Study of CP-violating asymmetries in $B^0 \to \pi^+\pi^-$, $K^+\pi^-$ decays


(The BABAR Collaboration)

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We present a measurement of the time-dependent $CP$-violating asymmetries in neutral $B$ decays to the $\pi^+\pi^-$ $CP$ eigenstate, and an updated measurement of the charge asymmetry in $B^0 \to K^+\pi^-$ decays. In a sample of 33 million $T(4S) \to B\bar{B}$ decays collected with the $\bar{B}A\bar{B}$ detector at the SLAC PEP-II asymmetric $B$ Factory, we find $65_{-11}^{+12} \pi^+\pi^-$ and $217 \pm 18 K^+\pi^-$ candidates and measure the asymmetry parameters $S_{\pi\pi} = 0.03_{-0.06}^{+0.11}, C_{\pi\pi} = -0.25_{-0.47}^{+0.49} \pm 0.14$, and $A_{K\pi} = -0.07 \pm 0.08 \pm 0.02$, where the first error is statistical and the second is systematic.

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In the Standard Model, all $CP$-violating effects arise from a single complex phase in the three-generation Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix $\lambda$. One of the central questions in particle physics is whether this mechanism is sufficient to explain the pattern of $CP$ violation observed in nature. Recent measurements of the parameter $\sin 2\beta$ by the BABAR $\mathbb{B}$ and BELLE $\mathbb{B}$ Collaborations establish that $CP$ symmetry is violated in the neutral $B$-meson system. In addition to measuring $\sin 2\beta$ more precisely, one of the primary goals of the $B$-Factory experiments in the future will be to measure the remaining angles ($\alpha$ and $\gamma$) and sides of the Unitarity Triangle in order to further test whether the Standard Model description of $CP$ violation is correct.

The study of $B$ decays to charmless hadronic two-body final states will play an increasingly important role in our understanding of $CP$ violation. In the Standard Model, the time-dependent $CP$-violating asymmetry in the reaction $B^0 \to \pi^+ \pi^-$ is related to the angle $\alpha$. In addition, observation of a significant rate asymmetry between $B^0 \to K^+ \pi^-$ and $\bar{B}^0 \to K^- \pi^+$ decays would be evidence for direct $CP$ violation, and ratios of branching fractions for various $\pi\pi$ and $K\pi$ decay modes are sensitive to the angle $\gamma$. Finally, branching fraction measurements provide critical tests of theoretical models that are needed to extract reliable information on $CP$ violation from the experimental observables.

The BABAR Collaboration recently reported measurements of branching fractions and charge asymmetries for several charmless two-body $B$ decays using a data set of 23 million $BB$ pairs $\mathbb{B}$. In this paper, using a data sample of approximately 33 million $BB$ pairs, we report a measurement of time-dependent $CP$-violating asymmetries in neutral $B$ decays to the $\pi^+\pi^-$ $CP$ eigenstate and an updated measurement of the charge asymmetry in $B^0 \to K^+\pi^-$ decays.

The time-dependent $CP$-violating asymmetry in the decay $B^0 \to \pi^+\pi^-$ arises from interference between mixing and decay amplitudes, and interference between the tree and penguin decay amplitudes. A $B^0\bar{B}^0$ pair produced in $T(4S)$ decay evolves in time in a coherent $P$-wave state until one of the two mesons decays. We reconstruct a sample of $B$ mesons ($B_{hh}$) decaying to the $h^+h'^-$ final state, where $h$ is a pion or kaon, and examine the remaining charged particles in each event to “tag” the flavor of the other $B$ meson ($B_{tag}$). The decay rate distribution $f_+(f_-)$ when $h^+h'^- = \pi^+\pi^-$ and $B_{tag} = B^0(\bar{B}^0)$ is given by $\mathbb{B}$

$$f_+ \pm = \frac{e^{-|\Delta t/\tau}|}{4\tau} \left[ 1 \pm S_{\pi\pi} \sin(\Delta m_d \Delta t) \mp C_{\pi\pi} \cos(\Delta m_d \Delta t) \right],$$

where $\tau$ is the $B^0$ lifetime, $\Delta m_d$ is the $B^0\bar{B}^0$ mixing frequency, and $\Delta t = t_{hh} - t_{tag}$ is the time between the $B_{hh}$ and $B_{tag}$ decays. The $CP$-violating parameters $S_{\pi\pi}$ and $C_{\pi\pi}$ are defined as

$$S_{\pi\pi} = \frac{2\Im \lambda}{1 + |\lambda|^2} \quad \text{and} \quad C_{\pi\pi} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}. \quad (2)$$

If the decay proceeds purely through the tree process $b \to uW^-$, the complex parameter $\lambda$ is directly related to CKM matrix elements,

$$\lambda(B \to \pi^+\pi^-) = \left( \frac{\bar{V}_{tb} V_{td}}{\bar{V}_{ub} V_{ud}} \right) \left( \frac{\bar{V}_{td} V_{ub}}{\bar{V}_{tb} V_{ud}} \right), \quad (3)$$

where we are assuming equal widths ($\Delta \Gamma_B = 0$) for the heavy and light mass eigenstates. Thus, at tree level in the Standard Model, $|\lambda| = 1$ and $\Im \lambda = \sin 2\alpha$, where $\alpha \equiv \arg [-V_{td} V_{ub}^*/V_{ub} V_{td}^*]$

Recent theoretical estimates indicate that the contribution of the gluonic penguin amplitude can be significant $\mathbb{B}$. The process $b \to dg$ carries the weak phase $\arg(V_{td} V_{ub}^*)$, which can modify both the magnitude and phase of $\lambda$. Thus, in general, $|\lambda| \neq 1$ and $\Im \lambda = |\lambda| \sin 2\alpha_{\text{eff}}$, where $\alpha_{\text{eff}}$ depends on the magnitudes and strong phases of the tree and penguin amplitudes. Several approaches have been proposed to obtain information on $\alpha$ in the presence of penguins $\mathbb{B}$.

In this analysis, we extract signal and background yields for $\pi^+\pi^-$, $K^+\pi^-$, and $K^+K^-$ decays $\mathbb{B}$, and the amplitudes of the $\pi\pi$ sine ($S_{\pi\pi}$) and cosine ($C_{\pi\pi}$) oscillation terms simultaneously from an unbinned maximum likelihood fit. We parameterize the $K\pi$ component in terms of the total yield and the $CP$-violating charge asymmetry

$$A_{K\pi} \equiv \frac{N_{K^-\pi^+} - N_{K^+\pi^-}}{N_{K^-\pi^+} + N_{K^+\pi^-}}. \quad (4)$$

The data sample used in this analysis consists of 33.7 fb$^{-1}$ collected with the BABAR detector at the SLAC PEP-II storage ring between October 1999 and June 2001. The PEP-II facility operates nominally at the $T(4S)$ resonance, providing collisions of 9.0 GeV electrons on 3.1 GeV positrons. The data set includes 30.4 fb$^{-1}$ collected in this configuration (on-resonance) and 3.3 fb$^{-1}$ collected below the $BB$ threshold (off-resonance) that are used for continuum background studies.

A detailed description of the BABAR detector is presented in Ref. $\mathbb{B}$. Charged particle (track) momenta are measured in a tracking system consisting of a 5-layer double-sided silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH) filled with a gas mixture of helium and isobutane. The SVT and DCH operate within a 1.5 T superconducting solenoidal magnet. The typical decay vertex resolution for fully reconstructed $B$ decays is approximately 65 $\mu$m along the center-of-mass (CM) boost direction. Photons are detected in an electromagnetic calorimeter (EMC) consisting of 6580 CsI(Tl) crystals arranged in barrel and forward endcap subdetectors.
The flux return for the solenoid is composed of multiple layers of iron and resistive plate chambers for the identification of muons and long-lived neutral hadrons.

Tracks from the $B_{hh}$ decay are identified as pions or kaons by the Cherenkov angle $\theta_c$ measured with a detector of internally reflected Cherenkov light (DIRC). The typical separation between pions and kaons varies from 8$\sigma$ at 2 GeV/c to 2.5$\sigma$ at 4 GeV/c, where $\sigma$ is the average resolution on $\theta_c$. Lower momentum kaons used in $B$ flavor tagging are identified with a combination of $B$ (for momenta down to 0.7 GeV/c) and measurements of ionization energy loss $dE/dx$ in the DCH and SVT.

Hadronic events are selected based on track multiplicity and event topology. We require at least three tracks in the laboratory polar angle region $0.41 < \theta_{lab} < 2.54$ satisfying the following requirements: transverse momentum greater than 100 MeV/c, at least 12 DCH hits, and originating from the interaction point within 10 cm in $z$ and 1.5 cm in $r-\phi$. Residual two-prong events from the reaction $e^+e^- \rightarrow l^+l^-(l=e,\mu,\tau)$ are suppressed by requiring the ratio of Fox-Wolfram moments $H_2/H_0$ to be less than 0.95 and the sphericity of the event to be greater than 0.01.

Candidate $B_{hh}$ decays are reconstructed from pairs of oppositely-charged tracks forming a good quality vertex, where the $B_{hh}$ four-vector is calculated assuming the pion mass for both tracks. We require each track to have $p_T > 0.1$ GeV/c to be greater than 0.95 and the sphericity of the event to be greater than 0.01.

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We define a beam-energy substituted mass $m_{ES} = \sqrt{E_0^2 - \mathbf{p}_F^2}$. The candidate energy is defined as $E_0 = (s/2 + \mathbf{p}_i \cdot \mathbf{p}_B)/E_i$, where $\sqrt{s}$ and $E_i$ are the total energies of the $e^+e^-$ system in the CM and laboratory frames, respectively, and $\mathbf{p}_i$ and $\mathbf{p}_B$ are the momentum vectors in the laboratory frame of the $e^+e^-$ system and the $B_{hh}$ candidate, respectively. Signal events are Gaussian distributed in $m_{ES}$ with a mean near the $B$ mass and a resolution of 2.6 MeV/c², dominated by the beam energy spread. The background shape is parameterized by a threshold function with a fixed endpoint given by the average beam energy.

We define a second kinematic variable $\Delta E$ as the difference between the energy of the $B_{hh}$ candidate in the CM frame and $\sqrt{s}/2$. The $\Delta E$ distribution is peaked near zero for $\pi^+\pi^-$ decays. For decays with one (two) kaons, the distribution is shifted relative to $\pi\pi$ on average by $-45$ MeV ($-91$ MeV), respectively, where the exact separation depends on the laboratory momentum of the kaon(s). The resolution on $\Delta E$ for signal decays is approximately 26 MeV. The background is parameterized by a quadratic function.

Candidate $h^+h^-$ pairs selected in the region $5.2 < m_{ES} < 5.3$ GeV/c² and $|\Delta E| < 0.15$ GeV are used to extract yields and $CP$-violating asymmetries with an unbinned maximum likelihood fit. The total number of events in the fit region satisfying all of the above criteria is 9741. A sideband region, defined as $5.20 < m_{ES} < 5.26$ GeV/c² and $|\Delta E| < 0.42$ GeV, is used to extract various background parameters.

The analysis method combines the techniques used to measure charmless two-body branching fractions and $\sin2\beta$. The primary issues in this analysis are determination of the $B_{tag}$ flavor, measurement of the distance $\Delta s$ between the $B_{hh}$ and $B_{tag}$ decay vertices, discrimination of signal from background, identification of pions and kaons, and extraction of yields and $CP$ asymmetries.

To determine the flavor of the $B_{tag}$ meson we use the same $B$-tagging algorithm used in the $\sin2\beta$ and $B^0-\bar{B}^0$ mixing analyses. The algorithm relies on the correlation between the flavor of the $b$ quark and the charge of the remaining tracks in the event after removal of the $B_{hh}$ candidate. We define five mutually exclusive tagging categories: Lepton, Kaon, NT1, NT2, and Untagged. Lepton tags rely on primary electrons and muons from semileptonic $B$ decays, while Kaon tags exploit the correlation in the process $b \rightarrow c \rightarrow s$ between the net kaon charge and the charge of the $b$ quark. The NT1 and NT2 categories are derived from a neural network that is sensitive to charge correlations between the parent $B$ and unidentified leptons and kaons, soft pions, or the charge and momentum of the track with the highest CM momentum. The addition of Untagged events improves the signal yield estimates and provides a larger sample for determining background shape parameters directly in the maximum likelihood fit.

The quality of tagging is expressed in terms of the effective efficiency $Q = \sum \epsilon_i D_i^2$, where $\epsilon_i$ is the fraction of events tagged in category $i$ and the dilution $D_i = 1 - 2w_i$ is related to the mistag fraction $w_i$. The statistical errors on $S_{\pi\pi}$ and $C_{\pi\pi}$ are proportional to $1/\sqrt{Q}$. Table summarizes the tagging performance in a data sample $B_{flav}$ of fully reconstructed neutral $B$ decays into $D^{(*)}-h^+$ ($h^+ = \pi^+, \rho^+, \phi^+$) and $J/\psi K^{*0}$ ($K^{*0} \rightarrow K^+\pi^-$) flavor eigenstates. We use the same tagging efficiencies and dilutions for signal $\pi\pi$, $K\pi$, and $KK$ decays. Separate background tagging efficiencies for each species are obtained from a fit to the $h^+h^-$ on-resonance sideband data and reported in Table.

The time difference $\Delta t$ is obtained from the measured distance between the $z$ position of the $B_{hh}$ and $B_{tag}$ decay vertices and the known boost of the $e^+e^-$ system.
TABLE I: Tagging efficiency $\epsilon$, average dilution $D = 1/2(D_{\mu^0} + D_{\mu^0})$, dilution difference $\Delta D = D_{\mu^0} - D_{\mu^0}$, and effective tagging efficiency $Q$ for signal events in each tagging category.

<table>
<thead>
<tr>
<th>Category</th>
<th>$\epsilon$ (%)</th>
<th>$D$ (%)</th>
<th>$\Delta D$ (%)</th>
<th>$Q$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>11.0 ± 0.3</td>
<td>82.3 ± 2.7</td>
<td>-2.1 ± 4.5</td>
<td>7.5 ± 0.5</td>
</tr>
<tr>
<td>Kaon</td>
<td>35.8 ± 0.5</td>
<td>64.8 ± 2.0</td>
<td>3.5 ± 3.1</td>
<td>15.0 ± 1.0</td>
</tr>
<tr>
<td>NT1</td>
<td>8.0 ± 0.3</td>
<td>55.6 ± 4.2</td>
<td>-12.1 ± 6.7</td>
<td>2.5 ± 0.4</td>
</tr>
<tr>
<td>NT2</td>
<td>13.9 ± 0.4</td>
<td>30.2 ± 3.8</td>
<td>9.0 ± 5.7</td>
<td>1.3 ± 0.3</td>
</tr>
<tr>
<td>Untagged</td>
<td>31.3 ± 0.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total $Q$</td>
<td>26.3 ± 1.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

TABLE II: Tagging efficiencies (%) for background events in each category.

<table>
<thead>
<tr>
<th>Category</th>
<th>$\epsilon(\pi\pi)$</th>
<th>$\epsilon(K\pi)$</th>
<th>$\epsilon(KK)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>1.0 ± 0.1</td>
<td>1.0 ± 0.1</td>
<td>1.5 ± 0.2</td>
</tr>
<tr>
<td>Kaon</td>
<td>26.0 ± 0.4</td>
<td>33.1 ± 0.6</td>
<td>23.5 ± 0.7</td>
</tr>
<tr>
<td>NT1</td>
<td>6.6 ± 0.2</td>
<td>5.4 ± 0.3</td>
<td>6.9 ± 0.4</td>
</tr>
<tr>
<td>NT2</td>
<td>17.6 ± 0.4</td>
<td>15.3 ± 0.5</td>
<td>19.7 ± 0.6</td>
</tr>
<tr>
<td>Untagged</td>
<td>48.9 ± 0.7</td>
<td>45.2 ± 0.6</td>
<td>48.3 ± 0.8</td>
</tr>
</tbody>
</table>

The $z$ position of the $B_{tag}$ vertex is determined with an iterative procedure that removes tracks with a large contribution to the total $\chi^2$ [9, 11]. An additional constraint is constructed from the three-momentum and vertex position of the $B_{hh}$ candidate, and the average $e^+e^-$ interaction point and boost. The typical $\Delta z$ resolution is 180 $\mu$m. We require $|\Delta t| < 17$ ps and $0.3 < \sigma_{\Delta t} < 3.0$ ps, where $\sigma_{\Delta t}$ is the error from the vertex fit. The resolution function for signal candidates is a sum of three Gaussians, identical to the one described in Ref. [9], with parameters determined from a fit to the $B_{tag}$ sample (including events in all five tagging categories). The background resolution function is parameterized as the sum of three Gaussians, with the parameters determined from a fit to the $h^+h^-$ on-resonance sideband data.

The data sample used in the fit contains 97% background, mostly due to random combinations of tracks produced in $e^+e^- \rightarrow q\bar{q}$ events. Discrimination of signal from background in the maximum likelihood fit is enhanced by the use of a Fisher discriminant $F$ [9]. The discriminating variables are constructed from the scalar sum of the CM momenta of all tracks and photons (excluding tracks from the $B_{hh}$ candidate) entering nine two-sided 10-degree concentric cones centered on the thrust axis of the $B_{hh}$ candidate. The distribution of $F$ for signal events is parameterized as a single Gaussian, with parameters determined from Monte Carlo simulated decays and validated with $B^- \rightarrow D^0\pi^-$ decays reconstructed in data. The background shape is parameterized as the sum of two Gaussians, with parameters determined directly in the maximum likelihood fit.

Identification of $h^+h^-$ tracks as pions or kaons is accomplished with the Cherenkov angle measurement from the DIRC. We construct Gaussian probability density functions (PDFs) from the difference between measured and expected values of $\theta_c$ for the pion or kaon hypothesis, normalized by the resolution. The DIRC performance is parameterized using a sample of $D^{*+} \rightarrow D^0\pi^+$, $D^0 \rightarrow K^-\pi^+$ decays reconstructed in data. Within the statistical precision of the control sample (approximately $10^5$ events), we find similar response for positively and negatively charged tracks and use a single parameterization for both.

We use an unimmed extended maximum likelihood fit to extract yields and CP parameters from the $B_{hh}$ sample. The likelihood for candidate $j$ tagged in category $c$ is obtained by summing the product of event yield $n_i$, tagging efficiency $\epsilon_{i,c}$, and probability $P_{i,c}$ over the eight possible signal and background hypotheses $i$ (referring to $\pi\pi$, $K^+\pi^-$, $K^-\pi^+$, and $KK$ decays),

$$L_c = \exp \left( -\sum_i n_i \epsilon_{i,c} \right) \prod_j \sum_i n_i \epsilon_{i,c} \mathcal{P}_{i,c}(\vec{x}_j|\vec{a}_j).$$

For the $K^+\pi^\mp$ hypotheses, the yield is parameterized as $n_i = N_{K\pi} (1 \pm A_{K\pi}) / 2$, where $N_{K\pi} = N_{K^+\pi^-} + N_{K^-\pi^+}$. We fix the tagging efficiencies $\epsilon_i$ to the values in Tables I and II. The probabilities $P_{i,c}$ are evaluated as the product of PDFs for each of the independent variables $\vec{x}_j = \{m_{ES}, \Delta E, \mathcal{F}, \theta_c^+, \theta_c^-, \Delta t\}$, where $\theta_c^+$ and $\theta_c^-$ are the Cherenkov angles for the positively and negatively charged tracks. The total likelihood $L$ is the product of likelihoods for each tagging category and the free parameters are determined by minimizing the quantity $\ln L$.

The $\Delta t$ PDF for signal $\pi^+\pi^-$ decays is given by Eq. 9, modified to include the dilution and dilution difference for each tagging category, and convolved with the signal resolution function. The $\Delta t$ PDF for signal $K\pi$ events takes into account $B^0\rightarrow\bar{B}^0$ mixing, depending on the charge of the kaon and the flavor of $B_{tag}$. We parameterize $B^0 \rightarrow K^+K^-$ decays as an exponential convolved with the resolution function.

There are 18 free parameters in the fit. In addition to the CP-violating parameters $S_{\pi\pi}$, $C_{\pi\pi}$, and $A_{K\pi}$, the fit determines signal and background yields (six parameters), the background $K\pi$ charge asymmetry, and eight parameters describing the background shapes in $m_{ES}$, $\Delta E$, and $\mathcal{F}$. We fix $\tau$ and $\Delta m_d$ to the world-average values [17].

In a sample of 33 million $B\bar{B}$ pairs we find $65^{+12}_{-11} \pi\pi$, $217 \pm 18 \ K\pi$, and $4.3^{+0.3}_{-0.4} K\bar{K}$ events. These yields are consistent with the branching fractions reported in
TABLE III: Central values and 90% C.L. intervals for $S_{\pi\pi}$, $C_{\pi\pi}$, and $A_{K\pi}$ from the maximum likelihood fit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Central Value</th>
<th>90% C.L. Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\pi\pi}$</td>
<td>$0.03^{+0.53}_{-0.36}$ ± 0.11</td>
<td>[-0.89, +0.85]</td>
</tr>
<tr>
<td>$C_{\pi\pi}$</td>
<td>$-0.25^{+0.42}_{-0.14}$ ± 0.14</td>
<td>[-1.0, +0.47]</td>
</tr>
<tr>
<td>$A_{K\pi}$</td>
<td>$-0.07$ ± 0.08 ± 0.02</td>
<td>[-0.21, +0.07]</td>
</tr>
</tbody>
</table>

Ref. [1], as well as measurements from other experiments [18, 19]. The results for $CP$-violating asymmetries are summarized in Table III. Statistical errors correspond to unit change in $\chi^2 \equiv -2 \ln(\mathcal{L})$. For each parameter, we also calculate the 90\% confidence level (C.L.) interval corresponding to a change in $\chi^2$ of 2.69, and taking into account the systematic error. The correlation between $S_{\pi\pi}$ and $C_{\pi\pi}$ is $-21\%$, while $A_{K\pi}$ is uncorrelated with either $S_{\pi\pi}$ or $C_{\pi\pi}$.

Figure 1 shows distributions of $m_{ES}$ and $\Delta E$ for events enhanced in signal decays based on likelihood ratios. We define $R_{sig} = \sum_i n_i P_i$ and $R_k = n_k P_k / \sum_i n_i P_i$, where $\sum_i (\sum_j)$ indicates a sum over signal (all) hypotheses, and $P_k$ indicates the probability for signal hypothesis $k$. The probabilities include the PDFs for $\theta_c$, $F$, and $m_{ES}(\Delta E)$ when plotting $\Delta E (m_{ES})$. The selection is defined by optimizing the signal significance with respect to $R_{sig}$ and $R_k$. The solid curve in each plot represents the fit projection after correcting for the efficiency of the additional selection (approximately 55\% for $\pi\pi$ and 85\% for $K\pi$).

Figure 2 shows the $\Delta t$ distributions and the asymmetry $A_{\pi\pi} (\Delta t) = (N_B^0 (\Delta t) - N_{\bar{B}\pi} (\Delta t)) / (N_B^0 (\Delta t) + N_{\bar{B}\pi} (\Delta t))$ for tagged events enhanced in signal $\pi\pi$ decays. The selection procedure is the same as Fig. 1, with the likelihoods defined including the PDFs for $\theta_c$, $F$, $m_{ES}$, and $\Delta E$. Approximately 24 $\pi\pi$, 22 $q\bar{q}$, and 5 $K\pi$ events satisfy the selection.

Systematic uncertainties on $S_{\pi\pi}$, $C_{\pi\pi}$, and $A_{K\pi}$ arise primarily from imperfect knowledge of the PDF shapes and uncertainties on tagging efficiencies, dilutions, $\tau$, and $\Delta m_d$. The total systematic error is calculated as the sum in quadrature of the individual uncertainties. The error on $A_{K\pi}$ is dominated by uncertainty in the mean of the $\Delta E$ PDF (0.01) and possible charge bias in track and $\theta_c$ reconstruction (0.01) [20]. Errors on $S_{\pi\pi}$ and $C_{\pi\pi}$ are dominated by the parameterization of $\Delta t$ resolution for signal and background ($\approx 0.07$ for $S_{\pi\pi}$, $\approx 0.03$ for $C_{\pi\pi}$), tagging (0.05), and, for $C_{\pi\pi}$ only, the mean of the $\Delta E$ PDF (0.1).

Extensive studies were performed to validate the fit technique. A large ensemble of Monte Carlo pseudo-experiments was generated from the nominal PDFs with the statistics observed in the full data set. Parameter errors and the maximum value of the likelihood obtained in the data fit are all consistent with expectations based on these pseudo-experiments, and all free parameters are unbiased. We have checked that consistent results are obtained when separating events by $B_{tag}$ flavor. As a
validation of the $\Delta t$ parameterization in data, we fit the full data set to simultaneously extract yields, background parameters, $\tau$, $\Delta m_d$, $S_{\pi\pi}$, and $C_{\pi\pi}$. We find $\tau = (1.52 \pm 0.12)$ ps and $\Delta m_d = (0.54 \pm 0.09) \text{ps}^{-1}$, and all other parameters are consistent with the nominal fit.

In summary, we have presented a measurement of time-dependent $CP$-violating asymmetries in $B^0 \rightarrow \pi^+\pi^-$ decays and an updated measurement of the charge asymmetry $A_{K\pi}$. The latter is consistent with our previous result reported in Ref. [4], as well as results from other experiments [21, 22]. We observe no evidence for direct $CP$ violation in the $K\pi$ mode and determine a 90% C.L. interval excluding a significant part of the allowed region. Although the current measurements of $S_{\pi\pi}$ and $C_{\pi\pi}$ do not significantly constrain the Unitarity Triangle, with the addition of more data and further improvements in detector performance and analysis techniques, future results will yield important information about $CP$ violation in the $B$-meson system.

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† Also with Università della Basilicata, Potenza, Italy
[10] When only one charge mode is given, conjugate decay modes are implied.
[12] The unit vector $\hat{z}$ is aligned along the detector axis in the electron beam direction.