Measuring Consumer Detriment under Conditions of Imperfect Information

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1 Introduction

This report aims to construct a simple model of consumer detriment that can be used to empirically estimate the loss in consumer surplus that arises when consumers have imperfect information on the terms of trade. In particular, we were asked by the OFT to derive a simple measure of consumer detriment in the following two cases.

- **Case 1:** Imperfect price information. Consumers cannot perfectly observe prices.
- **Case 2:** Imperfect output observation. Consumers lack relevant information about the characteristics (hereafter, referred to as “quality”) of the products sold in the market.

In this report we will treat the two cases separately, that is we shall assume that either consumers cannot observe the prices of a good of known quality or they observe prices but lack knowledge of the quality itself. This will allow us to better disentangle the effects of the two forms of imperfect information and to construct simple measures of consumer detriment in each of the two cases. Moreover, we shall treat quality as exogenous and focus on the effects of imperfect consumer information about the level of prices and outputs.

We define as **consumer detriment** (hereafter CD) the loss in consumer surplus that consumers experience due to the presence of imperfect information. That is, the consumer detriment is taken as the difference in consumer surplus between a situation where consumers are fully informed and a situation where consumers’ information is imperfect.\(^1\)

For the case when consumers are not perfectly informed about prices (imperfect price information) and gathering information is costly, the “law of one price”, according to which identical products must be sold at the same price, may not hold. Firms have incentives to create price dispersion in order to increase the consumers’ cost of finding better deals and enjoy some degree of monopoly power. In these circumstances, the consumer detriment arises because consumers may not buy the product at the cheapest price available in the market, or more precisely, at the price that would arise in the absence of imperfect information.

For the case when consumers cannot perfectly assess the quality of the product (imperfect quality information), the consumers’ choice over the quantity to purchase - at any given price - may not be appropriate for their real needs. Moreover, the price at which the product is sold in the market may differ from the perfect information price.

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\(^1\)The OFT defines CD as the potential to benefit consumers by: (i) providing more information, (ii) enhancing consumers’ understanding of, and propensity to act on, information, (iii) improving consumers’ access to, and the effectiveness of, redress.

The approach followed here was suggested by the OFT for operational convenience; it presumes that all the costs (e.g. search costs) that generate imperfect information can be avoided at no cost.
The main problem that is encountered in providing a measure of the consumer detriment that can be estimated with observed data relates to the difficulty of predicting what the level of prices and outputs would be if information were perfect. This is crucial since, as explained above, the level of consumer detriment depends on the characteristics of the allocation of resources under perfect information.

Ideally, one would like to be able to infer the perfect information allocation from the observation of the current set of prices and outputs. This requires making the following steps.

- Step 1) To identify how prices and outputs vary with the extent of the consumer information, for any market structure.
- Step 2) To estimate the level of precision in the consumers' information (i.e. how much consumers know about the distribution of prices in the economy or how much consumers overestimate/underestimate quality).
- Step 3) To characterize the market structure.

In the case of imperfect information about prices we conclude that Step 1 is unattainable within a reasonable level of reliability. Consequently, we have chosen to make an assumption on the allocation of resources under perfect information rather than trying to infer it from the observation of the current set of prices. In particular, we have provided a measure of consumer detriment under imperfect information about prices that is based on the assumption that under perfect information the market would behave competitively (either perfect competition or monopolistic competition).

To better understand our reasoning, consider how much consumers shop around before making their purchase decisions. This will depend on the individual’s search costs (opportunity cost of the time spent in gathering information, disutility of effort in information collection and processing, cost of delays, etc.). In turn, the price charged by each individual firm will be a function of the number of customers it expects to attract at that price as well as its own characteristics (for example, the level of marginal costs). Hence, there exists a link between the distribution of search cost in the economy and the level of prices, and this link will have different characteristics depending on the market structure and the degree of heterogeneity of consumers and firms. However, we cannot predict what the prices and hence the market structure would be if consumers were fully informed, since we are unable to perfectly assess the distribution of search costs in the economy and how much search costs influence consumers' willingness to shop around. Furthermore, minor changes in the assumptions about the distribution of search costs and the heterogeneity of consumers lead to profoundly different predictions.

Let us now turn to the case of imperfect quality observation. Here, it is still the case that the level of consumer detriment depends on the characteristics of
the allocation of resources under perfect information and that steps 1 to 3 would need to be made in order to infer it. For this case we will consider two alternative approaches that trade off reliability with obtaining suitable data. The first approach is based on the model set out in Consumer Detriment (OFT, 2000) according to which the level of imperfect information is reflected in the difference between marginal and average cost at the firm level. Aware of the simplified nature of the model, the OFT asked us to review it and to identify possible shortcomings. We will therefore discuss this model and attempt to provide a new version, which, while maintaining the same assumptions, overcomes some of the shortcomings of the original model. The second approach is based on the argument that trying to derive a theoretical link between variables like prices, outputs or costs, and the level of information precision may involve a high loss of generality. We will therefore treat the level of information precision as exogenous in the theoretical model and use economic models of firms’ behavior to predict the relationship between prices and information (Step 1), for any given market structure. We will then use an econometric method to measure the level of information precision in the hands of consumers (Step 2). Finally, we will suggest that the market structure is chosen on a case by case basis (Step 3).

The rest of the report is organized as follows.

In Section 2 we analyze the case of imperfect price information. In particular, in Section 2.1 we summarize some of the main findings of the economic literature on the effects of consumer’s imperfect information about prices. These findings are then used in Section 2.2 to derive a theoretically consistent measure of consumer detriment that can be empirically estimated.

In Section 3 we analyze the case of imperfect quality information. Section 3.1 briefly introduces the issue. A simple model of consumer detriment for this case was proposed in the OFT report on Consumer Detriment (2000), which we review in Section 3.2. First, maintaining the assumptions of the model we show that there is an important component of the consumer detriment that is missing. Second, we discuss the assumptions of the model and their shortcomings. In light of this Section 3.3 provides a revised version of the OFT model that corrects for the missing component and relaxes some of the restrictive assumptions of the original model. Further, we critically review it. In Section 3.4 we propose an alternative approach to calculate the consumer detriment for the case of imperfect quality information. In particular, we provide two formulations one for the short run equilibrium and one for the long run equilibrium. In Section 4 we discuss how to estimate the expressions provided in the report to measure consumer detriment. Finally, Section 5 concludes.
2 Imperfect information about prices

2.1 Introduction

The “law of one price”, according to which identical products (i.e., homogeneous products sold at the same location at a given point in time) must be sold at the same price, does not always hold. In fact, empirical evidence reveals that price differentials for relatively homogeneous goods are a widespread phenomenon.\(^2\)

Economists recognize that price dispersion for identical products may arise when consumers are not fully informed about prices in the market and gathering information is costly.\(^3\) Indeed, if information were costless, then rational consumers would search until they find the best terms of trades available in the market and perfect information would result. Instead, when information acquisition is costly, then consumers compare the expected marginal benefit from an additional piece of information with the marginal costs (opportunity cost of the time spent searching, disutility of effort in information gathering and processing, cost of delays, etc.) and some may thus stop searching before the best terms of trade are found.

In these circumstances, firms can enjoy some degree of monopoly power since the mobility of consumers is constrained by the cost of finding better deals. In particular, firms can exploit the lack of information on the part of consumers by charging higher prices for their products. Moreover, firms may even create noise in order to increase the costs for consumer to become perfectly informed. For example, Varian (1980) argues that periodic sales may serve precisely this purpose.

A brief look at the economic literature on imperfect information about prices reveals that the allocation of resources in the presence of search costs depends on a wide range of factors. For example, Diamond (1971) shows that if consumers have the same search costs, these costs are small and firms are identical, then each firm will act as a local monopolist and monopoly pricing results in equilibrium with zero price dispersion. However, price dispersion will result either when consumers are not identical (for example because they have different search costs, see Salop, 1977), or when firms have different cost structures (see for example Reinganum, 1979).\(^4\) In particular, Salop (1977) shows that if the elasticity of market demand is higher for those consumers with lower search costs, then it may be profitable for a monopolist to charge high prices to those consumers with high search costs and low prices to consumers with low search costs. A similar analysis is developed in Salop and Stiglitz (1977) for the case of competition and free entry where a direct link is shown between the level of search costs and the characteristics of the equilibrium in terms of price dispersion. In particular, if costs are low for a sufficient fraction of consumers in the

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\(^2\)See Pratt et al. (1979).

\(^3\)This point was first made by Stigler (1961). In his seminal paper he emphasizes that price dispersion is a measure of consumer ignorance in the market.

\(^4\)Other models show that price dispersion can arise when consumers differ ex post in the number of offers received (see for example, Butters 1977) or in their willingness to pay for the good (Diamond 1987).
market, then the competitive price will result; if instead search costs are particularly high then monopoly pricing emerges. Finally, for intermediate levels of search costs price dispersion will results in the range between the monopoly price and the competitive one and consumers who search more obtain a better deal.\footnote{This model for homogeneous good is generalised by Sudanand and Wilde (1982) to the case where consumers do not know the distribution of prices. See also Rob (1985) for the derivation of an equilibrium with identical firms and price dispersion in the presence of heterogeneous consumers.}

The above analyses suggest that it is not possible to derive a general formulation for the relationship between prices and consumers’ search costs. However, the level of prices in the market tend to increase with the cost of acquiring information.\footnote{For a simple proof of why this is the case, see Shy (1995).} This is because the higher the search costs, the lower the gain for consumers from searching for a lower price and the higher the degree of monopoly power that firms can exploit. Therefore, policies devoted to reducing consumers’ search costs can be desirable since they reduce the firms’ market power (and thus the level of prices) resulting from consumers’ imperfect information as well as the waste of resources spent in gathering information. Moreover, an increase in the percentage of informed consumers increases the level of effective competition in the market. This arises because it becomes less profitable for a firm to raise its price above the perfect information level since those consumers who are informed will move to the lowest available alternative. This “informational externality” amplifies the impact of a reduction in search costs on the level of prices and may lead to effective competition being restored when a sufficient number of consumers is informed.\footnote{See for example Salop and Stiglitz (1977) and Rob (1985).}

### 2.2 A measure of consumer detriment

The OFT proposed that, in order to model the consumer detriment, it would be convenient to define it as the difference in consumer surplus between the case of perfect information and that of imperfect information. This was acknowledged as a simplification which presumes, amongst other things, that all search costs can be avoided.

The main problem one encounters in providing a measure of CD for the case of imperfect information about prices is that the level of consumer detriment depends on what the allocation of resources (price and output) would be in the presence of perfect information (the benchmark), and this information is difficult to obtain. The approach we take here is to assume that in the presence of perfect information the market would be competitive (either perfect competition or monopolistic competition). Then, we construct a measure of CD that is based on the observed prices and on the level of marginal (or average) costs, where the latter convey information about the equilibrium price in presence of
perfect information. In this setting, any divergence of prices from the competitive (perfect information) level is attributed to the existence of imperfect information. Search costs allow the firms to enjoy some degree of monopoly power since the mobility of the consumers is constrained by the costs of finding a better deal.

An alternative is to build a framework to predict how equilibrium prices and outputs under perfect information translate in a world of imperfect information for any given market structure and then use empirical data to try to infer which market structure matches the prediction. We have chosen not to follow this approach because it is not robust to changes in the functional forms chosen to represent the distribution of search costs and the heterogeneity of consumers and firms, which could involve a high loss of generality. As briefly explained in the previous section, the characteristics of the equilibrium under price dispersion vary considerably according to consumers and firms’ characteristics as well as the distribution of search costs.

Further discussion on the pros and cons of our approach is developed at the end of this section.

**Perfect Information about prices**

Denote by \( p = D(Q, \cdot) \) the demand function and by \( p^* \) the equilibrium price, the corresponding level of output sold in the industry is given by \( Q^* = D^{-1}(p^*) \). The number of firms in the market under perfect information is denoted by \( m \); \( q_j^* \) is the level of output sold by the \( j \)-th firm in the market, where \( \sum_{j=1}^m q_j^* = Q^* \). All firms in the market under perfect information are identical. In this setting, denoting by \( AC(q_j) \) the average costs curve of the \( j \)-th firm, the firm’s equilibrium profit is

\[
\pi^*_j = [p^* - AC(q_j^*)] q_j^*
\]

where in light of the assumptions made, \( q_j^* = q^* \) and \( \pi^*_j = \pi^* \) for all \( j = 1, .., m \).

By now, we assume that the perfect information industry structure is one of perfect competition so that \( p^* = MC = \min AC \); the case of monopolistic competition will be further discussed.

In light of this we can calculate the consumer surplus in the presence of perfect information. Denoting by \( p^0 \), the level of price at which the demand tends to zero and linearly approximating the demand function, we obtain

\[
CS^* = \frac{1}{2} \sum_{j=1}^m (p^0 - p^*) q_j^*
\] (1)

**Imperfect Information about prices**
Now, consider the case where consumers are imperfectly informed about prices and price dispersion arises for homogeneous goods. Denote by \( p_i \), for \( i = 1, 2, \ldots, n \) the price charged by the \( i \)-th company (or \( i \)-th shop) under imperfect information, where

\[
p_1 \leq p_2 \leq \cdots \leq p_i \leq p_n
\]

and where \( n \) denotes the number of firms in the industry. Further, denote by \( q_i \) the quantity sold at price \( p_i \) by firm \( i \).

In order to measure the effect of imperfect price information on the level of consumer surplus we would need to know the characteristics of the demand schedule when consumers have positive search costs, for the amount of output consumers are willing to buy at any given price depends on the distribution of search costs. Under imperfect price information each consumer minimizes the total expected cost of buying a unit of a commodity and this includes both the price of the good and the search cost. It follows that a consumer’s reservation price depends on the cost of searching, the utility of the product to the consumer and the consumer’s knowledge about the distribution of prices. In this respect, it is difficult to build an expression for the level of consumer surplus under imperfect price observation that could hold with a reasonable level of generality.

In light of this, our approach is to capture the extent of the consumer detriment at the firm level and assume that \( n = m \). When compared to a situation where consumers are fully informed, imperfect price information generates two main effects on the equilibrium level of prices and outputs. First, those consumers who address firm \( i \) pay a price \( p_i \) rather than \( p^* \), where \( p_i \geq p^* \), over the \( q_i \) units of output they purchase. Second the number of units bought from each individual firm is likely to be lower than under perfect information \( (q_i \leq q^*) \) since the cost of buying (which includes the search costs) is higher. Hence, for each firm, over \( (q^* - q_i) \) units of outputs consumers bear a further loss. With the help of Figure 1, we can provide a measure of the consumer detriment that accounts for both these components: in particular, area \( A \) represents the first effect, while area \( B \) captures the second. The level of approximation is reflected in the fact that the marginal benefit function for the consumer under perfect and imperfect information will not be the same. Therefore, the areas in figure 1 measure imperfectly the level of consumer detriment at firm level. Moreover, the assumption that the number of firms is the same under perfect and imperfect information may involve some loss of generality, for it may be the case that the positive level of profits under imperfect information attracts new firms in the market. However, in the absence of a reliable relationship between the level of prices, the distribution of search costs and the number of firms in the industry, this simplification can hardly be avoided.

Following from the above discussion, the measure of consumer detriment that we suggest is given by
\[ CD = \sum_{i,j=1}^{n} \left[ (p_i - p^*)q_i + \frac{1}{2}(p_i - p^*)(q_j^* - q_i) \right] \] \hspace{1cm} (3)

Note that in case the total industry demand is rather inelastic, that is, the number of units consumers purchase is independent of the level of imperfect information, than the second effect described above would be irrelevant in aggregate. In this case, our measure of the consumer detriment would amount to

\[ CD = \sum_{i=1}^{n} [(p_i - p^*)q_i] \] \hspace{1cm} (4)

Rearranging expression (3) we obtain

\[ CD = \frac{1}{2} \sum_{i=1}^{n} \left( p_i q_i + p_i q_j^* - p^* q_j^* - p^* q_i \right) \] \hspace{1cm} (5)

Denoting by \( R_i \) and \( R_i^* \) the revenues of each individual firm under perfect and imperfect information, respectively, expression (5) yields:

\[ CD = \frac{1}{2} \sum_{i=1}^{n} \left( R_i - R_i^* + \frac{R_i^* p_i}{p^*} - \frac{R_i p^*}{p_i} \right) \]

which is equal to:

\[ CD = \frac{1}{2} \sum_{i=1}^{n} \left( R_i \left( \frac{p_i - p^*}{p_i} \right) + R_i^* \left( \frac{p_i - p^*}{p^*} \right) \right) \] \hspace{1cm} (6)

Taking into account that \( p^* \) is equal to marginal costs under perfect information and assuming that the firms’ cost structure under perfect and imperfect information does not significantly differ (we will relax this assumption later), we can rewrite expression (6) becomes

\[ CD = \frac{1}{2} \sum_{i=1}^{n} \left( R_i \left( \frac{p_i - MC_i}{p_i} \right) + R_i^* \left( \frac{p_i - MC_i}{MC} \right) \right) \] \hspace{1cm} (7)

Note that in case the number of units consumers purchase is independent of the level of imperfect information, then from expression (4) the level of CD would amount to only the first term in expression (7).

Moreover, under the assumption that all firms are identical and there are constant returns to scale, \( MC_i = AC_i \), expression becomes
\[ CD = \frac{1}{2} \sum_{i=1}^{n} \left( R\left( \frac{p_i - AC_i}{p_i} \right) + R^* \left( \frac{p_i - AC_i}{AC_i} \right) \right) \]

Multiplying the terms in the round bracket by \( q_i \) and noting that \( (p_i - AC_i)q_i = \pi_i \):

\[ CD = \frac{1}{2} \sum_{i=1}^{n} \left( \pi_i \left( 1 + \frac{R^*}{C_i} \right) \right) \tag{8} \]

where \( C_i = AC_i q_i \) is the total cost of firm \( i \).

In this case, our measure of the consumer detriment would amount to

Notice that the above formula also holds in the presence of monopolistic competition. In this case \( p^* = AC(q^*) \) while in the presence of imperfect information \( p_i \geq AC(q_i) \). Since consumer detriment is expressed at firm level, we can assume that \( q_i - q^* \) is small and approximate \( AC(q_i) \) with \( AC(q^*) \), which leads to (8).

Further, notice that since each price must lie between the competitive price and the monopoly price we can also find the lower and upper bound of the consumer detriment. Denote the former by \( \underline{CD} \) and the latter by \( \overline{CD} \), we obtain:

\[ \underline{CD} = 0 \]

since \( p_i = p^* \) for all \( i \), and, from (3):

\[ \overline{CD} = \left[ n \left( p^M - p^* \right) q^M + \frac{1}{2} \left[ n \left( p^M - p^* \right) (q^* -q^M) \right] \right] \tag{9} \]

where \( p^M \) is the monopoly price, \( Q^M \) the monopoly output and \( q^M = \frac{Q^M}{n} \) is the level of output produced by each individual firm. Expression (9) coincides with Harberger’s calculation of the consumer loss under monopoly pricing (since the welfare loss is equal to consumer detriment minus profits).\(^8\) In fact, setting \( p^* = MC = AC \), the term in the second square bracket is equal to \( \frac{\pi^M}{2} \), where \( \pi^M \) denotes the monopoly profits, and overall:

\[ \overline{CD} = \left[ \frac{\pi^M}{2} + \pi^M \right] = \frac{3\pi^M}{2} \]

\(^8\)From \( \varepsilon = \frac{p^M}{Q^M} \frac{\Delta p}{\Delta Q} \), where \( \Delta p = p^M - p^* \) and \( \Delta Q = Q^* - Q^M \), rewrite \( \Delta Q = \frac{\varepsilon Q^M \Delta p}{p} \) and substitute for \( \Delta Q \) in \( \frac{\Delta Q \Delta p}{2} \) to obtain: \( \frac{\Delta p \varepsilon Q^M \Delta p}{2p} \). Since, under monopoly \( \frac{\Delta p}{p} = \frac{\varepsilon}{\varepsilon} \) and since \( Q^M \Delta p = \pi^M \) (since \( p^* = AC \)) the result follows.
This results is in line with the study by Diamond (1971), who shows that imperfect price information can result in monopoly prices.

In the above formulation, we have considered the case where all firms have similar cost structure so that \( p^* \) is equal to the \( MC \) of a representative firm. This case is consistent with the findings of the economic literature on the equilibrium under imperfect information that shows how price dispersion may arise in the presence of heterogeneity between consumers even if all firms are identical. However, in her paper of 1987, Reinganum shows that under costly information acquisition, imperfect information may result in price dispersion also when firms have different marginal costs.\(^9\) This suggests that when the assumption of similar cost functions across firms is not confirmed by the data, the following alternative measure of consumer detriment can be used.

Denote by \( MC^* \) the marginal costs of a representative firm under perfect competition. Denote by \( MC_i \) the marginal costs of the \( i-th \) firm in the presence of imperfect competition. Further, denote by \( MC \) and \( MC \) the lower and upper-bound of \( MC_i \), that is: \( MC_i \in [MC, MC] \). We assume that the lowest marginal cost firm in the presence of imperfect information has the same marginal costs as the most efficient firm under imperfect information, that is: \( MC^* = MC \), which implies \( p^* = MC \). Then expression (6) becomes:

\[
CD = \sum_{i=1}^{n} \frac{1}{2} \left( R_i \left( 1 - \frac{MC}{p_i} \right) + R^* \left( \frac{p_i}{MC} - 1 \right) \right)
\]

Letting \( \alpha_i = \frac{MC_i}{MC} \), where \( \alpha \leq 1 \), we obtain:

\[
CD = \sum_{i=1}^{n} \frac{1}{2} \left( R_i \left( 1 - \alpha_i \frac{MC_i}{p_i} \right) + R^* \left( \frac{1}{\alpha} \frac{p_i}{MC_i} - 1 \right) \right)
\]

A CRITICAL REVIEW OF THE APPROACH

- The advantages of our formulation, which leads to equations (7), (8), (10), are that it does not rely on any assumption on the distribution of prices or of search costs among consumers. Moreover, it does not require the estimate of prices.

\(^9\) A priori we cannot say whether price dispersion will decrease, increase or remain constant over time. Whether or not price dispersion will persist in the long run depends on the incentives for firms to invest in R&D activities so as to reduce their costs. In particular, Reinganum shows that seller’s profits increase with a reduction in marginal costs (or average costs); moreover the marginal benefits are greater the lower the initial level of marginal costs. Since it is likely that marginal costs of cost reduction also increases as AC increases, it is not clear cut whether the optimal rate of AC reduction is increasing, decreasing or constant in AC. If it is increasing (decreasing) then cost dispersion is expected to reduce (increase) over time and so will price dispersion. If it is constant, then price dispersion will persist in the long run.
The level of approximation in our formulation is reflected in the fact that the CD, as stated, does not include the cost of searching and processing information. Thus, in this respect, our measure will underestimate the impact on consumer surplus of measures which reduce search costs. Moreover, the assumption that the number of firms is the same under perfect and imperfect information makes our measure of consumer detriment as representative of the loss in consumer surplus that arises because the firms in the market do not charge the perfect information price and therefore, in this respect, the CD estimated above is likely to overestimate CD in practice.

Although we have thought it to be inappropriate to postulate a functional form for the price-search costs relationship, it must be taken into account that such a relationship is likely to exist, as emphasized by the economic literature.\footnote{See for example Salop and Stiglitz (1977) and Rob (1985).} In particular, the level of price dispersion measured by the difference between the minimum and the maximum price charged, tends to increase with the costs of information gathering and individual prices also tend to increase in the level of search costs.\footnote{For a simple proof of why this is the case, see Shy (1995).} Since our measures of the consumer detriment are an increasing function of the general level of prices, CD is likely to increase with the level of search costs.

As explained in the previous section, our formulation does not allow us to disentangle the level of market power due to anti-competitive behavior from that related to the existence of imperfect information. This is because our benchmark case is either one of perfect competition or of monopolistic competition. This restriction was necessary in order to provide a proxy for the perfect information price ($p^\ast$). Therefore, it must be taken into account that our measures of CD could overestimate the loss in consumer surplus due to imperfect information whereas the perfect information structure were not competitive.

We have restricted the attention to the case of a homogenous product market. In fact, consumer detriment may arise also in the presence of heterogeneous products, for the price dispersion, which is natural in these forms of markets, may be excessive with respect to what would be justified by the existence of differences in product characteristics. However, the difficulty of “measuring” product characteristics as well plausible differences in the consumers’ tastes suggest that our simplification may be difficult to avoid.

**Estimation**

In the analysis above we have provided three alternative expressions for CD as given by formulas (7), (8), (10) and carefully explained the assumptions underlying each of them. In Section 4. we suggest how to estimate them.
3 Imperfect information about quality

3.1 Introduction

A second kind of consumer imperfect information about the terms of trade refers to the possibility that consumers lack detailed knowledge about the quality of the products sold in the market.

In his seminal paper, Spence (1977) shows that when consumers consistently underestimate the probability of product failure, even a competitive market will operate inefficiently.\textsuperscript{12} Subsequent analyses on the relationship between prices and imperfect quality information discuss the variety of factors that affect the performance of market under imperfect quality information. These include the market structure, the possibility of signalling quality through prices, advertising, warranties or investment in research and development and the level of consumers’ search costs as well as the firm’s reputational concern.\textsuperscript{13} For an excellent survey see Stiglitz (1987). It must be noted though that in practice price signalling provides consumers with imprecise information at best.\textsuperscript{14} Similarly, consumers relying on producer reputation or warranties may misestimate the value of the guarantee or service contract as a signal of reliability and choose inadequate or excessive protection.

A main insight of this literature is that the most serious cases of concern about imperfect quality information are related to those situations where consumers are unable to easily verify thorough experience the performance of the product attributes and producer’s reputation plays a limited role. Imperfect quality information is therefore a particularly severe problem for infrequently purchased experience goods and for credence goods, where consumers only have an imprecise estimate of quality obtained from sources such as direct observation, brand name, word-of-mouth communication and repeated purchase.\textsuperscript{15} Experience goods are defined as products whose quality can be ascertained only after purchase. Examples may include drinks, home maintenance and repairs. Credence goods are products whose quality cannot be verified by consumers even after purchase. Examples may include properties of food, drugs or cosmetics and some kinds of professional services like legal and medical services. Moreover, some credence goods, like medical and legal services or repair services, suffer from the problem that the seller is also the expert who determined how much of the service is needed. Thus, imperfect information about quality creates obvious incentives for opportunistic behavior by the sellers.\textsuperscript{16}

\textsuperscript{12} However, the first best allocation of resources can be achieved by imposing liability on the producer. In particular, if consumers are risk neutral, making producers liable for the full loss experienced by consumers when product fails yields a first best outcome. Instead, if consumers are risk averse, the optimal level of insurance does not guarantee enough incentives for product liability. Therefore, producer liability towards the consumers should be supplemented with producer liability towards the state.

\textsuperscript{13} See for example Bagwell and Riordan (1991).

\textsuperscript{14} See the empirical study by Hjorth-Andersen’s (1991).

\textsuperscript{15} For a more detailed discussion of the cases where imperfect quality information is likely to be a severe problem, see for example Ramsey (1985).

\textsuperscript{16} A study by the Federal Trade Commission (1980) on the optometry industry documents
The OFT model of consumer detriment, which was included in the *Consumer Detriment* report (2000), aims at calculating the loss in consumer surplus due to presence of imperfect information about quality. In view of the complicated nature of the task, the OFT asked us to help them to identify any shortcomings of their approach, which models CD in a similar way to monopoly rent, and to suggest ways in which it could be improved. In light of this, the next sections are devoted to review the OFT model, correct some of its shortcomings and then propose a new and alternative approach to measure CD.

### 3.2 Review of the OFT model

The OFT model, included in the *Consumer Detriment* report (2000), aims at calculating the loss in consumer surplus that arises in the presence of imperfect information about quality; quality is assumed exogenous. In particular, it considers the case where consumers overestimate the quality of the product sold in the market and hence there exists a positive difference between the observed level of demand and the level of demand that would arise in the absence of quality misperception. Concentrating on the case where consumers overestimate the quality of the product can appear unduly restrictive. However, we do agree with the OFT that this is the most relevant case, since when consumers underestimate the quality of the product, firms have strong incentives to convey the positive information about their products to consumers, via warranties, product liability self regulations and low prices to induce learning via repeated purchases. Moreover, in the case where consumers underestimate the quality of the product, there is less concern about consumer detriment. Indeed, consumers may benefit from firms charging prices that are lower than under perfect quality information, although the optimal level of output will not be produced.

In the setting, the main assumptions of the OFT model are the following.

A1 The market structure under perfect information and in the presence of the information shortfall is a monopoly.

A2 The profit of the monopolistic firm are not affected by the presence of the information shortfall. In particular, it is assumed that, when the observed demand lies above the true demand, the threat of entry induces the monopolistic firm to spend a fixed amount of resources in order to make entry unprofitable.17

A3 Under perfect information, the monopolistic firm enjoys constant returns to scale ($MC = AC$). Under imperfect information, average costs raise because of the fixed amount of resources spent by the incumbent to deter

17 Although it is argued that the model accounts for both the case where entry takes place and when it does not, the model is compatible only with the first situation since $q_d$ is taken as a measure of the incumbent’s output and not the total industry output.
entry. As a consequence \( AC > MC \) and the difference between \( AC \) and \( MC \) is taken as a measure of the information shortfall (distance between observed demand and true demand).

Under these assumptions, the equilibrium is characterized by a higher price \((p_d)\) and a higher quantity \((q_d)\) with respect to the case of perfect information \((p_m, q_m)\).

There are a number of criticisms which may be directed at the OFT model, and these follow below.

a) Given the assumptions in the model, there is an important component of the Consumer Detriment calculation that is missing.

In particular, the increase in price and output above the perfect information level produces two effects on consumer surplus. First, consumers pay a higher price \((p_d)\) rather than \(p_m\) over \(q_m\) units of output. Second, over \(q_d - q_m\) units of output consumers pay a price \(p_d\) that is greater than their willingness to pay for those units as given by the area underneath the true demand curve between \(q_d - q_m\). In the existing model the first component is not fully taken into account and therefore consumer detriment is under-estimated.

Following the OFT, the Consumer Detriment is defined as the difference in consumer surplus between the case of perfect information (and the true demand coincides with the observed demand) and that of imperfect information, which in the case analyzed by the OFT occurs when the observed demand lies above the true demand. With the help of Figure 2, which reproduces the case analyses in the OFT model, we can easily identify the consumer surplus in each of these two cases. In particular:

\[
\text{Consumer Surplus under (}p_m, q_m\text{)} = A + B + D - D = A + B \\
\text{Consumer Surplus under (}p_d, q_d\text{)} = A + B + D + F - (B + C + D + E + F) = A - C - E
\]

The consumer detriment \((CD)\), which is the loss in consumer surplus due to the existence of imperfect information is obtained by taking the difference between the consumer surplus under \((p_m, q_m)\) and the consumer surplus under \((p_d, q_d)\). This yields

\[CD = B + C + E\]

In the OFT model the consumer detriment is determined as the sum of the areas \(C\) and \(E\); consequently part of the loss in consumer surplus due to the increase in price over \(q_m\) units of output is missing (area \(B\)).

\[\text{\textsuperscript{18}}\text{Implicit in this formulation is the assumption that income effects are small so that we can use the Marshallian demand to measure consumer surplus.}\]

\[\text{\textsuperscript{19}}\text{Notice that in both cases the willingness to pay of consumers is given by the area underneath the true demand curve.}\]
b) The model restricts attention to the case of an incumbent monopolist (A1), and this seems too restrictive, because monopoly is unlikely to be the representative industry structure.

c) The model is based on the assumption that in the absence of the information shortfall, the monopolist’s marginal cost and average cost are constant (A3). As the information shortfall occurs, the incumbent incurs additional costs in order to protect its profits from the threat of entry. These costs are assumed to be independent of the quantity produced so that \( AC \) raise above \( MC \) and their divergence results in a measure of the extent of the information shortfall. However, there can be other factors that create a divergence between marginal costs and average costs and, in the present formulation, it would be impossible to distinguish them.

d) The entire model would equally apply to a situation where the monopolist faces a real increase in demand. As a result there is a clear risk of overestimating consumer detriment.

e) Consumer detriment, as given by expression (21) in the OFT model is a function of the price level \( p_d \), which may be difficult to estimate.

f) Equation (16) in the OFT model takes \( \Delta q_d \) as a proxy for \( q_d - q_m \); it implies that \( q_d - q_m \) is zero when \( p_d = p_m \). However, in the presence of an information shortfall, the level of equilibrium output may vary even when the price remains constant. Therefore, equation (16) involves some loss of generality.

3.3 A revised OFT model

As explained in the previous section, the Consumer Detriment amounts to the sum of the areas \( B, C \) and \( E \) in figure 1, that is:

\[
CD = \Delta p_d Q_d + \left( \frac{\Delta Q_d}{2} \right) \frac{\partial p}{\partial Q_d} \tag{11}
\]

where the first term is the sum of the areas \( B \) and \( C \), while the second term is the area \( E \) in figure 2. Now, consider the expression (16) in the OFT model, which represents an estimate of the elasticity of demand in absolute value. In the presence of \( n \) identical firms in the market, it becomes

\[
\varepsilon_d = \frac{\partial Q}{\partial p} \frac{p_d}{Q_d} \approx \frac{\Delta Q_d}{\Delta p_d} \frac{p_d}{Q_d} = \frac{(Q_d - Q_m)}{\Delta P_d} \frac{p_d}{Q_d} \approx \frac{(q_d - q_m)}{P_d} \frac{p_d}{q_m} \tag{12}
\]

In light of this we can rewrite (11) as follows

\[
CD = \Delta p_d Q_d + \frac{\Delta Q_d \Delta P_d}{2} \tag{13}
\]
or, equivalently

\[ CD = \Delta p_d Q_d + \frac{(\Delta p_d)^2 Q_d \varepsilon_d}{2p_d} \]  

(14)

Moreover, (12) implies

\[ \Delta Q_d = (Q_d - Q_m) = \frac{\varepsilon_d \Delta P_d Q_m}{p_d} \approx \frac{\varepsilon_d \Delta P_d Q_d}{p_d} \]  

(15)

Now, consider expression (15) in the OFT model, which is derived under the assumption that the individual firm’s profit are not affected by the existence of imperfect information since the individual firm incurs additional costs in order to deter entry.

\[ \Delta p_d = p_d - p_m = (AC_d - MC) + \pi_d \left( \frac{1}{q_d} - \frac{1}{q_m} \right) \]  

(16)

Substituting (12) into (16) and rearranging, we obtain expression (17) in the OFT model, that is

\[ \Delta p_d = \frac{(AC_d - MC)}{1 + \frac{\pi_d \pi_d}{p_q q_d}} \]  

(17)

Substituting for (17) into (14) (or, equivalently, substituting for 17 and 15 into 13):

\[ CD = \left( \frac{(AC_d Q_d - MCQ_d)}{1 + \frac{\pi_d \pi_d}{p_q q_d}} \right) + \frac{1}{2} \left( \frac{(AC_d Q_d - MCQ_d)}{1 + \frac{\pi_d \pi_d}{p_q q_d}} \right)^2 \frac{Q_d ^2 \varepsilon_d}{p_d Q_d} \]  

Moreover, since \( AC_d Q_d - MCQ_d = -pQ_d + AC_d Q_d + pQ_d - MCQ_d \), which in turns is equal to \( \frac{(p_d - MC)}{p_d} p_d Q - \sum_n \pi_d \), we obtain

\[ CD = \left( \frac{(p_d - MC)}{p_d} p_d Q - \sum_n \pi_d \right) + \frac{1}{2} \left( \frac{(p_d - MC)}{p_d} p_d Q - \sum_n \pi_d \right)^2 \frac{Q_d ^2 \varepsilon_d}{p_d Q_d} \]  

(18)

Assuming that each of the \( i-th \) firm in the market is profit maximizing\(^{20}\):

\[ \frac{d \pi_d}{d q_d} = p_d + q_d \frac{d p_d}{d Q_d} (1 + \lambda) - MC = 0 \]  

\(^{20}\)Notice that the OFT limit pricing model is based on the assumptions that firms set prices optimally (i.e. to maximise profits), but then engage in cost increasing activities to make entry unprofitable.
where $\lambda = \frac{d(q-d)}{dq}$ is the conjectural variation parameter, and $q_{-d}$ represents the output of all firms except firm $i$. From (19):

$$\frac{p_d - MC}{p_d} = -\frac{q_d}{p_d} \frac{dp_d}{dQ_d} (1 + \lambda)$$

which taking into account that $q_d = \frac{Q_d}{n}$, implies (for $\lambda \neq 1$)

$$\varepsilon_d = \frac{p_d}{p_d - MC} \frac{1 + \lambda}{n}$$

Substituting for the above into (18)

$$CD = \left( \frac{(p_d-MC)p_dQ - \sum_n \pi_d}{1 + \frac{\sum_n \pi_d}{p_dQd} p_d-MC(1 + \lambda)} \right) + \frac{1}{2} \left( \frac{(p_d-MC)p_dQ - \sum_n \pi_d}{1 + \frac{\sum_n \pi_d}{p_dQd} p_d-MC(1 + \lambda)} \right)^2 \frac{p_d}{p_d - MC} \frac{1 + \lambda}{np_dQd}$$

(20)

The use of conjectural variation allows us to adapt the model to different industry structures. In particular, the case of a monopoly analyzed by the OFT can be obtained by setting $n = 1$ and $\lambda = 0$. The resulting expression for the consumer detriment is the same as in the presence of collusive behavior between the $n$ firms in the market, as it can be noticed by setting $\lambda = n - 1$. The case of quantity competition à la Cournot can be analyzed by setting $\lambda = 0$. Finally, the case of $\lambda = -1$, which represents Bertrand competition leading to a perfectly competitive outcome with $p = MC$, cannot be analyzed in this setting for at the price equals to marginal costs firm make losses. However, expression (18) for $\lambda \rightarrow -1$, may help understanding the performance of competitive, although not perfectly, industries. In particular, the case of monopolistic competition can be analyzed from expression (18) under the condition $\pi = 0$.

**IMPROVEMENTS ON THE OFT MODEL**

With respect to expression (21) in the OFT model, the following improvements have been achieved:

- The measure of the consumer detriment has been adjusted for the missing component (see point a in the previous section) in the OFT model.
- The model has been extended to industry structures other than monopoly (see point b in the previous section).
- Expression (20) does not require a price estimate (see point c in the previous section), for prices appear only in the mark-up and in the total revenues, which can be estimated.

\[\text{Notice that the second term in the above expression coincides with expression (21) in the OFT model; the first term accounts for the missing component (point a above).}\]
LIMITS

- Criticisms (c), (d), (f) remain.

ESTIMATION

In this section we have provided a measure of consumer detriment for the case of imperfect information about quality, along the lines suggested by the OFT model. See Section 4 for a discussion on how to estimate expressions (18) and (20).

3.4 A new approach

When consumers are imperfectly informed about the quality of the products, the observed market demand and the demand of a fully informed agent do not coincide. In particular, when consumers overestimate quality, which is the case analyzed here, the true demand schedule lies below the observed demand schedule and consumer detriment may arise for reasons described in the previous section.

In order to provide a measure of consumer detriment for this case, we need to make the following steps (as anticipated in the introduction).

- **Step 1)** To draw a one to one correspondence between information structure and prices (output), for any market structure,
- **Step 2)** To estimate the information structure, that is the level of precision in the consumers’ information (i.e. how much consumers overestimate/underestimate quality).
- **Step 3)** To characterize the market structure.

In order to make Step 1, we assume that consumers can observe prices, so that price dispersion does not arise for identical products. Under this assumption, the equilibrium price in the market will mainly be dictated by the level of observed demand and we can use economic models to predict how the allocation of resources varies with the level of demand. This implies that if we are able to make Step 2, that is to estimate the divergence between the observed demand and the true demand, then we can determine the effect of imperfect information about prices and outputs, for any given market structure.

In order to determine the level of precision of consumers’ information (step 2), two alternative approaches can be followed. The first one consists of viewing the consumers’ level of information precision as dependent of some observable variables. Assuming a precise functional form for this relationship we can then use the observable variables to derive an estimate of the consumer’s level of information precision.\(^{22}\) For example, one could argue that the level of precision\

\(^{22}\)This is the approach followed by the OFT in the research paper *Consumer Detriment* (2000), where it is argued that the level of information precision can be measured by the difference between average and marginal costs.
on the product quality is (inversely) related to the level of informative advertis-
ing, according to some assumed function, and then use the data on advertising
to derive an estimate of the level of imperfect information. This approach would
reflect the so called partial view in the economic literature that sees advertising
as an informative device that helps consumers to make rational choices.

However, we believe that this approach might involve a high loss of precision
and could yield inappropriate policy implications. Our reasoning is based on
the following considerations. Advertising may in fact be informative but only on
the ‘positive’ quality of the good being advertised; negative information may be
withheld. Firms may also create ‘artificial’ product differentiation. Moreover,
the advertising of a product has strong psychological and sociological aspects
that go beyond optimal inference of objective quality. Advertising agencies
often appeal to the consumers’ desire for social recognition and a trendy life
style. This is the so called adverse view which argues that advertising fools
consumers and induces them to overestimate the quality of a product as well as
its level of differentiation.

In light of this it should appear clear that any attempt to specify a functional
form for the relationship between the level of advertising and the degree of
precision of the consumer’s information would be rather arbitrary and could
involve a high loss of accountability. Moreover, it would lead drastic policy
implications such as a suggestion that the consumer detriment due to imperfect
information could disappear if the optimal (arbitrary chosen though!) level of
advertising were imposed on firms.

Therefore, we prefer to suggest an alternative approach, which consists in
treating the level of imperfect information as exogenous in the theoretical model
and restricting ourselves to the derivation of the level of consumer detriment
given the level of imperfect information. Then, in Section 4 we describe a tech-
nique to measure the level of imperfect information empirically. This technique
is based on the presumption that cases where consumers under-estimate the
quality of a product are less likely to occur, for firms have incentives to avoid
it. Hence, either via informative advertising, or via provision of warranties or
even through voluntary certification of a minimum quality standard firms will
manage to convey the positive information about their products.

Finally, we characterize the market structure (Step 3) on a case by case basis
and use this to select the appropriate model of firms interaction.

3.4.1 A measure of consumer detriment

Denote by \( \theta \) the level of imperfect information, as given by the vertical difference
between the observed demand schedule and the true demand schedule, which
are assumed to be parallel. We assume that firms make their price and output
decisions on the basis of the observed demand and not the true demand. Further,
we assume that the structure of the market under perfect information is the same
as that under imperfect information. Under these assumptions we can calculate
the optimal price (quantities) firms will charge (supply) for any level of demand
and hence also for the case where the observed demand coincides with the true
demand.

We construct a quantity competition model with conjectural variations in order to formalize the relationship between the equilibrium price and the level of market demand. As is known, different values of the conjectural parameter represent different oligopoly models and are consistent with market performance ranging from perfect competition to monopoly. In this setting we consider both a short run equilibrium with no entry and a long run equilibrium when entry occurs.

case a Conjectural Variation model

- No entry

Consider an industry with \(n\) firms \((n \geq 1)\) supplying a homogeneous product. The profit function of each firm is

\[
\pi_i = p(Q, s, \theta)q_i - C_i(q_i)
\]

where \(p(Q, s, \theta)\) is the market demand, that depends on the total output \(Q\) on the true quality \(s\) and on the consumer’s over-estimate of quality \(\theta\). Moreover, \(Q = q_i + q_{-i}\), where \(q_i\) is the \(i\)-th firm’s level of output and \(q_{-i}\) is the total level of output of all firms except firm \(i\). It follows that the first order condition for profit maximization is:

\[
\frac{d\pi_i}{dq_i} = p(Q, s, \theta) - \frac{dC(q_i)}{dq_i} + \frac{dp}{dq_i}q_i = 0
\]

where:

\[
\frac{dp}{dq_i} = \frac{dp}{dQ} \left( \frac{dq_i}{dq_i} + \frac{dq_{-i}}{dq_i} \right)
\]

Let \(\lambda_i = \frac{dq_i}{dq_{-i}}\), where \(\lambda_i\) represent the conjectural variation term, for simplicity assumed to be the same across firms: \(\lambda_i = \lambda\) for \(i = 1, 2, \ldots, n\). From the first order condition, it follows

\[
\frac{p - MC_i}{p} = \frac{q_i}{p} \frac{dp}{dQ} \left( 1 + \lambda \right)
\]

Multiplying the right hand side term by \(\frac{Q}{\varepsilon_Q}\):

\[
\frac{p - MC_i}{p} = \frac{s_i}{\varepsilon_Q} \left( 1 + \lambda \right)
\]

where \(\varepsilon_Q = -\frac{dQ}{dp}\) is the elasticity of demand in absolute value and \(s_i = \frac{q_i}{Q}\) is the \(i\)-th firm’s market share.

We can easily obtain different market performance by letting \(\lambda\) assume the following values:
Case 1 $\lambda = -1$: Bertrand duopoly with $p - MC_i = 0$; that is a situation equivalent to perfect competition.

Case 2 $\lambda = 0$: Cournot duopoly $\frac{p - MC_i}{p} = \frac{1}{\epsilon Q}$ which is equal to $\frac{1}{ncQ}$ if all firms are identical.

Case 3 $\lambda = n - 1$: Co-operative behavior with identical firms: $\frac{p - MC_i}{p} = \frac{1}{\epsilon Q}$

Case 4 $\lambda = 0$ and $n = 1$: monopoly $\frac{p - MC_i}{p} = \frac{1}{\epsilon Q}$

We assume that firms set prices and outputs on the basis of the observed demand level. Therefore, the case where the difference between the observed demand and the true demand is equal to $\theta$ is equivalent - in terms of price and output - to a situation where the firms face an increase in demand equal to $\theta$. This allows us to calculate the consumer detriment by analyzing how prices and quantities vary with the level of demand. In particular, given the definition of CD, the sum of the areas $B$, $C$ and $E$ in Figure 3, yields:

$$CD = \Delta PQ - \Delta Q \left( \frac{\Delta Q \partial P}{2 \partial Q} \right)$$  \hspace{1cm} (22)

it follows that

$$CD = \Delta P \frac{\Delta Q}{\Delta \theta} \Delta \theta - \frac{1}{2} \left( \frac{\Delta Q}{\Delta \theta} \Delta \theta \right)^2 \frac{\partial P}{\partial Q}$$

where, since under perfect information, $\theta = 0$, in the above formula $\Delta \theta = \theta$. As $\theta$ increases the level of the observed demand raises as well as the level of imperfect information. Hence, this approach allows us to calculate the consumer detriment by analyzing how prices and quantities vary with the level of demand.

For simplicity we restrict the attention to a linear demand function, constant marginal costs and identical firms. In this setting, we obtain:

$$\frac{\partial q_i}{\partial \theta} = \frac{\pi q_i}{\pi q} = -\frac{p_0}{\partial \theta} \left( 1 + \lambda + n \right)$$

and

$$\frac{\partial Q_i}{\partial \theta} = \sum_i \frac{\partial q_i}{\partial \theta} = n \frac{\partial q_i}{\partial \theta} = -\frac{n p_0}{\partial \theta} \left( 1 + \lambda + n \right)$$  \hspace{1cm} (23)

Multiplying the numerator and denominator of (23) by $\frac{\partial Q}{\partial \theta}$ and making use of $\Delta Q = \frac{\partial Q_i}{\partial \theta} \Delta \theta$, we obtain
\[
\frac{\Delta Q}{Q} = \frac{n \varepsilon_\theta \varepsilon_Q}{(1 + \lambda + n)} \frac{\Delta \theta}{\theta}
\]  
(24)

where \( \varepsilon_\theta = \frac{\partial p}{\partial \theta} \) is the elasticity of prices with respect to the level of perceived quality and \( \varepsilon_Q = -\frac{\partial Q}{\partial p} \) is the elasticity of demand (in absolute value) with respect to the price.

Now consider the effect of a change in \( \theta \) on the equilibrium price level; this is given by:

\[
\frac{dp}{d\theta} = p_\theta + \frac{\partial p}{\partial Q} \frac{\partial Q_i}{\partial \theta}
\]

which in light of (23) yields

\[
\frac{dp}{d\theta} = \frac{p_\theta (1 + \lambda)}{(1 + \lambda + n)}
\]

and

\[
\Delta PQ = Q \frac{\partial p}{\partial \theta} \Delta \theta = Q \frac{p_\theta (1 + \lambda)}{(1 + \lambda + n)} \Delta \theta
\]

multiplying the above by \( \frac{\varepsilon_\theta}{\varepsilon_Q} \) :  

\[
\Delta PQ = Q \frac{\frac{pQ (1 + \lambda)}{(1 + \lambda + n)} \varepsilon_\theta}{\varepsilon_Q} \frac{\Delta \theta}{\theta}
\]  
(25)

Now, recall the formula for the consumer detriment (22), multiplying the second term in the above expression by \( \frac{Q^2}{p} \frac{\varepsilon_\theta}{\varepsilon_Q} \), we can rewrite (22) as follows:

\[
CD = \Delta PQ + \frac{1}{2} \left( \frac{\Delta Q}{Q} \right)^2 \frac{pQ}{pQ}
\]

Substituting for \( \Delta PQ \) from (25) and for \( \frac{\Delta Q}{Q} \) from (24) in the above formula and making use of \( \frac{\Delta \theta}{\theta} = 1 \), we obtain:

\[
CD = \frac{pQ (1 + \lambda) \varepsilon_\theta}{(1 + \lambda + n)} + \frac{1}{2} \left( \frac{n \varepsilon_\theta}{(1 + \lambda + n)} \right)^2 pQ \varepsilon_Q
\]  
(26)

where for \( \lambda \neq -1 \), \( \varepsilon_Q = \frac{p}{p-MC}s_i (1 + \lambda) \) from expression (21).
For different values of \( \lambda \) we obtain different values of \( CD \). In particular, for \( \lambda = 0 \) (Cournot competition)

\[
CD_{\lambda=0} = \frac{pQ\varepsilon_0}{(1+n)} + \frac{1}{2} \left( \frac{n\varepsilon_0}{(1+n)} \right)^2 pQ\varepsilon_Q
\]

Under monopoly \((\lambda = 0, n = 1)\)

\[
CD_{\lambda=0,n=1} = \frac{pQ\varepsilon_0}{2} + \frac{1}{2} \left( \frac{\varepsilon_0}{2} \right)^2 pQ\varepsilon_Q
\]

The above expression also represents the consumer detriment in case of collusion between identical firms (captured by \( \lambda = n - 1 \)).

In the case of Bertrand competition, i.e. perfect competition, we have:

\[
CD_{\lambda=-1} = \frac{1}{2} (\varepsilon_0)^2 pQ\varepsilon_Q \tag{27}
\]

**ENTRY**

In the previous section entry does not occur. This is a reasonable case when either sunk costs of entry are sufficiently high or when the level of imperfect information tends to disappear quickly, so that the original equilibrium is re-established. In this section we introduce the possibility that new firms enter the market, when there are sunk costs of entry equal to \( F \).

In order to derive the equilibrium prices and outputs under entry we need to calculate the equilibrium number of firms in the market, which further highlights the need to restrict attention to a very simple model, with linear demand, identical firms and constant average costs.

When entry is allowed, under the assumptions of our model (see Hamilton, 1999) for a more general case, the equilibrium prices decreases and the long run equilibrium is characterized by a level of price that is the same as under perfect information, and a greater level of output. In the appendix we prove that in this case the consumer detriment amounts to\(^{23}\)

\[
CD = \frac{1}{2} \varepsilon_Q \varepsilon_0^2 pQ \tag{28}
\]

Notice that the above equation is the same as expression (27), since the price does not vary and the increase in demand is totally reflected in the increase in the quantity produced. However, the value of \( pQ \) is clearly different.

**Review of the Model**

\(^{23}\)This expression represents the area ABC in figure 4.
Recall point (d) in the previous section. We believe that unless we are able to assess the extent by which the true demand diverges from the observed demand we may be mislead in the calculation of CD, in particular, we may measure as consumer detriment what is in fact is the effect of an increase in the real demand.

The current formulation is not based on any particular assumption on the market structure. We can use a case by case approach to evaluate the market structure and make an appropriate choice of $\lambda$, and then use the measure of consumer detriment that corresponds to that particular $\lambda$. However, the implicit assumption in this formulation is that the market structure does not vary with the structure of information.

The model assumes that imperfect information leads to a parallel shift in the observed demand and that marginal costs are constant. This was necessary in order to derive an estimable measure of consumer detriment.\footnote{See Hamilton (1999) for a more comprehensive theoretical case.}

In order to be able to measure the degree of information precision in the hands of consumers, according to the econometric method described in Section 4, we had to restrict attention to cases where under imperfect information consumers over-estimate the quality of the product. The model is therefore not appropriate to those cases where it is reasonable to assume that imperfect information can lead to under-estimation of quality (e.g. new products).

**Estimation**

In the analysis above we have provided two measures of the consumer detriment, one for the short run and one for the long run. These are given respectively by equations (26) and (28). See Section 4 for a discussion on how to estimate these equations.

### 4 Estimation approach

In this section we provide some insights as to how to estimate the expressions for the consumer detriment derived in the report.

- **Estimate of the markup** $\frac{p_i - MC}{p_i}$

  Nishimura et al. (1999) provide a method for estimating mark-up over marginal cost at the firm level, which can be used to estimate equations (7) (10) (18) and (20).\footnote{Note that, once we have estimated the markup we can easily obtain an estimate of $\frac{MC}{p_i}$ (for expression 7) and of $\frac{p_i - MC}{p_i}$ (for expression 10) as follows. From the markup $\frac{p_i - MC}{p_i}$, we can obtain an estimate of $\frac{MC}{p_i}$ by making use of the fact that $\frac{MC}{p_i} = 1 - \frac{p_i - MC}{p_i}$. Then, we can calculate $\frac{1}{p_i}$ to obtain $\frac{MC}{p_i}$ and hence $\frac{p_i - MC}{p_i}$, which is equal to $\frac{p_i}{\frac{MC}{p_i}} - 1$.} The method uses the following identity, based on two measures of the output elasticity (see Nishimura et al.,1999 p.1086):
\[
\mu([\alpha_K]_t + [\alpha_L]_t) = (1 + \frac{V_{t-1}}{Q_t})(1 - \eta(\frac{\Delta S_t}{S_{t-1}}))
\]  

(29)

Where \(\mu\) is the mark up, \(\alpha_j\) the Labour and Capital shares for \(j = L, K\), \(V_{t-1}\) is last period’s maximum output, \(Q_t\) is output and \(S_t\) is a measure of size of operation (for ease of exposition the firm specific index is suppressed). Taking logs of (29):

\[
\log(\mu) + \log([\alpha_K]_t + [\alpha_L]_t) = \log(1 + \frac{V_{t-1}}{Q_t}) + \log(1 - \eta(\frac{\Delta S_t}{S_{t-1}}))
\]

By approximating the logarithms on the right side using a first order Taylor’s series, assuming \(1 + \frac{V_{t-1}}{Q_t} \approx 0\) and \(1 - \eta(\frac{\Delta S_t}{S_{t-1}}) \approx 0\), it follows:

\[
\log(\mu) + \log([\alpha_K]_t + [\alpha_L]_t) = \frac{V_{t-1}}{Q_t} - \eta(\frac{\Delta S_t}{S_{t-1}})
\]

It is common to assume that the share depends on some other variables, such as the cycle of the economy or market specific factors, so that:

\[
\log(\mu) = \log(\mu_0) + \phi \log(x_t)
\]

Where \(x_t\) is a single variable or a vector of variables. Substituting out for \(\mu\):

\[
\log([\alpha_K]_t + [\alpha_L]_t) = \log(\mu_0) + \phi \log(x_t) + \frac{V_{t-1}}{Q_t} - \eta(\frac{\Delta S_t}{S_{t-1}})
\]

Nishimura et al. make the additional assumption that \(V_{t-1} \approx Q^*_t\), where \(Q^*_t\) may be measured by trend output. Hence, the equation to be estimated is:

\[
\log([\alpha_K]_t + [\alpha_L]_t) = \log(\mu_0) + \phi \log(x_t) + \frac{Q^*_t}{Q_t} - \eta(\frac{\Delta S_t}{S_{t-1}})
\]

The equation estimated by Nishimura et al. takes the following form:

\[
\log([\alpha_K]_{t}^{jk} + [\alpha_L]_{t}^{jk}) = \alpha_0^{jk} + \alpha_1^{jk}market_t^{jk} + \alpha_2^{jk}norcur_t^{jk} + \alpha_3^{jk}scale_t^{jk} + \sum_{t=1972}^{1994} \alpha_4^{jk} Year_t + u_t^{jk}
\]

where \emph{market} is the market condition of the firm measured by the ratio of net cash flow to the asset value, \emph{norcur} is \(\frac{Q^*_t}{Q_t}\) measured as the ratio of trend to actual sales, \emph{scale} is the growth rate of the firm’s scale, which is intended to be a determinant of managerial experience and \emph{Year} is a year dummy that is supposed to capture transitory technological shocks.

Nishimura et al. estimate the above model using a Panel of firms drawn from 21 industries over the time period 1971-1994. They conclude for the Japanese
case, that imperfect competition is the predominant market structure. Given ownership of UK firms, globalization and some similarities between Japan and the UK, then a similar conclusion might be relevant for the UK. Given the possibility of measurement error in all the variable the model is estimated by Instrumental Variables.\textsuperscript{26}

The estimated equation does not allow for unmodelled heterogeneity in the structure of the mark-up either across the sectors or the firms. Clearly, the measure of the mark-up might be suggestive of concentration or informational market power. It is possible to include firm specific and industry specific fixed effects (i.e., see the model on page 1098 of Nishimura \textit{et al.}, 1999). Nishimura \textit{et al.} also observed pro-cyclical behavior in the mark-up suggesting that such measures would require regular updating.

Finally, the theoretical analysis discussed is essentially based on comparative statics, which might be appropriate for a theoretical discourse of the type engaged in here. However, the process generating the data might well be dynamic. There is a plethora of literature that considers dynamic estimation of panels (Arrelano and Bond, 1991). In general such analysis assumes that the theoretical basis of the dynamic process derives from solutions to forward looking expectational models (Hansen and Sargent, 1982; West, 1995). In general, from where a dynamic derives is not an easy question to consider. This question has been partly addressed in the Panel context by Arrelano \textit{et al.} (1999), though Hunter and Ioannidis (2000) call into question the assumption that forward looking behavior can be simply identified from the existence of valid instruments.

One also needs to consider the appropriateness of selecting the difference operator and excluding long-run information or the appropriateness of the initial conditions (Blundel and Bond, 1998). The above paper considers what might be called a Panel Cointegration case (Engle and Granger, 1987), which implies that difference models are likely to be misspecified as under Cointegration there is a partial over-difference when the series are non-stationary. Should the relationship be static, then what is likely to be observed is the long-run behavior of the data.

**Data required for Estimating the Mark-up.**

In Appendix A, Nishimura \textit{et al.} (1999) describe carefully the calculations of the variables used in the study. This can largely be replicated in the UK, with the following adjustments. First, in estimating the capital stock $K$, they exclude land because of the huge discrepancy between book and market values. In the UK revaluation is permitted and therefore the exclusion is less necessary. Second, the estimation of the rental price of capital stock includes an economic rate of depreciation taken from KEO Data Base. This appears to be Japan specific and therefore we suggest to estimate depreciation rates with reference to accounting depreciation as a ratio to fixed assets.

\textsuperscript{26}Arrelano and Bond (1998) have a programme, DPD, which estimates dynamic panel data models and provides appropriate tests of instrument selection and Serial Correlation.
Critical review of the equations involving the estimate of the mark-up based on accounting information alone

Equations (7), (8) and (10) are similar equations in form.

Equation (7) has the advantage of only depending on the calculation of the mark-up based on the Nishimura estimator given above and knowledge of revenues $R_i$ and $R^*$ both calculated from total sales. In particular, in order to obtain a proxy for $R^*$, we can use two pieces of information: the mark-up and the ratio $\frac{AC_i}{MC_i}$. When the ranking of firms based on those pieces of information is coherent, then we select the revenue of the firm with the lowest $\frac{AC_i}{MC_i}$ ratio. When it is not coherent, then we look at a weighted average of these two measures to select the firm whose revenue is taken as proxy for $R^*$.

Moreover, if the suggestion in Nishimura et al. (1999) is valid, that $MC = AC$ for the effective range of the cost function generally observed, then equation (8) can be calculated directly from data on profits, sales and total costs.

Equation (10) can be given the following form:

$$CD = \sum_{i=1}^{n} \frac{1}{2} \left\{ R_i (1 - \alpha_i (1 - \mu_i)) + R^* \left( \frac{1}{\alpha_i (1 - \mu_i)} - 1 \right) \right\}$$

given the mark-up $\mu_i$, $R_i$, $R^*$ and a measure of $\alpha_i$ given by $\frac{AC_i}{MC_i}$, where $AC_i$ is a measure of the average costs of the most efficient firm that can be obtained by looking at the minimum cost producer in the industry. This is dependent on the accuracy of the measure of $\alpha_i$, which depends on proportionality between the ratio $\frac{AC_i}{MC_i}$ and $\frac{MC_i}{MC}$. The latter would appear less restrictive, than $MC = AC$ on which equation (8) is based. Hence, it would appear preferable to have some measure of the mark-up in the CD measure to be used. In this light, (7) appears the least restrictive, though it might well be compared with (10).

Equation (18) and (20) are based on the OFT definition and yield a correction of that formula. However, (18) requires in addition to the mark-up the calculation of the demand elasticity, which would involve estimating demand equations for each of the goods to be analyzed. Such analysis requires sales data and some measure of price. Should it be necessary to make such calculation, the degree of accuracy would be limited to an analysis at the level of CIB industrial classification aggregate data, or it requires the type of study that is suggested below for all goods in the economy. This leads us to consider (20), which still requires the mark-up, but also this depends on a measure of $\lambda$. Again, this calculation might be based on some pre-existing measure of concentration in the industries or it could be derived from the estimate of the markup. Equation (20) depends on profits, sales, $n$, $\lambda$ and $\mu_i$.

- Estimate of $\theta$, $\varepsilon_\theta$ for expressions (26) and (28).

In order to estimate $\theta$ and hence of $\varepsilon_\theta$ we need to obtain an estimate of the ‘imputed’ demand curve, that is the estimate of the price that - for the given
quantity sold - would prevail in the market if there was no overestimate of the quality embodied in the commodity. To this purpose, we suggest the following procedure, that is based on the explicit incorporation of quality into the price equation.

Following the approach taken by Houthakker (1951-1952) in order to incorporate the concept of quality into the consumption of a commodity we will describe consumption of commodity \( f \) by two variables: the physical quantity \( q_i \) and quality \( v_i \). The latter number which indicates the variety bought, is defined as the price per unit of 'that variety, under some basic price system. The cost of \( q_i \) units of quality \( v_i \) will then be \( \{q_i v_i\} \).

We seek to explain price changes which cause the cost of this consumption \( \{q_i, v_i\} \) to become \( q_i \{a_i + b_i v_i\} \) where \( a_i \) and \( b_i \) are constants such that \( a_i > 0 \) and \( a_i + b_i v_i > 0 \forall v_i \). That is, we decompose the observed price into 'quantity price' and 'quality price'. The former is identical to the price of the good in conventional consumer theory. The 'quality price' shows the price differential between different qualities. Under the basic price system \( a_i = 0 \) and \( b_i = 1 \).

The insight of Houthakker’s observation is that the price of the commodity could be divided into two ‘measurable’ subcomponents; a component that relates to the quantity consumed and another that relates to the level of ‘quality’ within the commodity. Thus, the price of a good can be defined as:

\[
p_i = \alpha_i + b_i v_i
\]

The parameter \( \alpha_i \) represents the 'quantity price' and the parameter \( b_i \) was labeled the 'quality price' (see figures 5 and 6).

Total consumer spending can thus be defined as

\[
M_i = (\alpha_i + b_i v_i)q_i
\]

the slope of the constraint is given by

\[
\left( \frac{\partial q_i}{\partial v_i} \right) = -\left[ b_i M_i / (\alpha_i + b_i v_i)^2 \right] < 0
\]

and its curvature is given by

\[
\left( \frac{\partial^2 q_i}{\partial v_i^2} \right) = \left[ 2M_i b_i^2 / (\alpha_i + b_i v_i)^3 \right] > 0
\]

Differentiating the constraint by \( \frac{\partial q_i}{\partial a_i} \) and \( \frac{\partial q_i}{\partial b_i} \), we obtain

\footnote{Note that the estimate of \( \epsilon_\theta \) can be obtained once estimated \( \theta \), since the expression \( p_\theta \) in \( \epsilon_\theta \) refers to the derivative of \( p \) with respect to \( \theta \). This is equal to 1 under the assumption that imperfect quality information leads to a parallel shift in the observed demand function.}
\[
\frac{dq_i}{db_i} = -[v_i M_i/(\alpha_i + b_i v_i)^2] < 0
\]

and
\[
\frac{dv_i}{db_i} = [(\alpha_i q_i^2 - M_i q_i)/(b_i q_i)^2] \text{normally} < 0
\]

Grilishes (1971) has shown that the use of the parameters \( a_i, b_i \) is the foundation for the hedonic technique of adjusting prices indices for variation in quality. This technique assumes that commodity prices \( p_{it} \) are a function of a set of ‘quality characteristics’ \( \{ X_{jit} \} \) where the index \( j \) lists the characteristics. When necessary due to quantitative limitations dummy variables are used for the \( X_{jit} \). Different functional forms are possible, however the semi-log functional form seems to be better in describing the data, that is

\[
\log(p_{it}) = \sum_{j=0}^{k} \alpha_i X_{jit} + u_{it}
\]

Houthakker’s \( b_{ij} = \frac{\partial p_{it}}{\partial v_{it}} \) correspond to the \( a_i \) coefficients in the above equation.

In a series of papers Ioannidis and Silver (2000) and Ioannidis et al. (2001) have estimated quality adjusted price indices for consumer durables such as TV’s and Video recorders. Their models included, in addition to the set of characteristics, further explanatory variables, such as the quantity sold, thus allowing for non-perfectly competitive markets. The estimated specifications are

\[
\log(p_{it}) = \sum_{j=0}^{k} \alpha_i X_{jit} + \gamma \{ z(q) \}_{it} + u_{it}
\]

where the vector of additional variables \( z \) includes quantities sold and other variables that used as proxies for the price cost-margin.

The presumption in this study is that consumers fail to appreciate correctly the ‘quality’ of the product, in fact they overestimate it, thus are paying higher than otherwise prices. In the light of the evident failure of consumers to optimize, it is desirable to recast the hedonic analysis away from the traditional functions towards frontiers. The econometric implication of the proposed reformulation from functions to frontiers is that the symmetrically distributed stochastic error term with zero mean is no longer the appropriate specification when analyzing this type of consumer behavior. By adopting frontier functions we allow for the possibility (as an ‘equilibrium’ position) that consumers will end up paying above the deterministic kernel of the price function due to their
postulated tendency to overestimate the ‘quality’ of the commodity, this deficiency may be due to the operating environment (‘exaggerating’ advertising, non-linear search cost, etc.). Stochastic frontier analysis (SFA) dates from a series of papers published in 1977 (Meeusen and van den Broeck, 1977, Aigner et al. 1977) and were applied to tests for allocative inefficiency in production. For a detailed summary and the theoretical foundations consult Kumbhakar and Lovell (2000).

For this study we propose the stochastic frontier model:

$$\log(p_{it}) = \sum_{j=0}^{k} \alpha_j X_{j,it} + \gamma\{z(q)\}_{it} + u_{it} - \theta_{it}$$

In this case $u_{it}$ is the two sided noise component and $\theta_{it}$ is the non-negative component that captures the quality overestimate (i.e. $\theta$ in the previous sections). Since the error term has two components this frontier model is often referred as the ‘composed error’ model. It is assumed that $E(u_{it}\theta_{it}) = 0$. The error term

$$e_{it} = u_{it} - \theta_{it}$$

is asymmetric and it does not possess zero mean, by construction. Estimation by OLS will not provide consistent estimates of the constant term, but the remaining estimated coefficients will be consistent. Schmidt and Lin (1984) suggest an asymptotic test for the existence of negative skewness in the OLS residuals. But in this case our interest lies in obtaining quantitative estimates of the ‘overestimation’ of quality rather than testing for its existence. To obtain them we require explicit modeling of the composite error structure and estimation by MLE.

Several options are available to us: a) normal, half-normal b) Normal, exponential and c) Normal, truncated normal.

The empirical evidence to date supports the adoption of relatively simple distribution such as half-normal and exponential, when modeling $\theta_{it}$. Estimation of the ‘inefficiency’ component can be undertaken using GMM and/or MLE techniques.\footnote{\textsuperscript{28}}

We postulate that the fitted values from the estimated equation, after the subtraction of the inefficiency component ($\theta_{it}$) will constitute our measure of prices that consumers would have paid had they not overestimated the quality of the commodity. Finally, note that in the hedonic regression one the $\gamma$ coefficients is associated with the quantities sold. This is taken to be equivalent to $\frac{\partial p}{\partial Q}$ in Section 3.4. We can then evaluate the elasticity $\varepsilon_Q$ at the point of interest.

\textbf{DATA REQUIREMENTS}

\footnote{\textsuperscript{28}For an in depth discussion consult Greene (1993,1997).}
This approach can only be implemented on a product basis than on an industry basis.\textsuperscript{29}

In estimating quality adjusted price indices Ioannidis and Silver used EPOS data (Electronic Point Of Sale). These data contain information for a broadly defined consumer durable such TV, Video recorder, PC, cars etc. The information consists of

a) the price per model,

b) the model’s technical characteristics (e.g.: The screen size (for TV’s), the Hard Disk capacity (for PC’s),

c) the brand name (e.g. SONY, for TVs), DELL, for PCs),

d) the number of units sold,

e) other information such stocks held etc.\textsuperscript{30}

5 Conclusions

In this report we have discussed different models and estimating techniques to measure the consumer detriment due to the presence of imperfect information about prices and quality. In particular, in Section 2 we have provided a simple measure of consumer detriment for the case of imperfect price information given by equations (7), (8) and (10). In Section 3 we have analyzed the case of imperfect quality information and discussed two alternative approaches to measure the consumer detriment for this case, leading respectively to equations (18), (20), and (26), (28). Some of these equations involve unobservable components. In particular, $R^*$ and $\theta$ can only be approximated from observed data.

For equations (7), (8) and (10) one can extract an estimate of $R^*$ using publicly available accounting data at the firm level, employing the method in Nishimura et al. (1999), as discussed in Section 4. In contrast, the estimate of $\theta$ in equations (26) and (28) requires EPOS type data by broadly defined product group. The econometric technique of stochastic frontier hedonic functions is suggested in order to obtain and estimate of $\theta$ and the other parameters involved in (26) and (28).

In principle, it is possible to estimate all of the equations discussed above. However, the data requirements, theoretical rigor and econometric complexity differ considerably. The quality of any results is likely to be affected by the assumptions and approximations made. As far as estimating the effect of imperfect information about price is concerned, then either (7) or (10) are preferred

\textsuperscript{29}However, note that the important methodological point in this section is the definition of ‘product’. As we refer in the text we have adopted the notion of ‘broadly defined products’. Such a product is, for example color televisions. This product is defined by a set of characteristics. One of then screen size covers the whole range of CTVs in the market. A producer may be in the TV market with a whole range of TV’s, that will all be accounted for as the index of screen sizes covers the ‘broadly defined product’. What is of importance in this case, is that there exists a characteristic that can span the whole of the ‘product’ range. In the example we offer, we also suggest that it is important to account for the quantities sold for each screen size by every firm.

\textsuperscript{30}For a full discussion please consult Ioannidis and Silver (2001).
to (8) as they are less restrictive; but they require estimates of the mark-up. From a theoretical perspective calculating consumer detriment due to imperfect information about quality is more satisfactory when either (26) or (28) are used. However, (26) and (28) rely on estimates of \( \theta \), a relatively complex econometric method and collection of data on product prices and sales. Should the mark-up have already been calculated, then (20) could be used directly to compute consumer detriment, though a simple calculation would rely on some strong assumptions about market structure.

6 Appendix

- ENTRY

In this appendix we derive the long run equilibrium for the model discussed in Section 3.4.1. Denote by \( F \) the level of sunk costs and by \( c \) the level of average costs. The demand function is given by

\[
p(Q, s, \theta) = a - bQ
\]

where \( a = A + \theta \), and \( \theta \) represents the vertical difference between the observed demand and the true demand.

From profit maximization and simple calculations we obtain the following equilibrium variables

\[
q = \frac{a - c}{b(n + 1 + \lambda)}
\]

\[
Q = \frac{n(a - c)}{b(n + 1 + \lambda)}
\]

\[
p = \frac{a(1 + \lambda) + nc}{(n + 1 + \lambda)}
\]

\[
p - c = \frac{(a - c)(1 + \lambda)}{(n + 1 + \lambda)}
\]

It follows that the level of profit of each of the firm in the market is given by

\[
\pi_i = \frac{(a - c)^2 (1 + \lambda)}{b(n + 1 + \lambda)^2}
\]

(30)

In this setting the long run equilibrium number of firms solves
\[ \pi_i = F \]

which in light of (30), yields an implicit expression for the number of firms, as given by

\[(n + 1 + \lambda)^2 = \frac{(a - c)^2(1 + \lambda)}{bF} \]  \hfill (31)

It follows that

\[ CD = \frac{\partial CD}{\partial \theta} \Delta \theta = \frac{dp}{d\theta} Q \Delta \theta + b \left( \frac{dQ}{d\theta} \right)^2 (\Delta \theta)^2 \]

where:

\[
\frac{dp}{d\theta} = \frac{\partial p}{\partial \theta} + \frac{\partial p}{\partial n} \frac{\partial n}{\partial \theta}
\]

\[
\frac{dQ}{d\theta} = \frac{\partial Q}{\partial \theta} + \frac{\partial Q}{\partial n} \frac{\partial n}{\partial \theta}
\]

and where from (31)

\[
\frac{\partial n}{\partial \theta} = \frac{(a - c)(\lambda + 1)}{(\lambda + 1 + n)bF}
\]

Simple calculations show that \( \frac{dp}{d\theta} = 0 \) and \( \frac{dQ}{d\theta} = \frac{1}{p} \), so that:

\[ CD = \frac{\partial CD}{\partial \theta} \Delta \theta = \frac{1}{2} \frac{dp}{d\theta} (\Delta \theta)^2 = \frac{1}{2} \frac{1}{b} (\Delta \theta)^2 \]

Finally, multiplying the right hand side of the above equation by \( \frac{p^2Q^2}{x^2} \theta^2 \rho \theta \rho \), and recalling that \( \rho = 1 \) and \( \Delta \theta = \theta \), we obtain:

\[ CD = \frac{1}{2} \frac{\xi Q^2 \theta^2 \rho Q}{x^2} \]

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References


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Figure 1: Consumer Detriment per individual firm
Figure 2: Consumer Detriment in the OFT model
Figure 3: Our approach: Consumer Detriment in the short run
Figure 4: Our approach: Consumer Detriment in the long run.

Figure 5: Price quality relationship
Figure 6: Price quality relationship against basic price system