

Optimal non-linear monetary policy rules

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Abstract

We propose a simply yet flexible framework for the analysis of optimal monetary policy rules that produces the type of non-linear responses derived in the literature as special cases. Perhaps more importantly, our framework suggests a richer set of non-linear responses than have been considered yet and thus may prompt further work in this area.

1) Introduction

Recent models of optimal monetary policy rules have allowed for non-linearity in the response of interest rates to inflation and output. In some of these models, non-linearity arises because the standard assumption of quadratic preferences has been amended to allow for asymmetry (eg Cukierman and Gerlach, 2003) or for zone-like preferences that might match the features of some inflation targeting regimes (Orphanides and Wieland, 2000). In other models, non-linearity arises because the aggregate supply or Phillips curve is non-linear (eg Nobay and Peel, 2000). In this paper, we propose a simple yet flexible framework that produces the type of non-linear responses derived in the literature as special cases. Perhaps more importantly,

our framework suggests a richer set of non-linear responses than have been considered yet and thus may prompt further work in this area.

The paper is structured as follows. The canonical linear Taylor rule model is briefly derived in section 2) as a benchmark. Section 3) considers a general model of policymakers' preferences and shows how this generalises and extends models proposed in the literature. Section 4) does the same for the case of a non-linear Phillips curve. Section 5) considers the interactions between non-quadratic preferences and a non-linear Phillips curve, showing how this implies a richer set of non-linear models than considered thus far in the literature. Section 6) concludes.

2) The Taylor rule

In this section we outline the derivation of the Taylor (1993) policy rule, as a foundation for the extensions in subsequent sections. We assume the economy comprises conventional aggregate demand and aggregate supply or Phillips curves. The aggregate demand curve is

$$(1) \quad y_t = -\rho(i_t - E_t \pi_{t+1}) + E_t y_{t+1} + \varepsilon_t^d$$

where y is the output gap, i is the nominal interest rate, π is the inflation rate, ε_t^d is an i.i.d demand shock and ρ is a positive coefficient. The Phillips curve is

$$(2) \quad \pi_t = \gamma y_t + \theta E_t \pi_{t+1} + \varepsilon_t^s$$

where ε_t^s is an i.i.d supply shock and γ and θ are positive coefficients.

We assume that policymakers choose the nominal interest rate at the beginning of period t on the basis of information available at the end of period $(t-1)$. Their optimisation problem is

$$(3) \quad \text{Min}_{\{i_t\}} E_{t-1} \sum_{j=0}^{\infty} \delta^j L_{t+j}$$

subject to (1) and (2), where δ is the discount factor and

$$(4) \quad L_t = \frac{1}{2}(\pi_t - \pi^*)^2 + \frac{\lambda}{2}y_t^2 + \frac{\mu}{2}(i_t - i^*)^2$$

is a conventional per-period quadratic loss function where π^* is the inflation target or desired inflation rate, i^* is the equilibrium or desired nominal interest rate and λ and μ are positive coefficients. Following the existing literature (eg Clarida et al, 1999) by solving this optimisation problem under discretion, the optimal monetary policy rule can be expressed as

$$(5) \quad i_t = i^* + \frac{\rho\lambda}{\mu}E_{t-1}y_t + \frac{\rho\gamma}{\mu}E_{t-1}(\pi_t - \pi^*)$$

This is the celebrated Taylor rule, with the familiar constant proportional response of interest rates to inflation and output.

3) Optimal monetary policy with non-quadratic preferences

We generalise the quadratic per-period loss function in (4) by assuming

$$(6) \quad L_t = \frac{1}{2} \frac{e^{\alpha_\pi(\pi_t - \pi^*)^{\beta_\pi}} - \alpha_\pi(\pi_t - \pi^*)^{\beta_\pi} - 1}{\beta_\pi \alpha_\pi^2} + \frac{\lambda}{2} \frac{e^{\alpha_y y_t^{\beta_y}} - \alpha_y y_t^{\beta_y} - 1}{\beta_y \alpha_y^2} + \frac{\mu}{2}(i_t - i^*)^2$$

where β_π and β_y are integers and α_π and α_y are real numbers. The optimal monetary policy rule then becomes

$$(7) \quad i_t = i^* + \frac{\rho\lambda}{\mu}E_{t-1}f(y_t; \beta_y, \alpha_y) + \frac{\rho\gamma}{\mu}E_{t-1}f((\pi_t - \pi^*); \beta_\pi, \alpha_\pi)$$

where $f(x; \beta, \alpha) = x^{\beta-1} \frac{e^{\alpha x^\beta} - 1}{\alpha}$. Equation (7) is a non-linear monetary policy rule that encompasses the various models that have been proposed in the literature with non-quadratic preferences as special cases.

The loss function in (6) can take on a number of forms depending on the values of the parameters. The loss function simplifies to the quadratic when $\alpha_\pi \rightarrow 0$, $\alpha_y \rightarrow 0$ and $\beta_\pi = \beta_y = 1$. In this case, we obtain the Taylor rule in (5). Figure 1) depicts $i_t - i^*$ as a function of the inflation gap $\pi_t - \pi^*$. In this case, the relationship is a straight line. Considering the response to inflation, if $\beta_\pi = 1$, we have the Linex preferences of Ruge-Murcia (2003) and Nobay and Peel (2003). The response of interest rates to inflation is a convex function of inflation if $\alpha_\pi > 0$ and a concave function if $\alpha_\pi < 0$. The relationships between $i_t - i^*$ and $\pi_t - \pi^*$ in these cases are also depicted in figure 1); the relationship is convex when $\alpha_\pi > 0$ as a greater concern for excessive inflation leads to a more vigorous adjustment of interest rates when inflation is above the target; the relationship is concave when $\alpha_\pi < 0$.

Linex preferences imply the optimal policy rule is asymmetric. An asymmetric response is arguably implausible, not least because many countries have a target zone for inflation rather than a point target (see Petursson, 2004, for details of the target zone in over 25 countries). Following Orphanides and Wieland (2000), a target zone implies a small response to inflation when this is clearly within the zone and stronger response when inflation is outside the zone. This type of response is obtained when $\beta_\pi > 1$. Interest rates are unresponsive to inflation in a zone around the desired level. Outside the zone, interest rates respond more strongly; the strength of the response is an increasing function of α_π . The response is symmetric if β_π is an even number (figure 2 illustrates this case). If β_π is an odd number, the response is asymmetric; the sign of the asymmetry is determined by α_π (there is stronger response when inflation is above rather than below the zone target if $\alpha_\pi > 0$). A higher value of β_π widens the zone.

This discussion has focussed on the response of interest rates to inflation. Similar considerations apply to the response to output. This response is convex if $\alpha_y > 0$ and concave if $\alpha_y < 0$. A convex response is consistent with arguments that policymakers are more averse to slumps than to booms (Cukierman and Gerlach, 2003). It has also been suggested that policymakers should only respond strongly to output when this is clearly some way from the equilibrium level (Meyer et al, 2001).

This suggests a zone-like response to output, which is obtained when $\beta_y > 1$. As with the response to inflation, this response is symmetric if β_y is even and asymmetric if β_y is odd.

4) Optimal monetary policy with a non-linear Phillips curve

In this section we revert to quadratic preferences but assume the Phillips curve is

$$(8) \quad \pi_t = \gamma y_t^{\beta_{pc}-1} \frac{e^{\alpha_{pc} y_t^{\beta_{pc}}} - 1}{\alpha_{pc}} + \theta E_t \pi_{t+1} + \varepsilon_t^s$$

The optimal monetary policy rule in this case is

$$(9) \quad i_t = i^* + \frac{\rho\lambda}{\mu} E_{t-1} y_t + \frac{\rho\gamma}{\mu} E_{t-1} g(y_t) E_{t-1} (\pi_t - \pi^*)$$

where $g(y; \beta_{pc}, \alpha_{pc}) = y^{\beta_{pc}-2} ((\beta_{pc} - 1) \frac{e^{\alpha_{pc} y^{\beta_{pc}}} - 1}{\alpha_{pc}} + \beta_{pc} y^{\beta_{pc}} e^{\alpha_{pc} y^{\beta_{pc}}})$ is proportional to the slope of the Phillips curve. The response of interest rates to inflation in (9) depends on the slope of the Phillips curve, and is thus a function of the output gap. The policy rule is a generalisation of the Taylor rule, which is obtained if the Phillips curve is linear, in which case $g(y_t) = 1$. The response of interest rates to inflation reflects the shape of the Phillips curve. The Phillips curve is convex if $\beta_{pc} = 1$ and $\alpha_{pc} > 0$; convexity was proposed by Laxton et al (1995) and is often assumed in recent models. The response of interest rates to inflation is a convex function of output in this case. The concave Phillips curve, suggested by Stiglitz (1997), is obtained where $\beta_{pc} = 1$ and $\alpha_{pc} < 0$; in this case the response to inflation is a concave function of output. Figure 3) illustrates the responses of interest rate to inflation in these cases. Finally, it has been suggested that the Phillips curve may be concave when the output gap is negative and convex when the output gap is positive (Dupasquier and Ricketts, 1998). The Phillips curve in (8) has this property when $\beta_{pc} > 1$. In this case, as figure 4) shows, the response to inflation is close to zero in a zone around the point where the output

gap is zero, but stronger when the output gap is larger in either direction. The Phillips curve is symmetric when β_{pc} is an even number; reflecting this, the response to symmetric. The Phillips curve is asymmetric when β_{pc} is odd; the response to inflation in this case is also an asymmetric function of output, with a stronger response when the output gap is positive if $\alpha_{pc} > 0$.

5) Optimal monetary policy with both non-quadratic preferences and a non-linear Phillips curve

Finally, we consider the case where preferences are non-quadratic and the Phillips curve is non-linear. Using (6) and (8), the optimal monetary rule in this case is

$$(10) \quad \hat{i}_t = i^* + \frac{\rho\lambda}{\mu} E_{t-1} f(y_t; \beta_y, \alpha_y) + \frac{\rho\gamma}{\mu} E_{t-1} g(y_t; \beta_{pc}, \alpha_{pc}) f(\pi_t - \pi^*; \beta_\pi, \alpha_\pi)$$

where the functions $f(\cdot)$ and $g(\cdot)$ are as defined above. In this policy rule, the second term reflects non-quadratic preferences over output while the third term reflects both non-quadratic preferences over inflation and a non-linear Phillips curve. This latter interaction suggests that the policy rule can take a number of forms beyond those suggested so far in the literature.

Figures 5) and 6) depict the relationship between $i_t - i^*$ and the output gap, for a given inflation gap, when there are zone-symmetric preferences over output and either a concave (figure 5) or a convex-concave (figure 6) Phillips curve. In both cases, there is a strong response to output when output is some way from equilibrium (the response is asymmetric in figure 5), reflecting the concavity of the Phillips curve). The most striking feature of these graphs is that there is a “perverse response”, whereby interest rates actually fall when output increases, for some values of the output gap. In figure 5), the perverse response occurs when the output gap is in a zone close to zero. In figure 6), the perverse response occurs when the output gap is

negative. These types of “perverse response” are analysed in more detail in Boinet and Martin (2006).

6) Conclusions

This paper has considered optimal monetary policy when non-quadratic preferences and a non-linear Phillips curve imply non-linear responses by policymakers. We propose a simply yet flexible framework that produces the type of non-linear responses derived in the literature as special cases. This framework suggests a richer set of non-linear responses than have been considered yet and thus may prompt further work in this area. The optimal policy rules suggested in this paper can be econometrically estimated. Ultimately, their value will be determined by empirical evidence.

References

Boinet, V, and C Martin, (2006), "The perverse Response of Interest Rates", unpublished paper, Brunel University

Cukierman, A, and S Gerlach, (2003), "The inflation bias revisited: theory and some international evidence," *Manchester School*, vol. 71(5), pages 541-565

Dupasquier, C and N Ricketts, (1998), "Non-linearities in the Output-Inflation Relationship", Bank of Canada Working Paper 98-14

Laxton, D, Meredith, G and Rose, D (1995), "Asymmetric Effects of Economic Activity on Inflation: Evidence and Policy Implications", *IMF Staff Papers*, Vol 42, No 2.

Meyer, L, E Swanson and V Wieland, (2001). "NAIRU Uncertainty and Nonlinear Policy Rules," *American Economic Review*, vol. 91(2), pages 226-231, May

Nobay, A. and D Peel, (2000). "Optimal monetary policy with a non-linear Phillips curve," *Economics Letters*, vol. 67(2), pages 159-164

Nobay A and D Peel, (2003). "Optimal Discretionary Monetary Policy in a Model of Asymmetric Central Bank Preferences," *Economic Journal*, vol. 113(489), pages 657-665

Orphanides, A, and V Wieland, (2000). "Inflation zone targeting," *European Economic Review*, vol. 44(7), pages 1351-1387

Petursson, T, (2004), "The effects of inflation targeting on macroeconomic performance", Economics Department, Central Bank Of Iceland

Ruge-Murcia, Francisco J, 2003. " Inflation Targeting under Asymmetric Preferences," *Journal of Money, Credit and Banking*, vol. 35(5), pages 763-85

Stiglitz, J, (1997), "Reflections on the Natural Rate Hypothesis", *Journal of Economic Perspectives*, 11, 3-10

Taylor, J. (1993). Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy*, 39, pp 195-214.

Figure 1) The response of interest rates with quadratic and Linex preferences

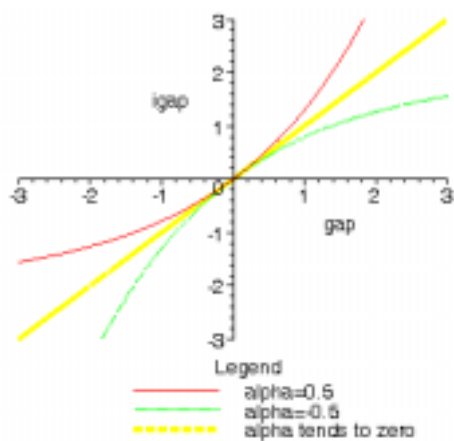


Figure 2) The response of interest rates with zone-symmetric preferences

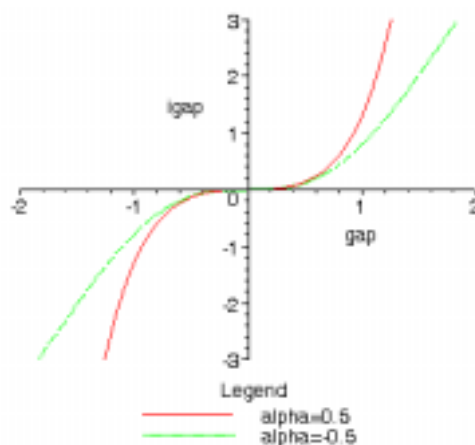


Figure 3) The response to inflation with linear, convex and concave Phillips curves

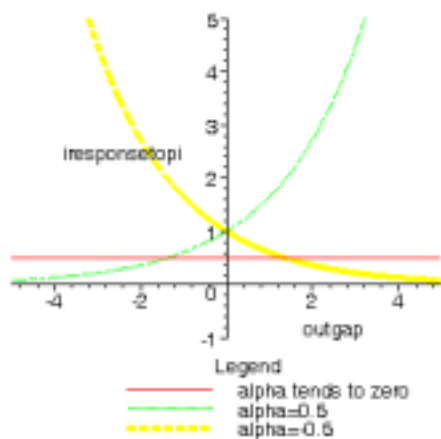


Figure 4) The response to inflation with a convex-concave Phillips curve

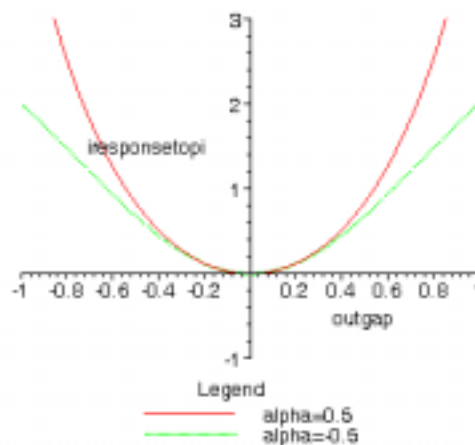


Figure 5) The response of interest rates with zone-symmetric preferences and a concave Phillips curve

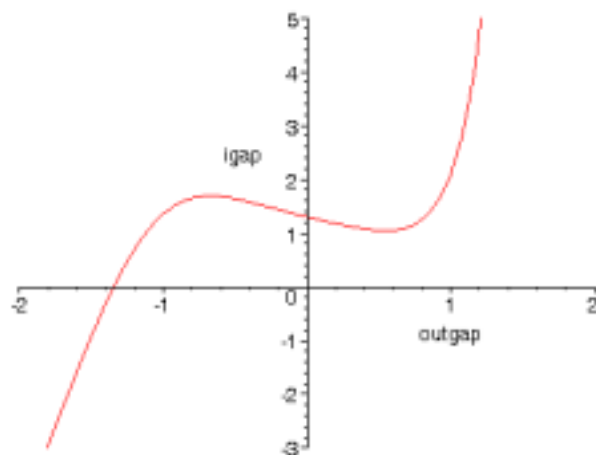


Figure 6) The response of interest rates with zone-symmetric preferences and a convex-concave Phillips curve

