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PERSISTENCE AND CYCLES IN THE US FEDERAL FUNDS RATE

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Abstract

This paper uses long-range dependence techniques to analyse two important features of the US Federal Funds effective rate, namely its persistence and cyclical behaviour. It examines annual, monthly, bi-weekly and weekly data, from 1954 until 2010. Two models are considered. One is based on an I(d) specification with AR(2) disturbances and the other on two fractional differencing structures, one at the zero and the other at a cyclical frequency. Thus, the two approaches differ in the way the cyclical component of the process is modelled. In both cases we obtain evidence of long memory and fractional integration. The in-sample goodness-of-fit analysis supports the second specification in the majority of cases. An out-of-sample forecasting experiment also suggests that the long-memory model with two fractional differencing parameters is the most adequate one, especially over long horizons.

Keywords: Federal Funds rate, persistence, cyclical behaviour, fractional integration

JEL classification: C32, E43

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1. Introduction

The Federal Funds rate is the interest rate at which depository institutions in the US lend each other overnight (normally without a collateral) balances held at the Federal Reserve System (the Fed), which are known as Federal Funds. Such deposits are held in order to satisfy the reserve requirements of the Fed. The rate is negotiated between banks, and its weighted average across all transactions is known as the Federal Funds effective rate. It tends to be more volatile at the end of the reserve maintenance period, the so-called settlement Wednesday, when the requirements have to be met.[1] The Federal Funds target rate is instead set by the Chairman of the Fed according to the directives of the Federal Open Market Committee (FOMC), which holds regular meetings (as well as additional ones when appropriate) to decide on this target. It is therefore a policy rate, used to influence the money supply, and to make the effective rate (which by contrast is determined by the interaction of demand and supply) follow it. Specifically, the Trading Desk of the Federal Reserve Bank of New York conducts open market operations on the basis of the agreed target. This is considered one of the most important indicators for financial markets, whose expectations can be inferred from the prices of option contracts on Federal Funds futures traded on the Chicago Board of Trade.

Given the fact that the Fed implements monetary policy by setting a target for the effective Federal Funds rate which also affects other linked interest rates and the real economy through various transmission channels, it is not surprising that both the theoretical and the empirical literature on this topic are extensive. Theoretical contributions include a well-known paper by Bernanke and Blinder (1988), who propose a model of monetary policy transmission which they then test in a follow-up study (Bernanke and Blinder, 1992) showing that the Federal Funds rate is very useful to forecast real

[1] In empirical studies, therefore, the series is often adjusted to eliminate this effect (see, e.g., Sarno and Thornton (2003).

macroeconomic variables, being a good indicator of monetary policy actions. Bartolini et al. (2002) instead develop a model of the interbank money market with an explicit role for central bank intervention and periodic reserve requirements that is consistent with the observed volatility pattern of the US Federal Funds rate.

On the empirical side, some papers examine the extent to which variables targeted by the Fed such as the output gap and inflation can explain the effective rate (see, e.g., Taylor, 1993 and Clarida et al., 2000); others analyse the daily market for Federal Funds (e.g., Hamilton, 1996, and Taylor, 2001). An influential study by Hamilton and Jorda (2002) introduced the autoregressive conditional hazard model for forecasting a discrete-valued time series such as the target; this specification is shown to outperform standard VAR models that are unable to differentiate between the effects of an increase in the target and those of an anticipated target decrease that did not take place. Other studies examine the predictive power of the effective rate of the target (Taylor, 2001) or other interest rates (e.g., Clarida et al., 2006). Sarno et al. (2005) provide the most extensive study of the forecasting performance of a variety of models of the Federal Funds rate proposed in the literature. They consider both univariate (randow walk, ARMA, EGARCH, Markov-switching etc.) and multivariate (M-TAR, BTAR, MS-VECM) specifications, and find that the best forecasting model is a univariate one using the current difference between the effective and the target rate to forecast the future effective rate (also, combination forecasts only yield marginal improvements). These findings are interpreted as suggesting that the Fed in fact follows a forward-looking interest rate rule.

Most of the models found in the literature to describe the behaviour of the Federal Funds rate (and of interest rates in general) assume nonstationarity and are based on first-differenced series. This is true, for instance of all the univariate specifications considered in Sarno et al. (2005), which imply that the series are I(1), without mean reversion and with permanent effects of shocks. This is a rather strong assumption that is not justified on theoretical grounds. The classic alternative is to assume that the Federal Funds rate and interest rates in general are stationary I(0) variables, and to model them as autoregressive processes with roots close to the unit circle, with the additional problem of the well-known low power of standard unit root tests. In this study we overcome this dichotomy by estimating fractional integration models allowing for both nonstationary and mean-reverting behaviour. Moreover, using recent techniques based on the concept of long-range dependence we explicitly model two well-known features of interest rates in general which also appear to characterise the Federal Funds rate, namely their persistence and cyclical behaviour, mostly overlooked in previous studies. In particular, we use fractional integration methods with multiple poles or singularities in the spectrum not constrained at the zero frequency as in the usual case, but allowing instead for poles at zero and non-zero (cyclical) frequencies. In this way we are able to capture the two aforementioned features of interest rates: their high degree of persistence (described by the pole in the spectrum at the zero frequency) and their cyclical pattern (described by the pole at the non-zero frequency).[2] Overall, our results confirm that both these stylised facts are important features of the stochastic behaviour of these series. Sensitivity to data frequency is then analysed by using annual, monthly, bi-weekly and weekly data, from 1955 until 2010.

The remainder of the paper is structured as follows. Section 2 describes the econometric approach. Section 3 presents the empirical results. Section 4 provides some concluding remarks.

2. Methodology

We consider the following model:

[2] Other sources of persistence/cyclical patterns are described by the short-run (ARMA) dynamics of the process.

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, \ldots, \tag{1}$$

$$(1 - L)^{d_1} (1 - 2\cos w_r L + L^2)^{d_2} x_t = u_t, \tag{2}$$

where y_t is the observed time series; β is a (kx1) vector of unknown parameters, and z_t is a (kx1) vector of deterministic terms, that might include, for example, an intercept (i.e. z_t = 1) or an intercept with a linear trend ($z_t = (1,t)^T$); L is the lag operator (i.e., $L^s x_t = x_{t-s}$); d_1 is the order of integration corresponding to the long-run or zero frequency; $w_r = 2\pi/r$, with r representing the number of periods per cycle; d_2 is the order of integration with respect to the non-zero (cyclical) frequency, and u_t is assumed to be an I(0) process, defined for the purposes of the present study as a covariance-stationary process, with a spectral density function that is positive and finite at any frequency on the spectrum. Note that d_1 and d_2 are allowed to be any real values and thus are not restricted to be integers.

The set-up described in (1) and (2) is fairly general, including the standard ARMA model (with or without trends), if $d_1 = d_2 = 0$ and u_t is weakly autocorrelated; the I(1) model if $d_1 = 1$ or, more generally, the ARIMA case if d_1 is an integer and $d_2 = 0$; the standard ARFIMA specification, if d_1 has a fractional value and $d_2 = 0$, along with other more complex representations.

We now focus on equation (2), and first assume that $d_2 = 0$. Then, for any $d_1 > 0$, the spectral density function of x_t is given by

$$f(\lambda) = \frac{\sigma^2}{2\pi} \left| g_u(\lambda) \right|^2 \left| 1 - e^{i\lambda} \right|^{-2 d_1},$$

where $g_u(\lambda)$ corresponds to the potential ARMA structure in u_t. It can be easily shown that this function $f(\lambda)$ contains a pole or singularity at the long-run or zero frequency, i.e.,

$$f(\lambda) \to \infty, \quad as \quad \lambda \to 0^+.$$

Further, note that the polynomial $(1-L)^{d_1}$ can be expressed in terms of its Binomial expansion, such that, for all real d_1,

$$(1-L)^{d_1} = \sum_{j=0}^{\infty} \binom{d_1}{j} (-1)^j L^j = 1 - d_1 L + \frac{d_1(d_1-1)}{2} L^2 - \ldots, \qquad (3)$$

implying that the higher the value of d_1 is, the higher the degree of dependence between observations distant in time will be. Thus, the parameter d_1 plays a crucial role in determining the degree of long-run persistence of the series. Examples of applications using this model can be found in Diebold and Rudebusch (1989), Sowell (1992), Baillie (1996) and Gil-Alana and Robinson (1997) among others.[3]

On the other hand, if $d_1 = 0$ in (2), then for any $d_2 \neq 0$, the process x_t has a spectral density function

$$f(\lambda) = \frac{\sigma^2}{2\pi} \left| g_u(\lambda) \right|^2 \left| 2\left(\cos(\lambda) - \cos(w_r)\right) \right|^{-2d_2},$$

which is characterised by a pole at a non-zero frequency, i.e.,

$$f(\lambda) \rightarrow \infty, \quad as \quad \lambda \rightarrow \lambda^*, \quad \lambda^* \in (0, \pi).$$

Moreover, the polynomial $(1 - 2\cos w_r L + L^2)^{d_2}$ can be expressed as a Gegenbauer polynomial, such that, defining $\mu = \cos w_r$, for all $d_2 \neq 0$,

$$(1 - 2\mu L + L^2)^{-d_2} = \sum_{j=0}^{\infty} C_{j,d_2}(\mu) L^j, \qquad (4)$$

where $C_{j,d_2}(\mu)$ are orthogonal Gegenbauer polynomial coefficients recursively defined as:

$$C_{0,d_2}(\mu) = 1,$$

$$C_{1,d_2}(\mu) = 2\mu d_2,$$

[3] Empirical studies estimating I(d) models of this form for interest rates include Lai (1997), Tsay (2000), Meade and Maier (2003) and Couchman, Gounder and Su (2006).

$$C_{j,d_2}(\mu) = 2\mu\left(\frac{d_2-1}{j}+1\right)C_{j-1,d_2}(\mu) - \left(2\frac{d_2-1}{j}+1\right)C_{j-2,d_2}(\mu), \quad j = 2, 3, \ldots$$

(see, inter alia, Magnus et al., 1966, or Rainville, 1960, for further details). Gray et al. (1989, 1994) showed that this process is stationary if $d_2 < 0.5$ for $|\mu = \cos w_r| < 1$ and if $d_2 < 0.25$ for $|\mu| = 1$. If $d_2 = 1$, the process is said to contain a unit root cycle (Ahtola and Tiao, 1987; Bierens, 2001); other applications using fractional values of d_2 can be found in Gil-Alana (2001), Anh, Knopova and Leonenko (2004) and Soares and Souza (2006).

In the empirical analysis we use a very general testing procedure to test the model given by equations (1) and (2). It was initially developed by Robinson (1994) on the basis of the Lagrange Multiplier (LM) principle that uses the Whittle function in the frequency domain. It can be applied to test the null hypothesis:

$$H_o: \; d \equiv (d_1, d_2)^T = (d_{1o}, d_{2o})^T \equiv d_o, \tag{5}$$

in (1) and (2) where d_{10} and d_{20} can be any real values, thus encompassing stationary and nonstationary hypotheses. The specific form of the test statistic (denoted by $\hat{R}$) is presented in the Appendix. Under very general regularity conditions, Robinson (1994) showed that for this particular version of his tests,

$$\hat{R} \rightarrow_d \chi_2^2, \quad as \quad T \rightarrow \infty, \tag{6}$$

where T indicates the sample size, and "$\rightarrow_d$" stands for convergence in distribution. Thus, unlike in other procedures, we are in a classical large-sample testing situation. A test of (5) will reject H_o against the alternative H_a: $d \neq d_o$ if $\hat{R} > \chi^2_{2,\alpha}$, where Prob ($\chi^2_2 > \chi^2_{2,\alpha}$) = α. Furthermore this test is the most efficient in the Pitman sense against local departures from the null, that is, if it is implemented against local departures of the form: H_a: $d = d_o + \delta T^{-}$

$^{1/2}$, for $\delta \neq 0$, the limit distribution is a $\chi_2^2(v)$, with a non-centrality parameter v that is optimal under Gaussianity of u_t.[4]

3. Empirical results

The series examined is the US Federal Funds effective rate, from 1954 till 2010, at annual, monthly, bi-weekly and weekly frequencies.

[Insert Figures 1 – 3 about here]

Figure 1 displays plots of the series at the four frequencies considered, the pattern being similar in all four cases. Figure 2 displays the correlograms; the two features mentioned above can clearly be seen: there is a slow decay in the sample autocorrelation values possibly due to persistence, and a cyclical pattern. The same two features are exhibited by the periodograms, displayed in Figure 3, with the highest peaks occurring at the smallest frequency (long-run persistence) and at frequency 7 corresponding to T/7 periods per cycle, namely to approximately 8 years in all cases.

First, we examine the degree of persistence considering only the long-run or zero frequency, that is, we specify a model such as (1) and (2) with $d_2 = 0$ a priori and z_t $(1,t)^T$, i.e.,

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, \ldots, \tag{7}$$

$$(1-L)^{d_1} x_t = u_t, \qquad t = 1, 2, \ldots, \tag{8}$$

with $x_t = 0$ for $t \leq 0$, under the assumption that the disturbance term u_t is white noise, AR(1) and AR(2) respectively. Higher AR orders and other MA (ARMA) structures were also considered, with similar results. We employ here a simple version of Robinson's (1994) procedure, testing H_o: $d_1 = d_{1o}$, for d_{1o}-values from 0 to 2 with 0.001 increments,

[4] Note, however, that Gaussianity is not necessary for the implementation of this procedure, a moment condition of only order 2 being required.

(i.e., d_{10} = 0, 0.001, 0.002, …, 1.999 and 2), and reporting the estimates of d_1 along with the 95% confidence intervals of the non-rejection values of d_1 based on the testing procedure.

[Insert Table 1 about here]

We obtain estimates for the three standard cases examined in the literature, i.e., with no regressors in the undifferenced regression (7) ($\beta_0 = \beta_1 = 0$); with an intercept (β_0 unknown and $\beta_1 = 0$); and with an intercept and a linear time trend (β_0 and β_1 unknown). The results for the time trend were found to be statistically insignificant in all cases, while the intercept was always significant. Thus, in what follows, we only consider the case of an intercept.[5] As already mentioned, Table 1 displays the estimates of d_1 based on the Whittle function in the frequency domain (Dahlhaus, 1989) along with the 95% confidence interval of the non-rejection values of d_1 using Robinson's (1994) method.

When u_t is assumed to be a white noise process, the results change substantially depending on the data frequency. In particular, for annual data the estimated value of d_1 is 0.937 and the I(1) null hypothesis cannot be rejected. It is rejected instead for monthly and bi-weekly data in favour of values of d_1 above 1. Finally, for weekly data, the estimated d_1 is smaller than 1 and statistically significant, implying mean reversion. When allowing for autocorrelated errors, if u_t is assumed to be AR(1) values of d_1 below 1 supporting mean reversion are obtained in the annual and monthly cases; for bi-weekly and weekly data, d_1 is instead slightly above 1 and the unit root null is rejected in favour of $d_1 > 1$ in the weekly case. Finally, if u_t is assumed to be AR(2) the unit root cannot be rejected in any single case and the estimated values of d_1 range between 0.722 (with annual data) and 1.045

[5] Note that with white noise u_t and for $t > 1$ this becomes the simple driftless random walk model.

(weekly data). The case of AR(2) disturbances is interesting because it allows to capture the cyclical pattern of the series through a short-memory I(0) process for u_t.[6]

Likelihood Ratio (LR) tests and other likelihood criteria (not reported) suggest that the model with AR(2) disturbances outperforms the others. These results, however, might be biased owing to the long memory in the cyclical structure of the series having been overlooked. Thus, we next consider a model such as (1) and (2) with z_t $(1,t)^T$, i.e., the null model now becomes

$$y_t \;=\; \beta_0 \;+\; \beta_1 t \;+\; x_t, \qquad (9)$$

$$(1 - L)^{d_1}(1 - 2\cos w_r L \;+\; L^2)^{d_2}\, x_t \;=\; u_t, \qquad (10)$$

again with I(0) (potentially ARMA) u_t. The results, for the case of an intercept, which is the most realistic one on the basis of the t-values (not reported), are displayed in Table 2.

[Insert Table 2 about here]

The estimated values of r and thus j = T/r (the number of periods per cycle) for the four series is now close to 8 years. Specifically, j is found to be 8 in the case of the annual data; 97 (and thus 97/12 = 8.089 years) for the monthly data; and 7.57 years (212/28 and 424/56) for bi-weekly and weekly data. This is consistent with the plots of the periodograms displayed in Figure 3. Focusing now on the fractional differencing parameters, it can be seen that d_1 is close to (although below) 1 and d_2 is slightly above 0 for the four series. For d_1 the unit root null is rejected in favour of mean reversion in the case of annual, bi-weekly and weekly data; however, for monthly data, even though d_1 is still below 1, the unit root null cannot be rejected at conventional significance levels. As for the cyclical fractional differencing parameter, d_2, is estimated to be 0.094 in the annual case and the I(0) null hypothesis cannot be rejected. In the remaining three cases, d_2 is significantly above 0 (thus displaying long memory), ranging from 0.145 (weekly data) to

[6] The estimates of the AR(2) coefficients (not reported) were in all cases in the complex plane, which is consistent with the cyclical pattern observed in the data.

0.234 (monthly data). Very similar values for d_1 and d_2 are obtained in the case of autocorrelated disturbances; LR and no-autocorrelation tests strongly support the white noise specification for u_t for each of the four series.[7]

Finally, we investigate which of the two specifications (the I(d) one with AR(2) disturbances or the one with the two fractional differencing structures) has a better in-sample performance, and also better forecasting properties. For the first of these two purposes we employ several goodness-of-fit measures based on the likelihood function. For the forecasting experiment, we use instead various statistics including the modified Diebold and Mariano (1995) (M-DM) statistic. Remember that the two models considered are:

$$y_t = \beta_0 + x_t; \qquad (1-L)^{d_1} x_t = u_t; \qquad u_t = \phi_1 u_{t-1} + \phi_2 u_{t-2} + \varepsilon_t, \qquad \text{(M1)}$$

and

$$y_t = \beta_0 + x_t; \quad (1-L)^{d_1}(1-2\cos w_r L + L^2)^{d_2} x_t = \varepsilon_t, \qquad \text{(M2)}$$

and therefore they differ in the way the cyclical component is modelled, model (M1) and (M2) adopting respectively an AR(2) process and a Gegenbauer (fractional) specification for the d_1-differenced (demeaned) series.

For the in-sample goodness of fit analysis we carry out first a Likelihood Ratio (LR) test noting that (M1) is nested in (M2). Thus, using in (M2) the equations given by (9) and (10) with $\beta_1 = 0$, $d_2 = 0$ and AR(2) u_t we obtain (M1). The results support the (M2) specification for three of the four series examined. Only for the annual data (M1) seems to be preferable at the 5% level. This is consistent with the results displayed in Tables 1 and 2, noting that the only confidence interval in Table 2 where $d_2 = 0$ is not excluded is

[7] We use here the Box-Pierce and Ljung-Box-Pierce statistics (Box and Pierce, 1970; Ljung and Box, 1978).

precisely that for the annual series. Other likelihood criteria (AIC and SIC) lead essentially to the same conclusions.[8]

Next we focus on the forecasting performance of the two models. For this purpose we calculate one- to twenty-step ahead forecasts over 20 periods for each of the four series at different data frequencies. The forecasts were constructed according to a recursive procedure conditionally upon information available up to the forecast date which changes recursively.

We computed the Root Mean Squared Errors (RMSE) and the Mean Absolute Deviation (MAD) for the two specifications of each series. The results (not reported here for reasons of space, but available from the authors upon request) indicate that the fractional structure outperforms the AR(2) model in practically all cases.

However, the above two criteria and other methods such as the Mean Absolute Prediction Error (MAPE), Mean Squared Error (MSE), etc., are purely descriptive devices.[9] Several statistical tests for comparing different forecasting models are now available. One of them, widely employed in the time series literature, is the asymptotic test for a zero expected loss differential due to Diebold and Mariano (1995).[10] Harvey, Leybourne and Newbold (1997) note that the Diebold-Mariano test statistic could be seriously over-sized as the prediction horizon increases, and therefore provide a modified Diebold-Mariano test statistic given by:

$$M - DM \;=\; DM\sqrt{\frac{n + 1 - 2h + h(h-1)/n}{n}},$$

[8] Note, however, that these criteria might not necessarily be the best criteria in applications involving fractional differences, as they focus on the short-term forecasting ability of the fitted model and may not give sufficient attention to the long-run properties of the fractional models (see, e.g. Hosking, 1981, 1984).

[9] The accuracy of different forecasting methods is a topic of continuing interest and research (see, e.g., Makridakis et al., 1998 and Makridakis and Hibon, 2000, for a review of the forecasting accuracy of competing forecasting models).

[10] An alternative approach is the bootstrap-based test of Ashley (1998), though his method is computationally more intensive.

where DM is the original Diebold-Mariano statistic, h is the prediction horizon and n is the time span for the predictions. Harvey et al. (1997) and Clark and McCracken (2001) show that this modified test statistic performs better than the DM test statistic, and also that the power of the test is improved when p-values are computed with a Student t-distribution.

We further evaluate the relative forecast performance of the different models by making pairwise comparisons based on the M-DM test statistic. We consider 5, 10, 15, 20 and 25-period ahead forecasts. The results are displayed in Table 3.

[Insert Table 3 about here]

They show that for the 5-step and 10-step ahead predictions it cannot be inferred that one model is statistically superior to the other. By contrast, over longer horizons there are several cases where the fractional model (M2) outperforms (M1). However, these forecasting methods may have very low power under some circumstances, especially in the case of non-linear models (see, e.g., Costantini and Künst, 2011). Thus, these results should be taken with caution.

4. Conclusions

This paper uses long-range dependence techniques to analyse two important features of the US Federal Funds effective rate, namely its persistence and cyclical behaviour. In particular, it examines annual, monthly, bi-weekly and weekly data, from 1954 until 2010. The main results are the following. When estimating a simple I(d) model, the estimates suggest that d is close to 1, in some cases below 1 indicating mean reversion, and in others above 1 implying a rejection of the I(1) hypothesis depending on the data frequency and the type of disturbances considered (white noise or AR(1)). If these are modelled as AR(2), which is highly plausible in view of the cyclical pattern of the series under examination, the results indicate that the I(1) null cannot be rejected at any of the four frequencies. The second model considered uses a Gegenbauer-type of process for the cyclical component, and therefore has two fractional differencing parameters, one corresponding to the long-run or zero frequency (d_1), and the other to the cyclical structure (d_2). When using this specification the results indicate that the order of integration at the zero frequency ranges between 0.802 (bi-weekly frequency) and 0.966 (monthly), whilst that of the cyclical component ranges between 0.094 (annual) and 0.234 (bi-weekly). Both the in-sample and out-of-sample evidence suggest that the long memory model with two fractional structures (one at zero and the other at the cyclical frequency) outperforms the other models.

Our results are not directly comparable to those of Sarno et al. (2005), who model the difference between the effective and the target rate, whilst we focus only on the former. Nevertheless, our analysis, based on letting the data speak by themselves to find the most suitable specification, produces valuable evidence for interest rate modelling, since it shows that an I(d) specification including a cyclical component outperforms both classical I(0) and simple I(d) models. This confirms the importance of adopting an econometric framework such as the one chosen here, which explicitly takes into account both persistence and cyclical patterns, to model the behaviour of the US Federal Funds effective rate and interest rates in general.

Appendix

The test statistic proposed by Robinson (1994) for testing H_o (5) in the model given by equations (1) and (2) is given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a},$$

where T is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_j^{*} \psi(\lambda_j) g_u(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g_u(\lambda_j; \hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T} \left(\sum_j^{*} \psi(\lambda_j) \psi(\lambda_j)' - \sum_j^{*} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \left(\sum_j^{*} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_j^{*} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j)' \right)$$

$$\psi(\lambda_j)' = [\psi_1(\lambda_j), \psi_2(\lambda_j)]; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g_u(\lambda_j; \hat{\tau}); \qquad \psi_1(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|;$$

$\psi_2(\lambda_j) = \log \left| 2 (\cos\lambda_j - \cos w_r) \right|$, with $\lambda_j = 2\pi j/T$, and the summation in * is over all frequencies which are bounded in the spectrum. $I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1-L)^{d_{1o}} (1 - 2\cos w_r L + L^2)^{d_{2o}} y_t - \hat{\beta}' \bar{z}_t$, with

$$\hat{\beta} = \left(\sum_{t=1}^{T} \bar{z}_t \bar{z}_t' \right)^{-1} \sum_{t=1}^{T} \bar{z}_t (1-L)^{d_{1o}} (1 - 2\cos w_r L + L^2)^{d_{2o}} y_t;$$

$\bar{z}_t = (1-L)^{d_{1o}} (1 - 2\cos w_r L + L^2)^{d_{2o}} z_t$, evaluated at $\lambda_j = 2\pi j/T$ and $\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau)$, with T^* as a suitable subset of the R^q Euclidean space. Finally, the function g_u above is a known function coming from the spectral density of u_t:

$$f(\lambda) = \frac{\sigma^2}{2\pi} g_u(\lambda; \tau), \qquad -\pi < \lambda \leq \pi.$$

Note that these tests are purely parametric and, therefore, they require specific modelling assumptions about the short-memory specification of u_t. Thus, if u_t is white noise, $g_u \equiv 1$, and if u_t is an AR process of the form $\phi(L)u_t = \varepsilon_t$, $g_u = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ.

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Figure 1: Original time series data

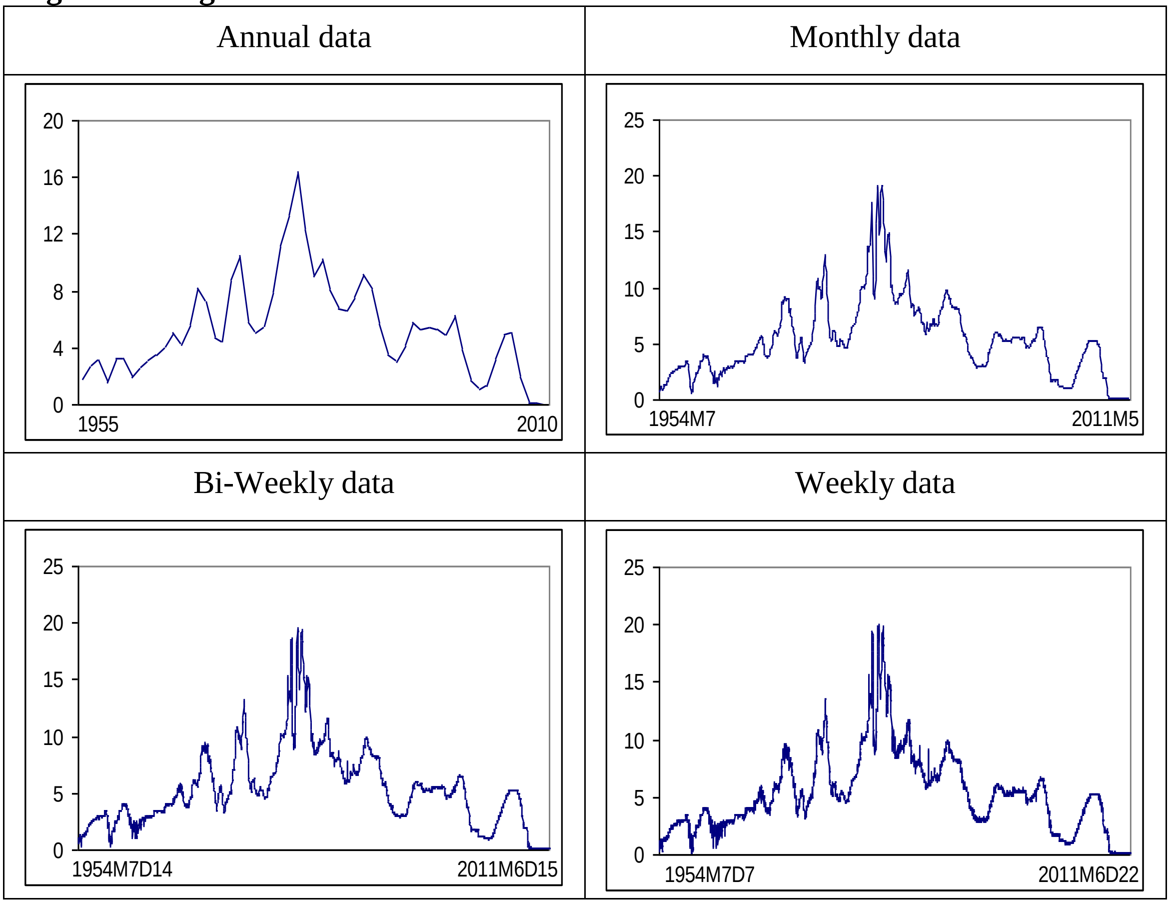
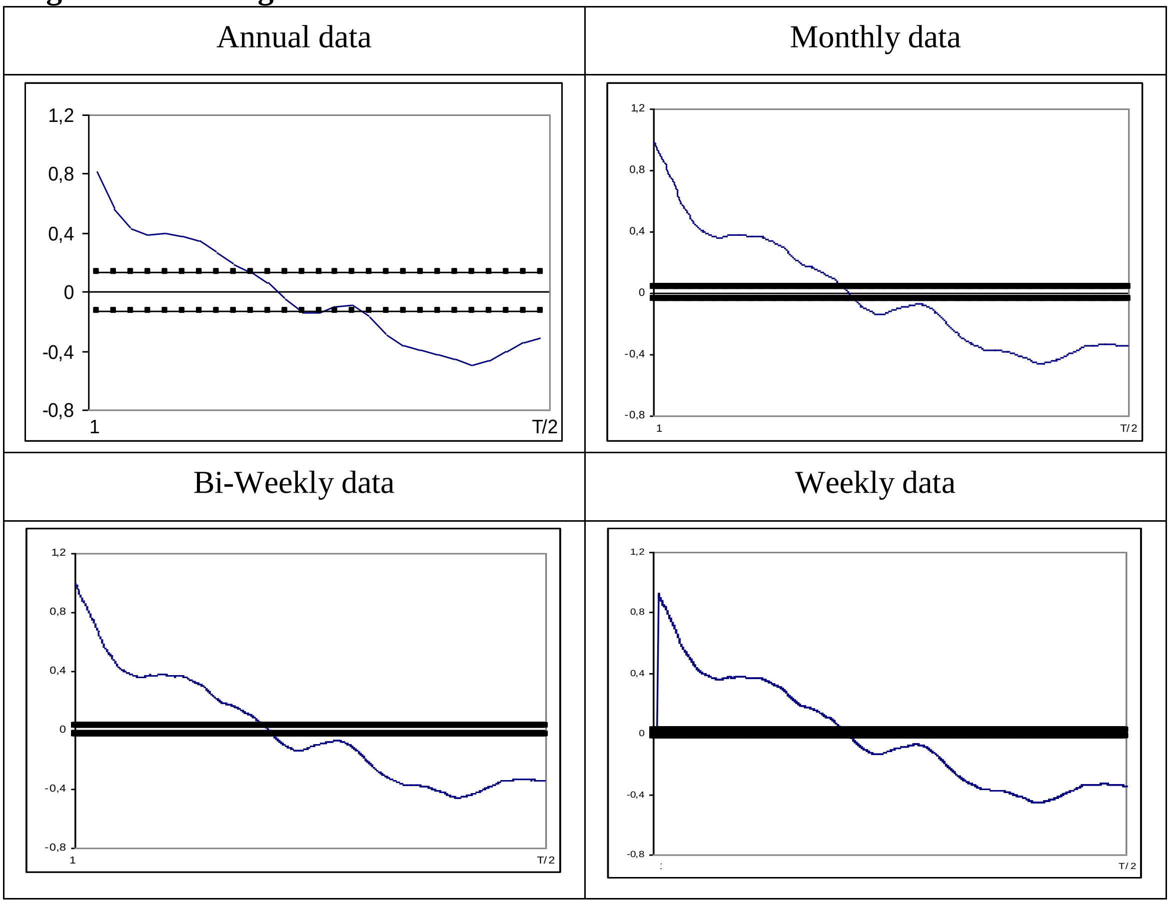
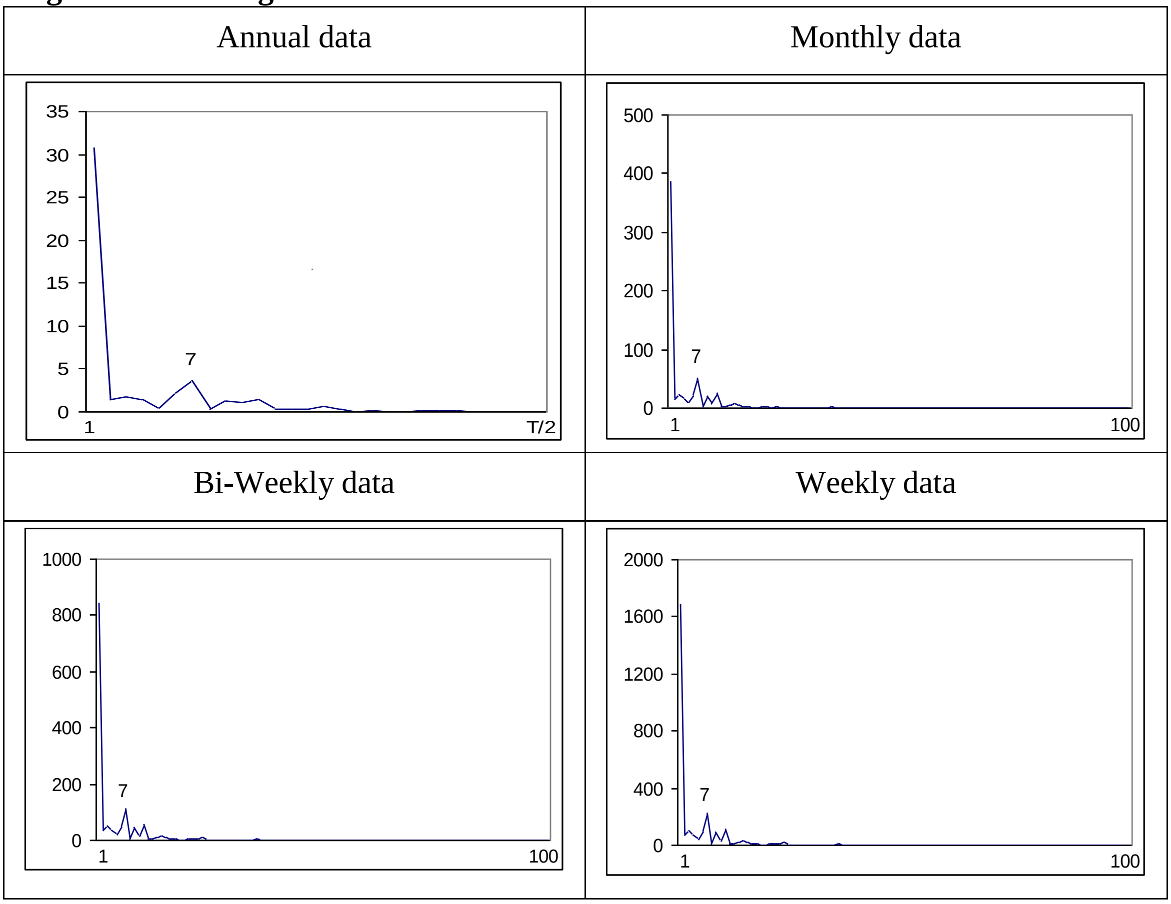


Figure 2: Correlogram of the time series



Note: The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.

Figure 3: Periodogram of the time series



Note: The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, $j = 1, \ldots, T/2$.

Table 1: Estimates of d and 95% confidence interval in an I(d) model with an intercept

	White noise	AR(1) disturbances	AR(2) disturbances
Annual	0.937 (0.704, 1.450)	0.544 (0.429, 0.700)	0.722 (0.334, 1.495)
Monthly	1.277 (1.189, 1.383)	0.821 (0.742, 0.913)	0.852 (0.679, 1.016)
Bi-Weekly	1.168 (1.122, 1.213)	1.025 (0.891, 1.146)	0.824 (0.633, 1.008)
Weekly	0.973 (0.954, 0.994)	1.086 (1.044, 1.127)	1.045 (0.984, 1.101)

The values are Whittle estimates of d in the frequency domain (Dahlhaus, 1989). Those in parentheses are the 95% confidence interval of the non-rejection values of d using Robinson (1994).

Table 2: Estimates of d_1 and d_2 in the model with two fractional structures

Frequency	r (j)	d_1	d_2
Annual	j = 7 (r = 8)	0.932 (0.561, 0.983)	0.094 (-0.008, 0.233)
Monthly	j = 683 (r = 97)	0.966 (0.895, 1.128)	0.145 (0.109, 0.217)
Bi-Weekly	j = 1486 (r = 212)	0.802 (0.661, 0.977)	0.234 (0.158, 0.299)
Weekly	j = 2973 (r = 424)	0.817 (0.722, 0.903)	0.156 (0.114, 0.198)

The values in parentheses in the third and fourth columns are the 95% confidence interval of the non-rejection values of d using Robinson (1994).

Table 3: Modified DM statistic: 5, 10, 15, 20 and 25-step ahead forecasts

(M1) vs (M2)	5	10	15	20	25
Annual	1.435	1.764	1.114	-1.698	**-4.311 (M2)**
Monthly	1.872	1.554	-1.050	**-3.564 (M2)**	**-12.344 (M2)**
Bi-weekly	1.115	1.355	**-3.211 (M2)**	**-5.667 (M2)**	**-10.093 (M2)**
Weekly	0.998	-0.065	-1.445	**-4.443 (M2)**	**-8.005 (M2)**

In bold the cases where model (M2) outperforms model (M2) in statistical terms.