

## On the influence of tube row number for mixed convection around micro tubes

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**Abstract:** A numerical simulation was performed on the heat transfer of mixed convection for fluid flowing across a micro-tube bundle by using Lattice Boltzmann Method. Firstly, the program code was validated by using a bench mark case of natural convection around a hot single tube inside a square enclosure. The local and averaged heat transfer coefficient of each tube in the bundle with various row numbers was calculated. Numerous cases have been simulated from a weak natural convection case (forced convection) to a pure natural convection case. The results indicate that the total averaged Nusselt number outside the tubes gradually decreases and becomes almost a constant with tube row number at low Reynolds number, which is different from the case of conventional scaled tube. The averaged Nusselt numbers and temperature fields for various situations were compared. The other influencing factors except of the tube row number on the heat transfer behavior of a tube bundle were also summarized and discussed.

**Keywords:** Lattice Boltzmann method; mixed convection; nature convection; micro-tube bundle; number of tube row

### 1. Introduction

The study on fluid flowing across a single cylinder, or a bundle of multiple cylinders, is a classic topic. Related research papers are common even in recent published literature on the use of various numerical methods, the simulated range of Reynolds number, the scale of tube and domain boundary, etc. (Shu et al., 2007, Yan and Zu, 2008, Yang et al., 2009, Hu, 2010, and Carmo and Meneghini, 2006). However, available papers for fluid flow in a non-isothermal field are still limited, especially for the case of a micro-tube bundle with mixed convection (Whitaker, 1972).

Fluid flowing across a tube bundle is common in many types of heat transmission equipment, including tubular heat exchangers, and power station boilers. The convectional sized shell and tube heat exchanger are, in general, operated in a turbulent flow region having a high heat transfer coefficient. The natural convection induced by the temperature difference of tube and shell-side fluid can be ignored. However, this may not hold for smaller tube diameters and dense parallelized

arrangement of tubes. In this case, the Reynolds number is small and natural convection due to the temperature difference in the shell sized fluid domain can play a role.

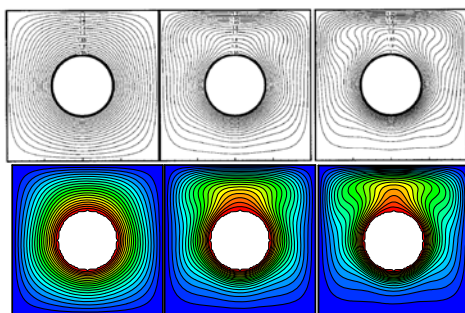
In fact, a temperature difference between different fluid parts in a heat exchanger is inevitable. Any heat transfer of forced convection is combined with some natural convection. The relative intensity of natural convection can be evaluated by using a ratio of Grashof number to the square of the Reynolds number, as used by Whitaker (1972). Accordingly, if the ratio of Grashof number to the square of the Reynolds number is less than or equal to 0.1, natural convection is negligible and it can be treated as pure forced convection. If the ratio is greater than or equal to 10, it is considered a pure natural convection. Between 0.1 and 10, the influence of forced convection and natural convection should both be taken into consideration (Sun et al., 2011, Herbert, 1972, and Zukauskas, 1972).

In this paper, a numerical simulation was performed on the heat transfer of mixed convection for fluid flowing across a micro-tube bundle using the Lattice Boltzmann Method. First, the program code was validated

by a benchmark case of natural convection in a square enclosure with a heated cylinder tube. Then several cases were simulated, varying the defined parameter from a weak natural convection case (forced convection) to a pure natural convection case. The other influencing factors on the heat transfer behavior of a tube bundle, in addition to the tube row number, were also summarized and discussed.

## 2. Verification of the program

In general, a simulation of natural convection heat transfer is more complicated than that of forced convection, because natural convection is a temperature coupled nonlinear dissipative system. In this paper, we only verify the correctness of natural convection, by using hot tube (dimensionless temperature given by unity one) in a square cavity enclosed with cold walls (dimensionless temperature given by zero). The computational domain is given by  $2.5 \times 2.5$ , and the uniform grid size by  $\Delta x = \Delta y = 0.025$ . The cases of  $Ra = 10^4$ ,  $Ra = 5 \times 10^4$  and  $Ra = 1 \times 10^5$  were simulated, specifically. The tube diameter is given by 1, the length or width of the square cavity is 2.5. For the velocity field, the initial velocities for all nodes in the computational domain is set to zero, and no-slip boundary conditions are adopted for the cavity walls. For the temperature field, the initial temperatures for all nodes in the computational domain except of the tube wall nodes is set to zero, and constant temperature boundaries are remained in the whole simulation.



**Fig.1.** The temperature fields around a heated cylinder in a square enclosure at various Ra numbers (from left  $10^4$ ,  $5 \times 10^4$ ,  $10^5$ ) (above Peng's (2004), bottom the present simulation)

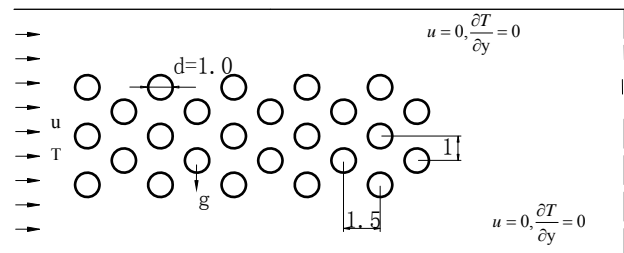
By comparing the local Nusselt number

around the tube surface, it is found that the obtained simulation result is consistent with the previous literature (Peng and Shu, 2004). Figure 1 shows the temperature fields of the previous and the present work, for comparison.

## 3. Model description and boundary conditions

Figure 2 shows the model of mixed convection flow across a tube bundle in two dimensions. The computational domain is  $40 \times 10$ , with a mesh size of  $800 \times 200$ . Each mesh has a length and a height of 0.05. Tube diameter is 1. The tube arrangement is shown in Figure 2, from top to bottom, and labeled left to right labeled by tube1 to tube 50.

The initial velocities for all nodes except the left boundary in the computational domain are zero. The initial velocity for the left inlet boundary nodes is assumed to be uniform. No-slip boundary conditions are used for the top and bottom boundaries, and the tube walls.



**Fig.2.** The simulation model of mixed convection flow over the tube bundle

For the temperature boundary conditions, the initial temperatures for all nodes except the tube wall in the computational domain are given by zero, the initial temperature for tube walls is given by unity one. The extrapolation boundary condition is used for right outlet boundary. Adiabatic boundary conditions are used for the top and bottom boundaries.

The Nusselt number,  $Nu$ , is one of the most important dimensionless parameters in describing the convective heat transport. The averaged,  $Nu$ , of the whole tube bundle is calculated to evaluate the heat transfer performance for each configuration including the effect of row number.

The Grashof, Reynolds and Rayleigh numbers were given, respectively, as below.

$$Gr = \frac{g\beta\Delta T l^3}{\nu^2}, Re = \frac{ul}{\nu}, \text{ and, } Ra = \frac{g\beta\Delta T l^3}{\nu\alpha}.$$

Where,  $\alpha$  is the thermal diffusion coefficient,  $\beta$  the thermal expansion,  $\nu$  the kinematic viscosity,  $l$  the length scale,  $\Delta T$  the temperature difference between tube and inlet fluid, and  $g$  the acceleration due to gravity.

## 4. Results and discussion

### 4.1 The influence of tube row number

The variations of the averaged Nusselt number of the tube bundle with the tube row number at different defined parameters are given in Figure 3. The figure indicates that the averaged Nusselt number of the whole tube bundle decreases with increasing tube row number, but the effect of tube row number weakens in different situations.

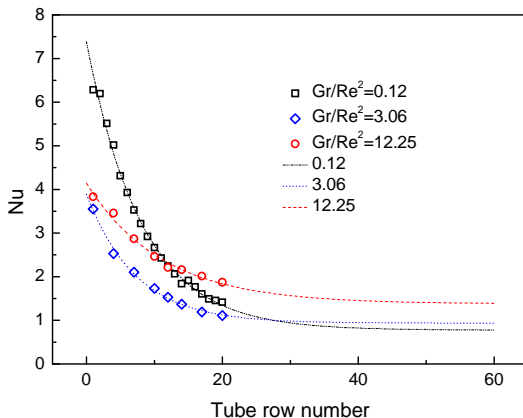


Fig. 3. The variation of the Nusselt number with the tube row number

At constant tube row number, the averaged,  $Nu$ , increases with the defined parameter ( $Gr/Re^2$ ). The variation of the averaged Nusselt number of the whole tube bundle can be correlated with the tube row number and the defined parameter ( $Gr/Re^2$ ), as given by Eq. 1.

$$Nu = 6.12n^{-0.45}(Gr/Re^2)^{-0.05} \quad (1)$$

The above equation can be used for values of the defined parameter ( $Gr/Re^2$ ) between 0.12 to 12.25, and tube row number.

The averaged negative and positive errors

are +20% and 20% respectively. The mean absolute error (MAE) defined in Eq.2 is approximately 19.45%, where  $M$  is the number of simulation cases.

$$MAE = \frac{1}{M} \sum \frac{|Nu_i - Nu_c|}{Nu_i} \times 100\% \quad (2)$$

### 4.2 The influence of Reynolds number

The temperature fields and the overall averaged Nusselt numbers of the tube bundle for different Reynolds numbers at the same Rayleigh number of  $10^4$  are given in Figure 4. It shows that the overall averaged Nusselt number increases as the Reynolds number increases at the same Rayleigh number, and the heat transfer performance of the tube at top row is worse than that of the tube at bottom row. The tubes in the top row are in a weaker region than bottom ones. With increasing Reynolds number, the wake region swings from upper-right to the right. As the Reynolds number increases, the relative role of natural convection gradually weakens and the role of forced convection enhanced. The wake boundary layer of the whole bundle begins to thin and the higher temperature region also shifts rightwards with the increasing of the Reynolds number.

### 4.3 The influence of Rayleigh number

Numerical simulations for the mixed convection around microtubes with a wide range of Rayleigh numbers from  $10^2$  to  $10^4$  at the same Reynolds number 143 were carried out.

The temperature field and overall average bundle Nusselt number for different Rayleigh numbers at the same Reynolds number of 143 were given in Figure 5. The overall averaged Nusselt number of the bundle increase as the Rayleigh number increases. The heat transfer performance of the right-hand rows is inferior to that of the leftmost ones, according to the temperature fields. As the Rayleigh number increases, the relative role of natural convection playing gradually enhances and the bundle's wake region swing to upward at  $Ra = 10^4$  obviously. The higher temperature region (marked in red) does not seem to move backwards with increasing the Reynolds

number, i.e., the overall heat transfer performance does not change much with Reynolds number.

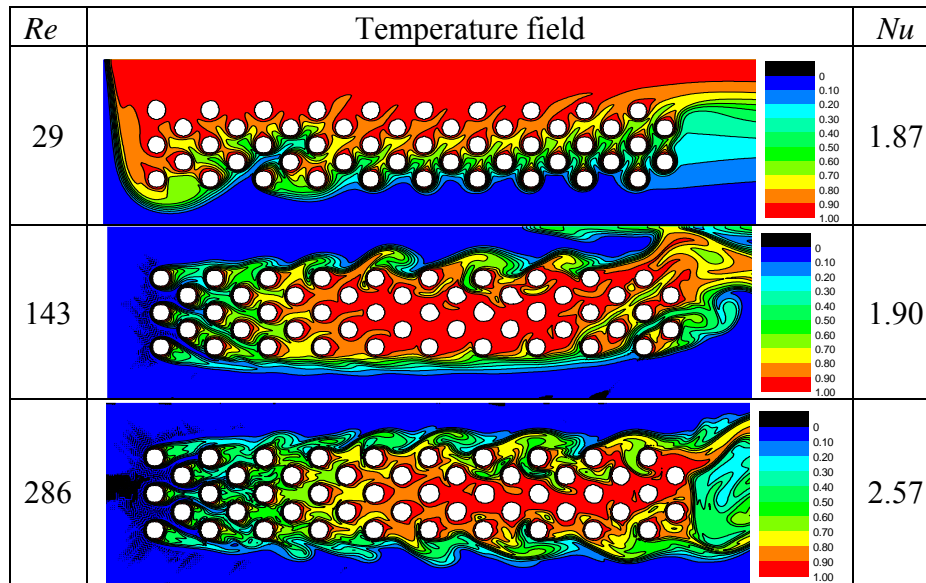


Fig. 4. The temperature field and the averaged  $Nu$  number for different  $Re$  number

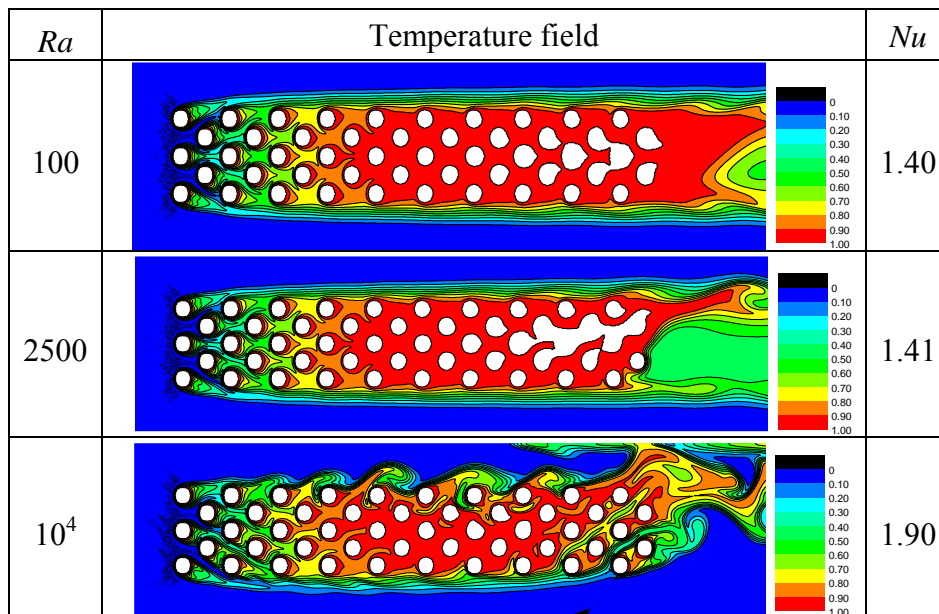


Fig. 5. The temperature field and the averaged  $Nu$  number for different  $Ra$  number

#### 4.4 The influence of $Gr/Re^2$

As defined in the previous works (Yang, et al., 2010 and Andreas, 1996, Krishne Gowda, et al., 1996.), the parameter,  $Gr/Re^2$ , a ratio of Grashof number to the square of the Reynolds number, can be used to evaluate the relative intensity of natural convection. It may

also be used as an empirical correlating independent. The temperature field and the averaged Nusselt number for different combination of Reynolds number and Rayleigh number, at the same  $Gr/Re^2$ , are shown in Figure 6.

The results reveal that even at the same

$Gr/Re^2$ , the temperature fields and averaged Nusselt numbers are much different for different combination of Reynolds number and Rayleigh number. The averaged Nusselt

numbers are affected by not only the Reynolds number, but also the Rayleigh number obviously.

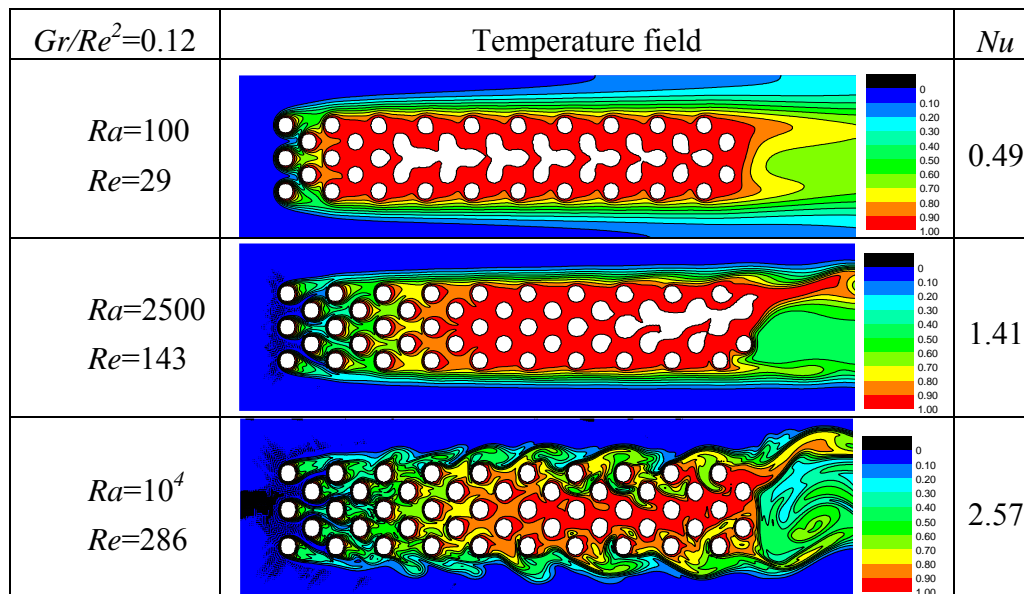


Fig. 6. The temperature field and the averaged  $Nu$  number for different  $Gr/Re^2$

## 5. Conclusions

After simulating numerous cases by varying the defined parameter from a weak natural convection case (forced convection) to a pure natural convection case, the following conclusions can be summarized:

(1) It is evident that with increasing row tube number, the averaged Nusselt number of the whole tube bundle decreases, and tends to converge to a constant at a large the row number. The convergent constant increases with the defined parameters, for instance, the averaged Nusselt number of the whole tube bundle is smaller for the case of  $Gr/Re^2 = 0.12$  than that of case of  $Gr/Re^2 = 12.25$ .

(2) The heat transfer performance for the tubes at back rows is worse than that of tubes at front ones as shown in the temperature field.

As the Reynolds number increases, the wake region that has a high temperature shrinks and moves backwards.

(3) Even at the same  $Gr/Re^2$ , the temperature fields and the averaged Nusselt

numbers are much different for various combinations of Reynolds number and Rayleigh number. The averaged Nusselt numbers are affected by both Reynolds number and Rayleigh number obviously, not only by the parameter of  $Gr/Re^2$ .

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