H_∞ State Estimation for Discrete-Time Complex Networks with Randomly Occurring Sensor Saturations and Randomly Varying Sensor Delays

Derui Ding, Zidong Wang, Bo Shen and Huisheng Shu

Abstract—**In this paper, the state estimation problem is investigated for a class of discrete time-delay nonlinear complex networks with randomly occurring phenomena from the sensor measurements. The randomly occurring phenomena include randomly occurring sensor saturations (ROSSs) and randomly varying sensor delays (RVSDs) that result typically from networked environments. A novel sensor model is proposed to describe the ROSSs and the RVSDs within a unified framework via two sets of Bernoulli distributed white sequences with known conditional probabilities. Rather than the commonly used Lipschitz-type function, a more general sector-like nonlinear function is employed to describe the nonlinearities existing in the network. The purpose of the addressed problem is to design a state estimator to estimate the network states through available output measurements such that, for all probabilistic sensor saturations and sensor delays, the dynamics of the estimation error is guaranteed to be exponentially mean-square stable and the effect from the exogenous disturbances to the estimation accuracy is attenuated at a given level by means of an H_∞-norm. In terms of a novel Lyapunov-Krasovskii functional and the Kronecker product, sufficient conditions are established under which the addressed state estimation problem is recast as solving a convex optimization problem via the semi-definite programme method. A simulation example is provided to show the usefulness of the proposed state estimation conditions.**

Index Terms—**Complex networks; state estimation; randomly occurring sensor saturations; randomly varying sensor delays**

I. Introduction

Complex networks are everywhere. Many phenomena in nature can be modeled as coupled networks such as brain structures, protein-protein interactions, social interactions, the Internet and the World Wide Web. All such networks can be represented in terms of nodes, edges and coupling strengths indicating complex connections between the nodes. Typical complex networks that have been thoroughly investigated include communication networks, social networks, electrical power grids, cellular and metabolic networks and the internet. Random graphs are known to be able to describe the large-scale networks with no explicit design principles, so the early study on complex networks has been the territory of graph theory since the seminal work in [6], [7]. Recently, due to the discovery of the "small-world" and "scale-free" properties [3], [30], the dynamical behaviors of complex networks have attracted an ever increasing research interest from a variety of communities such as mathematicians, statisticians, computer scientists and control engineers. As a result, a number of dynamics analysis issues have been extensively investigated for complex networks such as the stability and stabilization, synchronization, pinning control and spread mechanism, see e.g. [1], [2], [4], [8], [10], [12], [14], [17], [18], [20], [21], [23]–[26], [29], [31], [34], [35] and the references therein.

In the past decade, special attention has been focused on the stability and synchronization problems of various complex networks. Generally speaking, there are mainly two approaches that shed insightful lights on the stability and synchronization phenomenon in various real-world complex networks. The first one is the matrix eigenvalue analysis method that has been widely applied (see e.g. [14], [20], [21], [25], [26], [34]) for pinning control or impulsive control problems. For example, in [14], the important concept of virtual control has been proposed to show, in a nice way, that the pinned nodes "virtually" control other dynamical nodes through coupling and eventually lead to the synchronization of the whole network. The other approach is the linear matrix inequality (LMI) technology that has recently been adopted (see e.g. [5], [8], [10], [13], [16], [17], [35]) for complex networks with or without time-delays. For instance, in [5], the relationship between the stability of the whole network and the stability of its corresponding subsystems has been discussed, and the special decentralized control strategy has been employed to derive some necessary and sufficient conditions for the stability and stabilizability for linear networks. Furthermore, in [16], one of the first few attempts has been made to address the synchronization problem for stochastic discrete-time complex networks with time delays.

The vast literature on stability and synchronization problems of complex networks has implicitly assumed that the states of the complex networks under investigation are fully accessible. This is, unfortunately, not always the case in practice. For example, as a typical example of complex networks, the wireless sensor networks exhibit complicated coupling between the sensor nodes as well as network-induced phenomena such as random packet dropouts, random sensor saturations and random sensor communication delays. These phenomena, together with the large scale of the networks, often give rise

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D. Ding, Z. Wang, B. Shen and H. Shu are with the School of Information Science and Technology, Donghua University, Shanghai 200051, China. Z. Wang is also with the Department of Information Systems and Computing, Brunel University, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk)

to the unavailability of part of the sensors, i.e., only partial information from the sensor measurements is available. In this case, it becomes necessary to estimate the states of the nodes through partial but available measurements, and then use the estimated node states to carry out specified tasks such as dynamics analysis and synchronization control. The state estimation problem for various complex networks has recently drawn particular research attention, see e.g. [11], [16], [22]. For instance, in [16], by using a novel Lyapunov-Krasovskii functional and the Kronecker product, the state estimation problem has been studied for an array of discrete-time complex networks with the simultaneous presence of both the discrete and distributed time delays. Very recently, the state estimation problem over a finite-horizon has been investigated in [22] for a class of time-varying complex networks in terms of a new concept called H_∞-synchronization, and the estimator gain has been obtained by utilizing the recursive linear matrix inequalities (RLMIs).

For the purpose of estimating the network states in reality, the available measurement outputs are collected from all sensor that are then processed to minimize the effects from the possible noise and various kinds of incomplete information such as the missing measurement and communication delays. It is now well known that sensors cannot provide signals of unlimited amplitude due primarily to the physical or technological constraints. This phenomenon is referred to as the sensor saturation. Saturation brings in nonlinear characteristics that can severely restrict the application of traditional estimator design schemes. Specifically, this kind of characteristics not only degrades the estimation performance that can be achieved without saturation, it may also lead to undesirable oscillatory or even unstable behavior. Because of the practical importance of sensor saturations, much attention has been focused on the filtering and control problems for systems with sensor saturation [9], [32]. In most existing literature, the saturation is actually assumed to occur definitely. Such an assumption is, however, not always true. For example, in a network environment, the sensor saturations may occur in a probabilistic way and the saturation level may be randomly changeable as well. This is mainly due to the random occurrence of networked-induced phenomena such as random sensor failures leading to intermittent saturation, sensor aging resulting in changeable saturation level, sudden environment changes, etc. Such a phenomenon of sensor saturation, namely, randomly occurring sensor saturation (ROSS), has been largely overlooked in the area due probably to the difficulty in mathematical analysis. The main motivation of the present research is, therefore, to investigate how the ROSS phenomenon influences the performance of state estimation for complex networks.

In addition to the appearance of ROSSs, the sensor measurement delay serves another common phenomenon that occurs in a random way especially when the sensors are connected via communication networks. Sensor delays may be induced by a variety of reasons such as an asynchronous time-division-multiplexed network, intermittent sensor failures, and random congestion of packet transmissions, etc. Such a phenomenon is customarily referred to as the randomly varying sensor delays (RVSDs), see [33] and [27] for more details. In many cases, the RVSDs are a source of instability and performance deterioration of a complex network equipped with a large number of sensors. One of the most popular ways to describe the RVSDs is to use a Bernoulli distributed (binary switching) white sequence specified by a conditional probability distribution in the sensor output. This approach has first been proposed in [19] to deal with the optimal recursive estimation problem. Recently, it has been utilized in [27] for filtering problems and in [15] for control designs. Obviously, to reflect the network reality, it makes practical sense to consider both the ROSSs and RVSDs where their occurrence probabilities can be estimated via statistical tests. Up to now, to the best of the authors' knowledge, the estimation problem for complex networks with both the ROSSs and RVSDs remains an open yet challenging issue, and the main purpose of this paper to shorten such a gap. It is worth pointing out that the main difficulty lies in how to establish a unified framework to account for the two phenomena of ROSSs and RVSDs.

Summarizing the above discussions, the focus of this paper is on the state estimation problem for a class of discrete time-delay complex networks with randomly occurring phenomena including ROSSs and RVSDs that result typically from networked environments. Two sets of Bernoulli distributed white sequences with known conditional probabilities are introduced to describe the ROSSs and the RVSDs within a unified framework. A general sector-like nonlinear function is employed to describe the inherently nonlinear nature of the complex networks. By employing the Lyapunov stability theory combined with the stochastic analysis approach, a delay-dependent criterion is established that guarantees the existence of the desired estimator gains, and then the explicit expression of such estimator gains is characterized in terms of the solution to a convex optimization problem via the semi-definite programme method. Moreover, a simulation example is provided to show the effectiveness of the proposed estimator design scheme. The main contribution of this paper is mainly twofold: *1) A novel sensor model is established to account for both the ROSSs and RVSDs in a unified framework; and 2) based on this sensor model, the estimator design approach is proposed to ensure that the error dynamics is exponentially mean-square stable and the H_∞ performance constraint is satisfied.*

The rest of this paper is organized as follows. In Section II, a class of discrete time-delayed complex networks with both the ROSSs and RVSDs are presented. In Section III, by employing the Lyapunov stability theory, some sufficient conditions are established in the form of LMI and then the explicit expression of the estimator gains is given. In Section IV, an example is presented to demonstrate the effectiveness of the results obtained. Finally, conclusions are drawn in Section V.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The set of all positive integers is denoted by $\mathbb{N}$. $l_2([0,\infty);\mathbb{R}^n)$ is the space of square-summable n-dimensional vector functions over $[0,\infty)$. I denotes the identity matrix of compatible dimension. The notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of M. Sym$\{A\}$ denotes the symmetric matrix $A + A^T$. For a matrix $A \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalue of A, respectively. $\mathbb{E}\{x\}$ stands for the expectation of stochastic variable x. $||x||$ describes the Euclidean norm of a vector x. The shorthand $\mathrm{diag}\{M_1, M_2, \cdots, M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices M_1, ..., M_n. The symbol $\otimes$ denotes the Kronecker product. In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry.

II. Problem Formulation and Preliminaries

Consider the following array of discrete time-delayed complex networks consisting of N coupled nodes:

$$\begin{cases} x_i(k+1) = f(x_i(k)) + g(x_i(k-\tau(k))) \\ \qquad\qquad + \sum_{j=1}^{N} w_{ij}\Gamma x_j(k) + L_i v_1(k), \\ z_i(k) = M x_i(k), \\ x_i(s) = \psi_i(s), \forall s \in [-\bar{\tau}_M,\ 0], i = 1, 2, \cdots, N, \end{cases} \tag{1}$$

where $x_i(k) \in \mathbb{R}^n$ is the state vector of the ith node, $z_i(k) \in \mathbb{R}^r$ is the output of the ith node, and $v_1(k)$ is the disturbance input belonging to $l_2([0,\infty); \mathbb{R}^q)$. $\Gamma = \mathrm{diag}\{r_1, r_2, \cdots, r_n\}$ is the matrix linking the jth state variable if $r_j \neq 0$, and $W = (w_{ij})_{N\times N}$ is the coupled configuration matrix of the network with $w_{ij} \geq 0$ $(i \neq j)$ but not all zero. L_i and M are constant matrices with appropriate dimensions, and $\psi_i(s)$ is a given initial condition sequence.

The positive integer $\tau(k)$ describes the known time-varying delay satisfying $0 < \bar{\tau}_m \leq \tau(k) \leq \bar{\tau}_M$, where $\bar{\tau}_m$ and $\bar{\tau}_M$ are known positive integers representing the minimum and maximum delays, respectively. The nonlinear vector-valued functions f and $g : \mathbb{R}^n \mapsto \mathbb{R}^n$ are assumed to be continuous and satisfy $f(0) = 0$, $g(0) = 0$ and the following sector-bounded conditions

$$\begin{aligned} &[f(x) - f(y) - \phi_1^f(x-y)]^T[f(x) - f(y) - \phi_2^f(x-y)] \leq 0, \\ &[g(x) - g(y) - \phi_1^g(x-y)]^T[g(x) - g(y) - \phi_2^g(x-y)] \leq 0, \end{aligned} \tag{2}$$

for all $x, y \in \mathbb{R}^n$, where ϕ_1^f, ϕ_2^f, ϕ_1^g and ϕ_2^g are real matrices of appropriate dimensions.

In this paper, the N sensors with both saturations and delays are modeled by

$$\begin{aligned} y_i(k) =& \beta_i(k)[\alpha_i(k)\varrho(Cx_i(k)) + (1-\alpha_i(k))Cx_i(k)] \\ &+ (1-\beta_i(k))[\alpha_i(k)\varrho(Cx_i(k-d)) \\ &+ (1-\alpha_i(k))Cx_i(k-d)] + G_i v_2(k), \\ &i = 1, 2, \cdots, N, \end{aligned} \tag{3}$$

where $y_i(k) \in \mathbb{R}^m$ is the measurement output of the node i, $v_2(k)$ is the disturbance input which belongs to $l_2([0,\infty); \mathbb{R}^p)$, the sensor delay d is a scalar satisfying $0 < d \leq \bar{\tau}_M$, and G_i, C are known matrices with appropriate dimensions. The saturation function $\varrho : \mathbb{R}^m \mapsto \mathbb{R}^m$ is defined as

$$\varrho(x) = \begin{bmatrix} \varrho(x_1) & \varrho(x_2) & \cdots & \varrho(x_m) \end{bmatrix}^T, \tag{4}$$

where x_i is the ith element of the vector x and $\varrho(x_i) = \mathrm{sign}(x_i)\min\{1,\ |x_i|\}$. Here, the notation of "sign" denotes the signum function. Later, we will slightly abuse the notation by using ϱ to denote both the scalar valued and the vector valued saturation functions. Note that, without loss of generality, the saturation level is taken as unity. The variables $\alpha_i(k)$ and $\beta_i(k)$ $(i = 1, 2, \cdots, N)$ are Bernoulli distributed white sequences taking values on 0 and 1 with the following probabilities:

$$\begin{cases} \mathrm{Prob}\{\alpha_k^i = 1\} = \alpha_i \\ \mathrm{Prob}\{\alpha_k^i = 0\} = 1 - \alpha_i \end{cases} \text{and} \begin{cases} \mathrm{Prob}\{\beta_k^i = 1\} = \beta_i \\ \mathrm{Prob}\{\beta_k^i = 0\} = 1 - \beta_i \end{cases}$$

where $\alpha_i, \beta_i \in [0,1]$ are known constants. Throughout the paper, the stochastic variables $\alpha_i(k)$ and $\beta_i(k)$ are mutually independent in all i.

Remark 1: As is well known, in an abstract model for complex networks, the nodes in the same cluster usually possess the same attributes or properties. For example, in a homogeneous sensor network, the sensor nodes are typically identical in terms of battery energy and hardware complexity. An interesting topic for complex networks is to examine how the nodes interact each other to form rich dynamics through links according to a given topology dynamics. Therefore, it is reasonable to assume that the nodes' information is collected by means of the same type of measurements. Moreover, without loss of generality, the disturbances in measurements are set to be same. In the case of different kinds of disturbances, similar results can be obtained readily by using an augmented method.

Remark 2: Note that, the zero-row-sum property of the configuration matrix W is quite important for many traditional methods to deal with the dynamics analysis issues of complex networks. By assuming the zero-row-sum property, the eigenvalue-based matrix analysis methods could be employed to construct the difference of signals, see e.g [14], [16], [20]–[23], [25], [26], [34] for more details. These methods, however, are no longer valid for the problem addressed in this paper because of the sensor saturation phenomenon. One of the main contributions of this paper would be the development of a new methodology to deal with the phenomenon of both ROSSs and RVSDs without requiring the zero-row-sum property.

Remark 3: The measurement model proposed in (3) provides a novel unified framework to account for the phenomenon of both ROSSs and RVSDs. The stochastic variable $\alpha_i(k)$ characterizes the random nature of sensor saturation while the stochastic variable $\beta_i(k)$ is used to describe the phenomenon of the probabilistic sensor delay. By combining these two stochastic variables, model (3) represents the following four different phenomena: a) when $\beta_i(k) = 1$ and $\alpha_i(k) = 0$, sensor i works normally; b) when $\beta_i(k) = 1$ and $\alpha_i(k) = 1$, model (3) is reduced to $y_i(k) = \varrho(Cx_i(k))$ which means that the measurements received by sensor i are saturated; c) when $\beta_i(k) = 0$ and $\alpha_i(k) = 0$, it can be seen from model (3) that the measurements at previous d time-instant are employed by estimator i instead of the one at current time-instant; and d) when $\beta_i(k) = 0$ and $\alpha_i(k) = 1$, model (3) implies that the measurements are not only delayed but also saturated before they enter into the estimator i. In addition, it is easy to observe that the time-delay in the measurements takes random values as "0" when $\beta_i(k) = 1$ and "d" when $\beta_i(k) = 0$. Such kind of phenomenon is referred to as the randomly varying delays.

Based on the measurement $y_i(k)$, we construct the following state estimator for node i:

$$\begin{cases} \hat{x}_i(k+1) = f(\hat{x}_i(k)) + g(\hat{x}_i(k-\tau(k))) \\ \qquad\qquad + K_i(y_i(k) - C\hat{x}_i(k)), \\ \hat{z}_i(k) = M\hat{x}_i(k), \\ \hat{x}_i(s) = 0, \quad \forall s \in [-\bar{\tau}_M,\ 0], i = 1, 2, \cdots, N, \end{cases} \tag{5}$$

where $\hat{x}_i(k) \in \mathbb{R}^n$ is the estimate of the state $x_i(k)$, $\hat{z}_i(k) \in \mathbb{R}^r$ is the estimate of the output $z_i(k)$, and $K_i \in \mathbb{R}^{n\times m}$ is the estimator gain matrix to be designed.

For the purpose of simplicity, we introduce the following notations:

$$\begin{aligned}
&x_k = [\ x_1^T(k) \quad x_2^T(k) \quad \cdots \quad x_N^T(k)\ ]^T, \\
&\hat{x}_k = [\ \hat{x}_1^T(k) \quad \hat{x}_2^T(k) \quad \cdots \quad \hat{x}_N^T(k)\ ]^T, \\
&z_k = [\ z_1^T(k) \quad z_2^T(k) \quad \cdots \quad z_N^T(k)\ ]^T, \\
&\hat{z}_k = [\ \hat{z}_1^T(k) \quad \hat{z}_2^T(k) \quad \cdots \quad \hat{z}_N^T(k)\ ]^T, \quad \tilde{z}_k = z_k - \hat{z}_k, \\
&v_k = [\ v_1^T(k) \quad v_2^T(k)\ ]^T, \quad \tau_k = \tau(k), \quad e_k = x_k - \hat{x}_k, \\
&f(x_k) = [\ f^T(x_1(k)) \quad f^T(x_2(k)) \quad \cdots \quad f^T(x_N(k))\ ]^T, \\
&g(x_k) = [\ g^T(x_1(k)) \quad g^T(x_2(k)) \quad \cdots \quad g^T(x_N(k))\ ]^T, \\
&\vec{\varrho}(\tilde{C}x_k) = [\ \varrho^T(Cx_1(k)) \quad \varrho^T(Cx_2(k)) \quad \cdots \quad \varrho^T(Cx_N(k))\ ]^T, \\
&\tilde{f}_k = f(x_k) - f(\hat{x}_k), \quad \tilde{g}_k = g(x_k) - g(\hat{x}_k), \\
&K = \text{diag}\{K_1, K_2, \cdots, K_N\}, \quad \tilde{C} = I \otimes C, \\
&L = [\ L_1^T \quad L_2^T \quad \cdots \quad L_N^T\ ]^T, \quad \tilde{L} = [\ L \quad -KG\ ], \\
&G = [\ G_1^T \quad G_2^T \quad \cdots \quad G_N^T\ ]^T, \quad \tilde{M} = I \otimes M, \\
&E_i = \text{diag}\{\underbrace{0, \cdots, 0}_{i-1}, I, \underbrace{0, \cdots, 0}_{N-i}\}.
\end{aligned}$$

By using the Kronecker product, the error dynamics of the state estimation can be obtained from (1), (3) and (5) as follows:

$$\begin{cases} e_{k+1} = & \tilde{f}_k + \tilde{g}_{k-\tau_k} - K\tilde{C}e_k + (W \otimes \Gamma + K\tilde{C})x_k \\ & +\tilde{L}v_k - K\Bigg(\sum_{i=1}^{N} \alpha_i(k)\beta_i(k)E_i\vec{\varrho}(\tilde{C}x_k) \\ & + \sum_{i=1}^{N}(1-\alpha_i(k))\beta_i(k)E_i\tilde{C}x_k \\ & + \sum_{i=1}^{N}\alpha_i(k)(1-\beta_i(k))E_i\vec{\varrho}(\tilde{C}x_{k-d}) \\ & + \sum_{i=1}^{N}(1-\alpha_i(k))(1-\beta_i(k))E_i\tilde{C}x_{k-d}\Bigg), \\ \tilde{z}_k = & \tilde{M}e_k. \end{cases} \tag{6}$$

Then, by setting $\eta_k = [\ x_k^T \quad e_k^T\ ]^T$, we have the following augmented system

$$\begin{cases} \eta_{k+1} = & \mathcal{W}_1\eta_k + \mathcal{W}_2\eta_{k-d} + \vec{f}_k + \vec{g}_{k-\tau_k} \\ & +\mathcal{H}\tilde{\alpha}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k) + \mathcal{H}\tilde{\upsilon}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d}) \\ & + \sum_{i=1}^{N}(\tilde{\alpha}_k^i - \tilde{\alpha}_i)\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k) \\ & + \sum_{i=1}^{N}(\tilde{\beta}_k^i - \tilde{\beta}_i)\mathcal{G}_i\tilde{C}\mathcal{S}\eta_k \\ & + \sum_{i=1}^{N}(\tilde{\upsilon}_k^i - \tilde{\upsilon}_i)\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d}) \\ & + \sum_{i=1}^{N}(\tilde{\vartheta}_k^i - \tilde{\vartheta}_i)\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d} + \mathcal{L}v_k, \\ \tilde{z}_k = & \mathcal{M}\eta_k, \\ \eta_i = & [\psi_1^T(i), \psi_2^T(i), \cdots, \psi_N^T(i), \psi_1^T(i), \psi_2^T(i), \\ & \cdots, \psi_N^T(i)]^T, \forall i \in [-\bar{\tau}_M, 0] \end{cases} \tag{7}$$

where

$$\begin{aligned}
&\vec{f}_k = [\ f^T(x_k) \quad \tilde{f}_k^T\ ]^T, \quad \vec{g}_k = [\ g^T(x_k) \quad \tilde{g}_k^T\ ]^T, \\
&\tilde{\alpha}_k^i = \alpha_i(k)\beta_i(k), \quad \tilde{\upsilon}_k^i = \alpha_i(k)(1-\beta_i(k)), \\
&\tilde{\beta}_k^i = (1-\alpha_i(k))\beta_i(k), \quad \tilde{\vartheta}_k^i = (1-\alpha_i(k))(1-\beta_i(k)), \\
&\tilde{\alpha}_i = \alpha_i\beta_i, \quad \tilde{\alpha}^{\Lambda} = \text{diag}\{\tilde{\alpha}_1 I, \tilde{\alpha}_2 I, \cdots, \tilde{\alpha}_N I\}, \\
&\tilde{\beta}_i = \beta_i(1-\alpha_i), \quad \tilde{\beta}^{\Lambda} = \text{diag}\{\tilde{\beta}_1 I, \tilde{\beta}_2 I, \cdots, \tilde{\beta}_N I\}, \\
&\tilde{\upsilon}_i = \alpha_i(1-\beta_i), \quad \tilde{\upsilon}^{\Lambda} = \text{diag}\{\tilde{\upsilon}_1 I, \tilde{\upsilon}_2 I, \cdots, \tilde{\upsilon}_N I\}, \\
&\tilde{\vartheta}_i = (1-\alpha_i)(1-\beta_i), \quad \tilde{\vartheta}^{\Lambda} = \text{diag}\{\tilde{\vartheta}_1 I, \tilde{\vartheta}_2 I, \cdots, \tilde{\vartheta}_N I\}, \\
&\mathcal{H} = [\ 0 \quad -K^T\ ]^T, \quad \mathcal{G}_i = [\ 0 \quad -E_i^T K^T\ ]^T, \\
&\mathcal{M} = [\ 0 \quad \tilde{M}\ ], \quad \mathcal{S} = [\ I \quad 0\ ], \\
&\mathcal{W}_1 = \begin{bmatrix} W \otimes \Gamma & 0 \\ W \otimes \Gamma + K(I - \tilde{\beta}^{\Lambda})\tilde{C} & -K\tilde{C} \end{bmatrix}, \\
&\mathcal{W}_2 = \begin{bmatrix} 0 & 0 \\ -K\tilde{\vartheta}^{\Lambda}\tilde{C} & 0 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} L & 0 \\ L & -KG \end{bmatrix}.
\end{aligned}$$

As analyzed in [32], [36], the saturation function $\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)$ satisfies

$$[\vec{\varrho}(\tilde{C}\mathcal{S}\eta) - (I \otimes \Lambda)\tilde{C}\mathcal{S}\eta]^T[\vec{\varrho}(\tilde{C}\mathcal{S}\eta) - \tilde{C}\mathcal{S}\eta] \le 0, \tag{8}$$

where $\Lambda = \text{diag}\{\theta_1, \theta_2, \cdots, \theta_m\}$ and $0 \le \Lambda \le I$. Also, it follows from (2) that

$$\begin{aligned} &[\ \vec{f}_k - (I \otimes \phi_1^f)\eta_k\ ]^T[\ \vec{f}_k - (I \otimes \phi_2^f)\eta_k\ ] \le 0, \\ &[\ \vec{g}_k - (I \otimes \phi_1^g)\eta_k\ ]^T[\ \vec{g}_k - (I \otimes \phi_2^g)\eta_k\ ] \le 0. \end{aligned} \tag{9}$$

Definition 1: [28] The augmented system (7) with $v_k = 0$ is said to be exponentially mean-square stable if there exist constants $\varepsilon > 0$ and $0 < \hbar < 1$ such that

$$\mathbb{E}\{||\eta_k||^2\} \le \varepsilon\hbar^k \max_{i\in[-\bar{\tau}_M,\ 0]} \mathbb{E}\{||\eta_i||^2\},\ k \in \mathbb{N}.$$

The purpose of this paper is to design a set of state estimators of form (5) for the complex networks (1) with the sensor model (3) containing both ROSSs and RVSDs. More specifically, we are interested in looking for the parameters K_i $(i = 1, 2, \cdots, N)$ such that the following requirements are met simultaneously:

a) The zero-solution of the augmented system (7) with $v_k = 0$ is exponentially mean-square stable.

b) Under the zero-initial condition, for a given disturbance attenuation level $\gamma > 0$ and all nonzero v_k, the output error $\tilde{z}_k$ satisfies

$$\frac{1}{N}\sum_{k=0}^{\infty}\mathbb{E}\{||\tilde{z}_k||^2\} \le \gamma^2\sum_{k=0}^{\infty}||v_k||^2. \tag{10}$$

Remark 4: In terms of (6) and (10), it can be seen that the value of $\mathbb{E}\{||\tilde{z}_k||^2\}$ would become larger as the number of the nodes increases. Theoretically, the disturbance attenuation level γ for the overall network should account for the average disturbance rejection performance that is insensitive to the change of the number of the nodes in the estimator design. For this purpose, the term of $1/N$ is used to accommodate the average H_∞ index over the complex network so that the scalar γ reflects the practical significance of the H_∞ disturbance rejection level.

III. Main Results

In this section, the stability and the H_∞ performance are analyzed for the augmented system (7). A sufficient condition is given to guarantee that the augmented system (7) is exponentially mean-square stable and the H_∞ performance is achieved for all probabilistic sensor saturations and sensor delays. Then, the explicit expression of the desired estimator gains is proposed in terms of the solution to certain matrix inequalities derived according to the obtained condition.

Theorem 1: Let the estimator parameters K_i $(i = 1, 2, \cdots, N)$ and the diagonal matrix Λ be given. The zero-solution of the augmented system (7) with $v_k = 0$ is exponentially mean-square stable if there exist positive definite matrices Q_i $(i = 1, 2, \cdots, 6)$ and positive scalars λ_j $(j = 1, 2, \cdots, 4)$ satisfying

$$\Pi_1 = \begin{bmatrix} \Xi_{11} & \mathcal{W}_1^T P_1 \mathcal{W}_2 & 0 & \Xi_{14} & \mathcal{W}_1^T P_1 & \Xi_{16} & \mathcal{W}_1^T P_1 \mathcal{H}\tilde{\upsilon}^\Lambda \\ * & \Xi_{22} & 0 & \mathcal{W}_2^T P_1 & \mathcal{W}_2^T P_1 & \mathcal{W}_2^T P_1 \mathcal{H}\tilde{\alpha}^\Lambda & \Xi_{27} \\ * & * & \Xi_{33} & 0 & \lambda_2 \Phi_2^{gT} & 0 & 0 \\ * & * & * & \Xi_{44} & P_1 & P_1 \mathcal{H}\tilde{\alpha}^\Lambda & P_1 \mathcal{H}\tilde{\upsilon}^\Lambda \\ * & * & * & * & \Xi_{55} & P_1 \mathcal{H}\tilde{\alpha}^\Lambda & P_1 \mathcal{H}\tilde{\upsilon}^\Lambda \\ * & * & * & * & * & \Xi_{66} & \Xi_{67} \\ * & * & * & * & * & * & \Xi_{77} \end{bmatrix} < 0, \tag{11}$$

where

$$\begin{aligned}
\tilde{\beta}_i^* =& \beta_i(1-\alpha_i)[1-\beta_i(1-\alpha_i)], \\
\tilde{\vartheta}_i^* =& (1-\alpha_i)(1-\beta_i)[1-(1-\alpha_i)(1-\beta_i)], \\
\tilde{\upsilon}_i^* =& \alpha_i(1-\beta_i)[1-\alpha_i(1-\beta_i)], \quad \tilde{\alpha}_i^* = \alpha_i\beta_i(1-\alpha_i\beta_i), \\
\chi_i^{\tilde{\alpha}\tilde{\vartheta}} =& \chi_i^{\tilde{\beta}\tilde{\upsilon}} = \alpha_i\beta_i(1-\alpha_i)(1-\beta_i), \\
\chi_i^{\tilde{\alpha}\tilde{\beta}} =& \alpha_i\beta_i^2(1-\alpha_i), \quad \chi_i^{\tilde{\beta}\tilde{\vartheta}} = \beta_i(1-\alpha_i)^2(1-\beta_i), \\
\chi_i^{\tilde{\alpha}\tilde{\upsilon}} =& \alpha_i^2\beta_i(1-\beta_i), \quad \chi_i^{\tilde{\upsilon}\tilde{\vartheta}} = \alpha_i(1-\alpha_i)(1-\beta_i)^2, \\
\Phi_1^f =& I \otimes \mathrm{Sym}\{\frac{1}{2}\phi_1^{fT}\phi_2^f\}, \quad \Phi_2^f = I \otimes (\phi_1^f + \phi_2^f)/2, \\
\Phi_1^g =& I \otimes \mathrm{Sym}\{\frac{1}{2}\phi_1^{gT}\phi_2^g\}, \quad \Phi_2^g = I \otimes (\phi_1^g + \phi_2^g)/2, \\
P_1 =& \mathrm{diag}\{I \otimes Q_1,\ I \otimes Q_2\}, \quad \Xi_{33} = -P_3 - \lambda_2\Phi_1^g, \\
P_2 =& \mathrm{diag}\{I \otimes Q_3,\ I \otimes Q_4\}, \quad \Xi_{44} = P_1 - \lambda_1 I \\
P_3 =& \mathrm{diag}\{I \otimes Q_5,\ I \otimes Q_6\}, \quad \Xi_{55} = P_1 - \lambda_2 I, \\
\Xi_{11} =& \mathcal{W}_1^T P_1 \mathcal{W}_1 - P_1 + P_2 + (\bar{\tau}_M - \bar{\tau}_m + 1)P_3 \\
& - \lambda_1\Phi_1^f - \lambda_3\mathcal{S}^T\tilde{C}^T(I\otimes\Lambda)\tilde{C}\mathcal{S} \\
& + \sum_{i=1}^N (\tilde{\beta}_i^* + \chi_i^{\tilde{\alpha}\tilde{\beta}} + \chi_i^{\tilde{\beta}\tilde{\upsilon}} + \chi_i^{\tilde{\beta}\tilde{\vartheta}})\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1 \mathcal{G}_i\tilde{C}\mathcal{S}, \\
\Xi_{14} =& \mathcal{W}_1^T P_1 + \lambda_1\Phi_2^{fT}, \quad \Xi_{67} = \tilde{\alpha}^{\Lambda T}\mathcal{H}^T P_1 \mathcal{H}\tilde{\upsilon}^\Lambda, \\
\Xi_{16} =& \mathcal{W}_1^T P_1 \mathcal{H}\tilde{\alpha}^\Lambda + \lambda_3\mathcal{S}^T\tilde{C}^T(I\otimes\Lambda + I)/2, \\
\Xi_{22} =& \mathcal{W}_2^T P_1 \mathcal{W}_2 - P_2 - \lambda_4\mathcal{S}^T\tilde{C}^T(I\otimes\Lambda)\tilde{C}\mathcal{S} \\
& + \sum_{i=1}^N (\tilde{\vartheta}_i^* + \chi_i^{\tilde{\alpha}\tilde{\vartheta}} + \chi_i^{\tilde{\beta}\tilde{\vartheta}} + \chi_i^{\tilde{\upsilon}\tilde{\vartheta}})\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1 \mathcal{G}_i\tilde{C}\mathcal{S}, \\
\Xi_{27} =& \mathcal{W}_2^T P_1 \mathcal{H}\tilde{\upsilon}^\Lambda + \lambda_4\mathcal{S}^T\tilde{C}^T(I\otimes\Lambda + I)/2, \\
\Xi_{66} =& \tilde{\alpha}^{\Lambda T}\mathcal{H}^T P_1 \mathcal{H}\tilde{\alpha}^\Lambda - \lambda_3 I \\
& + \sum_{i=1}^N (\tilde{\alpha}_i^* + \chi_i^{\tilde{\alpha}\tilde{\beta}} + \chi_i^{\tilde{\alpha}\tilde{\upsilon}} + \chi_i^{\tilde{\alpha}\tilde{\vartheta}})\mathcal{G}_i^T P_1 \mathcal{G}_i, \\
\Xi_{77} =& \tilde{\upsilon}^{\Lambda T}\mathcal{H}^T P_1 \mathcal{H}\tilde{\upsilon}^\Lambda - \lambda_4 I \\
& + \sum_{i=1}^N (\tilde{\upsilon}_i^* + \chi_i^{\tilde{\alpha}\tilde{\upsilon}} + \chi_i^{\tilde{\beta}\tilde{\upsilon}} + \chi_i^{\tilde{\upsilon}\tilde{\vartheta}})\mathcal{G}_i^T P_1 \mathcal{G}_i.
\end{aligned}$$

Proof: Construct the following Lyapunov function for system (7):

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{12}$$

where

$$\begin{aligned}
V_1(k) &= \eta_k P_1 \eta_k + \sum_{i=k-d}^{k-1} \eta_i P_2 \eta_i, \\
V_2(k) &= \sum_{i=k-\tau_k}^{k-1} \eta_i P_3 \eta_i, \quad V_3(k) = \sum_{j=k-\bar{\tau}_M+1}^{k-\bar{\tau}_m} \sum_{i=j}^{k-1} \eta_i P_3 \eta_i.
\end{aligned}$$

Calculating the difference of $V_1(k)$ along the trajectory of system (7) with $v_k = 0$ and taking the mathematical expectation, we have

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_1(k)\} = \mathbb{E}\{V_1(k+1) - V_1(k)\}\\
&= \mathbb{E}\{\eta_{k+1}P_1\eta_{k+1} - \eta_k P_1\eta_k + \eta_k P_2\eta_k - \eta_{k-d}P_2\eta_{k-d}\}\\
&= \mathbb{E}\left\{\eta_k^T\mathcal{W}_1^T P_1\mathcal{W}_1\eta_k + \eta_{k-d}^T\mathcal{W}_2^T P_1\mathcal{W}_2\eta_{k-d}\right.\\
&\quad + \vec{f}_k^T P_1\vec{f}_k + \vec{g}_{k-\tau_k}^T P_1\vec{g}_{k-\tau_k}\\
&\quad + \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\tilde{\alpha}^{\Lambda T}\mathcal{H}^T P_1\mathcal{H}\tilde{\alpha}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)\\
&\quad + \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_{k-d})\tilde{\upsilon}^{\Lambda T}\mathcal{H}^T P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\quad + \sum\nolimits_{i=1}^N \tilde{\alpha}_i^*\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)\\
&\quad + \sum\nolimits_{i=1}^N \tilde{\beta}_i^*\eta_k^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_k\\
&\quad + \sum\nolimits_{i=1}^N \tilde{\upsilon}_i^*\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_{k-d})\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\quad + \sum\nolimits_{i=1}^N \tilde{\vartheta}_i^*\eta_{k-d}^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}\\
&\quad + 2\eta_k^T\mathcal{W}_1^T P_1\mathcal{W}_2\eta_{k-d} + 2\eta_k^T\mathcal{W}_1^T P_1\vec{f}_k\\
&\quad + 2\eta_k^T\mathcal{W}_1^T P_1\vec{g}_{k-\tau_k} + 2\eta_k^T\mathcal{W}_1^T P_1\mathcal{H}\tilde{\alpha}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)\\
&\quad + 2\eta_k^T\mathcal{W}_1^T P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d}) + 2\eta_{k-d}^T\mathcal{W}_2^T P_1\vec{f}_k\\
&\quad + 2\eta_{k-d}^T\mathcal{W}_2^T P_1\vec{g}_{k-\tau_k} + 2\eta_{k-d}^T\mathcal{W}_2^T P_1\mathcal{H}\tilde{\alpha}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)\\
&\quad + 2\eta_{k-d}^T\mathcal{W}_2^T P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d}) + 2\vec{f}_k^T P_1\vec{g}_{k-\tau_k}\\
&\quad + 2\vec{f}_k^T P_1\mathcal{H}\tilde{\alpha}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k) + 2\vec{f}_k^T P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\quad + 2\vec{g}_{k-\tau_k}^T P_1\mathcal{H}\tilde{\alpha}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)\\
&\quad + 2\vec{g}_{k-\tau_k}^T P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\quad + 2\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\tilde{\alpha}^{\Lambda T}\mathcal{H}^T P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\quad - 2\sum\nolimits_{i=1}^N \chi_i^{\tilde{\alpha}\tilde{\beta}}\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_k\\
&\quad - 2\sum\nolimits_{i=1}^N \chi_i^{\tilde{\alpha}\tilde{\upsilon}}\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\quad - 2\sum\nolimits_{i=1}^N \chi_i^{\tilde{\alpha}\tilde{\vartheta}}\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}\\
&\quad - 2\sum\nolimits_{i=1}^N \chi_i^{\tilde{\beta}\tilde{\upsilon}}\eta_k^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\quad - 2\sum\nolimits_{i=1}^N \chi_i^{\tilde{\beta}\tilde{\vartheta}}\eta_k^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}\\
&\quad - 2\sum\nolimits_{i=1}^N \chi_i^{\tilde{\upsilon}\tilde{\vartheta}}\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_{k-d})\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}\\
&\quad \left. -\eta_k P_1\eta_k + \eta_k P_2\eta_k - \eta_{k-d}P_2\eta_{k-d}\right\}.
\end{aligned} \tag{13}
$$

Similarly, we can derive

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_2(k)\} = \mathbb{E}\{V_2(k+1) - V_2(k)\}\\
&= \mathbb{E}\left\{\sum_{i=k-\tau_{k+1}+1}^{k}\eta_i P_3\eta_i - \sum_{i=k-\tau_k}^{k-1}\eta_i P_3\eta_i\right\}\\
&= \mathbb{E}\left\{\eta_k P_3\eta_k - \eta_{k-\tau_k}P_3\eta_{k-\tau_k}\right.\\
&\quad \left. + \sum_{i=k-\tau_{k+1}+1}^{k-1}\eta_i P_3\eta_i - \sum_{i=k-\tau_k+1}^{k-1}\eta_i P_3\eta_i\right\}\\
&= \mathbb{E}\left\{\eta_k P_3\eta_k - \eta_{k-\tau_k}P_3\eta_{k-\tau_k} + \sum_{i=k-\bar{\tau}_m+1}^{k-1}\eta_i P_3\eta_i\right.\\
&\quad \left. + \sum_{i=k-\tau_{k+1}+1}^{k-\bar{\tau}_m}\eta_i P_3\eta_i - \sum_{i=k-\tau_k+1}^{k-1}\eta_i P_3\eta_i\right\}\\
&\le \mathbb{E}\left\{\eta_k P_3\eta_k - \eta_{k-\tau_k}P_3\eta_{k-\tau_k} + \sum_{i=k-\bar{\tau}_M+1}^{k-\bar{\tau}_m}\eta_i P_3\eta_i\right\}.
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_3(k)\} = \mathbb{E}\{V_3(k+1) - V_3(k)\}\\
&= \mathbb{E}\left\{\sum_{j=k-\bar{\tau}_M+2}^{k-\bar{\tau}_m+1}\sum_{i=j}^{k}\eta_i P_3\eta_i - \sum_{j=k-\bar{\tau}_M+1}^{k-\bar{\tau}_m}\sum_{i=j}^{k-1}\eta_i P_3\eta_i\right\}\\
&= \mathbb{E}\left\{\sum_{j=k-\bar{\tau}_M+1}^{k-\bar{\tau}_m}\sum_{i=j+1}^{k}\eta_i P_3\eta_i - \sum_{j=k-\bar{\tau}_M+1}^{k-\bar{\tau}_m}\sum_{i=j}^{k-1}\eta_i P_3\eta_i\right\}\\
&= \mathbb{E}\left\{\sum_{j=k-\bar{\tau}_M+1}^{k-\bar{\tau}_m}(\eta_k P_3\eta_k - \eta_j P_3\eta_j)\right\}\\
&= \mathbb{E}\left\{(\bar{\tau}_M - \bar{\tau}_m)\eta_k P_3\eta_k - \sum_{i=k-\bar{\tau}_M+1}^{k-\bar{\tau}_m}\eta_i P_3\eta_i\right\}.
\end{aligned} \tag{15}
$$

From the elementary inequality $2a^Tb \le a^Ta + b^Tb$, it is straightforward to see that

$$
\begin{aligned}
&-2\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_k\\
&\le \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k) + \eta_k^T\mathcal{S}^T\tilde{\mathcal{G}}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_k, \qquad (16)\\
&-2\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\le \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)\\
&\quad + \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_{k-d})\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d}), \qquad (17)\\
&-2\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}\\
&\le \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k)\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)\\
&\quad + \eta_{k-d}^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}, \qquad (18)\\
&-2\eta_k^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\le \eta_k^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_k\\
&\quad + \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_{k-d})\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d}), \qquad (19)\\
&-2\eta_k^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}\\
&\le \eta_k^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_k\\
&\quad + \eta_{k-d}^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}, \qquad (20)\\
&-2\vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_{k-d})\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}\\
&\le \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_{k-d})\mathcal{G}_i^T P_1\mathcal{G}_i\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})\\
&\quad + \eta_{k-d}^T\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S}\eta_{k-d}. \qquad (21)
\end{aligned}
$$

Furthermore, in terms of (13)-(21), we can obtain

$$
\begin{aligned}
\mathbb{E}\{\Delta V(\eta_k)\} &= \mathbb{E}\{V(k+1) - V(k)\}\\
&= \sum_{i=1}^{3}\mathbb{E}\{\Delta V_i(k)\} \le \mathbb{E}\{\xi_k^T\tilde{\Pi}_1\xi_k\},
\end{aligned} \tag{22}
$$

where

$$
\begin{aligned}
\xi_k &= \left[\begin{array}{cccc}\eta_k^T & \eta_{k-d}^T & \eta_{k-\tau_k}^T & \vec{f}_k^T\end{array}\right.\\
&\qquad \left.\begin{array}{ccc}\vec{g}_{k-\tau_k} & \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_k) & \vec{\varrho}^T(\tilde{C}\mathcal{S}\eta_{k-d})\end{array}\right]^T,\\
\tilde{\Xi}_{11} &= \mathcal{W}_1^T P_1\mathcal{W}_1 - P_1 + P_2 + (\bar{\tau}_M - \bar{\tau}_m + 1)P_3\\
&\quad + \sum\nolimits_{i=1}^N(\tilde{\beta}_i^* + \chi_i^{\tilde{\alpha}\tilde{\beta}} + \chi_i^{\tilde{\beta}\tilde{\upsilon}} + \chi_i^{\tilde{\beta}\tilde{\vartheta}})\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S},\\
\tilde{\Xi}_{22} &= \mathcal{W}_2^T P_1\mathcal{W}_2 - P_2\\
&\quad + \sum\nolimits_{i=1}^N(\tilde{\vartheta}_i^* + \chi_i^{\tilde{\alpha}\tilde{\vartheta}} + \chi_i^{\tilde{\beta}\tilde{\vartheta}} + \chi_i^{\tilde{\upsilon}\tilde{\vartheta}})\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^T P_1\mathcal{G}_i\tilde{C}\mathcal{S},\\
\tilde{\Xi}_{66} &= \tilde{\alpha}^{\Lambda T}\mathcal{H}^T P_1\mathcal{H}\tilde{\alpha}^{\Lambda}\\
&\quad + \sum\nolimits_{i=1}^N(\tilde{\alpha}_i^* + \chi_i^{\tilde{\alpha}\tilde{\beta}} + \chi_i^{\tilde{\alpha}\tilde{\upsilon}} + \chi_i^{\tilde{\alpha}\tilde{\vartheta}})\mathcal{G}_i^T P_1\mathcal{G}_i,\\
\tilde{\Xi}_{77} &= \tilde{\upsilon}^{\Lambda T}\mathcal{H}^T P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\\
&\quad + \sum\nolimits_{i=1}^N(\tilde{\upsilon}_i^* + \chi_i^{\tilde{\alpha}\tilde{\upsilon}} + \chi_i^{\tilde{\beta}\tilde{\upsilon}} + \chi_i^{\tilde{\upsilon}\tilde{\vartheta}})\mathcal{G}_i^T P_1\mathcal{G}_i,
\end{aligned}
$$

$$\tilde{\Pi}_1=\begin{bmatrix}\tilde{\Xi}_{11} & \mathcal{W}_1^TP_1\mathcal{W}_2 & 0 & \mathcal{W}_1^TP_1 & \mathcal{W}_1^TP_1 & \mathcal{W}_1^TP_1\mathcal{H}\tilde{\alpha}^{\Lambda} & \mathcal{W}_1^TP_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\\ * & \tilde{\Xi}_{22} & 0 & \mathcal{W}_2^TP_1 & \mathcal{W}_2^TP_1 & \mathcal{W}_2^TP_1\mathcal{H}\tilde{\alpha}^{\Lambda} & \mathcal{W}_2^TP_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\\ * & * & -P_3 & 0 & 0 & 0 & 0\\ * & * & * & P_1 & P_1 & P_1\mathcal{H}\tilde{\alpha}^{\Lambda} & P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\\ * & * & * & * & P_1 & P_1\mathcal{H}\tilde{\alpha}^{\Lambda} & P_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\\ * & * & * & * & * & \tilde{\Xi}_{66} & \tilde{\alpha}^{\Lambda T}\mathcal{H}^TP_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\\ * & * & * & * & * & * & \tilde{\Xi}_{77}\end{bmatrix}.$$

Subsequently, from (8) and (9), it follows that

$$\begin{aligned}&\mathbb{E}\{\Delta V(\eta_k)\}\\ \leq&\mathbb{E}\Big\{\xi_k^T\tilde{\Pi}_1\xi_k-\lambda_1[\vec{f}_k-(I\otimes\phi_1^f)\eta_k]^T[\vec{f}_k-(I\otimes\phi_2^f)\eta_k]\\ &-\lambda_2[\vec{g}_{k-\tau_k}-(I\otimes\phi_1^g)\eta_{k-\tau_k}]^T[\vec{g}_{k-\tau_k}-(I\otimes\phi_2^g)\eta_{k-\tau_k}]\\ &-\lambda_3[\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)-(I\otimes\Lambda)\tilde{C}\mathcal{S}\eta_k]^T[\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)-\tilde{C}\mathcal{S}\eta_k]\\ &-\lambda_4[\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})-(I\otimes\Lambda)\tilde{C}\mathcal{S}\eta_{k-d}]^T\\ &\times[\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})-\tilde{C}\mathcal{S}\eta_{k-d}]\Big\}\\ \leq&\mathbb{E}\{\xi_k^T\Pi_1\xi_k\}.\end{aligned}\tag{23}$$

Since $\Pi_1<0$, there must exist a sufficiently small scalar $\varepsilon_0>0$ such that

$$\Pi_1+\varepsilon_0\text{diag}\{I,\ 0\}<0.\tag{24}$$

Then, it is easy to see from (23) and (24) that the following inequality holds:

$$\mathbb{E}\{\Delta V(\eta_k)\}\leq-\varepsilon_0\mathbb{E}\{||\eta_k||^2\}.\tag{25}$$

On the other hand, according to the definition of $V(k)$, one derives

$$\begin{aligned}\mathbb{E}\{V(k)\}\leq&\rho_1\mathbb{E}\{||\eta_k||^2\}+\rho_2\sum_{i=k-\bar{\tau}_M}^{k-1}\mathbb{E}\{||\eta_i||^2\}\\ &+\rho_3\sum_{i=k-\bar{\tau}_M}^{k-1}\mathbb{E}\{||\eta_i||^2\},\end{aligned}\tag{26}$$

where $\rho_1=\lambda_{\max}(P_1)$, $\rho_2=\lambda_{\max}(P_2)$ and $\rho_3=(\bar{\tau}_M-\bar{\tau}_m+1)\lambda_{\max}(P_3)$.

For any scalar $\mu>1$, together with (12), the above inequality implies that

$$\begin{aligned}&\mu^{k+1}\mathbb{E}\{V(k+1)\}-\mu^k\mathbb{E}\{V(k)\}\\ =&\mu^{k+1}\mathbb{E}\{\Delta V(k)\}+\mu^k(\mu-1)\mathbb{E}\{V(k)\}\\ \leq&[(\mu-1)\rho_1-\mu\varepsilon_0]\mu^k\mathbb{E}\{||\eta_k||^2\}\\ &+(\mu-1)(\rho_2+\rho_3)\sum_{i=k-\bar{\tau}_M}^{k-1}\mu^k\mathbb{E}\{||\eta_i||^2\}\end{aligned}\tag{27}$$

Then, along the similar line of proof of Theorem 1 in [26], we can achieve

$$\begin{aligned}\mu^k\mathbb{E}\{V(k)\}\leq&\mathbb{E}\{V(0)\}+(\omega_1(\mu)+\omega_2(\mu))\sum_{i=0}^{k-1}\mu^i\mathbb{E}\{||\eta_i||^2\}\\ &+\omega_2(\mu)\sum_{-\bar{\tau}_M\leq i\leq 0}\mathbb{E}\{||\eta_i||^2\},\end{aligned}\tag{28}$$

where $\omega_1(\mu)=(\mu-1)\rho_1-\mu\varepsilon_0$, $\omega_2(\mu)=\bar{\tau}_M\mu^{\bar{\tau}_M}(\mu-1)(\rho_2+\rho_3)$.

Let $\rho_0=\lambda_{\min}(P_1)$ and $\rho=\max\{\rho_1,\ \rho_2,\ \rho_3\}$. It is obvious from (12) that

$$\mathbb{E}\{V(k)\}\geq\rho_0\mathbb{E}\{||\eta_k||^2\}.\tag{29}$$

Meanwhile, it follows easily from (26) that

$$\mathbb{E}\{V(0)\}\leq\rho(2\bar{\tau}_M+1)\max_{-\bar{\tau}_M\leq i\leq 0}\mathbb{E}\{||\eta_i||^2\}.\tag{30}$$

In addition, it can be verified that there exists a scalar $\mu_0>1$ such that

$$\omega_1(\mu_0)+\omega_2(\mu_0)=0.\tag{31}$$

Therefore,it is not difficult to see from (28), (30), (31) that

$$\begin{aligned}\mu_0^k\mathbb{E}\{V(k)\}\leq&\rho(2\bar{\tau}_M+1)\max_{-\bar{\tau}_M\leq i\leq 0}\mathbb{E}\{||\eta_i||^2\}\\ &+\omega_2(\mu_0)\sum_{-\bar{\tau}_M\leq i\leq 0}\mathbb{E}\{||\eta_i||^2\},\end{aligned}\tag{32}$$

And then, it is obvious from (29) and (32) that

$$\begin{aligned}&\mathbb{E}\{||\eta_k||^2\}\\ \leq&\left(\frac{1}{\mu_0}\right)^k\frac{\rho(2\bar{\tau}_M+1)+\bar{\tau}_M\omega_2(\mu_0)}{\rho_0}\max_{-\bar{\tau}_M\leq i\leq 0}\mathbb{E}\{||\eta_i||^2\}.\end{aligned}\tag{33}$$

According to Definition 1, the augmented system (7) with $v_k=0$ is exponentially mean-square stable, which completes the proof. ■

Now, let us consider the H_∞ performance of the overall estimation process. In the following theorem, a sufficient condition is obtained that guarantees both the exponential mean-square stability and the H_∞ performance for the augmented system (7).

Theorem 2: Let the disturbance attenuation level $\gamma>0$, the estimator parameters K_i $(i=1,2,\cdots,N)$, and the diagonal matrix Λ be given. Then the augmented system (7) is exponentially stable in the mean square sense for $v_k=0$ and, under the zero initial condition, satisfies the H_∞ performance constraint (10) for all nonzero v_k, if there exist positive definite matrices $Q_i>0$ $(i=1,2,\cdots,6)$ and positive scalars λ_j

$(i=1,2,\cdots,4)$ satisfying

$$\Pi_2=\begin{bmatrix} \Xi_{11}^{*} & \mathcal{W}_1^TP_1\mathcal{W}_2 & 0 & \Xi_{14} & \mathcal{W}_1^TP_1 & \Xi_{16} & \mathcal{W}_1^TP_1\mathcal{H}\tilde{\upsilon}^{\Lambda} & \mathcal{W}_1^TP_1\mathcal{L}\\ * & \Xi_{22} & 0 & \mathcal{W}_2^TP_1 & \mathcal{W}_2^TP_1 & \mathcal{W}_2^TP_1\mathcal{H}\tilde{\alpha}^{\Lambda} & \Xi_{27} & \mathcal{W}_2^TP_1\mathcal{L}\\ * & * & \Xi_{33} & 0 & \lambda_2\Phi_2^{gT} & 0 & 0 & 0\\ * & * & * & \Xi_{44} & P_1 & P_1\mathcal{H}\tilde{\alpha}^{\Lambda} & P_1\mathcal{H}\tilde{\upsilon}^{\Lambda} & P_1\mathcal{L}\\ * & * & * & * & \Xi_{55} & P_1\mathcal{H}\tilde{\alpha}^{\Lambda} & P_1\mathcal{H}\tilde{\upsilon}^{\Lambda} & P_1\mathcal{L}\\ * & * & * & * & * & \Xi_{66} & \Xi_{67} & \tilde{\alpha}^{\Lambda}\mathcal{H}^TP_1\mathcal{L}\\ * & * & * & * & * & * & \Xi_{77} & \tilde{\upsilon}^{\Lambda}\mathcal{H}^TP_1\mathcal{L}\\ * & * & * & * & * & * & * & \mathcal{L}^TP_1\mathcal{L}-\gamma^2I \end{bmatrix}<0, \tag{34}$$

where

$$\begin{aligned}\Xi_{11}^{*}=&\mathcal{W}_1^TP_1\mathcal{W}_1-P_1+P_2+(\bar{\tau}_M-\bar{\tau}_m+1)P_3\\ &-\lambda_1\Phi_1^f-\lambda_3\mathcal{S}^T\tilde{C}^T(I\otimes\Lambda)\tilde{C}\mathcal{S}+\frac{1}{N}\mathcal{M}^T\mathcal{M}\\ &+\sum_{i=1}^{N}(\tilde{\beta}_i^{*}+\chi_i^{\tilde{\alpha}\tilde{\beta}}+\chi_i^{\tilde{\beta}\tilde{\upsilon}}+\chi_i^{\tilde{\beta}\tilde{\vartheta}})\mathcal{S}^T\tilde{C}^T\mathcal{G}_i^TP_1\mathcal{G}_i\tilde{C}\mathcal{S}\end{aligned}$$

and other parameters are defined as in Theorem 1.

Proof: According to Theorem 1, it is easily shown that the zero-solution of the system (7) with $v_k=0$ is exponentially stable in the mean square since the inequality (11) is implied by (34). It remains to show that, under zero-initial condition, the output error $\tilde{z}_k$ satisfies the H_∞ performance constraint (10).

Choosing the Lyapunov function similar to one in the proof of Theorem 1, we can calculate as follows:

$$\begin{aligned}&\mathbb{E}\{\Delta V(k)\}\\ \le\ &\mathbb{E}\{\xi_k^T\Pi_1\xi_k+2v_k^T\mathcal{L}^TP_1\mathcal{W}_1\eta_k\\ &+2v_k^T\mathcal{L}^TP_1\mathcal{W}_2\eta_{k-d}+2v_k^T\mathcal{L}^TP_1\vec{f}_k\\ &+2v_k^T\mathcal{L}^TP_1\vec{g}_{k-\tau_k}+2v_k^T\mathcal{L}^TP_1\mathcal{H}\tilde{\alpha}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_k)\\ &+2v_k^T\mathcal{L}^TP_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\vec{\varrho}(\tilde{C}\mathcal{S}\eta_{k-d})+v_k^T\mathcal{L}^TP_1\mathcal{L}v_k\}.\end{aligned} \tag{35}$$

where ξ_k and Π_1 are defined previously.

Setting $\tilde{\xi}_k=[\ \xi_k^T\quad v_k^T\ ]^T$, inequality (35) can be rewritten as

$$\mathbb{E}\{\Delta V(k)\}\le\mathbb{E}\left\{\tilde{\xi}_k^T\begin{bmatrix}\Pi_1 & \tilde{\mathcal{L}}^T\\ * & \mathcal{L}^TP_1\mathcal{L}\end{bmatrix}\tilde{\xi}_k\right\}, \tag{36}$$

where $\tilde{\mathcal{L}}=[\ \mathcal{L}^TP_1\mathcal{W}_1\quad \mathcal{L}^TP_1\mathcal{W}_2\quad 0\quad \mathcal{L}^TP_1\quad \mathcal{L}^TP_1\quad \mathcal{L}^TP_1\mathcal{H}\tilde{\alpha}^{\Lambda}\quad \mathcal{L}^TP_1\mathcal{H}\tilde{\upsilon}^{\Lambda}\ ]$

In order to analyze the H_∞ performance of the system (7), we introduce

$$\mathcal{J}(s)=\mathbb{E}\sum_{k=0}^{s}\left\{\frac{1}{N}||\tilde{z}_k||^2-\gamma^2||v_k||^2\right\} \tag{37}$$

where s is nonnegative integer.

Under the zero initial condition, one has

$$\begin{aligned}\mathcal{J}(s)=&\mathbb{E}\sum_{k=0}^{s}\left\{\frac{1}{N}||\tilde{z}_k||^2\right.\\ &\left.-\gamma^2||v_k||^2+\Delta V(k)\right\}-\mathbb{E}\{V(s+1)\}\\ \le&\mathbb{E}\sum_{k=0}^{s}\left\{\frac{1}{N}||\tilde{z}_k||^2-\gamma^2||v_k||^2+\Delta V(k)\right\}\\ \le&\mathbb{E}\sum_{k=0}^{s}\left\{\tilde{\xi}_k^T\Pi_2\tilde{\xi}_k\right\}<0.\end{aligned} \tag{38}$$

Letting $s\to\infty$, it follows from the above inequality that

$$\frac{1}{N}\sum_{k=0}^{\infty}\mathbb{E}\{||\tilde{z}_k||^2\}\le\gamma^2\sum_{k=0}^{\infty}||v_k||^2,$$

and the proof is now complete. ■

Up to now, the analysis problem of estimator performance has been solved. Finally, we are in a position to consider the H_∞ estimator design problem for the complex network (1). The following result can be easily accessible from Theorem 2, and the proof is therefore omitted.

Theorem 3: Let the disturbance attenuation level $\gamma>0$ and the diagonal matrix Λ be given. For the discrete time-delayed complex networks (1) with the sensor model (3) containing both ROSSs and RVSDs, the augmented system (7) is exponentially stable in the mean square sense for $v_k=0$ and satisfies the H_∞ performance constraint (10) under the zero initial condition for all nonzero v_k, if there exist positive definite matrices $Q_i>0$ $(i=1,2,\cdots,6)$, matrices Y_i $(i=1,2,\cdots,N)$ and positive scalars λ_j $(i=1,2,\cdots,4)$ satisfying

$$\begin{bmatrix}\bar{\Pi} & R^T & F_1^T & F_2^T & F_3^T & F_4^T\\ * & -\mathcal{Q}_2 & 0 & 0 & 0 & 0\\ * & * & -\mathcal{Q}_2 & 0 & 0 & 0\\ * & * & * & -\mathcal{Q}_2 & 0 & 0\\ * & * & * & * & -\mathcal{Q}_2 & 0\\ * & * & * & * & * & -\mathcal{Q}_2\end{bmatrix}<0, \tag{39}$$

where

$$\bar{\Pi}=\begin{bmatrix}\Pi_{11}&\Pi_{12}&\Pi_{13}&0&0&\Pi_{16}&\Pi_{17}&\Pi_{18}&\Pi_{19}&\Pi_{1,10}\\ *&\Pi_{22}&0&0&0&\Pi_{26}&\Pi_{27}&0&0&\Pi_{2,10}\\ *&*&\Pi_{33}&0&0&\Pi_{36}&\Pi_{37}&0&\Pi_{39}&\Pi_{3,10}\\ *&*&*&\Pi_{44}&0&0&0&0&0&0\\ *&*&*&*&\Pi_{55}&0&\Pi_{57}&0&0&0\\ *&*&*&*&*&\Pi_{66}&\Pi_{67}&\Pi_{68}&\Pi_{69}&\Pi_{6,10}\\ *&*&*&*&*&*&\Pi_{77}&\Pi_{78}&\Pi_{79}&\Pi_{7,10}\\ *&*&*&*&*&*&*&\Pi_{88}&0&\Pi_{8,10}\\ *&*&*&*&*&*&*&*&\Pi_{99}&\Pi_{9,10}\\ *&*&*&*&*&*&*&*&*&\Pi_{10,10}\end{bmatrix},$$

$$\begin{aligned}
\vec{Y}&=\text{diag}\{Y_1,Y_2,\cdots,Y_N\},\quad \mathcal{Q}_2=I\otimes Q_2\\
Z_1&=[\ \ell_1^{(1)}E_1\vec{Y}^T\quad \ell_2^{(1)}E_2\vec{Y}^T\quad\cdots\quad \ell_N^{(1)}E_N\vec{Y}^T\ ]^T,\\
Z_2&=[\ \ell_1^{(2)}E_1\vec{Y}^T\quad \ell_2^{(2)}E_2\vec{Y}^T\quad\cdots\quad \ell_N^{(2)}E_N\vec{Y}^T\ ]^T,\\
Z_3&=[\ \ell_1^{(3)}E_1\vec{Y}^T\quad \ell_2^{(3)}E_2\vec{Y}^T\quad\cdots\quad \ell_N^{(3)}E_N\vec{Y}^T\ ]^T,\\
Z_4&=[\ \ell_1^{(4)}E_1\vec{Y}^T\quad \ell_2^{(4)}E_2\vec{Y}^T\quad\cdots\quad \ell_N^{(4)}E_N\vec{Y}^T\ ]^T,\\
F_1&=[\ Z_1(I\otimes C)\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\ ],\\
F_2&=[\ 0\quad 0\quad Z_2(I\otimes C)\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\ ],\\
F_3&=[\ 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad Z_3\quad 0\quad 0\ ],\\
F_4&=[\ 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad Z_4\quad 0\ ],
\end{aligned}$$

$$\begin{aligned}
R&=[\ \vec{Y}(I-\tilde{\beta}^\Lambda)(I\otimes C)\quad -\vec{Y}(I\otimes C)\quad -\vec{Y}\tilde{\vartheta}^\Lambda(I\otimes C)\\
&\qquad 0\quad 0\quad 0\quad 0\quad -\vec{Y}\tilde{\alpha}^\Lambda\quad -\vec{Y}\tilde{\upsilon}^\Lambda\quad 0\quad -\vec{Y}G\ ],\\
\ell_i^{(1)}&=\sqrt{\tilde{\beta}_i^*+\chi_i^{\tilde{\alpha}\tilde{\beta}}+\chi_i^{\tilde{\beta}\tilde{\upsilon}}+\chi_i^{\tilde{\beta}\tilde{\vartheta}}},\\
\ell_i^{(2)}&=\sqrt{\tilde{\vartheta}_i^*+\chi_i^{\tilde{\alpha}\tilde{\vartheta}}+\chi_i^{\tilde{\beta}\tilde{\vartheta}}+\chi_i^{\tilde{\upsilon}\tilde{\vartheta}}},\\
\ell_i^{(3)}&=\sqrt{\tilde{\alpha}_i^*+\chi_i^{\tilde{\alpha}\tilde{\beta}}+\chi_i^{\tilde{\alpha}\tilde{\upsilon}}+\chi_i^{\tilde{\alpha}\tilde{\vartheta}}},\\
\ell_i^{(4)}&=\sqrt{\tilde{\upsilon}_i^*+\chi_i^{\tilde{\alpha}\tilde{\upsilon}}+\chi_i^{\tilde{\beta}\tilde{\upsilon}}+\chi_i^{\tilde{\upsilon}\tilde{\vartheta}}},\\
\Pi_{11}&=(W^TW)\otimes(\Gamma(Q_1+Q_2)\Gamma)-\lambda_3 I\otimes(C^T\Lambda C),\\
&\quad+\text{Sym}\{(W\otimes\Gamma)^T\vec{Y}(I-\tilde{\beta}^\Lambda)(I\otimes C)\}\\
&\quad-I\otimes(Q_1-Q_3-(\bar{\tau}_M-\bar{\tau}_m+1)Q_5)-\lambda_1\Phi_1^f\\
\Pi_{12}&=-(W\otimes\Gamma)^T\vec{Y}(I\otimes C),\\
\Pi_{13}&=-(W\otimes\Gamma)^T\vec{Y}\tilde{\vartheta}^\Lambda(I\otimes C),\\
\Pi_{16}&=[\ W^T\otimes(\Gamma Q_1)+\lambda_1\Phi_2^{fT}\\
&\qquad W^T\otimes(\Gamma Q_2)+(I\otimes C)^T(I-\tilde{\beta}^\Lambda)\vec{Y}^T\ ],\\
\Pi_{17}&=[\ W^T\otimes(\Gamma Q_1)\\
&\qquad W^T\otimes(\Gamma Q_2)+(I\otimes C)^T(I-\tilde{\beta}^\Lambda)\vec{Y}^T\ ],\\
\Pi_{18}&=-(W^T\otimes\Gamma)\vec{Y}^T\tilde{\alpha}^\Lambda+\lambda_3 I\otimes C^T(\Lambda+I)/2,\\
\Pi_{19}&=-(W^T\otimes\Gamma)\vec{Y}^T\tilde{\upsilon}^\Lambda,\\
\Pi_{1,10}&=[\ W^T\otimes(\Gamma(Q_1+Q_2))L+(I\otimes C)^T(I-\tilde{\beta}^\Lambda)\vec{Y}^TL\\
&\qquad -(W\otimes\Gamma)^T\vec{Y}G\ ],\\
\Pi_{22}&=-I\otimes(Q_2-Q_4-(\bar{\tau}_M-\bar{\tau}_m+1)Q_6)\\
&\qquad-\lambda_1\Phi_1^f+\frac{1}{N}(I\otimes(M^TM)),\\
\Pi_{26}&=[\ 0\quad (I\otimes C)^T\vec{Y}^T+\lambda_1\Phi_2^{fT}\ ],\\
\Pi_{27}&=[\ 0\quad (I\otimes C)^T\vec{Y}^T\ ],\\
\Pi_{2,10}&=[\ -(I\otimes C)^T\vec{Y}^TL\quad 0\ ],\\
\Pi_{33}&=-I\otimes Q_3-\lambda_4 I\otimes(C^T\Lambda C),\\
\Pi_{36}&=\Pi_{37}=[\ 0\quad -(I\otimes C)^T\tilde{\vartheta}^\Lambda\vec{Y}^T\ ],\\
\Pi_{39}&=\lambda_4 I\otimes C^T(\Lambda+I)/2,\\
\Pi_{3,10}&=[\ -(I\otimes C)^T\tilde{\vartheta}^\Lambda\vec{Y}^TL\quad 0\ ],\quad \Pi_{44}=-I\otimes Q_4,\\
\Pi_{55}&=-\text{diag}\{I\otimes Q_5,I\otimes Q_6\}-\lambda_2\Phi_1^g,\quad \Pi_{57}=\lambda_2\Phi_2^{gT},\\
\Pi_{66}&=\text{diag}\{I\otimes Q_1,I\otimes Q_2\}-\lambda_1 I,\\
\Pi_{67}&=\text{diag}\{I\otimes Q_1,I\otimes Q_2\},\\
\Pi_{68}&=\Pi_{78}=[\ 0\quad -\tilde{\alpha}^\Lambda\vec{Y}^T\ ]^T,\\
\Pi_{69}&=\Pi_{79}=[\ 0\quad -\tilde{\upsilon}^\Lambda\vec{Y}^T\ ]^T,\\
\Pi_{77}&=\text{diag}\{I\otimes Q_1,I\otimes Q_2\}-\lambda_2 I,\\
\Pi_{88}&=-\lambda_3 I,\quad \Pi_{8,10}=[\ -\tilde{\alpha}^\Lambda\vec{Y}^TL\quad 0\ ],\\
\Pi_{99}&=-\lambda_4 I,\quad \Pi_{9,10}=[\ -\tilde{\upsilon}^\Lambda\vec{Y}^TL\quad 0\ ],
\end{aligned}$$

$$\Pi_{6,10}=\Pi_{7,10}=\begin{bmatrix}(I\otimes Q_2)L&0\\(I\otimes Q_2)L&-\vec{Y}G\end{bmatrix},$$

$$\Pi_{10,10}=\begin{bmatrix}L^T(I\otimes(Q_1+Q_2))L-\gamma^2 I&-L^T\vec{Y}G\\ *&-\gamma^2 I\end{bmatrix},$$

and other parameters are defined as in Theorem 1. Moreover, if the above inequality is feasible, the desired state estimator gains can be determined by

$$K_i=Q_2^{-1}Y_i. \tag{40}$$

Remark 5: At present, a large number of results on complex networks have been available in the literature that require the symmetry and the zero-row-sum properties for the configuration matrix W. However, these results cannot be applied to the state estimation problem with sensor saturations considered in this paper. Our main results in Theorems 1-3 are applicable to a wide class of complex networks including both directed and undirected networks, and the measurement model in this paper is quite comprehensive that encompasses network-induced phenomena including both the randomly occurring sensor saturations and the randomly varying sensor delays. Comparing with existing literature, our results are obtained on a new state estimation problem for a more general model using in-depth stochastic analysis tools.

Remark 6: A general form of sector-like nonlinear function, instead of the commonly used Lipschitz-type function, is employed to describe the nonlinearities existing in the network.

The main results established contain all the information of the complex networks including the physical parameters, lower and upper bounds of the network state delay, the sensor delays, the H_∞ disturbance rejection attenuation level as well as the occurrence probabilities of the sensor saturations and sensor delays. Notice that the system governing the error dynamics involves both the fixed and varying time-delays. To reduce the possible design conservatism, a novel Lyapunov-Krasovskii functional is proposed in which the first component $V_1(k)$ is used to account for the fixed time-delay and other components are constructed to cater for the varying time-delays. In the next section, a simulation example is provided to show the usefulness of the proposed design procedure for the desired state estimators.

Remark 7: Note that, for the standard LMI system, the algorithm has a polynomial-time complexity. That is, the number $\mathcal{N}(\varepsilon)$ of flops needed to compute an ε-accurate solution is bounded by $O(\mathcal{M}\mathcal{N}^3 \log(\mathcal{V}/\varepsilon))$, where $\mathcal{M}$ is the total row size of the LMI system, $\mathcal{N}$ is the total number of scalar decision variables, $\mathcal{V}$ is a data-dependent scaling factor, and ε is relative accuracy set for algorithm. Let us now look at the H_∞ state estimation problem for the complex network (1) with the measurements (3), where the network size is N and the variable dimensions can be seen from $x_i(k) \in \mathbb{R}^n$, $y_i(k) \in \mathbb{R}^m$, $z_i(k) \in \mathbb{R}^r$, $\hat{x}_i(k) \in \mathbb{R}^n$ $(i = 1, 2, \cdots, N)$, $v_1(k) \in \mathbb{R}^q$, and $v_2(k) \in \mathbb{R}^p$. From Theorem 3, we have $\mathcal{M} = 8nN + 2mN + rN$ and $\mathcal{N} = 3n^2 + 3n + m^2N + 4$. Therefore, the computational complexity of the LMIs-based H_∞ state estimation algorithm can be represented as $O(n^7N + m^7N^4)$. Similarly, the computational complexity of the proposed condition in Theorem 1 is also $O(n^7N)$. Obviously, the computational complexity of the LMI-based algorithms depends polynomially on the network size and the variable dimensions. In order to reduce the computation burden, a possible way is to obtain the estimator gains node by node and then the computational complexity can be represented as $O(n^7N)$. Fortunately, research on LMI optimization is a very active area in the applied mathematics, optimization and the operations research community, and substantial speed-ups can be expected in the future.

IV. Numerical Examples

Consider a discrete time-delayed complex network (1) with three nodes. The coupling configuration matrix is assumed to be $W = (w_{ij})_{M\times M}$ with

$$W = \begin{bmatrix} -0.6 & 0.6 & 0 \\ 0.6 & -1.1 & 0.5 \\ 0 & 0.5 & -0.5 \end{bmatrix},$$

and the inner-coupling matrix is given as $\Gamma = \mathrm{diag}\{0.1, 0.1\}$. The disturbance matrices and the output matrix are as follows

$$L_1 = \begin{bmatrix} 0.04 & 0.03 \end{bmatrix}^T, \; L_2 = \begin{bmatrix} -0.02 & 0.04 \end{bmatrix}^T, \\ L_3 = \begin{bmatrix} 0.02 & -0.03 \end{bmatrix}^T, \; M = [0.70 \quad 0.65].$$

The nonlinear vector-valued functions $f(x_i(k))$ and $g(x_i(k))$ are chosen as

$$f(x_i(k)) = \begin{bmatrix} -0.6x_{i1}(k) + 0.3x_{i2}(k) + \tanh(0.3x_{i1}(k)) \\ 0.6x_{i2}(k) - \tanh(0.2x_{i2}(k)) \end{bmatrix},$$
$$g(x_i(k)) = \begin{bmatrix} 0.02x_{i1}(k) + 0.06x_{i2}(k) \\ -0.03x_{i1}(k) + 0.02x_{i2}(k) + \tanh(0.01x_{i1}(k)) \end{bmatrix}.$$

Then, it is easy to see that the constraint (2) can be met with

$$\phi_1^f = \begin{bmatrix} -0.6 & 0.3 \\ 0 & 0.4 \end{bmatrix}, \quad \phi_2^f = \begin{bmatrix} -0.3 & 0.3 \\ 0 & 0.6 \end{bmatrix},$$
$$\phi_1^g = \begin{bmatrix} 0.02 & 0.06 \\ -0.03 & 0.02 \end{bmatrix}, \quad \phi_2^g = \begin{bmatrix} 0.02 & 0.06 \\ -0.02 & 0.02 \end{bmatrix}.$$

Consider the sensors with both the ROSSs and RVSDs described by (3) with the following parameters:

$$G_1 = \begin{bmatrix} -0.03 \\ 0.02 \end{bmatrix}, \; G_2 = \begin{bmatrix} -0.02 \\ 0.04 \end{bmatrix},$$
$$G_3 = \begin{bmatrix} 0.06 \\ -0.02 \end{bmatrix}, \; C = \begin{bmatrix} 0.8 & 0.6 \\ 0.9 & -0.4 \end{bmatrix}.$$

In this example, the probabilities are taken as $\alpha_1 = 0.88$, $\alpha_2 = 0.85$, $\alpha_3 = 0.87$ and $\beta_1 = 0.91$, $\beta_2 = 0.92$, $\beta_3 = 0.9$, the delay parameters are chosen as $\bar{\tau}_m = 1$, $\bar{\tau}_M = 3$, the disturbance attenuation level is $\gamma = 0.92$, and the diagonal matrix is $\Lambda = \mathrm{diag}\{0.7, \; 0.7\}$. By using the Matlab (with YALMIP 3.0 and SeDuMi 1.1), we solve LMI (39) and obtain a set of feasible solutions as follows:

$$\lambda_1 = 14.6065, \quad Q_1 = \begin{bmatrix} 6.6408 & 2.5172 \\ 2.5172 & 1.3932 \end{bmatrix},$$
$$\lambda_2 = 53.3398, \quad Q_2 = \begin{bmatrix} 5.4508 & 1.9621 \\ 1.9621 & 3.0666 \end{bmatrix},$$
$$\lambda_3 = 0.3624, \quad Q_3 = \begin{bmatrix} 0.2706 & -0.0581 \\ -0.0581 & 0.1037 \end{bmatrix},$$
$$\lambda_4 = 2.1997, \quad Q_4 = \begin{bmatrix} 0.1516 & -0.1421 \\ -0.1421 & 0.3701 \end{bmatrix},$$
$$Q_5 = \begin{bmatrix} 0.5182 & -0.1538 \\ -0.1538 & 0.0527 \end{bmatrix},$$
$$Q_6 = \begin{bmatrix} 0.4129 & -0.1257 \\ -0.1257 & 0.0975 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} -0.0023 & -0.3538 \\ 0.3023 & -0.0403 \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} 0.0467 & -0.3267 \\ 0.3426 & -0.0630 \end{bmatrix},$$
$$Y_3 = \begin{bmatrix} -0.0162 & -0.3243 \\ 0.2813 & -0.0882 \end{bmatrix}.$$

Then, according to (40), the desired estimator parameters can be designed as

$$K_1 = \begin{bmatrix} -0.0466 & -0.0782 \\ 0.1284 & 0.0369 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.0411 & -0.0682 \\ 0.1380 & 0.0231 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -0.0468 & -0.0638 \\ 0.1216 & 0.0121 \end{bmatrix}.$$

In the simulation, the exogenous disturbance inputs are selected as

$$v_1(k) = 5\exp(-0.2k)\sin(k), \quad v_2(k) = \frac{6\cos(0.8k)}{k+1}.$$

The discrete time-varying delay $\tau(k)$ satisfies $\tau(k) = 2 + \sin(\pi k/2)$, and the constant delay is selected as $d = 1$. The initial values $\psi_i(k)$ $(i = 1,2,3;\ k = -3,-2,-1,0)$ are generated that obey uniform distribution over $[-1.4,\ 1.4]$. Simulation results are shown in Figs. 1-4, where Fig. 1-3 plot the actual measurements and ideal measurements for sensors 1-3, respectively, and Fig. 4 depicts the output errors. The simulation results have confirmed that the designed H_∞ estimator performs very well.

V. Conclusions

In this paper, we have investigated the H_∞ state estimation problem for a class of complex networks with time-varying delay and incomplete information. The considered incomplete information includes both the ROSSs and RVSDs. In order to take both the ROSSs and RVSDs into account in a unified way, a novel sensor model has been proposed by using two sets of Bernoulli distributed white sequences with known conditional probabilities. Then, some estimators have been designed such that the augmented system is exponentially mean-square stable and the estimation error satisfies the specified H_∞ performance requirement. Finally, the developed estimation approach has been demonstrated by a numerical simulation example. Further research topics include the extension of our results to more general complex networks with varying time-delays in measurements, and also to the network control systems with both ROSSs and RVSDs.

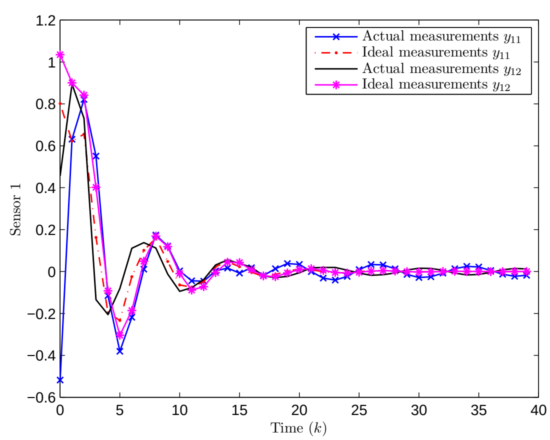

Fig. 1. Measurements from sensor 1

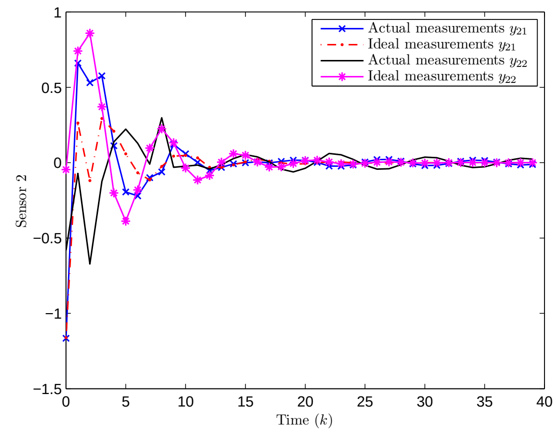

Fig. 2. Measurements from sensor 2

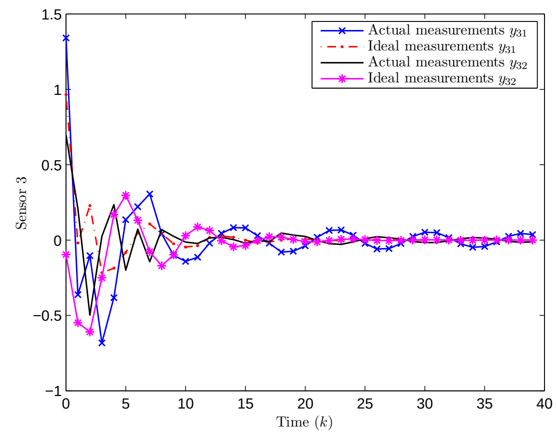

Fig. 3. Measurements from sensor 3

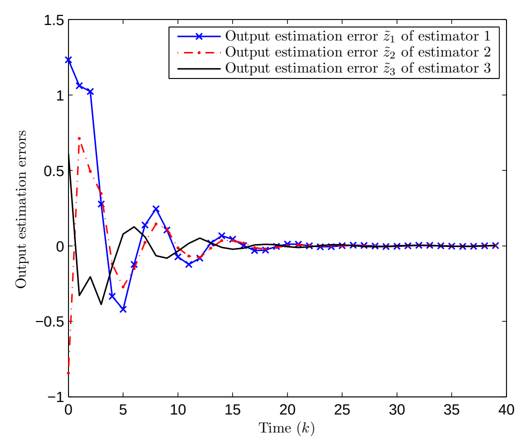

Fig. 4. Estimator errors $\tilde{z}_i(k)$ $(i = 1,2,3)$

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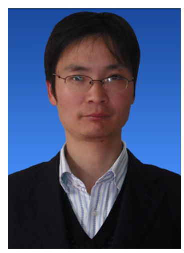

Derui Ding received the B.Sc. degree in industry engineering in 2004 and the M.Sc. degree in detection technology and automation equipments in 2007, both from Anhui Polytechnic University, Wuhui, China.

He is now a Lecturer with Anhui Polytechnic University and is currently pursuing the Ph.D. degree in control science and engineering with the Donghua University, Shanghai. His research interests include nonlinear stochastic control and filtering, as well as complex networks and sensor networks. He is an active reviewer for many international journals.

Zidong Wang (SM'03) was born in Jiangsu, China, in 1966. He received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently Professor of Dynamical Systems and Computing in the Department of Information Systems and Computing, Brunel University, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the UK. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published more than 100 papers in refereed international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves as an Associate Editor for 11 international journals, including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics - Part C. He is a Senior Member of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

Bo Shen received his B.Sc. degree in mathematics from Northwestern Polytechnical University, Xi'an, China, in 2003 and the Ph.D. degree in control theory and control engineering from Donghua University, Shanghai, China, in 2011.

He is currently a Lecturer with the School of Information Science and Technology, Donghua University, Shanghai, China. From 2009 to 2010, he was a Research Assistant in the Department of Electrical and Electronic Engineering, the University of Hong Kong, Hong Kong. From 2010 to 2011, he was a Visiting PhD Student in the Department of Information Systems and Computing, Brunel University, U.K. His research interests include nonlinear control and filtering, stochastic control and filtering, as well as complex networks and genetic regulatory networks. Dr. Shen is a very active reviewer for many international journals.

Huisheng Shu received his B.Sc. degree in mathematics in 1984 from Anhui Normal University, Wuhu, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in control theory and control engineering in 2005, both from Donghua University, Shanghai, China.

He is currently a Professor at Donghua University, Shanghai, China. He has published 16 papers in refereed international journals. His research interests include mathematical theory of stochastic systems, robust control and robust filtering.