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1

Introduction: Cointegration, Economic Equilibrium and the Long Run

The econometrician or statistician might be viewed as a forensic scientist, trying to detect from the splatter (of blood), a line through space from which it may be determined, how and by whom a crime was committed. The tools available to calculate and describe this evidence are estimators and tests, and then – conditional on the model selected – identification of the cause or the perpetrator of the crime.

At the very core of econometrics lies measurement, the quality of measurement and the existence of the measure. When a measure is considered then there is the practical question of whether measurement is feasible or not. Conventional statistical measurement and inference considered the behaviour of processes that are associated with distributions that are generally viewed as being fixed across the sample. When economists started to apply statistical measurement to economic data then the notion that the data were identically and independently distributed (IID) had to be rejected. Regression was used to measure the heterogeneity by estimating a mean conditional on exogenous information while the assumption that the data are independently and identically distributed (IID), was used to give structure to the unknown error in the model. Essentially some form of least squares regression became the method generally applied to explain economic phenomena, but in the early literature it is hard to find reference to the notion of non-stationarity. One exception is the book written by Herman Wold with Lars Jureen on the subject of demand analysis, which does consider the behaviour of stationary economic time series. However, Wold and Jureen (1953) analyzed data for the inter-war years, a period when price series fell in relative terms and growth of output was relatively stagnant. Hence, any question of how demand models might be derived when time series are non-stationary was apart from some exceptions ignored. It is of interest to note that, in a study of the demand for food, James Tobin estimated both a logarithmic inverse demand curve and in an attempt to remove serial correlation the same relationship in differences. The latter

equation became the basis of the Rotterdam model developed by Theil (1965) and Barten (1969). In the early 1970s, Box and Jenkins wrote a book that became highly influential in the statistical analysis of time series data. Box and Jenkins set out a methodology for building time series models, that firstly considers the appropriate degree of differencing required to render a series stationary, and then discusses the type of alternative models autoregressive (AR) or moving average (MA), or ARMA that might be used to estimate univariate time series and then considered the method of estimation. Fama (1970) suggests that the observation that financial time series follow random walks is consistent with the idea that markets were efficient. The random walk model implies that financial time series are non-stationary and, following Box and Jenkins, need to be differenced to make them stationary. The difference in the log of the share price approximates a return and when the financial market is efficient then returns are not supposed to be predictable.

The structure of time series models pre-dates Box and Jenkins. Yule (1927) first estimated AR processes and in 1929 Kolmogorov considered the behaviour of sums of independent random variables (see the discussion in Wold and Jureen (1953)). In the regression context, Sargan (1964) applied an MA error structure to a dynamic model of UK wage inflation. The Sargan model became the basis of most of the UK wage equations used in the large macroeconomic models (Wallis et al. 1984). In demand analysis, approximation rather than non-stationarity was behind differencing and developments in economic theory related to the structure of demand equations was more interested in issues of aggregation as compared with the possible time series structure of the data (Deaton and Muellbauer 1980). To difference time series became common practice in modelling univariate time series and this approach was also applied in finance where it was common to consider returns of different assets rather than share prices. The market model relates the return on a share to the return on the market. There was now a discrepancy between the methods applied in statistics and finance to time series data and the approach predominantly used by economists.

However, the first oil shock precipitated a crisis in macroeconomic model building. Most of the world's large macroeconomic models were unable to resolve many of the problems that ensued from this shock. Forecasts and policy simulations that provide the governments' predictions of the future and a practical tool for understanding the impact of policy on the economy were unable to explain what had happened and what policies might remedy the situation (Wallis et al. 1984). The UK Treasury's inability to forecast the balance of payments position led to the ludicrous situation of a developed economy being forced to borrow from the IMF – a remedy that would not have been sought had reasonable estimates been available of the true payments position. The whole approach to the econometric modelling of economic time series was in doubt.

Econometric modelling was criticized on three grounds – the specification of the models used, their forecast accuracy and their existence. The model building approach adopted at the London School of Economics (LSE) built on the methodology developed by Sargan (1964). The Sargan approach attempted to combine the lessons of conventional time series modelling by applying the difference operator to the dependent variable with the practical requirement of the economist that the model could be solved back to reveal relationships from which the levels of the data might be forecast. The LSE approach implied that economic time series were dynamic and best modelled as regressions that included an appropriate description of the dynamic process underlying the data. The approach reinforced the proposition that a valid regression was required to satisfy the Gauss–Markov conditions (Patterson 2000) and that any regression models estimated ought to be well specified. This became what has been called the Hendry methodology and in the UK and Europe this approach has provided a potent mechanism to generate reasonable approximations to many aggregate economic time series. In particular, the articles by Davidson et al. (1978) and Hendry and Mizon (1978) expound a single equation modelling methodology for consumption and money. Davidson et al. (1978) emphasize that correct specification follows from estimating general autoregressive distributed lag (ADL) models, states that the dynamic model explains the short-run behaviour of the stationary form of the data in differences, that any levels variables explain the long run and that the long run is associated with conventional economic theory. Hendry and Richard (1982, 1983) elaborated on these ideas further by explaining what an adequate approximation of the data is and how systems models are sequentially reduced into valid sub-models. The final important development that came out of this approach was the categorization of exogeneity into strict, weak, strong and super. As far as inference and the estimation of single equation regression models is concerned, weak exogeneity justified the use of contemporaneous variables such as income in consumption and money equations.

The LSE approach provided model builders with a methodology for estimating single equations by regression. Poor forecast performance was viewed as a sign of a poorly performing model and was viewed then as, correctable by valid model selection. In the US the failure of econometric model building was viewed as a failure of economic theory. Forecasts based on large macro models broke down because the postwar Keynesian consensus had broken down and the basis of failure was neoclassical monetary neutrality combined with hyper-rational agent behaviour. The Lucas critique suggested that the conventional macro models were unable to capture changes in agent responses to government policy, the deep parameters of the economic system. Models based on classical assumptions purported to show that monetary policy was not effective, while the notion that macroeconomic time series followed random walks was embedded in the article by Robert Hall (1978),

which showed that consumption followed a random walk.¹ In 1978 Sargent derived dynamic models based on rational expectations, which impose theoretical propositions about the underlying behaviour of agents on the short-run behaviour of the data. However, Sargent explicitly requires that the series are stationary for the solution to exist.² The literature derived from the neo-classical rational expectations solution to macro modelling has adopted two approaches to the problem of model specification. The first is to build dynamic models with data that are differenced and then to solve the expectations problem or to estimate the models using an unrestricted vector autoregressive (VAR) model. The former approach often uses Generalized Method of Moments to estimate the Euler equation via the errors in variables approach best explained by Wickens (1982). While Sims (1980) proposed the VAR methodology, first differencing the data to render it stationary and then estimating economic behaviour by systems of autoregressive models, suggesting that all the variables modelled are endogenous. Policy invariance is tested by looking at impulse responses and causal structure, rather than by deriving structural models.³

The LSE methodology assumed that long-run relationships existed and that conventional inference was valid irrespective of whether series are stationary or not. The rational expectations literature that transformed the data into differences risked the possibility that there may be over-differencing. Both approaches understood that time series modelling required dynamic models, the former assuming that conventional economic theory can be detected in terms of long-run relationships from the data, the latter approach that it cannot be. The idea that a correlation is not valid is best explained in Yule (1926) who considers a number of correlations that can only be viewed as nonsense. In particular, Yule found that the fall in Church of England marriages was positively correlated with the fall in the death rate between 1861 and 1913. This idea of nonsense correlation along with many of the problems associated with econometric modelling, including the appropriate measurement of expectations, was discussed by Keynes (1939).⁴ Keynes emphasizes the role of economics in statistical model building and explains that economists need to be looking at true causes as compared with correlations that derive from the dependence of variables on an underlying primary cause. In 1974 Granger and Newbold presented simulation results for nonsense regressions – relationships that are observed to be correlated, but cannot be. Granger and Newbold (1986) describe how univariate and multivariate economic time series ought to be modelled. Simulations presented in Granger and Newbold (1974, 1986) show that it is possible to run regressions on unrelated data and find significant relationships where there should be none. The 1974 article suggests that the discovery of an R^2 that exceeds the Durbin Watson (DW)

statistic ought to be indicative of the problem as then the DW statistic has to be less than one and as a result the model must suffer from significant serial correlation. The article appears to emphasize that badly misspecified models should be viewed with deep suspicion, because they may reveal relationships that are spurious. It is apparent that the econometrics profession had adopted this research agenda by building on one side of the Atlantic ADL models and on the other VARs in differences. However, the results associated with Granger and Newbold (1986) were somewhat subtler, in that when the data were generated via random walks with MA errors, spurious regressions could be observed with DW statistics in excess of one. Hence, the question of what determines a true regression relationship is further complicated by the existence of more complex explanations of individual time series.

This book considers methods by which it can be determined whether time series are stationary or non-stationary in differences, difference stationary or trend stationary or rendered stationary by subtracting from the non-stationary series some part of another series. The latter case is the cointegration case, which occurs when two or more series combine to produce stationary variables and a conventional regression equation between these variables has economic meaning in a long-run sense. This notion of cointegration is then developed in the context of multiple time series. A conclusion for the VAR methodology in differences is that when long-run behaviour exists, in terms of combinations of stationary variables in levels, the VAR is fundamentally misspecified. However, the generalization of the ADL to a system, can under the restrictions associated with cointegration provide a short-run explanation of the data, with long-run behaviour explained by restrictions on the levels in each equation.

In chapter 2, the characteristics of economic and financial time series are considered. The properties of the variance, covariance and autocovariance of stationary and non-stationarity time series are defined, in addition to the alternative definitions of stationarity. Time series models are defined for both their stationary and non-stationary representations. The statistical properties of the error are defined in terms of white noise residuals and the Wold decomposition. Non-invertibility, random walks and alternative notions of persistence are dealt with, as, before time series are modelled, they ought to be stationary. The proposition that a series is stationary needs to be tested and the data transformed to take account of non-stationarity or persistence. Having decided on the stationary form of the data, a time series model can be identified and estimated. Much of the existing literature handles persistence by first or second differencing data. The former is often appropriate for real variables such as output or employment, while second differences might often be required for nominal variables in economic models, GDP, sales and retail

prices or in finance, share prices, stock indices and dividends. Otherwise, fractional differencing might be required, with the resulting models being special cases of the autoregressive fractionally integrated moving average (ARFIMA) model.

In chapter 3, modelling non-stationary time series is handled in a single equation framework. When more than one series is analyzed, differencing might be more than is required. This occurs when series in combination are stationary (cointegration). Non-integer differencing is often required, in the case of series such as interest rates. Single equation models, which incorporate some different right-hand side variables in levels, are classified as error correction models. When the original data or their logarithms are non-stationary, cointegration may be observed when linear combinations of two or more levels variables are stationary. Then cointegration is valid when the relationships are bivariate or there is one cointegrating relationships in a system. When the regressors are exogenous, in a univariate time series context, the regressions can be viewed as ARMAX or ARMA models with exogenous variables.

In chapter 4, the multivariate time series model is developed from a stationary representation of the data that is known always to exist, the vector or VMA model in differences. The book explains the nature of multivariate time series under stationarity and then extends this to the cointegration case. We then explain how the VMA in differences can be transformed into an error correction model using the Granger representation theorem and the Smith–McMillan form developed by Yoo (1986). Cointegration is then described in terms of error correcting VARs or VECMs. A procedure for determining the existence of the VAR is described along with the Johansen approach to estimation and inference. The book explains the asymptotic theory that lies behind the Johansen test statistic. An application is developed based on the models of the UK effective exchange rate estimated by Hunter (1992), Johansen and Juselius (1992) and Hunter and Simpson (1995). Finally a number of alternative representations are developed and the question of multi-cointegration discussed.

In chapter 5, the exogeneity of variables in the VAR and the identification of long-run parameters are considered. Exogeneity is discussed in terms of the restrictions required for weak, strict and cointegrating exogeneity in the long run. Then alternative forms of exogeneity and causality are considered and the results associated with Hunter (1992) and Hunter and Simpson (1995) are presented. Identification is discussed in terms of conventional systems with $I(0)$ series, this approach is extended to show when the parameters can be identified via imposing the restrictions and solving out for the long-run parameters and their loadings. Identification is then discussed in terms of the results derived by Bauwens and Hunter (2000), Johansen (1995) and Boswijk

(1996). All three approaches are applied to the model estimated by Hunter (1992).

In chapter 6, more advanced topics are considered in some detail. Firstly, the $I(2)$ case, firstly using an extension to the Sargan–Bézout approach adopted by Hunter (1994), then in terms of the representation and test due to Johansen (1992) and Paruolo (1996), and finally the test procedures due to Johansen and Paruolo are applied to the exchange rate data in Hunter (1992). Fractional cointegration is briefly discussed in terms of the estimator due to Robinson and Marinucci (1998) and the test due to Robinson and Yajima (2002). Secondly, forecasting of non-stationary and stationary components is considered. The results produced by Lin and Tsay (1996) and Clements and Hendry (1995, 1998) are presented with a graphical analysis of the performance of the simulations developed by Lin and Tsay (1996). Finally, models with short-run structural equations are discussed – in particular, models with unit roots in the endogenous and exogenous processes. It is shown how to estimate models where the unit roots relate to the endogenous variables and then to the case associated with the exogenous variables.

In chapter 7, the reader is guided to further issues in the literature. Firstly, a plethora of articles on testing stationarity and non-stationarity has developed; the reader is directed where appropriate to the book by Patterson (2005). A condensed discussion of structural breaks is provided along with direction to appropriate references.

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