A Deflection, Buckling and Stress Investigation into the Telescopic Cantilever Beam

A Thesis submitted for the degree of Doctor of Philosophy

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ABSTRACT

The telescoping cantilever beam structure is applied in many different engineering sectors to achieve weight/space optimisation for structural integrity. There has been limited theory and analysis in the public domain of the stresses and deflections involved when applying a load to such a structure. This thesis proposes (a) The *Tip Reaction Model*, which adapts classical mechanics to predict deflection of a two and a three section steel telescoping cantilever beam; (b) An equation to determine the Critical buckling loads for a given configuration of the two section steel telescoping cantilever beam assembly derived from first principles, in particular the energy methods; and finally (c) the derivation of a design optimization methodology, to tackle localised buckling induced by shear, torsion and a combination of both, in the individual, constituent, hollow rectangular beam sections of the telescopic assembly. Bending stress and shear stress is numerically calculated for the same structure whilst subjected to inline and offset loading. An FEA model of the structure is solved to verify the previous deflection, stress and buckling predictions made numerically. Finally an experimental setup is conducted where deflections and stresses are measured whilst a two section assembly is subjected to various loading and boundary conditions. The results between the predicted theory, FEA and experimental setup are compared and discussed. The overall conclusion is that there is good correlation between the three sets of data.

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LIST OF SYMBOLS

$\boldsymbol{T_1}$, $\boldsymbol{T_2}$, $\boldsymbol{T_3}$**=** Stress Tensors acting on each face perpendicular to coordinate axes x_1, x_2, x_3

$\boldsymbol{T_i}$ = Universal stress tensor

T*= S**tress tensors distributed over surface ***S

n = Unit outward normal to the plane

h = Perpendicular distance from origin to plane ACB of tetrahedron

V= volume

∂W_E = External virtual work

∂W_I = Internal virtual work

U = Strain energy

U^*= Complementary strain energy

σ = Direct stress

e= Direct strain

$\prod$ =Total potential energy of the body

i, *j* = tensor notation

P = Applied tip load

P_{cr}= Critical buckling load

λ = Load displacement

δ = Lateral or out-of-plane displacement

I = Second moment of area

L = Length

E = Young’s modulus of elasticity

x=*z* = Arbitrary length

z’ = Non dimensional length parameter

ΔU_T = Change in total potential energy

ΔU_B = Change in bending strain energy

ΔU_P = Change in potential of external force or the work done by the load P

$w(x)$= $Y(z)$= Assumed deflection functions

$M(x)$= $M(z)$ = Bending moment functions

q = Shear flow

T = Torque

τ = Shear stress

F_y= Vertical force

A= Area

$$\mu = \left[\left(\frac{d_o}{d_e}\right) - 1\right]$$

$$\psi = \left[\left(\frac{h_o}{h_e}\right) - 1\right]$$

$$\eta = \left[\left(\frac{b_o}{b_e}\right) - 1\right]$$

L_1= Length of fixed-end section

α=overlap ratio

a_1=Overlap length

L_2= Length of free-end section

ϕ=Length variation ratio

w_1= w=Self weight of fixed-end section

γ = Self weight ratio

w_2= Self weight of free-end section

H= d= Depth

B = b =Breadth

T = t =Thickness

y = Overall Deflection

y_0 = Deflection of single fixed-end section cantilever

I_1= Second moments of area of fixed-end section

I_2= Second moments of area of overlap section

I_3= Second moments of area of free-end section

I_z= Second moments of area of section at length z from datum

I_e =Second moments of area of section at apex

I_o =Second moments of area of section at base

β= Second moment area ratio

F^V = Virtual force

M^V =Virtual moment

$d_{e=}$ diameter of apex

d_z =diameter at arbitrary length z from datum

d_o=diameter at base

$\sigma_{0\ =}$Stress at d_o

b_e=breadth at apex

b_z=breadth at arbitrary length z from datum

$b_{o=}$ breadth at base

h_e =height at apex

h_z=height at arbitrary length z from datum

h_o=height at base

f = Shape factor

Q = Geometrical coefficient

W = Weight of section

K = Buckling coefficients

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