The Effect on the Nusselt Number of the Non-linear Axial Temperature Distribution of Gas Flows through Commercial Microtubes

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Abstract The characteristics of nitrogen convective heat transfer through commercial stainless steel microtubes with inner diameter of 172 \( \mu \text{m} \) and 750 \( \mu \text{m} \) are investigated both experimentally and numerically. In this work is highlighted that the axial local gas bulk temperature distribution can present a strong non-linearity along the flow direction especially for microtubes having a small inner diameter and a thick solid wall. It is also demonstrated that the trend of the experimental Nusselt numbers as a function of the Reynolds number can be considered in good agreement with the conventional correlations if the average bulk temperature is calculated by taking into account the axial non-linearity of the gas bulk temperature. This fact explains the low values of the Nusselt numbers obtained in the previous experimental works appeared in literature where the convective heat transfer for gas flows through microtubes has been investigated assuming the gas bulk temperature distribution between the inlet and the outlet of the microtube as linear without verifying this hypothesis.

Keywords: Micro Flow, Convection, Conjugate heat transfer, Nusselt number

1. Introduction

During the last decades research in the field of Microfluidics has constantly increased due to the rapid growth of the technology applications of innovative microdevices, with typical dimensions ranging from 10 \( \mu \text{m} \) to 1 mm, in a variety of fields which span from bio-medicis to electronic components. Even if Microfluidics is now a consolidated discipline to some extent, topics such as the analysis of heat transfer mechanisms in microchannels have not been completely explored yet: whether the macroscale knowledge for single- and two-phase flow heat transfer is applicable to microscale is a significant open question still far from being completely answered. For this reason, in the last years a large amount of experimental analyses has addressed the study of the fluid-dynamical and heat transfer characteristics of single-phase flows in microchannels, as shown in the reviews by Morini (2004) and Hetsroni et al. (2005). A survey of these works reveals that there are comparatively few works focused on the experimental analysis of the single-phase convective heat transfer of gas flows through microchannels and these few results are far from univocal.

One of the first studies on microscale heat transfer was that of Wu and Little (1983), who experimentally investigated the trend of the Nusselt number of nitrogen flows through rounded rectangular microchannels as a function of the Reynolds number in laminar, transitional and turbulent regimes and compared their experimental results with the classical correlations proposed by Sieder and Tate (1936), Hausen (1959) and Dittus and Boelter (1930). Their results evidenced a strong disagreement with the classical correlations, so the authors proposed a specific correlation for the Nusselt number (\( Nu \)) for microchannels valid only for transitional and turbulent regimes (\( Re > 3000 \)):

\[
Nu = 0.00222Re^{3.09} Pr^{0.4} 
\] (1)

Choi et al. (1991) experimentally analyzed the forced convection of nitrogen flows in silica microtubes having inner diameters between 3 and 81 \( \mu \text{m} \). Their experimental results showed significant departures from the classical predictions, which gave rise to a new correlation for the Nusselt number in microtubes (\( 2500 < Re < 20000 \)):

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For the laminar regime Choi et al. (1991) obtained much lower values of the Nusselt number than those expected from classical predictions. The authors gave no justification for this and proposed a new correlation, valid for $Re < 2000$, on the basis of their own experimental results:

$$Nu = 3.82 \left(10^{-6}\right) Re^{1.96} Pr^{1/3}$$

(2)

Yu et al. (1995) investigated the convective heat transfer using nitrogen and water in the turbulent regime ($6000 < Re < 20000$) through microtubes having an inner diameter ranging between 19 and 102 $\mu$m. Also in this case, the Nusselt numbers in the turbulent regime were larger than expected, and the authors suggested the following specific correlation for the prediction of the Nusselt number in turbulent regime through microtubes:

$$Nu = 9.72 \left(10^{-4}\right) Re^{1.17} Pr^{1/3}$$

(3)

More recently, Hara et al. (2005) experimentally investigated the convective heat transfer in square minichannels having a hydraulic diameter between 0.3 and 2 mm for air flows under high pressure ratios. They reported that the Nusselt number in the turbulent regime can be larger or lower than the predicted values of the Dittus and Boelter correlation depending on the hydraulic diameter and on the length of the channel. In the laminar regime:

$$Nu = 0.007 Re^{1.2} Pr^{0.2}$$

(4)

More recently, Turner et al. (2007) have presented an experimental investigation of convective heat transfer for laminar nitrogen flow through a silicon rectangular microchannel ($D_h=95 \mu$m) with the aim to highlight the effects of the gas compressibility. Under the conditions of constant temperature gradient along the microchannel length, heat transfer experiments were conducted with laminar nitrogen gas flow, in which the outlet Mach number was between 0.10 and 0.42. They demonstrated that for compressible flow, the thermal boundary conditions of constant wall temperature gradient do not necessarily result in constant wall heat flux.

In spite of the large amount of correlations produced, if one compares the results of the various researchers, the mismatch among the data appears well evident both in the laminar and turbulent regimes, as shown in Fig. 1. It is apparent that the correlations for microtubes do not check with the Gnielinski correlation both in the laminar and transitional regimes. The strong dependence of the Nusselt number on the Reynolds number and the very low values of the Nusselt numbers encountered for low gas flow rates in microtubes seem to be two common results of the few experiments published so far. Unfortunately, the authors of these experimental works did not make any effort to give some physical explanation to these results, limiting themselves to arrange the data in new correlations.

From the above discussion, it is possible to conclude that for gas flows:

1. The agreement between the experimental Nusselt numbers in microtubes and the conventional correlations is not univocal both in the laminar and turbulent regime;
2. Few works have systematically investigated the influence of the main scaling effects on the convective heat transfer coefficients in microtubes.
3. Very low values of the Nusselt numbers have been obtained when the Reynolds number is lower than 1000, in complete disagreement with the conventional theory and, also, with the physical sense.

For these reasons, the objective of the present study is to investigate experimentally and numerically the convective heat transfer of nitrogen flows in commercial stainless steel microtubes having an inner diameter equal to 172 and 750 $\mu$m in both laminar and transitional regimes.
2. Experimental test rig

An experimental campaign devoted to characterize the behavior under forced convection of nitrogen flowing through rough commercial stainless steel microtubes. The main geometrical characteristics of the tested microtubes are summarized in Table 1. Two commercial stainless steel microtubes (Upchurch) having two different inner diameters and the same outlet diameter (1/16”) have been selected in order to check the effect of the reduction of the inner diameter on the Nusselt number when the wall thickness increases (conjugate effects).

Table 1
Geometrical characteristics of the tested microtubes.

<table>
<thead>
<tr>
<th>Tube</th>
<th>d (µm)</th>
<th>D (µm)</th>
<th>ε (µm)</th>
<th>L_e (mm)</th>
<th>L (mm)</th>
<th>ε/d (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>750</td>
<td>1588</td>
<td>4</td>
<td>500</td>
<td>475</td>
<td>0.8</td>
</tr>
<tr>
<td>#2</td>
<td>172</td>
<td>1588</td>
<td>3</td>
<td>100</td>
<td>75</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Fig. 2. Lay-out of the test rig.

The microtubes are mounted on the test rig shown in Figure 2. Nitrogen is stored in a high pressure flask (200 bar) and is brought by means of a valve to approximately 10 bars and room temperature before entering a 7 µm particle filter (Hamlet) to prevent possible impurities from clogging the microchannel. A three-way valve directs the flow to the proper flow sensors (Bronkhorst EL-Flow E7000). The gas then enters the test section, before which one arm of a differential pressure sensor is connected. After exiting the microtube, the gas is discharged to the atmosphere. The total pressure drop between the inlet and the outlet of the microtube is measured by means of a differential pressure transducer (Validyne DP15) with interchangeable sensing elements. To measure the temperature distribution along the external surface of the microtube a group of K-type calibrated thermocouples are glued onto it using an electrically non-conductive epoxy resin. In addition, at the inlet and outlet of the channel two additional thermocouples are inserted into the inlet and outlet plenums to measure the local fluid bulk temperature. The microtubes are heated resistively using a DC supply (HP6032A) which provides a constant electrical current. As the wall thickness of each microtube does not change along the axis and the material has homogeneous physical properties, the electrical resistance per unit length remains constant; this heating method is generally used in order to reproduce an uniform heat flux along the external walls of the tube (H boundary condition). An absolute pressure sensor (Validyne AP42) is used to monitor the atmospheric pressure at the exit of the test rig. The microtube is thermally insulated on the external surfaces with an Armaflex® (k_s=0.035 W/mK) layer and enclosed in an insulated box in order to decouple the test section from the room thermal conditions. An infrared thermo-camera is used (AVIO TVS 200EX) to check all the possible thermal bridges of the test section.

3. Nusselt number determination

For an H boundary condition (uniform axial imposed wall heat flux) the mean Nusselt number can be deducted from the experimental data by using the following equation:

\[
Nu = \frac{kd}{k_f} = \frac{d q_{w,i}}{k_f \left( \bar{T}_{w,i} - \bar{T}_b \right)}
\]

where \( \bar{T}_{w,i} \) is the average value of the inner wall temperature, \( \bar{T}_b \) is the average gas bulk temperature, \( k_f \) is the fluid thermal conductivity calculated at the fluid average bulk temperature, \( d \) is the inner diameter of the microtube and \( q_{w,i} \) is the heat flux at the inner wall of the microtube that can be calculated by means of an energy balance between the inlet and the outlet of the microtube:
\[ \dot{q}_{\text{in}} = \frac{\dot{m} \cdot c_{\text{pf}} \cdot (T_{b,\text{out}} - T_{b,\text{in}})}{\pi d L} \]  

(6)

where \( \dot{m} \) is the mass flow rate through the microtube and \( c_{\text{pf}} \) is the fluid specific heat calculated at the average bulk temperature and \( T_{b,\text{in}} \) and \( T_{b,\text{out}} \) are the value of the gas temperature measured by the K-type thermocouples inserted in the inlet and outlet plenums of the test section.

The inner wall temperature can be calculated by using the external average wall temperature \( (T_{w,\text{e}}) \) directly with a minor difference of the order of 0.15 K. More critical is the evaluation of the average value of the bulk temperature along the microtube as per Eq.(5). If the inner wall heat flux is uniform axially (ideal H boundary condition) a linear axial increase of the bulk gas temperature along the heated microtube is expected (Ozisik (1985)). For this reason, the following definition of the mean bulk temperature is adopted by most researchers for the analysis of micro convection when Joule heating is used for tubes:

\[ T_{\text{b}} = \frac{T_{b,\text{in}} + T_{b,\text{out}}}{2} \]  

(7)

However, there are some effects for which the use of Eq.(7) for the evaluation of the mean bulk temperature may induce large inaccuracies on the mean Nusselt number, which will be further discussed in the following.

To assess the accuracy of the experimental Nusselt numbers presented in this paper, the uncertainty associated with each measurement device used in the test rig is reported in Table 2. The uncertainty on the inner diameter evaluated through SEM imaging is of the order of ± 2%; the uncertainty on the microtube length is of the order of ± 0.3%. The roughness of the microtubes is deducted by superimposing a best-fitting circle over the corresponding tube contours in the SEM images. Apart from the uncertainty in the measurement of every single physical parameter, the sensitive coefficients which link these uncertainties to the final determination of the uncertainty on the Nusselt number can also play important roles. A comprehensive analysis of the Nusselt number dependence on the parameters to be measured was quantitatively carried out for a wide range of Reynolds numbers by Yang et al. (2011 b), who demonstrated that, under the operative conditions adopted in this work, without any underestimation of the outlet bulk temperature at small mass flow rates the uncertainty on the Nusselt number falls in the range 10%~18%. However, if the gas outlet temperature is underestimated by 1 K, the same uncertainty increases to the range 22%~50%.

### Table 2

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Range (0-FS)</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flowmeter (Bronkhorst EL-Flow E7000)</td>
<td>0-5000 (nml/min)</td>
<td>±0.6 % FS</td>
</tr>
<tr>
<td>Differential pressure transducer (Validyne DP15)</td>
<td>0-35 [kPa]</td>
<td>±0.5 % FS</td>
</tr>
<tr>
<td>Thermocouple (K type)</td>
<td>0-200 (°C)</td>
<td>±0.25 % FS</td>
</tr>
</tbody>
</table>

### 4. Results and discussion

#### 4.1 Axial distribution of bulk temperature

As discussed before, the gas local bulk temperature measurement still remains challenging for micro-fluidics. Some modern techniques for the analysis of liquid microflows, like the micro LIF-PIV technique, are not yet available for gas flows. The main consequence is that it becomes impossible to know experimentally the axial distribution of the bulk temperature of a gas flow through a heated microchannel; for this reason many researchers have used in the calculation of the Nusselt number the mean bulk temperature defined by Eq.(7) which means to assume implicitly that the axial gas bulk temperature distribution can be considered linear. Nevertheless, this hypothesis has not been verified, e.g. by using a numerical simulation for the specific case.

In order to verify the real distribution of the gas bulk temperature along the heated microtube, a CFD simulation of the forced convection through the microtube has been
conducted by using a finite volume commercial solver (FLUENT). The geometry of the microtube was represented and the grid resolution has been chosen so that the difference caused by grid resolution is at least one order of magnitude smaller than the experimental uncertainties. The calculation was carried out using the same conditions as in the experiment in terms of dimension of the microtube (inner and outer diameter, length), materials (stainless steel), position of fittings and position of the electrodes for the Joule heating of the test section.

The thermal boundary conditions are applied as follows. Along the heating length of the microtube the external wall temperature follows piecewise linearization based on the data measured from experiments, as shown in Figure 3 and 4 for microtube #1 and #2 respectively at specific Reynolds number. For the inlet and outlet fitting parts a constant heat loss coefficient is imposed, which is the average value measured in experiment for the test rig without gas flows. The front and back side surfaces of the microtubes are assumed adiabatic due to their small areas and negligible heat losses. The numerical model takes into account the effects of gas compressibility and viscous dissipation. As the bulk temperature gradient along the axis can be relatively large, the temperature dependence of the fluid properties is also considered.

The axial distribution of the gas bulk temperature within the two microtubes tested in this work has been obtained from the numerical simulation.

In Fig. 3 the axial distribution of the gas temperature is compared with the exit and inlet values of the bulk temperature measured by means of the thermocouples placed at the entrance and in the outlet plenum in the case of $Re =1400$ and for a heated power of 2.8 W. In the same Figure, the axial distribution of the external wall temperature is shown as a function of the non-dimensional axial coordinate ($x/L$).

It can be seen that the gas outlet temperature given by simulation is in agreement with the value obtained experimentally. However, the most important result shown in Fig. 3 is that the gas bulk temperature distribution is far from being linear throughout the full flow length. It is possible to note that in the first part of the tube there is a steep increase of the gas temperature, which tends to decrease the temperature difference between gas and wall as the flow develops. After this zone the temperature difference between the gas and the wall begins to stabilize around a small value. This fact demonstrates that the use of Eq.(7) for the calculation of the mean gas temperature must be considered with care in the case of gas flows because a simple arithmetic average of the inlet and outlet temperature under the conditions adopted in this work generates a large error on the Nusselt number.

A similar calculation has also been carried out for the smaller microtube #2 and the results are compared with experimental data obtained for $Re =600$ and an imposed heating power equal to 0.6 W in Figure 4; in this case, the non-linearity of the temperature axial distribution is more evident than for microtube #1.
In addition, in this case there is a significant temperature drop after the end of the heated section due to the presence of the outlet fitting which occupies the space between the position of the last thermocouple, where the temperature difference between the gas and the wall is negligible, to the end the microtube where the outlet bulk temperature is measured. As a result, along this small length the direction of heat flux is reversed (from the gas to the walls). If the value of the gas bulk temperature measured within the outlet plenum is used to determine the mean bulk temperature, a large underestimation may be introduced, even if the outlet gas temperature measurement itself is accurate.

The non-linearity of the axial bulk distribution highlights that in this case scaling effects like the conjugate heat transfer between the wall and the gas and the viscous dissipation can play an important role in the thermal behaviour of the system. Following the procedure described by Yang et al. (2011) it is possible to verify whether the main scaling effects can be considered negligible or not in the operative conditions adopted during these tests. By recalling the definition of the Mach number (compressibility effects), conduction parameter (conjugate effects) and Brinkman number (viscous effects):

\[
Ma_{avg} = \frac{4\pi d^2}{\pi d^2} \left(\frac{c_p}{c_p + c_v}\right)^{\frac{1}{y}} \frac{RT}{P_{avg}}
\]

\[
\lambda = \frac{k_c L}{k_{eq}} = 1 \left(\frac{D^2 - d^2}{2L} \frac{1}{Re Pr}\right)
\]

\[
Br = \frac{64\mu}{q_w} \left(\frac{nRT_{in}}{(P_{in} + P_{out})}\right)^2
\]

the range of variation of these parameters has been calculated and reported in Table 3 for the tested microtubes. The corresponding threshold values for each parameter are also shown in Table 3.

<table>
<thead>
<tr>
<th>Tube</th>
<th>d/L \times 10^3</th>
<th>Re</th>
<th>Ma_{avg} (&lt;0.3)</th>
<th>\lambda (&lt;0.01)</th>
<th>Pe (&lt;50)</th>
<th>Br \times 10^3 (&lt;12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>1.5</td>
<td>500-4000</td>
<td>0.03-0.18</td>
<td>0.001-0.008</td>
<td>350-2900</td>
<td>7-44</td>
</tr>
<tr>
<td>#2</td>
<td>1.72</td>
<td>400-3500</td>
<td>0.1-0.4</td>
<td>0.04-0.31</td>
<td>300-2400</td>
<td>26-38</td>
</tr>
</tbody>
</table>

In the case of microtube #2 where the non-linearity of the axial bulk temperature distribution is more evident the effects related to the conjugate heat transfer cannot be ignored (\(\lambda>0.01\)) as the effect of the viscous dissipation (Br>12\times10^{-3}) for the whole range of the tested Reynolds numbers. On the contrary, the effects related to the compressibility effects tend to become important only for large Reynolds numbers. This analysis explains for which effects the axial distribution of the bulk temperature can be strongly non-linear for microtube #2.

4.2 Determination of Nusselt number

The non-linear distribution of the bulk temperature may have a strong influence on the calculation of Nusselt number. In fact, the key factor in determining the value of the Nusselt number is the mean temperature difference between the bulk gas and the wall. Improper estimation of its value may totally change the determination of the Nusselt number. For example, in the case of microtube #1 considered previously at the Re number of 1400, the mean temperature difference between wall and gas to calculate the Nusselt number is greatly overestimated from 1.5 K (large Re) to 13.8 K (low Re). As this mean temperature difference is inversely proportional to the Nusselt number, this large overestimation reduces the value of Nusselt number by one order of magnitude. This can possibly explain the “extremely” small values of Nusselt number for gas micro convection in some early experimental results.

When the mean temperature difference is very small (of the order of 1 K), the values of the Nusselt number become highly sensitive to the errors introduced not only by the uncertainties of temperature sensors but also by the position of measurement as well as to the way of averaging the local temperature in order to obtain the mean value.

Another point to be raised is that the non-linear bulk temperature distribution may change the behavior of the Nusselt number compared with the assumption of linear axial bulk temperature. It is possible to use either analytical or numerical methods to reconstruct
this non-linear temperature distribution based on experimental data. As an example, in this work a curve fitting method was applied to follow the trend of the gas bulk temperature development in the non-linear first part of the heating length, by using the following expression:

\[ T_b(x) = a\left(\frac{x}{L}\right)^b + c \]  

(9)

where \( x \) is the axial position and \( a, b, c \) are mathematical parameters which can be determined from the experimental data. This method was applied to the experimental temperature data obtained for both microtube \#1 and microtube \#2 to consider the nonlinear bulk temperature distribution in the first stretch. When the fitting parameters are known for convection at a certain Reynolds number, the average bulk temperature is calculated through:

\[ T_{av} = \frac{1}{L} \int T_b(x) \, dx \]  

(10)

instead of Eq. (7), and the mean values for Nusselt number are obtained from Eqs. (5) and (6).

Figure 5 and 6 show the trend of the Nusselt number determined by considering the non-linear bulk temperature distribution in the first part of microtube (triangular symbols). The results are compared with the values obtained by calculating the mean bulk temperature of the gas flow using Eq.(7) (circular symbols).

It is well evident that for low Reynolds numbers the Nusselt number obtained by considering the axial bulk temperature distribution as linear is very small (lower than unity for \( Re < 1000 \)), whereas when the non-linear distribution of the bulk temperature distribution is accounted for the Nusselt number is always larger than 3 in the laminar regime, which is in agreement with the theory.

The results shown in Fig. 5 and 6 highlight that the large overestimation of the mean temperature difference between gas and wall when a linear distribution of gas temperature is assumed brings to an erroneous evaluation of the Nusselt numbers. This is true especially for microtubes with small inner diameters at low Reynolds numbers, where conjugate effects becomes stronger. In fact in these cases, a steeper non-linear increase of the gas bulk temperature in the first part of the microtube occurs. In Fig. 5 and 6 the correlations proposed by Gnielinski (1995) for macro-tubes and the correlations of Wu and Little (1983) (Eq.(1)), Choi et al. (1991) (Eq.(2-3)) and Yu et al. (1995) (Eq.(4)) for microtubes are shown together with the experimental results obtained in this work. It is interesting to note that the trend of Nusselt number based on the non-linear axial distribution of the gas bulk temperature is not in agreement with any of the correlations proposed for microtubes in either laminar or transitional regime. In fact, the trend of the experimental data follows Gnielinski’s correlation in both laminar and transitional regimes, which indicates that the conventional theory for forced convective heat transfer may also be considered to hold even for gas flows through micro-tubes with inner diameters down to 100 \( \mu \)m.
5. Conclusions

In this work the characteristics of gas convective heat transfer in commercial stainless steel microtubes have been investigated both experimentally and numerically. The main conclusions of this work can be summarized as follows:

- The axial bulk temperature distribution along the heating length of a microtube heated by Joule effect can be strongly non-linear for gas flows; by means of numerical simulations it has been demonstrated that a steep increase of temperature in the first part of the heating length takes place due to the thermal entrance region but also to the conjugate heat transfer between the gas and the solid walls which tends to increase the wall heat flux near the tube entrance.

- The assumption of a linear axial distribution of the bulk temperature must always be verified in experiments. In order to do this, an a-priori evaluation of the impact of the main scaling effects (especially compressibility, conjugate heat transfer and viscous dissipation) on the convective heat transfer has to be conducted. It has been demonstrated that the assumption of a linear distribution of the bulk temperature without verification can determine very low Nusselt numbers strongly dependent on the Reynolds number in laminar regime.

- It has been demonstrated that if the real axial distribution of the bulk temperature is used in the evaluation of the Nusselt number, the trend of the experimental Nusselt numbers is in good agreement with the conventional correlations both in the laminar and in transitional regimes.

6. Acknowledgements

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7. References


