Total Temperature Measurements of Laminar Gas Flow at Micro-tube Outlet: Cooled from the Wall

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Abstract   This paper presents experimental results on heat transfer characteristics of laminar gas flow in a micro-tube with constant wall temperature whose wall temperature is lower than the inlet temperature (cooled case).   The experiment was performed for nitrogen gas flow through micro-tubes with 163 and 243 μm in diameter and 50 mm in length.   The gas was heated in an upstream section of the micro-tube to T_{in}=315K, 335K and 355K.   The wall temperature was maintained at 305K by circulating water around the micro-tube.   The stagnation pressure was chosen in such a way that the exit Mach number ranges from 0.1 to 0.7.   The outlet pressure was fixed at the atmospheric condition.   The total temperature at the outlet, the inlet stagnation temperature, the mass flow rate and the inlet temperature were measured.   The numerical computations based on the Arbitrary – Lagrangian – Eulerian (ALE) method were also performed for the same conditions of the experiment for validation of numerical results.   The both results are in excellent agreement.   The total and bulk temperatures obtained by the present study are also compared with the temperature of the incompressible flow.

Keywords: Total temperature, Micro gas flow, Experimental heat transfer

1. Introduction

During the last two decades, flow and heat transfer through micro-geometrics have attracted an important research interest in micro-heat exchanger, micro-reactor and many other micro-thermal fluid devices.   In the case of the gas flows in the micro-systems, it is found that the flow accelerates due to the gas expansion and thermal energy converts to kinetic energy.   This results in a static temperature decrease of the gas [1].

Since the experimental work by Wu and Little [2], who measured the friction coefficient and Nusselt number for nitrogen, argon, and helium flows in silicon or glass micro-channels, many experimental and numerical investigations have been undertaken.   However, there seems to be few experimental investigations on the heat transfer of compressible gas flow through micro-geometries.

Asako [3], Asako and Toriyama [4] and Hong and Asako [5] performed numerical investigations to obtain the heat transfer characteristics of laminar gaseous flows in a micro-channel and in a micro-tube with constant wall temperature, whose wall temperature is lower or higher than the inlet temperature.   In the case of slow flow (Ma<0.3), identical heat transfer coefficients are obtained for both heated and cooled cases of incompressible flow.   However, in the case of fast flow (Ma>0.3), different heat transfer coefficients are obtained for each cooled and heated case.   The correlation for the prediction of the heat transfer rate of gaseous flow in a micro-channel and in a micro-tube has been proposed.   In order to validate numerical results by their studies [3-5], Hong et al. [6] measured total temperatures at a micro-tube outlet with constant wall temperature whose temperature is higher than the inlet temperature (heated case) in the laminar flow regime.

Turner et al. [7] performed experimental investigation to measure the inlet and outlet temperatures and the micro-channel wall temperature in thermal boundary conditions of

constant temperature gradient along the micro-channel length. Their measurement results were used to validate a two dimension numerical model for gaseous flow in a micro-channel.

As can be seen from the literature survey, there seems to be no experimental study on laminar heat transfer characteristics of micro-tubes with constant wall temperature whose temperature is lower than the inlet temperature (cooled case). This is the motivation of the present experimental study to measure the total temperature at the micro-tube outlet, the inlet stagnation temperature, the mass flow rate, and the inlet pressure to determine the quantitatively transferred amount of heat in a micro-tube with constant wall temperature.

2. Experimental setup

A schematic representation of the experimental setup used in the present study is shown in Fig. 1. The experiment was conducted with pressurized Nitrogen. Nitrogen from the tank passes through the regulator, the gas dryer and the mass flow meter (Kofloc 3300) which measures the mass flow rate. Then the gas flows into the heat transfer section and flows out to atmosphere. The gas flow is controlled by the regulator.

The detailed view of the heat transfer section is shown in Fig. 2. The heat transfer section consists of four components; the inlet section, the micro-tube, the water jacket, and outlet tube. The inlet tube is made of brass with nichrome wire curled around and is attached between the water jacket and the upstream chamber. The inner diameter of inlet tube is 8 mm. The gas was heated by heating the nichrome wire and the brass. The pressure transducer is attached to the upstream
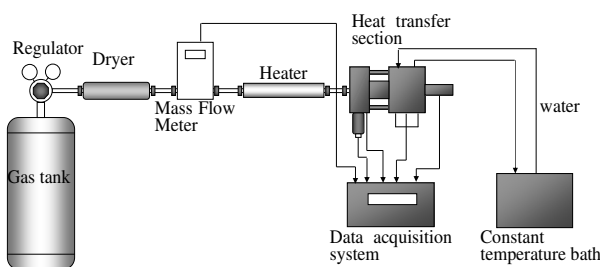
chamber of the inlet section. The thermo-couple is inserted into the inlet tube and gas temperature in the inlet section is measured. Silicone RTV is used to seal the connections to prevent the gas leakage. The outlet tube is also made of bakelite and is attached to the water jacket. This is to decrease heat loss and to measure the total temperature accurately. The inner diameter of the outlet tube is 2 mm. The heat transfer section is put on a foamed polystyrene stage to decrease the heat loss from the heat transfer section to the ground.

A SEM picture of the micro-tube cross section for the case of D=163μm is shown in Fig. 3. The micro-tube is made of SUS 304 stainless steel whose outer diameter is 402 μm and inner diameter is 163 μm and the length is 50mm. The wall thickness of micro-tube is 119.5 μm. The measurement of the inner diameter of the micro-tube will be stated later. The micro-tube is placed inside the water jacket to make the wall temperature constant. The water was circulated between the water jacket and a constant temperature bath. The water temperature in the water jacket was measured by three thermo-couples. These three thermo-couples were type-K sheathed thermo-couples of diameter of 0.5 mm. They were set at 5 mm, 22.5 mm and 45 mm from one end of the micro-tube in the axial direction and apart from 1mm in the radial direction.

Pressure transducer (Valcom VESX500G2) was attached to the upstream chamber to measure the inlet pressure of the micro-tube. The temperature of the gas near the inlet and the outlet of the micro-tube were measured as shown in Fig. 2. Then, bare wire type-K thermo-couples of 50 μm in end-diameter

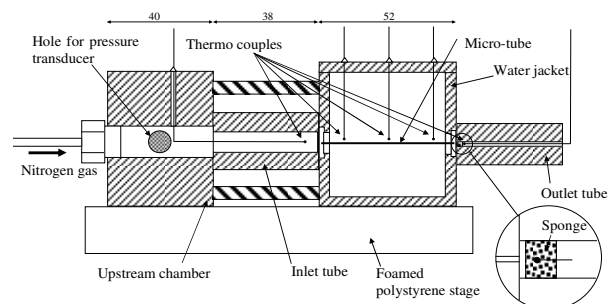Fig. 1 Schematic of experimental setup

Fig. 2 Detailed view of heat transfer section

Fig. 3   SEM picture of the micro-tube cross section

Table 1.   Measurement uncertainty

Measurement	Uncertainty
Pressure transducer (0-500 kPa)	$\pm$ 0.25 % of FS (1250 Pa)
Flow meter I (0-8.0$\times 10^{-6}$ kg/s)	$\pm$ 0.25 % of FS (2.0$\times 10^{-8}$ kg/s)
Flow meter II (0-1.3$\times 10^{-5}$ kg/s)	$\pm$ 0.25 % of FS (3.25$\times 10^{-8}$ kg/s)
Thermocouple (sheath type-K)	$\pm$ 0.1 K
Thermocouple (bare wire type-K)	$\pm$ 0.1 K

were used.   The uncertainty of the measurement is listed in Table 1.

The correlation between the total temperature and the static temperature is [3]

$$T_T = T_b + \frac{\int \rho u \frac{u^2}{2} dA}{\int \rho C_p u dA} \qquad (1)$$

where

$$T_b = \frac{\int \rho C_p u T dA}{\int \rho C_p u dA} \qquad (2)$$

$$T_T = \frac{\int \rho C_p u T dA + \int \rho u \frac{u^2}{2} dA}{\int \rho C_p u dA} \qquad (3)$$

Assuming the parabolic velocity profile, Eq. (1) at the outlet is rewritten as

$$T_T = T_b + \frac{U^2}{C_p} \qquad (4)$$

where U is the average velocity at a cross section.   Note that the kinetic temperature of flow with the parabolic velocity profile can be expressed by the second term of the right hand side of Eq. (4) and it differs from a form for flow with the uniform velocity profile.   The

average velocity can be obtained from

$$U = \frac{\dot{m} R T_b}{pA} \qquad (5)$$

where A is cross sectional area, p is the average pressure at the cross section, R is gas constant and $\dot{m}$ is mass flow rate.
Then, Mach number is

$$Ma = \frac{U}{\sqrt{\gamma R T_b}} \qquad (6)$$

where γ is specific heat ratio.

The cross sectional area ratio of the outlet tube and the micro-tube of D=243 μm is about 70 and the gas slows down in a sponge.   The sponge in the outlet was attached at the micro-tube exit.   It is made by Polyurethane cylinder of 2 mm in diameter and 4 mm in length.   Its coefficient of permeability is 6.4-10 cell/cm.   The distance between the thermo-couple and the micro-tube at exit is less than 1 mm.   The kinetic energy of the gas would be small enough in the down stream of the sponge.   Therefore, the gas temperature measured in the outlet tube can be considered as the total temperature.   The signals from the pressure transducer, the mass flow meter and thermocouples were connected to a data acquisition system (Eto Denki, CADAC21).

2.1 Inner diameter measurements of micro-tube

The inner diameter of the micro-tube was measured by flowing water in the tube. Details of the measurement are documented in our previous paper [1], and will not be given here.   The mass flow rate was measured by measuring the weight of the water over the period of around 300 s.   The inner diameter of the micro-tube was obtained by substituting the volumetric flow rate which was calculated from the mass flow rate, into the following equation.

$$\pi D^4 \rho g h - 64 \mu n D \dot{Q} = 128 \mu \ell \dot{Q} + \frac{16}{\pi} \kappa \rho \dot{Q}^2 \qquad (7)$$

Note that Eq. (7) was derived from the assumption of the Hagen-Poiseuille flow. The n and κ in Eq. (7) are the length correction and the kinetic energy correction coefficients and they are n=0.7 and κ=1.12. Measurements

Table 2   Measured tube diameter

Tube notation (μm)	Average diameter (μm)	Uncertainty (μm)
163	163.21	$\pm$ 0.9-1.06
243	242.92	$\pm$ 1.16-2.82

were conducted 6 times in an identical tube. An average of them was regarded as the inner diameter.   Then, the measured diameters are tabulated in Table 2.   The uncertainty for micro-tubes was also listed in the table.

3. Results and discussions

3.1 Experimental data

The experiments were conducted with two micro-tubes of 163 μm and 243μm in diameter. The temperature of the constant temperature bath was set to 305K and the inlet temperature was set to 315K, 335K and 355K, respectively. Heat transfer tests were conducted for 11 different gas flow conditions of D=163μm and 4 different gas flow conditions of D=243μm for each constant temperature case, respectively.   The experimental results for the case of $T_{stg} \approx$315K and $T_w \approx$305K are tabulated in Table 3.   When nitrogen temperature ranges from 200K to 350K, the viscosity, μ of nitrogen ranges from 1.260×10^{-5} to 2.000×10^{-5} (Pa·s).   The change in the value is relatively large.   The value of viscosity at the outlet bulk temperature was used for calculation of the Reynolds number.   And, the Mach number at the outlet is obtained from Eq. (6) under the assumptions that the pressure at the outlet of the tube is identical to the ambient pressure.   Then, the Reynolds number and outlet Mach number ranged from 520 to 3247 and from 0.145 to 0.734, respectively.   As shown in Table 3, the total temperatures at the micro-tube outlet are higher than the wall temperature, although there are no heat sources whose temperature is higher than the wall temperature around the outlet tube.   The similar results were obtained in our previous numerical studies [5]
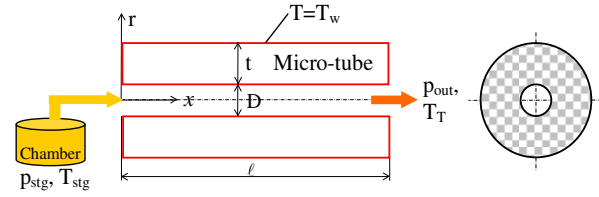

Fig. 4   A schematic diagram of problem

Table 3   Experimental results

#	D (μm)	p_{stg} (kPa)	T_w (K)	T_{stg} (K)	$T_{T,out}$ (K)	Re	Ma_{out}
1	163	150	305.1	315.5	305.1	520	0.145
2		175	305.0	315.1	305.3	768	0.212
3		200	305.0	315.2	305.9	1070	0.291
4		225	305.0	315.0	306.3	1370	0.365
5		250	305.1	315.1	306.8	1670	0.433
6		275	305.0	315.2	307.1	2010	0.505
7		300	305.0	315.1	307.3	2330	0.569
8		325	305.1	315.1	307.4	2620	0.622
9		350	305.0	315.2	307.4	2860	0.664
10		375	305.1	315.1	307.2	3090	0.701
11		400	305.1	315.1	306.9	3310	0.734
12	243	150	305.9	315.3	306.7	1299	0.240
13		175	305.9	315.3	307.7	1951	0.351
14		200	305.9	315.2	308.5	2596	0.451
15		225	305.9	315.2	309.2	3247	0.543

3.2 Numerical solutions

In order to compare with experimental results, numerical computations based on the Arbitrary-Lagrangian-Eulerian (ALE) method were conducted.   The reason we choose ALE method is that it is very stable for the computation of the high speed compressible flow.   The problem is modeled as a micro-tube with the wall thickness, t, its thermal conductivity, λ , and constant wall temperature, T_w at the outer surface of the tube as shown in Fig. 4.   A chamber at the stagnation temperature, T_{stg}, and pressure, p_{stg}, is attached to its upstream section.   The governing equations are identical to those documented in our previous works [4, 5] except the heat conduction in the wall.   Then, the governing equations are not given here.   In the present study, Knudsen number ($Kn=\sigma/D_h$, ratio of the gas mean free path to the characteristic length) that determines the degree of appropriateness of the continuum model is 4.2×10^{-4} for D=163μm under atmospheric pressure and room temperature.   Therefore, the slip on the

wall is assumed to be negligible. The computations were performed for 9 cases (6 cases for $D_i=163\mu m$ and $D_o=402\mu m$ and 3 cases for $D_i=243\mu m$ and $D_o=426\mu m$) for each wall temperature with axisymmetric conditions, respectively. The computational conditions and tube dimensions coincide with those of the experiments. The value of viscosity, μ at the outlet bulk temperature of the experiment was used for computations. The computational domain is divided into quadrilateral cells : 200 in the *x*-direction and 20 for the tube and 20 for the wall in the r-direction. Details of the numerical methodology are documented by Amdsen et al. [8] and Hong and Asako [5]. The mass flow rate, Reynolds and Mach number at the outlet numerically obtained coincide well with those obtained experimentally.

3.3 Bulk temperature

Attention will now be turned to the bulk temperature of an incompressible laminar flow. The bulk temperature of an incompressible laminar flow in a duct whose inner wall temperature is T_w is expressed as a function of mean Nusselt number and the location as (e. g., Burmeister [9])

$$T_b = T_w + (T_w - T_{in})\, e^{-4Nu_m X^*} \qquad (8)$$

where Nu_m is the mean Nusselt number of a simultaneously developing flow in a duct and X^* is the inverse of Graetz number, defined by

$$X^* = \frac{x}{D\,Re\,Pr} \qquad (9)$$

Note that Nu_m is also a function of X^*. The laminar heat transfer characteristics for duct flows have been investigated by many researchers and Nusselt number are reported in the literature (e. g., Shah et al. [10]). The numerically obtained Nu_m for simultaneously developing flow in tubes by Hwang and cited in Shah et al. [9] was used for the calculation of the static bulk temperature for incompressible flow. The calculated bulk temperature is normalized by $T_w - T_{in}$ for a tube and is listed in our previous report [3].

The correlation between the total temperature and the static temperature is expressed by Eq. (1). In the experiment, the
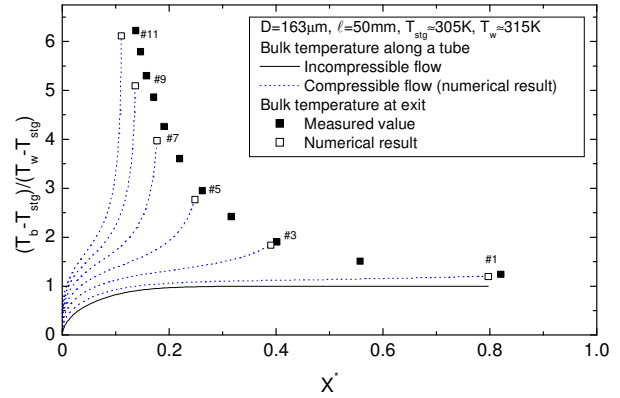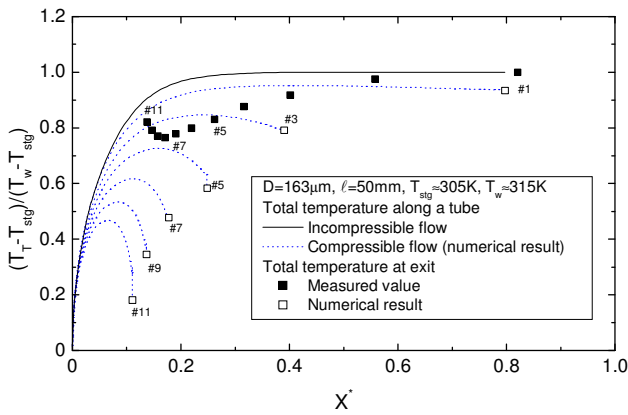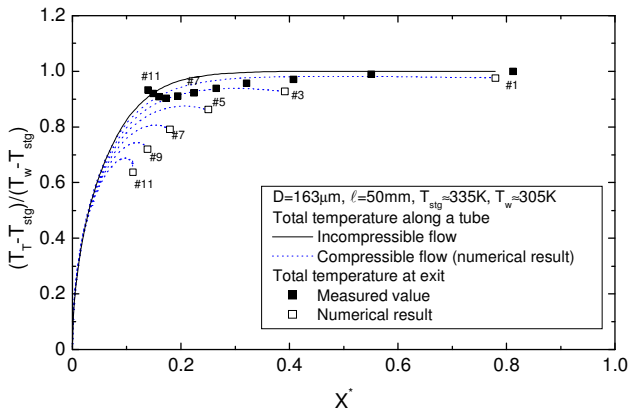

Fig. 5 Bulk temperature as a function of X^*.

total temperature at the tube outlet and also the flow rate were measured. The pressure at the tube outlet is assumed to be equal to the atmospheric pressure, then, the bulk temperature at the outlet of the micro-tube can be obtained by solving Eqs. (4) and (5). The normalized bulk temperature at the outlet of the micro-tube $(T_b - T_{stg})/(T_w - T_{stg})$ for $T_{in}\approx315K$ are plotted in Fig. 5. The numerically obtained bulk temperature is also plotted in the figure with those of bulk temperature for incompressible flow. The bulk temperature for the numerical data and incompressible flow are also normalized as $(T_b - T_{stg})/(T_w - T_{stg})$. In the case of slow (#1 and #2) flow, the numerically obtained normalized bulk temperature increases along the length to the downstream, and levels off and it has similar trend to that of incompressible flow. On the other hand, in the case of fast flow (#3 to #9), the numerically obtained normalized bulk temperature increases along the length to the downstream, and it takes a value greater than unity near the outlet due to the conversion of the thermal energy into the kinetic energy. The experimental data of the bulk temperature at the outlet of the micro-tube match well with those of numerical results. The qualitatively same tendency was obtained for cases of $T_{stg}\approx335K$ and 355K. Also, the qualitatively same tendency was obtained for cases of $D=243\mu m$.
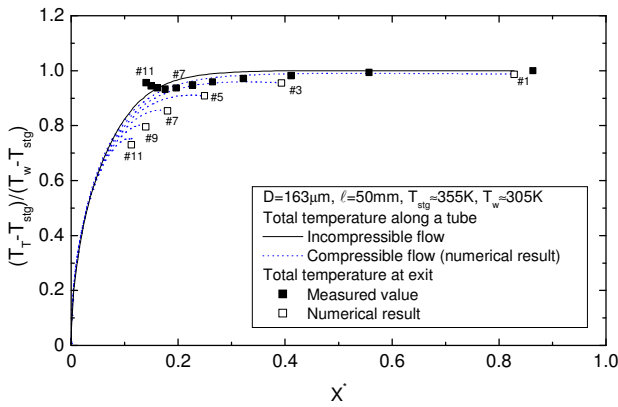
3.4 Total temperature

Attention will now be turned to the total

(a) $T_{stg} \approx 315K$ and $T_w = 305K$

(b) $T_{stg} \approx 335K$ and $T_w = 305K$

(c) $T_{stg} \approx 355K$ and $T_w = 305K$
Fig. 6   Total temperature as a function of X^*

temperature of the fluid, T_T. The experimentally measured total temperature for the cases of $T_{stg} \approx 315K$, 335K and 355K are plotted as a function of X^* in Figs. 6 (a) to (c) in the normalized form of $(T_T - T_{stg})/(T_w - T_{stg})$ for D=163µm, respectively. The experimentally measured total temperatures at the outlet of the micro-tube are also plotted in the figure. The numerically obtained total temperature is

lower than the bulk temperature of the incompressible flow. A part of the thermal energy converts into kinetic energy near the outlet when the flow is fast ($Ma_{out} > 0.3$). This results in a temperature drop and heat transfer from the wall near the outlet as shown Fig. 5. That is, there exists the heat transfer from the wall due to the temperature drop and it corresponds to the increment of the total temperature. The maximum increment of the total temperature is less than 10% of $T_w - T_{stg}$ in the case of $T_{stg} = 335K$ and $T_{stg} = 355K$. In the case of $T_{stg} = 315$ (Fig. 6 (a)), the maximum increment of numerical results reaches 50% of $T_w - T_{stg}$, however, the maximum increment of the experimental results is half of it. Although, there are some discrepancies between the numerical and experimental data, both data have similar trends, namely, it is obvious that the additional heat transfer from the wall was occurred.

In the case of large temperature difference between the inlet and wall temperature, the heat transfer from the wall due to the temperature drop is not so much. This fact indicates that the heat transfer of gaseous flow in a micro-tube can be predicted from correlation for the incompressible flow in a conventional-sized tube if the temperature difference, $T_w - T_{stg}$, is greater than 30K. However if the temperature difference, $T_w - T_{stg}$ is less than 30K, the heat transfer from the wall becomes relatively large and the heat transfer for the gaseous flow in a micro-tube cannot be predicted from correlation for the incompressible flow in a conventional-sized tube. As can be seen in Figs. 6 (b) to 6 (c), the values of the normalized total temperature are close to those of incompressible flow with increasing $T_w - T_{stg}$.

The ratio of the actual recovery and the dynamic temperature is defined as [3, 4 and 5]

$$\text{Recovery ratio} = \frac{T_T - T_{b,incomp}}{\int \rho \, u \frac{u^2}{2} dA \Big/ \int \rho \, C_p u dA} \quad (10)$$

where the denominator represents the dynamic temperature, T_k. Note that the temperature difference between the total temperature and the bulk temperature of the incompressible flow represents the recovery temperature.
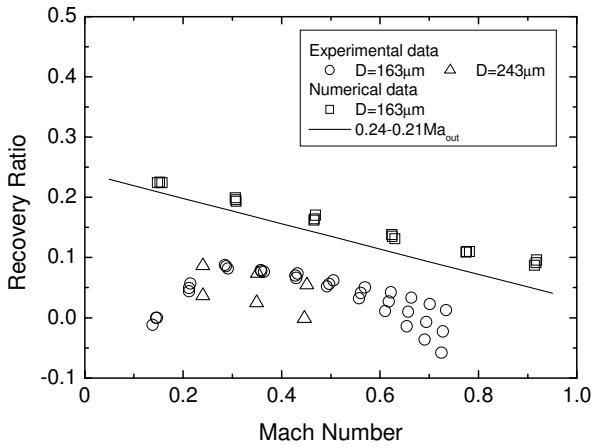
Fig. 7 Recovery ratio as a function of Ma_{out}

Eq. (10) can be rewritten as

$$T_T = T_{b,incomp} + \text{Recovery ratio} \times \frac{\int \rho u \frac{u^2}{2} dA}{\int \rho C_p u dA} \quad (11)$$

Then, the total temperature of the gaseous flow in micro-tubes can be predicted from the bulk temperature of the incompressible flow, the recovery ratio and the dynamic temperature. The recovery ratio calculated from Eq. (10) for both experimental and numerical cases of T_{in}=315K, 335K and 355K is plotted in Fig. 7 as the function of the Mach number at the outlet. The numerically obtained recovery ratio is independent of the wall temperature and decreases with increasing the Mach number at the outlet. The solid line in the figure represents the correlation of recovery ratio proposed by Hong and Asako [5] as

$$\text{Recovery ratio} = 0.24 + 0.21 Ma_{out} \quad (12)$$

Eq. (12) is in good agreement with the numerical results. The experimentally obtained recovery ratios depend on the wall temperature. The values of recovery ratio in some high flow cases are negative since total temperature was not accurately measured at the outlet of micro-tubes due to the heat loss into surroundings. However, they have similar trends with Eq. (12).

4. Conclusions

Experimental study to measure the total temperature at the outlet through a micro-tube with constant wall temperature was performed to determine the quantitatively transferred amount of heat and the local values of the temperature. The following conclusions are reached.

(1) Heat transfer characteristics of the gaseous flow are different from those of the liquid flow.
(2) In the case of fast flow (Ma_{out}>0.3), the total temperature is higher than the bulk temperature of incompressible flow since additional heat transfer from wall to gas occurs due to temperature falls by the thermal energy conversion into kinetic energy near the micro-tube outlet.
(3) The bulk temperatures obtained by the present experimental study were compared with numerical results. The results are in excellent agreement with them.
(4) The total temperature of gaseous flow in a micro-tube to determine the quantitatively transferred amount of heat can be predicted from the bulk temperature of incompressible flow, the recovery ratio and the dynamic temperature.

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