - v \right]^+ \right),$$

where  $[a]^+ = \max(a, 0)$ . In this situation, the set of optimal solutions is a closed interval whose left endpoint is the V@R. Further, we can minimize the CV@R by the following 2-stage stochastic program (see Schultz and Tiedemann, 2006)

$$\min_{x \in \mathcal{X}} \text{CV@R}(x) = \min_{x \in \mathcal{X}, v \in \mathbb{R}} \left\{ v + \frac{1}{1 - \beta} \mathbb{E}_{\omega \in \Omega} \left( \left[ c^T x + R(x, \omega) - v \right]^+ \right) \right\} 
= \min_{x \in \mathcal{X}, v \in \mathbb{R}, \Delta_{\omega} \ge 0} \left\{ v + \frac{1}{1 - \beta} \mathbb{E}_{\omega \in \Omega} \left( \Delta_{\omega} \right) \mid \Delta_{\omega} \ge c^T x + R(x, \omega) - v \right\},$$

with the last equation beeing true for finite  $\Omega$ .

#### 2.3. Requirements

As set out in chapter 1, our goal is to develop a methodology for quantitative decision support for SCND in manufacturing, with a focus on the early phases of the strategic decision process introduced in section 2.1. In the following section requirements for such a methodology are identified. As indicated by Vidal and Goetschalckx (1996, in Schmidt and Wilhelm 2000), some aspects of global logistics are difficult or even impossible to capture in a quantitative model, for example political stability. Following Schmidt and Wilhelm (2000), we limit the discussion to issues that appear to be quantifiable.

#### 2.3.1. Uncertainty Identification

As set out in section 2.1 and emphasised by numerous authors (see for example Bienstock and Shapiro, 1988; Eppen et al., 1989; Escudero et al., 1993; Goetschalckx and Fleischmann, 2008), uncertainty plays a crucial role in SCND. However, the list of potential uncertainties is extensive (see for example Chopra and Meindl, 2007, and chapter 3), which means that only a fraction of them can be considered in any model. Therefore, we adopt ideas from risk management to a qualitative methodology for uncertainty identification based on expert knowledge. This is presented in chapter 3 and used to identify relevant uncertainties, before developing a suitable model.

#### 2.3.2. Modelling Issues

Modelling techniques able to support SCND decision making are divided into two categories: optimisation and pure modelling approaches (Blackhurst et al., 2005). Characteristics of optimisation approaches include linear, (mixed) integer, and nonlinear programming, as well as deterministic and stochastic models. In contrast, pure modelling approaches like simulation do not include any optimisation elements and rely on decision makers to analyse results and identify improvements to be made. While simulation models are able to deal with elaborate detail, the successful applications of (stochastic) mixed integer programs in SCND (see for example Mitra et al. (2006) and section 4.1) prove that optimisation models are well qualified for these type of decision problems. Also, Shapiro (2007) emphasizes the importance of going beyond pure modelling approaches and using optimisation models to better support decisions. Therefore, we suggest using an optimisation model whose requirements are discussed as follows. Existing models and our proposed model are presented in chapter 4 and matched against these requirements.

#### Strategic Sub-Model

The key task of a SCND optimisation model is to design the supply chain by deciding on resource acquisitions and divestment options and the mission of each facility (Shapiro, 2007). This includes the selection of suppliers and the capacities of facilities and DCs (Vidal and Goetschalckx, 1997; Chopra and Meindl, 2007; Fleischmann et al., 2008; Simchi-Levi et al., 2008), where - depending on the modelling situation and application – continuous or discrete capacity can both be an appropriate choice (van Mieghem, 2003). Also, the technology used at each facility must be decided, and, thereby, which products are manufactured at each facility, which could also include make-or-buy decisions (Schmidt and Wilhelm, 2000; Chopra and Meindl, 2007; Shapiro, 2007; Goetschalckx and Fleischmann, 2008). As discussed in section 2.1, these decisions have great influence on manufacturing flexibility which should therefore be considered in the design (Goetschalckx and Fleischmann, 2008). Several authors (see for example Schmidt and Wilhelm, 2000; Meixell and Gargeya, 2005; Goetschalckx and Fleischmann, 2008) point out that the underlying supply chain structure for a SCND model should be holistic, that is, include suppliers, multiple manufacturing echelons, and customers. Also, it should feature a bill of material (BoM) structure, which is a directed graph describing the recipe for the manufactured products. Further, the design decisions need to be dynamic to develop a strategy adapted to a changing environment (Vidal and Goetschalckx, 1997; Schmidt and Wilhelm, 2000; Goetschalckx and Fleischmann, 2008).

#### **Tactical Sub-Model**

In the context of hierarchical planning, tactical decisions need to be anticipated to evaluate a supply chain design. Besides purchasing, production, labour, and the product flow through the network, this needs to include capacity utilisation and demand satisfaction (Vidal and Goetschalckx, 1997; Schmidt and Wilhelm, 2000; van Mieghem, 2003; Goetschalckx and Fleischmann, 2008). Also, any flexibility built into the network design should be exploited in these decisions (Chopra and Meindl, 2007). Further, detail is desirable, but has to be balanced carefully against the effort for acquiring the necessary data and computational complexity – as Vidal and Goetschalckx (1997) point out, strategic models always need a high degree of aggregation.

#### **Revenues and Costs**

As discussed in section 2.1, the aim of SCND is to determine the most effective longterm organisation of the company's supply chain, which means maximising profit while satisfying customer demand and responsiveness requirements. Therefore, an optimisation model needs to consider all relevant revenues and costs, and calculate performance measures in line with the company's accountancy standards. These should discount revenues and costs to their present value since SCND involves a long planing horizon (Chopra and Meindl, 2007; Goetschalckx and Fleischmann, 2008). In practice, nearly all costs follow economies of scale or include fixed costs, which therefore should not be neglected (Vidal and Goetschalckx, 1997; Chopra and Meindl, 2007; Shapiro, 2007). Also, supply chains are often international, which implies that export taxes, import tariffs, different income tax rates, duties, duty drawbacks, and exchange rates can have a significant effect on optimal design decisions (Schmidt and Wilhelm, 2000; Chopra and Meindl, 2007; Goetschalckx and Fleischmann, 2008).

#### The Role of Uncertainty and Risk in Decision Making

We have emphasised the importance of uncertainty in SCND and propose a methodology to identify relevant uncertainties in chapter 3. Obviously, an optimisation model should be consistent with these identified uncertainties and be able to deal with them in an appropriate way. Especially, it needs to include risk measures. However, few research exists on the choice of risk measures and in the lack of other criteria, we suggest that the chosen risk measure should at least fulfill the coherence as a set of desirable properties (see appendix A). Further, the model should explore the trade-off between risk and return (see section 2.2). However, in our opinion this does not mean that an optimisation model has to propose a single solution, optimal for a specific risk preference, but instead it should propose multiple efficient solutions, and leave the choice among them to the decision maker. Finally, a methodology to realistically represent uncertainty is needed as part of the model.

#### Flexibility of the Optimisation Model

As pointed out by Geoffrion and Powers (1995), a model for SCND needs to be flexible enough to adapt to evolving requirements in real world applications. Using a modern algebraic modelling language (see for example Kallrath, 2004), changing the model itself is simple. However, highly specialised solution algorithms, tailored to a specific problem structure, are far more complex to adapt to new situations, and requirements on a solution algorithm are therefore discussed next.

#### 2.3.3. Solution Methodology

Optimisation models that include all of the features described previously are very complex and challenging to solve, as they are large MIPs or SIPs. Further, real world uncertainty has many dimensions and, therefore, has to be considered in a multivariate setting. Typically, demand of different products at different markets in multiple time periods, and multiple exchange rates in multiple time periods are uncertain components.

When approximated by discrete probability distributions this means a large number of scenarios (realisations of the distribution) is needed for an accurate representation of uncertainty, which further increases the challenge to solve the decision problem. Moreover, the required flexibility of the optimisation model discussed previously implies that a solver has to be flexible enough to cope with new or modified parts of the model, such as strategic and tactical constraints, operational details, revenues and costs, risk measures in objectives and constraints, and uncertainties in different places. This can prohibit the utilisation of highly specialised algorithms. Solution methodologies able to deal with such decision problems in general, and the SCND model we propose in chapter 4 in particular, are discussed in chapter 5.

#### 2.4. Summary

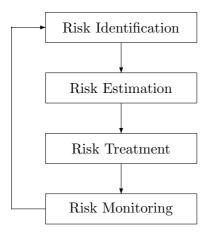
In this chapter we have refined the problem statement by defining SCND and related concepts, and introduced the relevant background to decision making under uncertainty. We have identified three requirements to support decision making in SCND: (i) a methodology to identify uncertainties relevant to the decision problem, (ii) an optimisation model, and (iii) an algorithm appropriate for this class of optimisation models. These three requirements are explored in the rest of this thesis and constitute the major focus of the research reported.

# 3. Uncertainty and Risk Identification for SCND

In this chapter we develop a process to identify and select uncertainties for a SCND project. We introduce the concept of risk management in section 3.1 to set the background and review the relevant literature in section 3.2. A new methodology for uncertainty selection is developed in section 3.3 and applied for a case study in section 3.4. The chapter is summarised in section 3.5.

# 3.1. Risk Management

In an organisation, risk management is a continuous and systematic process to identify and treat risks attached to the organisation's activities. It is a central part of strategic management and aims to secure the organisation's existence and long term success. A typical risk management process consists of the four phases (i) risk identification, (ii) risk estimation, (iii) risk treatment, and (iv) risk monitoring, which are cycled as shown in figure 3.1 (Norrman and Jansson, 2004; IRM, 2002; Hallikas et al., 2004; Locher et al., 2004; Manuj and Mentzer, 2008a; Vose, 2008).



**Figure 3.1.** Risk management process (adapted from Norrman and Jansson, 2004, and others).

Given the objectives of the decision maker, *risk identification* is the first step in the risk management process. Its aim is to identify an organisation's exposure to uncertainty, and, thereby, be able to mange risks proactively. Therefore, risk identification is an important stage of the risk management process, since unidentified risks remain neglected in the subsequent steps; hence, it should be approached in a methodical way. In a supply chain environment, risk identification must take into account the value stream of products and processes, assets and infrastructure, dependencies on other organisations, and the environment. The result of this process step is a list and description of the identified risks. (IRM, 2002; Hallikas et al., 2004; Norrman and Jansson, 2004; Peck, 2005; Vose, 2008)

The aim of risk estimation is to give a quantitative, semi-quantitative, or qualitative description of the probability and severity of each risk. This helps to focus attention on more severe risks and to choose suitable management options in the subsequent step. The results of risk estimation are commonly presented in a risk matrix, as shown in figure 2.2, page 10, which helps gaining an understanding of the organisation's risk exposure. Techniques used to estimate risks vary from expert opinions to sophisticated simulation or statistical models. The estimation of risks is further complicated by complex cause-and-effect relations, as the occurrence of one risk might increase or decrease another risk, often leading to feedback loops (Hallikas et al., 2002). (Hallikas et al., 2002; IRM, 2002; Hallikas et al., 2004; Manuj and Mentzer, 2008a; Vose, 2008)

Risk treatment is the process to manage the identified risks by reducing the probability of occurrence, reducing the consequence, or accepting the risk. Its goal is not necessarily to minimise risk, but to find an acceptable trade-off between risk and return for the organisation. Risk treatment actions generally include risk transfer, risk taking, risk elimination, risk reduction, and further analysis of individual risks. Also, risk treatment should define mitigation plans for reducing the consequences if an adverse event is realised, for example through business continuity management. (Hallikas et al., 2004; Norrman and Jansson, 2004; Manuj and Mentzer, 2008a; Vose, 2008)

The final phase of the risk management process is *risk monitoring*. It includes a reporting and review structure for identified and newly emerging risks, as well as controlling the implementation of risk treatment strategies. (IRM, 2002; Hallikas et al., 2004)

# 3.2. Literature Review: Analysis of Uncertainties Impacting SCND

In this section we discuss various research concerned with risk management in the context of supply chains. Our focus is on approaches that select a manageable number of uncertainties relevant to a SCND project. In section 3.2.1 we look into the literature

concerned with the risk management process in the context of supply chains. We give some examples of work focusing on the risk estimation step in section 3.2.3 and of work reporting practices in risk management in section 3.2.4. In section 3.2.5 we conclude with a summary of the literature and identify open research problems.

# 3.2.1. Supply Chain Risk Management

Miller (1992) is one of the first to take a holistic view at international business risk management, instead of focusing on single uncertainties. He provides a classification of risks as environmental, industry, and firm-specific risks. Environmental risks are further categorised as political, government policy, macroeconomic, social, and natural risks. Industry risks are categorised as input market, product market, and competitive risks, while firm risks include operating, liability, research and development, credit, and behavioral risks. Each category is illustrated by examples. Further, an overview of possibilities for organisations to deal with the identified risks is given.

Hallikas et al. (2002) are one of the first to describe a conceptual framework for risk management in network environments. The authors focus on subcontractors and aim to illustrate how small and medium-sized companies can analyse risks related to networking. Risks are discussed and characterised in two approaches: by hierarchical levels and by cause-and-effect relations. The hierarchical levels are 'risk clusters', 'effect factors', and 'causal factors'. For example, the risk cluster 'pricing' has underlying effect factors such as 'non-competitive prices' and 'price benefit of manufacturer not obtained', while a causal factor for 'non-competitive prices' could be 'company not able to achieve competitive cost efficiency'. However, these hierarchical levels often are not enough, as risk may consist of more complex cause-and-effect relationships. For example, the causal factor 'company not able to achieve competitive cost efficiency' could be the effect factor of a different cause, that might be part of the risk cluster 'manufacturing'. Hallikas et al. (2004) extend this work and investigate risk management in cooperative supplier networks.

Harland et al. (2003) review definitions and classifications of types of risk, based on which the authors present a risk management process. Identified risk categories are strategic, operations, supply, customer, asset impairment, competitive, reputation, financial, fiscal, regulatory, and legal risk.

Jüttner et al. (2003) identify research possibilities in supply chain risk management and propose that more empirically grounded research is needed to describe, explain, predict, and understand the risk management as it is currently practised in organisations. Jüttner (2005) identifies requirements on supply chain risk management from a practitioner perspective through an empirical survey. Thereby the author categorises risks via components of the supply chain and distinguishes supply, process and control, demand, and environmental risks.

Chopra and Sodhi (2004) investigate risk management strategies to prevent breakdowns of the supply chain. They categorise risks as disruptions, delays, systems, forecast, intellectual property, procurement, receivables, inventory, and capacity. Further, for each category they give examples and report empirical risk treatment strategies.

Christopher and Peck (2004) discuss strategies to improve the resilience of supply chains, while Peck (2005) identifies drivers of supply chain vulnerability through an empirical study and discusses according risk management strategies. In (Peck, 2006), the author investigates risk management in purchasing and supply and reports supplier assessment strategies.

Wagner and Bode (2006) make an empirical investigation into the vulnerability of supply chains and its relation to supply chain risk. Thereby, they identify drivers of supply chain vulnerability and correlate them to demand, supply, and catastrophic risks.

Manuj and Mentzer (2008a) present a risk management process with focus on global supply chains. Risks are categorised as supply, operational, demand, security, macro, policy, competitive, and resource risks, and examples of such risks are given. The authors discuss related risk management strategies in (Manuj and Mentzer, 2008b).

Rao and Goldsby (2009) review the literature on supply chain risk management and develop a classification of risks in supply chains, given as environmental, industry, organisational, problem specific, and decision maker risks. Each of these categories is comprehensively discussed, illustrated, and further sub-categorised.

#### 3.2.2. Risk Identification

Kouvelis et al. (2006) and Rao and Goldsby (2009) note that there is a substantial amount of literature dealing with supply chain risk management, but few explicitly address risk identification.

General techniques are proposed in some research. In (IRM, 2002) a list of possible methods is given that includes brainstorming, questionnaires, business studies, industry benchmarking, scenario analysis, risk assessment workshops, incident investigation, auditing and inspection, and hazard and operability studies. Vose (2008) suggests using a list of relevant topics to help participants of a risk identification workshop, while Harland et al. (2003) recommend to use brainstorming.

One of the few research concerned with risk identification is presented by Sodhi and Lee (2007). However, the authors do not describe a process, but have identified the risks themselves and present the resulting list of risks. This is for risks in the consumer electronics industry and categorised via two dimensions: strategic and operational risks versus supply-related, demand-related, and contextual risks. In each category, risks are described and illustrated by risk management strategies used by Samsung Electronics.

#### 3.2.3. Risk Estimation

The following publications are especially concerned with techniques for risk estimation. Pai et al. (2003) investigate inference techniques, Zsidisin et al. (2004) focus on the assessment of supply risks, and Tang (2006) gives an extensive review of quantitative models for managing supply chain risks. The textbook by Vose (2008) introduces basic quantitative methods in detail.

# 3.2.4. Practices in Risk Management

Finally, some research discusses practices in risk management. Johnson (2001) reports from the toy industry, as an industry facing rapid change and a high degree of demand uncertainty. Lee (2002) discusses strategies under supply and demand uncertainty of products. Finch (2004) shows through empirical studies that the risk of a large company is increased by having small- and medium-size enterprises as partners in critical positions in the supply chain. Norrman and Jansson (2004) report a case from Ericsson, where a fire at a sub-supplier causes huge disruptions. Ericsson's subsequent improvements to its risk management are discussed. Kleindorfer and Saad (2005) investigate risks arising from disruptions to normal activities and possible risk treatment strategies.

# 3.2.5. Summary of Uncertainty Selection Literature and Open Research Problems

We have seen in this section that there is an extensive literature on risk management in supply chains. However, as noticed by Kouvelis et al. (2006) and Rao and Goldsby (2009), and confirmed by the literature review, there is few research concerned with the process of risk identification. Some authors propose general techniques like the use of lists, brainstorming, and questionnaires, while other literature does not refer to a specific methodology but is helpful in this context by providing categorisations and examples of risks in supply chains.

Once risks are identified, a selection of a subset of these is not a direct concern of risk management. However, the literature agrees on using the risk estimation and its visualisation in a risk matrix (see figure 2.2, page 10), to focus attention on the most pressing risks.

Since no suitable methodology exists, we develop a process to identify and select relevant risks for SCND problems in the subsequent section, which sets out to answer the research question 'Which uncertainties need to be considered in SCND and how do we identify them?'

# 3.3. An Uncertainty Selection Process for SCND

We facilitate ideas from the reviewed literature to develop a process to identify and select uncertainties for a SCND project. The aim of the process is to identify the most important uncertainties to include in an optimisation model, while, at the same time, not selecting more than is manageable. It consists of a list of potential risks, a brainstorming session to complete this list, a risk estimation workshop, and a step to select uncertainties. The structure of the process is set out in figure 3.2 and the individual steps are described subsequently.

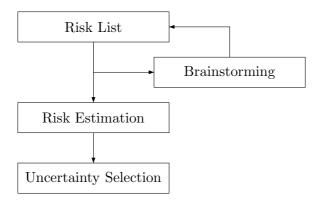


Figure 3.2. Process leading to uncertainty selection

The proposed process has three advantages over simply asking an expert to name the uncertainties to consider. (i) The results are more objective and especially less biased by recent developments or news coverage. (ii) Through the structured approach and the risk list, it is less likely that important risks slip. This can easily happen in SCND problems since they often involve new market and production regions, where issue are encountered that are not relevant in the existing production network. (iii) Undertaking a risk assessment in this early stage helps to manage risks in the further planning and implementation of the supply chain.

#### 3.3.1. Risk List

As a starting point for the uncertainty selection process, we compile a structured list of risks that can affect SCND, as suggested by Vose (2008). As noted by Hallikas et al. (2002), risks form a complex network of cause-and-effect relationships, which we break up at the point where it enters the supply chain. Consider for example the following chain: 'banking crises  $\rightarrow$  economic crisis  $\rightarrow$  low demand  $\rightarrow$  less production  $\rightarrow$  insolvency of supplier', which enters the supply chain at the event 'low demand'. Trying to list risks at an early point would include finding all reasonable causes for low demand or

even for economical crises – tasks predetermined to fail. However, exemplary causes are still listed in the risk list, as they are vital for understanding and estimating the identified risks. On the other hand, in the context of supply chain management under uncertainty, the effects of low demand are subject to a decision makers actions and should be part of a model, rather than be assumed as given.

Following this logic, we get a risk categorisation around elements of supply chains, similar to Jüttner (2005), with sub-categories from cause-and-effect relationships. The structure is shown in table 3.1, while the complete risk list is given in appendix B. Each risk in the risk list contains a name, a description, its category, and possible causes. The list is based on the examples found in the literature presented in section 3.2 and expert knowledge obtained through the brainstorming element of the process described subsequently.

Risk category	Description
Risks from suppliers and	Risks originating at a supplier.
procurement	
Risks from logistics	Risks from flows of goods in the network: suppliers to facilities, inter-facility, and facility to market. Includes storage at the origin and destination of the flow, stock turn, and handling.
Production risks	Risks originating at the production in the facilities.  Does not include production at a supplier.
Demand risks	All changes to demands of the products manufactured within the network.
Risks for profit and goals	Risks which influence financial and non-financial goals directly.
Risks from changing the network design	For example, during the acquisition or building of a new facility.
Network specific risks	Risks due to the interconnection of multiple suppliers, facilities, and markets.
Risks from the legal and social environment	Such as changes to laws and taxes.
Other risks	All other risks.

**Table 3.1.** Categories of the risk list

# 3.3.2. Brainstorming

Although the compiled risk list is extensive, in given situations it might be incomplete or require further detail. For example, this might be the case in specialised SCND projects concerning sourcing or distribution networks, or in tactical supply chain management. Therefore, we use a brainstorming session to identify further risks, as suggested by IRM (2002) and Harland et al. (2003). The brainstorming is executed with experts from the

project team realising the SCND project and – if possible – also experts running the existing and future supply chain. Its task is simply to name risks in the considered supply chain. The identified risks are added to the risk list, either as causes for existing risks or as new risks.

Besides completing the risk list, the brainstorming has two purposes: (i) The experts gain familiarity with the concepts of uncertainty and risk in supply chains, the structure of the risk list, and the identified risks. (ii) With the confidence that no risk has slipped, attention can focus on the estimation in the subsequent step of risk estimation. Also, motivation is increased when participants recognise their suggestions in the risk list.

This process step can be omitted for repeating SCND projects, especially if the same experts are involved.

#### 3.3.3. Risk Estimation

The central step of the uncertainty selection process is the risk estimation. While sophisticated models exist to estimate a risk's probability and severity, the aim of our process is only to identify a manageable set of relevant risks, which is subsequently investigated in depth in the optimisation model. Therefore, a simpler approach based on expert opinion is suitable.

We suggest a workshop with the participants described in the brainstorming step, where the task is to place the risks from the risk list in a risk matrix as in figure 2.2 (page 10). The participants estimate the probability and severity of the the risks relative to each other – that is, the risks are not given a numeric value, but risk A is placed above risk B, if it has a higher probability, and to the right, if its severity is higher. Consequently, the axes of the matrix are only semi-quantified by categories such as 'low', 'medium', and 'high'.

As pointed out by Miller (1992), the effect of a risk is different for different supply chains. For example, exchange rate uncertainty is of lower concern in a European supply chain in contrast to an inter-continental supply chain. Also, the relevance of supplier-related uncertainty depends on the types of products sourced or whether make-or-buy decisions are considered. Hence, the risk estimation is executed for every SCND project separately.

#### 3.3.4. Uncertainty Selection

The estimated risks in the risk matrix are identified with and grouped by their underlying uncertainty and uncertainties are selected in a discussion. In this step, the previous estimation helps to objectively choose the most important uncertainties to consider in the SCND project.

Not all uncertainties are suitable to include in an optimisation model; for example

a five day disruption at a supplier has significant effects on the operation of a supply chain, but the resulting 2% decrease of its annual delivery capacity is barely visible in a model with annual time periods. Other risks like the loss of prestige or intellectual property are difficult to capture in any quantitative model. Hence, at this stage a decision is made about which uncertainties to include in an optimisation model and which to consider otherwise, for example through simulation or qualitative assessment.

# 3.4. Uncertainty Identification for the Case Study

We have gone through the described uncertainty selection process for a SCND project similar to the network used in the case study in chapter 6. The supply chain consists of an intercontinental production network in the automotive industry, with suppliers, facilities, and markets in various emerging and developed economies.

Six uncertainties have been selected to consider in the optimisation model: uncertainty in demand, productivity, transport costs, labour costs, raw material costs, and exchange rates. Disruptions to production and supply were chosen to be analysed by a simulation model, which is not part of this thesis.

# 3.5. Summary

In this chapter we have introduced the concepts of risk management, reviewed related literature in the context of SCND, and shown that no suitable method exists to identify and select relevant uncertainties for a SCND optimisation model. Consequently, we have proposed a new process for risk identification, which is based on expert opinion and principal ideas from risk management. We have applied the uncertainty selection process in a case study and identified six uncertainties, which are integrated into a stochastic optimisation model described in the subsequent chapter. However, as discussed in section 3.3, risks have different effects on different supply chains, and therefore the selected risk are not universal but specific to the case study.

# 4. A Stochastic Optimisation Model for SCND under Uncertainty and Risk

In this chapter we introduce a stochastic optimisation model that meets the specifications for a decision support system identified in chapter 2 and that includes the uncertainties identified in chapter 3. In section 4.1 we review the relevant literature and consider the state of the art; taken together, these put the models described in this thesis in context. In section 4.2 the stochastic SCND model is described, while we provide details of a scenario generator for the uncertain parameters in section 4.3. The work reported in this chapter is based on Hollmann et al. (2011).

# 4.1. Literature Review: Quantitative Decision Support for SCND

The literature covering SCND includes a large volume of work reported under diverse topics, namely supply chain (network) design, facility location, capacity expansion problems, supply chain planning, and manufacturing flexibility.

In section 4.1.1 we discuss general introductions on the application of mathematical programming models to SCND and related literature reviews.

We review SCND models in section 4.1.2. These are characterised by a network structure with multiple nodes and multiple products, as well as capacity decisions on the nodes. The early studies of SCND (see for example Geoffrion and Graves, 1974) incorporate deterministic optimisation models while more recent studies take into account uncertainties by building stochastic programs (two-stage and multistage), (see for example Kall and Wallace, 1994; Birge and Louveaux, 1997). Since our aim is to develop a stochastic optimisation model, which follows as a natural extension of the deterministic case, both model classes are reviewed.

As set out in section 2.1, manufacturing flexibility is a topic closely related to SCND. In contrast to SCND, models concerned with manufacturing flexibility do not necessarily include a network structure or capacity decisions. Instead they mainly focus on allocation decisions or product-specific capacities. As this can also be part of the SCND decisions, an overview on manufacturing flexibility is given in section 4.1.3.

Within the hierarchy of supply chain planning, tactical and operational models over-

lap with strategic models; in section 4.1.4 a short introduction to the related literature is given.

Finally, we present our findings in the form of two summary tables in section 4.1.5. By critically examining these, we identify our research problem and the motivation for investigating it.

#### 4.1.1. Introductory Literature and Reviews

General introductions on the application of mathematical programming models to SCND can be found in Mitra et al. (2006); Snyder et al. (2006); Chopra and Meindl (2007); Shapiro (2007); Simchi-Levi et al. (2008). We discuss below a few further reviews, the first three of which focus on international aspects, and we note that the terms 'international' and 'global' are used interchangeably in the literature.

Vidal and Goetschalckx (1997) give a comprehensive summary of features of SCND models with emphasis on international aspects. As research directions, the authors suggest models that incorporate more of these features, but they also note that a model including all of them may not be practical. Hence, they conclude that interrelated, consistent models that exchange data are needed. This work is extended by Goetschalckx et al. (2002), where the authors focus on the integration of strategic and tactical models in international production networks.

The review by Meixell and Gargeya (2005) focuses on international aspects of SCND. Performance measures, the underlying network structure, special international aspects, and the types of decision variables are reported. For future research, the authors suggest including external suppliers and internal manufacturing, considering multiple echelons, developing performance measures – especially in an international context, and analysing industry-specific models.

Beamon (1998) classifies research into four categories: deterministic analytical models, stochastic analytical models, economic models, and simulation models. Besides these categories, models are summarised by the utilised performance measures and the types of strategic decision variables modelled. Identified research questions include performance measures, optimisation – especially in situations with decentralised decision making, some modelling issues, and classification of supply chains together with managerial insights on their behaviour.

Van Mieghem (2003) reviews the state of the art on strategic capacity management under uncertainty. This is categorised as static capacity investments, time, dynamic capacity investments, and capacity investments under risk aversion. Also, different model classes are explained, such as capacity expansion and facility location, and general modelling issues are discussed, namely capacity constraints, costs, inventory, and uncertainty.

Klibi et al. (2010) review SCND models under uncertainty by distinguishing between

the representation of uncertainty as random variables, as 'hazards' (high-impact extreme events), and as 'deep' uncertainty (hazardous events with unknown probability). Further, the authors investigate robustness, responsiveness, and resilience in the context of SCND. They conclude that most design models make too many assumptions and simplifications, and are therefore only of limited use in real-world business decision making. To overcome this, they propose research in SCND risk analysis, hazards modelling, scenario generation, performance measures, modeling of robustness, resilience, and responsiveness, and solution methods.

#### 4.1.2. SCND Models

#### **Deterministic SCND Models**

Geoffrion and Graves (1974) propose one of the early optimisation models for supply chain network design. The task is to optimally locate DCs between fixed facilities and markets and determine the connections between markets and DCs. All products for a specific market must be supplied from the same DC. The surrounding network structure is rather simple: there is just a single level in the DC-network, no time periods, but several products. Also, there is a single operational decision, the routing decision which is defined as the amount of a product shipped from a facility through a DC to a market. A solution technique based on Benders decomposition is developed, which proves to be very effective. The methodology is applied to a real life problem for a major food company. The result analysis concentrates on the performance of the proposed solution algorithm.

The model by Brown et al. (1987) distinguishes between facilities and production lines. While facilities can be opened or closed, lines must be assigned to opened facilities. The network has two stages (facilities and markets), a single time period, and multiple products. There are operational decisions on production amounts and network flows. The model is solved using a goal decomposition technique. The authors report the successful application of their model at a company in the food industry, where it significantly reduces planning time.

The model by Shulman (1991) is a two-echelon, multi time period, multi product network design model. The design decision is to associate a capacity from a finite set of possible capacities to each facility while the only operational decision is for production / distribution quantities. A solution technique using Lagrangian relaxation is developed and its computational performance is reported.

Arntzen et al. (1995) present a model for facility location in a multi level, multi product network, that includes an assignment of products to facilities with fixed set up costs. The operational part of the model covers production, inventory, transportation via different transportation modes, as well as auxiliary variables to calculate duty drawback and duty avoidance. The strategic decisions are static and the operational decisions dynamic; the objective is a weighted sum of the total costs and the total production and shipment time. The methodology is applied to a consumer electronics company, where it is used to (i) analyse the supply chain for new products, (ii) analyse sourcing strategies for commodity products, and (iii) to study supply chains of business units as well as company-wide. The results for several real life case studies are reported.

Dogan and Goetschalckx (1999) propose a network design model with multiple products and time periods. The structure is defined in six echelons as supplier – production – storage – finishing – storage – market, and the capacities for facilities and DCs are chosen from finite possibilities. Further, different machines can be installed at the facilities. The operational decisions are production, transportation, and inventory storage. The model is solved using an accelerated Benders approach and is applied in the packaging industry. The authors compare the results from a single time period model with results from three time periods with seasonal demand. They investigate the benefit of accounting for storage and find that there is a 2% reduction in costs. This can be explained by lower capacity requirements as the multi time period model is able to cover peak demands from inventory. However, demand used for the single time period model is from the peak season which explains the high capacity – using average demand instead might be the better option.

Yan et al. (2003) develop a static model where suppliers, facilities, and DCs are selected, and products are assigned to them. No operational details surpassing transportation are included. The distinguishing feature is the consideration of constraints of the type 'if a product is produced in at least x facilities, then at least y suppliers delivering the required raw materials must be selected'.

The model by Melo et al. (2005) is a multi level, multi time period, multi product capacity expansion problem, where the task is to decide the facilities' capacity. Capacity is modeled via a 0-1 variable, indicating whether a facility is operating or not, together with a continuous variable describing the actual capacity. A notable distinction is that the capacity is described via transfers from other facilities while the total network capacity is given. However, the authors demonstrate that this formulation can still be used for capacity expansion or reduction problems by using auxiliary facilities. Further, the authors mention the possibility to model capacities as a discrete choice, but this approach is not investigated. The operational decisions are procurement, routing, and inventory. The computational studies focus on computation times for alternative reformulations of the model, for relaxing of certain model aspects, and for varying the number of time periods, products, and markets.

Cordeau et al. (2006) develop a model for multi level, single time period, multi product network design. The decision is to select operating nodes from suppliers, facilities, and DCs. For each node an allocation decision is included, to decide whether

or not a product is assigned to the specific node. Also, for each transport link an allocation decision is included, indicating whether or not a product is shipped from a start node of the link to its destination node. The operational decisions are routing amounts per transportation mode for the links. A Benders decomposition approach is used to solve the problem and valid inequalities are given to strengthen the LP relaxation. The result analysis focuses on the performance of the proposed solution algorithm as well as the impact of the valid inequalities.

Fleischmann et al. (2006) present a model aimed at the automotive industry with three production lines at each facility: body assembly, paint shop, and assembly. Shipments between different facilities are not possible; hence, only one production stage is needed. Further, there are multiple time periods and multiple products. The strategic task is to choose the capacity from a finite set for each line at each facility, and to assign the products to facilities, while the operational decisions are production, transportation, and overtime. The authors demonstrate how the proposed model can be used to optimise the case study network and identify crucial issues in its design.

All presented deterministic SCND models are MIPs with 0-1 decisions for the network design. While the early papers still investigate specialised solution algorithms to deal with these types of problems, more recent research can focus on modelling aspects. This is due to the rapid improvements of general purpose MIP solvers such as CPLEX or Gurobi.

#### Stochastic SCND Models

Bienstock and Shapiro (1988) develop one of the first stochastic SCND models, which is in the context of energy networks. The underlying network design problem has a single production level, multiple time periods, and a single product. The capacity is modelled by continuous variables with piecewise linear costs, rather than by discrete expansion sizes. The only operational decision is the production volume. The stochastic model is two-stage with decisions taken here-and-now affecting the first x time periods, and the recourse stage being the remaining T-x time periods. Uncertainty comes from demand, air pollution control limits, the time of completion of a pipeline, and fuel prices. Example results for uncertainty in demand and environmental laws are shown, but only six scenarios are used. Several runs with different uncertainties are used to identify hedging strategies.

The model by Eppen et al. (1989) is a single production level, multi time period, and multi product capacity planning problem. Capacity is modeled via a finite set of possibilities and capacity changes are allowed only once for each facility. The operational decisions are production amounts and shortages. The stochastic model is two-stage where the first stage variables are the capacity acquisitions and the second stage are the operational decisions. Uncertainty comes from demand and sales contribution mar-

gin, and risk is measured by expected downside risk. Since the stochastic optimisation is based on only three scenarios, Monte Carlo simulation is used to provide a better estimate of the return distribution. The model is applied in the automotive industry.

Huchzermeier and Cohen (1996) present a stochastic dynamic programming model for SCND under exchange rate uncertainty. Design decisions include facility and market selection, as well as selection of transportation links. However, the optimisation only chooses from selected network configurations which reduces the state space from 2<sup>27</sup> to 16 states in the case study. The underlying network has only one production stage and operational decisions are limited to transportation of a single product. To account for international operations, individual taxes per country and exchange rates are included. The uncertainty in the latter is modeled as a stochastic diffusion process with three scenarios per time period. In the case study, financial hedging via future contracts is compared to operational hedging via switching network configurations. The applicability of the latter one might not be practical, since time periods represent quarter years which implies that production is switched between different global production facilities several times per year. Financial hedging on the other hand is reasonable for the considered time horizon of 15 months. In the case study results, operational hedging dominates financial hedging, measured as expected profit versus expected downside risk. These studies are extended in further research by the authors in Cohen and Huchzermeier (1999), where demand uncertainty is added to the model. Managerial insights are reported, which conclude that real options can improve both shareholder value and risk, they are more valuable than financial options, and combining both option types gives further benefits. Goh et al. (2007) formulate the same model as a multi-stage problem and propose a solution algorithm. However, neither computational performance nor case study results are reported.

The model by Vidal and Goetschalckx (2000) is a single echelon production network with one time period and multiple products. Facilities are selected by 0-1 decisions and there are allocation decisions, whether a supplier ships a certain good to a facility, as well as whether a facility produces a specific good. The operational decisions are production and routing with different transportation modes. Sensitivity analysis is used to deal with uncertainty, that is, the model is optimised as a deterministic model for each scenario separately and the optimal objective is reported. This is used to illustrate the effects of uncertainty in exchange rates, demand, supplier reliability, and procurement lead times on the minimum costs. Supplier reliability is included via a constraint, which ensures that the probability of being on time for all suppliers is at least a specified target value. Procurement lead times are included by cost factors for different transportation modes. In the case study, uncertainties are applied individually – only one parameter is assumed to be uncertain at a time while the rest are deterministic. In this situation, exchange rate has the largest effect on the optimal objective.

Dominguez-Ballesteros (2001) describes a two-stage stochastic model under demand uncertainty. The network includes multiple facilities, multiple products, multiple time periods with the strategic decisions on a subset of the operational time periods, and a single source of demand. Therefore, the operational decisions do not include transportation, but instead production, inventory, sales, unsatisfied demand, and demand carry-over. The strategic decisions are the selection of facility capacities from a set of finite possibilities, outsourced capacity, and inventory capacity. Risk is measured by CV@R and the trade-off between risk and return is explored, but result analysis is limited to reporting solutions and profit distributions.

Lucas et al. (2001) present a multi time period, multi product model, consisting of production facilities, packaging facilities, and DCs. Nodes can be opened or closed, and at operating nodes lines of different types can be installed. The operational decisions are production and transportation. The stochastic model is two-stage with the network design decisions taken here-and-now and the operational decisions being recourse. Demand is assumed to be uncertain although production efficiency and costs are mentioned as a source of uncertainty as well. In MirHassani et al. (2000) two solution approaches are proposed for this model: scenario analysis and a parallel Benders algorithm. The result analysis focuses on computational performance, but the model has also been applied in case studies in the consumer good as well as pharmaceutical industry. Earlier versions of this research also appear in Baricelli et al. (1996) and Lucas et al. (1997).

Tsiakis et al. (2001) present a distribution network design model where production facilities are assumed to be fixed. It has four levels with warehouses and distribution centres (smaller warehouses), one time period, and multiple products. Capacities of warehouses and DCs are continuous with a 0-1 variable indicating whether it is operating. Also, there are assignment decisions for warehouses to DCs, and DCs to markets. Production and transportation are the operational decisions, with transportation costs being piecewise linear to account for economies of scale. The stochastic model is two-stage with the network design decisions as first stage, the operational decisions as second stage variables, and uncertainty in demand. In the case study, uncertainty is represented by three scenarios and the expected value-solution and wait-and-see-solutions are compared to the here-and-now-solution via their expected costs.

Alonso-Ayuso et al. (2003a) present a comprehensive model for network design. It has multiple production levels, multiple products, as well as multiple time periods. In the strategic sub-model, facilities can be opened in the first time period only, and be expanded in the later periods by choosing capacities from a set of finite possibilities. Further, products are allocated to suppliers and facilities in the first time period. The operational decisions are production, inventory, and routing. The stochastic model is two-stage with the network design as here-and-now decisions, operational decisions

as recourse, and uncertainty in demand, product net profit, raw material costs, and production costs. In the case study, demand and raw material uncertainty is included via 23 scenarios. The analysis focuses on computational performance. In Alonso-Ayuso et al. (2007) the model is slightly extended with additional supplier selection, but no computational studies are reported. A simplified version is studied in Alonso-Ayuso et al. (2005a) with no suppliers, no product facility assignments, static capacity decisions and no BoM. Instead, the model includes V@R and probability of failure risk measures, but with the proposed solution algorithm only the expected value or probability of failure objective can be solved. In the case study, only the expected value objective is used and the analysis focuses on computational performance.

In Guillén et al. (2005), a two-stage stochastic program for SCND under demand uncertainty is presented. The model includes one echelon of facilities and one echelon of warehouses with multiple products, but no BoM structure. The first stage decisions are a static network design, consisting of facility and DC capacity, which is modeled by continuous values with binary decisions for fixed costs. Besides transportation and facility production, the operational second stage decisions include inventory levels at DCs, taken over multiple time periods. The objective function comprises three components: expected profit, expected downside risk, and demand satisfaction. In the analysis, the trade-offs between the objectives are explored.

Santoso et al. (2005) present a multi production echelon, single time period, multi product facility location problem, that is, the main decision is to decide which facilities are operational. This is evaluated against operational production / routing and shortage decisions. The stochastic model is two-stage with the network design as first stage and the operations as second stage. Production costs, transport costs, demand, supply, and capacity are all assumed to be uncertain. A computational study shows results from two real supply chain networks. In these studies, only demand and facility capacity are assumed to be uncertain, and the according scenarios are generated by sampling from independent log-normal distributions. The focus of the result analysis lies on the performance and quality of the solutions from the proposed approximation algorithm. A more detailed description of the approach and the computational results can be found in Santoso (2003).

Azaron et al. (2008) present a two-stage stochastic model with multiple echelons in the network, multiple products, but no BoM structure. Also, while mentioning a separation between facilities and DCs, they are handled identically, both on the strategic and the operational side. The model includes few details – the only strategic decision is whether facilities are opened; the operational decisions are transportation and shortages. Additionally, there is a possibility to expand facility capacity on the second stage. Further, the model is static with no time periods. The main focus of the paper is on a multi objective approach that combines expected costs, variance of costs, and probab-

ility of failure in a goal programming formulation. The model includes uncertainty in demand, the amount a supplier can deliver, and all second stage costs; however, in the case study only uncertainty in demand, supply, production costs, and capacity expansion costs are considered. These uncertainties are represented by eight scenarios. Given the static model, flexibility cannot be evaluated. The authors illustrate the trade-off in the different objectives, as well as the importance of accounting for uncertainty and risk in SCND, while the effect of different and importance of multiple uncertainties are not explored. Both risk measures facilitated seem unusual, as variance is not a downside risk measure, and penalising probability of failure does not take into account the failure's size. Also, both measures are computationally challenging, as variance gives a quadratic objective and probability of failure introduces 0-1 variables on the second stage. Additionally, the formulation uses seven parameters to parametrise the objective, which makes it hard to explore. The authors do not demonstrate the advantages of the proposed risk measures as compared to expected downside risk or CV@R.

The model by Bihlmaier et al. (2009) is a multi production level, multi time period, multi product network design problem for facilities. The capacity is chosen from a finite set and products are assigned to facilities. Operational decisions include production, routing, and shortages. Based on this, the authors develop different models: a deterministic model, which includes detailed work force planning and discrete choice of shift models, a second deterministic model where the shift models and work force are approximated by linear organisational capacity increases and decreases, and a two-stage stochastic model based on the latter deterministic model with demand uncertainty, strategic decisions on the first stage, and operational decisions on the second stage. The two deterministic models are compared in a case study where they give the same strategic decisions, which justifies the use of the approximation for the stochastic model.

The model by Kauder and Meyr (2009) is based on Fleischmann et al. (2006), with some simplifications on the details. This is combined with chaining strategies identified in Jordan and Graves (1995) and Graves and Tomlin (2003) (see section 4.1.3). To include these strategies, numerous constraints are added to the model. The model is analysed by a simulation approach, where the optimisation problem with and without the chaining constraints is solved as a deterministic model, and the behaviour of the strategic decision under seven further exchange rate and demand scenarios is simulated. In a regret approach this is compared to the optimal solution of the given scenario. The scenarios are rather simple with only one parameter being altered by a constant factor. While this helps in the analysis, it probably results in underestimating the value of flexibility since a more dynamic uncertainty would require more flexibility. The authors conclude that dedicated chain structures perform better under demand uncertainty and reasonably well under exchange rate uncertainty. The model is applied on data constructed similarly to the automotive application in Fleischmann et al. (2006).

Schütz et al. (2009) present a supply chain model for multiple levels and multiple echelons, based on Tomasgard and Høeg (2005). The design decision is to decide which processes to install at different facilities, where a process can apply to several products. Hence, one can think of each process as being a separate facility. The operational decisions are inventory at designated warehouse facilities, transportation, production, and shortages. The model can handle both, combining processes (as for example in the manufacturing industry) and splitting processes (as for example in the chemical industry). The design decisions are static, while the operational decisions are dynamic over multiple time periods. The results from this approach are compared to a simplified model with static operations, to analyse the benefits from the detailed operations. The stochastic model is two-stage with the design decisions on the first stage and operational decisions on the second stage. In theory, all model parameters are uncertain, except for the network design costs and BoM, but in the case study only demand is assumed to be uncertain. This is separated into two parts: short term uncertainty (seasonality) and long term uncertainty (trends). For long term demand uncertainty, a simple approach is used that samples from a uniform distribution. Short term uncertainty is modeled as a combination of an autoregressive forecasting process of first order, principle component analysis of the error term, and moment matching of the (uncorrelated) principle components based on Høyland et al. (2003). Results from an application in the Norwegian meat industry are reported, where the value of the stochastic solution is at least 16 \%, while the results from the approximated static solution are significantly worse. The improvement in the objectives mainly results from more installed capacity, which leads to higher flexibility.

Nickel et al. (2010) describe a multi-stage model that combines DC location with alternative investments in financial products and loans. Operational decisions are limited to shipments and shortages, and demand as well as return on investments are uncertain. The objective is a weighted sum of operational profit, financial returns, customer satisfaction, and expected downside risk. In computational studies, the value of the multi-stage stochastic solution (VMS) over deterministic optimisation is calculated; however, not as a rolling deterministic planning approach (see Kall and Wallace, 1994). Thereby the VMS is overestimated, which is on average less than 1% and never more than 3%. The trade-off between risk and return is not explored.

Stephan et al. (2011) develop a multi-stage model which is solved using stochastic dynamic programming, accelerated by dual reoptimisation and parallelisation. To cope with the increased complexity from the multistage approach, the network structure is relatively simple with a single echelon, multiple time periods, and fixed product to facility assignments. The strategic decision is to select facility capacities from discrete possibilities, while the recourse actions consist of a single production / transportation decision. The demand is uncertain and assumed to follow a Markovian model, which is

justified by a partial autocorrelation analysis of historical data. A main contribution is the rigorous analysis of the VMS over two-stage stochastic programming. Following a proposition for the VMS over deterministic optimisation from Kall and Wallace (1994), Stephan et al. (2011) use a rolling evaluation procedure where a two-stage model is solved after every uncertainty stage, as new information becomes available. However, given the inaccuracy of strategic planning data (in their case for the next six years), the reported VMSs of 0.02% to 0.55% seems rather small. An interesting insight is however, that as the VMS increases, less flexibility is available in the given assignments of products to facilities. This seems plausible since the less flexibility is available through shifting production between facilities, the more important it becomes to adapt the manufacturing capacity at each facility instead – which can be done best by a multistage optimisation.

All presented stochastic SCND models are SIPs with 0-1 variables for the network design at least on the first stage. These remain computationally challenging; therefore, several of the publications investigate specialised solution algorithms. These are reviewed in chapter 5 where we present our approach to this problem.

# 4.1.3. Manufacturing Flexibility

In-depth reviews and definitions of manufacturing flexibility can be found in various literature, such as Sethi and Sethi (1990); Gerwin (1993); Upton (1994); Toni and Tonchia (1998); Koste and Malhotra (1999). Further, Vokurka and O'Leary-Kelly (2000) and Hallgren and Olhager (2009) review empirical research on manufacturing flexibility. Jack and Raturi (2002) study volume flexibility through case studies, while Gupta et al. (1992) use an analytical approach. Bengtsson and Olhager (2002a) and Bengtsson and Olhager (2002b) use real options to evaluate manufacturing flexibility. The following approaches use quantitative methods from operational research; these are discussed in more depth.

Jordan and Graves (1995) investigate the benefits of process flexibility through a simulation approach in a network with one stage, one time period, and multiple products. The assignment of products to facilities is fixed and only production amounts are optimised for multiple demand scenarios. The assignments are represented by edges in a bipartite graph whose partition vertices are the products and facilities. A main contribution is the concept of chaining, where a chain is a connected sub-graph. The authors show through analytical and numerical studies, that the flexibility (measured as expected sales) of a Hamiltonian path is nearly as good as the flexibility of the fully connected bipartite graph. In this context, we note that a Hamiltonian path is a circular path that visits each of a graph's vertices exactly once. The authors show that chains have the greatest benefit when they are as large as possible. Furthermore, a measure is developed that indicates whether a given assignment is likely to be improved.

This work is extended by Graves and Tomlin (2003) for multi echelon networks. The authors introduce a measure of flexibility and propose flexibility guidelines that extend the single-stage chaining strategies to multi-stage networks.

The model by Jordan and Graves (1995) is extended to an optimisation model by Boyer and Leong (1996), which is used to investigate the relation between process and machine flexibility. Further, it is confirmed that an added link which closes a chain to a circle 'is an essential element in realising the full benefits of process flexibility', (Boyer and Leong, 1996).

Van Mieghem (1998) studies the simple situation where one company with two products can invest in product-specific and product-flexible machines in a static model. This can be solved analytically and the author investigates the effect of costs, sales margins, and demand uncertainty (especially its correlation) on the optimal capacity. Not surprisingly, with increased demand correlation the dedicated capacities increase while the flexible capacity decreases.

Chen et al. (2002) propose a multi-stage stochastic model that optimises product-flexible and product-dedicated capacity at a single facility. The benefits of flexible capacities in various situations are investigated.

Chandra et al. (2005) formulate a model for flexibility planning in the automotive industry. They investigate flexibility enablers such as product allocation, part commonality between products, and supply flexibility and demonstrate their benefits. The authors use a combination of Monte Carlo simulation, a genetic algorithm, and linear programming to solve the problem, while product allocation and commonality of parts are fixed.

Francas et al. (2009) extend the model by Jordan and Graves (1995) to a two-stage stochastic program with assignment decisions on the first stage, production amounts on the second stage, demand uncertainty, and the objective to minimise unsatisfied demand. This is used to investigate optimal assignment decisions under lifecycle demand and the authors conclude that chains still remain superior, although the benefit of additional flexibility decreases when the lifecycles of different products are out of phase as compared to when they are in phase.

Koberstein et al. (2011) incorporate different financial hedging instruments, namely forward contracts and put and call options, in a multi-stage stochastic program with product to facility assignments on the strategic level and transportation and shortages on the operational level. Demand and exchange rates are uncertain with the design decision in the first stage only, financial hedging as here-and-now decisions on every stage, and operational decisions as recourse. The objective includes expected profit and CV@R. The result analysis investigates the effect of including the financial hedging instruments. While it is clearly shown that for a given network design financial instru-

ments have a significant impact on risk and return, it is less obvious that they have a significant impact on the optimal network design.

# 4.1.4. Operational and Tactical Supply Chain Models

There is comprehensive research focusing on tactical and operational aspects of supply chain management. Some examples concerned with production planning or the integration of strategic and operational decisions are described below. Van Landeghem and Vanmaele (2002) propose a framework for Monte Carlo simulation in tactical supply chain planning under uncertainty, Alonso-Ayuso et al. (2005b) and Alonso-Ayuso et al. (2007) present a hierarchical suite of models, and Mula et al. (2006) provide a literature review for models under uncertainty at all hierarchy levels. In what follows we discuss case studies which use optimisation or simulation methods.

Escudero et al. (1993) describe two models for production planning under demand uncertainty with multiple products, time periods, and machines at a single facility. The first model decides on production volumes, inventory, lost demand, and amounts obtained from a secondary source, while the second model extends the secondary source by allowing for alternative supply. The models are described as multi-stage stochastic programs with various segmentation into here-and-now and recourse decisions, approximated using a three-stage scenario tree, and solved using Benders decomposition.

Escudero et al. (1999) formulate a two-stage stochastic model for the optimisation of the production in a given supply chain with multiple products. It covers many operational details, such as production, product substitution (if a component needed is not available, this might be substituted by a different suitable component), procurement, shipment, lost demand, backlogging of demand, and inventory. The uncertain parameters are production costs, procurement costs, and product demand with the decision variables for the first time period in the first stage and remaining time periods in the second stage. The authors suggest different objective functions: minimising expected costs, minimising expected maximum weighted product backlog, minimising expected total weighted product backlog, and minimising lost demand.

Bradley and Arntzen (1999) present a deterministic model that optimises (i) the capacity of a single facility with multiple production lines, and (ii) the production plan for multiple products. Capacity is chosen from discrete possibilities and the operational decisions include production, purchase, sales, overtime, and inventory. The authors show through computational studies that minimising unit costs or maximising equipment utilisation, instead of return on investment, leads to sub-optimal strategies.

Sabri and Beamon (2000) combine a simple strategic network design model with a detailed operational model in an iterative heuristic. The strategic model is static with multiple echelons and multiple products, the design decisions are facility selection, DC selection, and DC to market assignments, while the operational decisions include pro-

duction and transportation. The operational model is a heuristic model that specifies (s, S)-type inventory control strategies and includes uncertainty in production, delivery, and demand. In this context we note that a (s, S) inventory control states that S units are ordered whenever inventory falls below a level of s units. Large parts of this are handled by analytically solving sub-problems. The expected values from the operational model are then used as input for the strategic model in an iterative procedure. However, the benefits from including that much operational detail as compared to the strategic model are not thoroughly investigated.

Gupta and Maranas (2003) formulate a tactical two-stage model for supply chain production planning under demand uncertainty. Manufacturing quantities are first stage here-and-now decisions whereas transportation logistics make up the recourse decisions. A trade-off between customer satisfaction and costs is investigated.

Chen and Lee (2004) propose a production planning model in a multi echelon, multi product, and multi time period supply chain under uncertain demand and fuzzy product prices. They consider inventory and piecewise linear transportation costs. Numerous objectives, such as expected profits for multiple companies in the supply chain, inventory safety levels, customer service levels, and expected downside risk for selected objectives are investigated. A case study demonstrates that using the multiple objectives leads to a solution that counterbalances the differences between the members of the supply chain.

You and Grossmann (2008) present a supply chain model for the chemical industry. It includes network design decisions, but the focus is on lead times and operational details, such as inventory, production scheduling, and sequencing. By considering demand uncertainty, the resulting model becomes a two-stage mixed-integer nonlinear stochastic program. This is approximated, decomposed, and solved using a heuristic. This work is extended by You et al. (2009) and You and Grossmann (2010).

In section 4.1.2 we have reviewed an SCND model by Schütz et al. (2009); its operational sub-model is investigated by Schütz and Tomasgard (2009). This is a two-stage stochastic program which includes inventory, transportation, and shortfall. Decisions in the first time period are the first stage whereas the decisions in the remaining time periods are the second stage. The model is used to investigate the value of operational flexibility, defined as the ability to change production plans in a rolling planning methodology. Using case studies the authors demonstrate that operational flexibility is only beneficial when there is medium volume flexibility or medium delivery flexibility; when they are very low or very high, operational flexibility has no value.

## 4.1.5. Summary of SCND Literature and Open Research Problems

# **Summary of Essential Characteristics**

We have identified essential characteristics of SCND models and summarised the reviewed literature by these in tables 4.1 and 4.2.

Table 4.1 shows the general network design features of the deterministic and stochastic models. These are summarised in the following columns.

- 1. The reference is cited.
- 2. The network structure and types of nodes are listed, as well as which of these have capacity decisions.
- 3. The number of echelons (levels) in the network structure is given.
- 4. It is shown whether there is a BoM structure.
- 5. The number of time periods, first for the strategic decisions, and second for the operational decisions are supplied.
- 6. It is indicated how capacity is modeled.
- 7. The allocation decisions considered are given.
- 8. The operational decisions are shown. However, all models include transportation and/or production decisions and these are therefore omitted in the table.

The first part of this table summarises deterministic models, whereas the second part covers stochastic models. Abbreviations used in table 4.1 are summarised in table 4.3.

Table 4.2 extends the summary for stochastic models, to describe their additional features and summarises these in the following columns.

- 1. The reference is cited.
- 2. The modeling paradigm used to account for uncertainty is given.
- 3. The considered uncertainties are listed.
- 4. The distinction between first stage here-and-now decisions and second stage recourse decisions (respectively the according distinction for multi-stage models) is shown.
- 5. The modelling approaches to deal with risk are supplied.

Abbreviations used in table 4.2 are summarised in table 4.4.

(1) Authors	(2) Network	(3) Echelons	(4) BoM	(5) Periods	(6) Capacity	(7) Allocations	(8) Operational Decisions <sup><math>a</math></sup>
Geoffrion and Graves 1974	F-DC-M	3	_	1, 1	b	$\mathrm{DC} ext{-}\mathrm{M}^b$	
Brown et al. 1987	$\mathbf{F}^c ext{-}\mathrm{M}$	2	-	1, 1	b	$F-F^d$	
Shulman 1991	$\mathbf{F}$ -M	2	_	x, x	e		
Arntzen et al. 1995	$\mathbf{F}\text{-}\mathbf{M}$	X	$\checkmark$	1, x	b	F-P	Inventory, duty,
							transportation with modes.
Dogan and Goetschalckx 1999	S- $F$ - $DC$ - $M$	$6^e$	-	1, x	e		Inventory.
Yan et al. 2003	S-F-DC-M	4	✓	1, 1	b	S-P F-P DC-P	
Melo et al. 2005	$\mathbf{F}\text{-}\mathrm{M}$	37		V V	c+b	DC-F	Inventory.
Cordeau et al. 2006	S-F-DC-M	x x	_	x, x 1, 1	b	S-P	Transport mode
Cordeau et al. 2000	5-F-DO-M	A		1, 1	b	F-P DC-P S-F-P F-DC-P DC-M-P	selection.
Fleischmann et al. 2006	S- <b>F</b> -M	3	✓	x, x	е	F-P	Production, transportation, overtime.
Bienstock and Shapiro 1988	F	1	_	x, x	e+c		
Eppen et al. 1989	$\mathbf{F}$	1	_	x, x	e		Shortages.
Huch zermeier and Cohen $1996^f$	$S$ - $\mathbf{F}$ - $\mathbf{M}$	3		x, x	b	S-F F-M	Taxes.
Vidal and Goetschalckx 2000	$S$ - $\mathbf{F}$ - $M$	3	$\checkmark$	1, 1	b	S-F-P F-P	Transportation with modes.
Dominguez-Ballesteros 2001	F	1	-	x, x	e		Shortages, invent- ory, unsatisfied demand, demand carry-over.
Lucas et al. 2001	$\mathbf{F}\text{-}\mathbf{DC}\text{-}\mathbf{M}$	$4^g$	-	x, x	e		Shortages.
Tsiakis et al. 2001	F- <b>DC</b> -M	4	-	1, 1	c+b	DC-DC DC-M	Piecewise linear transport costs.
Alonso-Ayuso et al. 2003a	S- <b>F</b> -M	X	✓	x, x	е	F-P S-P	Inventory.
Guillén et al. 2005	$\mathbf{F}\text{-}\mathbf{DC}\text{-}\mathbf{M}$	3	_	1, x	c+b		Inventory.
Santoso et al. 2005	$S$ - $\mathbf{F}$ - $M$	x	_	1, 1	b		Shortages.
Azaron et al. 2008	S- <b>F</b> -M	X	-	1, 1	b		Shortages, capa- city expansion.
Bihlmaier et al. 2009	F-M	X	✓	x, x	e	F-P	Organisational capacity adap- tion, shortages.
Kauder and Meyr 2009	$S$ - $\mathbf{F}$ - $M$	3	_	x, x	e	F-P	Overtime.
Schütz et al. 2009	<b>F</b> -DC-M	x	$\checkmark^h$	1, x	b		Inventory, shortages.
Nickel et al. 2010	DC-M	2	-	x, x	b		Financial invest- ments, loans, shortages.
Stephan et al. 2011	$\mathbf{F}\text{-}\mathbf{M}$	2	_	x, x	e		
Our model	S-F-M	X	✓	x, x	c+b	F-P	Organisational capacity adaption, workforce, shortages.

<sup>&</sup>lt;sup>a</sup>All models include transportation and/or production decisions. These are omitted in the table.

**Table 4.1.** Literature overview – network features. Abbreviations are summarised in table 4.3.

 $<sup>^</sup>b1$ -to-many allocation: DC can supply several M, but each M must be supplied by a single DC.

<sup>&</sup>lt;sup>c</sup>Decision on facilities to be built and on lines to be built at each facility.

 $<sup>^</sup>d$ Many-to-1 assignment of lines to facilities: each line must be assigned to exactly 1 facility.

<sup>&</sup>lt;sup>e</sup>The structure is S-F-DC-F-DC-M.

<sup>&</sup>lt;sup>f</sup>Single product model.

 $<sup>{}^</sup>g{\rm The}$  structure is F-F-DC-M.

 $<sup>^</sup>h\mathrm{Includes}$  BoM and reverse BoM for combining and splitting processes.

(1)	(2)	(3)	(4)	(5)	(6)
Authors	Type	Uncertainty	HN Decisions	Recourse Decisions	Risk
Bienstock and Shapiro 1988	28	demand environmental laws (capacity) (prices)	first $x$ time periods	remaining time periods	
Eppen et al. 1989	2S+Sim	demand sales contribution	network design	operational decisions	EDR
Huchzermeier and Cohen 1996 Vidal and Goetschalckx 2000	StDP Sens	exchange rates exchange rates demand supplier reliability procurement lead times	network design	operational decisions all decisions	EDR
Dominguez-Ballesteros 2001 Lucas et al. 2001 Tsiakis et al. 2001 Alonso-Ayuso et al. 2003a	2S 2S 2S 2S	demand demand demand raw material costs (product net price) (production costs)	network design network design network design network design initial capacity	operational decisions operational decisions operational decisions dynamic capacity operational decisions	CV@R
Guillén et al. 2005	2S	demand	network design	operational decisions	EDR, shortages
Santoso et al. 2005	2S	demand facility capacity (transport costs) (supply)	network design	operational decisions	Ü
Azaron et al. 2008	2S	demand supply production costs capacity expansion costs (transport costs) (shortage costs)	network design	operational decisions	Var, PF
Bihlmaier et al. 2009	2S	demand	network design	operational decisions	
Kauder and Meyr 2009	Det+Sim	demand exchange rates	network design	operational decisions	
Schütz et al. 2009	2S	demand (all 2 <sup>nd</sup> stage parameters)	network design	operational decisions	
Nickel et al. 2010	MS	demand return of investments	network design financial investments loans	transportation	EDR
Stephan et al. 2011 Our model	StDP 2S+Sim	demand (Markovian) demand productivity exchange rates labour costs raw material costs transport costs	network design network design	operational decisions operational decisions	CV@R

**Table 4.2.** Literature overview – stochastic features. Abbreviations are summarised in table 4.4.

Columns	Notation	Meaning
2, 7	S	supplier
2, 7	F	facility
2, 7	DC	distribution centre
2, 7	$\mathbf{M}$	market
2, 7	P	product
2	$\operatorname{Bold}$	network nodes with capacity decisions; other nodes are fixed
3, 5	X	Model can handle an arbitrary number
5	$X_s, X_o$	$x_s$ : time periods on strategic level; $x_o$ : time periods on operational level
6	b	node capacity modeled as binary on/off decision
6	e	choice from multiple, discrete capacity possibilities, requiring
		multiple 0-1 variables
6	$^{\mathrm{c}}$	continuous capacity decision

**Table 4.3.** Abbreviations for network features used in table 4.1.

Columns	Notation	Meaning
2	2S	two-stage stochastic program
2	Det	deterministic optimisation of the HN decisions
2	MS	multi-stage stochastic program
2	$\operatorname{StDP}$	stochastic dynamic program
2	Sens	sensitivity analysis: optimisation for each scenario and re-
		porting the optimal objective under that scenario.
2	$\operatorname{Sim}$	simulation (ex-post evaluation) of the recourse decisions for
		a given solution of the HN decisions
3	(param)	model can handle uncertainty in <i>param</i> , but this is not in-
	(1 )	vestigated in the result analysis
4	$_{ m HN}$	here-and-now
6	EDR	expected downside risk, see section 2.2.2
6	shortages	expected value of relative, uncovered demand
6	$\mathbb{V}\mathrm{ar}$	variance, see section 2.2.2
6	PF	probability of failure to meet a given target objective, see
O .		section 2.2.2
6	CV@R	conditional value at risk, see section 2.2.2.
O	Cv@n	conditional value at 115K, see section 2.2.2.

**Table 4.4.** Abbreviations for stochastic features used in table 4.2.

#### **Identified Research Problems**

The literature review and the summary tables show that each of the requirements for a comprehensive SCND model (identified in section 2.3) appears in some of the research literature, but no work covers them altogether. In particular, there has been little research on multiple sources of uncertainty, coherent risk measures, and the resulting trade-off between risk and return. In the remainder of this chapter we present a model, which sets out to answer the research question 'What is an appropriate way to model the SCND problem?'

# 4.2. Model Description

# 4.2.1. Model Development

We develop a two-stage stochastic mixed integer programming problem with 0-1 and continuous decisions in the first stage and continuous decisions only in the second stage. The strategic network design decisions are in the first stage, while the operational second stage decisions follow naturally as recourse actions.

#### **Sets and Uncertainties**

The network underlying our model is defined in terms of suppliers, facilities, and markets. The facilities have multiple echelons which reflect the BoM structure of the products. This is a problem for a multinational company so that we have a set of currencies which are all eventually converted to the company's base currency. Further, we have a set of time periods and a set of different labour shifts. Uncertainty is captured by using discrete scenarios for demand, productivity, exchange rates, labour costs, raw material costs, and transport costs. This means, our model has uncertain parameters appearing in the right hand side, the technology matrix, as well as the objective.

## **Objective**

The objective has two components: maximising expected profit and minimising CV@R. By varying the weighting between these two, we are able to explore the trade-off between risk and return and, hence, to find solutions in accordance with the decision maker's risk preference. The expected profit is made up of revenues from sales less investment costs, operational costs, and penalties for unsatisfied demands. All monetary parameters are expressed as their net present value in the local currency, which is then converted to the base currency using defined exchange rates.

A general modelling paradigm underlying the cost structure is to use 0-1 and continuous variables together – this allows the inclusion of fixed costs and, hence, simple economies of scale.

## First Stage Strategic Decisions

The main decisions are to determine which facilities are open and what level of capacity should be installed. Associated with setting facility capacities there are decisions indicating whether capacities are increased or decreased between two time periods and, if so, by how much. Further, there are decisions to determine which products can be produced at which factory as well as determining which suppliers are operating. Finally, target levels for facility utilisation and labour levels are set. Deviations from these are penalised by second stage variables to account for exploiting flexibility. This reflects a company's desire not to shift production quantities rapidly between facilities, since a large variation of production amounts leads to organisational costs, penalties associated for breaking long term contracts with suppliers, redundancy payments, and recruitment costs.

#### **Second Stage Operational Decisions**

On the operational level, we have decisions about transport volumes. Based on these, production volumes, supplier purchases, as well as market deliveries and market short-falls are determined. Decisions on labour per shift at different variable cost rates for each shift reflect the increase of costs due to shift surcharges. Finally, we model by how much capacity consumption and labour requirements differ from their target levels.

#### **Constraints**

The following gives a brief outline of the constraints.

#### Constraints on first stage choices

Connect investments and divestments with the facility capacity via capacity balance equation.

Restrict facility capacity by facility choice.

Restrict divestment and investment of capacity by the corresponding choice decisions.

# Constraints connecting first stage and second stage decisions

Restrict supplier purchases by supplier choice.

Bound production capacity utilisation by facility capacity.

Restrict production by product-facility assignments.

Calculate deviations of capacity utilisation and labour requirement from target levels.

# Second stage recourse constraints

Network flow balance equations with BoM.

Connect labour per shift with total labour requirement.

Bound market sales by market demand.

CV@R calculation.

#### 4.2.2. Discussion of Model Features

#### Choice of Risk Measure

We use the CV@R as a risk measure for two reasons: most importantly, it as a coherent risk measure (see section 2.2.2 and appendix A) and therefore fulfills a set of commonly accepted desirable properties. Further, it is easily included in stochastic programs in a linear form; therefore, it does not require additional 0-1 variables.

# **Financial Hedging**

Our model includes multiple currencies which are exposed to exchange rate uncertainty. However, we do not include financial risk hedging instruments as these apply to shorter time scales and are therefore suitable for tactical rather than strategic models.

#### **Objective Features**

As mentioned in the problem description in section 1.1, the model presented in this paper was developed for an automobile manufacturer. For simplicity of notation we aggregate details in our model cost parameters. This is a simplification of the company's cost model which meets internal accounting requirements. These aggregations include international features such as import duties, export incentives, transfer prices, and taxes.

#### DCs and Storage

As our model is aimed at automotive supply chains, we do not consider DCs, as these do not play a crucial role in this industry. Further, in line with Fleischmann et al. (2006), our model does not allow any storage decisions at facilities. Stock levels are usually investigated by tactical and operational supply chain models.

# **Continuous Facility Capacity**

As discussed in section 2.1, the decision process of SCND consists of several phases with varying detail. The model presented in this chapter is designed for the early phases and, therefore, aims to identify promising network strategies. These are analysed in subsequent planning phases in more detail. As a consequence of this approach, we allow facility capacities to be continuous values rather than selecting from finite, discrete possibilities. This is appropriate for the given situation since an estimate of useful capacities is made first, before subsequently actual machinery setups are considered which require discrete choices. Also, continuous capacities give the model the possibility to react naturally to parameter values and uncertainty.

## Two-stage vs. Multistage

We model the decision process as two-stage, since we believe that this gives a good trade-off between accuracy and computational complexity. Multi-stage models have an advantage over two-stage models if decisions in earlier time periods significantly affect decisions in later periods.

However, we do not consider storage and, therefore, in the operational sub-model only labour and the use of flexibility have an influence over multiple time periods, since changing the number of labour or the facility utilisation between consecutive time periods generates costs. In the model this is approximated by deviations from a target level, which we believe is reasonably accurate and the resulting operations are more realistic than in the case where this aspect is neglected.

On the strategic level, the influence of decisions across time is more obvious, since investments and divestments affect the network design for all subsequent time periods. Few research exists on the benefit of multi-stage SCND over a rolling two-stage decision process; only Stephan et al. (2011) investigate this and in their case studies the difference is at most 0.55%. Therefore, we think that a two-stage model is sufficiently accurate.

#### 4.2.3. Model Formulation

In this section, a complete formulation of the mathematical programming model is given, where the following conventions are used for notations: sets and decision variables are denoted by upper case letters, set elements and parameters by lower case letters, uncertain parameters are bold, and parameters and decision variables are assumed to be zero for all non-defined index combinations. Further, units of measurement are given in square brackets, denoted as follows:

[CU] capacity units

[BC] monetary units in the base currency

[LC] monetary units in local currency

[PU] product units

[LH] labour hours

#### Sets

We have the following index sets:

S suppliers, indexed by s

F facilities, indexed by f

P products, indexed by p

M markets, indexed by m

Y shifts, indexed by y

C currencies, indexed by c

 $T = \{\underline{t}, \dots, \overline{t}\}$  time periods, indexed by t

 $\Omega$  scenarios, indexed by  $\omega$ 

In order to reflect the sparse structure present in supply chain networks, we define the following multi-dimensional sets:

- $PP \subseteq P \times P$  represents a product-product dependency, with  $(p_1, p_2) \in PP$  if and only if (iff) product  $p_2$  is needed to build product  $p_1$ . This represents the digraph of the BoM structure.
- $SP \subseteq S \times P$  represents a supplier-product dependency, with  $(s, p) \in SP$  iff product p can be delivered by supplier s.
- $FP \subseteq F \times P$  represents a facility-product dependency, with  $(f, p) \in FP$  iff product p can be produced at facility f.
- $MP \subseteq M \times P$  represents a market-product dependency, with  $(m, p) \in MP$  iff product p is sold at market m.
- $SFP \subseteq S \times F \times P$  represents a supplier-facility-product transport relation, with  $(s, f, p) \in SFP$  iff product p can be delivered from supplier s to facility f. Therefore, (s, f, p) must satisfy at least the following conditions

$$(s,p) \in SP$$
 and  $\exists p_2 \in P$  s.th.  $(f,p_2) \in FP \land (p_2,p) \in PP$ ,

since supplier s can only deliver product p to facility f if p is one of its products according to the supplier-product dependency SP, and if p is needed for the production of a product  $p_2$  which is manufactured at facility f.

 $FFP \subseteq F \times F \times P$  represents a facility-facility-product transport relation, with  $(f_s, f_d, p) \in FFP$  iff product p can be delivered from facility  $f_s$  to facility  $f_d$ . Therefore,  $(f_s, f_d, p)$  must satisfy at least the following conditions

$$(f_s, p) \in FP$$
 and  $\exists p_2 \in P$  s.th.  $(f_d, p_2) \in FP \land (p_2, p) \in PP$ ,

since the start-facility  $f_s$  can only deliver product p to the destination-facility  $f_d$  if p is one of its products according to the facility-product dependency FP, and if p is needed for the production of a product  $p_2$  which is manufactured at facility  $f_d$ .

 $FMP \subseteq F \times M \times P$  represents a facility-market-product transport relation, with  $(f, m, p) \in FMP$  iff product p can be delivered from facility f to market m. Therefore, (f, m, p) must satisfy at least the following conditions

$$(f,p) \in FP$$
 and  $(m,p) \in MP$ ,

since facility f can only deliver product p to the market m if p is one of its products according to the facility-product dependency FP, and if p is sold at market m according to the market-product dependency MP.

#### **Decision Variables**

According to the modelling paradigm of two-stage stochastic programming, we distinguish between two types of variables: first stage variables, which are not indexed over the scenario set  $\Omega$ , and second stage variables, which are indexed over  $\Omega$ .

# First Stage Strategic Decisions

 $SO_{st} \in \{0,1\}$ . Supplier s is operational (1) or not operational (0).

 $FO_{ft} \in \{0,1\}$ . Facility f is operational (1) or not operational (0).

 $C_{ft} \geq 0$ , [CU]. capacity of facility f.

 $IF_{ft}^+ \in \{0,1\}, t \in \{\underline{t}+1,\ldots,\overline{t}\}.$  Indicates if the capacity of facility f is increased between time period t-1 and time period t.

 $IF_{ft}^- \in \{0,1\}, t \in \{\underline{t}+1,\ldots,\overline{t}\}.$  Indicates if the capacity of facility f is decreased between time period t-1 and time period t.

 $IV_{ft}^+ \geq 0, t \in \{\underline{t}+1,\ldots,\overline{t}\}$ , [CU]. Amount by which the capacity is increased between time period t-1 and time period t.

 $IV_{ft}^- \geq 0, t \in \{\underline{t}+1, \dots, \overline{t}\}, [CU]$ . Amount by which the capacity is decreased between time period t-1 and time period t.

 $PF_{fpt} \in \{0,1\}, (f,p) \in FP$ . Product p is assigned to facility f between time period t-1 and time period t.

 $CP_{ft}^t \geq 0$ , [CU]. Target capacity consumption for production.

 $CL_{ft}^t \geq 0$ , [LH]. Target labour level.

 $V@R \in \mathbb{R}$ , [BC]. Value at Risk.

#### **Second Stage Operational Decisions**

 $TS_{sfpt\omega} \geq 0, (s, f, p) \in SFP$ , [PU]. Transport volume from supplier to facility.

 $TF_{f_sf_dpt\omega} \geq 0, (f_s, f_d, p) \in FFP,$  [PU]. Transport volume from facility to facility.

 $TM_{fmpt\omega} \geq 0, (f, m, p) \in FMP,$  [PU]. Transport volume from supplier to market.

 $CP_{ft\omega}^+ \geq 0$ , [CU]. Capacity consumption above target level.

 $CP_{ft\omega}^{-} \geq 0$ , [CU]. Capacity consumption below target level.

 $L_{fyt\omega} \geq 0$ , [LH]. Labour employed in shift mode y.

 $CL_{ft\omega}^+ \geq 0$ , [LH]. Labour above target level.

 $CL_{ft\omega}^- \geq 0$ , [LH]. Labour below target level.

 $\Delta_{\omega} \geq 0$ , [BC]. The maximum of zero and the difference between the profit in scenario  $\omega$  and V@R. Needed for CV@R calculation.

#### **Notation**

To make equations easier to read, we denote by  $PV_{fpt\omega}$  the production volume of product p at facility f which can be calculated as the total amount of product p being shipped from facility f to other facilities and to markets,

$$PV_{fpt\omega} = \sum_{f2} TF_{ff_2pt\omega} + \sum_{m} TM_{fmpt\omega}, \quad [PU], \ \forall (f,p) \in FP, t, \omega.$$

#### **Parameters**

## **Technology Parameters**

 $\overline{c}_f$  > 0, [CU]. Maximum facility capacity.

 $\underline{c}_f \geq 0$ , [CU]. Minimum facility capacity, if facility is operating.

 $\overline{c}_f^+$   $\geq 0$ , [CU]. Maximum facility capacity increase. Should satisfy  $\overline{c}_f^+ \leq \overline{c}_f$  since  $\overline{c}_f^+$  serves as big M in a constraint and the capacity increase cannot be more than the maximum capacity.

 $\overline{c_f}$   $\geq 0$ , [CU]. Maximum facility capacity decrease. Should satisfy  $\overline{c_f} \leq \overline{c_f}$  since  $\overline{c_f}$  serves as big M in a constraint and the capacity decrease cannot be more than the maximum capacity.

 $\underline{c}_f^+$   $\geq 0$ , [CU]. Minimum facility capacity increase, if the capacity is increased.

 $\underline{c}_f^ \geq 0$ , [CU]. Minimum facility capacity decrease, if the capacity is decreased.

 $\overline{s}_{sp}$  > 0,  $(s, p) \in SP$  [PU]. Maximum amount of product p supplier s can deliver.

 $cp_{fpt\omega} > 0, (f, p) \in FP$  [CU/PU]. Capacity consumption for producing product p.

 $cl_{fpt\omega} \ge 0, (f, p) \in FP$  [LH/PU]. Labour hours needed for producing product p.

 $d_{mpt\omega}$   $(m, p) \in MP$  [PU]. Demand of product p at market m.

 $bom_{p_1p_2} > 0, (p_1, p_2) \in PP$ , [PU<sub>2</sub>/PU<sub>1</sub>]. Amount of product  $p_2$  needed to produce one item of product  $p_1$ .

## **Objective Parameters**

- $\pi_{\omega} \in (0,1]$ . Scenario probability, which should satisfy  $\sum_{\omega} \pi_{\omega} = 1$ .
- $\alpha \in [0,1]$ . Weighting of the two objectives EP ( $\alpha = 1$ ) and CV@R ( $\alpha = 0$ ).
- $\beta \in [0,1]$ . Confidence level of the CV@R. Should be in (0.5,1) to measure the lower tail.
- $\rho_{mptc}$   $(m,p) \in MP$ , [LC/PU]. Revenue for selling product p at market m.
- $\sigma_{mptc}$   $(m,p) \in MP$ , [LC/PU]. Shortage penalty costs for for not being able to meet customer demand of product p at market m.
- $xr_{tc\omega}$  > 0, [LC/BC]. Exchange rate between local currency c and the base currency.
- $\gamma^{SF}_{stc\omega}$  [LC]. Fixed supplier costs.
- $\gamma_{sptc\omega}^{SV}$  [LC/PU]. Variable supplier costs. This includes the purchase costs for the products bought from the supplier.
- $\gamma_{ftc}^{FO}$  [LC]. Fixed facility costs.
- $\gamma^{C}_{ftc}$  [LC/CU]. Variable facility costs per capacity unit.
- $\gamma_{ftt_2c}^{IF^+}$  [LC]. Fixed costs in time period t for capacity increases in period  $t_2$ . Needs to satisfy  $\sum_{t_2} \gamma_{ftt_2c}^{IF^+} > 0$ .
- $\gamma_{ftt_2c}^{IF^-}$  [LC]. Fixed costs in time period t for capacity decreases in period  $t_2$ . Needs to satisfy  $\sum_{t_2} \gamma_{ftt_2c}^{IF^-} > 0$ .
- $\gamma_{ftt_2c}^{IV^+}$  [LC/CU]. Variable costs in time period t for capacity increases in time period  $t_2$ .
- $\gamma_{ftt_2c}^{IV^-}$  [LC/CU]. Variable costs in time period t for capacity decreases in time period  $t_2$ .
- $\gamma_{fptt_2c}^{PF}$   $(f,p) \in FP$ , [LC]. Fixed costs in time period t for assigning products to facilities in time period  $t_2$ .
- $\gamma_{fptc}^{PV}$   $(f,p) \in FP$ , [LC/PU]. Variable production costs.
- $\gamma_{ttc...}^{LF}$  [LC]. Fixed labour costs.
- $\gamma^{LV}_{fytc\omega}~~$  [LC/LH]. Variable labour costs per labour hour.
- $\gamma_{sfptc\omega}^{TS}$   $(s, f, p) \in SFP$ , [LC/PU]. Transport costs from supplier to facility.
- $\gamma_{f_sf_dptc\omega}^{TF}$   $(f_s, f_d, p) \in FFP$ , [LC/PU]. Transport costs from facility to facility.
- $\gamma_{fmptc\omega}^{TM}$   $(f, m, p) \in FMP$ , [LC/PU]. Transport costs from facility to market.
- $\gamma_{ftc}^{CP^+}$  [LC/CU]. Costs for capacity consumption above target level.
- $\gamma_{ftc}^{CP^-}$  [LC/CU]. Costs for capacity consumption below target level.
- $\gamma_{ftc}^{CL^+}$  [LC/LH]. Costs for labour above target level.

 $\gamma_{ftc}^{CL^-}$  [LC/LH]. Costs for labour below target level.

Note that the cost parameters for investment decisions  $IF^+, IF^-, IV^+, IV^-$ , and PF are indexed twice over time periods. This is needed to correctly calculate the net present value (NPV), since an investment decision with start of production in time period  $t_2$  can have investment costs in earlier time periods but also might have a residual book value at the end of the planning horizon. Also, especially product to facility assignments might cause additional fixed costs in following time periods, for example for maintenance or increased complexity.

## **Objective**

The goal is to maximise a weighted sum of the expected profit associated with the network design decision and its risk measured as CV@R. Profit has three elements: revenues from market sales of products, costs associated with the decisions, and penalty costs if market demand is not satisfied.

First, for time period t, currency c, and scenario  $\omega$ , we define the elements of the objective, which are all measured in local currency [LC]. We can calculate revenues as

$$\rho_{tc\omega} = \sum_{f,m,p} \rho_{mptc} T M_{fmpt\omega},$$

and shortage penalties as

$$\sigma_{tc\omega} = \sum_{m,p,t} \sigma_{mptc} (\boldsymbol{d}_{mpt\omega} - \sum_{f} TM_{fmpt\omega}).$$

Facility costs are the sum of the fixed facility costs for operating facilities and the capacity-dependent variable facility costs, expressed as

$$\gamma_{tc}^{F} = \sum_{f} \left( \gamma_{ftc}^{FO} F O_{ft} + \gamma_{ftc}^{C} C_{ft} \right).$$

Similarly, investment costs for capacity changes are calculated from fixed and variable costs for capacity decreases and increases as

$$\gamma_{tc}^{I} = \sum_{f,t_2} \left( \gamma_{ftt_2c}^{IF^+} IF_{ft_2c}^+ + \gamma_{ftt_2c}^{IF^-} IF_{ft_2c}^- + \gamma_{ftt_2c}^{IV^+} IV_{ft_2c}^+ + \gamma_{ftt_2c}^{IV^-} IV_{ft_2c}^- \right).$$

Supplier costs include fixed costs for operating suppliers plus variable costs per product bought. Note that this can also be used to express simple economies of scale in the purchase costs from suppliers.

$$\gamma_{tc\omega}^{S} = \sum_{s} \gamma_{stc\omega}^{SF} SO_{st} + \sum_{s,f,p} \gamma_{sptc\omega}^{SV} TS_{sfpt\omega}.$$

Production costs consist of fixed costs for assigning products to facilities, variable costs per produced product, and costs for production differing from the target level. This is expressed as

$$\gamma_{tc\omega}^{P} = \sum_{f,p,t_2} \gamma_{fptt_2c}^{PF} PF_{fpt_2} + \sum_{f,p} \gamma_{fptc}^{PV} PV_{fpt\omega} + \sum_{f} \left( \gamma_{ftc}^{CP^+} CP_{ft\omega}^+ + \gamma_{ftc}^{CP^-} CP_{ft\omega}^- \right).$$

Labour has fixed costs, variable costs per shift, and costs for labour differing from the target level:

$$\gamma_{tc\omega}^{L} = \sum_{f} \gamma_{ftc\omega}^{LF} F O_{ft} + \sum_{f,y} \gamma_{fytc\omega}^{LV} L_{fyt\omega} + \sum_{f} \left( \gamma_{ftc}^{CL^{+}} C L_{ft\omega}^{+} + \gamma_{ftc}^{CL^{-}} C L_{ft\omega}^{-} \right).$$

Finally, transport costs are defined as

$$\gamma_{tc\omega}^{T} = \sum_{s,f,p} \gamma_{sfptc\omega}^{TS} T S_{sfpt\omega} + \sum_{f_s,f_d,p} \gamma_{f_sf_dptc\omega}^{TF} T F_{f_sf_dpt\omega} + \sum_{f,m,p} \gamma_{fmptc\omega}^{TM} T M_{fmpt\omega}.$$

Now, the costs per local currencies can be defined as

$$\gamma^{LC}_{tc\omega} = \gamma^F_{tc} + \gamma^I_{tc} + \gamma^S_{tc\omega} + \gamma^P_{tc\omega} + \gamma^L_{tc\omega} + \gamma^T_{tc\omega}, \text{ [LC]}.$$

Summing over time periods while accounting for revenues, costs, and shortage penalties, we can exchange the profit from local currencies to the base currency which gives the profit per scenario in base currency,

$$\phi_{\omega} = \frac{1}{\boldsymbol{xr}_{tc\omega}} \sum_{t,c} \left( \rho_{tc\omega} - \gamma_{tc\omega}^{LC} - \sigma_{tc\omega} \right), \text{ [BC]}.$$

Finally, this gives the expected profit as the weighted sum of the profits per scenario:

$$\phi = \sum_{\omega} \pi_{\omega} \phi_{\omega}, \text{ [BC]}.$$

We include the CV@R in the objective by using the formulation by Rockafellar and Uryasev (2000) as introduced in section 2.2.3. Therefore, the model includes a constraint  $\Delta_{\omega} \geq V@R - \phi_{\omega}, \forall \omega$ , (see p. 61), which enables us to express the CV@R by minimising the term

$$\psi = -\left(V@R - \frac{1}{1-\beta}\sum_{\omega}\pi_{\omega}\Delta_{\omega}\right), \text{ [BC]}.$$

Now, our objective is to maximise the weighted sum of EP and CV@R:

maximise 
$$z = \alpha \phi - (1 - \alpha) \psi$$
  
=  $\sum_{\omega} \pi_{\omega} \left[ \alpha \phi_{\omega} + (1 - \alpha) \left( V@R - \frac{1}{1 - \beta} \Delta_{\omega} \right) \right], [BC]$ 

#### **Constraints**

We categorise the constraints into three groups: first stage constraints, which only contain first stage variables and are therefore independent of the scenario index  $\Omega$ ; linking constraints, which contain first as well as second stage variables; and operational constraints, which contain second stage variables only. Therefore, the first stage constraints are part of the master problem, while the linking and second stage constraints are part of the recourse problem.

#### **Constraints on First Stage Choices**

## Connect investments and divestments with capacity via capacity balance equation

The capacity increase minus the capacity decrease has to be equal to the difference between the capacity in time period t and the capacity in time period t-1. Note that for simplicity we assume the initial capacity in time period  $\underline{t}-1$  to be zero. This could of course be replaced by a parameter.

$$IV_{ft}^{+} - IV_{ft}^{-} = C_{ft} - \begin{cases} C_{f,t-1}, & t > \underline{t} \\ 0, & t = \underline{t} \end{cases}, \quad [CU], \forall f, t$$

Restrict facility capacity by facility choice The capacity of facility a f can only be greater than zero, if the facility is operating. Also, the capacity has to be at least the minimum capacity in this case.  $\overline{c}_f$  is the tightest possible big M since there is equality if f's capacity is at its maximum  $\overline{c}_f$ .

$$\underline{c}_f FO_{ft} \le C_{ft} \le \overline{c}_f FO_{ft}, \qquad [CU], \forall f, t$$

Restrict divestment and investment capacity by divestment and investment choice, respectively Facility capacity decreases  $IV_{ft}^-$  (increases  $IV_{ft}^+$ ) are connected with the according binary decision  $IF_{ft}^-$  ( $IF_{ft}^+$ ). Also, the minimum and maximum capacity decreases (increases) are established here.

$$\underline{c}_f^- I F_{ft}^- \leq I V_{ft}^- \leq \overline{c}_f^- I F_{ft}^- \qquad \text{and} \qquad \underline{c}_f^+ I F_{ft}^+ \leq I V_{ft}^+ \leq \overline{c}_f^+ I F_{ft}^+, \qquad [CU], \forall f, t \in \mathcal{C}_f$$

Constraints Connecting First Stage and Second Stage Decisions

Restrict supplier purchases by supplier choice Product p can only be shipped from supplier s to factories if s is operating.  $\overline{s}_{sp}$  is the tightest possible big M since this equation also establishes the supplier's product specific capacity.

$$\sum_{f} TS_{sfpt\omega} \le \overline{s}_{sp}SO_{st}, \qquad [PU], \forall s, p, t, \omega$$

Bound production capacity utilisation by facility capacity The capacity used to produce all products at facility f is bounded by the available capacity  $C_{ft}$ .

$$\sum_{p} c p_{fpt\omega} PV_{fpt\omega} \le C_{ft}, \qquad [CU], \forall f, t, \omega$$

Restrict production by product-facility assignments Product p can only be produced at facility f if p has been assigned to f.  $\frac{\overline{c}_f}{cp_{fpt\omega}}$  is the tightest big M we can use here: if the capacity of f is at its maximum  $\overline{c}_f$  and the production of p utilises f's capacity to the full, then there is equality in the constraint.

$$PV_{fpt\omega} \le \frac{\overline{c}_f}{cp_{fpt\omega}} \sum_{t_2=t}^t PF_{fpt_2}, \qquad [PU], \forall f, p, t, \omega$$

Calculate deviations of capacity utilisation and labour requirement from target levels Capacity consumption and labour requirements are linked to the target level as follows:

$$CP_{ft}^{t} + CP_{ft\omega}^{+} - CP_{ft\omega}^{-} = \sum_{p} \boldsymbol{c}\boldsymbol{p}_{fpt\omega}PV_{fpt\omega}, \qquad [CU], \forall f, t, \omega$$
$$CL_{ft}^{t} + CL_{ft\omega}^{+} - CL_{ft\omega}^{-} = \sum_{p} \boldsymbol{c}\boldsymbol{l}_{fpt\omega}PV_{fpt\omega}, \qquad [LH], \forall f, t, \omega$$

#### **Second Stage Recourse Constraints**

**Network flow balance equation with BoM** The network flow balance equation states that the amount of product p coming in at a facility f from all other factories and suppliers, has to be equal to the amount needed for products  $p_2$  produced at f. The outbound balance equation is satisfied via the definition of  $PV_{fp_2t\omega}$ .

$$\sum_{s} TS_{sfpt\omega} + \sum_{f_s} TF_{f_sfpt\omega} = \sum_{p_2} bom_{p_2p} PV_{fp_2t\omega}, \qquad [PU], \forall f, p, t, \omega$$

Connect labour per shift with total labour requirement The labour working in all shifts at facility f must meet the capacity required by production at f.

$$\sum_{y} L_{fyt\omega} = \sum_{p} c l_{fpt\omega} P V_{fpt\omega}, \qquad [LH], \forall f, t, \omega$$

**Bound market sales by market demand** The amount of a product p delivered to market m is limited by the demand of p at m. Note that shortages are allowed.

$$\sum_{f} TM_{fmpt\omega} \le \mathbf{d}_{mpt\omega}, \qquad [PU], \forall m, p, t, \omega$$

**Risk Calculation** Using the formulation by Rockafellar and Uryasev (2000), the CV@R can be calculated by using the following constraint, where  $\phi_{\omega}$  is the profit under scenario  $\omega$ .

$$\Delta_{\omega} \geq V@R - \phi_{\omega}, \quad [BC], \forall \omega$$

## 4.3. Random Parameters: Scenario Generation

In our model, uncertainty is present in demand, exchange rates, labour costs, productivity, supplier costs, and transport costs. These parameter uncertainties are captured through discrete scenarios created by scenario generators (see for example Di Domenica et al., 2007, 2009). The testing of scenario generators and their desirable properties, namely correctness, consistency, and stability, are discussed by Kaut and Wallace (2003) and Mitra et al. (2009). As scenario generation is not a main contribution of this thesis, we only briefly analyse the stability of the presented scenario generator in sectionsub:stabilityAnalysis.

We present a scenario generator that extends a moment matching algorithm (see Høyland et al., 2003; Date et al., 2008, and section 4.3.1) to multiple time periods by combining it with sampling. Thereby we generate vector-valued scenarios with a two-stage tree structure that captures correlations between components of the random vector. The scenario generator uses the company's forecast as the expected value since this is their best guess about future developments and is often based on sophisticated models and expert knowledge. Also, under this assumption the expected value problem matches a deterministic optimisation study undertaken by the company. The details of the scenario generator are described subsequently.

#### 4.3.1. Moment Matching

Assume our model has m-dimensional uncertainty represented by a random variable  $\tilde{\xi}: (\tilde{\Omega}, \tilde{\mathcal{F}}, \tilde{P}) \to (\mathbb{R}^m, \mathcal{B})$  where  $\mathcal{B}$  is the Borel  $\sigma$ -algebra on  $\mathbb{R}^m$  and let  $\Omega = \{1, \dots, N\}$ . The aim of moment matching scenario generation is to find a discrete random variable  $\xi: \Omega \to \mathbb{R}^m$  (the scenarios) that matches moments of  $\tilde{\xi}$ . Moment matching is suitable when the probability distribution of  $\tilde{\xi}$  is unknown, but observed realisation of  $\tilde{\xi}$  are given, since in this case the moments of  $\tilde{\xi}$  can be estimated.

We use the moment matching heuristic by Høyland et al. (2003), which approximates the first four marginal moments for each of the m components of  $\tilde{\xi}$  (expected value, variance, skewness, and curtosis) and the  $m \times m$  correlation matrix. In this approach, the goal is to find  $\xi$ , such that the Euclidean distance of the moments is less than some tolerance  $\varepsilon > 0$ :

$$\sum_{i=1}^{4} \left\| \mathbb{E}[\xi^{i}] - \mathbb{E}[\tilde{\xi}^{i}] \right\|^{2} + \sum_{k=1}^{m} \sum_{l=1}^{m} \left( \operatorname{corr}(\xi_{k}, \xi_{l}) - \operatorname{corr}(\tilde{\xi}_{k}, \tilde{\xi}_{l}) \right)^{2} \leq \varepsilon^{2}.$$

Here,  $\|.\|$  denotes the Euclidean norm and corr the correlation. The scenarios are constructed by an iterative procedure combining independent sampling, Cholesky decomposition of the covariance matrix, and cubic transformations. The resulting scenarios are all equally likely.

Date et al. (2008) present a different approach to moment matching scenario generation. Under the assumption that  $\tilde{\xi}$  is symmetric, it matches the first two moments exactly and approximates the fourth moments.

## 4.3.2. Bootstrapping

The main idea of (non-parametric) bootstrapping is to sample with replacement from historical data. In statistics, this is used to estimate a distribution of some statistic  $\theta$  on the random sample, such as the expected value, (Efron and Tibshirani, 1993; Cheng, 2006; Vose, 2008). The approach is set out in algorithm 4.1.

#### Algorithm 4.1 Bootstrapping

**Input:**  $N, B \in \mathbb{N}$ , historical data  $h1 - \tau, \dots, h_0$ .

- 1: **for** b = 1 **to** B **do**
- 2: Create a bootstrap sample  $s_b^1, \ldots, s_b^N$  by uniformly and independently sampling with replacement from  $h_{1-\tau}, \ldots, h_0$ .
- 3: Calculate the required statistic  $\hat{\theta}_b$  from the sample.
- 4: end for
- 5: **return** Use the empirical distribution of  $\tilde{\theta}_b$ ,  $b = 1, \ldots, B$  as probability distribution of the uncertainty in estimating the real statistic  $\theta$ .

An advantage of bootstrapping is that the generated scenarios are correct and consistent with the dataset in the sense of Zenios (2007). A disadvantage is that the values in the bootstrap samples are limited to those of the original sample  $h_{1-\tau}, \ldots, h_0$ . Bootstrapping is similar to the sampling involved in sample average approximation (see section 5.3.4).

## 4.3.3. Extended Moment Matching for Multi Time Period Scenarios

Our scenario generator is aimed at models where the recourse includes several time periods  $1, \ldots, T$ . Therefore, the uncertain vector  $\xi$  is grouped as  $\xi = (\xi_1, \ldots, \xi_T)^T$  with multivariate random variables  $\xi_i : \Omega \to \mathbb{R}^m, i = 1, \ldots, T$ . The main idea is to use

#### Algorithm 4.2 A multi time period scenario generator

```
Input: Desired number of scenarios N, number of future time periods T, time series of historical data h_{1-\tau}, \ldots, h_0.
```

Optional: base case scenario  $b = (b_1, \ldots, b_T)^T$ .

- 1: Calculate first 4 moments and correlation from  $h_{1-\tau}, \ldots, h_0$ .
- 2: **if** base case scenario b is given **then**
- 3: Replace first moment (mean) by 0.
- 4: **else**
- 5: Define b = 0.
- 6: end if
- 7: Use moment matching to generate N scenarios  $\tilde{s}^1, \dots, \tilde{s}^N$ .
- 8: **for** t = 1 **to** T **do**
- 9: Sample N times without replacement from  $\tilde{s}^1, \dots, \tilde{s}^N$  to receive scenarios  $s_t^1, \dots, s_t^N$ .
- 10: end for
- 11: **return** scenarios  $\xi(\omega) = (s_1^{\omega} + b_1, \dots, s_T^{\omega} + b_T)^T$ ,  $\omega \in \Omega = \{1, \dots, N\}$ , each with probability  $\frac{1}{N}$ .

moment matching to generate single time period scenarios and extend them by sampling without replacement to multi time period scenarios. The scenario generator is set out in algorithm 4.2. Given a time series of historical data, the moments are calculated and adjusted if a base case scenario is given. The moment matching scenario generator is applied once to generate N single period scenarios. Then, we sample N times without replacement for each time period, thereby creating a random permutation of the single period scenarios. Finally, a multi period scenario is the series of permutated single period scenarios. These are adjusted by the base case scenario to separate scenario generation from forecasting. If it is not given, the forecast trend of the scenarios is the mean of the historical data.

We observe that by sampling without replacement in step 9 we retain the moments of the generated scenarios. Many multi time period scenario generators combine all scenarios from one time period with all scenarios from other time periods, resulting in  $N^T$  scenarios. The sampling approach in this step guarantees that the number of scenarios is independent of the number of time periods.

#### 4.3.4. Autoregressive Transformation of the Time Series

For many applications, using direct observations as historical data gives crude results, as values observed several time periods earlier might be directly followed by recent values, therefore possibly leading to large jumps in the values. This is because the scenario generator itself assumes the values of two consecutive time periods of a scenario as independent, identically distributed. However, this can be changed by suitable transformations of the input data and appropriate backwards transformations of the

scenarios. For example, applying the scenario generator on

$$h'_{t} = h_{t} - h_{t-1}, \ t = 2 - \tau, \dots, 0$$

assumes a linear relationship between consecutive time periods. Using the back-transformation

$$h_t = h_{t-1} + h_t',$$

we see that this is an autoregressive process of order one if we interpret  $h'_t$  as the error term (Chatfield, 2003). In this sense, we generate scenarios for the error term  $h'_t$ . Alternatively,

$$h_t' = \frac{h_t}{h_{t-1}}$$

assumes a relative relationship, which – via logarithmic transformation – also gives an autoregressive model of order one. Similar transformations of order two or more are possible.

## 4.3.5. Example

A typical example with 20 scenarios for one uncertainty (m = 1) is given in figure 4.1. We apply relative transformation of the historical data  $h'_t = h_t/h_{t-1}$ , and have a constant base case scenario  $b_t = h_0$ , with the transformed base case scenario taking the value  $b'_t = 1$ . All elements of algorithm 4.2 applied to the transformed historical data are marked with a dash.

We observe a slight variation of  $\mathbb{E}[\xi_t]$  in figure 4.1. Using the back-transformation with  $\xi_0 = h_0$ , we get

$$\xi_t = \xi_t' \, \xi_{t-1} 
= \xi_t' \cdots \xi_1' \, h_0 
= (s_t' + b_t') \cdots (s_1' + b_1') h_0 
= \left( s_t' + \frac{b_t}{b_{t-1}} \right) \cdots \left( s_1' + \frac{b_1}{h_0} \right) h_0.$$

Therefore, by using the independence of the sampling in step 9 of algorithm 4.2, we get

$$\mathbb{E}[\xi_t] = \mathbb{E}\left[s_t' + \frac{b_t}{b_{t-1}}\right] \cdots \mathbb{E}\left[s_1' + \frac{b_1}{h_0}\right] h_0$$

$$= \frac{b_t}{b_{t-1}} \cdots \frac{b_1}{h_0} h_0$$

$$= b_t$$

$$= h_0.$$

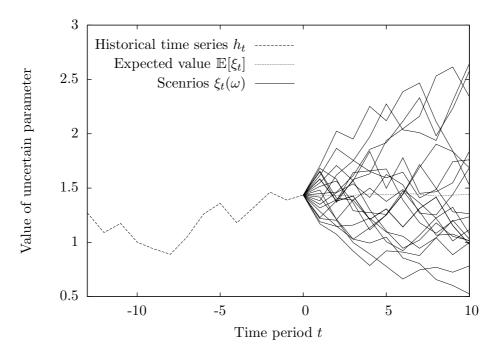


Figure 4.1. Example scenario tree with N=20 scenarios

Thereby we see, that the variation of  $\mathbb{E}[\xi_t]$  in the figure is due to numerical inaccuracies and the approximation of the mean in the moment matching algorithm.

# 4.4. Summary

In this chapter, we have given an extensive review of SCND literature and of related topics; we have shown that many of the desirable features identified in section 2.3 appear in some research, but no single study covers them altogether. We have developed an extensive model suitable for our application that includes many of the essential features. While the holistic network structure and detailed operations are important, its most distinguishing features are the inclusion of multiple sources of uncertainty which are identified in chapter 3, and the consideration of downside risk via a coherent risk measure. Finally, we have presented a scenario generator which captures the essential parameter uncertainties.

The resulting optimisation problem is computationally challenging and we investigate suitable solution techniques which are reported in chapter 5.

# 5. Solving Large Scale Two-Stage Stochastic Integer Programs

Stochastic programs are gaining increasing attention of the OR community. However, as the number of variables as well as constraints grow linearly with the number of scenarios, even with improved specialised solution techniques they remain computationally challenging – especially when some of the variables are integer. Hence, in problems with a large number of scenarios, which is required to have an accurate representation of real world uncertainty, even the most sophisticated solvers reach their limits. In this chapter we present a novel, iterative approach called adaptive scenario refinement (ASR) for solving large scale two-stage SIPs. The aim is to find good solutions for optimisation problems which are otherwise computationally intractable - rather than to find them fast. ASR starts on an examination set containing only one scenario, on which the here-and-now problem is solved. Then, scenarios are added iteratively to this set, based on regret and a distance measure, with the goal to add scenarios from regions of the probability space where more detail is beneficial. Therefore, numerous stochastic programs of increasing size are solved by this algorithm. ASR can be applied to any two-stage SIP, but it works best when the problem has relatively complete recourse (see section 2.2.3). The name 'adaptive scenario refinement' is loosely based on 'adaptive mesh refinement', a technique in numerical analysis that adaptively refines a mesh in areas where more details are needed.

This chapter is organised as follows: In section 5.1 we give notations and assumptions. In section 5.2 we introduce the concept of ex-ante decision making with ex-post evaluation, as this sets the background for the sampling methods in the literature review in section 5.3, as well as the ASR algorithm. We discuss known methods for handling large scale stochastic programs in section 5.3 and present the ASR heuristic in section 5.4. In section 5.5 we demonstrate that ASR is effective and well suited for the SCND model presented in chapter 4, as well as for stochastic programming benchmark problems taken from publicly available test libraries. Our findings are summarised in section 5.6.

## 5.1. Notations and Assumptions

The definitions and notations for two-stage stochastic programs introduced in section 2.2.3 are used throughout this chapter. Based on these, we introduce the following notations and assumptions.

## Support

The support of the probability measure P is defined as the smallest closed set whose probability is 1. If P is discrete, this is

$$\operatorname{supp}(P) = \overline{\{\omega \in \Omega | P(\omega) > 0\}}$$

where  $\bar{A}$  denotes the closure of a set A. Throughout this chapter we assume P to be discrete with finite support |supp(P)| = N. We call  $\omega \in \text{supp}(P)$  a scenario and, without loss of generality, we assume  $\Omega = \text{supp}(P)$ .

In this setting, the objective of the stochastic program (SP) becomes the finite sum

$$z_P(x) = c^T x + \sum_{\omega \in \Omega} P(\omega) R(x, \omega).$$

## Optimal Solutions $x_P^*$

By  $x_P^*$  we denote an optimal solution to (SP), that is,

$$x_P^* \in \operatorname*{arg\,min}_{x \in \mathcal{X}} z_P(x).$$

#### Reduced Number of Scenarios K

 $1 \le K \le N$  denotes the desired number of scenarios in the examination set.

## Dirac Probability Distribution $\mathbb{1}_{\tilde{\omega}}$

By  $\mathbb{1}_{\tilde{\omega}}$  we denote the Dirac probability distribution on  $(\Omega, \mathcal{F})$  that places all mass at  $\tilde{\omega}$ , defined via

$$\mathbb{1}_{\tilde{\omega}}(\omega) = \begin{cases} 1, & \text{if } \omega = \tilde{\omega} \\ 0, & \text{else} \end{cases}, \quad \text{for } \omega \in \Omega.$$

When used as an index, the probability distribution  $\mathbb{1}_{\tilde{\omega}}$  is abbreviated by  $\tilde{\omega}$ .

Note that 
$$z_{\tilde{\omega}}(x) = z_{\mathbb{1}_{\tilde{\omega}}}(x) = c^T x + R(x, \tilde{\omega})$$
 and  $P = \sum_{\tilde{\omega} \in \Omega} P(\omega) \mathbb{1}_{\tilde{\omega}}$ .

#### Distance $\delta$ on $\Omega$

Let  $\omega_1, \omega_2 \in \Omega$ . As introduced in section 2.2.3, the uncertainty of the stochastic program (SP) is represented by the random vector  $\xi : (\Omega, \mathcal{F}, P) \to (\mathbb{R}^m, \mathcal{B})$ . We define

 $\delta: \Omega \times \Omega \to \mathbb{R}$  as

$$\delta(\omega_1, \omega_2) = \left(\sum_{i=1}^{m} |\xi_i(\omega_1) - \xi_i(\omega_2)|^2\right)^{1/2},\,$$

that is, the Euclidean distance between  $\xi(\omega_1)$  and  $\xi(\omega_2)$ .  $\delta$  is a metric (or distance function) on  $\Omega$  if we assume  $\xi(\omega_1) \neq \xi(\omega_2)$  for all  $\omega_1 \neq \omega_2$ .

## 5.2. Ex-Ante Optimisation with Ex-Post Simulation

A general technique for investigating large scale stochastic programs is ex-ante optimisation with ex-post simulation (see Di Domenica et al., 2009), also referred to as in-sample optimisation with out-of-sample evaluation (see Kaut and Wallace, 2003). This method is set out in algorithm 5.1.

**Algorithm 5.1** Ex-ante optimisation with ex-post simulation (Di Domenica et al., 2009)

Input: (SP)

- 1: Apply scenario generator to generate probability distribution Q with K scenarios.
- 2: Ex-ante optimisation: solve  $\min_{x \in X} z_Q(x)$ .
- 3: Apply scenario generator to generate probability distribution P with N scenarios.
- 4: Ex-post evaluation: calculate  $z_P(x_O^*)$ .
- 5: **return**  $x_Q^*$ .

Two sets of scenario Q and P with K and N scenarios respectively are generated. The (SP) is solved using the smaller set and the first stage solution is evaluated on the larger set in a simulation approach. The interpretation of this is, that P is assumed to represent the 'real' uncertainty, usually with many scenarios, while Q is its approximation. Note that the scenario generators for P and Q do not need to be identical. Of course, the optimality gap  $z_P(x_Q^*) - z_P^*$  – and, therefore, the quality of this approach – depends not only on the problem (SP) itself, but also on the used scenario generators. Obviously, suitable methods like the (integer) L-shaped method (see section 5.3.2) can be used to solve the stochastic program.

This approach can be applied to our SCND model by running the scenario generator described in section 4.3 twice to generate two scenario sets with K and N scenarios, respectively. We refer to this approach as scenario generation (SCENGEN) and it is investigated in section 5.5.

#### Ex-Post Simulation with CV@R-Objective

As stated in section 2.2, decision making under uncertainty faces a fundamental tradeoff between risk and return, which can be explored by stochastic optimisation using a CV@R risk measure. Rockafellar and Uryasev (2000) show that, for a given feasible decision  $x \in \mathcal{X}$  of (SP), the CV@R at confidence level  $\beta \in (0,1)$  is calculated by the stochastic linear program

(CV@R) 
$$\text{CV@R}_P^{\beta}(x) = \min_{\nu, \phi_{\omega}, \omega \in \Omega} \nu + \frac{1}{1 - \beta} \sum_{\omega \in \Omega} P(\omega) \phi_{\omega}$$

$$\text{s.th. } \phi_{\omega} \ge z_{\omega}(x) - \nu, \ \omega \in \Omega,$$

$$\nu \in \mathbb{R}, \ \phi_{\omega} \ge 0, \ \omega \in \Omega.$$

Also, Fábián (2008) shows that minimising the CV@R in (SP) can be achieved by the stochastic program

(CP) 
$$\text{CV@R}_P^{\beta} = \min_{x \in \mathcal{X}, \nu \in \mathbb{R}} \nu + \frac{1}{1 - \beta} \mathbb{E}_P[S(x, \nu, \omega)]$$
s.th.  $Ax = b$ ,
$$x > 0, \nu \in \mathbb{R},$$

where

$$S(x, \nu, \omega) = \min_{y \in \mathcal{Y}, \phi \in \mathbb{R}} \phi$$
s.th.  $\phi \ge c^T x + q(\omega)^T y - \nu$ ,
$$W(\omega)y = h(\omega) - T(\omega)x$$
,
$$y \ge 0, \ \phi \ge 0$$
.

In ex-post evaluation we use  $z_P(x) = c^T x + \sum_{\omega \in \Omega} P(\omega) R(x, \omega) = \sum_{\omega \in \Omega} P(\omega) z_{\omega}(x)$ . This is also true for (CP), if  $\nu$  is fixed as a first stage decision to a feasible value – which is any value in  $\mathbb{R}$  since there are no constraints on  $\nu$ . However, the result does not give the CV@R, as it means solving

$$\min_{\phi_{\omega}, \omega \in \Omega} \left\{ \nu + (1 - \beta)^{-1} \mathbb{E}_{P}[\phi_{\omega}] \mid \phi_{\omega} \ge z_{\omega}(x) - \nu, \ \phi_{\omega} \ge 0, \ \omega \in \Omega \right\},\,$$

which differs from (CV@R^{\beta}\_{P}(x)), as  $\nu$  is fixed and not a variable.

Therefore, ex-post evaluation with a CV@R objective does not work directly as presented in algorithm 5.1. However, this problem is easy to overcome by adjusting the approach as set out in algorithm 5.2.

# Algorithm 5.2 Ex-ante optimisation with ex-post simulation for a CV@R objective

Input: (SP)

- 1: Apply scenario generator to generate probability distribution Q with K scenarios.
- 2: Ex-ante optimisation: solve  $\text{CV@R}_Q^\beta$  in (CP) with optimal solution  $X_Q^*$ . 3: Apply scenario generator to generate probability distribution P with N scenarios.
- 4: Ex-post evaluation: calculate  $z_P(x_O^*)$ .
- 5: CV@R calculation: solve  $\text{CV@R}_{\mathcal{P}}^{\beta}(x_O^*)$  via the problem (CV@R).
- 6: **return**  $x_O^*$ .

# 5.3. Literature Review: Algorithms for Solving Large Scale Stochastic Mixed Integer Programs

In this section, we review the literature on algorithms for solving large scale linear and mixed integer stochastic programs. We focus on approaches that are applied to or seem promising to apply to models for SCND under uncertainty, or that share features with ASR. We compare these algorithms to the requirements identified in section 2.3.3, that is, whether they are able to deal with large scale two-stage stochastic programs with mixed integer first stage, a large number of scenarios, uncertainty in the objective, right hand side, and technology matrix, as well as whether they are flexible enough to deal with evolving models.

For general introductions on stochastic programming, its theory, and solution algorithms, we refer to Kall and Wallace (1994); Birge and Louveaux (1997); Ruszczynski and Shapiro (2003b); Kall and Mayer (2010).

#### 5.3.1. The Deterministic Equivalent Model

Probably the most commonly used method to solve stochastic programs is to solve the deterministic equivalent model (DEM) with a state of the art solver like CPLEX, Gurobi, or XPRESS. However, this approach is not scalable and is not suitable for complex stochastic mixed integer programs with a larger number of scenarios; this is unfortunately the class of problem investigated by us.

#### 5.3.2. Decomposition Algorithms

#### Description of the Methodology

Due to their special structure, two-stage stochastic programs are naturally processed by decomposition approaches. There are two types of decomposition methods: primal decomposition and dual decomposition (Carge and Schultz, 1999).

Primal methods decompose the problem between the stages, that is, they solve the first stage problem in the decision variables x in one step, and the recourse problem  $R(x,\omega)$  in a separate step (Carøe and Schultz, 1999). Benders decomposition (also called the L-shaped method) is the most common primal decomposition method for linear stochastic programs. In appendix C we provide a description of the base algorithm; also see Birge and Louveaux (1997) for an introduction to Benders decomposition, and Zverovich et al. (2010) for a recent implementation. Stochastic programs with first stage integer variables and linear second stage can be solved by a branch-and-cut variation of Benders decomposition called the integer L-shaped method. See Laporte and Louveaux (1993) for an introduction and Escudero et al. (2007, 2009); Sherali and Zhu (2009) for recent advances.

Dual methods decompose the problem among the scenarios, that is, in a first step they relax the non-anticipativity constraints in the deterministic equivalent problem with explicit non-anticipativity constraints (DEX). Thereby, the problem decomposes into N separate problems, one for each scenario  $\omega \in \Omega$ . In a second step the solutions  $x_{\omega}$  for x are combined again (Carøe and Schultz, 1999). See Escudero et al. (2011) for recent advances, also applicable to multi-stage models.

#### Applications of Decomposition Algorithms in SCND

Many of the approaches for stochastic SCND reviewed in section 4.1.2 involve decomposition algorithms, tailored to the specific optimisation model. Bienstock and Shapiro (1988) and Lucas et al. (2001) use Benders decomposition while Santoso et al. (2005) accelerate Benders decomposition by trust regions, knapsack inequalities, upper bounding heuristics, cut strengthening, logistics constraints, and cut disaggregation to solve sub-problems within sample average approximation (see section 5.3.4). Based on this, Bihlmaier et al. (2009) use trust regions and an upper bounding heuristic to accelerate Benders decomposition. Alonso-Ayuso et al. (2003a) and Alonso-Ayuso et al. (2005a) propose a dual decomposition with a specialised branch and fix coordination algorithm from Alonso-Ayuso et al. (2003b); this can be applied to models with fixed recourse and pure integer first stage. Finally, Schütz et al. (2009) use the dual decomposition with Lagrangian relaxation as proposed in Carøe and Schultz (1999), to solve sub-problems within sample average approximation.

Table 5.1 shows the sizes of the according optimisation problems. By comparing these with the problem size of the SCND model presented in chapter 4 with the number of scenarios required to represent multiple sources of uncertainty (see table 6.2, p. 89), we see that this problem is too complex for the proposed approaches. Especially the number of 0-1 variables is significantly larger than in the other models. This is with the exception of Lucas et al. (2001). However, their model is an approximation where the first stage decision is to choose a network design from a given set of possibilities. Therefore, the solved optimisation problem is not a SCND model and has only 590 feasible first stage solutions. Also, many of the algorithms rely on specifics of the

Reference	0-1 Vars	Cont. Vars	Constraints	Scenarios
Bienstock and Shapiro 1988	165	2 200	1250	6
Lucas et al. 2001	590	5442096	580001	100
Santoso et al. 2005	140	1254860	469320	60
Bihlmaier et al. 2009				200
Alonso-Ayuso et al. 2003a	114	3634	3933	23
Schütz et al. 2009		28250	926000	60

**Table 5.1.** Problem sizes of SCND models solved by decomposition algorithms

An empty cell means that the according number is not reported in the reference. If multiple numbers are given by the reference, the largest is reported.

model structure, for example a pure integer first stage, uncertainty only in some places of the stochastic model, no risk objective, or no risk constraint. This means that they might not be flexible enough to cope with the evolving requirements from multiple application projects or are not applicable to our model at all.

## 5.3.3. Meta-Heuristic Approaches

Bianchi et al. (2009) present an extensive and recent survey on meta-heuristic approaches to stochastic integer programs, classifying the algorithms by the heuristics which are chosen. These are ant colony optimisation, evolutionary computation, simulated annealing, tabu search, stochastic partitioning methods, progressive hedging, rollout algorithms, particle swarm optimization, and variable neighbourhood search. However, of the nearly 50 reviewed publications, only five apply to two-stage stochastic programs and all of them are specialised to certain models that are not related to SCND.

Most recent meta-heuristic approaches include the following:

Lazić et al. (2010) present a variable neighbourhood decomposition search for general two-stage SIPs. However, the authors conclude that their algorithm 'usually requires much longer execution time than the deterministic equivalent solver' (Lazić et al., 2010), but becomes more effective when more of the decision variables are 0-1-variables. As this is not the case in our model, this solver is not suitable for us.

Crainic et al. (2011) propose a progressive hedging algorithm with dual decomposition tailored to stochastic network design, where the design decisions are to select arcs for operation in a given network. The algorithm is tailored to the specific problem structure – particularly it works only for pure binary first stage – and computational studies are reported with up to 96 scenarios. Therefore, it is not applicable to our model.

## 5.3.4. Sample Average Approximation

#### Description of the Methodology

The sample average approximation (SAA) is a Monte Carlo simulation-based approach that can be applied to any type of two-stage stochastic program, for example MIP, non-linear, and especially when  $\Omega$  is too large (or infinite) to evaluate the recourse function  $R(x,\omega)$  for all  $\omega \in \Omega$ . The main idea is to use Monte Carlo sampling as a scenario generator for Q and then to perform ex-ante optimisation with ex-post simulation several times, thereby providing estimators for the optimality gap  $z_P(x_{SAA}^*) - z_P^*$  and its variance. Sample average approximation is discussed in depth for example by Kleywegt et al. (2002); Shapiro (2003), and is set out in algorithm 5.3. L stochastic

```
Algorithm 5.3 Sample average approximation algorithm (Kleywegt et al., 2002)
```

```
Input: (SP), L \in \mathbb{N}, \tilde{K} \geq K.
```

- 1: Identically and independently sample  $\tilde{K}$  scenarios  $\tilde{\omega}_1, \dots, \tilde{\omega}_{\tilde{K}}$  from P.
- 2: Define a probability distribution  $\tilde{Q}$  on  $\{\tilde{\omega}_1, \dots, \tilde{\omega}_{\tilde{K}}\}$  via  $\tilde{Q} = \frac{1}{K} \sum_{k=1}^{\tilde{K}} \mathbb{1}_{\tilde{\omega}_k}$ .
- 3: for l = 1 to L do
- 4: Identically and independently sample K scenarios  $\omega_1^l, \ldots, \omega_K^l$  from P.
- 5: Define a probability distribution  $Q^l$  on  $\{\omega_1^l, \dots, \omega_K^l\}$  via  $Q^l = \frac{1}{K} \sum_{k=1}^K \mathbb{1}_{\omega_k}$ .
- 6: Ex-ante optimisation on  $Q^l$ : Solve the stochastic program  $\min_{x \in X} z_{Q^l}(x)$  and get solution  $x_{Q^l}^*$ .
- 7: Ex-post evaluation on  $\tilde{Q}$ : Calculate  $z_{\tilde{Q}}(x_{O^l}^*)$ .
- 8: end for
- 9: **return** a best solution  $x^* = x_{Q^i}^*$  with  $i \in \arg\min_{l \in \{1,...,L\}} \{z_{\tilde{Q}}(x_{Q^l}^*)\}$ .

programs are solved ex-ante, each on K scenarios which are independently sampled from the probability distribution P. The solutions are evaluated ex-post on a sampled probability distribution with  $\tilde{K}$  scenarios. Note that the evaluation of the  $x_{Q^l}^*$  using  $\tilde{Q}$ is only necessary if P is such that  $z_P(x)$  is not practical to calculate, for example due to a very large number of scenarios – otherwise P is used instead of  $\tilde{Q}$ .

Estimators for the optimality gap can be used to improve the algorithm, such that if the estimators are not sufficiently small, K,  $\tilde{K}$ , and/or L are increased. Also, under some regularity conditions and given that the set of feasible first stage solutions is finite, probabilistic bounds on the optimality gap are given in terms of K – however, for applications these are usually difficult to calculate and very conservative, that is, K is too large (Kleywegt et al., 2002; Shapiro, 2003).

A special case of sample average approximation is Monte Carlo importance sampling within Benders decomposition; see Infanger (1992) for a description of the linear case. Only one sample is taken (L=1), but still a confidence interval for the objective is provided. Thereby, the quality of a solution is evaluated and it can be improved by

taking a larger sample size K, if necessary. 'Importance' here refers to an improved sampling technique that reduces the estimator's variance as compared to 'crude' Monte Carlo sampling.

#### **Applications in SCND**

Linderoth et al. (2002) and Linderoth et al. (2006) study the behaviour of sample average approximation on test cases with up to  $6\times 10^{81}$  scenarios. Among the SCND models from section 4.1.2, Santoso et al. (2005) use sample average approximation with L=20, K=20, 30, 40, 60, and  $\tilde{K}=1000$  scenarios, while sampling from continuous log-normal distributions. Sub-problems are solved by accelerated Benders decomposition. Similarly, Schütz et al. (2009) use L=20, K=20, 40, 60, and  $\tilde{K}=N=1000,$  but solve the sub-problems by dual decomposition.

#### 5.3.5. Scenario Reduction

Given the stochastic program (SP) (or a multi-stage, possibly non-linear stochastic program) with probability distribution P, the aim of scenario reduction is to reduce the number of scenarios, such that the optimal objective of (SP) admits minimal changes (Heitsch and Römisch, 2007). Formally, the goal is to find a probability measure Q on  $(\Omega, \mathcal{F})$  with  $\operatorname{supp}(Q) \subseteq \Omega$ ,  $|\operatorname{supp}(Q)| \leq K$ , such that  $z_P(x_Q^*) - z_P^*$  is minimal. Obviously, this problem is not easy to solve as it includes solving the original stochastic program with the probability distribution P.

Therefore, an approach is developed by Dupačová et al. (2003) and improved by Heitsch and Römisch (2003), where the problem is approximated by finding Q, such that  $\mu(P,Q)$  is minimal. Here,  $\mu$  denotes a distance measure based on a Fortet-Mourier metric, which is chosen due to a stability result by Rachev and Römisch (2002). The special case of two-stage programs is discussed by Heitsch and Römisch (2007), where the problem becomes easier to handle, which allows for some parts of the algorithm to be calculated directly, rather than to be approximated. Note that the methodology is developed for convex stochastic programs; therefore, bounds on the gap do not hold for integer variables – but the algorithm still gives good results. In what follows, we discuss the scenario reduction approach for two-stage stochastic programs in more detail; this also constitutes part of our algorithm. Also, it is a suitable approach for our model and, therefore, included in the computational studies in section 5.5.

Other work on scenario reduction are considered below.

Henrion et al. (2008), based on work by Henrion et al. (2009), adapt the scenario reduction approach for linear stochastic programs to two-stage SIPs by using a discrepancy distance instead of the Fortet-Mourier metric. However, the approach is only applicable for stochastic programs with linear first stage, mixed integer recourse, and

randomness only in the right-hand side  $(h(\omega))$  in (SP). Also, the authors note that the complexity of the proposed algorithm quickly increases with the original number of scenarios N, the reduced number K, and the dimension m of the random vector  $\xi$ . Therefore, this approach is not investigated in the computational studies.

Karuppiah et al. (2010) present a different approach to scenario reduction for models where multiple uncertain parameters are stochastically independent. In this situation the set of all scenarios consists of the Cartesian product of the sets of individual scenarios. The goal then is to select a subset of minimal size from the combined scenarios, such that each individual scenarios keeps its marginal probability. This problem is solved as a MIP. As this approach is only applicable to certain types of stochastic programs, and especially not for the SCND model developed in chapter 4, it is not further investigated.

An application of scenario reduction to SCND is not reported in the literature.

#### **Optimal Redistribution**

A sub-problem of optimal scenario reduction is optimal redistribution. Denote the scenarios in  $\Omega$  by  $\omega_i$  with probabilities  $p_i = P(\omega_i), i = 1, ..., N$ . Given a examination set  $J \subset \{1, ..., N\}$  of scenarios, the goal of optimal redistribution is to determine probabilities  $q_j \geq 0, j \in J$ , such that the Fortet-Mourier distance  $\mu(P, Q)$  between P and the resulting probability measure  $Q = \sum_{j \in J} q_j \mathbb{1}_{\omega_j}$  is minimal.

As shown by Heitsch and Römisch (2007), this problem is solvable by simply adding the probability of deleted scenarios  $\omega_i$ ,  $i \in \{1, ..., N\} \setminus J$  to the probability of a closest scenario in the examination set J. This is set out in algorithm 5.4. Note that we only report the case where  $\mu$  is the Fortet-Mourier metric of order 1; the interested reader is referred to the cited references for cases of order greater than 1.

```
Algorithm 5.4 Algorithm of optimal redistribution (Heitsch and Römisch, 2007)
```

```
Input: \emptyset \neq J \subset \{1, \dots, N\}. Denote scenarios by \omega_i \in \Omega, i = 1, \dots, N.

1: q_j = P(\omega_j) for j \in J.

2: for i \in \{1, \dots, N\} \setminus J do

3: j \in \arg\min_{k \in J} \{\delta(\omega_i, \omega_k)\}.

4: q_j \leftarrow q_j + P(\omega_i).

5: end for

6: return probability distribution Q on \{\omega_j \mid j \in J\} defined as Q = \sum_{j \in J} q_j \mathbb{1}_{\omega_j}.
```

#### **Scenario Reduction Heuristics**

Using algorithm 5.4, the problem of optimal scenario reduction can be restated, but still is NP-hard. Therefore, the authors propose heuristics derived by extending the extreme cases where K = 1 (forward selection) and K = N - 1 (backward reduction).

Forward selection is set out in algorithm 5.5 and is investigated in the computational studies.

**Algorithm 5.5** Heuristic of scenario reduction with forward selection (Heitsch and Römisch, 2007)

```
Input: P

1: Let J^{0} = \emptyset.

2: for i = 1 to K do

3: Let

\bar{J}^{i-1} = \{1, \dots, N\} \setminus J^{i-1}

j_{i} \in \underset{j \in \bar{J}^{i-1}}{\min} \left\{ \sum_{k \in \bar{J}^{i-1} \setminus \{j\}} P(\omega_{k}) \min_{l \in J^{i-1} \cup \{j\}} \{\delta(\omega_{k}, \omega_{l})\} \right\}

J^{i} = J^{i-1} \cup \{j_{i}\}
```

- 4: end for
- 5: Do optimal redistribution on  $J^K$  as set out in algorithm 5.4 and get probability distribution Q.
- 6: return Q.

## 5.3.6. Iterative Approaches

Some iterative methods for multi-stage stochastic programs have been proposed: after solving the problem on a scenario tree, they add or remove some scenarios and solve the problem again with the idea to add detail to the tree exactly where needed (Kaut and Wallace, 2003). Dempster and Thompson (1999) uses expected value of perfect information as importance sampling criterion in a nested Benders decomposition while Casey and Sen (2005) uses dual variables from the current solutions. Both approaches only work for continuous linear stochastic programs and they cannot be easily transferred to two-stage problems: at a node of the multi-stage tree they either collapse all branching scenarios to a single expected value scenario, or they use all scenarios branching from that node. In a two-stage tree this means only having the possibility of solving the expected value problem or of solving the full problem (SP).

## 5.3.7. Summary of Solver Literature and Open Research Problems

We have shown that except for scenario reduction and scenario generation, none of the solution algorithms in the literature meets the requirements identified in section 2.3.3. Therefore, we present a novel solution heuristic in the subsequent section and compare it to the applicable existing methodologies in the computational studies in section 5.5.

# 5.4. Adaptive Scenario Refinement Algorithm

In this section, the adaptive scenario refinement (ASR) heuristic is presented. ASR iteratively solves the optimisation problem on an examination set of scenarios of increasing size, with the new scenario in each iteration being selected such that it lies in areas of the scenario set where improvements seem most likely. This is based on two criteria: the regret of the scenario and the probability it would get under optimal redistribution. For a feasible solution  $x \in \mathcal{X}$  of (SP) and scenario  $\omega \in \Omega$ , the regret is defined as

$$r_x(\omega) = z_\omega(x) - z_\omega^*,$$

that is, the difference of the objective on  $\omega$  under the current solution and the optimal wait-and-see objective of  $\omega$ . Thereby,  $z_{\omega}(x) - z_{\omega}(\tilde{x}) \leq r_x(\omega)$  for all feasible solutions  $\tilde{x} \in \mathcal{X}$  of (SP), which means that the objective of solution x under scenario  $\omega$  can be improved at most by  $r_x(\omega)$ . Conversely, a high regret of a scenario  $\omega$  indicates that under the current solution x the objective of  $\omega$  is much worse than it could be, indicating that such a scenario is desirable to add. The second criterion is the probability the scenario  $\omega$  would get by optimal redistribution if it was added to the examination set. The idea is that if  $\omega$  is close (in terms of the distance  $\delta$ ) to many already selected scenarios, this is lower than if  $\omega$  is far from any selected scenario. Thereby this criterion aims to ensure a certain diversity of the selected scenarios.

ASR is set out in algorithm 5.6. First, the wait-and-see problem is solved, the solutions are evaluated under all uncertainties, and a best solution is selected as the starting scenario. Then, the number of scenarios is iteratively increased, until the desired number K of scenarios is reached.

In each iteration, the regret is calculated and in step 6 the redistribution of probabilities is calculated, under the hypothesis that scenario  $\bar{\omega} \in \overline{\Psi}_k$  is added to the examination set  $\Psi_k$ . The selection of scenario k+1 in step 7 is the crucial step of the algorithm and follows the idea of balancing new probabilities and regret, as set out above. The new probability measure  $Q_{k+1}$  in step 8 is defined via the optimally redistributed probabilities.

In step 9 the here-and-now stochastic program is solved, which is by far the most time-consuming step of the algorithm. Any solver suitable for (SP) can be used and should be warm-started with  $x_k^*$  to improve the solution time. If this problem is infeasible, the algorithm terminates in step 10, as the set of feasible solutions will only become smaller if more sceanrios are added. Conversely, if the here-and-now problem is unbounded, the algorithm does not terminate, as the problem might become bounded by adding additional scenarios.

In step 11, the remaining recourse problems are solved. This is needed to calculate the regret in step 5 and the ex-post objective  $z_P(x_{k+1}^*)$  in step 14. If (SP) contains

## Algorithm 5.6 Adaptive scenario refinement algorithm

#### Input: (SP)

Note: When used as index, we abbreviate the probability distribution  $Q_k$  by the notation k. We write  $\overline{\Psi} = \Omega \setminus \Psi$  for an examination set  $\Psi \subset \Omega$ .

1: Solve the wait-and-see problem:

For each  $\omega \in \Omega$  solve  $\min_{x \in \mathcal{X}} z_{\omega}(x)$ .

Optimal solutions are denoted by  $x_{\omega}^*$  and optimal objectives by  $z_{\omega}^*$ .

2: Simulation:

Solve the recourse problems  $R(x_{\omega}^*, \tilde{\omega})$  for all  $\omega, \tilde{\omega} \in \Omega$  and calculate  $z_P(x_\omega^*) = c^T x_\omega^* + \sum_{\tilde{\omega} \in \Omega} P(\tilde{\omega}) R(x_\omega^*, \tilde{\omega}).$ 

- 3: Let  $\omega_1 \in \operatorname{arg\,min}_{\omega \in \Omega} \{ z_P(x_\omega^*) \}$ ,  $\Psi_1 = \{ \omega_1 \}$ , and  $Q_1 = \mathbb{1}_{\omega_1}$ .
- 4: **for** k = 1 **to** K 1 **do**
- Let  $r_k(\bar{\omega}) = z_{\bar{\omega}}(x_k^*) z_{\bar{\omega}}^*$  be the regret of unselected scenarios  $\bar{\omega} \in \overline{\Psi}_k$ .
- For  $\bar{\omega} \in \overline{\Psi}_k$  and  $\omega \in \Psi_k \cup \{\bar{\omega}\}$ , let  $\pi_k(\bar{\omega}, \omega)$  be the probability of  $\omega$  calculated by optimal redistribution in algorithm 5.4 on the scenario-set  $\Psi_k \cup \{\bar{\omega}\}\$ .
- 7:
- Let  $\omega_{k+1} = \arg\max_{\bar{\omega} \in \overline{\Psi}_k} \left\{ \pi_k(\bar{\omega}, \bar{\omega}) \, r_k(\bar{\omega}) \right\}$ . Let  $\Psi_{k+1} = \Psi_k \cup \left\{ \omega_{k+1} \right\}$  and define the probability distribution  $Q_{k+1}$  with support  $\Psi_{k+1}$  via  $Q_{k+1} = \sum_{\omega \in \Psi_{k+1}} \pi_k(\omega_{k+1}, \omega) \mathbb{1}_{\omega}$ .
- Ex-ante optimisation of (SP) on  $Q_{k+1}$ : 9:  $\min_{x \in \mathcal{X}} z_{k+1}(x)$  with optimal solution  $x_{k+1}^*$ .
- Stop if  $z_P(x_{k+1}^*) = \infty$ . The problem is infeasible. 10:
- 11: Ex-post simulation:

Solve the recourse problem  $R(x_{k+1}^*, \bar{\omega})$  for  $\bar{\omega} \in \overline{\Psi}_{k+1}$  and calculate  $z_P(x_{k+1}^*) = c^T x_{k+1}^* + \sum_{\omega \in \Omega} P(\omega) R(x_{k+1}^*, \omega).$  Stop if  $z_P(x_{k+1}^*) = -\infty$ . The problem is unbounded.

- 12:
- 13: end for
- 14: **return** a best solution  $x^* = x_i^*$  with  $i \in \arg\min_{i \in \{1,\dots,K\}} \{z_P(x_i^*)\}$ .

the calculation of a CV@R, steps 4 and 5 from algorithm 5.2 need to be executed here instead. If the result is unbounded, the algorithm terminates in step 12 since it has found a feasible first stage solution  $x_{k+1}^*$  with unbounded second stage. Conversely, if the second stage is infeasible, the algorithm does not terminate, as there might be a first stage solution which gives a feasible second stage.

As with any sampling procedure, the sequence of the ex-post objective-values  $z_P(x_k^*)$ ,  $k=1,\ldots,K$  is not necessarily monotonously decreasing. Therefore, the best solution found is returned in step 14, which might not be  $x_K^*$ . Obviously ASR converges to an optimal solution if we let  $K \to N$ .

## 5.5. Computational Studies

In this section we compare the results for adaptive scenario refinement (ASR) with scenario reduction (SCENRED) and, where available, also with different scenario sets from the scenario generator as ex-ante optimisation with ex-post evaluation (SCENGEN).

## 5.5.1. Description of Test Cases

We use test cases from the stochastic integer programming test problem library (SIPLIB) by Ahmed (2004), from the test-problem collection for stochastic linear programming (SLP) by Felt (2003), test cases (LSW) by Linderoth et al. (2001), and two test cases based on the SCND model described in chapter 4 – one with expected profit (EP) objective and one with CV@R objective. From the libraries, all test cases of two-stage stochastic linear or mixed integer programs with at least 25 scenarios and at most 4096 scenarios are used. A summary of the selected test cases is shown in table 5.2.

In sslp\_10\_50 integer constraints are relaxed to cope with computational complexity. Further, the number of scenarios in the LSW test cases is reduced from values as large as  $6 \times 10^{81}$  to values reasonable for ASR and SCENRED. For the SCND test cases, the scenario generator described in section 4.3 is used to generate scenario sets of arbitrary size. Obviously, the problems cannot be solved on the largest scenario set with 1000 scenarios – this is after all the reason to use approximation algorithms. Therefore, this set is used for ex-post evaluation only.

#### 5.5.2. Method of Investigation

#### **Optimality Gap**

Results from the algorithms are reported by their (relative) optimality gap

$$gap(x) = \frac{z_P(x) - z_P^*}{1 + |z_P^*|},$$

Test Suite	Test Case	Integer on Stage	Number of Scenarios	VSP
SIPLIB	dcap233	1, 2	200, 300, 500	$2.8 \times 10^{-2}$
SIPLIB	dcap243	1, 2	200, 300, 500	$2.4 \times 10^{-2}$
SIPLIB	dcap332	1, 2	200, 300, 500	$9.2 \times 10^{-2}$
SIPLIB	dcap342	1, 2	200, 300, 500	$2.2 \times 10^{-2}$
SIPLIB	$sslp\_5\_25$	1, 2	50, 100	0.0
SIPLIB	$sslp_10_50$	1	50, 100, 500, 1000, 2000	0.0
SIPLIB	$sslp_15_45$	1, 2	5, 10, 15	0.0
SCND	scnd_EP	1	$25, \ldots, 50, (1000)$	0.0
SCND	$\operatorname{scnd}_{\operatorname{L}\!\!\operatorname{CV}}$ R	1	$25, \ldots, 50, (1000)$	0.0
SLP	airlift	_	25	$1.8 \times 10^{-4}$
SLP	cargo	_	$2^i, i = 0, \dots, 12$	0.0
LSW	20term	_	1 024	$1.1 \times 10^{-2}$
LSW	$\operatorname{gbd}$	_	1024	$2.5 \times 10^{-2}$
LSW	lands	_	1 000	$2.8\times10^{-4}$
LSW	ssn	_	735	$2.0\times10^{-1}$
LSW	storm	_	625	$1.4\times10^{-4}$

Table 5.2. Summary of test cases

The value of stochastic programming (VSP) is introduced in section 5.5.2 and is given here for convenience.

where  $x \in \mathcal{X}$  denotes a feasible solution of (SP), and P is the probability distribution from the largest scenario set. As the SCND problems cannot be solved to optimality on the largest scenario set, the best feasible solution found in any investigation is used to approximate  $z_P^*$ .

#### Value of Stochastic Programming

In some of the test cases, one of the wait-and-see solutions is already optimal. To focus on the interesting results, we introduce a measure called (relative) value of stochastic programming (VSP), which we define as

$$z_{\text{VSP}} = \min_{\omega \in \Omega} \frac{z_P(x_\omega^*) - z_P^*}{1 + |z_P^*|}.$$

It gives the difference between the best wait-and-see objective and the optimal objective. Obviously,  $z_{\rm VSP} \geq 0$  and  $z_{\rm VSP} = 0$  if and only if one of the wait-and-see solutions  $x_{\omega}^*$  is an optimal solution to (SP). Problems with  $z_{\rm VSP} = 0$  are solved to optimality in step 1 of the ASR algorithm 5.6. Except for the SCND test cases, these are omitted in the result presentation.

#### **Computations**

The three approaches ASR, SCENRED, and SCENGEN are applied on the test cases in the following way:

**ASR and SCENRED** For each test case, the largest scenario set with N scenarios is selected. For  $K=1,\ldots,N$  ( $K=1,\ldots,50$  for SCND), ASR and SCENRED are applied as described in algorithm 5.6 and algorithm 5.5, respectively. The resulting optimality gaps are reported for each K.

**SCENGEN** For each test case the problem is solved on all scenario sets (except for the 1000 scenarios in SCND). Then, the solution is evaluated ex-post under the largest scenario set as set out in algorithm 5.1 and the optimality gap is reported.

#### Implementation

All three approaches are implemented in the FortSP stochastic programming solver system (see Ellison et al., 2010) and here-and-now problems are solved using an appropriate solver from the FortSP suite: Benders decomposition for linear programs, the integer L-shaped method for problems with integer variables on the first stage only, and the deterministic equivalent model for problems with integer variables on the second stage. CPLEX 12.1 is used to solve underlying linear and mixed integer programs. The default stopping criterion with a relative MIP gap  $10^{-4}$  is used, except for the SCND problems where it is set to  $10^{-2}$  due to computational complexity.

#### 5.5.3. Test Case Results

## **Stochastic Integer Programs**

Figures 5.1 and 5.2 show the relative optimality gap of the applicable algorithms for all SIP test cases with a non-zero value of stochastic programming. In the three test cases not shown, ASR finds an optimal solution in the first iteration while SCENRED always needs more than one scenario.

In the SIPLIB test cases shown in figure 5.1, ASR clearly outperforms the other approaches, as it always finds an optimal solution within the first 9 to 42 scenarios, while SCENRED needs 340 to 421 out of 500 scenarios. The SCENGEN approach has too few data points to make a reliable statement, but seems to perform rather worse than better than SCENRED.

The SCND test cases reported in figure 5.2 cannot be solved to optimality. Therefore, the best found solution is used to approximate the optimality gap, which in both cases is one of the wait-and-see solutions. Therefore, ASR finds an optimal solution with the first scenario. For expected profit optimisation the SCENRED algorithm performs

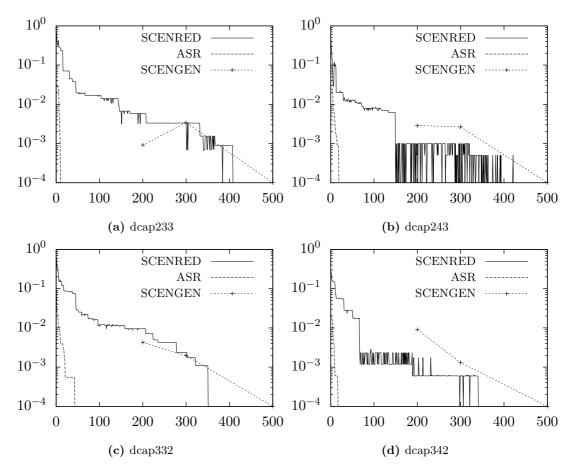


Figure 5.1. Relative optimality gap for SIPLIB test cases

The x-axis shows the number of scenarios K, the y-axis the relative gap.

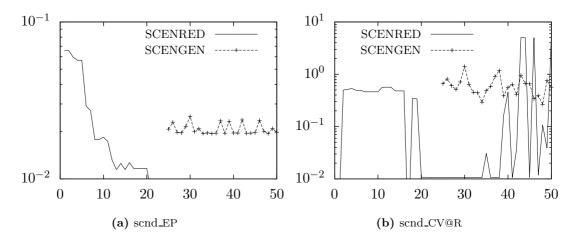


Figure 5.2. Relative optimality gap for SCND test cases

The x-axis shows the number of scenarios K, the y-axis the relative gap. ASR is omitted since the VSP is less than the optimality gap.

good as well, with all solutions within the MIP gap after 21 scenarios. However, for CV@R optimisation the results look worse, since the objective does not seem to converge to 0. Instead, the gap varies between  $1\,\%$  and  $500\,\%$ . The SCENGEN approach has a gap of around  $2\,\%$  for all scenarios when optimising expected profit, and between  $10\,\%$  and  $110\,\%$  for CV@R optimisation. In both cases, SCENGEN does not show any sign of converging to the optimal value.

## Stochastic Linear Programs

Figure 5.3 shows the relative optimality gap of the applicable algorithms for the test cases of linear stochastic programs with a non-zero value of stochastic programming. In the remaining test case not shown, ASR finds an optimal solution in the first iteration while SCENRED needs more than one scenario.

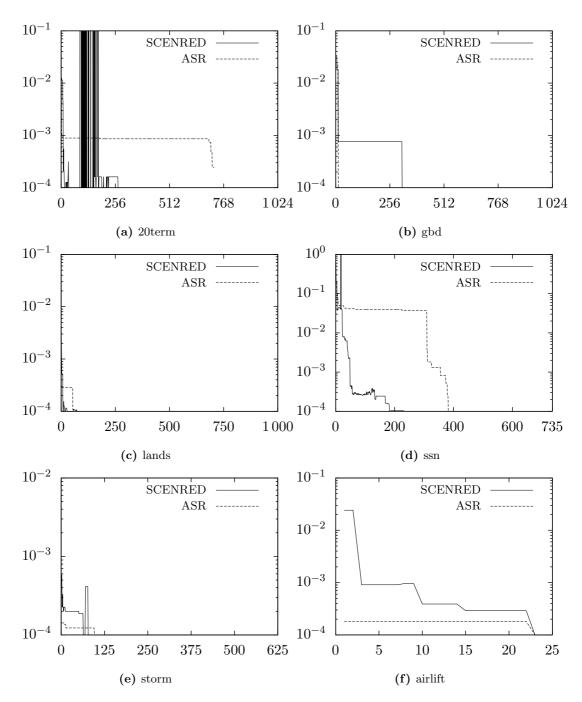
For the LSW and SLP test cases shown in figure 5.3, results are mixed. In the test cases gbd and airlift, ASR is better than SCENRED, while in lands and storm they both perform very good. In the 20term and ssn test cases, ASR performs worse. Notable about these problems is that in both the objective consists only of penalty costs for some scenarios on the second stage with no first stage costs, which means there is little information in the regret for ASR to work with.

## 5.5.4. Advantages of ASR

Similar to sample average approximation, the benefit of the ASR heuristic is not that it is fast – in fact if we want to solve a stochastic program with  $K \leq N$  scenarios using ASR, this involves solving K here-and-now problems on examination sets of increasing size  $1, \ldots, K$ , as well as solving the wait-and-see problem on N scenarios with  $N^2$  recourse evaluations. Therefore, it cannot be faster than any exact method that solves the problem on K scenarios directly. However, the benefit of ASR is illustrated by the SCND test case. In this situation, the problem cannot be solved with  $N=1\,000$  scenarios as the problem is too large. Instead, it can only be solved with K=50 scenarios in reasonable time, even with a state-of-the-art integer L-shaped solver. In this situation, good approximations are needed. Therefore, the aim is not to solve the problem on K or N scenarios as fast as possible, but to select K scenarios, such that the resulting solution is as good as possible when evaluated under all N scenarios.

# 5.6. Summary

As we have seen, the SCENGEN solution approach gives the worst results, which emphasises the importance of generating a large number of scenarios for the resulting stochastic program. The ASR method did significantly better than SCENRED in 12 out



**Figure 5.3.** Relative optimality gap for LSW and SLP test cases for linear stochastic programs

The x-axis shows the number of scenarios K, the y-axis the relative gap.

of 16 cases, about equally good in two cases, and worse in two cases, both of which did not include integer variables. However, most importantly, ASR was the best algorithm in the SCND test cases. Therefore, ASR is the solution algorithm we adopt for the SCND case study in the subsequent chapter.

# 6. A SCND Case Study and Results Analysis

In this chapter we apply the developed methodology to a case study network and investigate its benefits and implications. This combines the stochastic SCND model introduced in chapter 4 with the corresponding scenario generator, as well as the identified uncertainties from chapter 3 as they are part of the model. Also, we use the ASR heuristic from chapter 5 to solve the optimisation problems.

We describe the case study network in section 6.1 and the scenarios in section 6.2. We discuss the case study results in section 6.3 and show the reliability of the results by demonstrating the stability of the scenario generator in section 6.4. Finally, we summarise the chapter in section 6.5.

# 6.1. The Case Study Network

We apply the SCND methodology to a production network in the automotive industry. To maintain confidentiality we use simulated but representative data; the general structure is retained.

In the case study we have ten products whose assembly structure is shown in figure 6.1. The main product groups are the power train (PT), car bodies, and assembled cars. Car bodies and assembled cars are further categorised as small (S) and large (L). The supply chain network is distributed over the four currency regions Euro (EUR), Brazilian Real (BRL), Chinese Yuan (CNY), and US Dollar (USD), with the network as set out in figure 6.2. The first three regions have a production structure as shown in figure 6.1, whereas there is no production in the US. The EUR is the company's base currency. The set sizes are shown in table 6.1. The dimensions of the first and second stage for a single scenario, as well as of the deterministic equivalent model (DEM) with 50 and 1000 scenarios are summarised in table 6.2.

For the purpose of illustration, we have decided for the case study to allow the first stage decisions to be set in the first time period only. To account for this, we assume constant expected demand over all time periods.

CV@R is measured at 90 % confidence level.

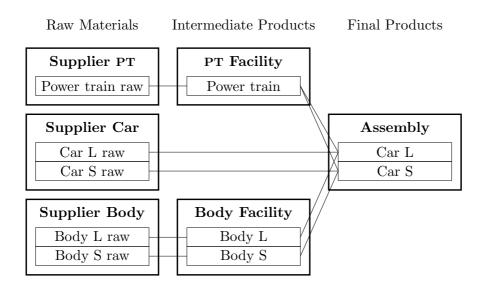


Figure 6.1. BoM and production structure

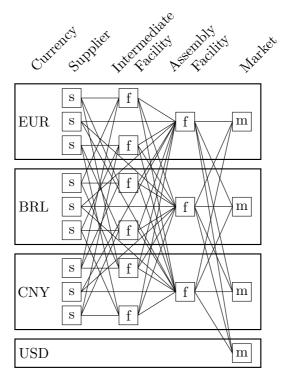


Figure 6.2. Network structure with currencies

Set	Name	Size
S	Suppliers	9
$\mathbf{F}$	Facilities	9
M	Markets	4
P	Products	10
Y	Shifts	3
$\mathbf{C}$	Currencies	4
$\mathbf{T}$	Time periods	10
N	Scenarios	1000

Table 6.1. Sets and case study sizes

	1 <sup>st</sup> stage	2 <sup>nd</sup> stage	DEM 50	DEM 1 000
0-1 variables	510	0	510	510
Continuous variables	451	1681	84501	1681451
Constraints	270	891	44820	891270

Table 6.2. Case study model sizes

Second stage sizes are for a single scenario and the deterministic equivalent model (DEM) sizes for 50 and 1000 scenarios, respectively.

## 6.2. Scenario Generation for Random Parameters

For the multiple uncertain parameters included in our model we assume a correlation structure of the uncertainties as set out in figure 6.3.

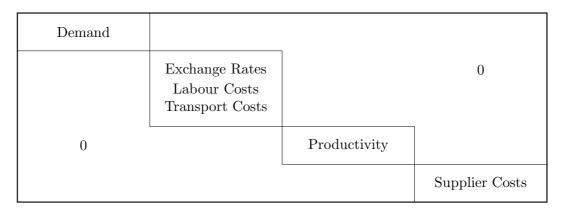


Figure 6.3. Correlation structure of uncertainties

We apply the relative autoregressive transformation of order 1 (see section 4.3) to the historical data and construct 1000 scenarios by fitting the scenario generator described in section 4.3. However, two of the uncertain parameters, namely demand and productivity, require further discussion since these include some expert intervention.

#### 6.2.1. Demand Scenarios

Given historical data of demand and the company's demand forecast, we assume the uncertainty to be in the company's forecast. Therefore, we generate scenarios for the forecasting error instead of for the demand. This approach has the advantage that we do not claim to have any expert knowledge or insight into the demand forecast or market and sales strategy – which is a core expertise of any company. Instead we just objectively measure, how good they can predict their demand. This increases the acceptance of the scenarios and, therefore, of the whole study.

While not explicitly modeled in the scenario generator, the following conceptual decomposition of demand uncertainty is useful in the later discussion. For a product  $p \in P$ , market  $m \in M$ , time period  $t \in T$ , and scenario  $\omega \in \Omega$ , we can rewrite the demand  $d_{mpt\omega}$  as

$$d_{mpt\omega} = dt_{t\omega} \, dp_{pt\omega} \, dm_{mpt\omega}.$$

Here

$$dt_{t\omega} = \sum_{m,p} d_{mpt\omega}$$

denotes the total demand, which is used to represent *volume uncertainty* in overall demand.

$$dp_{pt\omega} = \frac{1}{dt_{t\omega}} \sum_{m} d_{mpt\omega}$$

is the share of product p on the total demand and is used to represent  $product \ mix$  uncertainty. Finally,

$$dm_{mpt\omega} = \frac{1}{\sum_{m} d_{mpt\omega}} d_{mpt\omega}$$

is the share of market m on the demand of product p and is interpreted as market location uncertainty.

#### 6.2.2. Productivity Scenarios

The uncertainty in productivity is more difficult to capture: the uncertainty is especially high for new production facilities – but for new facilities there is also a lack of historical data. Our approach is to assume a learning curve for the capacity consumed by the production of each product (see for example Chase et al., 2006). Learning curves assume a logarithmic decline in capacity consumption over the cumulated production amount, where with each doubling of production the capacity consumption is reduced by a factor  $L \in (0,1]$ . As this is not possible to include in a linear optimisation problem we use a slight variation, where the decline is over time instead. This gives a capacity

consumption cf(t) in time period  $t \in \{\underline{t}, \dots, \overline{t}\}$  as

$$cf(t) = cf_t \cdot L^{\log_2(t - \underline{t} + 1)},$$

where  $cf_{\underline{t}}$  is the initial capacity consumption in the first time period  $\underline{t}$ , and  $L \in (0,1]$  is the learning rate.

Uncertainty in productivity is then modelled as uncertainty in the learning rate L. This is generated by moment matching without sampling, since no multi time period uncertainty is needed in this situation.

## 6.2.3. Single Period Scenario Results

To get an impression of the generated scenarios, we show the box plots of the single time period scenarios  $\tilde{s}$  from the moment matching algorithm, as introduced in algorithm 4.2. Except for the productivity scenarios, we use the relative transformation with a base case scenario; therefore these uncertainties have mean 1 plus the base case value. The results for the generated scenarios are shown as box plots in figures 6.4 – 6.7 for 1 000 generated scenarios. Readers not familiar with the definition of box plots are referred to appendix D.

As we can see from figure 6.4, demand uncertainty is similar across the markets and products, with slightly higher uncertainty in China and for large cars. Demand scenarios have mean 1 to comply with the situation that we allow the first stage decisions to be set in the first time period only.

Figure 6.5 shows the economical uncertainties. Obviously, transport costs are subject to huge variation and also they increase on average by 12% per year. Among the exchange rates, the BRL has the highest variability towards the EUR while USD and CNY behave similarly – which reflects the fact that the CNY is closely linked to the USD. As expected, labour costs are most volatile in China with a high annual increase of 7.5%. In Europe most scenario values are within a vary narrow range, but some extreme cases also appear. Labour costs uncertainty in Brazil has lower variation – this is due to fewer available historical data.

Supplier cost uncertainty shown in figure 6.6 behaves similar in the different countries, with slightly higher variation in China than in Brazil, and with Europe the least volatile. Supplier costs have mean 1 when considered as real costs after inflation.

Finally, figure 6.7 shows the productivity uncertainty for the learning rate. There is nearly no uncertainty in Europe and 75% of the scenario values in Brazil are between 0.98 and 1.00 with few values below 0.95. In China, productivity uncertainty is higher with 50% of the scenario values between 0.95 and 0.98 and some values as low as 0.85.

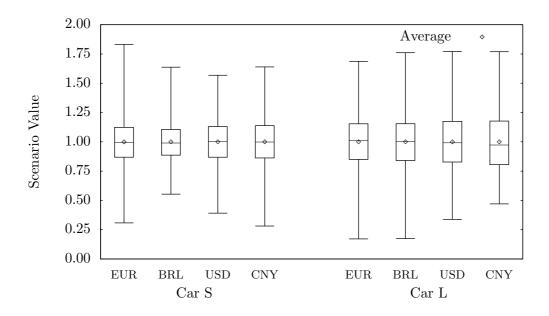
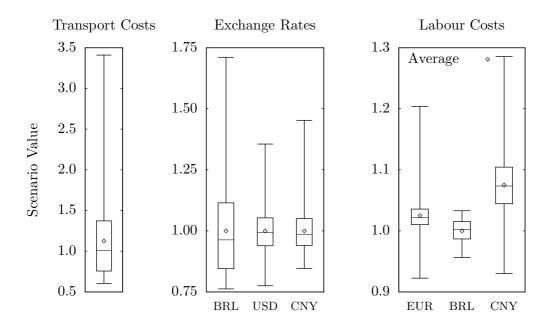
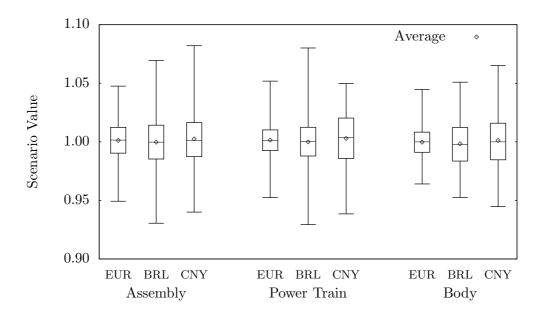


Figure 6.4. Box plots of demand scenarios for products at markets



**Figure 6.5.** Box plots of scenarios for transport costs, exchange rates towards the EUR, and labour costs



**Figure 6.6.** Box plots of scenarios for supplier costs of different raw materials in different countries

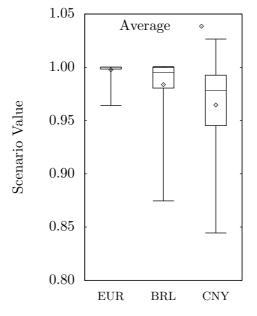


Figure 6.7. Box plots of scenarios for productivity in different countries

#### 6.3. SCND Case Study Result Analysis

We study the effects of introducing multiple parameter uncertainties and illustrate its importance by studying the behaviour of the case study network under various uncertainties. This is achieved by the following method of investigation, belonging to the general concepts of combining ex-ante decision making with ex-post evaluation (see section 5.2). In the ex-ante decision making phase we select a subset of parameters to be represented by their probability distribution, replace all other uncertainties by their expected value, and solve the resulting here-and-now optimisation problem. In the ex-post evaluation phase we select a second subset of parameters to be represented by their statistical distribution and replace the remaining parameters by their expected value. Next, the first stage decision variables are set to the optimal solution values from the ex-ante optimisation, and then we solve the second stage recourse problem.

To explore the trade-off between risk and return, we use as objective either expected profit or CV@R. Also, we compare the results to deterministic profit maximisation. By utilising the notations of table 6.3, we specify an investigation for expected profit maximisation and CV@R minimisation as  $\mathcal{Z}_{u_a}^{u_p}$  and  $\mathcal{R}_{u_a}^{u_p}$ , with  $u_a, u_p \in \{A, D, X, P, T, L, S, \emptyset\}$ , where  $u_a$  is the uncertainty set for the ex-ante phase and  $u_p$  the uncertainty set for the ex-post phase. As the first stage decisions do not depend on the ex-post evaluation, these are denoted by  $\mathcal{Z}_{u_a}$  and  $\mathcal{R}_{u_a}$ , respectively.

Notation	Meaning
$A = (D_{\omega}, X_{\omega}, P_{\omega}, T_{\omega}, L_{\omega}, S_{\omega})$	all uncertainties simultaneously
$D = (D_{\omega}, \bar{X}, \bar{P}, \bar{T}, \bar{L}, \bar{S})$	demand uncertainty
$X = (\bar{D}, X_{\omega}, \bar{P}, \bar{T}, \bar{L}, \bar{S})$	exchange rate uncertainty
$P = (\bar{D}, \bar{X}, P_{\omega}, \bar{T}, \bar{L}, \bar{S})$	productivity uncertainty
$T = (\bar{D}, \bar{X}, \bar{P}, T_{\omega}, \bar{L}, \bar{S})$	transport cost uncertainty
$L = (\bar{D}, \bar{X}, \bar{P}, \bar{T}, L_{\omega}, \bar{S})$	labour cost uncertainty
$S = (\bar{D}, \bar{X}, \bar{P}, \bar{T}, \bar{L}, S_{\omega})$	supplier cost uncertainty
$\emptyset = (\bar{D}, \bar{X}, \bar{P}, \bar{T}, \bar{L}, \bar{S})$	no uncertainty
$\overline{\mathcal{Z}}$	expected profit maximisation as objective
$\mathcal R$	CV@R minimisation as objective

Table 6.3. Notations for objectives and uncertainties

denotes expected value and  $\omega$  uncertainty via scenarios  $\omega \in \Omega$ .

In section 6.3.1 we investigate all uncertainties, that is,  $\mathcal{Z}_A^A$  and  $\mathcal{R}_A^A$ , and compare these to the expected value problem  $\mathcal{Z}_{\emptyset}^A$ . In section 6.3.2, we investigate the importance of accounting for multiple uncertainties in ex-ante decision making when faced by multiple uncertainties in ex-post evaluation. Therefore, we look at the investigations  $\mathcal{Z}_{u_a}^A$ ,  $\mathcal{R}_{u_a}^A$ , and  $\mathcal{Z}_{\emptyset}^A$  with  $u_a \in \{A, D, X, P, T, L, S\}$ . In section 6.3.3 we investigate

the situations  $\mathcal{Z}_A^{u_p}$ ,  $\mathcal{R}_A^{u_p}$ , and  $\mathcal{Z}_{\emptyset}^{u_p}$  with  $u_p \in \{A, D, X, P, T, L, S\}$ . This highlights the effects of different uncertainties on given network designs.

We solve the here-and-now problems using the ASR algorithm described in chapter 5 with K=50 out of  $N=1\,000$  scenarios and the MIP gap set to 1%. Ex-post evaluation is performed on all 1000 scenarios. All objective values are scaled such that the expected value of the deterministic solution is set to act as a reference point with 100 units of profit.

# 6.3.1. The Trade-Off Between Risk and Return with Multiple Sources of Uncertainty

We analyse the investigations  $\mathcal{Z}_A^A$ ,  $\mathcal{R}_A^A$ , and  $\mathcal{Z}_\emptyset^A$  with the results from the stochastic optimisations shown in a risk return diagram in figure 6.8. We also give five solutions on the efficient frontier by varying the weight  $\beta = i/6, i = 1...5$ . Note that only four solutions are visible since  $\beta = 1/6$  and  $\beta = 2/6$  give the same solution, which is the bottom left point of the efficient solutions shown. Given that we wish to have high profit and low CV@R, solutions in the top left corner of the risk return diagram are efficient.

From the risk return diagram, we can get a first impression of how the different solutions behave. The value of the stochastic solution depends on the weighting of expected profit and CV@R in the objective, that is, the risk preference of the decision maker. For the extreme points, this is the vertical difference 28.8 between  $\mathcal{Z}_{\emptyset}^{A}$  and  $\mathcal{Z}_{A}^{A}$ , and the horizontal difference 123.7 between  $\mathcal{Z}_{\emptyset}^{A}$  and  $\mathcal{R}_{A}^{A}$ . Also, the diagram illustrates the trade-off between risk and return: the CV@R from  $\mathcal{R}_{A}^{A}$  is 8.9 lower than the one from  $\mathcal{Z}_{A}^{A}$ . However, this risk reduction comes at a cost, which is 17.0 of the maximum possible expected profit.

Figure 6.9 shows the box plots of profit distributions for the different investigations, as well as the inverse of the cumulative distribution function. This illustrates the situation in the risk return diagram:  $\mathcal{Z}_A^A$  has high potential when it comes to the upside of the profit distribution but also significant risk on the downside, even with a possibility of losses.  $\mathcal{R}_A^A$  in contrast results in less losses but at the expense of heaving a worse upside tail. Hence, we again see the trade-off between risk and return in those two solutions.  $\mathcal{Z}_{\emptyset}^A$  has a poor downside tail distribution while producing a good upside tail distribution. We observe that  $\mathcal{Z}_A^A$  already significantly reduces the risk when compared to the deterministic optimisation  $\mathcal{Z}_{\emptyset}^A$ .

For further investigation we describe the network design decisions of the solutions. The facility capacities from the different solutions are shown in figure 6.10. While solution  $\mathcal{Z}_{\emptyset}$  installs all capacities in Brazil,  $\mathcal{Z}_{A}$  allocates all capacities in Europe.  $\mathcal{R}_{A}$  splits capacities between Europe and China for the assembly and power train facilities, but has production in Europe only for the body shop. This different behaviour is due

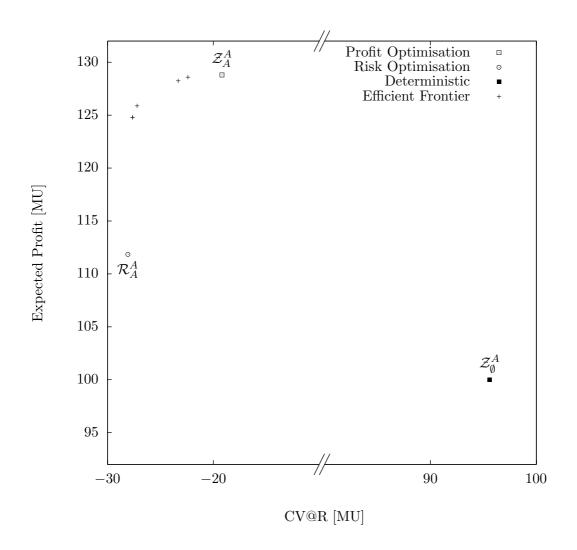


Figure 6.8. Risk return diagram The CV@R-axis is split.

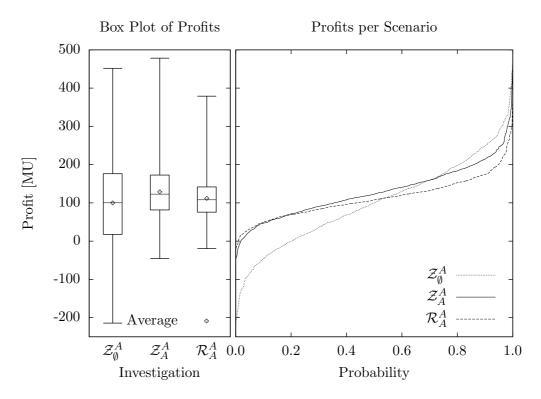


Figure 6.9. Box plot of profit distributions and profits per scenario

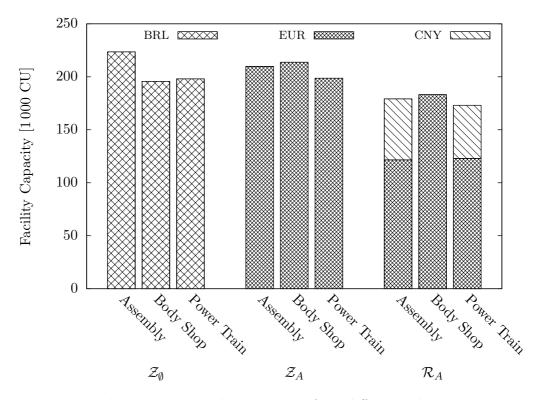


Figure 6.10. Facility capacities from different solutions

Ex-post optimisation Supplier	EU	$\mathcal{Z}_{\emptyset}$ BR	CN	EU	$\mathcal{Z}_A$ BR	CN	EU	$\mathcal{R}_A$ BR	CN
Assembly raw	_	✓	_	✓	_	_	✓	_	$\checkmark$
Body raw	_	$\checkmark$	_	$\checkmark$	_	_	$\checkmark$	_	_
Power train raw	$\checkmark$	_	_	$\checkmark$	_	_	$\checkmark$	_	_

Table 6.4. Suppliers operating

to the fact that body shops have high fixed costs which makes them expensive to build and maintain.

Operating suppliers are shown in table 6.4. Raw materials are locally sourced, except for the power train raw materials in  $\mathcal{Z}_{\emptyset}$  and in  $\mathcal{R}_A$ . In  $\mathcal{R}_A$  they are not split – as is the production – but they are still located at the larger production facility.

All facilities that are open are assigned all of the appropriate products. Labour and production base levels are difficult to compare due to differing productivity. However, in general, base levels are equal to labour and production requirements in  $\mathcal{Z}_{\emptyset}$  and to the expected requirements in  $\mathcal{Z}_A$ , but about 10% lower than the requirements in  $\mathcal{R}_A$  since this decreases the costs in the worst cases.

Figure 6.11 shows the components contributing to the objective with the box plots showing the corresponding range of values. Supplier costs form the biggest part, fol-

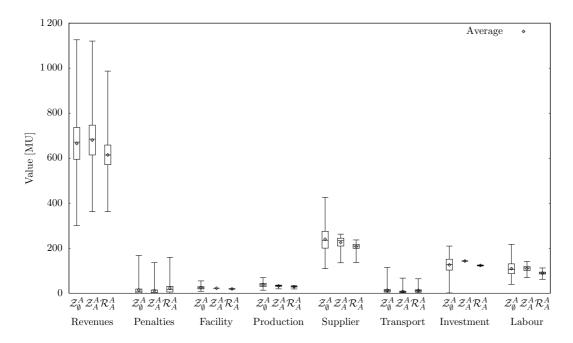


Figure 6.11. Box plot break down of revenues and costs

lowed by investment and labour costs. As ex-post evaluation on the given first stage solutions, exchange rates have an effect on all objective components, except for those that are incurred in EUR. Demand uncertainty has an effect on all variable parts, which means it does not apply to facility and investment costs. Productivity has an influence on production and labour costs while labour, supplier, and transport cost uncertainty have only effects on their corresponding cost component.

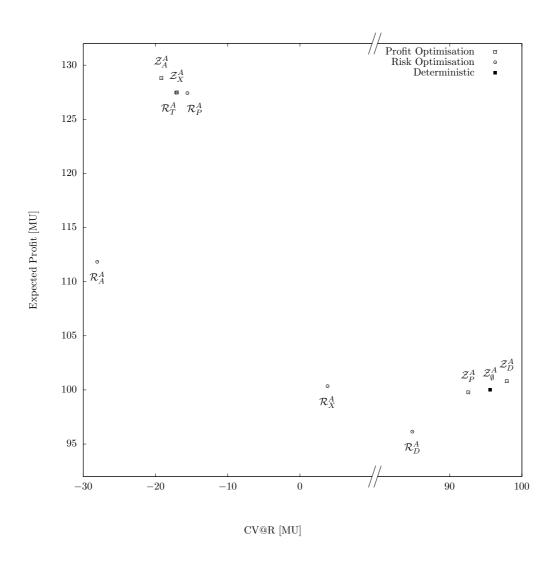
#### 6.3.2. The Effects of Multiple Uncertainties

We analyse the benefits of accounting for multiple uncertainties by comparing the solutions from ex-ante optimisation under single uncertainties with ex-ante optimisation under all uncertainties, when faced with all uncertainties in ex-post evaluation. The corresponding investigations are  $\mathcal{Z}_{u_a}^A$ ,  $\mathcal{R}_{u_a}^A$ , and  $\mathcal{Z}_{\emptyset}^A$  for  $u_a \in \{A, D, X, P, T, L, S\}$ . Their risks and returns are shown in figure 6.12, but the investigations  $\mathcal{Z}_T^A$ ,  $\mathcal{Z}_L^A$ ,  $\mathcal{R}_L^A$ ,  $\mathcal{Z}_S^A$ , and  $\mathcal{R}_S^A$  are omitted, as their first stage decisions are the same as or very similar to the solution  $\mathcal{Z}_{\emptyset}$ . The according expected profit and CV@R for all solutions are given in table 6.5.

	$\mathcal{Z}$	$u_a$	$\mathcal{R}$	$\dot{A}$
$u_a$	EP	CV@R	EP	CV@R
A	128.80	-19.19	111.84	-28.07
X	127.48	-17.04	100.35	3.83
D	100.81	97.90	96.15	84.81
P	99.77	92.57	127.42	-15.58
${\rm T}$	100.00	95.59	127.47	-17.16
$\mathbf{S}$	100.00	95.59	100.00	95.59
$\mathbf{L}$	100.00	95.59	100.05	95.49
Ø	100.00	95.59		

**Table 6.5.** Risk and return for all solutions

Surprisingly, only three of the single-uncertainty solutions are close to the efficient frontier, namely  $\mathcal{Z}_X^A$ ,  $\mathcal{R}_T^A$ , and  $\mathcal{R}_P^A$ .  $\mathcal{Z}_X^A$  leads to the best solutions from the single-uncertainty optimisations, with the profit only 1.0% worse than  $\mathcal{Z}_A^A$  but still 11.0% higher risk. While  $\mathcal{R}_P^A$  and  $\mathcal{R}_T^A$  are nearby, these solutions are completely misleading: although the aim of risk optimisation is a low CV@R at the cost of a low expected profit, these solutions actually have a high expected profit at the cost of being risky. This is a consequence of ex-post evaluation; for  $\mathcal{R}_P$  it can be explained since productivity uncertainty is lower in Europe than in the other two countries and for  $\mathcal{R}_T$  by the fact that having all production at the largest market requires the least transports. Hence, both solutions install facility capacity in Europe and are thereby similar to  $\mathcal{Z}_A$ .  $\mathcal{R}_X$  is the solution most distinct from the other results. It installs the assembly and power train facilities in China, and the body shop facility in Europe. However, this solution is far from being efficient. The solutions  $\mathcal{Z}_D$  and  $\mathcal{R}_D$  are very similar to the deterministic

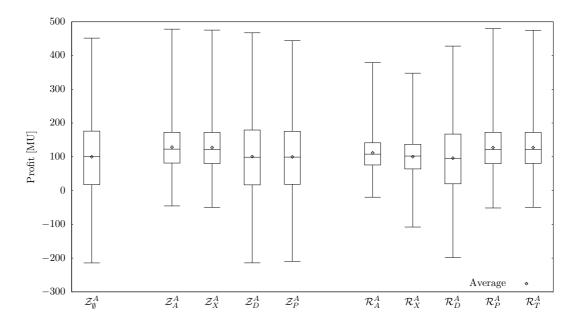


 $\textbf{Figure 6.12.} \ \, \text{Risk return diagram}$ 

The CV@R-axis is split. Solutions not shown are very close to  $\mathcal{Z}_{\emptyset}^{A}$ .

solution  $\mathcal{Z}_{\emptyset}$ , with all facilities in Brazil. The main difference is, that  $\mathcal{Z}_{D}$  installs about 5% more, and  $\mathcal{R}_{D}$  about 10% less capacity than  $\mathcal{Z}_{\emptyset}$ . All other solutions are equal or nearly equal to the deterministic solution  $\mathcal{Z}_{\emptyset}$ .

The differences in downside risks and upside gains become even more apparent when looking at the box plots of the profit distributions in figure 6.13. Again, the investigations  $\mathcal{Z}_T^A$ ,  $\mathcal{Z}_L^A$ ,  $\mathcal{R}_L^A$ ,  $\mathcal{Z}_S^A$ , and  $\mathcal{R}_S^A$  are omitted as they give the same results as  $\mathcal{Z}_{\emptyset}^A$ .



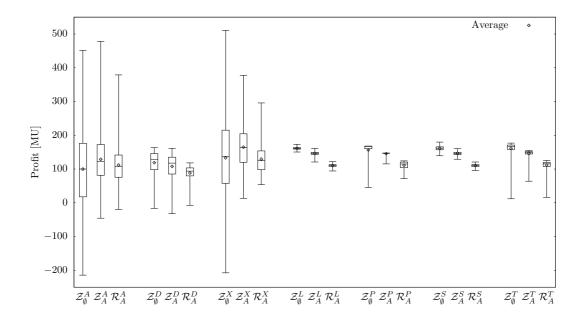
**Figure 6.13.** Box plots of profit distributions for ex-post evaluations under all uncertainties

#### 6.3.3. Risk Hedging Strategies in SCND

Figure 6.14 shows the box plots of the profit distributions for the solutions  $\mathcal{Z}_A$ ,  $\mathcal{R}_A$ , and  $\mathcal{Z}_{\emptyset}$  with ex-post evaluation under different uncertainties. Since the other solutions studied in section 6.3.2 behave similarly to one of the above three investigations, we focus on discussing these only.

#### **Exchange Rate Uncertainty**

For these network designs we observe that exchange rate uncertainty has the biggest effect in ex-post evaluation; in what follows we show how we can hedge against this uncertainty. The deterministic investigation  $\mathcal{Z}_{\emptyset}^{X}$  has a high exposure to exchange rate uncertainty, as all of the facilities are located in Brazil. The Brazilian market being the smallest of all four means that only a small proportion of production costs are covered by local revenues, while the remaining costs are exchanged into the EUR as the company's



**Figure 6.14.** Box plots of profit distributions for ex-ante decisions under all uncertainties

base currency. Similarly, the revenues in USD and CNY are also exchanged into the EUR. In  $\mathcal{Z}_A^X$ , all facilities are located in Europe which leads to lower uncertainty in profits since production costs are not exposed to exchange rate uncertainties, although the revenues in foreign currencies are. In  $\mathcal{R}_A^X$  facilities are Europe as well as in China, which further reduces the effect of exchange rate uncertainty. This is because in China revenues are set off against production costs, and since the CNY closely traces the USD, a similar effect applies for revenues in the US. Also, with assembly suppliers in Europe and China, sourcing can partly be adapted to take advantage of fluctuating exchange rates. In this study, clearly we have not considered multinational corporate tax planning or financial hedging instruments.

#### **Demand Uncertainty**

The next major influence on profit is due to demand uncertainty for which we distinguish in the discussion between market location uncertainty, product mix uncertainty, and volume uncertainty, as described in section 4.3. Market mix uncertainty influences transportation costs and import duties. In our case study, transportation costs are relatively low and duties are not considered and, thereby, the risk from market location uncertainty is low. Flexible production systems, that is, production systems that are able to manufacture multiple products efficiently, hedge against product mix uncertainty. Since in the solutions each operating facility is able to manufacture every

product, product mix uncertainty has an equal effect on all investigations. Finally, volume uncertainty is much harder to hedge against as this requires the production costs to scale with the load. Unfortunately, in the automotive industry expensive machinery is needed which mainly prohibits this strategy. Still, the flexibility of labour costs is different from country to country: changes to the labour force in Europe can be expensive, while in emerging countries companies are able to reduce labour numbers during economic down turns. In summary, this explains why the effect of demand uncertainty in  $\mathcal{Z}_{\emptyset}^{D}$  and  $\mathcal{Z}_{A}^{D}$  is similar, and even the added flexibility in  $\mathcal{R}_{A}^{D}$  is not able to decrease it significantly. On the other hand, this solution has lower capacity which leads to lost opportunities on the upside.

#### **Productivity Uncertainty**

Productivity uncertainty in Europe is lower than in emerging countries – especially for European companies which have been established for several years, in contrast to new facilities in low cost countries. Hence, avoiding new facilities in emerging markets and keeping production in Europe is an obvious possibility to hedge against productivity uncertainty. This is reflected in the investigations  $\mathcal{Z}_{\emptyset}^{P}$ ,  $\mathcal{Z}_{A}^{P}$ , and  $\mathcal{R}_{A}^{P}$ .

#### **Labour Cost Uncertainty**

The results from the ex-post evaluation under labour cost uncertainty reflect the uncertainty in the labour cost scenarios. This uncertainty has the smallest effect on profits from all considered uncertainties.

#### Supplier Cost Uncertainty

In our model, supplier cost uncertainty is strongly driven by steel costs, which makes their behaviour correlate similarly around the world. This explains why their effect is similar in all solutions, and why it is hard to hedge against these.

#### **Transport Cost Uncertainty**

Finally, transport cost uncertainty has a relatively large influence on the total profit, given its rather small share on all costs (see figure 6.11). This is because transport costs are the most volatile of all considered uncertainties, given the large variation in oil prices.  $\mathcal{Z}_A^T$  has the least exposure since production is in Europe, close to one of the main markets.  $\mathcal{R}_A^T$  has a slightly higher exposure because production is split across Europe and China and therefore, intercontinental transport between the production echelons are needed, which increases transport costs and hence the exposure to transport cost uncertainty.  $\mathcal{Z}_\emptyset^T$  instead produces in Brazil, which is a small market and power train

raw materials are sourced in Europe. Therefore, many long trips are needed, which explains the high effect of transport cost uncertainty in this investigation.

#### 6.4. Stability Analysis of the Scenario Generator

#### 6.4.1. Notation

In this section, we use the notation as introduced in section 5.1. Further, we label the probability distribution  $Q_i$  as i, when used as an index.

#### 6.4.2. In-Sample and Out-of-Sample Stability

As discussed by Kaut and Wallace (2003) and Mitra et al. (2009), most scenario generators are non-deterministic and hence generate different scenario trees in each run. Therefore, the so called 'stability' is a desirable property of scenario generators. It is tested for a specific optimisation problem by generating multiple probability distributions  $Q_i$ , i = 1, ..., I with the same input and solving the resulting optimisation problems  $z_i^* = \min_{x \in \mathcal{X}} z_i(x)$ . We can then distinguish two types of stability: in-sample and out-of-sample. The scenario generator is considered *in-sample stable*, if we have

$$z_i(x_i^*) \approx z_j(x_i^*), \quad \forall i, j \in \{1, \dots, I\},$$

that is, if we receive approximately the same optimal objective value for each optimisation run. The scenario generator is considered *out-of-sample stable*, if we have

$$z_P(x_i^*) \approx z_P(x_j^*), \quad \forall i, j \in \{1, \dots, I\},$$

where P is the probability distribution of the 'true' uncertainty. However, P is usually not known in applications in SCND. Therefore, we use the following test for *empirical* out-of-sample stability instead:

$$z_k(x_l^*) \approx z_i(x_j^*), \quad \forall k, l, i, j \in \{1, \dots, I\}.$$

Here, a scenario genrator is considered emprically out-of-sample stable, if all optimal solutions  $x_i^*$  have approximately the same objective value under each probability distribution  $Q_j$ .

Note that all stability properties are considered in respect of a particular optimisation problem.

#### 6.4.3. Test Case

The scenario generator used in our approach is non-deterministic for two reasons. (1) The single-period scenarios in the moment matching approach are based on a random sample and (2) we use sampling to extend the single-period scenarios to multiperiod scenarios. To analyse the stability of the results, we run multiple experiments on a fixed supply chain using multiple scenario-trees. For this purpose, we use the supply chain described in section 6.1, but reduce the computational effort by only considering the final assemply and according raw materials. Based on the same parameters as shown in section 6.2, we generate five scenario-trees, each containing 1 000 scenarios. As in the case study in section 6.3, we optimise the supply chain for each scenario-tree using the ASR heuristic with 50 scenarios for the stochastic investigations  $\mathcal{Z}_A^A$  and  $\mathcal{R}_A^A$ .

#### 6.4.4. Test Case Results

Tables 6.6 and 6.7 show the results of the stability analysis for expected profit and CV@R objective, respectively. The values  $z_i(x_j^*), i, j = 1, ..., 5$  are shown, where the column indicates the scenario set j for ex-ante optimisation while the row indicates the scenario set i for ex-post simulation. All numbers are scaled, such that the average of each table is 100.

		Scen	nario se	t j for o	ptimisa	tion
		1	2	3	4	5
$\begin{bmatrix} t & i \\ on \end{bmatrix}$	1	99.8	99.7	99.5	99.5	99.7
set latic	2	100.1	100.0	99.8	99.8	100.0
nario simul	3	100.5	100.3	100.2	100.2	100.4
cenario r simu	4	99.9	99.7	99.6	99.6	99.8
Scondon	5	100.5	100.5	100.2	100.3	100.5

**Table 6.6.** Stability results for expected profit objective

		Scei	nario set	t j for o	ptimisa	tion
		1	2	3	4	5
t i	1	100.1	100.1	99.7	99.5	100.1
set lati	2	99.7	99.7	99.7	99.8	99.7
rio nu	3	99.2	99.2	99.5	98.8	99.3
senari r simu	4	101.6	101.6	101.8	101.7	101.6
$S_{cc}$ for	5	99.5	99.5	99.5	99.4	99.5

Table 6.7. Stability results for CV@R objective

The in-sample stability is expressed by the diagonal elements  $z_i(x_i^*), i = 1, ..., 5$  of the tables. Clearly, this is very good for the expected profit objective, as all values

are within [99.6, 100.5], that is, they are all within the relative MIP gap of 1%, used to terminate the optimisations. Also, for the CV@R objective all objective values are within the range [99.5, 101.7], which is still very narrow. Therefore, we conclude that the scenario generator is in-sample stable for the SCND test case.

For empirical out-of-sample stability we look at the whole tables. With the expected profit objective, all objective values are in the range [99.5, 100.5] and are therefore again within the relative MIP gap. As expected, the optimal objective values are less stable for the CV@R objective, since the CV@R is driven by the extreme realisations the probability distributions, which are less stable to genearate. However, all values are in the range [98.8, 101.8], which is still reasonably small. Therefore, we conclude that the scenario generator is also empirically out-of-sample stable for the SCND test case.

Further, if we look at rows and columns of the tables separately, we see that the main driver for the variation in the out-of-sample objectives is the stability of the objective value, rather than the solution quality. For the expected profit objective, we have

$$\max_{k} z_i(x_k^*) - \min_{j} z_i(x_j^*) \le 0.3$$

in each row i, while

$$\max_{i} z_{i}(x_{j}^{*}) - \min_{l} z_{l}(x_{j}^{*}) \in [0.7, 0.8]$$

for all columns j. Similarly, for the CV@R objective we have

$$\max_{k} z_{i}(x_{k}^{*}) - \min_{j} z_{i}(x_{j}^{*}) \le 0.7$$

in each row i and

$$\max_{i} z_{i}(x_{j}^{*}) - \min_{l} z_{l}(x_{j}^{*}) \in [2.3, 2.9]$$

for all columns j. Therefore, if we optimise for a given probability distribution  $Q_i$ , the optimal solution  $x_i^*$  is also within the MIP gap for all other probability distributions  $Q_j, j = 1, \ldots, 5$ . This suggests that, while we cannot predict the optimal objective  $z_P(x_P^*)$  by using  $z_i(x_i^*)$  exactly (especially in the case of CV@R optimisation), we can still assume that the optimal solution  $x_i^*$  is nearly optimal for  $z_P$ , that is,  $z_P(x_P^*) \approx z_P(x_i^*)$ .

## 6.5. Summary

In this chapter we have presented a SCND case study application of the methodology developed in the previous chapters. The supply chain is typical for a company in the automotive industry, with multiple products, potential production in Europe, Brazil, and China, as well as markets in Europe, Brazil, China, and the United States.

We have seen that there is a significant trade-off between risk and return, indicating that it is essential to take risk measures into account in the design of supply chain networks. More generally, stochastic programming solutions are superior to the expected value solution, demonstrating that it is important to consider uncertainties.

We have investigated the results from ex-ante optimisation under single sources of uncertainties when faced with multiple uncertainties in ex-post evaluation, as they would in the real world. Thereby we have seen that only expected profit optimisation under exchange rate uncertainty gives nearly optimal results; in particular no solution came even close to the optimal CV@R. This highlights the importance of considering multiple instead of single sources of uncertainty, especially in the face of risk.

Finally, we have studied the effects of different uncertainties on given network designs. While in the literature most work on SCND focuses on demand uncertainty, our study suggests that exchange rate uncertainty has a larger effect on profits and can also be better hedged against. Both aspects make it the most important uncertainty to consider. However, while exchange rate uncertainty is the main influence on the expected profit, the CV@R is determined by the combination of the worst case behaviours of multiple uncertainties together.

Overall, we have demonstrated that the consideration of multiple uncertainties and risk is (i) applicable to real world SCND applications and (ii) superior to the most common stochastic programming approach, where expected profit is optimised under demand uncertainty. Further, the results are reliable and reproducable as we have demonstrated the stability of the scenario generator.

## 7. Discussion and Conclusions

#### 7.1. Summary of Findings and Contribution

In this thesis we have investigated the question of how to support decision making in SCND with a quantitative methodology.

In chapter 2 we have investigated the methodological requirements for quantitative decision support in SCND. We have identified three key issues: (i) We have high-lighted that uncertainty and risk play an important role and it is necessary to identify a manageable number of uncertainties. (ii) The optimisation model should include comprehensive strategic and tactical stages, with detailed cost calculation in line with accountancy standards and the multiple sources of uncertainty and risk previously identified. (iii) A solution approach is required with which the resulting large scale SIP can be investigated, but which is still flexible enough to cope with evolving model structures in various real world applications.

In chapter 3 we have investigated the first problem of identifying a set of relevant uncertainties for a SCND project, that are included in an optimisation model. We have proposed a new process for uncertainty selection based on concepts from the risk management literature. This is a structured, qualitative approach which aims to objectively capture expert opinion. For a case study we have considered an international production network in the automotive industry and have selected six sources of uncertainty: demand, productivity, transport costs, labour costs, raw material costs, and exchange rates. Our approach of identifying the critical uncertainties and its inclusion in the decision modeling paradigm is novel and makes contribution to knowledge.

In chapter 4 we have investigated the modeling of SCND under uncertainty. We have developed a two-stage SIP which includes the six uncertainties identified earlier, a holistic network structure, and detailed operations, especially in regards to exploiting flexibility. Since the decisions are made under uncertainty, the profit for a given network design is itself uncertain. To explore the resulting trade-off between risk and return, risk has been measured using CV@R, since it is a coherent risk measure that has the added benefit of leading to a linear optimisation model. Further, we have proposed a scenario generator to represent the uncertainty, which is an extension of a moment matching method by Høyland et al. (2003). Thereby we have generated multi time period scenarios while capturing the correlation structure of the multivariate uncertainty. To the

best of our knowledge, our model is different to other approaches found in the literature and makes novel contributions in the following ways. (i) We have included multiple sources of uncertainties in the model. (ii) Our study has considered a coherent measure of risk to explore the trade-off between risk and return. (iii) We have constructed practical and realistic scenarios by incorporating correlations between parameters and including dynamic variation over time. (iv) The logical structure of the optimisation model captures more detail than has previously been reported in the literature. Although many of these features (i) - (iv) appear in some research, no comprehensive study covering them altogether has previously been reported. Our development of this comprehensive model bringing in diverse features also makes contribution to knowledge.

In chapter 5 we have investigated solution methodologies suitable for the proposed model, which is a SIP with a large number of scenarios. We have proposed a new solution heuristic called adaptive scenario refinement (ASR), that iteratively adds scenarios based on a maximum regret criterion. This has been implemented in the FortSP stochastic programming solver system (see Ellison et al., 2010). We have carried out an empirical study comparing the solution quality of ASR to other approaches reported in the literature. Publicly available linear and mixed integer test cases for stochastic programming, as well as the developed SCND model have been used. Thereby, we have demonstrated that ASR gives an approximation of the optimal solution superior to the other approaches in the majority of the cases, particularly for the SCND problem.

Having developed a comprehensive methodology for SCND in the previous chapters, we have investigated its benefits and implications in chapter 6. Therefore, we have analysed the results of a case study based on a typical production network in the automotive industry, with multiple products, potential suppliers and production in Europe, Brazil, and China, as well as markets in Europe, Brazil, China, and the United States. The focus has been to analyse the effects and benefits of the key features of the optimisation model, that is, multiple sources of uncertainty and the CV@R risk measure.

Thereby, we have investigated the problem of SCND in a novel way, since each of the reviewed research does not consider all of the following aspects. (i) We have studied the effects of uncertainties on supply chains and according hedging strategies. Most research focuses on algorithmic performance instead, and only the value of the stochastic solution is reported as an indicator for the importance of accounting for uncertainties. (ii) We have explored the trade-off between risk and return, which is often neglected, even if a risk measure is included. (iii) We have included multiple sources of uncertainty in the case study and have investigated the benefits of this. Azaron et al. (2008) is the only publication reporting a case study with more than two sources of uncertainty being considered simultaneously. However, their investigation is limited by considering only eight scenarios and the benefits from accounting for multiple uncertainties are not explored.

Through our approach we have gained the following insights. We have demonstrated that there is a considerable trade-off between risk and return in the design of a supply chain and, hence, that risk measures should not be neglected. We have shown that optimisation results can be significantly improved by accounting for multiple sources of uncertainty simultaneously; especially in the case of risk minimisation. While in the literature most work on SCND focuses on demand uncertainty, our study has suggested that for multi national supply chains, exchange rate uncertainty is more important for the choice of optimal strategies. Additionally, the effect of exchange rate uncertainty on the network can be significantly influenced by an appropriate network strategy, making it even more important to consider.

In conclusion, we have developed a novel and comprehensive approach to SCND, consisting of uncertainty identification, a stochastic optimisation model, and an appropriate solution algorithm. We have applied this approach to a case study in an innovative way and, thus, demonstrated its applicability and benefits, while gaining new insights into the design of supply chains. The SCND model and the use of the coherent downside risk measure in the stochastic program are innovative and novel; these and the ASR solution algorithm taken together make contributions to knowledge.

### 7.2. Suggestions for Future Research

In chapter 3 we have presented an approach for uncertainty identification in the context of supply chains. We have seen in the case study in chapter 6 that not all of the identified uncertainties had a significant impact on the supply chain performance or its optimal design. Therefore, we think that further research is necessary to analyse the effects of various sources of uncertainty on the SCND, to better understand their different impacts. Also, not all uncertainties are suitable for an optimisation model. To take these into account during the decision process, the integration of stochastic optimisation with other approaches, such as simulation or qualitative models, is an interesting field of research.

In the modelling of SCND we suggest three issues that could be addressed by further research. (i) By using a two-stage model, we have approximated the underlying multi-stage decision process. The quality of this approximation in SCND has only very recently been addressed for the first time by Stephan et al. (2011). However, due to the computational complexity only few details are included in their model, it considers uncertainty in demand only, and no risk. Therefore, more research in the trade-off between detail in the decision process, detail in the model, and detail in the uncertainty representation and risk is an interesting direction for future work. (ii) While our model includes many features of strategic and tactical sub-models compared to other stochastic SCND models, it still neglects many aspects, such as storage and seasonality

in demand. Bihlmaier et al. (2009) present one of the few research investigating the 'right' amount of detail in SCND modeling, with a focus on labour costs. This allows for further research into this matter. (iii) We have used CV@R to measure risk since it is coherent and results in a linear optimisation problem. However, numerous other risk measures exist that could be used instead. Future research could investigate computational performance of different risk measures, as well as characteristics of the return distribution. However, the latter issue is difficult to approach, since this evaluation depends on the risk preferences of the decisions maker.

A topic often neglected in publications on SCND under uncertainty is scenario generation. Also outside the domain of SCND, little research is reported on this and even fewer scenario generators are applicable to generate two-stage scenario trees over multiple time periods. Further, the question of how many scenarios are needed for an accurate enough representation of multivariate uncertainty remains challenging.

We think that the investigations on the effects of uncertainty on SCND could be extended in the following two ways. (i) The question of which uncertainties to include could be further investigated. We have included six uncertainties in the model and shown that this is superior to optimisation under single sources of uncertainty. However, including two or three sources of uncertainty might be enough to gain most of the benefits from the six uncertainties – the question then remains, which two (or three) to include. On the other hand, by including additional uncertainties we might find that even six sources are not enough. (ii) There is little research reporting managerial insights and strategies for SCND under uncertainty. We think that more research into this direction would propagate the application of stochastic programming for SCND in industry by revealing the benefits of including uncertainty.

Solution algorithms for two-stage stochastic programs and SIPs are an important topic and there is already a wide research community dedicated to it. However, many of the algorithms in the SCND literature are highly specialised and cannot be easily applied to other models. We feel that to expand the application of stochastic programming in industry, more general purpose stochastic solvers, modeling systems, and scenario generators need to be developed.

We feel that iterative solution algorithms are an interesting field of research. In the ASR algorithm different criteria for the selection of new scenarios could be explored, for example to ensure a better spread of the scenarios. However, the most important issue would be an estimate of the gap, which is especially difficult to establish in the non-convex case of SIPs.

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# **Appendix**

# A. On the Coherence of Risk Measures

In this appendix we introduce the coherence property of risk measures, since the selection of the CV@R as risk measure is based in large part on this. Coherence was first introduced by Artzner et al. (1999) and is a set of four desirable properties that should be fulfilled by risk measure to comply with intuition. See Roman and Mitra (2009) for a summary of this property for various risk measures.

Using the notations from section 2.2.2, let  $(\Omega, \mathcal{F}, P)$  be a probability space and  $\mathcal{L}$  be a linear space of real valued, measurable functions on  $(\Omega, \mathcal{F}, P)$ , which are assumed to represent portfolio return distributions. A risk measure  $\rho : \mathcal{L} \to \mathbb{R}$  is defined to be coherent, if it fulfills the following four properties:

$$\rho(\xi + c) = \rho(\xi) - c, \qquad \forall \xi \in \mathcal{L}, c \in \mathbb{R}, \tag{T}$$

$$\rho(\xi + \zeta) \le \rho(\xi) + \rho(\zeta), \qquad \forall \xi, \zeta \in \mathcal{L}, \tag{S}$$

$$\rho(\lambda \, \xi) = \lambda \rho(\xi), \qquad \forall \xi \in \mathcal{L}, \lambda \ge 0, \tag{PH}$$

$$\xi \le \zeta \Rightarrow \rho(\xi) \ge \rho(\zeta), \qquad \forall \xi, \zeta \in \mathcal{L}.$$
 (M)

(T) is the transitivity property, stating that if a sure gain c is added to a portfolio, its risk is decreased by c. (S) is called subadditivity and ensures that diversification reduces risk (Acerbi et al., 2001). Property (PH) is called positive homogeneity and its interpretation is that if an amount  $\lambda$  is invested in a portfolio, the risk is  $\lambda$  times the risk of investing the amount 1. Finally, (M) is a monotonicity property: if the return of  $\xi$  is always less than the return of  $\zeta$ , then  $\xi$  has higher risk than  $\zeta$ .

# B. A Risk List for SCND

In this appendix we give a complete description of the risk list introduced in section 3.3. The list is structured as set out in table 3.1, with sub-categories related to cause-and-effect relationships. The risks in each category are summarised in tables B.1 - B.9.

Risk	Description
Delay at a supplier Temporary breakdown of a supplier or location of a supplier Permanent breakdown of a supplier or location of a supplier Delivery has to be (partially) re-	A delivery leaves a supplier too late.  A supplier is not able to supply for a limited time period, e.g. due to strike action.  A supplier is not able to supply permanently or for an indefinite time, e.g. due to insolvency or war.  E.g. unusable or wrong goods.
ordered New location of a supplier Missing suppliers	Due to a new supplier location, an existing facility is located sub-optimal.  Not enough sufficiently qualified suppliers available.

Table B.1. List of risks from suppliers and procurement

Risk	Description
Delays of delivery during transportation	E.g. due to a traffic jam or storm.
Delivery has to be (partially) re- ordered	E.g. due to unusable or wrong goods, theft, or accident.
A route of transportation is permanently not available	E.g. due to laws in a country.
Storage risks	Risks during storage and related handling of a good.

Table B.2. List of risks from logistics

Risk	Description
Shifting markets	Demand shifts from one market to a different market.
Changes in the demand mix	Demand mix changes.
Local decrease of demand	Compared to the forecast demand.
Global decrease of demand	Compared to the forecast demand.
Local increase of demand	Compared to the forecast demand.
Global increase of demand	Compared to the forecast demand.

Table B.3. List of demand risks

Risk	Description	
Production quality	E.g. faulty components or products.	
Productivity	Decrease in productivity of a single facility or the whole network.	
Capacity shortages	The available production capacity in a facility or the network	
	does not suffice for the demand.	
Temporary blackout of part of a	A part of a facility (e.g. a production line) is interrupted for a	
facility	limited time period.	
Temporary blackout of a facility	The whole production at a facility is interrupted for a limited	
	time period.	
Permanent blackout of a facility	The whole production at a facility is stopped permanently or for	
	an indefinite time period, e.g. due to a fire or war.	

Table B.4. List of production risks

Risk	Description
Interface risks	Problems at the connection between old and new parts of the network, particularly for network flows such as information, goods, and money.
Barriers for entering new markets	E.g. obstruction from governments to protect local businesses.
Technical equipment and labour	Problems during acquisition or putting into operation of equipment and labour.
Buildings and land	Problems during acquisition and putting into operation of buildings and land.
Missing suppliers	Not enough sufficiently qualified suppliers available.

Table B.5. List of risks from changing the network design

Risk	Description
Flow of information	Risks concerning the flow of information.
Increased cycle time	Longer cycle times in the network, e.g. due to centralisation or globalisation.
Dependencies	Problems originating from dependencies on other locations or suppliers.

Table B.6. List of network specific risks

Risk	Description
Local content laws	Change of laws.
Possibilities for working time	Due to laws or agreements with the works council some working
	times become less attractive.
Environmental laws	E.g. emissions of plants or products.
Interference of governments	E.g. subsidies.

 ${\bf Table~B.7.}$  List of risks from the legal and social environment

Risk	Description				
Financial risks					
Interest rate risk	Change of interest rate.				
Inflation risk	Change of inflation rate.				
Exchange rate risk	Change of exchange rate.				
	Decreased revenues				
Decreased net price	Price of the sold products decreases.				
Debit failures	E.g. leasing customers.				
Decreased sales	See also demand risks.				
	Increased costs				
Investment costs	E.g. for a new facility or production line.				
Cost of materials	Costs of materials increase, e.g. intermediate products from sup				
	pliers, raw materials, or imputed costs for products from other				
	plants.				
Energy costs	Energy costs increase more than expected.				
Taxes and duties	Increases in various taxes and duties.				
Maintenance and repairs	Costs for maintenance, repairs of buildings, and land increase.				
Production costs	Increase in production costs.				
Transport costs	Transport costs increase more than expected.				
Labour costs increase more than expected.					
Physical losses					
Loss of investment goods	E.g. due to natural disasters, war, terror attacks, or man-made disasters.				
Injuries or loss of human lives	E.g. due to accidents and natural or man-made disasters.				
Other losses					
Loss of prestige	E.g. due to child labour at a supplier.				
Loss of intellectual property and know-how	E.g. through theft or failed cooperation.				

Table B.8. List of risks for profit and goals

Risk	Description
Product development	Problems during development of a new product, e.g. leading to a delayed start of production.
Environment within the company	Risks which originate in the fact, that the production network is part of a larger company. E.g. risks from other parts of the company, management, shareholders, or lack of control.

Table B.9. List of other risks

# C. Benders Decomposition

Benders decomposition (also called L-shaped method) is among the most common techniques to solve linear stochastic programs exactly. In its simplest form, for (SP) with relatively complete fixed recourse and  $\mathcal{X} = \mathbb{R}^{n_1}, \mathcal{Y} = \mathbb{R}^{n_2}$ , it is set out in algorithm C.1 (Birge and Louveaux, 1997). Notations are used as explained in chapter 5. The constraints  $E_l x + \theta \ge e_l$  are called *optimality cuts*. In case (SP) is not relatively complete, Benders decomposition can be extended such that it involves solving a second set of recourse linear programs to generate *feasibility cuts*.

#### Algorithm C.1 Benders decomposition algorithm (Birge and Louveaux, 1997)

**Input:** (SP) linear with relatively complete fixed recourse.

- 1: Set  $s = \nu = 0$ .
- 2: repeat
- 3: Set  $\nu = \nu + 1$ . Solve the linear program:

$$\begin{aligned} & \text{if } s > 0 \\ & \min z = c^T x + \theta \\ & \text{s.th.} \quad Ax = b \\ & E_l x + \theta \geq e_l, \\ & x \geq 0, \ \theta \in \mathbb{R}. \end{aligned} \end{aligned} \qquad \begin{aligned} & \text{if } s = 0 \\ & \min z = c^T x \\ & \text{s.th.} \quad Ax = b \\ & x \geq 0. \end{aligned}$$

- 4: Let  $(x^{\nu}, \theta^{\nu})$  be an optimal solution with  $\theta^{\nu} = -\infty$  for s = 0.
- 5: for  $\omega \in \Omega$  do
- 6: Solve the linear program:

min 
$$w = q(\omega)^T y$$
  
s.th.  $Wy = h(\omega) - T(\omega)x^{\nu}$ ,  
 $y > 0$ .

- 7: Let  $\pi^{\nu}(\omega)$  be the simplex multipliers associated with the optimal solution.
- 8: end for
- 9: Let

$$E_{s+1} = \sum_{\omega \in \Omega} P(\omega) (\pi^{\nu} (\omega))^{T} T(\omega), \qquad e_{s+1} = \sum_{\omega \in \Omega} P(\omega) (\pi^{\nu} (\omega))^{T} h(\omega),$$

$$w^{\nu} = e_{s+1} - E_{s+1} x^{\nu}, \qquad s = s+1.$$

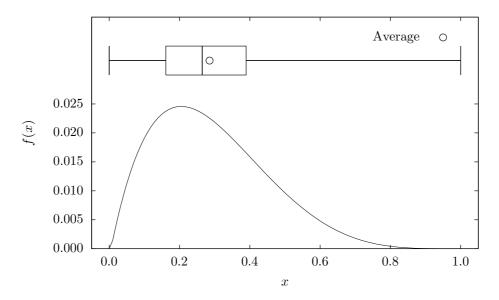
- 10: until  $\theta^{\nu} > w^{\nu}$
- 11: **return** optimal solution  $x^{\nu}$ .

## D. Box Plots

A box plot is a compact graphical representation of the quartiles and expected value of a probability distribution. The spacings between the different parts of the box plot indicate the degree of spread and skewness of the probability distribution. Its advantage over a histogram is that it takes up less space and, thereby, helps comparing different probability distributions.

Let  $\xi:(\Omega,\mathcal{F},P)\to(\mathbb{R},\mathcal{B})$  be a random variable with  $\mathcal{B}$  the Borel  $\sigma$ -algebra on  $\mathbb{R}$ . Let F be the distribution function of  $\xi$  and f its probability density function (if it exists). The  $i^{\text{th}}$  quartile  $q_i$  is defined as  $q_i=F^{-1}(0.25i),\ i=0,\ldots,4$ . Thereby,  $q_0$  is the minimum of the support of  $\xi$ ,  $q_4$  its maximum,  $q_1$  the lower quartile,  $q_3$  the upper quartile, and  $q_2$  the median. Note that, in the case where F is not continuous,  $F^{-1}(y)$  might not exist. Various definitions are proposed for the quartiles in this situation, for example via the left-sided inverse  $F^{-1}(y)=\sup\{x\mid F(x)\leq y\}$ . Also,  $q_0$  and  $q_4$  are infinite, if the support of  $\xi$  is unbounded.

A box plot is a diagram of the five quartiles and the expected value. The lower and upper quartiles are represented by a box, the median by a line within that box, and minimum and maximum by lines outside the box. A typical example of a probability density function and its box plot are shown in figure D.1.



**Figure D.1.** Example of a probability density function f and its box plot