Measurement of $B^0 - \bar{B}^0$ Flavor Oscillations in Hadronic $B^0$ Decays

Flavor oscillations of neutral $B$ mesons have been studied in $e^+e^-$ annihilation data collected with the BABAR detector at center-of-mass energies near the $\Upsilon(4S)$ resonance. The data sample used for this purpose consists of events in which one $B^0$ meson is reconstructed in a hadronic decay mode, while the flavor of the recoiling $B^0$ is determined with a tagging algorithm that exploits the correlation between the flavor of the heavy quark and the charges of its decay products. From the time development of the observed mixed and unmixed final states we determine the $B^0$-$\bar{B}^0$ oscillation frequency $\Delta m_d$ to be $0.516 \pm 0.016(\text{stat}) \pm 0.010(\text{syst})$ ps$^{-1}$.

PACS numbers: 12.15.Hh, 11.30.Er, 13.25.Hw

In the Standard Model, $B^0$-$\bar{B}^0$ mixing occurs through second-order weak diagrams involving the exchange of up-type quarks, with the top quark contributing the dominant amplitude. A measurement of the mass change of up-type quarks, with the top quark contributing the dominant amplitude.
difference $\Delta m_d$ between the mass eigenstates is therefore sensitive to the value of the Cabibbo-Kobayashi-Maskawa matrix element $V_{ud}$. The phenomenon of particle–antiparticle mixing in the neutral $B$ meson system was first seen almost fifteen years ago. The oscillation frequency $\Delta m_d$ has been measured with both time-integrated and time-dependent techniques.

In this Letter we present a measurement of time-dependent mixing based on a sample of 32 million $B\bar{B}$ pairs recorded at the $\Upsilon(4S)$ resonance with the BABAR detector at the Stanford Linear Accelerator Center. This study and a related CP asymmetry measurement are described in more detail in Ref.\[3\]. At the PEP-II asymmetric-energy $e^+e^-$ collider, the $\Upsilon(4S)$ provides a source of $B^0\bar{B}^0$ pairs moving along the $e^-$ beam direction (z axis) with a known Lorentz boost of $\beta\gamma = 0.55$, allowing a novel technique for determining $\Delta m_d$.

The $B^0\bar{B}^0$ mixing probability, for a given $B^0$ lifetime $\tau$, is a function of $\Delta m_d$ and the proper decay-time difference $\Delta t$ between the two neutral $B$ mesons produced in a coherent $P$-wave state in the $\Upsilon(4S)$ event. The result is a time-dependent probability to observe unmixed $(+)$, $B^0\bar{B}^0$, or mixed $(-)$, $B^0\bar{B}^0$ and $\bar{B}^0B^0$, events:

$$\text{Prob}(B^0\bar{B}^0 \rightarrow B^0\bar{B}^0, B^0\bar{B}^0 \text{ or } \bar{B}^0B^0) \propto e^{-|\Delta t|/\tau} \left(1 \pm \cos \Delta m_d \Delta t\right). \quad (1)$$

The effect can be measured by reconstructing one $B$ in a flavor eigenstate, referred to as $B_{\text{tag}}$, while the remaining charged particles originating from the decay of the other $B$, referred to as $B_{\text{rec}}$, are used to identify, or “tag”, its flavor as a $B^0$ or a $\bar{B}^0$. The charges of identified leptons and kaons are the primary indicators, although other information in the event can also be used to identify the flavor of $B_{\text{tag}}$. The time difference $\Delta t = t_{\text{rec}} - t_{\text{tag}} \simeq \Delta s/\beta\gamma c$ is determined from the separation $\Delta s$ of the decay vertices for the flavor-eigenstate and tagging $B$ decays along the boost direction. The tagging and vertexing algorithms used in this analysis are nearly identical to those employed for CP violation studies, in which one $B$ is fully reconstructed in a CP eigenstate.

The value of $\Delta m_d$ is extracted from a tagged flavor-eigenstate $B^0$ sample with a simultaneous unbinned maximum likelihood fit to the $\Delta t$ distributions of mixed and unmixed events. There are two principal experimental factors that complicate the probability distribution given by Eq.\[1\]. First, the tagging algorithm, which classifies events into categories $i$ depending on the source of the available tagging information, incorrectly identifies the flavor of $B_{\text{tag}}$ with a probability $w_i$. This mistag rate reduces the observed amplitude of the oscillation by a factor $(1 - 2w_i)$. Second, the resolution for $\Delta t$ is comparable to the oscillation period and must be well understood. The probability density functions (PDFs) for the unmixed and mixed signal events, $\mathcal{H}_{\pm,\text{sig}}$, can be expressed as the convolution of the underlying $\Delta t$ distribution for the $i^{th}$ tagging category,

$$h_{\pm}(\Delta t; \Delta m_d, w_i) = e^{-|\Delta t|/\tau} \left[1 \pm (1 - 2w_i) \cos \Delta m_d \Delta t\right],$$

with a $\Delta t$ resolution function containing parameters $\hat{a}_j$. A log-likelihood function is then constructed by summing $\ln \mathcal{H}_{\pm,\text{sig}}$ over all events within each of the tagging categories. The likelihood is maximized to extract simultaneously the mistag rates $w_i$, the resolution function parameters $\hat{a}_j$, and the mixing parameter $\Delta m_d$.

The BABAR detector is described in detail elsewhere. Charged particles are detected and their momenta measured by a combination of a 40-layer drift chamber and a five-layer silicon vertex tracker (SVT) embedded in a 1.5-T solenoidal magnetic field. $B_{\text{rec}}$ decay vertices are reconstructed with a resolution of typically 65 $\mu$m along the boost direction. A detector of internally reflected Cherenkov radiation (DIRC) is used for charged hadron identification. Kaons are identified with a neural network based on the likelihood ratios in the SVT and DCH, derived from $dE/dx$ measurements, and in the DIRC, calculated by comparing the observed and expected pattern of Cherenkov light for either kaons or pions. A finely segmented CsI(Tl) electromagnetic calorimeter (EMC) is used to detect photons and neutral hadrons, and to identify electrons. Electron candidates are required to have a ratio of EMC energy to track momentum, an EMC cluster shape, DCH $dE/dx$, and DIRC Cherenkov angle consistent with expectation. The instrumented flux return (IFR) is segmented and contains resistive plate chambers for muon and neutral hadron identification. Muon candidates are required to have IFR hits located along the extrapolated DCH track, an IFR penetration length, and an energy deposit in the EMC consistent with the muon hypothesis.

Neutral $B$ mesons are reconstructed in a sample of multihadron events in the flavor eigenstate decay modes $D^+\pi^-, D^+\pi^+, D^+\bar{\rho}^+, D^+\rho^- a_1^+$ and $J/\psi K^{*0}$. The decay channels $K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0$ and $K^{*0}\pi^-$ are used to reconstruct $D^0$ candidates, while the modes $K^+\pi^-\pi^0$ and $K^{*0}\pi-$ are used for $D^-$ candidates. Charged $D^{*-}$ candidates are formed by combining a $D^0$ with a soft $\pi^-$. Finally, the $B^0$ candidates are formed by combining a $D^{*-}$ or $D^-$ candidate with a $\pi^+$, $\rho^+$ ($\rho^+ \rightarrow \pi^+\pi^0$ or $a_1^+ (a_1^+ \rightarrow \pi^+\pi^-\pi^0)$); likewise, $B^0 \rightarrow J/\psi K^{*0}$ candidates are reconstructed from combinations of $J/\psi$ candidates, in the decay modes $e^+e^-$ and $\mu^+\mu^-$, with a $K^{*0}$ ($K^{*0} \rightarrow K^+\pi^-)$.

The selection and reconstruction of these decay chains and the selection of multihadron events, including continuum suppression, is described in more detail in Ref.\[3\].

Neutral $B$ candidates are identified by the difference $\Delta E$ between the energy of the candidate and the beam energy $\sqrt{s}/2$ in the center-of-mass frame, and the beam-energy substituted mass $m_{\text{BSS}}$, calculated from $\sqrt{s}/2$ and
the reconstructed momentum of the $B$ candidate. Candidates are selected by requiring $m_{ES} > 5.2$ GeV/c$^2$ and $\Delta E$ within $\pm 2.5$ standard deviations of 0 (typically $|\Delta E| < 40$ MeV). When multiple candidates in a given event are selected (with probability of about 0.25%), only the one with the smallest $|\Delta E|$ is retained.

After the daughter tracks of the $B_{rec}$ are removed, the remaining tracks are analyzed to determine the flavor of the $B_{tag}$. For this purpose, we use the flavor tagging information carried by primary leptons from semileptonic $B$ decays, charged kaons, soft pions from $D^*$ decays, and high momentum charged particles, to assign an event to a single tagging category.

Events are assigned a Lepton tag if they contain an identified lepton with a center-of-mass momentum greater than 1.0 or 1.1 GeV/c for electrons and muons, respectively. The momentum requirement selects mostly primary leptons by suppressing opposite-sign leptons from semileptonic charm decays. If the sum of charges of all identified kaons is non-zero, the event is assigned a Kaon tag. The final two tags involve a multivariable analysis based on a neural network, which is trained to identify primary leptons, kaons, and soft pions, and the momentum and charge of the track with the maximum center-of-mass momentum. Depending on the output of the neural net, events are assigned either an NT1 (more certain) tag, an NT2 (less certain) tag, or are not tagged at all (about 30% of all events) and excluded from the analysis.

Tagging assignments are made mutually exclusive by the hierarchical use of the tags. Events with a Lepton tag and no conflicting Kaon tag are assigned to the Lepton category. If no Lepton tag exists, but the event has a Kaon tag, it is assigned to the Kaon category. Otherwise events with neural network tags are assigned to corresponding neural network categories.

The decay time difference $\Delta t$ between $B$ decays is determined from the measured separation $\Delta z = z_{rec} - z_{tag}$ along the $z$ axis between the reconstructed $B_{rec}(z_{rec})$ and flavor-tagging decay $B_{tag}(z_{tag})$ vertex. This measured $\Delta z$ is converted into $\Delta t$ with the use of the known $\Upsilon(4S)$ boost, including a correction on an event-by-event basis for the direction of the $B$ mesons with respect to the $z$ direction in the $\Upsilon(4S)$ frame. The $\Delta t$ resolution is dominated by the $z$ resolution of the tag vertex position. Reconstruction of the $B_{tag}$ decay vertex starts with all tracks in the event except those incorporated in $B_{rec}$. In order to reduce the contamination from $D$ meson decay products, those identified as kaons are also excluded. An additional constraint is provided by the calculated $B_{tag}$ production point and three-momentum, determined from the momentum of the $B_{rec}$ candidate, its decay vertex, the average position of the interaction point, and the $\Upsilon(4S)$ boost. Tracks with a large contribution to the $\chi^2$ are iteratively removed from the fit until those remaining ($\geq 1$) have a reasonable fit probability or all tracks are removed. Only events with a reconstructed $B_{tag}$ vertex, $|\Delta t| < 20$ ps and $\sigma_{\Delta t} < 1.4$ ps are retained (about 84%), where $\sigma_{\Delta t}$ is the measurement error derived from the vertex fits.

![Distribution of $m_{ES}$ for all $B^0$ candidates with a flavor tag and a reconstructed tag vertex.](image)

The distribution of $m_{ES}$ for the selected candidates is shown in Fig. 1, where the result of a fit with a Gaussian distribution for the signal and an ARGUS function for the background is also displayed. The fitted number of signal events and their purity (for $m_{ES} > 5.27$ GeV/c$^2$) are $6347 \pm 89$ and $(85.8 \pm 0.5)$%, respectively. The sample composition by tagging category is given in Table I.

<table>
<thead>
<tr>
<th>Category</th>
<th>Tagged</th>
<th>Purity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>1097 ± 34</td>
<td>96.0 ± 0.7</td>
</tr>
<tr>
<td>Kaon</td>
<td>3156 ± 63</td>
<td>84.6 ± 0.7</td>
</tr>
<tr>
<td>NT1</td>
<td>798 ± 31</td>
<td>88.9 ± 1.2</td>
</tr>
<tr>
<td>NT2</td>
<td>1293 ± 43</td>
<td>79.4 ± 1.3</td>
</tr>
</tbody>
</table>

In the likelihood fit, the $\Delta t$ resolution function is approximated by a sum of three Gaussian distributions (core, tail, and outlier) with different means and widths, $R(\delta t; \hat{a}) = \sum_{k=1}^{2} \frac{f_k}{S_k \sigma_{\Delta t} \sqrt{2\pi}} \exp \left( -\frac{(\delta t - b_k \sigma_{\Delta t})^2}{2(S_k \sigma_{\Delta t})^2} \right) + \frac{f_3}{\sigma_3 \sqrt{2\pi}} \exp \left( -\frac{\delta t^2}{2\sigma_3^2} \right)$, where $\delta t = \Delta t - \Delta t_{true}$. The sum of the fractions $f_k$ is constrained to unity. For the core and tail Gaussians, the widths $\sigma_k = S_k \times \sigma_{\Delta t}$ are the event-by-event measurement errors scaled by a common factor $S_k$. The scale factor of the tail Gaussian is fixed to the Monte Carlo value since it is strongly correlated with the other resolution function parameters. The third Gaussian, with
a fixed width of \( \sigma_3 = 8 \text{ ps} \), accounts for outlier events with incorrectly reconstructed vertices (less than 1% of events). A separate core bias coefficient \( b_{1,i} \) is allowed for each tagging category \( i \) to account for small shifts due to inclusion of charm decay products in the tag vertex, while a common bias coefficient \( b_2 \) is used for the tail component. These offsets are proportional to \( \sigma_{\Delta t} \) since both the size of the bias and the resolution for \( z_{\text{tag}} \) depend kinematically on the polar angle of the flight direction of the charm daughter. The tail and outlier fractions and the scale factors are assumed to be the same for all decay modes, since the \( z_{\text{tag}} \) measurement dominates the resolution for \( \Delta t \). This assumption is confirmed by Monte Carlo studies. Separate resolution parameters are used for two different data-reconstruction periods, referred to as Run1 and Run2, which mainly differ in vertex performance and tracking efficiency.

In the presence of backgrounds, additional terms must be added to the signal PDF \( \mathcal{H}_{\pm,\text{sig}} \) for each background source,

\[
\mathcal{H}_{\pm,i} = f_{i,\text{sig}} \mathcal{H}_{\pm,\text{sig}} + \sum_{j=bkgd} f_{i,j} \mathcal{B}_{\pm,i,j}(\Delta t; \hat{b}_{\pm,i,j}),
\]

where the background PDFs \( \mathcal{B}_{\pm,i,j} \) provide an empirical description for the \( \Delta t \) distribution of the background events in each tagging category \( i \) of the sample. The fraction of background events for each tagging category and background source is given by \( f_{i,j} \), while \( \hat{b}_{\pm,i,j} \) are parameters used to characterize each source of background by tagging category for mixed and unmixed events. The signal probability \( f_{i,\text{sig}} \) is determined from the measured event \( m_{ES} \) on the basis of a separate fit to the observed \( m_{ES} \) distribution in tagging category \( i \). The sum of signal and background fractions is forced to unity.

The \( \Delta t \) distributions of the background are described with a zero lifetime component and a non-oscillatory component with non-zero lifetime. We fit for separate resolution function parameters for the signal and the background to minimize correlations. The likelihood fit involves a total of 44 parameters, including \( \Delta m_d \), the average mistag fraction and the difference between \( B^0 \) and \( \bar{B}^0 \) mistags for each tagging category (8), parameters for the signal \( \Delta t \) resolution (16), and parameters for background time dependence (5), \( \Delta t \) resolution (6), and effective dilutions (8). The value of \( \Delta m_d \) was kept hidden throughout the analysis until all analysis details and the systematic errors were finalized, to eliminate possible experimenter’s bias.

The results from the likelihood fit to the tagged \( B^0 \) sample are summarized in Table I. The probability to obtain a likelihood smaller than that observed is 44%, evaluated with a parameterized Monte Carlo technique. The value of \( \Delta m_d \) given by the fit, prior to final corrections, is \( \Delta m_{d,\text{fit}} = 0.525 \pm 0.016 \text{ ps}^{-1} \). One method for displaying the result of the full likelihood fit is to use the observed mixing asymmetry,

\[
A_{\text{mix}}(\Delta t) = \frac{N_{\text{unmixed}}(\Delta t) - N_{\text{mixed}}(\Delta t)}{N_{\text{unmixed}}(\Delta t) + N_{\text{mixed}}(\Delta t)}.
\]

If the flavor tagging and \( \Delta t \) determination were perfect, the asymmetry as a function of \( \Delta t \) would be a cosine with unit amplitude. The amplitude is diluted by the mistag

---

**TABLE II: Results for \( \Delta m_d \) and a subset of the parameters obtained from the likelihood fit to the \( \Delta t \) distributions.** \( \Delta m_d \) includes small corrections described in the text.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Value</th>
<th>Parameter</th>
<th>Fit Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta m_d )</td>
<td>0.516 ± 0.016</td>
<td>( w ) (Lepton)</td>
<td>0.079 ± 0.014</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( w ) (NT1)</td>
<td>0.219 ± 0.022</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( w ) (NT2)</td>
<td>0.344 ± 0.020</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( S_i ) (Run1)</td>
<td>1.37 ± 0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( S_i ) (Run2)</td>
<td>1.19 ± 0.11</td>
</tr>
<tr>
<td>( f_2 ) (Run1)</td>
<td>0.014 ± 0.010</td>
<td>( f_2 ) (Run2)</td>
<td>0.015 ± 0.010</td>
</tr>
<tr>
<td>( f_3 ) (Run1)</td>
<td>0.008 ± 0.004</td>
<td>( f_3 ) (Run2)</td>
<td>0.000 ± 0.014</td>
</tr>
</tbody>
</table>

**FIG. 2:** Distributions of \( \Delta t \) in data for the selected a) unmixed and b) mixed events (\( m_{ES}(B_\text{rec}) > 5.27 \text{ GeV}/c^2 \)), with projections of the likelihood fit (solid) and the contribution of the background (dashed) overlaid. The time-dependent mixing asymmetry \( A_{\text{mix}}(\Delta t) \) is shown in c).
probability and the experimental resolution for $\Delta t$. The observed $\Delta t$ distributions of both the mixed and unmixed events, and their asymmetry $A_{mix}$ are shown along with projections of the likelihood fit result in Fig. 2.

Since the parameters of the $\Delta t$ resolution for both signal and backgrounds are free parameters in the fit, their contribution to the uncertainty on $\Delta m_d$ is included as part of the statistical error. Remaining systematic errors arise from the choice of the signal $\Delta t$ resolution description, its capability to handle outliers and various worst-case SVT misalignment scenarios ($\pm 0.005 \text{ ps}^{-1}$), and by approximations and uncertainties in the $\Delta z$ to $\Delta t$ conversion due to the knowledge of the absolute $z$ scale of the BABAR detector and PEP-II boost (less than $\pm 0.002 \text{ ps}^{-1}$). Systematic errors due to background include the choice of its $\Delta t$ distribution and resolution description ($\pm 0.002 \text{ ps}^{-1}$), variation of the sum of background fractions from the separate $m_{ES}$ fits, and the uncertainty on the magnitude of the small $B^+$ component of the signal ($\pm 0.002 \text{ ps}^{-1}$). A correction of $-0.002 \text{ ps}^{-1}$, derived from data, is made to account for the small variation of the background composition as a function of $m_{ES}$, which affects the background $\Delta t$ distribution. The statistical error ($\pm 0.002 \text{ ps}^{-1}$) on this extrapolation from the sideband to the signal region is included as a systematic uncertainty. An additional correction of $-0.007 \text{ ps}^{-1}$ is applied for a bias observed in fully simulated Monte Carlo events. The bias is mainly due to correlations between the mistag rate and the $\Delta t$ resolution that are not explicitly incorporated into the likelihood function. The systematic error assigned to this correction includes contributions from the statistical precision of the Monte Carlo study ($\pm 0.003 \text{ ps}^{-1}$), model variations due to uncertain branching fractions and lifetimes of the tag-side $D$ mesons and the assumed fraction of wrong-sign kaons produced in $B$ decays ($\pm 0.001 \text{ ps}^{-1}$), and variation of the requirement on the maximum allowed value of $\sigma_{gt}$ ($\pm 0.003 \text{ ps}^{-1}$). Finally, the variation of the fixed $B^0$ lifetime within the known errors leads to a systematic uncertainty of $\pm 0.006 \text{ ps}^{-1}$.

In conclusion, a new technique involving the time-difference distribution of a tagged sample of fully-reconstructed neutral $B$ decays has been used to determine the $B^0$-$\bar{B}^0$ mixing frequency $\Delta m_d$ to be

$$\Delta m_d = 0.516 \pm 0.016(\text{stat}) \pm 0.010(\text{syst}) \text{ ps}^{-1}.$$ 

This is one of the single most precise measurements available. Moreover, the error on $\Delta m_d$ is still dominated by the size of the reconstructed $B^0$ sample, leaving substantial room for further improvement as more data are accumulated. The result is consistent with the current world average [4] and a recent BABAR measurement with a dilepton sample [10]. The analysis shares the same flavor-eigenstate sample as used for the determination of $\sin 2\beta$, thereby providing an essential validation for the reported $\sin 2\beta$ result [4].

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF (Germany), INFN (Italy), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the Swiss NSF, A. P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

---

[1] The symbol $B^0$ refers to the $B_d$ meson; charge conjugate modes are implied throughout this paper.