## THE DEVELOPMENT OF A BEARING OF HIGH

STIFFNESS AND A WIDE SPEED RANGE.

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## TEXT IN ORIGINAL IS CLOSE TO THE EDGE OF THE PAGE

## TO MY WIFE AND CHIIDREN WHO

MADE ALL THIS WORK WORTHWHILE

$$
\begin{aligned}
& \text { إلى زو } \\
& \text { جـعلوا مذا الـعـلـــرى النـور }
\end{aligned}
$$

"Summary"

The thesis describes work carried out in the development of a bearing for a wide speed range, and of high stiffness. The intended speed range is from zero to 9000 rpm , and the intended load capacity is approximately $6.6 \times 10^{4} \mathrm{~N}(6.7$ tonne) in the low speed range and $4 \times 10^{3} \mathrm{~N}$ ( 0.41 tonne) in the high speed range. The intended stiffness is approximately $8.75 \times 10^{8} \mathrm{Nm}^{-1}$ (2300 tonne in ${ }^{-1}$ ) in the low speed range and $3.5 \times 10^{8} \mathrm{Nm}^{-1}$ ( 905 tonne in ${ }^{-1}$ ) in the high speed range.

It appeared that the bearing would have to work hydrostatically at low speed, and would, of course, become self generating at high speed. The bearing which has been designed is of three-lobe form with the intention of enhancing stiffness and stability in the selfgenerating regime. High pressure inlets with no pockets are provided in the mid lobe positions, and low pressure inlets are provided at the apices of the lobes.

Two development bearings of 135 mm bore and 127 mm width have been made. A bearing-spindle system has been investigated in two full scale test rigs. In the first test rig in which there was no rotation of the journal, the hydrostatic performance of the bearing and the hydrostatic performance of the oil control devices were tested. These tests showed the bearing to be satisfactory with respect to load capacity, stiffness and with respect to the operation of the constant volume flow valves which were chosen as the control devices. The second test rig was built to represent an actual
arrangement for a machine tool spindle. The three lobe bearing in both externally pressurized and self-generating regimes has been tested over a speed range from zero to 7000rpm, which was the limit for the available drive, and with two oils of different viscosities. The results from these tests showed that the behaviour of the bearing was satisfactory up to the highest speed (7000rpm) and revealed that spindle deflection had a significant effect upon overall stiffness. Typically at 7000 rpm , the power loss was 8.6 kW , and the oil temperature rise through the bearing was $39^{\circ} \mathrm{C}$.

In addition to the design study and the test work, theoretical studies have been carried out. In the initial design stage an approximate analytical prediction was made of bearing stiffness in the hydrostatic regime in order to establish that feasible and convenient values for supply pressure, oil flow quantity and pumping power were possible. An approximate prediction was also made (and this drew upon published results for circular bore bearings) of the stiffness of the bearing in the hydrodynamic regime. The evaluations from these approximate analyses were made by means of two programmes written for the purpose. Later, to provide more realistic solutions, Reynolds equation was expressed in finite difference form, and a computer programme was written for the numerical solution of Reynolds equation. Various versions of this programme have been developed for several particular applications. The applications which have been investigated are:

1) A programme for the self-generating regime. In one version of the programe the boundary conditions at the low pressure grooves are assigned to zero dimensionless pressure (i.e. the conventional boundary conditions at the inlet grooves), and in another version the boundary conditions are assigned to non-zero dimensionless pressure. The listings (i.e.
programmes themselves) and a sample of the outputs are given in Appendices. Outputs from these programmes have been compared with test results of the three-lobe bearings, and with the published results for circular bore bearings.
II) A programme for the self-generating regime of full circular
bore journal bearings. The results from this programe have
been compared with published results for full circular bore
bearings ( 360 degree bearings).
III) A programme in which the oil flow quantity is fixed to a precalculated oil flow for a stationary journal centred within its bearing. This programme has been applied to hydrostatic operation without journal rotation, and to the operation with journal rotation in which self-generating might be present to limited or to dominant extents. In one version of the programme the fixed oil flow is supplied at mid-lobe positions and in another version the fixed oil flow is supplied at the apices of the lobes. Outputs from these versions are compared with test results and with calculated results from (I).

The dimensionless outputs of these programmes have been used in a further programme "The Design Programme" which gives values of power loss, oil flow, oil temperature rise, load and stiffness for a bearing specified dimensionally.

The question always arises in a development of a programme whether all errors have been eliminated. As a confirmation of the correctness of the programmes they have been used to give predictions
which should correspond with published results. A close agreement has been found.

The principal use of the programmes has been to give calculated values to set besides the test results, and also to predict the behavlour of a wider range of three lobe bearings than could be tested experimentally. The comparison between test and numerical prediction has been found to be satisfactory. Suggestions are made for future work. In the present work there is only a brief discussion of the stability characteristics of the three-lobe journal bearings, and a fuller discussion is outstanding. It is also suggested that a head with a stiffer shaft should be designed, and possibly run ing in $a$ bearing with modified geometry to give an even higher stiffness together with a reduction in oil temperature rise.

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## NOMENCLATURE

C Bearing diametral clearance or the
lobe clearance.
$\overline{\mathbf{C}}$ Maximum diametral clearance m
$\overline{\bar{C}} \quad$ Minimum diametral clearance m
$C_{M}$ Mean diametral clearance m
$C_{t}$ Power constant $\quad \mathrm{Nm} \mathrm{s} \mathrm{s}^{-1}$
D Bearing Diameter or lobe diameter m
$\mathrm{F}_{\mathrm{H}}$ Horizontal Component of force N
$F_{R}$ Radial force N
$\mathrm{F}_{\mathrm{t}}$ Total force N
$F_{v} \quad$ Vertical component of force $N$
H* Dimensionless oil film thickness, $h / C$
Hp Pumping power KW
Hs Shear Power Loss KW
K Specific heat
cal $\mathrm{gm}^{-10} \mathrm{C}^{-1}$
L Bearing Length
m

N Shaft speed r.p.m.
$P_{i} \quad$ Pressure at bearing inlets $\quad \mathrm{Nm}^{-2}$
Po Pressure at zero journal eccentricity $\mathrm{Nm}^{-2}$
$P_{i, j}^{*}$ Dimensionless pressure, $\frac{P_{i, j}}{P_{0}}$
$P_{s}$ Supply pressure (i.e. pump pressure) $\mathrm{Nm}^{-2}$
$P_{\theta}$ Pressure along the direction of rotation $\mathrm{Nm}^{-2}$
$Q_{\text {A, a Axial flow }}$
litres $s^{-1}$
Qc Induced pressure circumferential flow
Iitres $s^{-1}$
$Q_{h}$ Hydrodynamic flow
litres $s^{-1}$

| $Q_{n}$ | Velocity Induced flow | Iitres $\mathrm{s}^{-1}$ |
| :---: | :---: | :---: |
| $\mathrm{Q}_{\mathrm{p}}$ | Zero speed flow | litres $s^{-1}$ |
| $Q_{t}$ | Total flow | litres $s^{-1}$ |
| Q | Flow through constant flow device | litres $s^{-1}$ |
| S | Duty Parameter $* \frac{\mu \text { NLD }}{W}(\mathrm{D} / \mathrm{C})^{2}$ | litres $\mathrm{s}^{-1}$ |
| Sp | Speed Paramter, $\frac{N}{P_{O}}\left(\frac{D}{C}\right)^{2}$ |  |
| $T$ | O11 Temperature | ${ }^{\circ} \mathrm{C}$ |
| U | Surface speed | $m s^{-1}$ |
| W | Bearing load | N |
| $W_{x}$ | Load component in the direction of the |  |
|  | attitude line | N |
| $W_{Y}$ | Load component in the direction perpendicular |  |
|  | on the attitude direction | N |
| $\bar{W}_{t}$ | $\text { Dimensionless load, } \frac{W}{L D \mathrm{P}_{0}}$ |  |
| X* | Dimensionless distance. $\quad$ /D |  |
| Z* | Dimensionless distance, $z / L$ |  |
| d | Journal diameter | m |
| fo | Heat loss factor |  |
| h | Oil film thickness | m |
| $\mathrm{h}_{\min }$ | Minimum oil film thickness | m |
| $\kappa$ | Number of iterations |  |
| $\ell$ | Axial length of oil inlets | m |
| n | Number of mesh points in the direction |  |
|  | of rotation |  |
| m | Number of mesh points across the bearing |  |
| $q_{a}$ | Axial flow coefficient, $6 \mu \quad \mathrm{Q}_{\mathrm{A}} / \mathrm{C}^{3} \mathrm{P}_{0}$ |  |

$q_{c} \quad$ Induced pressure circumferential flow
coefficient

$$
\frac{12 \mu Q_{c}}{c^{3} P_{0}}
$$

$q_{h}$ Hydrodynamic flow coefficient, $\frac{4 Q_{h}}{\pi D L C N}$
$q_{n}$ velocity induced flow coefficient $\frac{2 Q_{n}}{C \Pi \square N L}$
$q_{\rho}$ Pressure flow coefficient
$x \quad$ Bearing distance measured in the direction of rotation

Bearing distance measured perpendicular m
to the direction of rotation
a The axial groove angle degree
$\beta$ Load angle for each lobe
Y Load angle for the complete bearing degree
$\delta$ Bearing eccentricity m
$\delta_{1}, \delta_{2}$ and $\delta_{3}$ Lobes eccentricity m
$\Delta_{r}$ Bearing offset m
$\Delta x$ and $\Delta y$ Dimensionless and height of the
mesh square set
$\Delta T \quad$ Temperature rise
$\varepsilon_{0}$ Offset ratio,
$\frac{2 \Delta r}{c}$
$\epsilon_{B} \quad$ Bearing eccentricity ratio $\frac{2 \delta}{C_{\text {min }}}$
$\varepsilon \quad$ Eccentricity ratio, $\varepsilon_{B}\left(1-\varepsilon_{0}\right)$
$\varepsilon$ I Eccentricity ratio of the bottom lobe
$\varepsilon_{j}$ Eccentricity ratio of the left lobe
$\varepsilon_{\mathrm{K}} \quad$ Eccentricity ratio of the right lobe
$\theta$ Lobe angle degree
$\theta_{I}, \theta_{J}$ and $\theta_{K}$ Angle between the position of minimum film thickness and the starting of the lobe in the direction of rotation. degree
$\lambda \quad$ Bearing stiffness $\mathrm{Nm}^{-1}$
$\lambda$ Dimensionless stiffness
$\mu$ Dynamic viscosity of the lubricant poise
$\bar{\mu} \quad$ Dimensionless viscosity
$\rho$ Oil density $\quad \mathrm{gm} \mathrm{cm}^{-3}$
「 Shear stress
$\phi$ Bearing attitude angle degree
$\Phi_{\text {I The bottom lobe attitude angle degree }}$
$\phi_{J}$ The left lobe attitude angle degree
$\phi_{\mathrm{K}}$ The right lobe attitude angle degree
$\Omega \quad$ Optimum over-relaxation factor

## SUBSCRIPTS

HI Horizontal
VL Vertical
1 Reference to the count in the axial direction

I,J,K Refer to the considered lobe

Reference to the count in the direction of rotation
Max Maximum

Min
Minimum
o Point of zero eccentricity
1,2 Inlet and outlet

## TERMINOLOGY

The following conditions are met in this thesis:
i) No rotation of journal, controlled high pressure oil only.
ii) With rotating journal, controlled high pressure only.

1ii) With rotating fournal, uncontrolled low pressure ofl only.

Conditions i) and ii) will be referred to as being hydrostatic or externally pressurized, where in condition ii) the fournal speed with other factors is sufficient to produce very considerable selfgenerating pressure the term externally pressurized will only be used. Condition iij) will be as hydrodynamic or self-generating.

CHAPTER 1.

INTRODUCTION

## INIRODUCTION

### 1.1 General

The problems associated with friction, wear and the introduction of a lubricant between moving surfaces, have been appreciated for a long time. The subject of lubrication received much attention since the later part of the 19th Century when Reynolds published the first theorectical analysis for the fluid film problem. Reynolds' analysis was based on the experimental work carried out by Tower (refs. 1,2,3).

In this investigation, lubrication of journal bearings may be classified, as self-generating bearings* (hydrodynamic bearings) or externally pressurized bearings (hydrostatic bearings). In the hydrodynamic bearings the creation of a fluid film between the bearing and journal surfaces depends on the relative velocity and the presence of a wedge shape between the two surfaces. Hydrodynamic journal bearings have received much attention and a list of references covering the theory and test results are given in References 1 to 11.

The main advantages of self-generating bearings are high load capacity at high speed and eccentricities, high overload tolerance and the absence of a high pressure supply line. However, there are some applications where hydrodynamic bearings fail to provide the required load and stiffness characteristics especially at low speeds and at low values of eccentricity.

[^0]
## 1.2.

 Externally Pressurized BearingsIn contrast with the self-generating bearings, in the externally pressurized bearings the load carrying capacity is produced by externally pressurized fluid in the gap between the sliding surfaces. Hence the main advantage of externally pressurized bearings (i.e. high stiffness, high load carrying capacity at low values of eccentricity and zero speed, accurate journal position, low friction coefficients and consequently a minimum starting torque and wear) is dependent as much upon the characteristics of the high pressure system (i.e. pressure, flow and method of compensation) as upon the bearing configuration. Specific bearing requirements within a wide range can be met by appropriate design.

The main disadvantage of externally pressurized bearings is the need of a high pressure supply line with means for providing volume flows to the inlets which are substantially constant and independent of the condition of operation of the bearing. A failure in the high pressure supply may lead to a complete failure of the bearing system.

Hydrostatic bearings, (i.e. externally pressurized bearings), have been known since 1862, when the Frenchman, Girard invented the first hydrostatic bearing (ref. 11). The next publication on the subject of hydrostatic bearings was by Lord Rayleigh who described
a hydrostatic thrust bearing and solved the equation for load, flow and frictional torque (ref. 11). The name "hydrostatic bearing" became adopted after Fuller (12) had presented an analysis of a pressurized pad bearing, which he described as an oil lift.

Oil lifts are normally used with large rotors to reduce the starting torque. The analysis had been made for a single pad of 180 degree arc, and was simplified by considering the case of an infinitely long bearing, so that there was no axial flow of the oil to be considered.

### 1.3. The Conventional Externally Pressurized Journal Bearings.

The idea of the conventional externally pressurized journal bearing, in which high pressure pockets have been introduced around the circumference, was described by Shaw and Macks (13). The pockets may contain up to $90 \%$ of the bearing surface area, so the load capacity may be increased, while the power loss due to the shaft rotation may be decreased; but at high speeds, the turbulence within the pockets increases the power loss.

The first analysis and performance characteristics for the conventional externally pressurized journal bearing was presented by Raimondi and Boyd (14), for a bearing which was compensated by an orifice or by capillary tube.

In the last 25 years different approaches for the analysis of conventional hydrostatic journal bearings have been described in the literature (15-24); the analyses are generally restricted to bearings which are compensated by capillary tubes or by orifices. With these devices the flow has to be defined in terms of the supply pressure and the downstream pressure. In general the results of the analysis have been expressed as mathematical relations without any numerical evaluation for specific applications. There has been little experimental verification of the expressions and the majority of the experimental work which has been reported relates to a small diameter bearing with the shaft stationary or rotating slowly.

In addition to the analytical approaches various numerical solutions for the hydrostatic bearings have been published, references (25-28). These methods would be quite impracticable but for the electronic computer.

### 1.4. External Compensation

In the simple hydrodynamic bearing which in general has a circular bore the only oil supply requirement is to fill the convergent space between journal and bearing with oil. In practice, supply pressure from $1.5 \times 10^{5} \mathrm{Nm}^{-2}(20 \mathrm{psi})$ to $7.5 \times 10^{5} \mathrm{Nm}^{-2}$ (100 psi) is used, Reference (29). In some instances arrangements are made within the bearing to permit additional flows of oil, which do not pass through the confine.of journal and bearing, and whose function is cooling rather than lubrication. However, it may be said that the behaviour of hydrodynamic bearings is more dependent upon bearing configurations than upon the oil supply system. In contrast, the behaviour of hydrostatic and externally pressurized bearings is dependent as much upon the oil supply system as upon the configuration of the bearing itself.

In a hydrostatic bearing, to create the bearing load carrying capacity, there must be a pressure drop between the supply pressure, and the pressure of the pad. Furthermore, to develop a restoring force when the journal is displaced it is necessary that the pressure at a pad should rise when the journal approaches the pad. For this to happen the rate of flow through the inlets should be held substantially constant irrespective of the pressure downstream of the control means . This requirement can be satisfied, in principle, by
using orifices, capillary tubes, constant flow devices, or diaphragm valves. These devices are described in Table 1.1.

The control devices have received much attention because it has been realised that the control devices (compensation devices) have a considerable effect on the performance of externally pressurized bearings. In particular, the stiffness is directly dependent on the type of compensation device.

Loeb and Rippel (30), Malonski and Loeb (31), have investigated the effect of the capillary, orifice, and flow control valves as compensation devices, on the stiffness of the lubricant film; as might be expected they found a constant flow device, which might be a constant flow valve or constant displacement pump , to produce the greater the stiffness. Also they found that the performance of a constant flow control valve is independent of the pressure ratio (i.e. the ratio between the pad pressure and the pump pressure at zero eccentricity)

Mohsin (32), De Gast (33), and Morsi (34), have described control devices which cause the flow rate to increase as the load increases. By thismeans, in principle, the bearing stiffness may be made infinite over a limited range of conditions.

The different methods of compensation applicable for the externally pressurized bearings have been described in several references summarized by O'Donoghue and Hooke (35), where the advantages, disadvantages and the limitation in using each method are described. In references (36-38) several applications where no external compensation is used are described.
Table 1.1 Comparison between different types of compensating devices

| Compensation device |  | Description | Formula defining the flow | Advantage | Disadvantage |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Capillary <br> tubes |  | A simple tube having a large length diameter ratio. Capillaries are available as glass or hypodermic. | $Q^{c}=\frac{\mathrm{Ps}-\mathrm{Pi}}{\mathrm{Kc} \eta}$ <br> (for Laminar flow) <br> $K_{c}$ is the <br> capillary <br> constant. | Simplicity and bearing load and stiffness are independent of oil viscosity. | Low stiffness and high power loss due to the required pressure drop. |

Table 1.1 Comparison between different types of compensating devices

| Compensation device |  | Description | Formula defining the flow | Advantage | Disadvantage |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Orifices |  |  | $\ell_{0}=\frac{\pi d^{2}}{4} \mathrm{Cd} \sqrt{\frac{2}{\rho}(\mathrm{ps}-\mathrm{pi})}$ <br> Cd: is the discharge coefficient. | Can be accommodated in small space and leads to slightly higher stiffness than the capillary. | Greater flow variatior the flow characteristid change as the viscosity changes, increased tendency to blockage, and difficult to predict the exact performance |

Table 1.1 Comparison between different types of compensating devices

| Compensation device. |  | Description | Formula defining the flow. | Advantages | Disadvantages |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Constant <br> flow <br> control |  | The constant flow may be achieved by using a fixed displacement pump or by spool valve. The spool is controlled by a low modulus spring ensuring a constant force within the limits of its movement. | - | High stiffness, low power loss, stiffness independent of the supply pressure and pressure ratio and the valve may be adjusted to cope with different conditions. | The bearing stiffness is dependent on the oil viscosity and temperature variation. |

Table 1.1. Comparison between different types of compensating devices.

| Compensation device |  | Description | Formula defining the flow | Advantages | Disadvantages |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Diaphragm <br> Valves |  | A diaphragm is used to control the flow rate to the bearing. The fluid is supplied through a control pad and its flow rate depends on the gap between the diaphragm and pad. | - | High stiffness, and no sticking. | Needs a separate design for each case and may lead to a negative stiffness. |

### 1.5. Applications of Externally Pressurized Bearings

The advantages of the externally pressurized bearings, as described above, have led to theirchoice for some applications in which they have been used successfully, references (39-41). Uses of externally pressurized bearings in the machine tool industry are discussed in references (42-49). The uses include applications to slide ways and to spindles. The use of externally pressurized bearings in particular gives enhanced finish, permits smaller tolerances, and allows items of precision to be obtained at moderate cutting speeds when they are demanded by the material.

Compressible fluids (gas), such as air, have been used as a lubricant, and externally pressurized gas bearings have been developed for machine tools of very high speed, where finishing to ultra-tolerances is required. Gas lubricated bearings have been fully described by Grassam and Powell (50), and the various applications of the externally pressurized gas lubricated bearings have been presented and discussed by Decker (51) and Wunch and Nimo (52).

### 1.6. Externally Pressurized Bearing System.

It has been stated previously in paragraph 1.3, that most of the literature which has been published in the field of externally pressurized bearings has concentrated on the analysis of a single bearing, without reference to the behaviour of the complete machine tool spindle and without reference to specified design requirements. Little experimental test results have been published for externally pressurized bearings.

Kher and Cowely (53), have given an approximate theoretical analysis and have described some test results for a capillary compensated externally pressurized journal bearing. The analysis is simplified by considering the short bearing circumstance which has been solved for a stationary journal. Two pocket bearings of 50 m m diameter circular bore were tested over a limited range of speed ( 1900 rpm ).

Decker and Shapiro (54-55) have presented a theoretical study of the development of a hydrostatic bearing for a milling spindle. The maximum speed of the spindle was 2000 rpm , and the maximum radial load was 8910 N. Results for pumping power, shearing power loss, temperature rise and stiffness are presented. In the absence of experimental verification it is difficult to judge how far these results would hold in practice.

Davies and Andvig (56) have presented results obtained from tests on an orifice compensated hydrostatic journal bearing with four pockets under the action of steady radial loads which produce severe bending of the shaft. The bending was produced with a shaft of 76 mm diameter and the overhang of load was 65 mm . The results are presented in terms of journal eccentricity, and it has been deduced by the writer that loadstypically of 3400 N were employed. An important conclusion of Davies and Andvig is that the severe shaft bending had no serious effect upon the performance of the hydrostatic bearing.

Recently, Parsiegla (57), in his paper presented at the 6th

Conference on "The Optimization Problems in Design and Production of Machines", has presented an experimental investigation of hydrostatic bearings for heavy duty machine tools with high rotational speeds. A system of two conventional hydrostatic journal bearings and one thrust bearing were used for a machine tool spindle of 130 m m diameter. Measurements of power loss and temperature rise are presented for speeds up to $10^{4} \mathrm{rpm}$. To minimize power loss, a very thin oil ( $\mu=2.5 \mathrm{Cp}$ at $40^{\circ} \mathrm{C}$ ) was used. At $10^{4} \mathrm{rpm}$ the power loss due to the rotation of the shaft was approximately 70 KW , however, it was noted that the load capacity was reduced because of the low viscosity actually within the bearing. At speeds greater than 4000 rpm it was noted that the power loss exceeded the calculated power loss and this was attributed to the development of turbulence in the pockets. There was no provision for supplying the bearing with low pressure oil, and the hydrodynamic operation at high speed depended upon the externally pressurized supply. It seems reasonable to suppose that hydrodynamic operation was severely affected detrimentally by the presence of the pockets.
1.7. The object of the present investigation.

The purpose of the present investigation is to develop a single journal bearing for a wide speed range and high stiffness. The development work was originated by an enquiry for a machine tool bearing of 152 mm diameter, and with the specification given in table 1.2.

The preliminary analysis showed that the rolling element bearings cannot match the required load, diameter, stiffness and running speed.

Rolling bearings have well known limited working capabilities, they cannot withstand static loads for a long time, or shock and impulse loads; they are not suitable for working at high temperatures. The higher stiffnes exists only with pre-loading which reduces the working life (58-60).

It seemed possible, from a feasibility study, that the specification of table 1.2 could be met by a journal bearing which operates hydrostatically at low speed, and which becomes selfgenerating as the speed of rotation is increased. In a selfgenerating bearing of circular bore (Figure l.la) the stiffness of the bearing arises in the locality of the position of minimum film thickness, and the remainder of the surface contributes little to the stiffness but a great deal to the shearing loss within the bearing. However, if the journal bearing is a multi-lobe bearing (Figure l.b), then there is a position of film convergence at each lobe, and each lobe contributes to the overall stiffness in the selfgenerating regime. By adopting a multi-lobe journal bearing it was considered that the required stiffness in the self-generating regime could be obtained at a lower power loss and oil temperature rise than would be possible with a bearing of circular bore. This concept reacted upon the dessign of the bearing in its hydrostatic aspect.

In a journal bearing which is entirely hydrostatic the advantages of the high pressure pockets (Figure l.lc) are well known, but as reported in references $(61,62,63)$, the consequent removal of surface greatly diminishes the load carrying capacity of the bearing as a

Table 1.2. Specification.

| Regime | Speed <br> rpm | Max. <br> Bearing <br> Pressure $\mathrm{Nm}^{-2}$ | Torque <br> Nm | Power <br> kw | Deflection m | Mean <br> Stiffness $\mathrm{Nm}^{-1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Low speed | 8-40 | $3.86 \times 10^{6}$ | $6.1 \times 10^{4}$ | - | unimportant | - |
|  | 40-840 | $3.86 \times 10^{6}$ | - | 22 | " | - |
| Finishing | 840-1800 | $1.18 \times 10^{6}$ | - | - | $2.5 \times 10^{-5}$ | $8 \times 10^{8}$ |
| High Speed | 1800-9000 | $2.34 \times 10^{5}$ | - | - | $1.25 \times 10^{-5}$ | $3.2 \times 10^{8}$ |

For the above bearing pressuresmaximum loads for a bearing of 135 mm diameter and 127 mm long become $6.6 \times 10^{4} \mathrm{~N}, 6.6 \times 10^{4} \mathrm{~N}, 2.0 \times 10^{4} \mathrm{~N}$ and $4.0 \times 10^{3} \mathrm{~N}$ respectively.


Figure 1.1. Hydrodynamic pressure distribution around three different bearings.
self-generating bearing. Furthermore, in reference (64) it has been shown that the turbulence within the pockets increases the power loss, and the resisting torque due to the pocket friction is not always negligible and becomes dominant at high rotational speeds. The dynamic behaviour of the pocket bearings has been investigated by Optiz et al (65), Davies and Leonard (66) and Leonard and Rowe (67). It has been reported that the hydrostatic bearings with high pressure pockets are liable to dynamic instability as selfgenerating bearings, and the bearing damping is reduced by incorporating the pockets. Consequently, it was decided that the journal bearing would be designed with three-lobes, and without pockets. A high pressure oil inlet for the hydrostatic regime is provided at the centre of each lobe, and a low pressure inlet is provided for the self-generating regime at each conjunction of the lobes. The required characteristics in the hydrostatic regime would be obtained by employing a sufficiently high inlet pressure. Multilobe bearings have been analysed by Pinkus (68-70). They have been proposed as having good anti-whirl and anti-vibration properties and as having high stiffness and damping characteristics, references (71-72).

The proposed bearing will be protected from severe damage should the high pressure supply fail due to blockage or pipe failure because of the continuous supply to it of low pressure oil.

In the present investigation a system of bearings is proposed to support a machine tool spindle. A three-lobe journal bearing for hydrostatic and hydrodynamic operation which is compensated by constant volume flow valves is used as the main front bearing to
provide the required load capacity and stiffness. Rolling elements are chosen for tail and thrust bearing, both for economy of construction and to reduce the total power loss of the spindle.

The bearing spindle system has been investigated in two full scale test rigs. In the first test rig the hydrostatic performance of the three-lobe bearing was tested but without rotation of the journal. Also the characteristics of the compensation devices were tested. The second test rig was built to represent an actual arrangement for a machine tool spindle. The three-lobe bearing in both externally pressurized and self-generating regimes has been tested over a speed range from 0 to 7000 rpm , and with two oils of different viscosities. Measurements have been made of power loss, oil flow rate, oil temperature rise and journal displacement in both vertical and horizontal directions under the influence of a steady vertical load.

Two bearings of the three-lobe type have been machined with different lobe clearances, offset ratios and axial groove dimensions. These bearings have been tested in the second test rig, and the second bearing was also tested with high pressure oil admitted at the conjunctions rather than at the centre of the lobes. The method used to produce the three-lobe bearings is described. A different method for producing the three-lobe bearing has been examined experimentally and the results are outlined.

The behaviour of the spindle assembly has been investigated with special reference to the bending of the spindle and the stiffness of
the tail bearing assembly. With the aid of a computer programme for calculating spindle deflection, several arrangements for the complete assembly have been examined, and alternative arrangements of tail and thrust bearing have been outlined.

In addition to the test work and the design study mentioned above, a theoretical investigation has been carried out, and the theoretical predictions have been compared with test results and published data. The initial theoretical work was based upon the analytical solution of Reynolds' equation and was limited to the infinitely wide bearing instance. A three-lobe externally pressurized bearing with a stationary journal was considered. In addition an approximate analytical study of the three-lobe bearing in the hydrodynamic regime was carried out. Originally the concept was quantified on the basis of these approximate analytical studies. Later Reynolds' equation was expressed in finite difference form and a computer programme for its solution was written. Programmes of this kind have been applied to the following circumstances.
I. Self-generating regime with a vertical load directed towards the centre of a lobe.
II. As above, but with a vertical load directed towards the conjunction between the lobes.
III) Self-generating regime with dimensionless pressure at the inlet grooves assigned to non-zero values.
IV. Externally pressurized regime with a vertical load as (I) and (II).
V. Externally pressurized regime with the high pressure oil admitted at the conjunction of the lobes.
VI. Self-generating regime for the full circular bore bearing (360 ${ }^{\circ}$ journal bearing).

The principal output of these programmes is :-
a) Dimensionless pressure
b) Duty parameter
c) Attitude angle
d) Load angle
e) Hydrodynamic flow coefficient
f) Pressure flow coefficient
g) Total flow coefficient
h) Dimensionless pressure at each inlet
i) Flow coefficient
j) load factor
k) load angle

Hydrodynamic regime
externally pressurized regime

A further computer programme which is called "The Design Programme" has been written, where in this programme the dimensionless output of the finite difference programmes may be used as an input for the calculation of actual oil flow, temperature rise, power loss, actual load and actual stiffness. As appropriate this programme has been run in connection with test bearings, but has also been run in connection with bearings which are cited in the literature. This has been done in particular to seek assurance that the programmes are correct. The programme has also been used for calculation of a small bearing which is of interest in an other connection.


#### Abstract

All the computer programes which have been used in this investigation have been written by the author except the computer programe which has been used to calculate the spindle deflection. The computer programmes' list as well as a sample of the output are presented in the Appendices.


Comparison between theory and experiment is presented. Modification for the test rig* and computer programmes for future investigation have been outlined. Figure 1.2 shows a flow chart for the sequence of the investigation.

* The writer designed, drew, assembled and commissioned the rig himself. He also carried out some machining and fitting.


## Functional Specification

of the Bearing.
\{Chapter 1\}

Conceptual Design
\{Chapter 1\}

Preliminary analysis for the externally pressurized regime, (i.e., Infinite Iong Bearing solution), to estimate the oil flow, pressure and the pump power.
\{Chapter 2\}

## Self Generating

\{Chapter 3\}
I) Finite difference solution of Reynolds equation for the three lobe journal bearings.
II) The output of the finite difference calculations is used in the Design Programme for the calculation of oil flow, oil outlet temperature, power loss, load and stiffness.




Figure 1.2 Flow Chart for the Sequence of the Investigation.

## 2. ANALYSIS OF A THREE-LOBE EXTERNALLY PRESSURIZED JOURNAL BEARING

### 2.1 Introduction

In paragraph 1.7 the conceptual design of a three-lobe bearing intended to meet the functional specification was described. To design an actual test bearing the concept has to be quantified, and in this chapter the quantification with relation to the hydrostatic mode of operation of the bearing, with no journal rotation, will be described. In the first instance the bearing is treated as if it were infinitely long. This treatment takes no cognizance of the axial oil flow which existswith an actual bearing, and narrow bearing theory is used to estimate the axial oil flow.

The discussion of the problem in this chapter is based upon elementary theory and is approximate. In later chapters less approximate treatments based upon finite difference solutions are presented for the hydrostatic (Chapter 5) and hydrodynamic (Chapter 3) regimes. The predictions which are required are:

Pressure distribution, the supply pressure, resultant force, stiffness, oil flow and pump power.

With reference to the present approximate treatment the appropriate expressions are described as functions of the gap between journal and a single lobe with the journal symmetrically disposed. The treatment of stiffness is based upon a journal displaced from its central position within the complete bearing so that different gaps obtain at each lobe. The fournal can then no longer be symmetrically disposed with respect to each lobe, however the departures from symmetry are small, and in the analysis symmetry is assumed.

The circumferential oil flow is based upon the solution for an infinitely wide bearing. To obtain a very approximate estimate for the side leakage it has been assumed that one half of the maximum pressure calculated for the infinitely wide lobe exists along the centre line of the lobe, and that the pressure falls off linearly to the sides to give constant axial pressure gradient. This is an arbitrary procedure but the sum of the flows so calculated is reasonably close to the ofl flow found experimentally, and to the more precise calculation of oil flow by the finite difference method.

The initial calculation will now be presented and discussed with reference to the choice of dimensions for an actual test bearing.

### 2.2 Infinitely Wide Bearing Solution

The general arrangement of the bearing is illustrated in Figure 2.1. The high pressure oil is admitted to each lobe through a series of holes, which in the analysis are treated as if they provided a uniform supply of oil over the full width of the lobe. The oil is assumed to flow circumferentially into axial grooves of zero pressure, and in consequence each lobe is taken as being isolated without any interaction with another.

### 2.3 Single Lobe

The coordinates of the analysis are indicated in Figure 2.2. In the steady state condition and, when the bearing is considered to be infinitely wide, the Navier, Stoke's equation may be taken in the approximate form

$$
\begin{equation*}
\frac{\partial P}{\partial x}=\mu \frac{\partial_{2} U}{\partial y^{2}} \tag{2.3.1}
\end{equation*}
$$



FIG.2.1 Arrangement of externally pressurized inlets.


Repeated integration with respect to $y$, with the following boundary conditions, $U=0$ when $y=h$ and $U=0$ when $y=0$, leads to

$$
\begin{equation*}
U=\frac{1}{2 \mu} y(y-h) \frac{\partial P}{\partial x} \tag{2.3.2}
\end{equation*}
$$

The circumferential oil flow to right and left of the oil inlet may be expressed as

$$
\begin{equation*}
q_{c}=2 L \int_{0}^{h} U d y \tag{2.3.3}
\end{equation*}
$$

Substitution from(2.3.2) into equation(2.3.3) and integration leads to

$$
\begin{equation*}
\frac{\partial P}{\partial \theta}=-\frac{D}{L} \cdot 3 \mu \cdot q_{c} \cdot \frac{1}{h^{3}} \tag{2.3.4}
\end{equation*}
$$

The oil film thickness may be expressed as

$$
\begin{equation*}
h=C / 2(1-\varepsilon \cos \theta) \tag{2.3.5}
\end{equation*}
$$

where,
$\varepsilon=\frac{\text { the distance between the journal centre and the centre of the pad }}{\text { the lobe radial clearance }}=\frac{\delta}{c} \cdot$
From(2.3.5) and(2.3.4) it follows that

$$
\frac{\partial P}{\partial \theta}=-\frac{24 D q_{c} \mu}{L C^{3}}\left(\frac{1}{1-\varepsilon \cos \theta}\right)^{3}
$$

and it follows that

$$
\underset{\theta}{p}=-\frac{24 D q_{c}^{\mu}}{L c^{3}} \int\left(\frac{1}{1-\varepsilon \cos \theta}\right)^{3} d \theta
$$

It is convenient to write the above equation in the form

$$
\begin{equation*}
\frac{P_{\theta} / 24 \mu D q_{c}}{L C^{3}}=\int\left(\frac{}{1-\varepsilon \cos \theta}\right)^{3} d \theta \tag{2.3.6}
\end{equation*}
$$

The integration of equation(2.3.6) is carried out by using the Sommerfeld variable defined by

$$
\cos \gamma=\frac{-\varepsilon+\cos \theta}{1-\varepsilon \cos \theta}
$$

The result of integration is given by


The constant of integration is to be evaluated from:
$P=0$ at $\theta=\theta_{1}$ where $\theta_{1}$ defines an extremity of the lobe. The associated value of $\gamma$ will be denoted by $\gamma_{1}$.

It then follows that

$$
S=\frac{1}{\left(1-\varepsilon^{2}\right)^{5 / 2}}\left[\gamma_{1}\left(1+\frac{\varepsilon^{2}}{2}\right)+2 \varepsilon \sin \gamma_{1}+\frac{\varepsilon^{2}}{2} \sin \gamma_{1} \cos \gamma_{1}\right] \text { (2.3.8) }
$$

In Figure 2.3, $S$ is plotted as a function of $\varepsilon$, for various values of $\theta_{1}$. At the inlet $P=P_{0}, \quad \theta=0, \quad \cos \gamma=1, \quad \sin \gamma=0$ and from (2.3.7)

$$
{ }^{\mathrm{P}_{\mathrm{o}}}\left(\frac{24 \mu \mathrm{Dq}}{\mathrm{~L} \mathrm{c}^{3}}\right)=\mathrm{s}
$$

and the pressure distribution along the lobe is given by

$$
\begin{align*}
& \mathrm{P}_{\theta} /\left[24 \mu D q_{c} / L C^{3}\right]  \tag{2.3.10}\\
& =\frac{1}{\left(1-\varepsilon^{2}\right)^{\frac{3}{2}}}\left[\begin{array}{l}
\left(1+\varepsilon^{2} / 2\right)\left(\gamma_{1}-\gamma\right)+2 \varepsilon\left(\sin \gamma_{1}-\sin \gamma\right)+\frac{\varepsilon^{2}}{2} \\
\left.\left(\sin _{\gamma_{1}} \cos \gamma_{1}-\sin \sin _{\gamma} \cos \gamma\right)\right]
\end{array}\right.
\end{align*}
$$



FIG.2.3 Variation of pressure parameter with lobe eccentricity ratio.

### 2.4 The Eydrostatic Force of a Single Lobe

The hydrostatic force generated by a lobe may be evaluated from

$$
\begin{align*}
F & =2 \int_{0}^{\theta_{1}} L R P_{\theta} \cos \theta d \theta  \tag{2.4.1}\\
& =L D{ }_{0}^{\theta_{1}}\left[P_{\theta} \sin \theta\right]-\int_{0}^{\theta_{1}} \frac{\partial P}{\partial \theta} \sin \theta d \theta .
\end{align*}
$$

The pressure $P_{\theta}$ is equal to zero at $\theta=\theta_{1}$ and $\sin \theta$ is zero at $\theta=0$, thus the hydrostatic force per lobe is expressed by

$$
\begin{equation*}
F=\frac{24 \mu D^{2} q_{c}}{C^{3}} \int_{0}^{\theta} \frac{1}{(1-\varepsilon \cos \theta)^{3}} \sin \theta d \theta \tag{2.4.2}
\end{equation*}
$$

Equation(2.4.2) is integrated by using the Sommerfeld substitution and the result is

$$
\begin{equation*}
F=\frac{24 \mu D^{2} q}{C^{3}} \frac{1}{\left(1-\varepsilon^{2}\right)^{2}}\left[-\cos \gamma-\frac{\varepsilon}{4} \cos 2 \gamma\right]_{0}^{\gamma} \tag{2.4.3}
\end{equation*}
$$

By substituting from equation (2.3.9) into (2.4.3) and rearranging, the force is given by

$$
\begin{equation*}
F=L D P_{0} E \tag{2.4.4}
\end{equation*}
$$

where

$$
E=\left(1-\varepsilon^{2}\right)^{\frac{1}{2}}\left[\frac{-\left(1+\cos \gamma_{1}\right)-\varepsilon / 4\left(1+\cos 2 \gamma_{1}\right)}{\gamma_{1}\left(1+\varepsilon^{2} / 2\right)+2 \varepsilon \cos \gamma_{1}+\varepsilon 2 / 2 \sin \gamma_{1} \cdot \cos \gamma_{1}}\right]
$$

In Figure 2.4, E is plotted as function of $\varepsilon$ for various values of $\theta_{1}$.


### 2.5 The Complete Bearing

The calculation of the total force, total oil flow and overall stiffness require that the eccentricity ratio at each lobe should be related to the eccentricity of the journal within the complete bearing. With reference to Figure $2.5 a$ the journal centre is at the centre of the bearing at 0 , and is displaced by $\Delta r$ from the centres of the lobes at $O_{I, J, K}$. In Figure $2.5 b$ the journal centre is at $O_{1}$, and is displaced from the bearing centre at $O$ by the distance $\delta$. The eccentricity ratio with reference to each lobe is given by

$$
\begin{aligned}
& \varepsilon_{I}=\frac{2 \delta_{I}}{C}=\frac{2\left\{\delta^{2}+\Delta r^{2}+2 \cdot \delta \cdot \Delta r \cdot \cos \phi\right\}^{\frac{1}{2}}}{C} \\
& \varepsilon_{J}=\frac{2 \delta_{J}}{C}=\frac{2 \delta_{J}}{C}=\frac{2\left\{\delta^{2}+\Delta_{r}^{2}-2 \cdot \delta \cdot \Delta_{r} \cos (\Pi / 3-\phi)\right\}^{\frac{1}{2}}}{C} \\
& \varepsilon_{K}=\frac{2 \delta_{K}}{C}=\frac{2\left\{\delta^{2}+\Delta_{r}^{2}-2 \cdot \delta \cdot \Delta_{r} \cos \left(\Pi / I_{3}+\phi\right)\right\}^{\frac{1}{2}}}{C}
\end{aligned}
$$

When $\phi$ is zero (i.e. the centre of the journal moves vertically downward) the eccentricity ratios for the lobes are given by

$$
\begin{aligned}
& \varepsilon_{I}=\frac{2 \delta_{I}}{C}=\frac{2\left(\delta+\Delta_{r}\right)}{C} \\
& \varepsilon_{J}=\varepsilon_{K}=\frac{2 \delta_{J, K}}{C}=\frac{2\left\{\delta^{2}+\Delta_{r}^{2}-\delta . \Delta_{r}\right\}^{\frac{1}{2}}}{C}
\end{aligned}
$$


(b)

FIG.2.5 Displacements relationships.
and when $\phi$ is 180 degrees (i.e. the centre of the journal moves vertically upward), the eccentricity ratios are given by

$$
\begin{aligned}
& \varepsilon_{I}=\frac{2 \delta_{I}}{C}=\frac{2\left(\Delta_{r}-\delta\right)}{C} \\
& \varepsilon_{J}=\varepsilon_{K}=\frac{2 \delta_{j, k}}{C}=\frac{2\left\{\Delta_{r}+\delta^{2}+\delta . \Delta_{r}\right\}^{\frac{1}{2}}}{C}
\end{aligned}
$$

### 2.6 The Total Force

If the load acts in an arbitrary direction, then the force from each lobe will be such that the vector sum of the forces will balance the applied load.

The total hydrostatic force is given by

$$
\begin{equation*}
\vec{F}_{t}=\sum_{I-K} \vec{F} \tag{2.6.1}
\end{equation*}
$$

When the external load is applied vertically downward the total force is given by

$$
\begin{equation*}
F_{t}=\left\{F_{I}-\left(F_{J}+F_{K}\right) \sin 30\right\} \tag{2.6.2}
\end{equation*}
$$

By substitution from (2.4.4) it follows that

$$
\begin{align*}
F_{t} & =L D\left[\left(P_{o} E\right)_{I}-\left\{\left(P_{O} E\right)_{J}+\left(P_{o} E\right)_{K}\right\}_{\sin 30}\right] \\
& =L D P_{o I}\left[E_{I}-\left\{\frac{\left(P_{O} E\right)_{J}}{P_{o I}}+\frac{\left(P_{o} E\right)_{K}}{P_{o I}}\right\} \sin 30\right] .
\end{align*}
$$

from(2.3.9) and(2.3.6) it follows that

$$
\begin{equation*}
F_{t}=L D P_{o I}\left[E_{I}-\left\{E_{I} \frac{S_{J}}{S_{I}}+E_{K} \frac{S_{K}}{S_{I}}\right\} \sin 30\right] \tag{2.6.4}
\end{equation*}
$$

When the external load is considered to be applied from the opposite direction the total hydrostatic force is given by

$$
\begin{equation*}
F_{t}=L D F_{J, K}\left[\left(E_{J}+E_{K}\right) \sin 30-E_{I} \frac{S_{I}}{S_{J}}\right] \tag{2.6.5}
\end{equation*}
$$

### 2.7 The Oil Flow

### 2.7.1 The Circumferential Oil Flow

When the shaft is stationary by symmetry the flow divides equally to left and right, and therefore the rate of flow in the circumferential direction for one lobe is given by expression(2.3.9) which will be written as

$$
\begin{equation*}
q_{c}=\left[P_{0} \quad L \quad C^{3} / 24 \mu \mathrm{D}\right] \cdot \frac{1}{S} \tag{2.7.1}
\end{equation*}
$$

2.7.2 The Axial Flow

It is clear from Figure 2.2 that the oil flows out of a hydrostatic lobe from the sides as well as in the circumferential direction.

The approporiate form of the approximate Navier, Stoke's equation in the axial direction is

$$
\begin{equation*}
\frac{\partial y w}{\partial y^{2}}=\frac{1}{\mu} \frac{\partial P}{\partial z} \tag{2.7.2}
\end{equation*}
$$

Repeated integration of equation(2.7.2) with respect to $y$, with the following boundary conditions:
$w=y$ at $y=h$ and $w=0$ at $y=0$
leads to

$$
\begin{equation*}
w=\frac{1}{2 \mu} \frac{\partial P}{\partial z}(y-h) y \tag{2,7.3}
\end{equation*}
$$

The total axial flow from one lobe is given by

$$
\begin{equation*}
q_{a}=4 \int_{0}^{\theta} \int_{0}^{h_{1}} R w d \theta d y \tag{2.7.4}
\end{equation*}
$$

which, from (2.7.3) may be written as

$$
\begin{equation*}
q_{a}=-4 R / 2 \mu \frac{\partial P}{\partial z} \int_{0}^{\theta_{1}} d \theta \int_{0}^{h}(y-h) y d y \tag{2.7.5}
\end{equation*}
$$

After integration and substituting for the oil film thickness by C/2 (1- $-\cos \theta$ ), equation (2.7.5) may be expressed as

$$
\begin{equation*}
q_{a}=-\frac{4 R}{2 \mu} \frac{C^{3}}{8 \times 6} \frac{\partial P}{\partial z} \int_{0}^{\theta_{1}}(1-\varepsilon \cos \theta)^{3} d \theta \tag{2.7.6}
\end{equation*}
$$

The pressure at the lobe centre line is deemed to fall linearly to zero at the extremities of the bearing, however the pressure gradient $\frac{\partial P}{\partial z} \quad$ is to be represented by

$$
\begin{equation*}
\frac{\partial P}{\partial z}= \pm \frac{P_{o}}{2} \tag{2.7.7}
\end{equation*}
$$

After substituting from 2.7.7) in 2.7.6), and integrating the axial oil flow for one lobe is given by

$$
\begin{equation*}
q_{a}=-\frac{D C^{3} P_{o}}{48 \mu L}|K|_{0}^{\theta_{1}} \tag{2.7.8}
\end{equation*}
$$

where

$$
K=\left[\theta\left(1+3 / 2 \varepsilon^{2}+\varepsilon^{3} \sin \theta+3 / 4 \varepsilon^{2} \sin 2 \theta-3 \varepsilon \sin \theta-\frac{1}{3} \varepsilon^{3} \sin 3 \theta\right]_{0}^{\theta_{1}}\right.
$$

In Figure 2.6, $K$ is plotted as a function of $\varepsilon$ for one value of $\theta_{1}$.

### 2.7.3 The Total Oil Flow

The total oil flow out of one lobe will be taken as

$$
\begin{equation*}
q_{t}=q_{c}+q_{a} \tag{2.7.9}
\end{equation*}
$$

and the total flow out of the bearing as

$$
\begin{equation*}
q_{t}=q_{t I}+q_{t J}+q_{t K} \tag{2.7.10}
\end{equation*}
$$

In an externally pressurized bearing the oil is admitted to the bearing via a constant flow device, however the oil flow is the same for each pad and the total oil flow is taken as

$$
\begin{equation*}
Q_{t}=3 q_{t} \tag{2.7.11}
\end{equation*}
$$



FIG.2.6 Axial flow parameter versus eccentricity ratio.
2.8.1 Single Lobe Stiffness

The stiffness of the externally pressurized bearing will be taken as the negative derivative of the force with respect to the oil film thickness. Because the load-displacement relationship is approximately linear the stiffness given by the derivation and the secent stiffness, are essentially coincident.

The stiffness is given by

$$
\begin{equation*}
\lambda \simeq \frac{\mathrm{dF}}{\mathrm{~d} \delta} \propto \frac{\mathrm{dF}}{\mathrm{c} / 2 \mathrm{~d} \varepsilon} \tag{2.8.1}
\end{equation*}
$$

and it follows from equation(2.4.3) that

$$
\begin{aligned}
\lambda /\left|48 \mu q_{c} D^{2} / C^{4}\right|= & \left.-\frac{1}{\left(1-\varepsilon^{2}\right)^{3}} \right\rvert\,\left\{-\varepsilon(4+\varepsilon)+\varepsilon\left(4 \cos \gamma_{1}-\varepsilon \cos 2 \gamma_{1}\right)\right\} \\
& \left.+\left(1-\varepsilon^{2}\right)\left\{-0.25-\sin \gamma_{1}-\frac{1}{2} \varepsilon \sin 2 \gamma_{1}+\frac{1}{4} \cos 2 \gamma_{1}\right\} \right\rvert\,=\lambda^{-}
\end{aligned}
$$

In Figure $2.7 \bar{\lambda}$ is shown as a function of $\varepsilon$ for various values of $\theta_{1}\left(\gamma_{1}\right)$.

### 2.8.2 Total Stiffness

To calculate the total bearing stiffness resulting from the stiffness of each lobe, (Figure 2.8)the stiffness of each lobe is represented by springs of stiffness $\lambda_{I}$, $\lambda_{J}$ ard $\lambda_{K}$ respectively. When the journal centre is displaced from 0 to 0 , each spring will be displaced a distance $\delta_{I}, \delta_{J}$ and $\delta_{K}$ respectively.

The stiffness for the complete bearing is given by

$$
\begin{equation*}
\lambda_{e}=\lambda_{I}+\lambda_{J} \sin ^{2} 30+\lambda_{K} \sin ^{2} 30 \tag{2.8.3}
\end{equation*}
$$

The direction for the load with relation to the pad repeats itself every $60^{\circ}$ as the load is rotated. Consequently, the above expression of stiffness $\lambda_{e}$ is valid for 6 specific directions of the load within the bearing. It is highly improbable that the stiffness will vary significantly as the load is swung through $60^{\circ}$, and it is considered that the above expression sufficiently represents the stiffness of the bearing for all directions of the load.

### 2.9 Compensation Method.

In paragraph 1.4 various methods of external compensation for the externally pressurized bearings were discussed. In the present investigation the high pressure oil was supplied to the high pressure inlets via individual control valves. Volume control valves were chosen in contrast with orifices or capillaries to minimise the loss and therefore the pump power, to minimise the pressure of the pump, and also to provide a greater stiffness. Furthermore, by using the control valves, any change in bearing dimension, oil flow and the supply pressure can be accomodated by changing the setting of the valves whereas otherwise new orifices or capillaries would be required.

### 2.10 Pump Power

The pumping power required to pump the oil to the high pressure inlet is given by

$$
\begin{equation*}
H_{p}=\frac{P_{s} Q_{t}}{C_{t}} \tag{2.10.1}
\end{equation*}
$$

$$
\begin{equation*}
P_{s}=P_{o}+\Delta P \tag{2.10.2}
\end{equation*}
$$



FIG.2.7 Variation of the stiffness parameter with the lobe eccentricity ratio.


FIG. 2.8 Stiffness Analogy

## where

$\Delta \mathrm{P}$ : is the pressure drop in the control valve

### 2.11 Results of the Computations

The purpose of these initial calculations was to choose dimensions for the first test bearing* The hydrostatic calculations alone are insufficient because the bearing has also to be suitable for high speed of rotation (i.e. choice of dimensions has to be made with reference to both hydrostatic and hydrodynamic predictions). The hydrodynamic predictions are described in the next chapter, and the choice of dimensions is described. Here the appropriate calculations and curves, required for the choice, but pertaining only to the hydrostatic regime, are presented. In nondimensional form the complete cross section of journal and bearing is defined uniquely by ( $\varepsilon$ ) and the lobe diametral clearance when the journal is centred within the bearing. The addition of an actual diameter, and an actual length define the dimensions of the bearing in all respects. The results of calculations which will be presented relate to a journal of nominal diameter 135 mm (5.30in), and to a bearing length of 127 mm 5 in ).

The choice of journal diameter follows from the specifications of Chapter 1 , and the length of the bearing was an initial guess which in fact happened to remain a suitable length with respect to load carrying capacity after the assessment had been completed.

The inputs to the computer programme were chosen to include appropriate quantities which appear in the specification of the bearing. The inputs of that kind are:
bearing load, journal displacement and journal diameter.

* The calculations were carried out by means of a computer programme which embodies the expressions already developed.

The inputs are completed by bearing length, oil viscosity and the distance between the centre of each lobe and the centre of the bearing ( $\Delta r)$, and the lobe diametral clearance (c).

The outputs from the programe are:
The overall stiffness
(eq п. 2.8.3)
The total oil flow (i.e. the sum of the axial and circumferential flows (eqns 2.7.1.
2.7 .8 and 2.7 .10$)$

The inlet pressure at each pad
(eq n. 2.3.9)
The supply pressure which is taken to be the greatest of the three inlet pressures
(eq n. 2.3.9)
The pumping power
(eqn. 2.10.1)

It is sufficient to take the supply pressure as being equal to the greatest inlet pressure because the minimum pressure drop across a control valve necessary for it to perform its constant flow function is small in comparison with the inlet pressure (~10\%).

The computer programme itself is given in Appendix (A1.2). Calculations were carried out for the following values of the variables: Load: $2.232 \times 10^{4}, 2.932 \times 10^{4}, 2.632 \times 10^{4}$ and $4.332 \times 10^{4} \mathrm{~N}$. Journal displacement: $2.545 \times 10^{-5}, 5.089 \times 10^{-5}, 7.634 \times 10^{-5}$ and $10.178 \times 10^{-5} \mathrm{~m}$. Lobe Diametral Clearance: $2.05 \times 10^{-4}, 2.3 \times 10^{-4}, 2.8 \times 10^{-4}, 3.05 \times 10^{-4}$ and $2.33 \times 10^{-4} \mathrm{~m}$.
Bearing offset $(\Delta r)=5.86 \times 10^{-5} \mathrm{~m}$, Oil viscosity $=0.2$ poise.

Typical plots of stiffness, pump power, supply pressure and oil flow as functions of lobe diametral clearance are given in Figure 2.9. In Figure 2.10 in which supply pressure and the geometry of the bearing


#### Abstract

are constant, bearing load and stiffness are shown as functions of the displacement of the journal from its central position in the bearing.


Figure 2.9 shows that as the lobe diametral clearance is increased, the oil flow increases and also the pumping power, and that the stiffness falls substantially. Also it may be seen from figure 2.9 that the functional specification stiffness $\left(8 \times 10^{8} \mathrm{Nm}^{-1}\right)$ can be obtained with a diametrical clearance of $2.8 \times 10^{-4} \mathrm{~m}$, with a supply pressure of $6.1 \times 10^{6} \mathrm{Nm}^{-2}$, with a total volume flow of $1.2 \times 10^{-2} 1 \mathrm{~s}^{-1}$ with a pumping power of 0.8 kw . However, the final choice of the bearing dimensions should be made to give a compromise solution for the requirements of both the hydrostatic and the hydrodynamic regimes and this will be discussed in Chapter 3.



FIG. 2.10
Variation of load and stiffness with the bearing eccentricity ratio.

## 3. NUMERCIAL SOLUTION FOR THE THREE-LOBE SELF-GENERATING JOURNAL BEARING

### 3.1 Introduction

In paragraph 2.11 the point was made that the dimensions of the bearing have to be chosen with reference to both hydrostatic and hydrodynamic regimes and the hydrostatic characteristics of the bearing were derived in an approximate manner. In the historical sequence of the work an approximate analysis was made of the hydrodynamic characteristics of the bearing, and the dimensions of the first test bearing were chosen with reference to this approximate treatment. However, the degree of approximation in the hydrodynamic instance is large with respect to power loss, oil flow, and with respect to viscosity. It has seemed to be preferable therefore to commence this chapter with the better discussion of the hydrodynamic regime based upon a finite difference treatment, rather than with a discussion of the very approximate treatment. However, at the end of the chapter the approximate treatment is briefly described, and its predictions are compared with those of the finite difference calculations.

### 3.2 Finite Difference Solution

In principle the numerical solution of Reynolds' equation may be performed either by using the finite difference formulation or by the finite element technique (Ref.81). The advantage of the finite element method is that irregular mesh elements can be used, however in the context of bearings there is no need for irregular elements, and the finite difference formulation is to be preferred because it leads to quicker solutions.

The assumptions of the finite difference solution are the usual assumptions implicit in Reynolds' equation including constant viscosity. A calculation commences by inserting some reasonable value for the viscosity, however the intention is that the constant viscosity should be the viscosity at the outlet of the bearing. The calculation approaches the desired viscosity by an iteration in which viscosity, power loss, oil flow and outlet temperature all become self-consistent within a limit set within the programme. Each calculation within an iteration comprises the solution of typically 648 simultaneous equations which are solved by an over relaxation technique.

As is usual in bearing calculations, there is a mathematical solution containing negative pressures and a more realistic solution which has none. When a negative pressure arises at a mesh point that pressure is set to zero before the point is again invoked within the calculation. In that way the solution is constrained towards the realistic solution. It leads not only to zero pressure at the trailing edge of the film but also to zero pressure gradient. A criterion has to be provided for completion of the iterations, and the criterion which has been used is that the dimensionless pressures of the final iteration should differ by no more than 0.0001 from those of the penultimate iteration. To put this into perspective the maximum dimensionless pressure is typically 60. In precise terms a dimensionless pressure less than -0.0001 would be set to zero whereas a dimensionless pressure of -0.0001 would stand.

The required form of Reynolds' equation is,

$$
\begin{equation*}
\partial / \partial x\left(\frac{1}{\mu} h^{3} \frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\mu} h^{3} \frac{\partial P}{\partial z}\right)=6 U \frac{d h}{d x} \tag{3.1}
\end{equation*}
$$

The following dimensionless forms are introduced.

$$
\begin{array}{ll}
X^{*}=\mathrm{x} / \mathrm{D} & , \quad Z^{*}=z / \ell \\
\mathrm{H}^{*}=\mathrm{h} / \mathrm{C} & ,
\end{array}
$$

and in dimensionless form equation (3.1) becomes

$$
\begin{equation*}
\partial / \partial X^{*}\left(H^{*} 3 \frac{\partial P *}{\partial X^{*}}\right)+(D / L)^{2} \frac{\partial}{\partial \bar{Z} *}\left(H^{*} \frac{\partial P *}{\partial Z}\right)=6 \Pi \frac{d H^{*}}{d X^{*}} \tag{3.2}
\end{equation*}
$$

With reference to Figure 3.1 the individual terms, in finite difference form become respectively $\partial / \partial X^{*}\left(H^{*} \frac{\partial P^{*}}{\partial X^{*}}\right)_{j} \approx \frac{\left(P_{i, j+2}^{*}-P_{i, j}^{*}\right) \times H_{i, j+1}^{3 *}-\left(P_{i, j}^{*}-P_{i, j-2}^{*}\right) \times H_{i, j-1}^{* 3}}{\Delta X^{* 2}}$ (3.3)
$\partial / \partial Z^{*}\left(H^{*} \frac{\partial P^{*}}{\partial Z^{*}}\right) \approx \frac{\left.\left(P_{i+1, j}^{*}-P_{i, j}^{*}\right) \times H_{i+1, j^{*}}^{-\left(P_{i, j}^{*}-P_{i-1, j}^{*}\right.}\right) \times H_{i-1, j}^{*}}{\Delta Z^{* 2}}$ (3.4)
$\left(\frac{d H}{d X}\right)_{j}^{*}=-\frac{1}{2} \varepsilon \sin \theta$
$\left(\frac{d H}{d X}\right)_{j}^{\star}=\frac{H_{i, j+1}^{\star}-H_{i, j-1}^{*}}{\Delta X^{*}}$


Values for $H$ are calculated for each value of $j$
Values for $P$ are calculated for each value of $i$ but only for odd values of $j$.

FIG.3.1 Finite difference representation.

By substituting (3.3), (3.4) and (3.6) into (3.2) and solving for $P_{i, j}$, the finite difference equation becomes
$\left\{\begin{array}{l}\left\{\begin{array}{l}\left.\left\{\pi\left(\frac{H_{i, j-1}-H_{i, j+1}}{\Delta x}\right)\right\}+\left\{P_{i, j+2}\right)\left(H_{i, j+1}^{3}\right) / \Delta x^{2}\right\} \\ \\ +\left\{\left(P_{i, j-2}\right)\left(H_{i, j-1}^{3}\right) / \Delta x^{2}\right\}\end{array}\right. \\ \left\{\begin{array}{l}\frac{\left.\left.(D / L)^{2} \times\left(P_{i+1, j}\right)\left(H_{i+1, j}^{3} / \Delta Z^{2}\right)\right\}+\left\{(D / L)^{2}\left(P_{i-1, j}\right) / \Delta Z^{2}\right)\right\}}{\left(H_{i, j+1}^{3}\right) / \Delta x^{2}+\left(H_{i, j-1}^{3} / \Delta x^{2}\right)} \\ \left.+(D / L)^{2} \times\left(H_{i+1, j}^{3} / \Delta Z^{2}\right)+(D / L)^{2} H_{i-1, j}^{3} / \Delta Z^{2}\right)\end{array}\right.\end{array}\right]_{K .7}^{*}$

For a mesh of $i=1$ to $m+1$ and $j=1$ to $n+1$ where $n$ is even, and with the assumption that pressures at the boundary are known, then there are $(n-1) \times(m-1)$ linear simultaneous equations to be solved. These equations are of the form
$\left\{P_{i, j}\right\}_{K+1}^{*}=\left\{C_{1}+C_{2} P_{i, j+2}+C_{3} P_{i, j-2}+C_{4} P_{i+1, j}+C_{5} P_{i-1, j}\right\}_{K}^{*} \quad(3.8)$
where $K$ relates to the sequence of iterations and denotes that the values of $P_{i, j}$ are those already calculated, and $K+1$ denotes the current value of $\mathrm{P}_{\mathrm{i}, \mathrm{j}}$,
$C_{1}=\frac{6 \pi\left(\frac{H_{i, j-1}-H_{i, j+1}}{\Delta X}\right)^{*}}{A}, C_{2}=\frac{\left(H_{i, j+1}^{3}\right)^{*} / \Delta X^{2}}{A}, C_{3}=\frac{\left(\stackrel{H}{i, j-1}^{*}\right) / \Delta X^{*}}{A}$
$C_{4}=\frac{\left.(D / L)^{2}{ }_{H}^{t_{i+1, j}}\right) / \Delta \stackrel{\star}{Z}^{2}}{A}, C_{5}=\frac{(D / L)^{2}\left(\stackrel{H}{H}_{i+1, j}\right) / \Delta Z^{*}}{A}$ and
$A=\left(\stackrel{\star}{H}_{i, j+1} / \Delta\right.$ X $\left.^{\star}\right)+\left(\stackrel{*}{H}_{i, j-1} / \Delta X^{*} 2\right)+\left({ }^{D} / L\right)^{2}\left(\stackrel{H}{H}_{i+i, j}^{3} / \Delta Z^{*} 2\right)+\left({ }^{D} / L\right)^{2}\left(H_{i-1, j}^{3} / \Delta Z^{*} 2\right)$

The set of simultaneous equations, in each of which there are only five nonzero coefficients, are solved by a modified Gauss-Seidel technique. The condition for the procedure to converge is that in each equation the coefficients on the leading diagonal of the array should exceed the sum of all other coefficients on the line. The modification of the Gauss-Seidel method is the speeding up of convergence by use of an overrelaxation factor $(\Omega)$ so that equation (3.8) becomes,

$$
\begin{align*}
\left\{P_{i, j}\right\}_{k+1}^{*} & =\Omega\left\{C_{1}+C_{2} P_{i, j+2}+C_{3} P_{i, j-2}+C_{4} P_{i+1, j}+C_{5} P_{i-1}\right\}_{k}^{*} \\
& +(1-\Omega)\left\{P_{i, j}\right\}_{k}^{*} \tag{3.9}
\end{align*}
$$

The over relaxation factor ( $\Omega$ ) is given by (Ref. 85 )

$$
\begin{equation*}
\Omega=2 \frac{1-\left(1-\zeta^{2}\right)^{\frac{1}{2}}}{\zeta^{2}} \tag{3.10}
\end{equation*}
$$

where, $\zeta=1-\frac{\pi^{2}\left[4+(\Pi D / L)^{2}\right.}{2\left[m^{2}+(\mathrm{Dn} / L)^{2}\right.}$

The iteration procedures starts with the $P^{\prime}$ s around the boundary set to boundary values (which might be zero) and with all internal P's set to zero. The solution was performed for boundary values at the axial grooves between 0 and 5.

The particular grid employed in the calculations for each lobe is shown in Figure 3.2, $i=1$ to $29(m=28), j=1$ to $51(n=50)$ so that there are $27 \times 24=648$ equations to be solved. The pressures around the boundary are zero except in the axial grooves which may be taken as any appropriate value. The pressures must be symmetrical about the line $1=15$, and

the symmetry could be used to reduce the number of equations to be solved. However a test in which that was done gave no reduction in time of computation, and consequently the full set of equations has been retained with the advantage that, as a check upon the programme, the output can be inspected for the symmetry which must exist.

The symmetry of the output may be seen in Figure 3.10 which is typical. To provide for the axial grooves which separate the lobes, the grooves in each lobe were allocated $20^{\circ}$, and the dimensionless pressure in these grooves may be taken as zero or any dimensionless value which is expressed by $P^{*}=\frac{P}{\mu N} \frac{(C)^{2}}{D}$.

### 3.3 Method of Solution for Complete Bearing

With reference to Figure 3.3 the centre of the journal is displaced from the centre of the bearing by a distance $\delta$, and $\boldsymbol{\delta}$ is included in the input to the calculation as the dimensionless displacement given by $2 \delta / C_{\text {min }}=\varepsilon_{B}$.

The journal is fixed at given eccentricity by assuming an attitude angle $\varnothing$ which is also included in the input. The forces due to the hydrodynamic pressures at each lobe are then evaluated, and the resultant force in the vertical, and in the horizontal direction are calculated. The total hydrodynamic force and the angle between the vertical direction and the direction of the total force are also calculated, and then the journal displacements in the direction of the total force and in the direction perpendicular to the total force direction are calculated. $\varnothing$ is then increased by a small increment (i.e. $\pm 10^{\circ}$ ), the calculation is repeated until $\varnothing$ is equal to $\pm 60^{\circ}$ and then $\varepsilon_{B}$ is then increased by a small increment (i.e. +o.1), the calculation is repeated and stiffness


FIG.3.3 Bearing geometry.
coefficient is evaluated as change in dimensionless vertical load. change in displacement

A conceptual flow chart of the routine used for the solution of the self-generating three lobe journal bearing is shown in Figure 3.4.

To revert now to individual lobes, the expression for the film thickness at each lobe, and the position of minimum film thickness at each lobe, in terms of $\varepsilon_{B^{\prime}} \varnothing$ and $\varepsilon_{0}$ are given in Figure 3.5.

Film thickness together with an initial value of viscosity, which is to be taken as constant, permits the calculation of the pressures at each lobe. With reference to Figure 3.6 the hydrodynamic forces in the direction of the lobe attitude line, and in the direction perpendicular to the lobe attitude line may be calculated from:

$$
\begin{align*}
& \left(W_{x}^{*}\right)=\int_{0}^{\theta} \int_{0}^{L} P_{\theta}^{*} \cos \theta d \theta  \tag{3.11}\\
& \left(W_{y}^{*}\right)=\int_{0}^{\theta} \int_{0}^{L} P_{\theta}^{*} \sin \theta d \theta \tag{3.12}
\end{align*}
$$

In numerical form equations (3.11) and (3.12) become

$$
\begin{align*}
& \left(W_{x}^{*}\right)_{I, J, K}=\sum_{i=1}^{m+1} \sum_{j=1}^{n+1} P_{i, j}^{*}, \Delta X^{*} \cdot \Delta Z^{*} \cdot \cos \theta_{j}  \tag{3.13}\\
& \left(W_{y}^{*}\right)_{I, J, K}=\sum_{i=1}^{m+1} \sum_{j=1}^{n+1} P_{i, j,}^{*} \Delta X^{*} \cdot \Delta Z^{*} \cdot \sin \theta_{j} \tag{3.14}
\end{align*}
$$



FIG. 3.4
Flow chart for the hydrodynamic computer programme.


FIG.3.5 Bearing geometry.


FIG. 3.6
Hydrodynamic forces relationship.
and the resultant force is given by

$$
\begin{equation*}
\left(\mathrm{F}_{\mathrm{t}}^{*}\right)_{\text {I.J.K }}=\left\{\left(\mathrm{W}_{\mathrm{x}}\right)^{2}+\left(\mathrm{W}_{\mathrm{y}}\right)^{2}\right\}^{\frac{1}{2}} \tag{3.15}
\end{equation*}
$$

and the angle between the direction of the force and the displacement is given by

$$
\begin{equation*}
\beta_{I, J, K}=\sin ^{-1}\left(W_{y} / F_{t}\right)_{I, J, K} \tag{3.16}
\end{equation*}
$$

The force ( $F_{t}$ ) is then decomposed into a horizontal and into a vertical force at each lobe. The total force is then given from:

$$
\begin{equation*}
\mathrm{F}_{\mathrm{T}}^{*}=\left(\mathrm{F}_{\mathrm{V} t}^{*}+\mathrm{F}_{\mathrm{Ht}}^{*}\right)^{\frac{1}{2}} \tag{3.17}
\end{equation*}
$$

and the angle between the vertical direction and the direction of the total force is given by

$$
\begin{equation*}
\gamma=\tan ^{-1}\left(-\mathrm{F}_{\mathrm{Ht}}^{*} / \mathrm{F}_{\mathrm{Vt}}^{*}\right) \tag{3.18}
\end{equation*}
$$

where $\vec{F}_{V t}=F_{V I}+F_{V j}+F_{V k}$ and $\vec{F}_{H t}=F_{H I}+\vec{F}_{H J}+F_{H K}$ The bearing duty parameter is given by:

$$
\begin{equation*}
\mathrm{S}=\frac{1}{\mathrm{~F}_{\mathrm{T}}^{\star}} \tag{3.19}
\end{equation*}
$$

Where $S=\frac{\mu N}{P} \quad(D / C)^{2}$ and $P=F_{T} / L D$

## 3.4

 StiffnessFor a steady state running condition the lubricant film stiffness in the radial direction will be defined by

$$
\begin{equation*}
\lambda=\frac{\mathrm{dF}_{\mathrm{R}}}{\mathrm{dh}_{\min }}=\frac{\mathrm{dF}_{\mathrm{T}} \cos \Phi}{\mathrm{dF}_{\min }} \tag{3.20}
\end{equation*}
$$

since

$$
\begin{equation*}
F_{T}=\frac{\mu D L N}{S}(D / C)^{2} \tag{3.21}
\end{equation*}
$$

and

$$
\begin{align*}
& \mathrm{h}_{\min }=\mathrm{C} / 2(1-\varepsilon)  \tag{3.22}\\
& \lambda=\frac{2 \mu \mathrm{DLN}}{\mathrm{C}}(\mathrm{D} / \mathrm{C})^{2} \frac{\mathrm{~d}\left(\frac{\cos \phi}{\mathrm{~S}}\right)}{\mathrm{d} \varepsilon} \tag{3.23}
\end{align*}
$$

and the stiffness coefficient is given by

$$
\begin{equation*}
\lambda^{*}=\frac{\lambda}{2 \mu D L N /(D /)^{2}}=\frac{d\left(\frac{\cos \phi}{S}\right)}{d \varepsilon} \tag{3.24}
\end{equation*}
$$

where $\varepsilon=\varepsilon_{B}\left(1-\varepsilon_{0}\right)$

The stiffness coefficient has been calculated from the numerical solution of Reynolds equation. A typical calculation of the stiffness coefficient for the three lobe bearing is given in Figure 3.15. The calculations have been carried up to eccentricity ratios larger than those appropriate in practical use of the bearing, and with reference to the functional specifications of the three lobe bearing, the maximum eccentricity ratio $\left(\varepsilon_{B}\right)$ would be 0.17 (the eccentricity ratio is calculated for a lobe clearance of $2.6 \times 10^{-4} \mathrm{~m}$ ).

### 3.5 Oil Flow

The oil flow fed to a bearing serves a dual purpose. Whilst its. primary functions is to act as a lubricant in the creation of a load carrying film, it is also called upon to carry away the heat generated in the bearing. As a consequence, the rate of flow of oil through a bearing is an important consideration in bearing design and operation, and has to be known so that the oil system can be designed.

Oil is normally supplied to the inlet grooves under pressure, and with the journal stationary there would be a flow of oil through the clearances of the bearing. This kind of flow will be called the pressure flow. There would be also a flow of oil if oil were supplied at zero pressure, and the journal is rotating. This flow is due to the pumping action of the moving surface, and must also escape axially from the bearing. This flow will be called the hydrodynamic flow. The total oil flow will be regarded as the sum of the pressure flow (Poiseuille flow) and the hydrodynamic flow (Couette flow).

In the description of the finite difference technique in paragraph 3.2. it was mentioned that the pressures around the boundary of the film do not have to be zero but may be finite. The finite difference method may be used to calculate the hydrodynamic oil flow by inserting zero dimensionless pressure along the boundary which is the boundary of the axial oil inlet grooves. Also the finite difference method may be used to calculate the total oil flow (i.e. hydrodynamic oil flow and pressure oil flow) by inserting the pressure of the inlet oil along the boundary of the axial grooves. The oil inlet pressure in dimensionless
form is given by

$$
\begin{equation*}
P^{*}=P / \mu N(C / D)^{2} \tag{3.25}
\end{equation*}
$$

where P: is the oil pressure at the inlet (Figure 3.7).

From this point of view the total side-leakage oil flow or the hydrodynamic oil flow from one lobe and from both the bearing ends is given by

$$
\begin{equation*}
Q_{t, h}=2 \int \frac{1}{12 \mu} h^{3} \frac{\partial P}{\partial z} d x \tag{3.26}
\end{equation*}
$$

where $\frac{d p i s}{d z}$ axial pressure gradient at the ends of the bearing. The above expression may be used for the calculation of the total oil flow or for the hydrodynamic oil only. Subscript $t$ denotes the total oil flow (i.e. $p * \neq 0$ ) whereas subscript $h$ denotes the hydrodynamic oil flow (i.e. $\mathrm{P}^{*}=0$ ).

It is convenient to express the oil flow in terms of a dimensionless flow coefficient which relates the oil flow to the Petroff oil flow i.e.

$$
\begin{equation*}
Q_{t, h}=q \cdot \frac{\pi}{4} \cdot N \cdot D \cdot L \cdot C . \tag{3.27}
\end{equation*}
$$

By equating equation (3.26) and (3.27), the oil flow coefficient is given by

$$
\begin{equation*}
\left(q_{t, h}\right)_{I . J . K}=\frac{2}{3 \Pi} \cdot \frac{1}{N D L C} \int \frac{1}{\mu} h^{3} \frac{\partial P}{\partial z} d x \tag{3.28}
\end{equation*}
$$



FIG.3.7 Boundary conditions for the calculation of the total oil flow.

After substituting dimensionless forms for $\frac{\partial P}{\partial z}, h^{3}, \mu$ and $d x$ and after rearranging the oil flow coefficient is given by

$$
\begin{equation*}
\left(q_{t, h}\right)_{I, J, K}=\frac{2}{3 \pi} \cdot(D / L)^{2} \int H^{*} 3 \frac{\partial P^{*}}{\partial z^{*}} \cdot d x^{*} \tag{3.29}
\end{equation*}
$$

In numerical form equation(3.29) becomes

$$
\begin{equation*}
\left(q_{t, h_{I . J . K}}\right)^{(2 / 3 \pi)(D / L)^{2} \sum_{i=1}^{n+1} H_{m, i}^{*}\left(\frac{P_{m-1, j^{-P}}^{m+1, j}}{2 . \Delta Z}\right)^{*} \Delta X^{*} . . . . . .} \tag{3.30}
\end{equation*}
$$

The calculation of $q_{t, h}$ from expression (3.30) depends upon a prior solution of Reynolds equation (3.7) to give the pressure distribution over the complete bearing. The calculations of $q_{t, h}$ is incorporated in the same computer programme. The term 'solution of Reynolds equation' will denote the calculation of the pressure distribution, and the oil flow coefficient by the finite difference procedure.

The dimensionless flow coefficient ( $q_{t, h}$ ) has been calculated separately for each lobe and have been summed to give the oil flow coefficient of the complete bearing which may be expressed as

$$
\begin{equation*}
q_{o(t, h)}=\sum_{I, J, K}\left(q_{t, h}\right) \tag{3.31}
\end{equation*}
$$

Reynolds equation has been solved for inlet pressures ( $P$ *) of $0,0.5$, 1,2,3 and 5. Typical results of calculation are shown in Figure 3.8. At $P^{*}=0$, the calculated flowcoefficient is the hyarodynamic flow coefficient $\left(q_{0}\right)_{h}$ and where $p * \neq 0$, the calculated flow coefficient
is the total flow coefficient $\left(q_{0}{ }_{\prime} t^{\prime}\right.$ which comprises the hydrodynamic flow coefficient and the pressure flow coefficient ( $q_{0}$ ) $p^{\text {. . It may be }}$ seen from Figure 3.8 that the pressure flow coefficient varies linearly with the dimensionless inlet pressure, and consequently, in general for any value of $\varepsilon_{B}$ (or duty parameter) it is necesssary only to solve Reynolds equation for two values of $P^{*}$. Also it may be seen that at higher values of offset ratio the pressure flow coefficient is increasing marginally as the dimensionless inlet pressure increases. The notion was proposed at the start of this paragraph that the pressure flow depends only upon the inlet pressure and the bearing clearance, and clearly Figure 3.8 is consistent with that notion. An alternative way for the calculation of the pressure flow is considered below.

It would appear from Figure 3.8 that at low offset ratios the pressure flow is in general substantially greater than the hydrodynamic flow, however with normal values of inlet pressure, viscosity, clearance and speed the value of the dimensionless inlet pressure does not rise above a value of two in practice. The results of the oil flow coefficient calculations have been used to calculate the actual oil flow, the power loss and the temperature rise. Typical results of calculations are listed in Table 3.1 where it may be seen that the dimensionless inlet pressure decreases as the speed and the eccentricity ratio increase.

A second way in which total oil flows can be assessed is to calculate the pressure flow from the solution of Reynolds equation when the shaft is stationary. For a stationary shaft Reynolds equation reduces


FIG.3.8 Variation of the total oil flow coefficient with the inlet dimensionless pressure.

TABLE 3.1
Power loss, oil flow, temperature rise and dimensionless inlet pressure for the three-
lobe Journal bearing.
$L=1.27 \times 10^{-1} \mathrm{~m}$
$D=1.35 \times 10^{-1} \mathrm{~m}$
$C=3 \times 10^{-4} \mathrm{~m}$
$\varepsilon_{0}=0.4$
$\mathrm{T}_{1}=40^{\circ} \mathrm{C}$
$P=1.47 \times 10^{5} \mathrm{Nm}^{-2}$
$\mu=33 \mathrm{C} . \mathrm{P}$. at $40^{\circ} \mathrm{C}$

$$
\varepsilon_{B}=0
$$

| N rpm | $1 \mathrm{Q}_{\mathrm{h}}^{-1}$ | $1 \mathrm{Q}_{\mathrm{s}}-1$ | $\begin{aligned} & Q_{t} \\ & 1 \mathrm{~s}^{-1} \end{aligned}$ | $\xrightarrow[\mathrm{ch}]{\mathrm{H}_{\text {c }}}$ | $\mathrm{T}_{2} \mathrm{O}_{\mathrm{C}}$ | $P^{*}=\frac{P}{\mu N}(C / D)^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 | 0.0066 | 0.0214 | 0.028 | 0.49 | 51 | 2.06 |
| 3000 | 0.0198 | C. 0382 | 0.058 | 2.48 | 65 | 1.14 |
| 5000 | 0.033 | 0.0525 | 0.855 | 5 | 74 | 0.9 |
| 7000 | 0.0462 | 0.0658 | 0.112 | 7.86 | 79.5 | 0.737 |
| 9000 | 0.0594 | 0.0796 | 0.139 | 10.77 | 84 | 0.64 |

$\varepsilon_{B}=0.6$

| $\mathrm{N}$ <br> rpm | $1 \mathrm{~s}^{\mathrm{h}^{\text {a }}}$ | $\mathrm{C}_{\mathrm{p}} \mathrm{s}^{-1}$ | $Q_{t}$ $1 s^{-1}$ | $\begin{gathered} \mathrm{H}_{\mathrm{S}} \\ \mathrm{kw} \end{gathered}$ | T C $\mathrm{O}^{\text {C }}$ - | $P^{*}=\frac{P}{\mu N}(C / D)^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 | 0.016 | 0.0265 | 0.0425 | 0.56 | 48.5 | 1.87 |
| 3000 | 0.048 | 0.0438 | 0.0918 | 3.06 | 60 | 0.97 |
| 5000 | 0.080 | $0.058 \%$ | 0.1387 | 6.34 | 67 | 0.71 |
| 7000 | 0.112 | 0.069 | 0.181 | 10.5 | 73 | 0.615 |
| 9000 | 0.144 | 0.0820 | 0.226 | 14.72 | 77 | 0.524 |

to the form

$$
\begin{equation*}
\frac{\partial}{\partial x}\left(h^{3} \frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial z}\left(h^{3} \frac{\partial P}{\partial z}\right)=0 \tag{3.32}
\end{equation*}
$$

The solution of equation(3.32) is also required in connection with the hydrostatic aspect of the bearing, and this solution is discussed in Chapter 5. Here it is sufficient to note that the pressure flow coefficient is given by

$$
\begin{equation*}
\left(q_{P}\right)_{I, J \text { and } K}=\int h^{3} \frac{\partial P}{\partial z} d x \tag{3.33}
\end{equation*}
$$

The dimensionless pressure flow coefficient has been calculated separately for each lobe and have been summed to give the oil flow coefficient of the complete bearing which may be expressed as

$$
\begin{equation*}
q_{o p}=\sum_{I, J} \text { and } K q_{p} \tag{3.34}
\end{equation*}
$$

and the pressure flow is calculated from

$$
\begin{equation*}
Q_{p}=\frac{P C^{3} q_{o p}}{6 \mu} \tag{3.35}
\end{equation*}
$$

In expression (3.33) the axial pressure gradient at the end of the bearing denoted by $\frac{d P}{d z}$, is actually calculated from the solution of equation (3.32).

The pressure flow was measured experimentally, simply by collecting the oil flow out of the bearing when the shaft is stationary. In such tests the only thing which can vary is oil viscosity and from
expression (3.35) one would expect $Q_{p} x \mu$ to be constant. The experiment gave a value of $\operatorname{lpx} \mu$ of 8.36 dyne cm whereas the calculated value is 9.24. The two results are in close agreement. However, the pressure flow as measured or calculated in the context of a stationary shaft is not necessarily the pressure flow which will arise when the shaft is rotating, because the temperature is then influenced additionally by the rotation of the shaft. It was found that if the pressure flow with a rotating shaft is based upon oil outlet temperature then the calculation leads to a pressure flow somewhat in excess of the pressure flow as derived by subtracting the calculated hydrodynamic flow from the calculated total oil flow or from the measured total flow. For instance at a speed of 6000 rpm , the calculated total oil flow is $0.1151 s^{-1}$, the calculated hydrodynamic oil flow is $0.0381 \mathrm{~s}^{-1}$, which gives a pressure flow of $0.0771 \mathrm{~s}^{-1}$. The measured total oil flow was $0.1061 s^{-1}$ which gives a pressure flow of $0.0681 \mathrm{~s}^{-1}$. while the calculated pressure flow from expressure (3.35) is $0.1281 \mathbf{s}^{-1}$. The trend shown by the results is to be expected because clearly the mean oil temperature to be associated with the pressure flow is somewhere between inlet and outlet temperatures. It has been found from the experimental tests which are reported in Chapter 6, that agreement between the calculated pressure flows and the experimentally derived pressure flows is obtained by taking a mean temperature given by

$$
\begin{equation*}
T_{m}=T_{1}+\left(T_{2}-T_{1}\right) \times 0.65 \tag{3.36}
\end{equation*}
$$

Semi empirical expressions are frequently used for the calculation of the pressure flow, e.g. Ref 11 and Ref 70 give the following
expressions for the calculations of the pressure oil flow.

$$
\begin{align*}
& Q_{p}=\frac{C^{3} P D M}{96 \mu L}\left(q_{0, g}^{*}+\frac{46}{D(1-\ell / L}\right)  \tag{ref:11}\\
& Q_{p}=\frac{P C^{3} V}{24 \mu}
\end{align*}
$$

(ref: 70)

The first expression was modified to take into account the presence of tha three axial grooves and also to take into account the change in the oil film thickness within each lobe. In Ref. 11 it has been pointed out that it is often difficult to give precise values to the viscosity when the bearing is hot, but a thermocouple placed at the inlet manifold and not in the bearing spill should give tolerable accuracy. However, a viscosity associated with the mean temperature given by equation (3.36) has been used in the semi-empirical formula for the calculation of the pressure flow.

Quite clearly there are several ways of dealing with the pressure flow and the results given by the various methods in one particular instance are listed in Table 3.2. In that table line A gives the calculated hydrodynamic oil flow, line B gives the calculated total oil flow and line C is pressure flow. Line D gives the pressure flow as calculated with a stationary shaft and using the mean oil temperature given by expression(3.36) while line $F$ gives the pressure flow by using the oil outlet temperature. Line E gives the total ofl flow as implied by lines $A$ and $D$. Line $G$ gives the pressure flow as calculated from semi-empirical expression (Ref. 11) and by using the mean oil temperature given by expression(3.36) while line H gives the

TABLE 3.2 Calculated oil flow for the three lobe Journal bearing*

| Line | Type of Flow | Origin | $\begin{aligned} & \text { Oil Flow } \\ & 1 \mathrm{~s}^{-1} \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| A | Hydrodynamic flow | Eqn. 3.27 | 0.038 |
| B | Total oil flow | Eqn. 3.27 | 0.115 |
| C | Pressure flow | $\mathrm{B}-\mathrm{A}$ | 0.077 |
| D | Pressure flow | $\begin{gathered} \text { Eqns. } 3.35- \\ 3.36 \end{gathered}$ | 0.0735 |
| E | Total oil flow | A+D | 0.1115 |
| F | Pressure flow | Eqn. 3.35 | 0.128 |
| G | Pressure flow** | Refll +eqn. 3.36 | 0.060 |
| H | Total oil flow | $A+G$ | 0.098 |
| I | Measured flow | Chapter 6 | 0.106 |


** The expression given in ref. 11 has been adapted to be used for the three lobe bearing.
total oil flow as implied by lines $A$ and $G$. Line $I$ gives the measured total oil flow. It may be seen from the table that the same result is obtained whether the total oil flow is calculated directly as in line $B$ or is obtained by summing the hydrodynamic and pressure flows as in line $E$ provided that the mean temperature is used in calculating the latter. It may also be seen that the pressure flow given by the modified semi-empirical expression is in substantial agreement with the other calculated pressure flows. The calculated total oil flow given by line $B$ and line $E$ are greater than the total measured oil flow by approximately $8.5 \%$ which gives a reasonable tolerance for the design of the oil system. However, by using the previous methods for the calculations of the oil flow to calculate the power loss and the oil temperature rise, a very close agreement between the calculated and the measured (Chapter 6) power loss and oil temperature is achieved.

The question arises, which is the preferred method for calculating total oil flow. Because of its simplicity of conception the one step method of equation(3.27) is preferred. It is of interest to compare the pressure distribution given by this one step method with the pressure distribution given when the hydrodynamic flow alone is considered (eqn. 3.7). The comparison is shown in Figure 3.9 which shows that both calculations place the peak pressure in the same position but at a typical eccentricity ratio of 0.4 , the peak pressure for the total calculation is slightly higher than that for the hydrodynamic flow alone.

### 3.6 Results of the Numerical Computations

The validity of the computer programme and its accuracy has been


FIG.3.9 Pressure distribution at the axial mid-plane.
examined by using the computer programme to solve Reynolds equation for a journal bearing of full circular bore. Table 3.3 compares the result of the present programme with results previously published in Refs.1,2, and very close agreement is obtained. Also it has been pointed out that the solution of Reynolds equation for the pressure distribution was carried over the full mesh and the symmetry of the pressure distribution around the bearing centre can be used as a check upon calculation. In Figure 3.10 a complete output for the pressure distribution is given and it may be seen that the pressure is symmetrical around the bearing axial mid-plane. As a further check upon the accuracy of the computer programme, Figure 3.11 shows the pressure distribution at the axial mid-plane. It may be seen that the pressure film is terminated beyond the position of the minimum film thickness. Also it may be seen from Figure 3.11 that the condition of zero pressure and zero pressure gradient at the trailing edge has been satisfied in the solution. The computer routine has written in Fortran VI language and it has been performed on the I.C.I. 1900 Computer, and on the C.D.C. 7600 computer.

All of the input and the output of the finite difference programme is in dimensionless form. The output comprises the load angle $\gamma$, the attitude angle $\varnothing$, duty parameter, the hydrodynamic oil flow coefficient, the pressure oil flow coefficient, the total oil flow coefficient and the dimensionless pressure at the mesh points.

The computer programme was performed for the following values of the variables

$$
\begin{aligned}
& \varepsilon_{0}: 0.3,0.4,0.5, \ldots \ldots \ldots . . . \\
& \varepsilon_{B}: 0,0.1,0.2, \ldots \ldots \ldots . . . . \text {, and } 0.9 .
\end{aligned}
$$




 70.086



,


P*: 0, 1, 2 and 5.

The results of calculation are presented in Tables 3.4-3.8. Table 3.4 relates to a load acting towards the centre of a lobe, and Table 3.5 relates to a load acting towards a conjunction of adjacent lobes. Tables 3.6 and 3.7 relate to calculation in which a finite pressure has been entered at the oil inlets. Journal position is given by Table 3.6 and the total oil flow coefficient for axial grooves of 30 degree width are given in Table 3.7. In Table 3.8 pressure flow coefficients as calculated from expression(3.32) are given.

Typical results from Tables $3.4-3.5$ will be discussed and compared with available results for other bearing designs. Figure 3.12 shows the variation of the hydrodynamic flow coefficient with the eccentricity ratio. It may be seen from Figure 3.12, at high values of eccentricity the hydrodynamic flow coefficient decreases as the offset ratio increases, however in real terms the increase of the offset ratio leads to an increase in the lobe diametral clearance and it will be shown in paragraph 3.9 that the actual hydrodynamic flow increases as the offset ratio increases. It may also be seen from Figure 3.12 that at low values of eccentricity the flow coefficient increases as the offset ratio increases. This trend in the characteristics of the hydrodynamic flow coefficient is attributable to the finite lobe eccentricity which exist at zero bearing eccentricity. As the offset ratio reaches zero (i.e. full circular bore with three groove bearing) the hydrodynamic action is diminished and the hydrodynamic flow coefficient becomes zero. Another important factor in the hydrodynamic
*the flow coefficient relates to the axial flow and in this instance no pressures are developed, and in principal there is no axial flow.

Table 3.4. Results of Computation for* the three-lobe bearings


| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | Ф | S | $q_{h}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\frac{1}{2}$ | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | $\begin{aligned} & 30 \\ & 32 \\ & 35 \\ & 38 \\ & 38 \\ & 38 \\ & 40 \\ & 42 \\ & 42 \end{aligned}$ | $\begin{aligned} & 0.4 \\ & 0.2 \\ & 0.129 \\ & 0.091 \\ & 0.0625 \\ & 0.0454 \\ & 0.0345 \\ & 0.0263 \\ & 0.0172 \end{aligned}$ | 0.328 0.328 0.3285 0.33 0.33 0.332 0.333 0.334 0.335 |
| 0.8 | 1 | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | $\begin{aligned} & 30 \\ & 33 \\ & 35 \\ & 37 \\ & 40 \\ & 40 \\ & 41 \\ & 42 \\ & 42 \end{aligned}$ | $\begin{aligned} & 0.286 \\ & 0.139 \\ & 0.088 \\ & 0.0625 \\ & 0.0454 \\ & 0.033 \\ & 0.025 \\ & 0.0189 \\ & 0.015 \end{aligned}$ | 0.16 0.161 0.161 0.161 0.162 0.162 0.163 0.164 0.1645 |

Table 3.4. continued Results of Computation for the Threelobe Journal Bearings

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | Ф | S | ${ }^{9} \mathrm{~h}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.7 | $\frac{1}{2}$ | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | $\begin{aligned} & 40 \\ & 42 \\ & 42 \\ & 45 \\ & 45 \\ & 46 \\ & 47 \\ & 47 \\ & 46 \end{aligned}$ | $\begin{aligned} & 0.884 \\ & 0.426 \\ & 0.27 \\ & 0.188 \\ & 0.134 \\ & 0.096 \\ & 0.071 \\ & 0.0476 \\ & 0.03 \end{aligned}$ | $\begin{aligned} & 0.305 \\ & 0.306 \\ & 0.308 \\ & 0.312 \\ & 0.316 \\ & 0.32 \\ & 0.336 \\ & 0.33 \\ & 0.337 \end{aligned}$ |
|  | 1 | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | 42 <br> 44 <br> 45 <br> 47 <br> 47 <br> 47 <br> 47 <br> 47 <br> 45 | $\begin{aligned} & 0.556 \\ & 0.263 \\ & 0.168 \\ & 0.118 \\ & 0.087 \\ & 0.0625 \\ & 0.0454 \\ & 0.033 \\ & 0.02 \end{aligned}$ | $\begin{aligned} & 0.152 \\ & 0.154 \\ & 0.155 \\ & 0.157 \\ & 0.159 \\ & 0.162 \\ & 0.164 \\ & 0.166 \\ & 0.172 \end{aligned}$ |
| 0.6 | $\frac{1}{2}$ | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | 43 <br> 50 <br> 51 <br> 51 <br> 52 <br> 52 <br> 51 <br> 50 <br> 47 | $\begin{aligned} & 1.37 \\ & 0.65 \\ & 0.4 \\ & 0.26 \\ & 0.203 \\ & 0.167 \\ & 0.1124 \\ & 0.066 \\ & 0.037 \end{aligned}$ | $\begin{aligned} & 0.272 \\ & 0.276 \\ & 0.282 \\ & 0.29 \\ & 0.3 \\ & 0.31 \\ & 0.32 \\ & 0.334 \\ & 0.353 \end{aligned}$ |

Table 3.4 continued Results of Computation for the Three-lobe Journal Bearings.

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | $\Phi$ | S | $\mathrm{q}_{\mathrm{h}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.6 | 1 | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | $\begin{aligned} & 50 \\ & 52 \\ & 53 \\ & 53 \\ & 53 \\ & 52 \\ & 51 \\ & 50 \\ & 45 \end{aligned}$ | $\begin{aligned} & \hline 1 \\ & 0.76 \\ & 0.36 \\ & 0.22 \\ & 0.16 \\ & 0.112 \\ & 0.0855 \\ & 0.069 \\ & 0.0417 \\ & 0.022 \end{aligned}$ | $\begin{aligned} & 0.139 \\ & 0.142 \\ & 0.145 \\ & 0.15 \\ & 0.154 \\ & 0.16 \\ & 0.166 \\ & 0.173 \\ & 0.191 \end{aligned}$ |
| 0.5 | $\frac{1}{2}$ | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | $\begin{aligned} & 57 \\ & 58 \\ & 59 \\ & 58 \\ & 58 \\ & 57 \\ & 55 \\ & 53 \\ & 45 \end{aligned}$ | $\begin{aligned} & 2.04 \\ & 0.995 \\ & 0.62 \\ & 0.425 \\ & 0.298 \\ & 0.204 \\ & 0.137 \\ & 0.095 \\ & 0.048 \end{aligned}$ | $\begin{aligned} & 0.235 \\ & 0.240 \\ & 0.253 \\ & 0.268 \\ & 0.283 \\ & 0.310 \\ & 0.323 \\ & 0.357 \\ & 0.415 \end{aligned}$ |
|  | 1 | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | $\begin{aligned} & 55 \\ & 55 \\ & 60 \\ & 60 \\ & 55 \\ & 55 \\ & 55 \\ & 53 \\ & 45 \end{aligned}$ | $\begin{aligned} & 1.11 \\ & 0.502 \\ & 0.323 \\ & 0.22 \\ & 0.156 \\ & 0.112 \\ & 0.076 \\ & 0.056 \\ & 0.0255 \end{aligned}$ | $\begin{aligned} & 0.122 \\ & 0.127 \\ & 0.134 \\ & 0.143 \\ & 0.152 \\ & 0.162 \\ & 0.173 \\ & 0.193 \\ & 0.22 \end{aligned}$ |

Table 3.4 continued Results of Computation for the Three-lobe Journal Bearings.



Table 3.4 continued Results of Computation for the Three-lobe Journal Bearings.


| $\varepsilon_{0}$ | $L^{1}$ D | $\varepsilon_{\mathrm{B}}$ | Ф | S | ${ }^{\text {q }}$ h |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | $\frac{1}{2}$ | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | $\begin{aligned} & 73 \\ & 70 \\ & 70 \\ & 65 \\ & 62 \\ & 58 \\ & 50 \\ & 47 \\ & 34 \end{aligned}$ | $\begin{aligned} & 3.76 \\ & 1.7 \\ & 1.063 \\ & 0.655 \\ & 0.48 \\ & 0.356 \\ & 0.20 \\ & 0.111 \\ & 0.039 \end{aligned}$ | $\begin{aligned} & 0.154 \\ & 0.177 \\ & 0.214 \\ & 0.244 \\ & 0.30 \\ & 0.361 \\ & 0.43 \\ & 0.50 \\ & 0.574 \end{aligned}$ |
|  | 1 | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | 73 <br> 70 <br> 70 <br> 64 <br> 60 <br> 54 <br> 50 <br> 42 <br> 35 | $\begin{aligned} & 1.81 \\ & 0.855 \\ & 0.48 \\ & 0.31 \\ & 0.25 \\ & 0.161 \\ & 0.104 \\ & 0.059 \\ & 0.025 \end{aligned}$ | $\begin{aligned} & 0.083 \\ & 0.099 \\ & 0.12 \\ & 0.14 \\ & 0.174 \\ & 0.215 \\ & 0.244 \\ & 0.290 \\ & 0.321 \end{aligned}$ |

Table 3.5 Results of Computation for the Three Lobe Journal Bearing*


| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | $\phi$ | s | $\mathrm{q}_{\mathrm{h}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.8 | $\frac{1}{2}$ | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \end{aligned}$ | $\begin{aligned} & 25 \\ & 22 \\ & 20 \end{aligned}$ | $\begin{aligned} & 0.455 \\ & 0.23 \\ & 0.149 \end{aligned}$ | $\begin{aligned} & 0.327 \\ & 0.328 \\ & 0.329 \end{aligned}$ |
|  | 1 | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \end{aligned}$ | $\begin{aligned} & 27 \\ & 25 \\ & 22 \\ & 20 \end{aligned}$ | $\begin{aligned} & 0.303 \\ & 0.15 \\ & 0.10 \\ & 0.073 \end{aligned}$ | $\begin{aligned} & 0.161 \\ & 0.161 \\ & 0.161 \\ & 0.161 \end{aligned}$ |
| 0.7 | $\frac{1}{2}$ | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \end{aligned}$ | $\begin{aligned} & 35 \\ & 30 \\ & 27 \\ & 24 \\ & 22 \\ & 18 \end{aligned}$ | $\begin{aligned} & 0.91 \\ & 0.455 \\ & 0.294 \\ & 0.208 \\ & 0.16 \\ & 0.125 \end{aligned}$ | $\begin{aligned} & 0.304 \\ & 0.305 \\ & 0.308 \\ & 0.312 \\ & 0.317 \\ & 0.322 \end{aligned}$ |
|  | 1 | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \end{aligned}$ | $\begin{aligned} & 37 \\ & 34 \\ & 31 \\ & 27 \\ & 25 \end{aligned}$ | $\begin{aligned} & 0.568 \\ & 0.28 \\ & 0.184 \\ & 0.132 \\ & 0.1 \end{aligned}$ | $\begin{aligned} & 0.153 \\ & 0.154 \\ & 0.155 \\ & 0.158 \\ & 0.16 \end{aligned}$ |
| 0.6 | $\frac{1}{2}$ | $\begin{aligned} & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \end{aligned}$ | $\begin{aligned} & 43 \\ & 40 \\ & 35 \\ & 32 \\ & 28 \end{aligned}$ | $\begin{aligned} & 1.36 \\ & 0.654 \\ & 0.425 \\ & 0.293 \\ & 0.218 \end{aligned}$ | $\begin{aligned} & 0.272 \\ & 0.276 \\ & 0.283 \\ & 0.293 \\ & 0.305 \end{aligned}$ |

* The results which are listed in this table are obtained from the solution of Reynolds equation (3.7) for zero dimensionless pressure at the axial grooves.

Table 3.5 cont. Results of Computation for the Three Lobe Journal Bearing


| $\varepsilon_{0}$ | $\mathrm{~L} / \mathrm{D}$ | $\varepsilon_{\mathrm{B}}$ | $\phi$ | s | $\mathrm{q}_{\mathrm{h}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.6 | $\frac{1}{2}$ | 0.6 | 25 | 0.157 | 0.319 |
|  |  | 0.7 | 22 | 0.113 | 0.33 |
|  |  |  | 0.1 | 47 | 0.78 |

Table 3.5 cont. Results of Computation for the Three Lobe Journal Bearing


| $\varepsilon_{0}$ | $\mathrm{~L} / \mathrm{D}$ | $\varepsilon_{\mathrm{B}}$ | $\phi$ | s | $\mathrm{q}_{\mathrm{h}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5 |  | 0.6 | 36 | 0.125 | 0.191 |
|  |  | 1 | 0.7 | 31 | 0.09 |

Table 3.6 Results of Computation for the Three Lobe Journal Bearing

$\mathrm{P}^{*}=1.0$

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | $\phi$ | s | $\mathrm{q}_{\mathrm{t}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | 60 <br> 61 <br> 61 <br> 55 | $$ | $\begin{aligned} & 0.58 \\ & 0.604 \\ & 0.667 \\ & 0.77 \\ & 0.9 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | 60 <br> 60 <br> 55 <br> 53 | $\begin{gathered} - \\ 0.54 \\ 0.225 \\ 0.105 \\ 0.066 \end{gathered}$ | $\begin{aligned} & 0.204 \\ & 0.212 \\ & 0.24 \\ & 0.33 \\ & 0.36 \end{aligned}$ |

$P^{*}$ is the dimensionless pressure at the axial grooves

Table 3.7 Results of Computation of Three Lobe Journal Bearing

| $\varepsilon_{0}$ | L/D | $E_{B}$ | p* | $q_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.5 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.219 \\ & 0.240 \\ & 0.268 \\ & 0.310 \\ & 0.357 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.1191 \\ & 0.127 \\ & 0.143 \\ & 0.162 \\ & 0.193 \end{aligned}$ |
| 0.5 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 0.964 \\ & 1.005 \\ & 1.144 \\ & 1.361 \\ & 1.646 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 0.303 \\ & 0.321 \\ & 0.383 \\ & 0.47 \\ & 0.577 \end{aligned}$ |
| 0.5 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 2.153 \\ & 2.249 \\ & 2.51 \\ & 2.95 \\ & 3.55 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 0.621 \\ & 0.663 \\ & 0.761 \\ & 0.91 \\ & 1.108 \end{aligned}$ |

Table 3.7 cont. Results of Computation of the Three Lobe Journal Bearing
$\alpha=30^{\circ}$

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | $\mathrm{p}^{*}$ | $q_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.4 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $0$ | $\begin{aligned} & 0.179 \\ & 0.208 \\ & 0.243 \\ & 0.306 \\ & 0.414 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.098 \\ & 0.115 \\ & 0.138 \\ & 0.170 \\ & 0.230 \end{aligned}$ |
| 0.4 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 2.79 \\ & 2.93 \\ & 3.35 \\ & 4.0 \\ & 4.94 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 0.83 \\ & 0.875 \\ & 1.123 \\ & 1.251 \\ & 1.558 \end{aligned}$ |
| 0.3 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.1362 \\ & 0.177 \\ & 0.243 \\ & 0.365 \\ & 0.50 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.075 \\ & 0.111 \\ & 0.142 \\ & 0.218 \\ & 0.290 \end{aligned}$ |
| 0.3 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 3.6 \\ & 3.78 \end{aligned}$ |

Table 3.7 cont. Results of Computation of the Three Lobe Journal
Bearing

$$
\alpha=30^{\circ}
$$

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | $\mathrm{p}^{*}$ | $q_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.3 | $\frac{1}{2}$ | 0.4 | 5 | 4.35 |
|  |  | 0.6 | 5 | 5.34 |
|  |  | 0.8 | 5 | 6.73 |
|  | 1 | 0 | 5 | 1.1 |
|  |  | 0.2 . | 5 | 1.156 |
|  |  | 0.4 | 5 | 1.364 |
|  |  | 0.6 | 5 | 1.7 |
|  |  | 0.8 | 5 | 2.14 |
| 0.6 | $\frac{1}{2}$ | 0 | 0 | 0.270 |
|  |  | 0.2 | 0 | 0.276 |
|  |  | 0.4 | 0 | 0.29 |
|  |  | 0.6 | 0 | 0.31 |
|  |  | 0.8 | 0 | 0.334 |
|  | 1 | 0 | 0 | 0.134 |
|  |  | 0.2 | 0 | 0.142 |
|  |  | 0.4 | 0 | 0.150 |
|  |  | 0.6 | 0 | 0.160 |
|  |  | 0.8 | 0 | 0.173 |
| 0.7 | $\frac{1}{2}$ | 0 | 0 | 0.300 |
|  |  | 0.2 | 0 | 0.306 |
|  |  | 0.4 | 0 | 0.312 |
|  |  | 0.6 | 0 | 0.32 |
|  |  | 0.8 | 0 | 0.33 |
|  | 1 | 0 | 0 | 0.150 |
|  |  | 0.2 | 0 | 0.154 |
|  |  | 0.4 | 0 | 0.157 |
|  |  | 0.6 | 0 | 0.162 |
|  |  | 0.8 | 0 | 0.166 |
| 0.8 | $\frac{1}{2}$ | 0 | 0 | 0.326 |
|  |  | 0.2 | 0 | 0.328 |
|  |  | 0.4 | 0 |  |

Table 3.7 cont. Results of Computation of the Three Lobe Journal Bearing

$$
\alpha=30^{\circ}
$$

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | p* | $q_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.8 | $\frac{1}{2}$ | 0.6 | 0 | 0.332 |
|  |  | 0.8 | 0 | 0.334 |
|  | 1 | 0 | 0 | 0.164 |
|  |  | 0.2 | 0 | 0.164 |
|  |  | 0.4 | 0 | 0.165 |
|  |  | 0.6 | 0 | 0.196 |
|  |  | 0.8 | 0 | 0.199 |
| 0.6 | $\frac{1}{2}$ | 0 | 3.0 | 0.961 |
|  |  | 0.2 | 3.0 | 0.985 |
|  |  | 0.4 | 3.0 | 1.041 |
|  |  | 0.6 | 3.0 | 1.2 |
|  |  | 0.8 | 3.0 | 1.394 |
|  | 1. | 0 | 3.0 | 0.328 |
|  |  | 0.2 | 3.0 | 0.337 |
|  |  | 0.4 | 3.0 | 0.368 |
|  |  | 0.6 | 3.0 | 0.427 |
|  |  | 0.8 | 3.0 | 0.504 |
| 0.7 | $\frac{1}{2}$ | 0 | 3.0 | 0.811 |
|  |  | 0.2 | 3.0 | 0.823 |
|  |  | 0.4 | 3.0 | 0.858 |
|  |  | 0.6 | 3.0 | 0.918 |
|  |  | 0.8 | 3.0 | 1.003 |
|  | 1 | 0 | 3.0 | 0.291 |
|  |  | 0.2 | 3.0 | 0.295 |
|  |  | 0.4 | 3.0 | 0.308 |
|  |  | 0.6 | 3.0 | 0.329 |
|  |  | 0.8 | 3.0 | 0.363 |
| 0.8 | $\frac{1}{2}$ | 0 | 3.0 | 0.695 |
|  |  | 0.2 | 3.0 | 0.699 |
|  |  | 0.4 | 3.0 | 0.710 |
|  |  | 0.6 | 3.0 | 0.731 |
|  |  | 0.8 | 3.0 | 0.76 |

Table 3.7 cont. Results of Computation of the Three Lobe Journal Bearing
$\alpha=30^{\circ}$

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | p* | $q_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 3.0 \\ & 3.0 \\ & 3.0 \\ & 3.0 \\ & 3.0 \end{aligned}$ | $\begin{aligned} & 0.263 \\ & 0.264 \\ & 0.278 \\ & 0.279 \\ & 0.285 \end{aligned}$ |
| 0.45 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 2.45 \\ & 2.68 \\ & 2.92 \\ & 3.46 \\ & 4.19 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 0.713 \\ & 0.75 \\ & 0.883 \\ & 1.07 \\ & 1.3 \end{aligned}$ |
| 0.45 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.198 \\ & 0.218 \\ & 0.279 \\ & 0.374 \\ & 0.47 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.109 \\ & 0.12 \\ & 0.141 \\ & 0.165 \\ & 0.222 \end{aligned}$ |
| 0.35 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 3.18 \\ & 3.34 \\ & 3.85 \\ & 4.63 \\ & 5.99 \end{aligned}$ |

Table 3.7 cont. Results of Computation of the Three Lobe Journal Bearing

$$
\alpha=30^{\circ}
$$

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | $\mathrm{p}^{*}$ | $q_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.35 | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \end{aligned}$ | $\begin{aligned} & 0.96 \\ & 1.006 \\ & 1.18 \\ & 1.46 \\ & 1.84 \end{aligned}$ |
|  | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.158 \\ & 0.192 \\ & 0.303 \\ & 0.43 \\ & 0.549 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0.087 \\ & 0.11 \\ & 0.140 \\ & 0.20 \\ & 0.260 \end{aligned}$ |
|  |  | . |  |  |

Table 3:8 Results of Computation for the Three Lobe Journal

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | ${ }^{\text {q }}$ p |
| :---: | :---: | :---: | :---: |
| 0.5 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 1.149 \\ & 1.187 \\ & 1.298 \\ & 1.388 \\ & 1.76 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0.704 \\ & 0.73 \\ & 0.8 \\ & 0.921 \\ & 1.1 \end{aligned}$ |
| 0.4 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 1.48 \\ & 1.53 \\ & 1.71 \\ & 2.0 \\ & 2.43 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0.926 \\ & 0.962 \\ & 1.08 \\ & 1.27 \\ & 1.54 \end{aligned}$ |
| 0.3 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 1.86 \\ & 1.94 \\ & 2.2 \\ & 2.64 \\ & 3.58 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 1.194 \\ & 1.25 \\ & 1.42 \\ & 1.71 \\ & 2.1 \end{aligned}$ |

Table 3.8 Results of Computation for the Three Lobe Journal Bearing.

| $\varepsilon_{0}$ | L/D | $\varepsilon_{B}$ | $\mathrm{q}_{\mathrm{p}}$ |
| :---: | :---: | :---: | :---: |
| 0.5 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 1.34 \\ & 1.4 \\ & 1.52 \\ & 1.79 \\ & 2.07 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 0.783 \\ & 0.81 \\ & 0.89 \\ & 1.131 \\ & 1.23 \end{aligned}$ |
| 0.4 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 1.74 \\ & 1.806 \\ & 2.016 \\ & 2.37 \\ & 2.89 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 1.038 \\ & 1.082 \\ & 1.212 \\ & 1.432 \\ & 1.752 \end{aligned}$ |
| 0.3 | $\frac{1}{2}$ | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 2.21 \\ & 2.31 \\ & 2.52 \\ & 3.15 \\ & 3.93 \end{aligned}$ |
|  | 1 | $\begin{gathered} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \end{gathered}$ | $\begin{aligned} & 1.35 \\ & 1.41 \\ & 1.6 \\ & 1.938 \\ & 2.41 \end{aligned}$ |

flow coefficient is that at higher offset ratios (i.e. $\varepsilon_{0}>0.6$ ) the hydrodynamic flow coefficient is nearly constant for eccentricity ratio up to 0.9. The implication of the hydrodynamic flow coefficient for the bearing performance will be examined in paragraph 3.9.

The locus of journal centre is shown in Figure 3.13. At low values of eccentricity all three lobes contribute to the hydrodynamic performance of the bearing so that they all have a marked effect on the attitude angle which is lower than the angle of the other circular bearings ( $E$ and $F$ of the Figure). At high values of eccentricity the hydrodynamic contribution of the two upper lobes decreases. For instance at $\varepsilon_{B}=0.3$ the generated dimensionless hydrodynamic force at the bottom and the upper lobes is $1.133,0.093$ and 0.228 respectively while at $\varepsilon_{B}=0.9$, the dimensionless hydrodynamic forces become 11.28 , 0.046 and 0.467 respectively. Consequently it can be seen that as the eccentricity increase the hydrodynamic effect of the upper lobes decreases, so that the attitude angle tends to increase. Also it may be seen that the increase of the offset ratio has a marked effect on decreasing the attitude angle.

The other important dimensionless parameters which are used to describe the bearing performance are the duty parameter and the stiffness coefficient. In Figure 3.14 the duty parameter is plotted against the bearing eccentricity ratio and in Figure 3.15 the stiffness coefficient as defined in paragraph 3.4 is shown. Quite clearly, from Figure 3.14 and 3.15, increasing the offset ratio leads to a decrease in the duty parameter (i.e. increasing the load capacity) and leads to an increase in the stiffness coefficient. However, because the offset ratio and lobe clearance are not variable independently, the changes in duty parameter and stiffness coefficient do not necessarily imply similar


FIG. 3.12
Variation of the hydrodynamic flow coefficient with the eccentricity ratio.


FIG. 3.13
Variation of attitude angle with the eccentricity ratio.


FIG. 3.14
Variation of the duty parameter with the eccentricity ratio.


FIG. 3.15
Variation of the stiffness coefficient with the eccentricity ratio.
changes in actual load capacity and in actual stiffness. The trend in these and other physical quantities must be found by applying the non-dimensional relationship to bearings which are defined in terms of actual dimensions and to oils of specified viscosity. The calculations pertaining to such specific circumstances are presented and discussed in paragraph 3.9.

Figures 3.16 and 3.17 show the variation of the duty parameter and the attitude angle with the eccentricity ratio for a vertical load directed towards the centre of a lobe and for a vertical load directed towards the conjunction of adjacent lobes. It may be seen from Figure 3.16 that the duty parameter is nearly independent of the load direction while it can be seen from Figure 3.17 that there is a big difference in the attitude angle.

### 3.7 Power Loss

The total friction force acting upon the journal is given essentially by

$$
\begin{equation*}
F=\int_{0}^{L} \int_{0}^{2 \pi} \frac{\mu U R d \theta}{h} d y \tag{3.37}
\end{equation*}
$$

For a three lobe journal bearing and with the journal centred in the bearing (Figure 3.18) the force due to all three lobes is given by

$$
\begin{equation*}
F=6 \int_{0}^{L} \int_{\Pi}^{2 \pi / 3} \frac{\mu U R d \theta}{h} d y \tag{3.38}
\end{equation*}
$$



FIG. 3.16
Variation of the duty parameter with the eccentricity ratio.


FIG.3.17 Locus of shaft centre.


FIG. 3.18
Power loss analogy for a three-lobe bearing.

The oil film thickness is expressed by:

$$
\begin{equation*}
h=C / 2\left(1+\varepsilon_{0} \cos \theta\right) \tag{3.39}
\end{equation*}
$$

With viscosity treated as being constant it follows from (3.38) and (3.39) that

$$
\begin{equation*}
F=61 \frac{U \mu D}{C} \int_{\Pi}^{2 \pi / 3} \frac{d \theta}{\left(1+\varepsilon_{0} \cos \theta\right)} \tag{3.40}
\end{equation*}
$$

Expression (3.40) may be integrated by using the Sommerfeld substitution which is given in Chapter 2. The result is

$$
\begin{equation*}
F=\frac{6 \mathrm{UDL}}{C\left(1-\varepsilon_{0}^{2}\right)^{\frac{1}{2}}}\left\{\cos ^{-1} \frac{-1+\varepsilon_{0}}{1-\varepsilon_{0}}-\cos ^{-1} \frac{-0.5+\varepsilon_{0}}{1-0.5 \varepsilon_{0}}\right\} \tag{3.41}
\end{equation*}
$$

and the power loss is given by

$$
\begin{equation*}
H_{S}=\frac{6 \pi^{2} \mu L N^{2} D^{3}}{C\left(1-\varepsilon_{0}^{2}\right)^{\frac{1}{2}}}\left\{\cos ^{-1} \frac{-1+\varepsilon_{0}}{1-\varepsilon_{0}}-\cos ^{-1} \frac{-0.5+\varepsilon_{0}}{1-0.5 \varepsilon_{0}}\right\} \tag{3.42}
\end{equation*}
$$

Equation(3.42) may be used with any system of units. However, when $\mu$ is given in poise, $L$ in $G, D$ in $\mathrm{cm}, \mathrm{N}$ in r.p.m., Ccm , the power loss in KW is given by

$$
\begin{equation*}
H_{S}=1.646 \times 10^{-12} \frac{\mu L N^{2} D^{3}}{C} \frac{1}{\left(1-\varepsilon_{0}^{2}\right)^{\frac{1}{2}}}\left\{\cos ^{-1} \frac{-1+\varepsilon_{0}}{1-\varepsilon_{0}}-\cos ^{-1} \frac{-0.5+\varepsilon_{0}}{1-0.5 \varepsilon_{0}}\right\} \tag{3.43}
\end{equation*}
$$

For a circular bore bearing $\varepsilon_{0}=0$, and equation(3.42) reduces to

$$
\begin{equation*}
H_{p}=\frac{6 \pi^{2} \mu L N^{2} D^{3}}{C} \cdot \frac{\pi}{3}=\frac{2 \pi^{3} \mu L N^{2} D^{3}}{C} \tag{3.44}
\end{equation*}
$$

which is the Petroff's loss for a circular bearing. The ratio between the power loss of the three lobe bearing and the Petroff power loss of a circular bore bearing which has a diametral clearance equal to the lobe diametral clearances of the three lobe bearing is defined as the power loss factor, and is given by

$$
j_{s}=\frac{H_{s}}{H_{p}}=\frac{3}{\pi}\left\{\cos ^{-1} \frac{-1+\varepsilon_{0}}{1-\varepsilon_{0}}-\cos ^{-1} \frac{-0.5+\varepsilon_{0}}{1-0.5 \varepsilon_{0}} \times \frac{1}{\left(1-\varepsilon_{0}^{2}\right)^{\frac{1}{2}}}\right\}(3.45)
$$

In Figure 3.19 the power loss factor is plotted against the offset ratio. The relation shows that for the same lobe clearance (i.e. the clearance of the circular bore bearing is equal to the lobe clearance of the three lobe bearing), and the same viscosity, the power loss of the three lobe bearing is greater than the power loss of the circular bore bearing by the factor Js. However, because the offset ratio and lobe clearance are not variable independently, the changes in the offset ratio do not necessarily imply that the actual power loss will follow the power loss factor. The power losses ina particular bearing of circular bore and in a particular three lobe bearing will now be discussed. In the first instance the power loss of a circular bore bearing will be considered. The computer programme has been used to calculate the power loss simply


FIG. 3.19
Variation of the power loss factor with the offset ratio.
by setting the offset ratio $\varepsilon_{0}=0$. In Figure $3.20 a$ the calculated power loss is shown as functions of speed. It has previously been shown in paragraph 3.6 that the flow coefficients given by the computer programme are substantially the same as those given in the literature. It follows from this that at any particular speed the oil outlettemperature given by the 'design programme', which will be discussed in paragraph 3.8, is the same as the temperature arising from the published values of flow coefficient. Because of this, it is valid in comparing the calculated power loss as given by the programe with power loss found from published power loss coefficient to take as the oil viscosity that found by means of the design programme.

The power loss of the three-lobe bearing will now be considered. The calculation of the power loss as given by the programme for an offset ratio of $\varepsilon_{0}=0.35$ is shown in Figure $3.20 b$ together with the measured power loss as reported in Chapter 6. The calculated power loss is somewhat greater than that measured at low speed but measured and calculated losses become substantially equal at a speed of 5000 rpm.

There is no unique basis for comparing the power loss of a circular bore and a three-lobe bearing, but a reasonable basis seems to be between bearings of equal minimum clearance. The diametral clearance of the circular bore bearing of Figure 3.20 a is the same as the minimum clearance of the three-lobe bearing of Figure 3.20b. In Figure 3.20c the power loss of that particular circular bore bearing and of that particular three-lobe bearing are compared, and it may be seen that the circular bore bearing gives somewhat less power loss over

FIG.3.20 Variation of power loss with Journal speed.
speed range. This is usually so as may be seen from Figure 3.20d in which calculated results are compared in the instance that the three lobe bearing has an offset ratio of 0.8 . In this instance a smaller power loss is indicated for the circular bore bearing over the full speed range. The general conclusion from this comparison is there is no major difference in power loss between circular and three lobe bearings of equal minimum clearance. However there are substantial differences in other respects and these will be discussed as they arise.

### 3.8 The Design Programme, "Computation of actual oil flow, temperature

rise, power loss, load capacity and stiffness".
A second computer programe has been written which is in the nature of a design programme and which will be termed "The Design Programme". The input to this programme includes the dimensionless output of the finite difference programe together with dimensions and actual viscosity which completely described a real bearing situation. The output of the design programme is the actual total oil flow, the actual hydrodynamic oil flow, the actual pressure oil flow, power loss, oil temperature rise, outlet viscosity, load capacity and stiffness. The design programme includes an iteration routine by which temperature rise and outlet viscosity are brought to self consistent values. Flow and power loss are calculated on the basis that the outlet viscosity pertains throughout the bearing.

The inputs to the programe are given below, and numerical values which are given relate to the bearings which have been tested. Numerical values are not given for oil viscosity, flow coefficient, duty parameter and stiffness coefficient because different values are required for
each combination of minimum clearance and offset ratio
however the numerical values which appear in brackets against these quantities are typical values just to indicate the magnitude of the quantities.

The inputs to the programme are:

```
Shaft diameter: \(1.35 \times 10^{-1} \mathrm{~m}\).
Bearing axial length: \(1.27 \times 10^{-1} \mathrm{~m}\).
Oil grooves axial length: \(1.07 \times 10^{-1} \mathrm{~m}\).
Oil grooves circumferential length: \(3.5 \times 10^{-2}\) and \(1.75 \times 10^{-2} \mathrm{~m}\).
Minimum clearance: \(1.016 \times 10^{-4}, 1.27 \times 10^{-4}, 1.52 \times 10^{-4}, 1.78 \times 10^{-4}\),
and \(2.03 \times 10^{-4} \mathrm{~m}\).
Oil inlet temperature: 30,40 and \(50^{\circ} \mathrm{C}\).
Oil viscosity at the inlet temperature: Tellus 37 oil ( 0.53
to 0.22 poise). VIT. 150 oil (2.19 to 0.736 poise).
Pump pressure: \(1.47 \times 10^{5} \mathrm{Nm}^{-2}\)
Journal Speed: \(10^{3}, 2.10^{3}, 3 \times 10^{3}, \ldots . . . . .\). and \(10^{4}\) r.p.m.
Journal displacement: \(1.27 \times 10^{-5} \mathrm{~m}\).
Offset ratio: \(0.0 .35,0.45,0.6,0.7\) and 0.8 .
Hydrodynamic oil flow coefficient: (from O to O.26).
Total oil flow coefficient: \(\quad\left\{q_{h}+\left(0.023\right.\right.\) to \(\left.\left.0.5 x p^{*}\right)\right\}\) (paragraph 3.5)
Duty parameter: (from 2.5 to 0.13 )
Stiffness coefficient: (from 210 to 5)
```

The iterative routine itself may be described as follows:
(1) For the considered oil, calculate the factor a by substituting the oil viscosity at 30 and $50^{\circ} \mathrm{C}$ into the following expression:

$$
\mu_{50}=\mu_{30} \exp ^{-a(50-30)}
$$

(2)

```
For the considered speed, enter an arbitrary outlet temperature, T T
Calculate the oil viscosity at the outlet from the following
expression:
```

$\mu_{2}=\mu_{1} \exp ^{-a\left(T_{2}-T_{1}\right)}$
The oil flow is calculated within the programe by two methods,
either of which may be chosen for a particular run as follows:
. calculate the dimensionless inlet pressure ( $P^{*}$ ) from expression
(3.25).
. calculate the total oil flow from expression (3.27) by using the
total oil flow coefficient which is calculated from expression
(3.31).

OR
. calculate the pressure oil flow by using the semi-empirical expression
. calculate the hydrodynamic oil flow from expression(3.27)
. calculate the total oil flow as the sum of the pressure flow and the hydrodynamic flow.

Calculate the shear power loss from expression(3.43). Calculate the oil outlet temperature from the following expression:
$H_{s}=P_{K Q_{t}}\left(T_{2}-T_{1}\right)$.
where,
$\mathrm{H}_{\mathrm{S}}$ is the shear power loss which is calculated in step (5). KW. $Q_{t}$ is the total oil flow which is calculated step (4), $\mathrm{Cm}^{3} \mathrm{~S}^{-1}$. $K$ specific heat of the oil, $\simeq 0.47 \mathrm{Cal} \mathrm{qm}^{-1} 0^{0} \mathrm{C}^{-1}$.
$p$ oil density, $\propto 0.88$.
$\mathrm{T}_{1,2}$ oil inlet and outlet temperatures respectively, ${ }^{\circ} \mathrm{C}$.

If the calculated outlet temperature ( $T_{2}$ ) in step (6) is equal to the outlet temperature which is entered in step (2), or there is a difference within $\pm 2^{\circ} \mathrm{C}$, the procedure is completed and the calculations may be carried out for another speed. However, if the difference is greater than $\pm 2^{\circ} \mathrm{C}$, within the programme itself a new trial value is found for $T_{2}$ and the iteration is repeated.
(8) The outputs of the computer programme are:

The oil outlet temperature ( $T_{2}$ ), ${ }^{\circ} \mathrm{C}$.
The oil outlet viscosity ( $\mu_{2}$ ), poise.
The hydrodynamic oil flow, $1 \bar{s}^{1}$.
The pressure oil flow, $1 \mathrm{~s}^{-1}$.
The total oil flow, $1 s^{-1}$.
The power loss, Kw.
Load carrying capacity, N.
Bearing stiffness, $\mathrm{Nm}^{-1}$.

The computer programme may be used for any bearing of any dimensions providing that the oil flow coefficient, the stiffness coefficient and the duty parameter of that bearing are entered in the input. The computer programme itself as well as a sample of the output are given in Appendix A2.2.

### 3.9 Results of the Design Programme Computation

The validity of the design programme has been examined by comparing the quantities predicted with experimental measurements. Such comparisons have been made in the context of large turbine bearings, and in the context of test bearingsof the present work. In Table 3.9 a comparison is made between calculated and experimental results for a 19 " $\times 10$ " $\times 0.023$ " bearing of circular bore. For convenience three equally spaced axial grooves
were assumed in the calculation where in the actual bearing the axial grooves were not equally spaced. However the table shows a good agreement and provides substantial evidence of correctness of the design programme and the calculation of the flow coefficient. In Figure 3.21a comparison is made between the prediction of the design programe and the experimental results from the test rig which will be discussed in Chapter 6. There is close agreement.

The trends predicted by the design programme will be described now. The variation of power loss with journal speed is shown in Figure 3.22. The results are plotted for minimum clearances of $1.016,1.52$ and $2.03 \times 10^{-4} \mathrm{~m}$. It may be seen that the power loss increases as the offset ratio increases and also the power loss increases marginally as the minimum clearance increases. Also it may be seen that the dependence of power loss upon the offset ratio $\left(\varepsilon_{0}\right)$ decreases as the minimum clearance increases.

The variation of the oil temperature rise with the journal speed is shown in Figure 3.23. It is quite clear that as the offset ratio increases the oil temperature rise decreases over the full range of minimu clearances which have been used in the calculations. Figure 3.24 shows the variation of the oil flow with the fournal speed. It may be seen that the oil flow is increased as the offset ratio and the minimum clearance increases.

Figures 3.25 and 3.26 show the variation of the bearing load and the bearing stiffness with the journal speed. In Figures $3.25 \mathrm{a}, \mathrm{b}$ and $3.26 \mathrm{a}, \mathrm{b}$, it is quite clearly shown that the increase in the offset ratio has a marked effect on the bearing load and on the bearing stiffness and it

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{16}{|l|}{Table 2.9 The Design Programme Results and the Test Results for a $19 \times 10 \times 0.023^{\prime \prime}$ Turbine Journal Bearing.} <br>
\hline \multicolumn{8}{|l|}{The Design Programme} \& \multicolumn{8}{|l|}{10
Refs. Smith, 82
DUFFIN and Private Conmunication} <br>
\hline $$
\begin{gathered}
\mathrm{N} \\
\mathrm{rpm}
\end{gathered}
$$ \& $\varepsilon$ \& $$
\begin{aligned}
& \mathrm{T}_{1} \\
& { }^{\circ} \mathrm{C}
\end{aligned}
$$ \& Px10 ${ }^{-5}$ \& $\stackrel{Q}{\ell s^{-1}}$ \& T

${ }^{\circ} \mathrm{C}$ \& $$
\begin{gathered}
\mu_{2} \\
\text { c.p. }
\end{gathered}
$$ \& H

K.W.

K. \& N \& \begin{tabular}{c}
W/LD** <br>
(1m <br>
<br>
\hline

 \& 

T <br>
<br>
<br>
\hline

 \& 

$\mathrm{Px10}$ <br>
$\mathrm{Nm}^{-2}$ <br>
\hline

 \& $\stackrel{Q}{\ell s^{-1}}$ \& 

T 20 <br>
<br>
<br>
\hline

\end{tabular} \& \[

$$
\begin{gathered}
\mu_{2} \\
\text { C.P. }
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\mathrm{H}_{\mathrm{s}} \\
\mathrm{~K} . \mathrm{W} .
\end{gathered}
$$
\] <br>

\hline 1000 \& 0.4 \& 50 \& 0.74 \& 1.012 \& 5.9 \& 19 \& 15.54 \& 1000 \& $4 \times 10^{5}$ \& 50 \& 0.74 \& 0.872 \& 58.33 \& 19.4 \& 15 <br>
\hline 2000 \& " \& " \& " \& 1.631 \& 67 \& 14 \& 47.37 \& 2000 \& $\cdots$ \& " \& " \& 1.63 \& 67.78 \& 14 \& 54 <br>
\hline 3000 * \& " \& " \& " \& \& 74 \& 12 \& 91.5 \& 3000* \& " \& " \& " \& 2.88 \& 78.9 \& 9.7 \& 150.5 <br>
\hline
\end{tabular}

At Journal speed of 3000 rpm , the calculated results from the design programme is for a laminar oil flow in the bearing
clearance, while the test results obtained at 3000 rpm were observed with the presence of turblent flow in the bearing
The load on projected area was used with the test results because the value of the eccentricity ratio has not been
mentioned in the quoted references.


FIG.3.21 Variation of power loss, oil temperature rise, and oil flow with the Journal speed.

 $\stackrel{3}{0}$


Variation of the oil temperature rise with the Journal speed.

FIG.3.24 Variation of the oil flow with the Journal speed.

t-s 7 'MOTA TITO


U





FIG.3.26 (a) Variation of stiffness with the Journal speed.
may be seen that the load capacity and the bearing stiffness increases as the offset ratio increases over the full range of speeds. At higher values of clearances (Figure $3.25 c-3.26 c$ ) there are small increases in the load capacity and bearing stiffness due to the increase in the offset ratio. It also may be seen from Figures $3.25 a, b, c$ and $3.26 a, b, c$ that the load capacity and the bearing stiffness decrease as the minimum clearance increases.

The use of these predictions in the design of a bearing will now be discussed. The results which are plotted in Figures 3.22-3.26 show that for a bearing of high load carrying capacity and high stiffness, the bearing should be designed with high offset ratio and with small minimum clearance. The increase in the offset ratio leads to a big decrease in the oil temperature rise, but consequently the power loss is increased. The small minimum clearance leads to an increase in the oil temperature rise but the power loss is decreased. However, the choice of the offset ratio and the minimum clearance should be made to give a compromise solution for the required load capacity, bearing stiffness and for reasonable power loss and oil temperature rise. The predictions of the design programme show that the functional spacification of Table 1.1 can be met by a bearing of offset ratio of 0.45 , such a bearing would have a minimum clearance of $1.43 \times 10^{-4} \mathrm{~m}$ and a lobe clearance of $2.6 \times 10^{-4} \mathrm{~m}$. At 9000 rpm and for oil inlet temperature of $40^{\circ} \mathrm{C}$, the power loss is 11 KW , and the oil temperature rise is $51^{\circ} \mathrm{C}$. A three lobe journal bearing was machined to have an offset ratio of 0.45, lobe diametral clearance of $2.6 \times 10^{-4}$ and a minimum clearance of $1.43 \times 10^{-4} \mathrm{~m}$, with the intention that different clearences may be tested by reducing the journal diameter. It should be pointed out that in Chapter 2 the approximate hydrostatic analysis has shown that such
dimensions are satisfactory for the hydrostatic performance.

### 3.10 An Approximate Assessment of the Bearing Stiffness

The choice of dimensions for the original test bearing was based upon assessment of power loss, temperature rise and bearing stiffness made by approximate and elementary methods. All of these are now superseded by the computer programmes which have been described, but there is some value in describing the approximate treatment of stiffness and its results as a check upon the results from the numerical solution. In a bearing of full circular bore stiffness is substantially determined within a relatively small area dispose mainly ahead of the position of minimum film thickness. In elementary treatment of stiffness it was supposed that each lobe would encompass the area of significance about each position of minimum film thickness, and that in consequence the three lobe bearing could be treated in stiffness as three bearings of full circular bore placed one against another and with their centres suitably displaced.

In Figure 3.27 a journal is shown at eccentricity $\Delta x$ in a bearing of full circular bore. It is running at this eccentricity, and attitude angle $\varnothing$ due to the external load F. (in the multi-lobe bearing the force $F$ is created by the hydrodynamic pressures associated with the other lobes). By considering that under the influence of an external load with components $P_{x l}$ and $P_{Y l}$, the centre of the journal is further displaced by a distance $\delta r$ but that the attitude angle $\varnothing$ remains unchanged, the components of the displacement are obviously

$$
\begin{align*}
& \Delta x_{1}=\delta r \cos \phi  \tag{3.46}\\
& \Delta y_{1}=\delta r(-\sin \phi)
\end{align*}
$$


(a) Circular bore Journal bearing.

(b) Multi-lobe bearing.

FIG. 3.27
Force-displacemer: relationship for Jou:nal bearing.

It is clear from figure 3.27b, that this displacement $\delta$ w with respect to the next lobe will produce components of displacement.

$$
\begin{align*}
& \Delta \mathrm{x}_{2}=\delta r[\cos (\phi+120)]  \tag{3.47}\\
& \Delta \mathrm{y}_{2}=\delta r[\sin (\phi+120)]
\end{align*}
$$

and with respect to the third lobe

$$
\begin{align*}
& \Delta_{\mathbf{x}_{3}}=\delta_{r}[\cos (\phi+240)]  \tag{3.48}\\
& \Delta_{y_{3}}=\delta_{r}[-\sin (\phi+240)]
\end{align*}
$$

Therefore

$$
\begin{align*}
& \Delta x_{1}=\delta_{r}[\cos \phi] \\
& \Delta y_{1}=\delta_{r}[-\sin \phi] \\
& \Delta x_{2}=\delta_{r}\left[-\frac{1}{2} \cos \phi-\frac{\sqrt{3}}{2} \sin \phi\right]  \tag{3.49}\\
& \Delta y_{2}=\delta r\left[\frac{1}{2} \sin \phi-\frac{\sqrt{3}}{2} \cos \phi\right] \\
& \Delta x_{3}=\delta_{r}\left[-\frac{1}{2} \cos \phi+\frac{\sqrt{3}}{2} \sin \phi\right] \\
& \Delta y_{3}=\delta_{r}\left[\frac{1}{2} \sin \phi+\frac{\sqrt{3}}{2} \cos \phi\right]
\end{align*}
$$

From the theory of the dynamic characteristics of bearings, the incremental forces $P_{x 1}$ and $P_{Y I}$ are related to the displacement they produce by expressions of the form.

$$
P_{x l} / F=a_{11} \frac{\Delta x_{1}}{C / 2}+a_{12} \frac{\Delta y_{1}}{C / 2}
$$

$$
\begin{equation*}
\mathrm{P}_{\mathrm{Yl}} / \mathrm{F}=\mathrm{a}_{21} \frac{\Delta \mathrm{x}_{1}}{\mathrm{C} / 2}+\mathrm{a}_{22} \frac{\Delta \mathrm{y}_{1}}{\mathrm{C} / 2} \tag{3.50}
\end{equation*}
$$

where $a_{11}, a_{12}, a_{21}, a_{22}$ are the displacement coefficients (Ref. lo )

Consequently,

$$
\begin{align*}
& P_{x 1} / F=\frac{\delta r}{C / 2}\left[a_{11} \cos \phi-a_{12} \sin \phi\right] \\
& P_{y 1} / F=\frac{\delta r}{C / 2}\left[a_{21} \cos \phi-a_{22} \sin \phi\right] \\
& P_{x 2} / F=\frac{\delta r}{C / 2}\left[a_{11}\left(-\frac{1}{2} \cos \phi-\frac{\sqrt{3}}{2} \sin \phi\right)+a_{12}\left(\frac{1}{2} \sin \phi-\frac{\sqrt{3}}{2} \cos \phi\right)\right] \\
& P_{y 2} / F=\frac{\delta r}{C / 2}\left[a_{21}\left(-\frac{1}{2} \cos \phi-\frac{\sqrt{3}}{2} \sin \phi\right)+a_{22}\left(\frac{1}{2} \sin \phi-\frac{\sqrt{3}}{2} \cos \phi\right)\right]  \tag{3.51}\\
& P_{x 3} / F=\frac{\delta r}{C / 2}\left[a_{11}\left(-\frac{1}{2} \cos \phi+\frac{\sqrt{3}}{2} \sin \phi\right)+a_{12}\left(\frac{1}{2} \sin \phi-\frac{\sqrt{3}}{2} \cos \phi\right)\right] \\
& P_{y 3} / F=\frac{\delta r}{C / 2}\left[a_{21}\left(-\frac{1}{2} \cos \phi+\frac{\sqrt{3}}{2} \cos \phi\right)+a_{22}\left(\frac{1}{2} \sin \phi+\frac{\sqrt{3}}{2} \cos \phi\right)\right]
\end{align*}
$$

the total force in the $x_{1}$ direction is given by:

$$
\begin{equation*}
\left(P_{x 1}\right)_{\text {total }}=P_{x 1}-\frac{1}{2} P_{x 2}-\frac{1}{2} P_{x 3}-\frac{\sqrt{3}}{2} P_{y 3}+\frac{\sqrt{3}}{2} P_{y 3} \tag{3.52}
\end{equation*}
$$

Similarly,
${ }^{\left(P_{y 1}\right)_{\text {total }}=} P_{y 1}-\frac{1}{2} P_{y 2}-\frac{1}{2} P_{y 3}+\frac{\sqrt{3}}{2} P_{x 2}-\frac{\sqrt{3}}{2} P_{x 3}$

After substitution for $P_{x:}, P_{Y l}, \ldots$. etc, and rearranging, the result is given by

$$
\begin{align*}
& \left(P_{x 1}\right)_{\text {total }} / F=\frac{3}{2} \frac{\delta r}{C / 2}\left[a_{11} \cos \phi+a_{12} \sin \phi+a_{21} \sin \phi+a_{22} \cos \phi\right] \\
& \left(P_{Y 1}\right)_{\text {total }} / F=\frac{3}{2} \frac{\delta r}{C / 2}\left[a_{11}(-\sin \phi)+a_{12}(-\cos \phi)+a_{21} \cos \phi+a_{22}(-\sin \phi]\right. \tag{3.54}
\end{align*}
$$

The force $F$ is calculated from the following expression

$$
\begin{equation*}
F=\frac{\mu D L N}{S}(D / C)^{2} \tag{3.55}
\end{equation*}
$$

The calculated stiffness by using expression (3.54) and the calculated stiffness from the numerical solution are plotted in figure 3.28. It may be seen that there is satisfactory agreement between both methods of calculation. Furthermore, as expected the stiffness given by the approximate method is greater than the stiffness given by the numerical solution because in the approximate solution each lobe was considered to be as a full circular bore bearing whilsthe actual lobe in the numerical solution was of $100^{\circ}$ arc.


FIG.3.28
Variation of bearing stiffness with the Journal speed.

## C H A PTER 4

## 4. PRELIMINARY EXPERIMENTAL WORK

### 4.1 Introduction

The literature on hydrostatic bearings covers bearings with pockets, and with compensation by capillary or by orifices means, however nothing has been found in the literature upon a three lobe hydrostatic bearing, nor upon hydrostatic bearings of any kind which have no pockets, and which are compensated by constant volume valves. Because of this there seemed to be good reason in the first instance to investigate in a relatively simple rig, and without rotation of the journal, the hydrostatic behaviour of the bearing. Such a rig would provide a screening test before proceeding to the far greater elaboration required for high speed testing. It also seemed reasonable to suppose that a nonrotating journal would behave similarly to a journal running so slowly that little self-generation is present.

### 4.2 Test Rig

The bearing as arranged for hydrostatic test is shown in Figure 4.1. The rig does not permit continuous rotation, but the effect of applying the force in different directions with respect to the lobes may be tested by rotating the bearing in its frame. The force is applied by a hydraulic ram, and the force is calculated from the cross-sectional area, and the ram pressure. The movement of the journal with respect to the bearing is measured with dial gauges capable of reading down to $0.00254 \mathrm{~mm} .(0.0001$ inch $)$, and four dial test indicators, two at each end, are mounted to detect the vertical displacement as well as the horizontal displacement. The absence of metal to metal contact between journal and bearing was monitored by measuring the electrical resistance between journal and bearing. High pressure oil was supplied from the high pressure pump to the high pressure inlets via individual volume

| 1 | Steel housing | 5 | Dial gauge |
| :---: | :--- | :--- | :--- |
| 2 | Brass shell | 6 | Loading arms |
| 3 | Journal | 7 | Hydraulic ram |
| 4 | Dial gauge |  |  |

FIG.4.1
Test rig for external pressurization.
control valves. Volume control valves were chosen for the reasons which are reported in Chapter 1 and Chapter 2. The supply pressure from the pump was measured, oil temperature and pressure were measured downstream of each of the the volume control valves. A photograph of the test rig with its instrumentation is shown in Figure 4.2. The rod to be seen in the left was provided against the possibility that the journal would be angularly unstable within the bearing. However, no need arose to use the rod.

### 4.3 The Bearing

The concept of the bearing which was used in the experimental work is illustrated by Figure 4.3. The bearing was made by splitting a brass cylinder into three segments, by reassembing the segments but separated by shims, by boring the assembly circular, by removing the shims and finally by turning the outer surface circular (Figure 4.4). The thickness of shims ( $t$ ) was calculated from $t=2 \Delta r$ sin 60 to provide the required offset of the centres of the lobes. Particular care had to be taken with the abutments of the three segments so that no bending was introduced on tightening (Appendix A4). A high pressure oil inlet for the hydrostatic regime is provided at the centre of each lobe, and a low pressure inlet is provided for the self-generating regime at each conjunction of the lobes. The form achieved by the process is shown by the roundness chart of Figure 4.5. The departures from the intended form are $0.00254 \pi m(\sim 5 \%)$.

### 4.4 Test Procedures

Before experimental testing was commenced the flow characteristics of each valve were tested by measuring volume flows at different valve settings, at different supply pressures and at different oil temperatures. Details are given in Appendix A7. It was confirmed that the valves do deliver a constant volume flow.


FIG.4.2 HYDROSTATIG TEST RIG.


FIG. 4.3

(c) The outer surface turned circular and the assembly fitted into steel housing.


The bypass of the high pressure pump, and the values were adjusted to give the desired supply pressure and the desired volume flow. In the static rig no oil cooler was provided, and in consequence the oil temperature rose over the full sequence of the testing. However, within a test at particular values of supply pressure and volume flow, the oil temperature rise was small so that a particular oil temperature could be attributed to each individual test.

Horizontal load was applied to the journal by means of the hydraulic jack, the position of the journal in the bore of the bearing was measured both horizontally and vertically by means of dial gauges, and the pressure downstream of a constant flow valve, which is the inlet pressure to a lobe, was measured. It was found by careful attention to the symmetry of loading that the rig could be adjusted so that the journal remained parallel to the axis of the bearing as the journal moved. The criterion is obviously equality of dial gauge measurements at each end of the bearing. A small vertical displacement of the journal was noticed at the start of loading, but it did not increase as the load was increased.

Results were obtained with lobe diametral clearances from $2.6 \times 10^{-4}$ to $3.7 \times 10^{-4} \mathrm{~m}(0.0104$ to 0.0144 inch). The dimensions of the test bearing are given in Table 4.1. The different diametral clearances were obtained by machining the journal.

### 4.5 Test Results

The tests were carried out with an oil which has a viscosity of 20 cp . at $75^{\circ} \mathrm{C}$ (Shell Vitria 150). Conditions of test were supply pressures from $1.03 \times 10^{7} \mathrm{Nm}^{-2}$ to $2.06 \times 10^{7} \mathrm{Nm}^{-2}$ (from 1500 to 3000 psi), total volume flow from 0.1 to 0.2 litres per second. Pumping powers were from 2.8

| Designa- <br> tion <br> of <br> Bearing | m | m | d | $\Delta_{\mathrm{r}}$ | C | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $1.352 \times 10^{-1}$ | $1.349 \times 10^{-1}$ | $5.75 \times 10^{-5}$ | $2.6 \times 10^{-4}$ | $1.45 \times 10^{-4}$ | $2.025 \times 10^{-4}$ |
| 1 | m | m | C |  |  |  |
| 2 | $352 \times 10^{-1}$ | $1.3487 \times 10^{-1}$ | $5.75 \times 10^{-5}$ | $3.0 \times 10^{-4}$ | $1.85 \times 10^{-4}$ | $2.425 \times 10^{-4}$ |
| 3 | $1.352 \times 10^{-1}$ | $1.348 \times 10^{-1}$ | $5.75 \times 10^{-5}$ | $3.7 \times 10^{-4}$ | $2.55 \times 10^{-4}$ | $3.175 \times 10^{-4}$ |

${ }^{2} / \mathrm{L} / \mathrm{D}=0.94$

TABLE 4.1 Dimensions of test bearings.
to 6.6 KW , oil temperatures at inlet were from 40 to $70^{\circ} \mathrm{C}$. Stiffnesses were found typically of $1 \times 10^{9} \mathrm{Nm}^{-1}\left(\sim 5.7 \times 10^{6} 1 \mathrm{~b} \mathrm{in}^{-1}\right)$, and there was no metal to metal contact up to the maximum load which was $5.6 \times 10^{4} \mathrm{~N}$ (~5.7 tons).

Specific test results are shown in Figures 4.6 to 4.10. In Figure 4.6 journal displacement is shown as a function of load for three rates of oil flow. In all tests the supply pressure was $1.7 \times 10^{7} \mathrm{Nam}^{-2}$ ( 2500 psi ), and at no load the ratio between the supply pressure and the downstream pressure was approximately 0.6. The results show that the displacement load relationship is not exactly linear and that an increase in volume flow produces an increase in stiffness.

Figure 4.7 shows a plot of journal displacement and of the downstream pressure versus the external load. The circular points are for a load directed towards the conjunction of the lobes and the triangular points are for a load directed towards the centre of a lobe. The difference in the measured displacement in both directions is insignificant and the only difference was in the downstream pressure which as is to be expected has a greater value when the load is applied through the centre of the lobe.

In Figure 4.8 the lobe downstream pressure and the rate of flow are shown as functions of the external load. The oil flow was not measured directly but measured from the calibration table which is given in Appendix A7. However, in the proper bearing rig flow meters were fitted in the high pressure lines and the measurements then made confirmed the correctness of the procedure adopted in Figure 4.8.

In Figure 4.9 the bearing stiffness is plotted versus the bearing pressure ratio (i.e. the ratio between the downstream pressure at zero


FIG.4.6
Load displacement relationship for different rates of flow.


FIG.4.7 Load displacement, and load pressure relationship for two different direction of loading.


FIG.4.8 Variation of lobes pressure and oil flow with the external load.


FIG.4.9 Variation of stiffness with the pressure ratio.
load and the pump pressure). For constant valve settings and constant viscosity the pressure ratio was adjusted to the desired value by adjusting the supply pressure. It may be seen from Figure 4.9 that the bearing stiffness is independent of the pressure ratio and in consequence the stiffness is independent of the supply pressure. The minimum pressure drop across a valve during this test was $7 \times 10^{5} \mathrm{Nm}^{-2}(\sim 100 \mathrm{psi})$.

### 4.6 Discussion

From the tests which have been carried out with the three lobe bearing, the experimentally determined performance substantially matches or exceeds the functional specification of Table l.l. For instance, the functional specification calls for a stiffness in finishing of $8 \times 10^{8} \mathrm{Nm}^{-1}$ at low speed. It may be seen from Figure 4.10 that this stiffness can be obtained with a diametral clearance of $3.3 \times 10^{-4} \mathrm{~m}$, with a supply pressure of $1.7 \times 10^{7} \mathrm{Nm}^{-2}$, with a total volume flow of 0.31 litres per second and with a pumping power of 3.8 KW . Also it may be seen that at a diametral clearance of $3 \times 10^{-4} \mathrm{~m}$, a stiffness of $1 \times 10^{9} \mathrm{Nm}^{-1}$ was achieved. In addition the functional specification calls for a pressure on projected area of $3.86 \times 10^{6} \mathrm{Nm}^{-2}$. From Figure 4.6 a pressure of $3.73 \times 10^{6} \mathrm{Nm}^{-2}$ on projected area was achieved experimentally without metal to metal contact. It is concluded that the bearing configuration which has been tested is capable of satisfying the slow speed specification of the functional specification.

The hydrodynamic predictions for the bearing and journal dimensions of the hydrostatic tests were reviewed to see if an acceptable hydrodynamic performance would be likely. The hydrostatic tests have covered diametral clearances of $2.6,3$ and $3.7 \times 10^{-4} \mathrm{~m}$. It has been predicted

* The actual load is $6.4 \times 10^{4} \mathrm{~N}$.


FIG. 4.10
Variation of stiffness, ofl flow, and pumping power with the diametral clearance.

```
in Chapter 3 that under hydrodynamic conditions the power loss increases
marginally as the diametral clearance is increased, that the oil
temperature rise falls, and that the stiffness falls marginally.
However, because the predicted hydrodynamic stiffness exceeds the
stiffness required by the specification the fall in stiffness as the
diametral clearance is increased is of no consequence in the hydro-
dynamic regime, and the lesser rise in oil temperature favours
a larger rather than a smaller diametral clearance, but in the hydro-
static regime the stiffness with a diametral clearance of 3.7\times10 -4}\textrm{m
is too low, and consequently a maximum diametral clearance between 3
and 3.7\times10-4m had to be set for the hydrodynamic tests. Another consid-
eration is that it is easy to increase diametral clearance by reducing
journal diameter while to decrease the clearance requires the building
up of the journal. The final dimensions for hydrodynamic testing was
a diametral clearance of 2.6\times10 -4 m, with the intention of employing a
greater diametral clearance when a knowledge existed of the actual
hydrodynamic performance of the initial clearance.
```


## CHAPTER 5

NUMERICAL ANALYSIS OF THE THREE-LOBE

EXTERNALLY PRESSURIZED JOURNAL BEARING .

## 5. NUMERICAL ANALYSIS OF THE THREE-LOBE EXTERNALLY PRESSURIZED JOURNAL BEARING .

### 5.1. Introduction

In Chapter 2 an approximate analysis has been described for the externally pressurized circumstances with zero shaft speed. The analysis has provided a reasonable prediction for the steady-state parameters, (i.e. oil flow rate, load capacity, stiffness, supply pressure and pumping power). In Chapter 2, the analysis was simplified by reducing the Navier Stoke's equation to the case of the infinitely wide journal bearing with zero speed of rotation. The pressure between the adjacent lobes was assumed to be zero, and the oil was considered to flow in the circumferential direction and then out of the bearing through the low pressure grooves. Consequently the steady state parameters (i.e. load and stiffness) were considered to be dependent only on the circumferential flow. Furthermore, the problem was simplified by taking the journal as symmetrically disposed with respect to each lobe. To obtain a closer approximation to the oil flow out of the bearing, the axial flow from both ends of the bearing was calculated on the basis of a linear pressure drop along the bearing width and a constant pressure around the lobe arc equal to half the inlet pressure. The results from these approximate calculations have been described in Chapter 2.

It is to be expected that an actual bearing would depart substantially from these simplifying assumptions. In an actual bearing the pressure at the low pressure grooves is not zero, and it was found from the experimental measurements that the pressure at the axial grooves is
about 20\% of the pressure at the high pressure inlets.

In this chapter a numerical solution based upon the finite difference formulation of the Reynolds eqn. will be described. This numerical solution is distinguished from the numerical solution described in Chapter 3, in that in the present instance the following circumstances obtain:-
i) A condition which has to be satisfied is that the oil flow rate at each high pressure inlet has to be the same. This is in contrast with the condition in Chapter 3 in which the only boundary condition specified was the dimensionless pressure at the boundary of the lobe.
ii) The pressure at the high pressure inlets must be finite and becomes a specified boundary condition. Pressures are expressed non-dimensionally as $P^{*}=P / P_{0}$, where $P_{0}$ is in essence the $\left(\varepsilon_{8}=0\right)$ pressure downstream of the control valves. When the journal is centred within the bearing the pressure downstream of each control valve must be the same at all the lobes. In the computer programme the dimensionless pressure downstream of the control valve of each lobe may be set as input from some small finite value to unity. The oil flow rate, in dimensionless form, is then calculated as a function of the dimensionless pressure at the high pressure inlets.

When the journal is displaced within the bearing the dimensionless pressures downstream of each control valve are no longer equal, but the nondimensional oil flow remains the same as it was when

[^1]the journal was in its central position. There is an iteration procedure within the computer programme by which the dimensionless pressure downstream of each control valve is found such that the dimensionless flow through each control valve is the same, and is also equal to the dimensionless flow through each control valve when the journal was in its central position. The iteration procedure starts with the input of the same dimensionless pressure downstream of each valve as has been described in the context of the centralized journal, i.e. each calculation starts with a centralized journal and then proceeds to the displacement of the journal.

The displacement of the journal is specified within the programme, and the journal is displaced successively to $\varepsilon_{B}=0,0.1,0.2, \ldots$, 0.9. With stationary journal the load and displacement are co-linear. With journal rotation this is no longer so. The computer programme is arranged to maintain the vertical direction of journal displacement, and outputs the departure from co-linearity of the load.
iii) Reynolds' equation is to be solved both for the zero speed and finite journal speed circumstances. In the former instance the right hand side of Reynolds' equation is zero (eqn. 5.1 below).
iv) The calculation of the oil flow includes the pressure induced flow in the axial direction, the pressure induced flow in the circumferential direction and the shear flow in the circumferential direction whereas in Chapter 3 the axial flow only has been calculated.

Because of the possibility of circumferential oil flow from one lobe to another, Reynolds' equation has to be solved for the bearing as a whole in contrast with the lobe by lobe treatment of Chapter 3. The finite difference mesh has to cover all the lobes.

### 5.2. Finite Difference Solution.

The solution of Reynolds equation will be performed by using the finite difference formulation as has been described in paragraph 3.2 (Chapter 3). Also the assumptions of the finite difference solution are the usual assumptions implicit in Reynolds equation including constant viscosity which will be treated as the outlet viscosity. When the journal speed is not zero, the pressure distribution solution may yield negative pressures. Negative pressures are set to zero as was described in paragraph 3.2.

The required form of Reynolds equation is,

$$
\begin{equation*}
\frac{\partial}{\partial X}\left(\frac{1}{\mu} h^{3} \frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\mu} h^{3} \frac{\partial P}{\partial z}\right)=6 U \frac{d h}{d x} \tag{5.1}
\end{equation*}
$$

The dimensionless forms which have been used in the numerical solution of Chapter 3 will be used here except that the dimensionless pressure will be defined as

$$
\begin{equation*}
P^{*}=\frac{P}{P_{0}} \tag{5.2}
\end{equation*}
$$

where, $P_{0}$ is the $\left(\mathcal{E}_{B}=0\right)$ pressure downsteam of the control valves. When the journal is in its central position (i.e. $\varepsilon_{B}=0$ ), the highest value for $P$ at a high pressure inlet with the pressure drop across the control device may be used as the supply pressure.

In dimensionless form equation (5.1) becomes

$$
\begin{equation*}
\frac{\partial}{\partial X^{*}}\left(H^{*} 3 \quad \frac{\partial P^{*}}{\partial X^{*}}\right)+\left(\frac{D}{L}\right)^{2} \frac{\partial}{\partial Z^{*}}\left(H^{*} \frac{\partial P^{*}}{\partial Z^{*}}\right)=6 \pi S_{p} \frac{d H^{*}}{d X^{*}} \tag{5.3}
\end{equation*}
$$

where,

$$
\begin{equation*}
s p=\frac{\mu N}{P_{0}}(D / C)^{2} \tag{5.4}
\end{equation*}
$$

The speed parameter ( $S p$ ) is related to the running speed of the externally pressurized bearing, to the lobe. pressure, to the outlet viscosity and to the lobe diameter - clearance ratio.

When $N=0$, then $S p=0$ and the right hand term of equation (5.3) is zero. Equation (5.3) has been transformed to a finite difference form by the same procedure used with equation (3.2) and the finite difference equation becomes:

For a mesh of $i=1$ to $m+1$ and $j=1$ to $n+1$ where $n$ is even and with the assumption that pressures at the boundary are known, then there are $(n-1) \times(m-1)$ linear simultaneous equations which have to be solved by the over relaxation technique which has been previously used in Chapter 3.


#### Abstract

The iteration procedure starts with the P's around the boundary set to zero except at the high pressure inlets where the dimensionless boundary pressure is set to a finite value $0<P^{*} \leq 1.0$.


The particular grid employed in the calculations for the complete bearing is shown in figure 5.1. Because the axial grooves do not break through over the full axial length of the bearing, the flow from one lobe to the adjacent lobes depends upon the pressure distribution in these adjacent lobes so that the three lobes are being treated here simultaneously, whereas in Chapter 3 each lobe has been treated separately and the pressure distribution was calculated for each lobe independently. In figure 5.1, $i=1$ to 29 ( $m=28$ ), $j=1$ to $181(n=180)$ so that there are $28 \times 89=2403$ equations to be solved.
5.3. Method of Solution for Complete Bearing. To solve equation (5.5), the boundary conditions around each lobe must be stated. To find the pressure at each inlet as well as the pressure at each point of the grid when the shaft is displaced within the bearing, equation (5.5) is solved typically by setting the dimensionless pressure $\mathrm{P}^{*}$ for zero journal eccentricity to unity at all the bearing inlets. The pressure at each grid point is then calculated. The oil flow out from each lobe is calculated and is considered as the reference flow. Because the journal is central within the bearing, then the pressure in all the lobes is the same and there is no pressure induced flow nor velocity induced flow in the circumferential direction to give a net delivery of oi.1 from one lobe to another. All oil must leave each lobe axially (Fig.5.2a

Figure 5.1.

Now, when the solution is performed for an eccentricity ratio greater than zero, the solution yields a flow rate out of the lobe which is initially different from the flow calculated at $\mathcal{E}_{B}=0$ (i.e., the reference flow). With reference to figures 5.2 b and 5.5 , this flow is given by

$$
\begin{equation*}
\left(Q_{v}\right)_{I, J, K}=\left\{2 Q_{a}+Q_{c l}+Q_{c 2}+Q_{n \text { out }}-Q_{n \text { in }}\right\}_{I, J, K} \tag{5.6}
\end{equation*}
$$

Where:
$Q_{a}$ : is the flow out from the lobe through the bearing side (pressure induced flow in the axial direction).
$Q_{C l, 2}:$ is the flow from the lobe to the adjacent lobes (pressure
induced flow in the circumferential direction).
$Q_{n}$ out $:$ is the velocity induced flow from the considered lobe to the downstream lobe.
$Q_{n}$ in is the velocity induced flow from the upstream lobe to the
considered lobe. $Q_{v}$ : is the oil flow through the constant flow valve.

The dimensionless pressure at each inlet is then corrected and the solution is repeated until the calculated flow for each lobe is equal with the flow calculated at zero eccentricity ratio. The forces due to the hydrostatic and hydrodynamic pressures are then calculated, and the resultant force in the vertical, and in the horizontal direction are calculated at each lobe. The total force and the angle between the vertical direction and the direction of the total force are also calculated.


Figure 5.2. Oil Flow Dattern.

A conceptual flow chart of the routine used for the solution is shown in figure 5.3 and may be described as follows:
(1) The dimensionless pressure at the high pressure inlets is given a value of 1.0 (i.e. $P / p_{0}=1.0$ ).
(2) For zero eccentricity ratio (i.e. $\varepsilon_{B}=0$ ), solve equation (5.5) for the pressure distribution.
(3) Calculate the oil fow out of each lobe and retain this flow as the reference flow.
(4) For the given eccentricity ratio and displacement direction, resolve equation (5.5). Calculate the generated forces in the direction of the attitude line and in the direction perpendicular on the altitude line, then calculate the load factor.

Two computer programmes have been written for the solution of Reynolds equation for the externally pressurized regime. In both instances the load in the journal is assumed to be towards the centre of a lobe, or to be reversed and to be towards the conjunction of each lobe. In



Figure 5.3. Flow Chart for the hybrid bearing computer programme.
the first computer programme, high pressure oil is admitted to the high pressure inlets at the centre of each lobe (figure 5.4., A, B and $C$ ), and in the second programme, metered high pressure oil is admitted to the "Low pressure inlets" at the conjunction of the lobes. Through a pressure of time the second programme has not been used comprehensively, and its only use has been the calculation of the pressure flow coefficient for various dimensionless speeds of rotation in the self generating regime as has been described in paragraph 3.5 in Chapeter 3. In this chapter the output of the first programme will be discussed.
5.4. Oil Flow
5.4.1. Axial Pressure induced flow

The oil flow out of a bearing lobe from both ends and which is induced by the pressure difference is given by

$$
\begin{equation*}
Q_{a}=2 \int \frac{1}{12 \mu} h^{3} \frac{\partial P}{\partial z} d x \tag{5.7}
\end{equation*}
$$

where $\frac{\partial P}{\partial z}$ is the axial pressure gradient at the ends of the bearing.

After substituting dimensionless forms for $\frac{\partial P}{\partial z}, h^{3}, \mu$ and $d x$ and after rearranging the axial pressure induced flow coefficient is given by

$$
\begin{equation*}
\left(q_{a}\right)_{I, J, K}=\left(D / L_{L}\right) \int H^{* 3} \cdot \frac{\partial P^{*}}{\partial z^{*}} \cdot d x^{*} \tag{5.8}
\end{equation*}
$$

In numerical form equation (5.8) becomes

$$
\begin{equation*}
\left(q_{a}\right)_{I, J, K}=(D / L) \sum_{j=1,61,121}^{n / 3+1,2 n / 3^{+1} \text { and } n+1} H_{m, j}^{* 3} \frac{P_{m-1, j^{-P} m+1, j}^{*}}{2 . \Delta Z} \cdot \Delta x^{*} \tag{5.9}
\end{equation*}
$$



Figure 5.5. The flow pattern out and into the bearing lobe.

The calculation of $q_{a}$ from expression (5.9) depends upon prior solution of Reynolds equation (5.5) to give the pressure distribution over the complete bearing. The calculation of $q_{a}$ is incorporated in the same computer programme.

The oil flow out of each lobe may be expressed as

$$
\begin{equation*}
\left(Q_{a}\right)_{I, J, K}=\frac{P_{O} c^{3}}{6 \mu} \cdot q_{a} \tag{5.10}
\end{equation*}
$$

### 5.4.2. Circumferential Pressure Induced Flow

When the bearing eccentricity is zero the pressure in all the bearing lobes is similar, and there will be no flow from one lobe to the adjacent lobes. When the journal is displaced within the bearing a differential pressure between the bearing lobes will exist and leads to a flow from one lobe to the adjacent lobes. This flow is called the circumferential pressure induced flow and is given by

$$
\begin{equation*}
Q_{c}=2 \int \frac{1}{12 \mu} h^{3} \frac{\partial P}{\partial X} d z \tag{5.11}
\end{equation*}
$$

where $\frac{\partial P}{\partial x}$ is the circumferential pressure gradient at the conjunction with the adjacent lobes.

After substituting dimensionless forms for $\frac{\partial P}{\partial x}, h^{3}, \mu$ and $d z$ and after rearranging the circumferential pressure induced flow coefficient is given by

$$
\begin{equation*}
\left(q_{C}\right)_{I, J, K}=(L / D) \int H^{* 3} \cdot \frac{\partial P^{*}}{\partial x^{*}} \cdot d z^{*} \tag{5.12}
\end{equation*}
$$

In numerical form equation (5.12) becomes

$$
\begin{equation*}
\left(q_{C}\right)_{I, J, K}=(I / D) \sum_{i=1}^{m+1} H_{n, i}^{* 3}\left(\frac{P_{n-1, i^{-P} n+1, i}}{2 . \Delta X}\right)^{*} \cdot \Delta Z * \tag{5.13}
\end{equation*}
$$

The calculation of $q_{C}$ from expression (5.13) also depends upon prior solution of Reynolds equation (5.5) to give the pressure distribution over the complete bearing. Because the pressure difference and the oil film thickness do not have to be the same at both the right and left sides of the lobe (Lines $A-A$ and $B-B$ in figure 5.1), the circumferential pressure induced flow coefficient $\left(q_{C}\right)$ is calculated separately at each side of the lobe and that is incorporated in the same computer programme. The value of $q_{c}$ is positive when the flow of lubricant is flow out of the lobe and is negative when the lubricant flows into the considered lobe.
5.4.3. Velocity Induced Flow The flow in the direction of rotation due to the rotation of the journal is equal to the shear flow and is given by

$$
\begin{equation*}
Q_{n}=\frac{h}{2} U L \tag{5.14}
\end{equation*}
$$

After substituting dimensionless form for $h$ and after rearranging, the velocity induced flow coefficient is given by

$$
\begin{equation*}
q_{n}=\frac{2 Q_{n}}{\Pi D N L C}=H^{*} \tag{5.15}
\end{equation*}
$$

Where

$$
\begin{aligned}
& \mathrm{H}^{*} \text { is the dimensionless oil film thickness at the lobe sides } \\
& \text { (lines } A-A \text { and } B-B \text { in figure } 5.1 \text { ), } \\
& q_{n} \text { is the velocity induced flow coefficient. }
\end{aligned}
$$

### 5.4.4. The Total Oil Flow

The quantity of oil which is delivered to an externally pressurized bearing is held constant whether the journal is stationary or rotating at any speed. The oil flow rate which is calculated for a concentric stationary journal is the oil flow rate which will be maintained constant at any bearing eccentricity and at any speed by the compensation device (in this investigation the compensation device is a constant volume flow valve). However, for a concentric journal within the bearing there will be no net flow in the circumferential direction, and the only flow out of the bearing is the axial pressure induced flow which is calculated at zero bearing eccentricity from expression (5.10) for each lobe.

The total flow rate through the compensating devices and out of the bearing is given by

$$
\begin{equation*}
Q_{t}=3 Q_{v}=\sum_{I, J, K} Q_{a} \tag{5.16}
\end{equation*}
$$

where at $\varepsilon_{B}=0,\left(Q_{a}\right)_{I}=\left(Q_{a}\right)=\left(Q_{J}\right)_{K}$

### 5.5. Load and Load Factors*

From the obtained pressure distribution, the load components in the direction of attitude line and the direction perpendicular on the attitude line at each lobe are found from

$$
\left.\begin{array}{l}
\left(W_{X}^{X_{I, J, K}}\right.  \tag{5.17}\\
\\
\left(W_{Y}\right)_{I, J, K}=\int P \cdot d x \cdot d z \cdot \cos \theta \\
\end{array}\right\}
$$

*for notation, see figure 3.6.

After substituting dimensionless forms for $P, d x$ and $d y$ and after rearranging, expression (5.17) may be written as:

$$
\left.\begin{array}{l}
\left(W_{X}\right)_{I, J, K} /\left(P_{O}\right.  \tag{5.18}\\
L D)=\int P^{*} \cdot d x^{*} \cdot \mathrm{~d} z_{*}^{*} \cos \theta \\
\left(W_{Y}\right)_{I, J, K} /\left(P_{0}\right. \\
L D)=\int P^{*} \cdot \mathrm{~d} x^{*} \cdot \mathrm{~d} z_{\cdot}^{*} \cdot \sin \theta
\end{array}\right\}
$$

where

$$
{ }^{\left(W^{\bullet}\right)_{I, J, K}=(W)_{I, J, K} /\left(P_{\max } L D\right) ~}
$$

In numerical form expression (5.18) becomes

$$
\begin{align*}
& \left(W_{X, J, K}^{*}\right)_{I, J}^{m+1} \sum_{i=1}^{n / 3+1,2 n / 3+1, n+1} P_{i, j, \Delta X^{*} \cdot \Delta Z^{*} \cdot \cos \theta_{j}} \\
& \left(W_{Y}^{*}\right)_{I, J, K}=\sum_{i=1}^{m+1} \frac{n / 3+1,2 n / 3+1, n+1}{j=1,61,121} P_{i, j}^{*} . \Delta X^{*} \cdot \Delta Z^{*} \cdot \sin \theta_{j} \tag{5.19}
\end{align*}
$$

and the resultant dimensionless load is given by

$$
\begin{equation*}
\left(W^{\bullet}\right)_{I, J, K}=\left\{\left(W_{x}^{\prime}\right)_{I, J, K}^{2}+\left(W_{Y}^{\prime}\right)_{I, J, K}^{2}\right\}^{\frac{1}{2}} \tag{5.20}
\end{equation*}
$$

and the angle between the direction of the load and the displacement for each lobe is given by

$$
\begin{equation*}
\beta_{I, J, K}=\sin ^{-1}\left(W_{Y}^{\prime} / W^{\prime}\right)_{I, J, K} \tag{5.21}
\end{equation*}
$$

The $\left(W^{\prime}\right)_{I, J, K}$ is then decomposed into a horizontal and into a vertical force at each lobe. The load factor is then given by

$$
\begin{equation*}
\left(W_{t}^{*}\right)=\left(W_{v t}^{-2}+W_{H t}^{-2}\right)^{\frac{1}{2}} \tag{5.22}
\end{equation*}
$$

Where, $W_{v t}^{\sim}$ is the vector sum of the vertical dimensionless load at each lobe and $W_{H t}^{*}$ is the vector sum of the horizontal dimensionless loads at each lobe.

The angle between the vertical direction and the direction of the total load is given by

$$
\begin{equation*}
\gamma=\tan ^{-1}\left(-W_{H t}^{-} / W_{V t}^{-}\right) \tag{5.23}
\end{equation*}
$$

$W_{t}$ is called the load factor and includes both the hydrodynamic and the hydrostatic components of load.

When the journal is stationary the load factor comprises only the hydrostatic effect.

In both instances the bearing load is given by

$$
\begin{equation*}
W_{t}=W_{t}^{-} \cdot L D P_{0} \tag{5.24}
\end{equation*}
$$

The load factor $\left(W_{t}\right)$ is calculated in the same computer programme.

### 5.6. Bearing Stiffness

The bearing stiffness has been defined as:

$$
\lambda=\frac{\text { change in the magnitude of the load }}{\text { change in the magnitude of the displacement }}
$$

or

$$
\begin{equation*}
\lambda=\frac{\mathrm{d} \mathrm{w}_{\mathrm{t}}}{\mathrm{c} / 2 \mathrm{~d} \varepsilon} \tag{5.25}
\end{equation*}
$$

Since,

$$
\begin{equation*}
W_{t}=W_{t}^{\prime} \cdot L D P_{0} \tag{5.26}
\end{equation*}
$$

then,

$$
\begin{equation*}
\lambda=\frac{2 L D P_{0}}{C} \frac{d W_{t}^{-}}{d \varepsilon} \tag{5.27}
\end{equation*}
$$

and the stiffness coefficient is given by

$$
\begin{equation*}
\lambda^{\prime}=\lambda \frac{C}{2 L D P_{0}}=\frac{d W_{セ}^{-}}{d \varepsilon} \tag{5.28}
\end{equation*}
$$

The values of the load factor ${\underset{t}{\prime}}_{\prime}$ which is evaluated at a series
of values of eccentricity ratios is used to calculate the stiffness parameter $\lambda^{*}$.
5.7. Results of the Numerical Solution

The computer routines for the solutions of Reynold's equations for the externally pressurized circumstances have been written in Fortran VI and it has been performed on the I.C.L. 1900 Computer and on the C.D.C. 7600 Computer. The computer routines and samples of the output are presented in APF endix A.3.

All the inputs and the outputs of the finite difference programmes are in dimensionless form. The output comprises the load angle $\gamma$, the attitude angle $\phi$, the load factor $W_{t}^{-}$(Figure 3.6), the oil flow coefficient, the pressure distribution at the mesh points, the oil film thickness at the mesh points and the pressure at the bearing inlets. The computer programme was run for the following values of the input variables:

```
\(\varepsilon_{0}: 0.3,0.4,0.5,0.6,0.7\)
\(\varepsilon_{B}=0,0.1,0.2, \ldots . . . . ., 0.9\).
    \(\phi: 0\) and 180 degrees.
    p*: 1
    \(\mathrm{sp}: 0,0.01,0.03,0.05\), and 0.1.
    \(I / D: 1.0\).
```

Results of calculation are presented in tables 5.1 to 5.9 . For tables 5.1 to 5.5 the journal displacement is directed towards the centre of a lobe whereas in table 5.6 to table 5.9 the journal displacement is directed towards a conjunction of adjacent lobes. Table 5.1 presents results for a stationary journal ( $\mathrm{Sp}=0$ ) whereas in tables 5.2 to 5.5 the speed parameter is increased, table by table from O.Ol to O.1. Table 5.6 is related to a stationary journal.

Tables 5.1 to 5.9 are all calculated for a mean value of the three dimensionless inlet pressure of l.O. In tables 5.7 to 5. 9 the speed parameter is also increased table by table from 0.O1 to 0.1. In general the mean value of the three dimensionless pressure at the inlets was specified, and the programme calculated the individual dimensionless inlet pressure together with the common flow coefficient pertaining to each inlet. However, the calculation for tables 5.6 to 5.9 has been carried out for only a few values of eccentricity because a more refined routine is required to adjust the dimensionless pressure at each inlet.

Typical results from the computer outputs and from tables 5.1 to 5.9 will now be discussed. Figure 5.6 shows the pressure distribution at the axial mid-plane when the journal is stationary. It may be seen, at zero bearing eccentricity, that the pressure distribution is symmetrical about the inlet. Although not shown in the figure, the calculated pressures were the same at each lobe. For instance at mesh point of $i=15$ and $j=5$, the dimensionless pressure was 0.84576 , and for the upper lobes at similar points the pressures were 0.84484 and 0.84492 . When the journal is displaced within the bearing the pressure distribution for the bottom lobe is shown by the line $B$, and the distribution for the upper lobes by line $C$. Line $B$ is symmetrical, but as is to be expected, line $C$ is not. At $\varepsilon_{B}=O$ (line $A$ ) it may be seen that the pressure at the axial grooves ( $j=31,91$ and 151) is approximately $25 \%$ of the pressure at the high pressure inlets $(j=1,181,61$ and 121). The test results show that this ratio is approximately 20\%, however, it

Table 5.1 Results of Computation for the Threelobe Externally Pressurezed Journal Bearing •
$L / D=1 \quad P_{\varepsilon=0}=1$
$S p=0$


| $\varepsilon_{0}$ | $\varepsilon_{B}$ | $\gamma$ | $W_{t}^{-}$ | $\mathrm{p} / \mathrm{p}$ 。 | $\mathrm{q}_{\mathrm{a}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | $\begin{aligned} & 0 \\ & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \end{aligned}$ | $\begin{gathered} 0 \\ 1.5 \\ -0.35 \\ -0.3 \\ -0.19 \\ -0.16 \\ 0.11 \\ 0 \\ 0 \\ 0 \end{gathered}$ | 0.121 <br> 0.305 <br> 0.526 <br> 0.84 <br> 1.35 <br> 2.18 <br> 4.0 <br> 9.6 <br> 41.3 | 1.0 <br> 1.23 <br> 1.67 <br> 2.31 <br> 3.36 <br> 5.3 <br> 8.95 <br> 17.93 <br> 50.7 <br> 295 | $0.176$ |
| 0.4 | 0.1 <br> 0.2 <br> 0.3 <br> 0.4 <br> 0.5 <br> 06. <br> 0.7 <br> 0.8 <br> 0.9 | 0 -0.8 -0.45 -0.34 -0.27 0.21 0 0.1 0.07 0.065 | $\begin{aligned} & 0.134 \\ & 0.292 \\ & 0.499 \\ & 0.795 \\ & 1.21 \\ & 2.08 \\ & 3.75 \\ & 8.93 \\ & 36.12 \end{aligned}$ | 1.0 1.28 1.69 2.32 3.37 5.1 9.0 17.93 50.2 276.5 | $0.116$ |
|  |  |  |  |  |  |

Table 5.1 continued - Results of Computation for the Three-lobe Externally Pressurized Journal Bearing .


Table 5.2. Results of Computation for the Three-lobe Externally Pressurized Journal

## Bearing

$L / D=1 \quad P_{\varepsilon=0}=1$
$S p=0.01$


| $\varepsilon_{0}$ | $\varepsilon_{B}$ | $\gamma$ | $W_{t}^{*}$ | $\mathrm{P} / \mathrm{P}_{0}$ | $q_{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | 0 | 0 | 0 | 1.0 | 0.176 |
|  | 0.1 | 8 | 0.129 | 1.23 | " |
|  | 0.2 | 5.5 | 0.313 | 1.67 | " |
|  | 0.3 | 5.5 | 0.533 | 2.3 | " |
|  | 0.4 | 5.3 | 0.85 | 3.34 | " |
|  | 0.5 | 5.0 | 1.34 | 5.26 | " |
|  | 0.6 | 5.0 | 2.2 | 8.95 | " |
|  | 0.7 | 4 | 4.0 | 17.9 | " |
|  | 0.8 | 2 | 9.61 | 50.6 | " |
|  | 0.9 | 1 | 41.4 | 295 | " |
| 0.4 | 0 | 0 | 0 | 1.0 | 0.116 |
|  | 0.1 | 7.0 | 0.14 | 1.27 | " |
|  | 0.2 | 7.0 | 0.3 | 1.67 | " |
|  | 0.3 | 7.0 | 0.51 | 2.3 | " |
|  | 0.4 | 7.0 | 0.81 | 3.35 | " |
|  | 0.5 | 7.0 | 1.23 | 5.0 | " |
|  | 0.6 | 6.0 | 2.0 | 8.8 | " |
|  | 0.7 | 5 | 3.75 | 18 | " |
|  | 0.8 | 3 | 8.9 | 50 | " |
|  | 0.9 | 1.2 | 36.4 | 278 | " |

Table 5.2 continued - Results of Computation for the Three-lobe
Externally Pressurized Journal Bearing .

$$
L / D=1 \quad P_{\varepsilon=0}^{*}=1 \quad S p=0.01 \quad . \phi=0
$$

| $\varepsilon_{0}$ | $\varepsilon_{B}$ | $\gamma$ | $W_{t}^{*}$ | $\mathrm{P} / \mathrm{P}_{0}$ | $q_{\text {a }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | 0.1 <br> 0.2 <br> 0.3 <br> 0.4 <br> 0.5 <br> 0.6 <br> 0.7 <br> 0.8 <br> 0.9 | 0 12.6 12 10 10 10 8 6 4 2 | $\begin{gathered} 0 \\ 0.115 \\ 0.26 \\ 0.47 \\ 0.71 \\ 1.1 \\ 1.8 \\ 3.41 \\ 7.6 \\ 31.2 \end{gathered}$ | $1.0$ $1.21$ $1.59$ <br> 2.24 <br> 3.15 <br> 4.86 <br> 8.0 <br> 17.45 <br> 46.0 <br> 259 | $0.069$ |
| 0.6 | 0.1 <br> 0.2 <br> 0.3 <br> 0.4 <br> 0.5 <br> 0.6 <br> 0.7 <br> 0.8 <br> 0.9 | 16.6 <br> 16.2 <br> 15.4 <br> 14.77 <br> 12.8 <br> 11 <br> 8.42 <br> 5.2 <br> 2.3 | 0 0.113 0.245 0.42 0.65 1.037 1.65 2.94 6.64 24.1 | $\begin{aligned} & 1.0 \\ & 1.21 \\ & 1.58 \\ & 2.17 \\ & 3.11 \\ & 4.92 \\ & 8.26 \\ & 16.5 \\ & 44.38 \\ & 221 \end{aligned}$ | $0.038$ |
|  |  |  |  |  |  |

Table 5.2 continued - Results of Computation for the Three-lobe
Externally Pressurized Journal Bearing .
$L_{/ D}=1 \quad P_{\varepsilon=0}=1$


Table 5.3. Results of Computation for the Three-lobe Externally Pressurized Journal Bearing
$L / D=1 \quad P_{\varepsilon=0}=1$
$\mathrm{Sp}=0.03$


| $\varepsilon_{0}$ | $\varepsilon_{B}$ | $\gamma$ | $W_{t}^{-}$ | $\mathrm{P} / \mathrm{P}_{0}$ | $q_{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | 0 | 0 | 0 | 1.0 | 0.176 |
|  | 0.1 | 19 | 0.144 | 1.21 | " |
|  | 0.2 | 16.4 | 0.336 | 1.65 | " |
|  | 0.3 | 16.4 | 0.562 | 2.26 | " |
|  | 0.4 | 15.7 | 0.885 | 3.32 | " |
|  | 0.5 | 14.4 | 1.39 | 5.2 | " |
|  | 0.6 | 12.7 | 2.22 | 8.86 | " |
|  | 0.7 | 9.7 | 4.0 | 17.6 | " |
|  | 0.8 | 7.4 | 9.16 | 46.8 | " |
|  | 0.9 | 3 | 38.6 | 273 | " |
| 0.4 | 0 | 0 | 0 | 1.0 | 0.116 |
|  | 0.1 | 23.34 | 0.14 | 1.23 | " |
|  | 0.2 | 20.77 | 0.334 | 1.66 | " |
|  | 0.3 | 21.c | 0.554 | 2.27 | " |
|  | 0.4 | 20.3 | 0.86 | 3.3 | " |
|  | 0.5 | 19 | 1.29 | 5.0 | " |
|  | 0.6 | 15.65 | 2.06 | 8.36 | " |
|  | 0.7 | 12.65 | 3.44 | 15.6 | " |
|  | 0.8 | 8.7 | 7.27 | 39.66 | " |
|  | 0.9 | 4.0 | 28.3 | 214.76 | " |

Table 5.3. continued - Results of Computation for the Three-lobe Externally Pressurized Journal Bearing .


Table 5.4. Results of Computation for the Three-lobe Externally Pressurized Journal Bearing .


| $\varepsilon_{\delta}$ | $\varepsilon_{B}$ | $\gamma$ | $W_{t}^{\prime}$ | P* | $q_{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | 0 | $\bigcirc$ | 0 | 1.0 | 0.176 |
|  | 0.1 | 27.7 | 0.163 | 1.22 | " |
|  | 0.2 | 27.3 | 0.354 | 1.6 | " |
|  | 0.3 | 27 | 0.59 | 2.2 | " |
|  | 0.4 | 25 | 0.926 | 3.22 | " |
|  | 0.5 | 22 | 1.4 | 4.86 | " |
|  | 0.6 | 19 | 2.15 | 7.9 | " |
|  | 0.7 | 17 | 3.87 | 15.4 | " |
|  | 0.8 | 11 | 8.15 | 40.3 | " |
|  | 0.9 | 5 | 31.8 | 22.3 | " |
| 0.4 | o | 0 | $\bigcirc$ | 1.0 | 0.116 |
|  | 0.1 | 33 | 0.172 | 1.22 | " |
|  | 0.2 | 31.5 | 0.382 | 1.64 | " |
|  | 0.3 | 31.5 | 0.61 | 2.19 | " |
|  | 0.4 | 30 | 0.913 | 3.0 | " |
|  | 0.5 | 28 | 1.28 | 4.26 | " |
|  | 0.6 | 26 | 1.85 | 6.52 | " |
|  | 0.7 | 22 | 3.0 | 12.2 | " |
|  | 0.8 | 17 | 5.8 | 29.3 | " |
|  | 0.9 | 9 | 19.2 | 14.2 | " |

Table 5.4. continued - Results of Computation for the Three-lobe
Externally Pressurized Journal Bearing .


Table 5.5. Results of Computation for the Three-lobe Externally Pressurized Journal Bearing .

$$
L_{/ D}=1 \quad P_{\varepsilon=0}
$$

$S p=0.1$


| $\varepsilon_{0}$ | $\varepsilon_{B}$ | $\gamma$ | $W_{t}$ | p* | $\mathrm{q}_{\mathrm{a}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | 0 | 0 | 0 | 1.0 | 0.176 |
|  | 0.1 | 42 | 0.222 | 1.21 | " |
|  | 0.2 | 42 | 0.472 | 1.58 | " |
|  | 0.3 | 40 | 0.735 | 2.0 | " |
|  | 0.4 | 40 | 1.0 | 2.64 | " |
|  | 0.5 | 36 | 1.47 | 3.8 | " |
|  | 0.6 | 34 | 2.15 | 6.0 | " |
|  | 0.7 | 32 | 3.4 | 10.6 | " |
|  | 0.8 | 28 | 5.75 | 23.2 | " |
|  | 0.9 | 18 | 16.5 | 104.7 | " |
| 0.4 | 0 | 0 | 0 | 1.0 | 0.116 |
|  | 0.1 | 48 | 0.25 | 1.21 | " |
|  | 0.2 | 46 | 0.50 | 1.51 | " |
|  | 0.3 | 46 | 0.74 | 1.82 | " |
|  | 0.4 | 47 | 1.02 | 2.2 | " |
|  | 0.5 | 50 | 1.34 | 2.55 | " |
|  | 0.6 | 51 | 1.81 | 3.3 | " |
|  | 0.7 | 54 | 2.48 | 4.31 | " |
|  | 0.8 | 65 | 3.38 | 3.4 | " |

Table 5.5. continued - Results of Computation for the Three-lobe

Externally Pressurized Journal Bearing. .


Table 5.6. Results of Computation for the Three-hole Externally Pressurized Journal Bearing .


Table 5.7. Results of Computation for the Three Lobe Externally Pressurized Journal Bearing.

$$
\mathrm{L} / \mathrm{D}=1 \quad \mathrm{P}^{*} \varepsilon_{\varepsilon=0}=1
$$

$$
\mathrm{sp}=0.01
$$



| $\varepsilon_{0}$ | $\varepsilon_{B}$ | $\gamma$ | $W_{t}^{*}$ | P* | $q_{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | $\begin{array}{r} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \end{array}$ | 0 <br> 4 <br> 5 <br> 5 <br> 5 <br> 5 | $\begin{gathered} 0 \\ 0.15 \\ 0.28 \\ 0.43 \\ 0.6 \\ 0.738 \end{gathered}$ | $\begin{aligned} & 1.0 \\ & 1.18 \\ & 1.35 \\ & 1.58 \\ & 1.85 \\ & 2.17 \end{aligned}$ | 0.176 |
| 0.4 | $\begin{gathered} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{gathered}$ | 0 <br> 7.5 <br> 7 <br> 6 <br> 6 | $\begin{gathered} 0 \\ 0.13 \\ 0.26 \\ 0.4 \\ 0.71 \end{gathered}$ | $\begin{aligned} & 1.0 \\ & 1.16 \\ & 1.36 \\ & 1.6 \\ & 1.8 \end{aligned}$ | $0.116$ |
| 0.5 | $\begin{array}{r} 0 \\ 0.1 \\ 0.2 \\ 0.3 \end{array}$ | $\begin{aligned} & 0 \\ & 10.86 \\ & 9.4 \\ & 7.76 \end{aligned}$ | $\begin{gathered} 0 \\ 0.12 \\ 0.24 \\ 0.388 \end{gathered}$ | $\begin{aligned} & 1.0 \\ & 1.15 \\ & 1.35 \\ & 1.62 \end{aligned}$ | $0.069$ |
|  |  |  |  |  |  |

Table 5.8. Results of Computation for the Three Lobe Externally Pressurized Journal Bearing.

$$
L / D=1 \quad P_{\varepsilon=0}^{*}=1 \quad S p=0.03
$$

$\phi=180^{\circ}$

| $\varepsilon_{0}$ | $\varepsilon_{B}$ | $\gamma$ | $W_{t}^{*}$ | P* | $q_{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | $\begin{gathered} 0 \\ 0.1 \\ 0.3 \end{gathered}$ | $\begin{array}{r} 0 \\ 15 \\ 13 \end{array}$ | $\begin{gathered} 0 \\ 0.155 \\ 0.494 \end{gathered}$ | $\begin{aligned} & 1.0 \\ & 1.19 \\ & 1.73 \end{aligned}$ | $0.176$ |
| 0.4 | 0.1 <br> 0.2 <br> 0.3 | 0 <br> 19 <br> 17 <br> 15 | $\begin{gathered} 0 \\ 0.154 \\ 0.32 \\ 0.48 \end{gathered}$ | $\begin{aligned} & 1.0 \\ & 1.22 \\ & 1.5 \\ & 1.7 \end{aligned}$ | $0.116$ |
| 0.5 | $\begin{gathered} 0 \\ 0.1 \\ 0.2 \\ 0.3 \end{gathered}$ | 21.4 <br> 20.64 <br> 21 | $\begin{gathered} 0 \\ 0.161 \\ 0.322 \\ 0.47 \end{gathered}$ | $\begin{aligned} & 1.0 \\ & 1.25 \\ & 1.55 \\ & 1.6 \end{aligned}$ | $0.069$ |
| 0.6 | $\begin{array}{r} 0 \\ 0.1 \\ 0.2 \\ 0.3 \end{array}$ | 29 <br> 27 <br> 24 | $\begin{gathered} 0 \\ 0.147 \\ 0.293 \\ 0.44 \end{gathered}$ | $\begin{aligned} & 1.0 \\ & 1.24 \\ & 1.54 \\ & 1.9 \end{aligned}$ | $0.038$ |
|  |  |  | . |  | 1 |

Table 5.9. Results of Computation for the Three-Lobe Externally Pressurized Journal Bearings.

$$
\mathrm{L} / \mathrm{D}=1 \quad \mathrm{P}_{E=0}^{*}=1 \quad \mathrm{Sp}=0.05
$$

| $\varepsilon_{0}$ | $\varepsilon_{B}$ | $\gamma$ | $w_{t}^{*}$ | P* | $\mathrm{q}_{\mathrm{a}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3 | $\begin{gathered} 0 \\ 0.1 \\ 0.2 \end{gathered}$ | $\begin{gathered} 0 \\ 19 \\ 20 \end{gathered}$ | 0 <br> 0.19 <br> $0 \% 62$ | $\begin{aligned} & 1.0 \\ & 1.26 \\ & 1.53 \end{aligned}$ | $0.176$ |
| 0.4 | $\begin{gathered} 0 \\ 0.1 \\ 0.3 \end{gathered}$ | 24 <br> 21 | $\begin{aligned} & 0 \\ & 0.194 \\ & 0.6 \end{aligned}$ | $\begin{aligned} & 1.0 \\ & 1.3 \\ & 2.0 \end{aligned}$ | $0.069$ |
| 0.5 | $\begin{gathered} 0 \\ 0.1 \\ 0.2 \\ 0.3 \end{gathered}$ | 0 <br> 32 <br> 28.8 $24.7$ | $\begin{aligned} & 0 \\ & 0.183 \\ & 0.372 \\ & 0.6 \end{aligned}$ | $\begin{aligned} & 1.0 \\ & 1.3 \\ & 1.65 \\ & 2.13 \end{aligned}$ | $0.069$ |
| 0.6 | $\begin{gathered} 0 \\ 0.1 \\ 0.2 \\ 0.3 \end{gathered}$ | 47 <br> 44 <br> 39 | $\begin{aligned} & 0 \\ & 0.076 \\ & 0.193 \\ & 0.32 \end{aligned}$ | $\begin{aligned} & 1.0 \\ & 1.0 \\ & 1.12 \\ & 1.38 \end{aligned}$ | $0.038$ |
|  |  |  |  |  |  |


may be mentioned that in the computer programme, the oil grooves are not represented and the bearing surface is assumed to be continued.

Figures 5.7 and 5.8 relate to a rotating journal and are respectively for speed parameter of 0.03 and 0.1 . With reference to figure 5.7 it may be seen that the pressure distribution is no longer symmetric about the lobe (i.e. $\theta=60^{\circ}$ ), but the maximum dimensionless pressure for all $\varepsilon_{B}$ is still at the position of the high pressure inlets (i.e. $\theta=60^{\circ}$ ). Also it may be seen that as the bearing eccentricity ratio ( $\varepsilon_{B}$ ) increases the hydrodynamic effect increases, and negative pressures start to develop at the trailing end of the lobes, and as the eccentricity increases, the pressure distribution curve is terminated before the end of the lobe circumferential length. With reference to Fig. 5.8, at a speed parameter of 0.1 , the bearing is behaving as a self generating bearing. The pressure distribution curves are similar to those plotted in Chapter 3. Also it may be seen that the maximum pressure position is no longer located at the high pressure inlets but the high pressure positions are now located before the minimum film thickness as has been found in the self generating analysis.

Figure 5.9, which relates to a stationary journal, shows the variation of the dimensionless pressure at high pressure inlets with the bearing eccentricity ratio. Line A is drawn for a vertical load directed towards the centre of the bottom lobe (i.e., towards the high pressure inlet). Line $B$ is drawn for a vertical load directed towards the conjunction of the lobes and the journal movement is not directly against the high pressure inlets. It may be seen that for the case represented by line $A$ the dimensionless pressure $P *$ increases



Figure 5.9. Dimensionless Inlet Pressure Bearing Eccentricity Characteristics.
rapidly as the journal moves towards the high pressure inlet. The oil flow through the bearing is determined by the setting of the constant volume valves, and is independent of whether the journal is rotating or not, loaded or not. However, with the journal central and for a particular value of $P^{*}$ the oil flow required to establish that pressure will be a function of the way in which the space between journal and the lobes varies (i.e. function of the offset ratio $\varepsilon_{0}$ ). In figure 5.10 the variation of the oil flow coefficient against offset ratio is plotted for a central journal. It may be seen that the oil flow coefficient decreases as the offset ratio increases, however, in real terms the increase of the offset ratio leads to an increase in the lobe diametral clearance so that the actual oil flow may be increased as the offset ratio increases.

The effect of the variation of bearing eccentricity ratio on load factor is shown in figure 5.11. The results obtained are for zero speed and for various offset ratios. It may be seen that the load factor increases as the eccentricity ratio increases but the load factor decreases as the offset ratio increases. The decrease in the load factor as the offset ratio increases may be attributed to the departure from a uniform oil film thickness around the journal which will affect the pressure distribution.


Figure 5.10. Oil Flow Coefficient - Bearing Offset Ratio Characteristics.


Figure 5.11. Load Factor - Bearing Eccentricity Characteristics.

In figure 5.12 the effect of the variation of speed on load factor is shown. It is quite clear that as the speed parameter increases the hydrodynamic effects on the load parameter increases, so that the load factor $w_{t}$ and the bearing load increase as the speed increases. Also it may be seen from figure 5.12 that an increase in eccentricity ratio leads to an increase in hydrodynamic effects which is typically what has been found by the hydrodynamic solution in chapter 3. In figure 5.13 the load factor is plotted against the speed parameter for two different . . offset ratios. As the speed parameter increases it may be seen that the effect of offset ratios upon the load factor decreases. However, at speed parameters beyond 0.09 a cross-over occurs. This trend in the characteristics of load factor is attributed to the increase in the hydrodynamic effects as the speed parameter and the offset ratio increases; also this trend in the characteristics of the load factor is consistent with results which have been discussed in Chapter 3 , where it has been shown that the bearing duty parameter (i.e. bearing load) increases as the offset ratio increases.

Figure 5.14 shows the variation of the stiffness coefficient $\bar{\lambda}$ as defined by expression (5.28) with the bearing eccentricity ratio. Quite clearly, from figure 5.14 increasing the eccentricity ratio increases the stiffness coefficient, also


Figure 5.12. Load Factor - Speed Parameter Characteristics.


Figure 5.13. Load factor - Speed Parameter Characteristics.


Figure 5.14. Stiffness Coefficient - Eccentricity Ratio Characteristics.
the stiffness coefficient increases with the offset ratio. However, because the stiffness coefficient is dependent on the lobe clearance, and because the offset ratio and lobe clearance are not variable independently the change in the stiffness coefficient does not necessarily imply similar changes in actual stiffness.

The effect of speed parameter on the load angle $Y$, is shown in figure 5.15. When the speed parameter is zero (i.e. stationary journal) the load angle $Y$ is zero for all the eccentricity ratio values, and the load is in the same sense as the journal displacement. When the speed parameter increases, the angle between the vertical direction of the journal displacement and the load direction increases. Also it may be seen that at speed parameters beyond 0.85 a crossover occurs which may be attributed to the increase in the hydrodynamic effects.

In figure 5.16 it may be seen that an increase in the offset ratio increases the load angle. In both figures 5.15 and 5.16 the load angles are comparable to the load angles for the self generating circumstances which have been reported in chapter 3.


Figure 5.15. Load Angle - Speed Parameter Characteristics.


Figure 5.16. Load Angle - Speed Parameter Characteristics.

### 5.8. Design procedure

In this paragraph a numerical example will be worked in order to show how to predict the performance of a three lobe bearing from the results which have been presented. The numerical values which are used in this example relate to the bearing which has been tested. Also, the load and the displacement values which are used in this example relate to the functional specification given in table 1.1. The numerical values are:

$$
\begin{aligned}
& D=135 \mathrm{~mm} \quad L=127 \mathrm{~mm} \quad C=0.26 \mathrm{~mm} \quad \delta=0.0254 \mathrm{~m} \mathrm{~m} \\
& \varepsilon_{0}=0.45 \quad W_{t}=22270 \mathrm{~N} \quad \mathrm{~N}=0 \rightarrow 5000 \mathrm{rpm} \\
& \text { Oil: Tellus } 37
\end{aligned}
$$

The eccentricity ratio $E=\frac{2 \delta}{C}=\frac{2 \times 0.0254}{0.26}=0.1954$

The bearing eccentricity ratio $\varepsilon_{B}=\frac{\varepsilon}{\left(1-\varepsilon_{0}\right)}=\frac{0.1954}{(1-0.45)}=0.355$

The load factor $W_{t}^{*}=0.62$
(Figure 5.11)
$P_{0}=W_{t} / L D W_{t}^{-}=\frac{22270}{0.127 \times 0.135 \times 0.25}=2.1 \times 10^{6} \mathrm{Nm}^{-2}$

The oil flow coefficient, $q_{a}=0.09$
(Figure 5.10)

$$
\begin{aligned}
& \mu=0.22 \text { at } 50^{\circ} G^{-} \quad \begin{array}{c}
\text { (The operating temperature is } \\
\text { considered to be } \left.50^{\circ} \mathrm{C}\right)
\end{array} \\
& Q_{a}=\frac{q_{a} P_{0} C^{3}}{6 \mu}=\frac{0.09 \times 2.1 \times 10^{6} \times(0.026)^{3} 981000}{6 \times 0.22 \times 1000 \times 9.81 \times 10^{4}}=0.025 \mathrm{ls}^{-1} \\
& \text { The total oil flow }=3 \times Q_{a}=0.075 \mathrm{\ell s}^{-1}(1.0 \mathrm{G.P.M.)} \\
& \text { From table } 5.1 \text { and for } \varepsilon_{B}=0.355 \text { then } \mathrm{P}^{*}=2.77
\end{aligned}
$$

The pressure at the highest pressure inlet for bearing eccentricity ratio of $0.355=2.1 \times 10^{6} \times 2.77=5.82 \times 10^{6} \mathrm{NM}^{-2}$

The supply pressure, $\mathrm{Ps}=5.82 \times 10^{6}+$ the pressure drop across constant volume flow valve.

$$
P s=5.82 \times 10^{6}+6.67 \times 10^{5}=6.487 \times 10^{6} \mathrm{NM}^{-2}
$$

By considering the shaft is rotated at 1000 rpm:-

The speed parameter, $S p=\frac{\mu N}{P_{0}}(D / C)^{2}$

$$
=\frac{0.22 \times \frac{1000}{60} \times 9.81}{2.1 \times 10^{2} \times 981000}\left(\frac{13.5}{0.026}\right)=0.044
$$

The load factor $W_{t}^{\prime}$ (from tables 5.2 to 5.5 ) $=0.7$.

The load which may be carried by the journal at $\mathcal{E}_{B}=0.355$ and

Ns 1000 rpm is $=\frac{0.70}{0.62} \times 22270=24942 \mathrm{~N}^{*}$

The load angle $\gamma=35$ degrees (tables 5.3 and 5.4)

In chapter 3 , the validity of the computer programmes which have been used for the self generating regime have been examined by using the author's programmes to solve problems where the results are published. With respect to the programmes which have been used in this chapter, there is nothing which has been published about the three lobe kearings as externally pressurized bearings; also there is nothing in the literature about the analysis of externally pressurized bearings which are compensated by the constant volume flow valves. However, the validity of the computer programmes which

The calculated load of 24942 N with the displacement of 0.0254 mm implies a stiffness of $9.82 \times 10^{8} \mathrm{Nm}^{-1}$.
have been used in this chapter has already been discussed in paragraph 5.7 with the discussion of the computer programme results. In paragraph 5.7 it has been shown that the solutions produce a symmetric pressure distribution for all the three lobes, and also it has been shown that by increasing the speed parameter the pressure distribution becomes similar to the pressure distribution of the self generating regime. The load angle of a stationary journal is fourd to be zero; however, in paragraph 5.7 it has been shown that as the speed parameter increases the load angle approaches the load angle of the self generating circumstance. Also the load factor shows a similar behaviour with the speed parameter . All these comparisons between the output of present computer programme and the output of the computer programme used in chapter 3 , with the results of the numerical example may show the logic and the correctness of the results obtained from the present programme.

As a further check upon the computer programme, the calculated oil flow is compared with the measured oil flow. In table 5.9 the calculated oil flow for an offset ratio of 0.45 and different conditions of operation are compared with the measured oll flow at the same operating conditions. Quite clearly the results show that a very close agreement exists which again substantiates the validity of the programme.

A comparison between the results of this analysis, the approximate analysis (chapter 2) and the tests results (chapters 4 and 6) will be presented in chapter 8.
Table 5.10. Comparison between the calculated and measured Oil Flow for the Externally Pressurized regime.


### 6.1. Introduction

The success or otherwise of the bearing which has been described in meeting both the low and high speed aspects of the specification can only be established by an experimental investigation in which the behaviour of the bearing assembly is tested over a wide range of operation. A full scale test rig for the hydrostatic and hydrodynamic investigation of the three lobe journal bearing has been built. Speeds up to 7000 rpm have been reached. Two oils of different viscosities have been used and two different bearing shells have been machined and used in test.
6.2. Test Rig and Instrumentation ${ }^{*}$

A layout of the test rig is shown in Figure 6.1. The shaft ${ }^{1}$ is supported by the wide speed range bearing ${ }^{2}$ (i.e. three-lobe bearing), and by the rolling element thrust ${ }^{4}$ with the tall bearing ${ }^{3}$ in the housing ${ }^{5}$ on the right hand side. A hydraulic ram, which is not shown, is used to apply a vertical load via the slave bearing ${ }^{7}$ fitted to the overhang of the shaft. The conception of the rig is that it should correspond as closely as is practical with an actual machine tool spindle, for instance the tail and thrust races have been chosen with machine tool duty in mind, and are mist lubricated through the nozzle ${ }^{8}$. The displacement of the journal with respect to the bearing is measured with inductive displacement transducers ${ }^{9}$ at each end of the bearing. There are two pairs of transducers arranged to measure vertical and horizontal displacements. Measurements hava been made at each end by transferring the transducers. Thermocouples ${ }^{10}$ are provided to measure oil and bearing metal temperature. Flow and pressure at both high pressure and low pressure inlets were measured, but the flow was not measured simultaneously because to reduce the provision of

[^2] He also carried out some machining and fitting.

instruments, the moving of an instrument from one position to another was accepted. Photographs of the test rig with instrumentation are shown in Figures 6.2 and 6.3.

### 6.2.1. The Three Lobe Bearing

Two test bearings have been machined to provide different diametral clearance* and different dimensions for the axial grooves where the low pressure oil for hydrodynamic operation is delivered. A high pressure oil inlet for the hydrostatic regime is provided at the centre of each lobe. The main dimensions of the test bearings are shown by Figure 6.4. A bearing is also shown by the photograph of Figure 6.5. The machining of a bearing is described in detail in Appendix A.4., and in Appendix A. 5 an alternative method is described which has been tested as a process, but no actual bearing machined by the process has been made or tested.

### 6.2.2. The Design of the Shaft

In the design of the test shaft it was considered that the shaft should be of a form as close as possible to an actual machine spindle. The shaft is illustrated in Figure 6.6. The shaft was designed to provide an extension to carry a slave bearing for overhung loading of the test bearing ${ }^{7}$. (Figure 6.1).

Evaluations by conventional means were made of the bending stress, shear stress and deflection in the shaft. The stresses are low, but at the position of maximum stress, where the dimensions are reduced for the thrust bearing, a very generous filet is provided to avoid any possibility of a fatigue failure.

In a machine tool application the precise axial position of the cutting tool could be important, and with the notion of minimizing *In principle the same effect could have been achieved by reducing the diameter of the shaft, but it would have been difficult to modify the first bearing in other respects.



FIG.6.3 TEST RIG AND INSTRUMENTATIONS


FIG. 6.4 Three lobe-bearing.


FIG. 6.5 TEST BEARING.
First bearing - the lines in the bore and score marks caused by detritus in the oil ways. The second bearing is completely undamaged.

the effect upon this precison of the thermal expansion of the shaft, it was decided to fit the thrust bearing as close as is possible to the three lobe bearing. This gave rise to a reduced diameter of the tail to permit the fitting of the thrust. However, as will be described later, this was probably a mistake, as it makes the shaft more flexible than would otherwise be so. (The thrust and tail bearings are shown at 4 and 3 respectively - Figure 6.1).

The shaft was roughly machined from normalized En8 steel, the shaft was heat treated for stress relieving ( 1.5 hours at $690^{\circ} \mathrm{C}$, furnace cooled. Shaft rotated in heat treatment) and then it was finally machined to the required dimensions. The positions of the main journal bearing, the thrust bearing and the tail bearing were ground. Details of the manufacturing process are given in Figure 6.6. The machined shaft is shown by the photographs in Figures 6.7 and 6.8.

### 6.2.3 The Design of the Thrust Bearing

In the design of the thrust bearing, no consideration was given to a specific thrust load and the main criteria, for the design of the thrust bearing was the speed limit and the shaft diameter. It was found that an ordinary roller or ball thrust is not suitable for the required speeds, and that a single row deep groove ball bearing is better suited to accommodate axial loads at high speeds. A deep groove ball bearing of 80 mm . bore was used; this bore diameter is the greatest diameter which is compatible with the maximum speed.

### 6.2.4 Design of the Tail Bearing

The required load to be supported at the position of the tail bearing was calculated and it was found that the load carrying capacity of the bearing does not introduce a problem. However, for the required running accuracy of a machine tool it is advisable to use cylindrical


FIG.6.7 TEST SHAFT WITH MOUNTED ROLLING ELEMENTS.


FIg.6.8 TEST SHAFT WITH THE THRUST AND TAIL BEARINGS HOUSING.
roller bearings. To achieve the speeds of the specification given in Table l. 1 , a cylindrical roller bearing of 60 mm bore has been used. Figure 6.7 shows the tail bearing, thrust bearing and the slave bearing mounted on the test shaft.
6.2.5 The Design of the Rolling Bearings Housing

A steel housing to house the thrust bearing and the tail bearing was designed with the following functions in mind:
i) The outer ring of the thrust ball bearing is a clearance fit with the housing. The magnitude of that radial clearance must be greater than the shaft deflection at that point under the maximum load so that the bearing is subjected only to thrust loads.
ii) The outer ring of the tail bearing is an interference fit with the housing. However, dismounting the housing from the shaft could be easily made as the inner and outer rings of the tail bearing are separable. The roller tail bearing provides no constraint of axial movements due to thermal expansion.

1ii) The cover plates (i.e. seals), have been machined with a sufficient clearance to give an adequate venting area for the air of the mist lubrication. The flow of air is a provision against the ingress of dirt. Figure 6.8 shows the housing assembled to the shaft, while Figure 6.9 shows the manufacturing drawing for the housing.
6.2.6 The Design of the Rolling Bearings Lubricating System

For high speed bearings, or where high bearing temperatures prevail, the main task of lubricant is to remove heat, and several methods of lubrication are available. One of these is mist lubrication in which

a suspension of oil in air is forced through the bearings.

The oil mist consists of atomised oil in air and it is conveyed to the lubricating point where it is completely or partially precipitated. The size of the spray nozzles, quantity of air, quantity of oil, and the air pressure, are supplied by the manufacturer.

Two oils of different viscosities have been used in the mist lubricator. The full specifications are presented in Appendix 6.

### 6.2.7 Design of the Loading System

To apply a radial load on the three lobe bearing, a hydraulic ram is employed to apply radial loads via a slave bearing, to the overhang of the shaft (Figures 6.10-6.12).

In Figure 6.10 a self-aligning spherical roller bearing ${ }^{1}$, is mounted in a bearing housing ${ }^{2}$ and the housing is held coaxial with the shaft ${ }^{3}$ by the rod ${ }^{4}$. The load is applied through a hydraulic ram ${ }^{5}$ of known area and pressure. The loading bearing itself is lubricated through the mist fitting ${ }^{6}$ which is connected to the mist lubricator by the plastic tube ${ }^{7}$. The loading bearing was considered to be a sacrificial bearing, but in fact there has been no failure of it.

### 6.2.8 The Design of Oil Circuit for the Externally Pressurized and Self-Generating Operation.

The oil circuit is illustrated by Figure 6.13. The high and low pressure circuits are arranged so that they can be operated individually or simultaneously. However, an oil cooler is provided only in the low pressure circuit, but the circuit is arranged so that oil from the cooler is passed directly back to the sump if the low pressure supply to the bearing is not required, but it is required to remove

THIRD ANGLE PROJECTION

$\qquad$



FIG.6.13
Self-generating and externally pressurized oil circuit.
heat generated in the high pressure circuit. The diagram shows four positions for flow measurement. However, only two flow meters were available, and they were moved from position to position as was appropriate for the test to be carried out.

### 6.2.9 Calibration of the Compensation Devices

The flow characteristics of each valve were tested by measuring volume flows at different valve settings, at different supply pressures, at different back pressures and at different oil temperatures between $22 \& 80^{\circ} \mathrm{C}$.

The valves were calibrated by subjecting each valve to varying back pressures whilst the pump line pressure was held constant. The flow rates were measured by using the "STOP WATCH AND BUCKET TECHNIQUE". The results show that the valves are working correctly. The variation of the oil viscosity between $250 C . S$ and l7C.S. has resulted in a flow variation of 6\%. However, by considering that some difference may occur due to the timing, the flow variation may be considered negligible.

The valve controlled the flow rate by varying the area of an orifice within the valve. To nulify the effects of fluctuating back pressure the valve contains a spring controlled by-pass loop.

The full calibration results are given in Appendix A.7, while in Figure 6.14 a calibration chart for the oil flow against the valve setting is shown.

### 6.2.10 Calibration of the Flowmeters

The flow meters which were used are of the turbine type, with generation of frequency signal which is directly proportional to the speed of rotation (i.e. the flow through the pipe line) so for high viscosity oil such meters indicate flow rates less than the true flow rates, and it is necessary to calibrate them. The basis of calibration was the


FIG.6.14 VFS $1 / 100$ adjuster calibration curve.
flow associated with particular settings of the constant valves which had already been calibrated over the full range of viscosities by the "bucket and stop watch" technique (Appendix 7). But because the calibration was carried out in the high pressure line with constant flow valves already present, the opportunity was taken to check the calibration of the valves simultaneously. A calibration chart is given in Figure 6.15 where the dotted lines show the flow as measured by "bucket and stop watch" with the settings of the constant volume valve. Quite clearly it would be unrealistic to rely on the nominal reading of the flow meter at temperatures below $60^{\circ} \mathrm{C}$, and that was not done. In that range, the high pressure oil flow was based upon the calibration chart of Figure 6.14, while the total oil flow from the bearing was measured by taking the drain to a bucket, and by using the "bucket and stop watch" technique. No internal inconsistency has ever arisen to bring the calibration of the flow meters into doubt.
6.2.11. Measurements of the Shaft Displacements and the Journal
Centre Attitude.

The displacements of the journal with respect to the bearing were measured with inductive displacement transducers at each end of the bearing. The transducers were matched in pairs to increase the linearity of the output, and each pair is connected to a carrier frequency amplifier. The output was displayed on a digital millivolt meter. The electrical output from the amplifiers is either a voltage between $\pm 10 \mathrm{~V}$ or a current between $\pm 20 \mathrm{ma}$. The output signal for a given displacement depends upon the iritial gap between shaft and transducers and upon the gain of the amplifiers which can be adjusted internally. There are no external knobs for the adjustment of the gain so that gain remains constant until it is deliberately changed by internal adjustment.


FIG. 6.15
Variations of the flow-meter reading with the oil temperature.

The output signal can be brought to zero at will by connecting a potentiometer of a $100 \Omega$ to a zero external terminals, and in general the output was brought to zero with the journal in unloaded condition. The measuring circuit is shown by Figures 6.16 and 6.17 .

### 6.2.12. Calibration of the Transducers.

To calibrate the inductive transducers, the high pressure system was engaged to locate the journal centrally within the bearing. The transducers were set by feeler gauges with an initial gap ranging between 0.1524 mm and 0.4 mm . Dial gauges were positioned as near as possible from the transducers. A load was applied to the shaft and the deflection of the shaft was measured by the dial gauges and the change in the output of the transducers was noted. This procedure was repeated at working temperatures ranging from $25^{\circ} \mathrm{C}$ and up to $85^{\circ} \mathrm{C}$, the transducers were recalibrated from time to time (2 times a week) to ensure the accuracy of the results. Typical calibration charts are given in Figure 6.18. The difference between the calibration is attributed to the different gain settings.
6.2.13. Pressure Measurements.

The supply pressure of the high pressure pump, the pressures at the high pressure oil inlets and the pressures at the low pressure inlets were measured by Bourdon gauges which were dead weight calibrated.

### 6.2.14. Temperature Measurements

To monitor the temperatures of the oil at the bearing inlets, outlets, and bearing metal temperature, thermocouples were provided in 10 positions. The thermocouples were connected to a 'read out' unit with builtin compensations. Thermometer pockets were provided in the high pressure line and these pockets contained thermocouples and mercury in glass thermometers. This dual provision provided a check upon the read out unit.


FIG. 6.16
The inductive transducers circuit for displacements measurements.

The transducers are set permanently at A.A and B.B.
The transducers are moved to $C . C$ and D.D, as the tests require.


FIG.6.17 Transducers arrangements of the Journal.


FIG. 6.18
Calibration Chart for the displacement transducers.

### 6.2.15 The Metallic Contact Monitoring

To monitor any occurence there might be of metal to metal contact between journal and bearing, and also to provide a convenient means of judging the maximum allowable load, the electrical resistance between journal and bearing was measured with a meter which imposed less than 3 volts across the oil film. To permit these measurements of resistance the parallel paths such as through the housing of tail and thrust bearing were insulated from the main frame.

### 6.2.16 Drive Unit

In the early stages of the design, the very approximate estimation of the power loss suggested that a power of approximately 40 KW would be required to rotate the test shaft at 9000 rpm . (The more refined calculations of Chapter 3, and the experimental test results show that early estimate to be too large by a factor of approximately 2). The provision of a variable speed electrical drive of that power was too expensive to contemplate, and it was decided to use an existing $22 \mathrm{KW} / 1500 \mathrm{rpm}$ electrical motor together with a set of timing pulleys and belt to provide various speeds in the range 720 to 4000 rpm . The speed of the motor together with limitation of pulleys and belt limited the shaft speed with the motor to 4000 xpm . In addition the starting torque imposed a limitation on the speed at which the shaft can be driven. It was seen that these limitations could be over-come by use of a petrol engine which could be run at 4000 rpm , and which has a clutch to relieve the starting torque problem. Later a petrol engine drive was provided with belt connection to the shaft and speeds up to 7000 mpm were obtained. The limitation on speed then became the maximum safe velocity of the belt itself. Because the test results up to 7000rpm give confidence that may be reasonably extrapolated to 9000 rpm ,
and because the time and additional cost which would be involved in reaching 9000 rpm , the maximum test speed of 7000 rpm was deemed to be adequate. The use of a belt drive directly to the end of the shaft inevitably introduces some vibrations, but the vibrations was never excessive. Some form of inline drive is obviously to be preferred.

### 6.2.17 Selection of the Oil

In the early approximate design investigation, Shell Oil Vitria 150 with kinematic viscosity of 150 centistokes at $40^{\circ} \mathrm{C}$ had been assumed, and this oil is one of the oils which has been used in the test. In the test programme the disadvantage of an excessive high outlet temperature with this oil emerged, whilst other aspects of the test results indicated that it would be permissible to use an oil of significantly lower viscosity. For instance, in paragraph 6.3.1 to 6.3.6 oil outlet temperatures of $72^{\circ} \mathrm{C}$ and $95^{\circ} \mathrm{C}$ are reported at 4000 rpm in externally pressurized and in hydrodynamic tests respectively. Because oil temperature rise is approximately proportional to the rotational speed it is quite clear that an excessive oil outlet temperature would arise had the test been continued to higher rotational speeds. Theory indicates that the oil temperature rise is reduced when an oil of lower viscosity is employed. A second oil of lower viscosity "Shell Tellus 37" was employed in a further series of tests. The viscosity-temperature characteristics of these oils are given in Figures 6.19 to 6.20.

### 6.2.18 Power Loss Measurements

The power absorbed by a bearing has been measured in the usual way from measurements of oil flow and of oil temperature rise as the oil passes through the bearing. However a ccrrection should be added to the


FIG.6. 19
Typical viscosity-temperature characteristics of Shell Vitria 150


FIG.6.20 Typical viscosity-temperature characteristics of Shell Tellus 37.
power loss given by that calculation which does not include the heat fluxes of conduction along the journal and of heat transfer to the air from the pedestal and journal. The correction is relatively small and approximate means for its assessment will suffice.

If the bearing and journal satisfied the assumptions of Newton's Law of cooling then a quantity $k$ can be defined which is given by

$$
\mathrm{K}=c_{T} / \ln \left\{\frac{\theta-\theta_{e}}{\theta_{1}-\theta_{e}}\right\}
$$

where $k$ is the heat transfer coefficient, $\theta$ is the metal temperature at time $T, \theta_{1}$ is the metal temperature at time $0, \theta_{e}$ is the ambient temperature which is taken as $22^{\circ} \mathrm{C}$ and C is the thermal capacity of the metal which may be calculated from

$$
c=m C_{p}:\left(\theta_{1}-\theta_{e}\right)
$$

where, $C_{p}$ is the specific heat for steel and $m$ is the mass of the bearing and pedestal metal.

The method employed to determine the correction was to take a cooling curve of the bearing pedestal immediately following the arrest of the fournal, and the stopping of the oil flow. Typical cooling curves are shown in Figure 6.21. These cooling curves show that the assumptions of Newton's law of cooling do not hold because the curvature of the cooling curve is opposite to that which is required by Newton's Law of cooling. Nevertheless, the end points of the cooling curves have been used for the estimation of $k$, and of the heat loss to the surroundings. The thermal capacity $C$ was taken to be the thermal capacity of the bearing and its pedestal and that portion of the


FIG.6.21 Variation of bearing metal temperature with the time.


#### Abstract

journal within the bearing. The cooling curves of Figure 6.21 on the above approximate basis gave heat fluxes of 0.397 and 0.584 KW respectively which are respectively $9 \%$ and $15 \%$ of the heat lost by way of the oil itself. These percentages are consistent with a value quoted in the literature, e.g. references 7 and 84 which give $15 \%$ as a correction factor for the heat lost by other means, and for the purpose of this investigation a uniform and conservative correction of $20 \%$ has been applied to the heat loss by way of the oil in arriving at the total power loss.


### 6.3 Tests and Test Results of the Externally Pressurized Operation

 The following results relate to the bearing being supplied only with high pressure oil via constant volume control valves, and with shaft rotation in the range 0 to 4000 rpm when the VIT 150011 is used and in the range 0 to 6000 rpm when Tellus 37 oil is used.
### 6.3.1 Static Stiffness <br> 6.3.1a Test with VIT 150 Oil

The salient dimensions of the journal and bearing used in the tests are given in Table 6.1. The first test carried out in the main test rig was to measure the hydrostatic gtiffness of the bearing, in much the same way as it had been measured in the static rig, and to compare the results with those previously obtained and which are reported in Chapter 4. A vertical load was applied to the shaft over-hang by the hydraulic ram and the displacement of the journal was measured at both ends, both by dial gauges and the inductive transducers which were mounted at positions $B-B$ and $D-D$. Figure 6.22 shows the measured displacement along the axial length of the journal, the displacements at the AFI, MID, and the FORE planes are deduced

|  | Journal Diame ter $d$ $m$ |  <br> m | Offset <br> $\Delta r$ |  |  |  |  |  | L/D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1.33 \mathrm{x} \times 10^{-1}$ | $1.349 \times 10^{-1}$ | - | $5.75 \times 10^{-5}$ | - | $2.6 \times 10^{-4}$ | $2.05 \times 10^{-4}$ | $145 \times 10^{-4}$ | 16 | 0.95 |

TABLE 6.1 Dimensions of the original test bearing.

from the measured displacements at $B$ and $D$. It may be seen from Figure 6.22 that the displacement varies along the axial length of the bearing. The measured displacement load relationships are shown by Figure 6.23. The magnitude of the displacements are plotted but the displacements of line AFT were in fact of opposite sign to those of lines $M I D$ and $F O R E ;$ the change in sign reflects the bending of the shaft itself. However, line MID, which is the true mean line has been used to derive a mean stiffness which is $1.125 \times 10^{9} \mathrm{Nm}^{-1}$, and which is substantially the stiffness as found in the static rig ( $1.4 \times 10^{9} \mathrm{Nm}^{-1}$ ). The journal did not tilt in the static rig, and consequently this substantial agreement shows that the tilting of the fournal in the bearing, which arose because of the bending of the shaft, has little effect upon the mean stiffness.

The journal displacement was measured in the vertical direction and the horizontal direction. The displacements in the horizontal plare at the AFT, MID, and FORE planes were deduced from the measured displacements, and from the disposition of the planes of measurement as shown by Figure 6.24.

However, for a stationary shaft, the displacement of the journal in the horizontal direction is negligible compared with that in the vertical direction. The results are shown in Figure 6.25 where the displacements are given in the form of a polar diagram.

### 6.3.1b Test with Tellus 37 Oil

The static stiffness of the bearing was measured in much the same way as it had been measured in paragraph 6.3.1a. The measured displacement load relationships are shown by Figure 6.26. Also Figure 6.26 shows the measured displacements which have been reported in paragraph 6.3.1a.


FIG. 6.23
Load-displacement relationship (experimental)


FIG.6. 24
Shaft displacement in the horizontal plane.


It may be seen that with the Shell VIT. 150 oil and at an operating temperature of $50^{\circ} \mathrm{C}(\mu \mathrm{m} 6 \mathrm{CP})$ the static stiffness at the journal mid-plane was $1.125 \times 10^{9} \mathrm{Nm}^{-1}$. By using the Tellus 37 oil and at an operating temperature of $65^{\circ} \mathrm{C}(\mu=13 \mathrm{Cp})$ the static stiffness at the journal mid-plane becomes $8.98 \times 10^{8} \mathrm{Nm}^{-1}$. However, the reduction in the oil viscosity by $80 \%$ leads to a reduction in the static stiffness by 20\%. Also it may be seen from Figure 6.26 that at operating temperatures between 35 and $42^{\circ} \mathrm{C}$ the static stiffness given by the low viscosity oil is much the same as that given by a high viscosity oil.

### 6.3.2. Static Deflection of the Test Shaft

The unexpected change in the sign of the deflection over the length of the bearing suggested that the deflection of the shaft should be measured at as many axial positions as were available. The measured displacements are shown by the circular points in Figure 6.27, and the points suggest a significant bending of the shaft under the moment and shear forces which result from the load. A computer programme* existed for calculating shaft displacements at speed under the influence of any number of out of balance forces and out of balance moments. The programme is based upon the transfer matrix method of Myklestadt and Prohl. This programme was adapted to calculate the bending of the static shaft by entering an insignificant speed of rotation. The result of the calculation is given by the dotted line of Figure 6.27. The tail bearing was assumed to be connected to ground by a spring of stiffness $3 \times 10^{8} \mathrm{Nm}^{-1}$, and the journal bearing was represented by 3 springs disposed along its length with stiffnesses adjusted so

[^3]

FIG. 6.26
Displacement-load relationship for externally pressurized operation (experimental).

that together their effect was equivalent to the measured hydrostatic stiffness of the bearing itself. It may be seen from Figure 6.27 that there is substantial agreement between observed and calculated displacement and it is concluded that the bending of the shaft makes a considerable contribution to the total deflection at the loading position. The programme has been used to investigate different shaft, tail and thrust bearing arrangements and the results are presented and discussed in Chapter 8. The computer programme itself with a sample of the output are presented in Appendix A8.
6.3.3 The Load Speed Relationship of the Externally Pressurized Operation
6.3.3a. Tests with VIT 150 Oil

With shaft rotation the journal displacement at both the bearing ends was measured by the inductive displacement transducers. The transducers were first mounted at positions $C-C$ and $D-D$ and then moved to positions $A-A$ and $B-B$ (Figures 6.17, 6.22 and 6.24). During each test the speed was brought to zero and a check upon the calibration of the inductive transducers against the dial gauges was made. The journal displacements at the AFT, MID and FORE plane positions were deduced from the measured displacements in their planes of measurements. The results are shown in Figures 6.28 and 6.29. The displacements at the AFT plane are not plotted because they were very small $\left(\sim 5 \times 10^{-6} \mathrm{~m}\right)$. The test results are shown in the form of polar diagrams in Figure 6.28. It may be seen from Figures 6.28 and 6.29 that the bearing displacement decreases as the speed increases and that the bearing attitude angle changes marginally as the speed was increased from 1000 to 4000 rpm.

### 6.3.3b Tests with Tellus 37 Oil

With shaft rotation the journal displacement was measured in much the same way as described in paragraph 6.3.a, except that in

the present measurements the maximum speed is 6000 rpm instead of 4000 rpm . The results of measurements are shown in Figure 6.30. It may be seen at 4000 rpm and with a radial force of $3 \times 10^{4} \mathrm{~N}$, that the radial displacement at the journal mid-plane is $2.65 \times 10^{-5} \mathrm{~m}$ (i.e. the bearing stiffness is $1.132 \times 10^{9} \mathrm{Nm}^{-1}$ ). The oil outlet temperature was $59^{\circ} \mathrm{C}$. In paragraph 6.3.3b (Figure 6.29) it has been shown that by using a high viscosity Oil the radial displacement at the journal mid-plane was $2.55 \times 10^{-5} \mathrm{~m}$ (i.e. the bearing stiffness is $1.176 \times 10^{9} \mathrm{Nm}^{-1}$ ). The oil outlet temperature was $72^{\circ} \mathrm{C}$. However, it may be seen from this example that with shaft rotation there is negligible difference between the stiffness obtained by using an oil of high viscosity and the stiffness obtained by using an oil of low viscosity, but it is quite clear that there is a big difference in the oil outlet temperature which favours the use of the low. viscosity oil. Also it may be seen from Figure 6.30 that the journal displacement decreases as the speed increases.

### 6.3.4 Power Loss, Oil Flow and Oil Temperature Rise for the Externally Pressurized Operation

6.3.4a Tests with the VIT 150 Oil

With a constant oil flow the variation of power loss and of ofl temperature rise with fournal speed are given respectively in Figures 6.31 and 6.32. Power loss and ofl temperature rise are given for various speeds as functions of ofl inlet temperature in Figure 6.33. All measurements of power loss have been derived from measurements of oil flow, and oil temperature rise. Corrections were made for the heating of the oil due to the fall in its pressure, and for heat losses to the surroundings. The pressure correction was made by measuring the temperare rise of the ofl through the bearing when the journal was stationary and this temperature rise was subsequently


FIG. 6.30
Displacement-speed characteristics for the externally pressurized operation.
deducted from the measured temperature rise when the journal was rotating. The measured temperature rise was approximately $4^{\circ} \mathrm{C}$ which is in substantial agreement with a predicted temperature rise of $5^{\circ} \mathrm{C}$ as calculated from the condition of constant total enthalpy of the oil. The correction for heat loss to the surroundings was found from cooling curves with the journal stationary and with no oil flow (paragraph 6.2.18). This correction was found to be $20 \%$ of the total heat loss, and has been applied in the results.

It may be seen from Figure 6.31 that there is no change in the slope of the $\log -\log$ plot of power loss versus speed up to 4000 rpm , which suggests no departure from laminar flow in the oil film. The plotted power loss is for the no load condition; limited tests have been carried out in the loaded condition and the results show that as the load increases there is a slight increase in the oil outlet temperature. In considering this trend it must be recalled that the oil flow was held constant.

The power loss-speed characteristics of Figure 6.31 is expressed by Hs $=3 \mathrm{~N}^{0.89}$. The index is lower than is usual, and the low value of the index is attributed to the volume flow of oil being held constant in contrast with the usual hydrodynamic circumstance in which the oil is freely available. An effect of the constant volume of ofl flow can also be seen in the trend of oil outlet temperature with load. The differences between these present results, and the corresponding results with full oil supply, are described and discussed more fully in paragraphs 6.3.6-6.4.22 following the presentation of the full flow results.



FIG. 6.32
Oil outlet temperature-speed characteristics for the externally pressurized operation.


FIG.6.33
Power loss, and oil temperature rise variation with the oil inlet temperature.

### 6.3.4b Tests with the Tellus 37 Oil

With a constant oil flow the variation of power loss and of oil temperature rise with journal speed are given in Figures 6.34 and 6.35. It may be seen from Figure 6.34 that there is no change in the slope of the log-log plot of power loss versus speeds up to 6000 rpm , which suggests no departure from the laminar flow in the oil film. At 6000 rpm the Reynolds' number for the bearing is approximately 486 while the critical Reynolds' number is $973^{*}$. There is some evidence that a departure from laminar conditions occurs at the critical Taylor number (Refs. $82-83$ ) and the linearity of the log-log plot against speed up to a speed of 6000 rpm is perhaps to be expected. If the onset of turbulence is delayed until the critical Taylor number is reached then the flow would not become turbulent before a speed of approximately 8400 rpm .

The power loss curves of Figure 6.34 were obtained without journal load. However, because the volume flow of oil is constant, any variation in power loss with journal load would become manifest through a change in oil temperature rise. Several measurements of oil temperature rise were made both with and without journal load and no influence of load upon temperature rise was found. However, it should be mentioned that at the maximum load the bearing eccentricity ratio ( $\varepsilon_{B}$ ) at the journal mid-plane was 0.3. It is concluded under conditions of externally pressurized operation up to a lobe eccentricity of 0.3 that the power loss is independent of the load.

The power loss speed characteristics of Figure 6.34 is expressed by Hs $=1.15 \mathrm{~N}^{1.212}$, where Hs is in KW and N is in rpm. Again, as was mentioned in paragraph 6.3.4a, the index is lower than is usual,

[^4]

FIG. 6.34
Power loss-speed characteristics for externally pressurized operation.
and the low value of the index is attributed to the volume flow of oil being held constant. It may be seen from Figure 6.34 that the measured power loss at 4000 rpm is 6.2 KW , while the measured power loss with the first oil was 10.3KW (Figure 6.31). (1.e. reduction of power loss by $\sim 40 \%$ ) resulting from the use of low viscosity oil. However, it must be recalled that the drop in the bearing stiffness was approximately 4\% at 4000rpm. Quite clearly there is very considerable advantage in the particular instances of this paragraph in the use of the low viscosity oil.

Figure 6.35 shows the variation of the oil temperature rise with the journal speed, while Figures 6.36 and 6.37 show the measured power loss and the measured oil temperature rise with two oils of different viscosities. It may be seen from Figures 6.36 and 6.37 that there is a considerable improvement in the measured power loss and the temperature rise by using low viscosity oil.

### 6.3.5 Load Speed Relationship in Hydrodynamic Operation <br> 6.3.5a Tests with VIT 150 Oil

The journal displacement with respect to the bearing was measured over a range of speeds from 1000 to 4000 rpm . Constant vertical loads of $5.4 \times 10^{3} ; 10.8 \times 10^{3}, 16.2 \times 10^{3}$ and $21.6 \times 10^{3} \mathrm{~N}$ were applied at the shaft overhang. The equivalent loads at the bearing mid-plane are $7.5 \times 10^{3}, 1.5 \times 10^{4}, 2.25 \times 10^{4}$ and $3 \times 10^{4} \mathrm{~N}$ respectively.

The procedure used in bringing the journal to the desired test conditions was to float the journal hydrostatically, to bring the shaft to speed, to admit low pressure oil, and then to switch off the
$0 i 1$ Temperature Rise, ${ }^{\circ} \mathrm{C}$


FIG.6. 35
Oil temperature rise-speed characteristics for the externally pressurized operation.


FIG. 6.36
Power loss-speed characteristics for the externally pressurized regime.


FIG.6.37
Oil temperature rise-speed characteristics
for the externally pressurized regime.
high pressure supply of oil. This sequence was followed in reverse order at the end of a test. It was noticed that there was little Change in journal attitude when the transfer was made from high pressure oil only to low pressure oil, and such changes as there were diminished as the speed of the journal was increased.

Table 6.2 lists the measurements of journal displacement together with the deduced values of journal displacement at the fore and midplane of the bearing. It may be seen that the fore-plane and midplane values are in substantially constant proportion, and may therefore be presented graphically simple by adopting appropriate scale. This is done in Figure 6.38 in which the journal radial displacement at the bearing mid-plane and at the bearing fore-plane is shown as a function of the external load at the journal mid-plane. It may be seen that the journal displacement decreased as the speed was increased (i.e. higher load carrying capacity). In Figure 6.39 the results of the measured displacements, load, speed and oil outlet temperature are plotted in a dimensionless form. The bearing duty parameter, which is based upon lobe dimensions, is calculated from $\frac{\mu N L D}{W}(D / C)^{2}$ and $W$ is load at the journal mid-plane and $\mu$ is the oil viscosity at the outlet. The results of the calculated duty parameter are plotted as a function of the kearing eccentricity ratio at both the fore and mid-plane of the bearing. In Figure 6.40 the duty parameter is plotted against the speed.

In Figure 6.41 the positior of the centre of the journal with relation to the centre of the bearing is shown. It may be seen at constant load


For notation see figures 6.21 and 6.23 .


FIG. 6.38
Load-displacement characteristics for the hydrodynamic operation (experimental)


TiG.6.39
Duty parameter variation with eccentricity ratio (experimental)


FIG. 6.40
Variation of the duty parameter with the Journal speed (experimental)
(1)

## that as the speed increases the attitude angle decreases.

### 6.3.5b Tests with Tellus 37011

The journal displacement with respect to the bearing was measured over a range of speeds from 1000 to 6000 rpm. The procedure used in these tests has been described in paragraph 6.3.5a. In Figure 6.42 the fournal radial displacement at the bearing mid-plane and at the bearing fore-plane is shown as a function of the journal speed. It may be seen at constant load of $7.5 \times 10^{3} \mathrm{~N}$, that the displacements at the journal mid-plane are $2.85 \times 10^{-5}, 2.4 \times 10^{-5}, 2.0 \times 10^{-5}$ and $1.75 \times 10^{-5} \mathrm{~m}$ at speeds of $10^{3}, 2 \times 10^{3}, 3 \times 10^{3}$ and $4 \times 10^{4} \mathrm{rpm}$ respectively. The corresponding stiffnesses are $2.63 \times 10^{8}, 3.125 \times 10^{8}, 3.75 \times 10^{8}$ and $4.286 \times 10^{8}$ $\mathrm{Nm}^{-1}$ respectively. The required stiffness of the functional specification is $3.51 \times 10^{8} \mathrm{Nm}^{-1}$ which can be obtained at journal speeds over 2000rpm. From the results which have been reported in paragraph 6.3.5a the displacement at the journal mid-plane was $2.44 \times 10^{-5}$, $1.75 \times 10^{-5}$ and $1.45 \times 10^{-5} \mathrm{~m}$ at journal speeds of 1000,3000 and 4000 rpm respectively. However it may be seen from these values that by using a low viscosity oil the journal displacement has been increased by

18\% (i.e. the bearing stiffness has been reduced by 18\%), but because the stiffness obtained with the low viscosity oil exceeds the stiffness required by the specification the fall in stiffness as the oil viscosity is decreased is of no consequence in the hydrodynamic performance of the bearing.

In Figure 6.43 and 6.44 the results of the measured displacements, load, speed and oil outlet temperature are plotted in a dimensionless


FIG. 6.42
Displacement speed characteristics for the selfgenerating regime.


FIG. 6.43
Variation of duty parameter with the eccentricity ratio for the self-generating operation.


FIG. 6.44
Duty parameter variation with speed for the self-generating regime.
form. The bearing duty parameter which is based upon lobe dimensions is calculated from $\frac{\mu N L D}{W}(D / C)^{2}$ and $W$ is load at the journal midplane and $\mu$ is the oil viscosity at the outlet.
6.3.6 Power Loss, Oil Flow and Temperature Rise in Hydrodynamic Operation with Low Pressure Oil only
6.3.6a Tests with VIT 150 Oil

The variation of power loss, oil flow and oil temperature rise with journal speed are presented in Figures 6.45 and 6.46.

In these tests oil flow was measured directly by bucket and stop watch; the flow meter was operating in an inaccurate region because of high oil viscosity and because of low flow.

The results of the measured power loss are shown in Figure 6.45 in which the circular points show the measured power loss as calculated from oil flow and temperature rise. The triangular points have been adjusted by $20 \%$ to take into account the heat loss from the rig. The power loss is plotted against speed on a log-log basis and the experimental points lie on a straight line. The dotted extrapolation indicates the power loss to be expected at high speeds for the same type of oil at the same inlet temperature. The exponent in the power loss-speed relationship is now 1.8 in contrast with the exponent of 0.89 which was found with a controlled volume flow of ofl (paragraph 6.3.4a Figure 6.31). Limited observations were made of the variation of the power loss with external load and the results showed that as the load increased the oil outlet temperature fell, oil flow increased, and the power loss also. For instance at a load


FIG. 6.45
Variation of the power loss with the Journal speed for the self-generating regime.
of $1.5 \times 10^{4} \mathrm{~N}$, a speed of 3000 rpm , the power loss increased by $13 \%$, over the power loss at zero load.

Figure 6.46 shows the variation of the oil outlet temperature and the oil flow with the journal speed. It may be seen that the oil flow is generally very low and that may explain why the oil outlet temperature in the hydrodynamic operation is higher than the oil outlet temperatures which were measured with controlled oil flow. For instance it may be deduced from figure 6.32 for which the oil flow was $0.19 \mathrm{Ls}^{-1}$ that at 2000 rpm the temperature rise is $14^{\circ} \mathrm{C}$ at an inlet temperature of $30^{\circ} \mathrm{C}$, while in the present instance the oil temperature rise is $30^{\circ} \mathrm{C}$ with an oil flow of $0.015 \mathrm{Ls}^{-1}$.
6.3.6b. Tests with Tellus 37 Oil.

The variation of Power Loss, Oil Flow and Oil Temperature rise with journal speed are presented in Figures 6.47 and 6.48. The tests have been carried out in the same way as explained in paragraph 6.3.6a, except that the oil flow has been measured by the flow meter.

The results of the measured power loss are shown in Figure 6.47 in which the circular points show the measured power loss as calculated from oil flow and temperature rise. The triangular points show the measured power loss which has been reported in paragraph 6.3.6a by using an oil of high viscosity. However, it may be seen that there is no difference in the power loss because as the oil viscosity decreased the oil flow is increased and the temperature rise is decreased. With high viscosity oil the oil flow is decreased and the temperature rise is increased but $Q x \Delta T$ is constant in both cases.


FIG.6.46
Variation of outlet temperature and oil flow with speed for hydrodynamic operation.


FIG.6.47
Power loss-speed characteristics for the selfgenerating regime.

This is in contrast with the externally pressurized instance with its constant volume flow in which a change in oil viscosity did lead to a change in power loss. Figure 6.48 shows the variation of the oil temperature rise and the oil flow with journal speed. It may be seen that the oil flow is generally low and the oil flow is increased by increasing the bearing load. The consequence of increasing the oil flow under loading on the temperature rise and power loss will be dealt with in the next section. Figure 6.49 shows the variation of the oil temperature rise and the oil flow with the journal speed. The results are plotted for the two types of oil which have been used in the tests. However, it may be seen that there is considerable difference in the oil flow and the oil temperature rise, and by using an oil of low viscosity, the oil flow increases and the oil temperature rise decreases.

### 6.4. Tests and Test Results with the Second Bearing.

After the investigation with the first bearing had been completed, a new bearing was machined with a larger lobe diametral clearance and with large axial grooves. This was done for two reasons, firstly to give a more comprehensive picture of the three-lobe bearings in general, together with a more adequate comparison of experimental with theory, and secondly to increase the oil flow in the hydrodynamic regime and to reduce the oil temperature rise so that it becomes more realistic to imagine the bearing being able to run at speeds somewhat higher than have been reached in the present tests.

The machining procedure of the second bearing is given in Appendix A4.


Journal Speed $\times 1 \overline{0}^{3}$, rpm

FIG. 6.48
Oil temperature rise and oil flow variation with speed for the self-generating regime.


FIG. 6.49
Oil flow and oil temperature rise variation with speed for the self-generating regime.

### 6.4.1 Tests and Test Results of the Externally Pressurized Operation The salient dimensions of the journal and bearing used in the tests are given in Table 6.3.

The following results relate to the bearing being supplied only with high pressure oil via the constant volume control valves and with shaft rotation in the range 0 to 7000 rpm . However, the high pressure Oil was supplied in the first instance at mid-lobe position as was the design intention, and secondly the high pressure oil, still via the control valves was supplied to the axial grooves between the pads which by design intention are the low pressure oil inlets.

Oil Shell Tellus 37 was used.

### 6.4.11 Load-displacement Relationship of the Externally Pressurized <br> Operation

In the first instance the static stiffness of the bearing and the load speed relationship was measured in much the same way as has been described in paragraphs 6.3.1a, $b$ and 6.3.2.a, $b$. In the second instance the same procedure was adapted, but of course with the change in the position of the oil supply.

Figure 6.50 shows the variation of the journal displacement with the external load at the journal mid-plane. The results obtained in paragraphs 6.3 .16 and 6.3 .3 b are also plotted in the same figure. Quite clearly Figure 6.50 shows the expected result
that as the lobe clearance increase:s the journal displacement increases. At zero speed the stiffness falls from $9.98 \times 10^{8} \mathrm{Nm}^{-1}$ to $8.33 \times 10^{8} \mathrm{Nm}^{-1}$. (The specifications of Table 1.1 calls for a stiffness of $8.0 \times 10^{8} \mathrm{Nm}^{-1}$ ). Also it. may be seen that at the journal mid-plane

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| Lobe <br> Diameter <br> D <br> mm | Shaf <br> Diameter <br> d <br> mm | mm | Off-set <br> $\Delta r$ <br> mm |  | Lobe <br> Diameter <br> Cl earance <br> C <br> mm | Max. <br> Clearance <br> C Max <br> mm | Min. <br> Clearance <br> C Min <br> mm |  | L/ ${ }_{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 134.01 | 133.685 | 133.905 | 0.0587 | 133.877 | 0.325 | 0.246 | 0.192 | 30 | 0.95 |



Variation of the Journal displacement with the external load for the externally pressurized operation.
and for a load of $3 \times 10^{4} \mathrm{~N}$, when the lobe clearance is increased from $2.6 \times 10^{-4} \mathrm{~m}$ to $3.25 \times 10^{-4} \mathrm{~m}$, the bearing stiffness falls from $1.13 \times 10^{9} \mathrm{Nm}^{-1}$ to $9.375 \times 10^{8} \mathrm{Nm}^{-1}$. In figure 6.51 the test results are shown in the form of polar diagrams. It may be seen that the bearing attitude angle changed marginally as the speed was increased from 3000 to 5000 rpm .

In the second instance a new feature of the bearing has been tested. In figure 6.52 the load-displacement relationship is shown and it may be seen that for the same oil flow and the same inlet pressure, the bearing stiffness (i.e, force/displacement) has improved by admitting the oil at the axial grooves. It should be mentioned that the axial groove for each lobe is of 30 degree width in the circumferential direction and does not break through at the ends of the bearing. The increase in stiffness is probably due to the inlet grooves acting as a pressure pocket. The disadvantage of this method of pressurization is that as the fournal speed is increased the pressure at the bearing inlet decreases and the oil flow must be increased to keep a positive pressure at the bearing inlets.

### 6.4.12 Power Loss, Oil Flow and Oil Temperature Rise for the Externally Pressurized Operation.

The power loss-speed characteristic is shown in figure 6.53. Lines $A$ and $B$ are both for the second bearing ( $C=3.25 \times 10^{-4} \mathrm{~m}$ ) but for different oil flows. It may be seen that the lines are substantially parallel. Line $C$ is for a lobe diametrical clearance of $2.6 \times 10^{-4} \mathrm{~m}$, (i.e, the original bearing). At low speeds the increase in the lobe clearance leads to a decrease in the shear power loss but as the speed increases line $A$ and line $C$ approach each other and at a speed of 6000 rpm the power loss of both lines becomes substantially the same.
Eccentricity Ratio, $\mathcal{E}_{B}$

(a) Mid-plane

> Second Bearing
> Tellus 37 oil
> $\mathrm{C}=3.25 \times 10^{-4} \mathrm{~m}$
> $\mathrm{P}_{\mathrm{s}}=1.36 \times 10^{7} \mathrm{Nm}^{-2}$
> $\mathrm{Q}=0.191 \mathrm{~s}^{-1}$
> $\mathrm{~T}_{1}=40{ }^{\circ} \mathrm{C}$
> $\mu=0.33$ poise



FIG. 6.52
Variation of the Journal displacement with the external load.


FIG.6.53
Variation of power loss with Journal speed (externally pressurized).

Also it may be seen for the same oil flow that the index of the power loss increases as the lobe clearance increases (lines A \& C). Figure 6.54 shows the oil temperature rise-speed characteristic. The characteristics of the original bearing is also shown, and it may be seen that the increase in the lobe clearance has produced a small decrease in the oil temperature rise. That the decrease in the oil temperature rise is only marginal is attributed to that the oil flow was held constant over the full speed range and the oil flow was also held constant for the lobe clearances which have been tested.

Figure 6.55 shows the power loss-speed characteristics for the two different ways in which the high pressure oil was admitted. The triangular points represent the measured power loss when the high pressure oil is admitted at the original high pressure inlets, while the circular points represent the power loss as measured when the high pressure oil is admitted at the axial grooves. It may be seen that the power loss is decreased when the high pressure oil is admitted to the axial grooves, but the difference in the power loss becomes insignificant as the rotational speed is increased. The decrease of the power loss may be understood from the following discussion. When the high pressure oil is admitted at the original high pressure inlets a large proportion of this relatively cold oil is forced to flow in the direction of the journal rotation and tends to decrease the oil film temperature. The decrease in the oil film temperature results in a higher viscosity and somewhat greater power loss. When the high pressure oil is admitted at the axial grooves, a great proportion of the oil escapes out of the bearing from both ends of the axial grooves, so that the oil film temperature becomes somewhat higher than in the previous instance, and the power loss becomes somewhat smaller than the power loss of the original method of pressurization.


FIG. 6.54
Oil temperature rise-speed characteristics
for the externally pressurized operation.


FIG. 6.55
Power loss speed characteristics for the externally pressurized operation.

The oil which escapes out of the axial grooves is mixed with the film oil and acts as a cooling media so that the measured oil outlet temperature In the second case is lower than that measured with the original method in which the oil was admitted. When the rotational speed of the journal increases a great proportion of the oil which is supplied at the axial grooves is forced to flow in the direction of rotation due to the pumping action of the journal and consequently as the speed increases the difference in the power loss decreases.

### 6.4.2 Tests and Test Results of the Self Generating Operation.

The following results relate to the bearing being supplied only with low pressure 011 and with shaft rotation in the range 1000 to 7000 rpm . 011 Shell Tellus 37 was used.

## 6.4 .21 <br> Load-Speed Characteristics of the Self Generating Operation.

The test results for the self generating operation are presented in figures 6.56-6.58. In figures 6.59-6.60 the results have been compared with the test results which have been reported in paragraph 6.3 and as expected the increase in the lobe clearance has resulted in an increase in the journal displacement and consequently the bearing stiffness is decreased.

Power Loss, Oil Temperature Rise and Oil Flow-Speed Characteristics.

The power loss, oil flow and oil temperature rise have been measured in much the same way as has been discussed in paragraph 6.3. Furthermore in the tests with the second bearing the relation between bearing load, power loss, oil temperature rise and oil flow has been investigated as well as the relation between the supply pressure and the oil flow. Also the pressure and the hydrodynamic oilflows have been measured.


Figure 6.56. Variation of the journal displacement with the speed (self generating).


Figure 6.57. Duty Parameter - Speed Characteristics for the self generating operation.


FIG.6.58
Variation of the duty parameter with the eccentricity ratio.


FIG. 6.59
Variation of Journal displacement with the Journal speed (self-generating).


FIG. 6.60
Variation of the duty parameter with the eccentricity ratio.

In figures 6.61-6.62, the measured power loss, oil temperature rise and oil flow are plotted versus the journal speed. In figure 6.63 the power loss, oil temperature rise and oil flow at no load of the original bearing are plotted with measured results of the second bearing at no load. It may be seen that the increase in the lobe clearance has increased the power loss marginally and has decreased the oil temperature rise. The decrease may be attributed to the increase in the oil flow.

### 6.4.23. Power Loss, Oil Temperature rise and Oil flow-load. Characteristics.

The variation of power loss, oil flow and oil temperature rise with journal load for the second bearing are shown in figures 6.646.65. Varying the journal load between zero and 22500 N at journal speeds of 3000 rpm and $4000 \mathrm{rpm} .$, resulted in a slight increase in the power loss with increase of the load. The oil flow increased, also the oil temperature rise decreased, but again started to increase as the load was increased over $1.5 \times 10^{4} \mathrm{~N}$.
6.4.24. Oil Flow-Inlet Pressure Characteristics.

The effect of varying the oil inlet pressure on the oil flow is shown in figure 6.66, for a journal speed of 4000 rpm and zero journal speed. The measurements were made with no load on the journal and with, a journal load of $1.5 \times 10^{4} \mathrm{~N}$. At no journal load, increasing the oil inlet pressure resulted in a substantial increase in the oil flow, and it is quite clear that the pressure flow is exceeding the hydrodynamic flow. At a journal load of $1.5 \times 10^{4} \mathrm{~N}$, increasing the oil inlet pressure resulted in a substantial increase


FIG. 6.61
Variation of the power loss with Journal speed (self-generating).


FIG.6.62
Variation of oil temperature rise and oil flow with the Journal speed.


FIG. 6.63 Variation of power loss, oil temperature rise, and oil flow with the Journal speed (self-generating).


FIG. 6.64
Power loss, oil temperature rise and oil
flow-load characteristics.


FIG. 6.65
Power loss, oil temperature rise, and oil flowload characteristics.

in the oil flow, but the hydrodynamic oil flow is also increased and becomes the predominant oil flow. In figure 6.6, the pressure flow at 4000 rpm was $0.038 \mathrm{ls}^{-1}$, while at zero speed the measured pressure flow was $0.025 \mathrm{ls}^{-1}$; however, it may be seen that the pressure flow increases as the speed and the oil temperature increases, as has been considered in chapter 3.

### 6.5. Discussion of Test Results

The discussion of test results which is given below relates principally to the tests which have been carried out on a 135 mm diameter x 127 m three lobe journal bearing which has a lobe clearance of 0.26 mm . and an offset ratio of 0.45 (the first bearing). The test results from the second bearing are introduced in the discussion of the effect of the lobe clearance on the bearing performance.

### 6.5.1. Externally Pressurized Operation with Stationary Shaft.

The tests which have been carried out with three lobe bearings and without shaft rotation have shown clearly that a stiffer spindle should be used if the capability of the bearing is to be realised fully. Despite the bending of the shaft the experimentally determined performance measured at the journal mid-plane, substantially matches or exceeds the functional specification of table 1.1; for instance the stiffness at the bearing mid-plane is $1.125 \times 10^{9} \mathrm{Nm}^{-1}$ (figure 6.23), which is substantially the stiffness as found in the static test rig (chapter 4), which exceeds the functional specification requirements for a stiffness in finishing of $8 \times 10^{8} \mathrm{Nm}^{-1}$ at low speed.

The above results were obtained with Shell VIT. 150 oil ( $v=150 \mathrm{CS}$. at $40^{\circ} \mathrm{C}$ ), but tests were carried out also with Shell Tellus 37 oil ( $v=37 \mathrm{CS}$ at $40^{\circ} \mathrm{C}$ ). With this lower viscosity oil the bearing stiffness at the bearing mid plane was found to be $9.98 \times 10^{8}$ (figure 6.26), which is approximately $88.6 \%$ of the stiffness with the first oil.

Increasing the lobe diameteral clearance from $2.6 \times 10^{-4} \mathrm{~m}$ to $3.25^{-4} \mathrm{~m}$ (the offset ratio decreased from 0.45 to 0.35 ) resulted in a decrease in the bearing stiffness from $8.98 \times 10^{8} \mathrm{Nm}^{-1}$ to $8.33 \times 10^{8} \mathrm{Nm}^{-1}$ (figures 6.26 and 6.50).
6.5.2. Externally Pressurized Operation with Rotating Shaft

Increasing the journal speed from zero to 4000 rpm resulted in a small increase in the bearing stiffness. For instance the bearing stiffness increased from $1.125 \times 10^{9} \mathrm{Nm}^{-1}$ at zero speed to $1.176 \times 10^{9}$ $\mathrm{Nm}^{-1}$ at 4000 rpm (figure 6.29). Also increasing the journal speed resulted in an increase in the power loss and in the oil temperature rise. At 4000 rpm the power loss was $10.3 \mathrm{~kW} .$, and the oil temperature rise was $32^{\circ} \mathrm{C}$ (the oil inlet temperature was $40^{\circ} \mathrm{C}-\mathrm{VIT}$. 150 oil).

Increasing the oil inlet temperature resulted in a decrease in the power loss and in an increase in the oil outlet temperature, whereas, by using a lower viscosity oil (Tellus 37) the power loss and the oil outlet temperature substantially decreased. At 4000 rpm the power loss decreased from 10.3 KW to 6.2 kW , and the oil temperature rise decreased from $32^{\circ} \mathrm{C}$ to $19^{\circ} \mathrm{C}$ (figures 6.36 and 6.37).

Increasing the lobe clearance resulted in a marginal decrease in the power loss particularly at low speeds but as the speed increases
the power loss is much the same as found with the original bearing. Also, increasing the lobe clearance has little effect on the oil temperature rise (figures 6.53-6.54).

### 6.5.3. Self Generating Operation

In the self generating regime and with Shell VIT. 150 oil, and at a speed of 4000 rpm and a load of $7.5 \times 10^{3} \mathrm{~N}$, the journal displacement at the bearing mid plane was found to be $1.45 \times 10^{-5} \mathrm{~m}$, which corresponds to a stiffness of $5.19 \times 10^{8} \mathrm{Nm}^{-1}$. Also at 4000 rpm and with no load the power loss was found to be 3.2 kW. , the oil flow $0.03 \ell \mathrm{~s}^{-1}$ and the oil temperature rise was found to be $57^{\circ} \mathrm{C}$.

By using a lower viscosity oil and at 4000 rpm , the journal displacement at the bearing mid-plane increased from $1.45 \times 10^{-5} \mathrm{~m}$ to $1.71 \times 10^{-5} \mathrm{~m}$ (the stiffness is descreased from $5.19 \times 10^{8}$ to $4.26 \times 10^{8} \mathrm{Nm}^{-1}$ Also, at 4000 rpm the oil temperature rise decreased from $57^{\circ} \mathrm{C}$ to $40^{\circ} \mathrm{C}$, but because the oil flow had increased from $0.03 \ell s^{-1}$ to $0.039 \ell s^{-1}$, the power loss is much the same as the power loss measured with the first oil.

Increasing the lobe diameteral clearance from $2.6 \times 10^{-4} \mathrm{~m}$ to $3.25 \times 10^{-4} \mathrm{~m}$ decreased the bearing stiffness from $4.26 \times 10^{8} \mathrm{Nm}^{-1}$ to $3.58 \times 10^{8} \mathrm{Nm}^{-1}$, the power loss was little affected, but the oil flow increased and at 4000 rpm the oil temperature rise had decreased from $40^{\circ} \mathrm{C}$ to $33^{\circ} \mathrm{C}$ (figures 6.56 to 6.63 ).

Varying the journal load between zero and $2.25 \times 10^{4} \mathrm{~N}$ at journal speeds of 3000 and 4000 rpm, resulted in an increase in the power loss with increase of journal load. The oil flow increased and the oil
temperature rise decreased initially with load, reaching a minimum at a load of $1.5 \times 10^{4} \mathrm{~N}$, then increased but still remained lower than the oil temperature rise at zero load (figures 6.64 and 6.65).

Increasing the oil inlet pressure resulted in a substantial increase in the oil flow. At no load the pressure flow represents a substantial proportion of the total oil flow, whereas at a load of $1.5 \times 10^{4} \mathrm{~N}$ the hydrodynamic oil flow became dominant.

The test results are summarized in table 6.4 , and the following salient factors are drawn from the table:
I. The maximum stiffness at zero speed is $1.125 \times 10^{9} \mathrm{Nm}^{-1}$ and the minimum stiffness is $8.98 \times 10^{8} \mathrm{Nm}^{-1}$.
II. In the externally pressurized regime and at 4000 rpm the maximum stiffness is $1.176 \times 10^{9} \mathrm{Nm}^{-1}$, the minimum stiffness is $9.375 \times 10^{8} \mathrm{Nm}^{-1}$, the maximum power loss is 10.3 Kw , the minimum is $5.5 \mathrm{Kw} .$, the maximum oil temperature rise is $32^{\circ} \mathrm{C}$ and the minimum is $17.5^{\circ} \mathrm{C}$.
III. In the self generating regime and at 4000 rpm , the maximum stiffness is $4.286 \times 10^{8} \mathrm{Nm}^{-1}$, the minimum is $3.21 \times 10^{8} \mathrm{Nm}^{-1}$, the power loss is 3 Kw ., the maximum oil temperature rise is $51^{\circ} \mathrm{C}$ and the minimum is $29^{\circ} \mathrm{C}$.
IV. At 6000 rpm and with externally pressurized operation the power loss, oil temperature rise and the bearing stiffness are $10.5 \mathrm{Kw} .$, $32^{\circ} \mathrm{C}$ and $1.39 \times 10^{9} \mathrm{Nm}^{-1}$ respectively, whereas with the selfgenerating operation these became $6.7 \mathrm{Kw} .46^{\circ} \mathrm{C}$ and $5.34 \times 10^{8} \mathrm{Nm}^{-1}$ respectively.
TABLE 6.4. Test Results and Functional Specification.

| Regime | Test Results |  |  |  |  |  |  | Functional <br> specification <br> stiffness $\mathrm{Nm}^{-1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Test * Condition | $\begin{aligned} & \text { Speed } \\ & \text { r.p.m. } \end{aligned}$ | Power Loss Kw. | $\begin{aligned} & \text { Oil flow } \\ & \ell s^{-1} \end{aligned}$ | Oil Temp. <br> Rise, ${ }^{\circ} \mathrm{C}$ | Load <br> N | $\begin{aligned} & \text { Stiffness } \\ & \mathrm{Nm}^{-1} \end{aligned}$ |  |
| External | A | 0 | (3) ** | 0.2 | 4 | $3 \times 10^{4}$ | $1.125 \times 10^{9}$ | $8 \times 10^{8}$ |
| Pressure | A | 4000 | 10.3(3) | " | 32 | $3 \times 10^{4}$ | $1.176 \times 10^{9}$ | " |
| " | B | 0 | - (3) | " | 4 | $3 \times 10^{4}$ | $8.98 \times 10^{8}$ | " |
| " | B | 4000 | 6.2 (3) | " | 19 | $3 \times 10^{4}$ | $1.132 \times 10^{9}$ | " |
| " | B | 6000 | 10.5(3) | " | 32 | " | $1.39 \times 10^{9}$ | " |
| " | C | 0 | - (3) | " | 4 | $3 \times 10^{4}$ | $8.33 \times 10^{8}$ | " |
| " | C | 4000 | 5.5 (3) | " | 17.5 | " | $9.375 \times 10^{8}$ | " |
| " | c | 6000 | 10 (3) | " | 30 | - | - | " |
| Self-Gen. | A | 2000 |  | 0.015 | 23 | $7.5 \times 10^{3}$ | $3.52 \times 10^{8}$ | $3.51 \times 10^{8}$ |

Test Results and Functional Specification.
TABLE 6.3. continued.

| Regime | Test Results |  |  |  |  |  |  | Functional <br> specification <br> stiffness $\mathrm{Nm}^{-1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Test * condition | Speed r.p.m. | Power loss Kw. | $\begin{aligned} & \text { Oil flow } \\ & \ell s^{-1} \end{aligned}$ | Oil Temp. <br> Rise. ${ }^{\circ} \mathrm{C}$ | $\begin{gathered} \text { Load } \\ \text { N } \end{gathered}$ | $\begin{aligned} & \text { Stiffness } \\ & \mathrm{Nm}^{-1} \end{aligned}$ |  |
| Self-Gen. | A | 4000 | 3 | 0.03 | 51 | $7.5 \times 10^{3}$ | $5.19 \times 10^{8}$ | $3.51 \times 10^{8}$ |
| " | B | 2000 | - | 0.029 | 20 | $7.5 \times 10^{3}$ | $3.125 \times 10^{8}$ | " |
| " | B | 4000 | 3 | 0.055 | 35 | " | $4.286 \times 10^{8}$ | " |
| " | B | 6000 | 6.7 | 0.088 | 46 | " | $5.34 \times 10^{8}$ | " |
| " | C | 2000 | 1.25 | 0.035 | 16 | " | $2.5 \times 10^{8}$ | " |
| " | C | 4000 | 3.7 | 0.075 | 29 | " | $3.21 \times 10^{8}$ | " |
| " | C | 6000 | 6.8 | 0.11 | 37 | " | $4.41 \times 10^{8}$ | " |
| " | c | 7000 | 8.6 | 0.125 | 39 | - | - | " |

Table 6.3. continued
** Test Condition
$D=135 \mathrm{~m}$
$\mathrm{L}=127 \mathrm{~mm}$
$C=0.26 \mathrm{~mm}$.
$\varepsilon_{0}=0.45$
$\alpha=16^{\circ}$

Oil: Shell VIT. 150.

Test Condition B: As above except that,
Oil: Shell Tellus 37.

Test Condition $C$ :

| $D=134 \mathrm{~mm}$ | $L=127 \mathrm{~mm} \quad C=0.325 \mathrm{~mm}$ |
| :--- | :--- |
| $\varepsilon_{0}=0.35$ | $\alpha=30^{\circ}$ |
| Oil: Shell Tellus 37. |  |

* Figure in brackets is the pumping power.


## CHAPTER 7

ON THE DYNAMIC CHARACTERISTICS OF THREE-LOBE
JOURNAL BEARINGS FOR SELF-GENERATING AND EXTERNALLY PRESSURIZATION.

## ON THE DYNAMIC CHARACTERISTICS OF THREE-LOBE JOURNAL BEARINGS FOR SELF-GENERATING AND EXTERNALLY PRESSURIZATION.

7.1. Introduction

Throughout Chapters 2 to 6, the experimental work and the theoretical analysis have been devoted to the steady state running of the three-lobe journal bearingsin both the self-generating and the externally pressurized regimes. Dynamic characteristics have not been calculated comprehensively nor have any tests been carried out. The object of this chapter is to outline how the dynamic characteristics might be calculated, and to comment upon the dynamic implications of such relevant information as has been gathered.

In this chapter a general discussion on the dynamic characteristics of journal bearings is briefly presented with special reference to the method used for the calculation of the dynamic coefficients. The dynamic characteristics of the control device,(i.e. constant volume flow valves) will also be considered. The way in which the computer programmes might be modified to yield the dynamic coefficients is outlined. In chapter 8, the required modifications of the test rig will be discussed.
7.2. Dynamic Characteristics of Journal Bearings.

One of the vibration problems which are associated with journal bearings is oil whirl. Oil whirl is a self excited vibration in which
the vibration is almost exactly half the journal rotation speed. Another type of the self excited vibration is known as oil whip (ref. 2) or as low-frequency whirl (ref. 10) which is a shaft vibration occurring at the natural frequency of the shaft when the rotating speed of the shaft is approximately two or more times the natural frequency. Oil whirl has been the subject of many investigations (references 71 to 80 ). These kinds of vibration are normally associated with high-speed lightly loaded bearings. The bearing dynamic characteristics play an important role in the stability of high speed machinery. However, stability is determined by both the dynamic characteristics of the rotating parts and by the dynamic characteristics of the journal bearings. Consequently, several bearing designs have been introduced to reduce and to delay the tendency for a shaft to whirl in a bearing. These designs include elliptical bearings, pressure bearings, longitudinal-groove bearings, pivoted-shoe bearings, nut cracker type bearings and the three-lobe bearings (reference 58). In all these designs the bearing bore is far from a complete circle and the surface is always interrupted by axial grooves, which may be used to admit the oil or to act as relief grooves.

In references (10-74-76) the dynamic characteristics of full circular bore journal bearings are presented. In the analysis, it is assumed that the dynamic characteristics with respect to a journal motion of a small amplitude can be represented by linear force displacement, and linear force velocity relations. It is assumed that the force displacement terms can be calculated, as for zero squeeze velocity, by differentia tion of the conventional steady running characteristics, and that the
force velocity terms can be calculated, as for zero displacement, from the squeeze effects in the film with boundaries fixed as for steady running conditions.

With this assumption and with reference to figure 7.1 the dynamic characteristics can be expressed in the dimensionless form.

$$
\begin{align*}
& P_{x}^{*}=a_{11} x^{*}+a_{12} y^{*}+b_{11}\left(x^{*} / w\right)+b_{12}\left(y^{*} / w\right)  \tag{7.1}\\
& P_{y}^{*}=a_{21} x^{*}+a_{22} y^{*}+b_{21}\left(x^{*} / w^{\prime}\right)+b_{22}\left(y^{*} / w\right) \tag{7.2}
\end{align*}
$$

where,

$$
\begin{aligned}
& P_{x}^{*}=\frac{P_{x}}{W}, P_{y}^{*}=\frac{P_{y}}{W} \\
& x^{*}=\frac{x}{C}, y^{*}=\frac{y}{C} \\
& x^{*} / W_{W}=\frac{\dot{x}}{C W}, Y^{*} /{ }_{w}=\frac{\dot{y}}{C W} \\
& a_{11}, \ldots \ldots \text { and } a_{22}: \text { are the displacement coefficients. } \\
& b_{11}, \ldots \ldots \text { and } b_{22}: \text { are the velocity coefficients } \\
& P_{x} \text { and } P_{y} \text {, are the disturbance forces. } \\
& W: \text { is the steady load } \\
& C: \text { is the lobe radial. clearance, }
\end{aligned}
$$

and
$w$ : is the angular velocity of the journal.

In references ( $10-74-76$ ) the displacement coefficients of the full circular bore bearings have been calculated from the steady running characteristics and the velocity coefficients have been calculated from the results which have been obtained by solving Reynolds' equation

which includes the squeeze term as will be explained below.

For the three-lobe bearing, it has been shown in chapter 3 that at higher values of eccentricity ratios the calculated attitude angle and duty parameter are nearly the same as for a circular bore bearing. However, because both the velocity coefficients and the displacement coefficients are functions of the attitude angle and the duty parameter, the dynamic characteristics may be expected to be the same as those calculated for a circular bore bearing. With smaller values of eccentricities, substantial pressure is created in both the bottom and side lobes with consequent effects on the attitude and duty parameter. In Chapter 3 it has also been shown that at small values of eccentricity there is substantial difference between the three-lobe bearings and the full circular bore bearings with respect to the attitude and the duty parameter and this difference increases as the offset ratio increases. In consequence, for a lightly load three-lobe bearing, the dynamic characteristics may be expected to differ from those calculated for a circular bore bearing.

In chapter 3, Reynolds' equation for the steady running condition of the three-lobe bearings has been solved. The attitude angle has been found for a vertical load where the results for different values of $L / D, \varepsilon_{0}, \varepsilon_{B}$ are tabulated in tables 3.4 and 3.5. Those results may be differentiated by a method similar to the method described in references (74-76) to find the displacement coefficients for the threelobe bearings.

For the calculation of the velocity coefficients Reynolds' equation, which must now include the squeeze term, may be written as

$$
\begin{equation*}
\partial / \partial x\left(\frac{h^{3}}{\mu} \cdot \frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^{3}}{\mu} \cdot \frac{\partial P}{\partial z}\right)=6 U \frac{d h}{d x}+12 \frac{d h}{d t} \tag{7.3}
\end{equation*}
$$

Equation (7.3) may be expressed in a dimensionless form and then this equation may be transformed into a finite difference form by the same procedure as has been described in chapter 3.

In reference (10), it has been pointed out that the solution of equation (7.3) for zero velocity and by using the conventional assumptions of boundary conditions which have been described in chapter 3, will give dynamic characteristics which are linear and which are consistent in their force-displacement terms with the terms obtained by differentiation of the steady-running characteristics of the bearing. When the squeeze term is not zero the conventional assumptions no longer define the actual boundary conditions, but the effect of using the conventional assumptions on the dynamic characteristics is, however, very small for perturbing velocities of small amplitude. However, equation (7.3) can be solved by using the same boundary conditions which have been described in chapter 3. The Computer programme itself can be used and the only modification required in the computer routine is to include the squeeze term in the main finite difference equation (equation 3.7).

The velocity coefficients may be calculated from the outputs of the solution of equation (7.3) by a differentiation procedure similar to that described in references (74-76). However, because
in Chapter 3 it has been shown that at small and moderate values of eccentricity the three-lobe bearings has attitude-duty parameter-stiffness characteristics better than that of circular bore bearings or of longitudinal groove bearings or of elliptical bearings, it may be expected that the three-lobe bearings will have improved dynamic and damping characteristics.

The same procedure can also be used with respect of equation (5.5) to calculate the dynamic characteristics of the three-lobe bearings with external pressurization.
7.3. Dynamic Characteristics of the Compensation Devices.

The performance of the three-lobe bearing as an externally pressurized bearing depends on the characteristics of the external compensation device. In the experimental work which has been described in chapters 4 and 6 , the bearing has been compensated by constant volume flow valves. Also in the analysis which has been described in chapter 5, the calculations were based upon a constant volume flow delivered to each lobe. Throughout the experimental investigations only a constant undirectional load has been applied on the test shaft. With a constant load it has been found that the control valves perform satisfactorily, and deliver a constant volume flow providing that there is a minimum pressure difference of $6.67 \times 10^{5} \mathrm{Nm}^{-2}$ (100 psi) across the valves.

If a machine tool instance is considered in which a milling cutter with 12 cutting edges is rotating at 1000 rpm , then the load on the bearing will change 12,000 times per minute so that the
pressure in the bearing lobes and at each high pressure inlet wilfluctuate at the same rate. In Appendix A7 it has been shown that the flow through the valve is kept constant by movement of a spool. However, when the pressure is fluctuating at 12,000 times a minute, the spools would also have to move at the same rate to keep the oil flow constant. The constant volume flow valves have not been subject to any dynamic loading so that the dynamic response of these valves is unknown yet. But, if the test results prove that these control devices do not perform satisfactorily under fluctuating load, the constant volume flow valves may be replaced by a constant volume flow pump which would be insensitive to the fluctuations of the load.

CHAPTER 8

Discussion, Conclusions and Recommendations.

### 8.1. Introduction

In Chapters 2, 3 and 5 the theoretical analysis of the three lobe bearing has been presented and discussed. The comparisons between the results of the theoretical analysis and the test results which have been reported in Chapters 4 and 6 were made when it was necessary to make an immediate comparison between theory and experiment. In this chapter a more comprehensive discussion and comparison will be made. Suggestions are made for future work. These include a re-designed system permitting a stiffer shaft to be used, bearings of modified geometry aimed at higher stiffness, lower power loss, and a lower oil temperature rise, and test and theoretical work to explore more thoroughly than has been possible in this thesis, the dynamic characteristics and stability of the bearing.

### 8.2. Externally Pressurized Operation.

The results of the approximate analysis which has been reported in Chapter 2 and the results of the numerical analysis which have been reported in Chapter 5, will be compared with the test results which have been reported in Chapters 4 and 6 .

Figure 8.1 shows the calculated total oil flow and the calculated lobe pressure as functions of lobe diameteral clearance. Line $A$ and line $C$ have been calculated from the results of the numerical solution (paragraph 5.8, Chapter 5), whereas lines $B$ and $D$ have been calculated from the approximate analysis (paragraph 2.11, Chapter 2). It may be seen that lines A and B are in substantial
agreement. However, this agreement is perhaps fortuitous because the approximate calculation in line $B$ includes both the circumferential flow as would be obtained in infinitely wide bearing together with a side leakage flow based upon very approximate assumption. In fact the oil flow should be the side leakage flow alone and on that basis the substantial agreement between the two curves would no longer exist. With reference to lines $C$ and $D$, both methods predict the lobe pressure correctly in order of magnitude but the more exact numerical solution shows a fall in lobe pressure with increased lobe diameteral clearance, whereas the approximate method has the opposite trend. Line $C$, the numerical solution, shows the trend also shown by the test results (figure 4.7, Chapter 4), and the trend which one would expect.

In table 8.1 calculated and measured flows are compared. Both the measured and calculated oil flows were carried out for an unloaded journal which is centred in the bearing. It may be seen that there is a good agreement between the measured and numerically calculated oil flow whereas the oil flow calculated by the approximate method is approximately 70\% of the measured oil flow.

Figure 8.2 shows the variation of the measured and calculated load factor with the bearing eccentricity ratio. The load factor has been calculated numerically in Chapter 5. At low values of eccentricity, there is a good agreement between test and theory but at higher values of eccentricity the theory predicts a somewhat higher load factor.

The variation of the load angle with the speed parameter is shown in figure 8.3. The solid line relates to the numerically


| Conditions of Measurement and calculation |  |  |  |  |  | $\begin{aligned} & \text { Measured } \\ & Q_{t} \\ & \ell_{s^{-1}} \end{aligned}$ | Calculated - Approx. |  | Calculated Numerical |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{\circ}$ 。 | $\mathrm{C} \times 10^{4}, \mathrm{M}$ | $\mathrm{P}_{\mathrm{s}}, \mathrm{NM}^{-2}$ | $\mathrm{P}, \mathrm{NM}^{-2}$ | P/Ps | $\mu$,Poise |  | $Q_{c} \ell^{-1}$ | $Q_{t} \ell^{-1}$ | $Q_{t} \ell^{-1}$ |
| 0.45 | 2.6 | $1.32 \times 10^{7}$ | $6.65 \times 10^{6}$ | 0.5 | 0.273 | 0.186 | 0.109 | 0.135 | 0.2 |
| " | " | $1.0 \times 10^{7}$ | $5.7 \times 10^{6}$ | 0.57 | 0.167 | 0.261 | 0.153 | 0.186 | 0.272 |
| " | " | $1.33 \times 10^{7}$ | $6.4 \times 10^{6}$ | 0.48 | 0.22 | 0.261 | 0.131 | 0.159 | 0.240 |
| " | " | $6.67 \times 10^{6}$ | $4.3 \times 10^{6}$ | 0.65 | 0.163 | 0.186 | 0.118 | 0.145 | 0.207 |
| " | " | 1. U $\times 10^{6}$ | $4.24 \times 10^{6}$ | 0.26 | 0.167 | 0.186 | 0.119 | 0.146 | 0.206 |




Figure 8.2. Load Factor - Bearing Eccentricity Ratio
calculated load angle $(\gamma)$ whereas the circular points relate to the measured load angle at the bearing mid plane. In deducing the the speed parameter from the test results, oil viscosity was taken as the viscosity at the oil outlet. Despite the assumptions which have been used in the numerical solution, the variation of journal displacement along the axial width of the bearing in the test, and the assumption that the effective viscosity is the viscosity at oil outlet, it may be seen from figure 8.3 , that there is a good agreement between test and theory.
8.3. Power Loss and Oil Temperature Rise in the Externally

Pressurized Operation.

The computer programme which has been described in Chapter 3 (the design programme), has been used to calculate the power loss and the oil temperature rise in the externally pressurized regime, simply by inserting the total oil flow rate and by setting the offset ratio to the required value.

The power loss and the oil temperature rise in the externally pressurized regime have been calculated by setting the total oil flow at $0.2 \ell s^{-1}$. This value has been chosen because it has been commonly used in the experimental work. In figure 8.4 the calculated power loss for the first bearing, together with the measured power loss as reported in Chapter 6 are shown as functions of journal speed. The circular points relate to the power loss as deduced from measurements of oil flow and of oil temperature rise, whereas the triangular points relate to the total power loss which has been found to be 1.2 x the deduced power loss (Chapter 6, paragraph 6.2.18). It may be seen that measured and calculated power losses are substantially the same.


Figure 8.3. Load Angle - Speed Parameter Characteristics.


Figure 8.5 shows the variation of both the calculated and measured oil temperature rise with the journal speed. The results are plotted for oil inlet temperatures of 30,40 and $50^{\circ} \mathrm{C}$. It may be seen that the maximum difference between measured and calculated oil temperature rise is approximately $\pm 3^{\circ} \mathrm{C}$, and a close agreement exists over the full speed range and for all the oil inlet temperatures.

Calculated and measured power loss and oil temperature rise for the second bearing are shown in figure 8.6. as functions of journal speed. The calculated and measured power losses are much the same up to a speed of 5000 rpm , whereas at speeds over 5000 rpm the calculation gives somewhat less power loss, but the maximum difference does not exceed 15\%.

In oraer to consider this difference between calculated anc measured power loss further, the Taylor's criterion for the onset of Taylor's vortices has been calculated for the bearing. The critical Reynolds' number is given by $\operatorname{Re}=41.1(D / C)^{\frac{1}{2}}$, where $41.1(D / C)^{\frac{1}{2}}$ is the onset of Taylor's vortices and pertains to a journal running centrally in a circular bore bearing. In the instance of the three-lobe bearing, the clearance to be taken in the calculation of Reynolds' number and the viscosity of the fluid which is to be taken have to be decided. The decisions made are that the clearance is the lobe clearance, and that the oil viscosity is the viscosity of the oil at outlet. The Reynold's numbers calculated in that way are shown as a function of journal speed in figure 8.7. On the same figure the Reynolds' numbers corresponding with Taylor's criterion are marked as transition Reynolds' numbers, and it may be seen that a transition is not to be



Figure 8.6. Variation of Power Loss and Oil Temperature Rise with journal speed.


Figure 8.7. Reynolds' Number - Journal Speed Characteristics.
expected at speeds below 7000 rpm. This is a speed higher than the speed at which the calculated power loss falls below the measured power loss, and it is concluded that the difference is not due to a transition from laminar to turbulent flow.

### 8.4. Self-generating Operation

Figure 8.8 shows a comparison between the measured duty parameter and the calculated duty parameter from the numerical solution. The results of the measured displacements, load, speed and oil outlet temperature are used to calculate the duty parameter (Chapter 6) and the duty parameter was plotted as a function of the bearing eccentricity ratio at both the fore and mid plane of the bearing. This was done because the bending of the shaft brings ambiguities into the eccentricity ratio pertaining to the test results. This has been dealt with by plotting the test results for the eccentricity ratios on both of the fore and mid plane. It may be seen from figure 8.8 that the calculated duty parameter is between the measured duty parameters. At low values of eccentricity the measured duty parameter at the bearing mid plane is close to the calculated duty parameter, whereas at higher values of eccentricities the duty parameter measured at the bearing fore plane becomes closer to the calculated line, but generally there is a reasonable agreement between test and theory.

In figure 8.9, measured and calculated attitude angle are shown as functions of the duty parameter. The calculated curve pertain to an offset ratio of 0.45 , the offset ratio of the test


Figure 8.8. Duty Parameter versus Eccentricity Ratio:


FIG. 8.9 Attitude Angle ( $\varnothing$ ) Versus Duty Parameter (s).
bearing, and the calculation was drawn from the results of numerical calculation (Table 3.5). The figures show a substantial difference between calculated and observed attitude angle which is due to the bend of the shaft, which increases the vertical displacement and consequently decreases the attitude angle.

Figure 8.10 shows the variation of the hydrodynamic oil flow coefficient with the duty parameter. The solid line relates to the numerically calculated flow coefficient whereas the points relate to the oil flow coefficient as found from the experiments. It may be seen that a close agreement is obtained between test and theory.

Figure 8.11 shows the variation of both the measured and the calculated stiffness with the journal speed. The measured stiffness at the bearing mid plane is reproduced from figure 6.42 , whereas the calculated stiffness is obtained from the design programe simply by running the programme for a journal displacement of $1.27 \times 10^{-5} \mathrm{~m}$ and for an offset ratio of 0.45 . The measured stiffness is somewhat greater than the stiffness as predicted by the design programme but the maximum difference between measured and calculated stiffness is approximately 20\%.

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FIG.8.10 Hydrodynamic Flow Coefficient Versus Duty Parameter


FIG.8.11 Stiffness Versus Journal Speed


Figure 8.12. Power Loss, Oil Temperature Rise and Oil Flow Versus Journal Speed.


FIG.8.13 Power Loss, Oil Temperature Rise and Oil Flow Versus Journal Speed.


#### Abstract

onset of Taylor's vortices (figure 8.14) indicatesthat the transition occurs at speeds exceeding the maximum test speed. However, there is an indication that transition may occur before the maximum speed of the specification ( 9000 rpm ) is reached.


In table 8.2 the measured and calculated performance parameters are listed with the type of agreement which has been achieved between test and theory. It may be seen that a close agreement has been achieved between test and theory for the parameters which have not been affected by the bend of the shaft whereas a reasonable agreement has been obtained when the test results have been affected by the bend of the shaft. A big difference between test and theory was only found with respect to the measured attitude angle.

### 8.5. Modification of the Test Spindle Assembly.

The design of the spindle used in the test was greatly influenced by the choice of rolling bearing elements for tail and for thrust. This choice has the advantages of cheapness and of minimum power loss, but the rolling elements capable of running up to 9000 rpm , at the required duty, demanded that the diameter of spindle should be substantially reduced outside of the main bearing itself. As has been shown in Chapter 6, the spindle itself bent considerably at the greatest test load and this bending has the effect of reducing spindle stiffness well below the inherent stiffness of the bearing itself, and also it has the effect of greatly complicating the interpretation of bearing behaviour with respect to such quantities as eccentricity ratio and attitude angle.


Fig314 Reynolds Number Versus Journal Speed.

## TABLE 8.2 Agreement Between Test and Theory

| Parameter | Selfgenerating | Externally <br> pressurized |
| :---: | :---: | :---: |
| Power loss <br> Oil Temperature Rise <br> Oil Flow <br> Eydrodynamic Oil Flow Coefficient <br> Total Oil Flow Coefficient (Chapter 6) <br> Pressure Oil Flow Coefficient (Chapter 6) <br> Stiffness <br> Load Factor <br> Duty Parameter <br> Attitude Angle <br> Load Angle <br> Journal Deflection (Chapter 6) |  |  |

* $/ \sqrt{c l o s e}$ agreement
/ Reasonable agreement
X Big difference

The existence of a computer programme for the calculation of the elastic bending of the shaft, and which has already been referred to in Chapter 6, makes it easy to consider the effects of various redesigns of shaft. The redesigns which have been considered are shown from $B$ to $D$ in figure 8.15 and the deflection of the redesigns, together with the actual test shaft which has been used (A) are given in figure 8.16. It can be seen that the significant matter is to maintain the diameter of the spindle at its full value right down to the tail bearing. In the machine tool context the problem then arises of differential thermal expansion of the shaft relative to the frame of the machine from the thrust face to the cutting tool. In design $D$ a sliding thrust bearing close to the main bearing is suggested.

### 8.6. Three Lobe Bearings Versus Circular Bore Bearings

The question always arises, is a multi lobe bearing necessary or would a bearing of plain circular bore suffice? To find an answer to this question a comprehensive study of the behaviour of journal bearings which have an offset ratio between zero (circular bore bearings and circular bore bearings with three axial grooves) and 0.8 (three lobe bearings) has been presented in paragraph 3.9 where it has been shown that for bearings of the same minimum clearance the three lobe bearings have the following advantages over the full circular bore bearings. The advantages are:-
I. Higher stiffness. This may be seen from figure 3.26 a in which the stiffness increases as the offset ratio is increased (an offset ratio for zero is a bearing of circular bore).

II. Great load capacity. It may be seen from Figure 3.25a that the load given at constant minimum film thickness increases with offset ratio.
III. Lower oil temperature rise. It may be seen from Figure 3.23 that the oil temperature decreases as the offset ratio increases.

The above advantages exist together with a power loss in the three lobe bearing which is substantially the same as the power loss of a bearing of circular bore (3.22). The power loss of the three lobe bearings becomes significantly greater than the power loss of circular bore bearings at offset ratios exceeding 0.6 .

### 8.7. Three lobe bearings for Very High Speed

It became of interest in another context to employ the analysis for a three lobe bearing to predict the behaviour of bearings for a journal of 5.08 cm (2") run up to speeds between $2 \times 10^{4} \mathrm{rpm}$ to $3 \times 10^{4} \mathrm{rpm}$. The results of calculation are presented in figure 8.17 which also includes a result for a bearing of circular bore $\left(\varepsilon_{0}=0\right)$. Quite clearly figure 8.17 shows that to achieve a higher bearing load with the minimum possible oil temperature rise, a three lobe bearing with high offset ratio has to be used.

### 8.8. CONCLUSION

The initial objective of the work described in this thesis. was to establish the feasibility of a wide speed range bearing which satisfies the specification given in table 1.1. With the exception


Fig.8.17 Power Loss, O.l Temp Rise \& Bearing Load Versus Journal Speed.
of the speed range of the test rig which has been restricted to 7000 rpm , instead of the 9000 rpm in the specification, feasibility has been established. Tests have been carried out with two bearings of different detailed design and with two oils of different viscosity. In general the preferred arrangement is the first bearing with its high offset ratio combined with the oil of lower viscosity, and with both high and low pressure oil inlets at the conjunction of the lobes as has been tested with the second bearing.

The test work has been paralleled by analytical and computational work which has been checked against the test results and also against published material, particularly upon bearings of circular bore. The computer programmes which have been developed have allowed predictiong to be made for a wider range of variablesthan it has been possible to test. A suggestion from this computer work is that the optimum three lobe bearing with respect of combining stiffness with minimum oil temperature rise and with minimum power loss would be with an offset ratio of 0.6 and with the least possible minimum clearance.

There are, of course, aspects both of the bearing as a bearing and as a bearing in a machine tool spindle context which have not been tested. The loading of the bearing has been limited to a steady load and the field of the dynamic performance of the bearing has not been investigated. The test speed was limited to 7000 rpm due to the use णf a belt drive, and it is suggested, particularly if higher speeds are to be attempted, that the final drive of the shaft should be in line. The fitting of a roller element thrust close to the main bearings has produced difficult shaft deflection. A fitting of a thrust in that position is necessary, and clearly much greater
consideration than has been possible should be given to the provision of a sliding element thrust which can be combined with maintenance of full spindle diameter back to the tail bearing. It is possible that the thrust could be combined with the main bearing by making the main bearing of double conical form. The test bearings have been made by elaborate machining and the elastic distortion method, which has only been briefly described, might be pursued. Clearly there is nothing sacrosanct about the precise way in which the profile of the bearings differ from the circle in the sense that the fourier components provided by the machining method might be replaced, equally effectively, by a different set provided by various means of elastic deformation. By adapting the computer programmes which have been developed, the characteristics of bearing profiles produced by more convenient routes could be predicted.

## REFERENCES

1. CAMERON, A.
2. PINKUS, O. and STERNLICHT, B.
3. FULLER, D.D.
4. BURWEL工, J.T.
5. BOYD, J., and ROBERTSON, B.P.
G. WILCOCK, D.F.
'Turbulence in high-speed journal bearings'. Trans. of the A.S.M.E., August 1950, (825-834).
'An experimental investigation of temperature
effects in journal bearings', Proc. of the Inst.
Mech.Engrs., Lubrication \& Wear, 2nd Convention
$(1957)$, p.l11.
'A Thermodynamic analysis of journal bearings', Proc. Inst. Mech. Engrs., 1966-1967, Vol. 181, (Pt. 30), (117-126).
'A theoretical and experimental investigation of the effect of elastic distortions on the performance of journal bearings', Proc. Inst.Mech.Engrs., 1967-68, Vol. 182, pt. 3N, (192-200).
6. SMITH, D.M. 'Journal bearings in turbo-machinery', Champion and Hall Ltd., 1969.
7. CAMERON, A.
8. FULLER D.D.
9. SHAW, M.C. and MACKS, E.F.
10. RAIMONDI, A.A. and BOYD, J.
11. MORI, $H$ and YABE, H.
'A theoretical investigation on hydrostatic bearing', Bull, J.SME. 6 (22), 1963, (354-363).
'Design of hydrostatic bearings', Machine Design, 35 (Dec.5, 1963), 158-162.
12. STANSFIELD, F.M.
13. OPTIZ, H.
14. ROWE,W.B., and O'DONOGHUE, J.P.
'Hydrostatic bearing design', Tribology 2,No. 1, (Feb. 1969), 25-71.
15. GHIGLIOZZA, R.M. MICHELINI, R.M. and ROSSI, L.
'Basic Lubrication Theory', Longman, 1971.
'Hydrostatic lubrication, Part II - oil lefts', Machine Design, July 1947, pp 49-52.
'Analysis and lubrication of bearings', 1949, 287, (McGraw-Hill, New York and London).
'An analysis of orifice and capillary compensated hydrostatic journal bearings', A.S.M.E. - A.S.L.E. Ft. Conf. Lubric. Baltimore, 1954 (October).
16. RIPPEI, H.C.
'The design of hydrostatic journal bearings', Proc. 8th Int. Mach. Tool Des. Res. Conf. (Sept. 1967), Pt. 1, 419,(Pergamon Press, Oxford).
'Pressure pad bearings', Conf. on lubrication and wear, Inst. Mech.Engrs. Session 2, Fluid Film Lub., paper 8, London, sep. 67.
'A design method for externally pressurized journal bearings', Tribology (August 1970), 168-174.
17. DAVIES, P.B.
'A general analysis of multi-recess hydrostatic journal bearings', Proc. Inst.Mech.Engrs., 184, (Part 1), 1969-1970, (827-838).
18. GHOSH, B.
'An exact analysis of a hydrostatic journal bearing with a large circumferential sill', Wear, 21 (1972), 367-375.
19. GHOSH, B. 'Load and flow characteristics of a capillary compensated hydrostatic journal bearing', Wear, 23 (1973), 377-386.
20. CUSANA, C., and CONRY, T.F.
21. CASTEILI, V., and SHAPIRO, W.
22. O'DONOGHUE, J.P. ROWE, W.B., and HOOKE, C.J.

O'DONOGHUE,J.P., ROWE, W.B., and HOOKE, C.J.
28. O'DONOGHUE, J.P., ROWE, W.B., and HOOKE, C.J.
'Design of Multi-Reces journal bearings for minimum total power loss', Trans. of A.S.M.E. Journal of Engineering for Industry, Feb. 1974.
'Improved method for numerical solution of the incompressible fluid film lubrication problem', Journal of lubrication technology, Trans. A.S.M.E., Series F, Vol.89, No. 2, April 1967, p.211-218.
'A comparison of four different computing methods for hydrostatic bearings', Tribology Convention, I.Mech.E., Brighton, May 1970.
'Computer Analysis of externally pressurized journal bearings', Inst. Mech.Engrs., Proc., Vol. 184, Pt.3L, 1967-1970, Tribology Convention, 1970, Paper 7, p. 48-53.
'A solution using the super-position technique for externally pressurized multi-recess journal bearings, including hydrodynamic effects', Proc., Inst. Mech. Engrs., 1970/71, 185.
29. ROSENBLATT, M., 'Oil flow, Key Factor in Sleeve Bearing and WILCOCK, D.F. Performance', Trans. A.S.M.E., 74, 849-865, (1952).
30. LOEB, A.M., and RIPPEL, H.C.
'The advantage of flow control valves over orifice and capillaries in compensating hydrostatic bearings', Journal of Franklin Institute, October 1957, pp 339-341.
31. MALONOSKI, A.B., 'The effect of the method of compensation on LOEB, A.M.
hydrostatic bearing stiffness', J. Basic Engng., Trans. AM Soc. Mech.Engrs. 1961, 83, 179.
32. MOHSIN, M.E. 'The use of controlled restrictions for compensating hydrostatic bearings', Advances in MTDR, 1963, (Pergamon Press, Oxford).
33. DE GAST, J.G.C. 'A new type of controlled restrictor for double film hydrostatic bearings and its application to high precision machine tools', Proc. of the 7th Int. MTDR. Conference, 1966, p.273.
34. MORSI, S.A. 'Tapered spool controller for pressurized oil film bearings', Proc. Inst. Mech.Engrs., Vol. 184 Pt.1, No. 21, (1969-1970).
35. O'DONOGHUE J.P., 'Design of inherently stable hydrostatic bearings', and HOOKE, C.J. Inst. of Mech.Eng., Tribology Convention, paper 23, Proc. Inst. Mech.Engrs. 1967-1968, 182 Pt. 3N.
36. O'DONOGHUE, J.P. 'Tapered lands hydrostatic oil bearings', and LEWIS, G.K.
37. MANAM, J., 'Tapered lands hydrostatic journal bearings', FOWLER, J.H., and CARPENTER, A.L.

Proc. Inst. Mech.Engrs. 1964-1965, Vol. 179, Pt. 3F, paper No. 22.
38. O'DONOGHUE, J.P. 'Compensation Methods for externally pressurized and ROWE, W.B. bearings', Engrs. Digest, 30, (1969), 49.
39. RIPPEL, H.C. 'Review of hydrostatic lubrication', Franklin Inst. sponsored seminar on computer aided design of hydrostatic bearing system, 1967 (13th December), London.
40. ROYIE, J.K., 'Application of Automatic Control to pressurized HOWARTH, R.B., and oil film bearings', Proc. Inst.Mech.Engrs. 176, CASELY-HAYFORD,E.L.

No. 22, (1962).
41. REDDECLIFFE, J.M., 'Hydrostatic bearings for cryogenic rocket and VOHR, J. engine turbo pumps', J.Lubrication Technology, Trans. A.M. Soc. Mech.Engrs. 1969, 91 (Series F), 557.
42. HAHN, R.S.
'Some advantages of hydrostatics in machine tools', ASLE annual meeting, Chicago, 26-28, May 1964.
43. ENNIS, B., and COWLEY, A.
'Applications of sealed hydrostatic bearing to the vertical slide of an horizontal drilling machine', Int. MTDR, (1964), 265-288.
44. RIPPEL, H.C. 'Role of hydrostatic bearings in manufacture'. Horizons in manufacturing seminar, Inst. of Science \& Technology, University of Michigan, Ann Arbor, 9 May, 1967.
45. ROWE, W.B.
46. RIPPEL, H.C., and FULLER, D.D.
'Experience with four types of grinding machine spindles', Proc. 8th MTDR Conference, University of Manchester, 12-5, Sept. 1967.

```
'Advantages offered by hydrostatic bearings in machine tool applications', Franklin Institute Research Laboratories, Report No. 32TR, 68-5, presented at ASME Design Engineering Conference, Chicago 1968.
```

'The practical use and advantages of externally pressurized liquid bearings', Proc. of Industrial lubrication and Tribology Symposium, London (1969).
48. MOHSIN, M.E., and HODGSON, B.
ROWE, W.B., and $O^{\prime} D O N O G H U E, J . P$.
'A hydrostatically lubricated ram for horizontal milling and boring machines', Proc. of 4 th Intn. MTDR Conf., September 1963, pp 111-138, Pergamon Press, 1964.
49. FAVARETO, M., and RAZELLI, G.
'Design of hydrostatic journal bearings under impulsive loads', I.M.T.D.R. (1974), 299-316.
'GAS Lubricated Bearings', Butterworths, London (1964).
51. DECKER, 0.
'Potential of gas lubricated bearings in ultra-
precision machine tool applications', Franklin
Inst. Research Laboratories, report No. $32 T \mathrm{R}$
68-7, presented at ASME Design Engineering Conf.,
Chicago, April 1968.
52. WUNCH, M.L., and NIMO, W.H.
'Industrial applications of gas bearings in the
U.K.', paper C.18, Conference on externally
pressurized bearings, London (November 1971).
53. KHER, A.K., and COWLEY, A.
'The design and performance characteristics of a capillary compensated hydrostatic journal bearings', Proc. 8th M.T.D.R. Conf., University of Manchester, 12-15 September, 1967.
54. DECKER, O., and 'Computer aided design of hydrostatic bearings for SHAPIRO, W.

DECKER, O., and SHAPIRO, W.
'Computer aided design of hydrostatic bearings for machine tool applications', Part II, Applications, Proc. of 9th I.M.T.D.R. Conf. Sept. 1968, Pergamon Press, 1969.
56. DAVIES, P.B., and ANDVIG, T.A.
'The Behaviour of a multi-recess hydrostatic journal bearing in the presence of severe shaft bending', presented at 9th Int. M.T.D.R. Conf., University of Birmingham, 1968.
57. PARSIEGLA, K.
'Hydrostatic bearings in heavy duty machine tools with high rotational speed', presented at 6th Conference on the "Optimization problems in design
and production of machines", held at INSTYTUT
KONSTUKCJI IEKSPLOR TACJI MASZYN i INSTYTUT
TECHNOLOGII BUDOWY MASZYN POLITECHNIKI WROCLAWSKIGJ,
POLAND, 21-25 September 1978.
58. WILCOCK, D.F. and BOOSER, E.R.
'Bearing design and application', McGrawHill Book Company, 1957.
'Bearings in machine tools', Publication No. $2580^{I I_{E}}$.
60. SKF
61. SHELDON, D.F., O'DONOGHUE, J.P.
'A hydrodynamic pocket journal bearing', Proc. Instn.Mech.Engrs. 1970, 184 (Part 3L), 70-74.
'slot-entry bearing for hydrodynamic and hydrostatic operation', Journal of Mech.Engr. Science, Vol. 18, Nov. 1976.
63. ROWE, W.B., KOSHAL, D., and STOUT, K.J.
64. SHINKLE, J.N. and HORNUNG, K.G.
65. OPTIZ, H., BOTTCHER, R., and EFFENBERGER, W.
'Frictional characteristics of liquid Hydrostatic journal bearings', Journal of Basic Engineering, Transactions of the ABME, March 1965, (163-169).
'Investigation of recessed hydrostatic and slot-entry journal bearings for hybrid hydrodynamic and hydrostatic operation'. Wear, 43, (1977), 55-69.

[^6]66. DAVIES, P.B., and LEONARD, R.
'Stability of multi-recess hydrostatic journal bearings', Journal of Mech. Engineering Science, Vol. 12, No. 2, 1970.
67. LEONARD, R., and ROWE, W.B.
'Dynamic force coefficient and the mechanism of instability in hydrostatic journal bearings', Wear, 23, (1973) 277-282.
'Analysis of Elliptical Bearings', Trans. ASME, Vol. 78, July 1956.
'Power loss in elliptical and 3-lobe bearings', Trans. ASME, Vol. 78, July 1956.
'Analysis and characteristics of three-lobe Bearings', Trans. ASME, Ser D.Vol. 79, March 1959.
71. POPE, A.W., and HEALY, S.P.,
'ANTI-vibration Journal Bearings', Proc. Inst. Mech.Engrs. 1966-1967, Vol. 181, Pt. 3B, 98-115.
72. GLIENICKE, J. 'Experimental investigation of the stiffness and damping coefficients of turbine bearings and their applications to instability prediction', Proc. Inst. Mech.Engrs., 1966-67, Vol. 181, Pt 3B, 116-129.
'Oil Whirl of Flexible Rotors', Proc. Inst. Mech. Engrs. Vol. 176, No. 22 (1962).
74. MORRISON, D. 'Influence of Plain journal bearings on the Whirling Action of an Elastic Rotor', Proc. Inst. Mech. Engrs., Vol. 176, No. 22 (1962).
75. HOLMES, R. 'Oil Whirl Characteristics of a Rigid Rotor in $360^{\circ}$ Journal Bearings', Proc. Inst. Mech. Engrs. Vol. 177, No. 11 (1963), 291-307.
76. SMITH, D.M.

```
'Dynamic Characteristics of Turbine Journal
Bearings'. Inst. Mech.Engrs. - Lubrication
and Wear Convention (1963), 72-86.
```

'Instability Phenomena due to Circular Bearing Oil Films'. Journal Mechanical Engineering Science, Vol. 8 No. 4 (1966), 419-425.
78. BONES, J.A. and HANNAM, R.G.
'Whirling of Shafts with Asymmetric Stiffness', Journal Mechanical Engineering Science, Vol. 8, No. 4 (1966), 437-447.
79. BADGLEY, R.H. and BOOKER, J.P.

```
'Turborotor Instability: Effect of initial
transients on Plane Motion'. Journal of
Lubrication Technology (October 1969), 625-633.
```

80. AKERS, A., MICHAELSON, S., and CAMERON, A.
```
'Stability Contours for a Whirling Finite Journal Bearing'. Trans. of ASME, Paper No. 70, Lubs. 3, (1970), 1-7.
```

81. HEATH, H.H., and COWKING, W.
'MELBA: A suite of computer programmes for analysis of self-acting plain bearings'. GEC journal of Science \& Technology, Vol. 45, No.2, (1979), 83-90.
82. PRIVATE COMMUNICATION 'Steady load and Dynamic Characteristics of a 19" dia. x 10" three-hand Bearing'.
83. DUFFIN, S., and JOHNSON, B.T.
'Some Experimental and Theoretical Studies of Journal Bearings for Large TurbineGenerator Sets'. Proc. Inst. Mech. Engrs. 1966-67, Vol. 181, Pt. 3B.
84. DOWSON, D., HUDSON, J.D., HUNTER, B., and MARCH, C.M.
'An Experimental Investigation of the
Thermal Equilibrium of Steadily loaded
Journal Bearings'. Proc. Inst. Mech. Engrs.
1966-1967, Vol. 181, Pt. 3B.
85. FORSYTHE, S.E. and WASON, W.R.
86. WILCOCK, D.F.
87. HAHN, H.W.
88. LOEB, A.M. and RIPPEL, H.C.
'Dynamically Loaded Bearings of Finite Length' 1957, I.Mech.E. Conf. on Lubrication and Wear, p.101.
'Determination of Optimum Proportions for Hydrostatic Bearings'. A.S.L.E. Trans., 1958.
'On the Optimization of the Stiffness of Externally Pressurized Bearings'. Journal of Basic Engineering, March 1962.
'Characteristics of an Externally Pressurised Bearing Having Variable External Flow Restrictions'. Trans. A.S.M.E. Ser.D, June 1963, p. 291.
'Journal Bearing Dynamic Characteristics Effect of Inertia of Lubricant'. Proc. Inst. Mech.Engrs. 1964-1965, Vol. 179, Pt.3j.
'Interface Restrictor Hydrostatic Bearing'. International Mach. Tool Des. Research \& Design, 1965.
89. HUGGINS, N.J.
90. NEALE, M.J.
91. SMALLEY, A.J., and MCCALLIN, H.
'Tests on 24-4n Diameter Journal Bearing:
Transition from Laminar to turbulent flow'. Proc. Inst. Mech.Engrs., 1966-1967, Vol.181, Pt. 3B.
'Selection of Bearings'. Proc. Inst. Mech. Engrs. 1967-68, Vol. 182, Pt. 3A.
'The Effect of Journal Misallignment on the Performance of a Journal Bearing under Steady Running Conditions'. Proc. Inst. Mech.Engrs. 1966-67, Vol. 181, Pt. 3B.
'The Influence of Viscosity Variation with. the Temperature on Journing Bearing Performance'. Proc. Inst. Mech.Engrs., 1966-67, Vol. 181, Pt.3B.
'Recent Developments in Fluid Film Lubrication Theory'. Proc. Inst. Mech.Engrs., 1967-68, Vol. 182, Pt. 3A.
'Load Capacity, Stiffness and flow characteristics of a hydrostatically lubricated six-pocket Journal Bearing supporting a rotating spindle'. Proc. Inst. Mech.Engrs., 1967-68, Vol. 182,Pt 3N.
'Solution for Flow in a Bearing Groove'. Proc. Inst. Mech.Engrs., 1967-68, Vol. 182, Pt. 3N.
92. COWLEY, A., and KHER, A.K.
93. GHIGLIAZZA, R., and MICHELINI, R.C.
94. O'DONOGHUE, J.P. ROWE, W.B., and HOOKE, C.J.
95. HELLER, S., and SHAPIRO, W.
96. O'DONOGHUE, J.P., ROWE, W.B., and HOOKE, C.J.
97. ROWE, W.B., $0^{\prime}$ DONOGHUE, J.P., and CAMERON, A.
98. KHER, A.K., and COWLEY, A.
'The Dynamic Characteristics of a
hydrostatically supported spindle
bearing system'. Advanced Mach. Tool
Des., Res., 1968.
'Friction in Externally Pressurized Journal Bearings'. Wear, 12, (1968) 241-256.
'Design of Hydrostatic Bearings using an Operating Parameter'. Wear, 14 (1969), 355-362.
'A numerical solution for the incompressible hybrid journal bearing with covation'. Journal of Lubrication Technology, July 1969.
'Some Tolerance effects in Hydrostatic Bearings'. Advanced Mach. Tools Des., Res., 1970.
```
'Optimization of Externally Pressurized Bearings for minimum power and low temperature rise.' Tribology, August 1970.
```

[^7]22. O'DONOGHUE, J.P., HOOKE, C.J., and COOK, P.J.
'Analysis of the squeeze film in a multi-recess hydrostatic journal bearing'. Inst. Mech. Engrs., (45-171).
'Diaphragm Valves for Controlling
Opposed Pad Hydrostatic Bearings'.
Proc. Inst. Mech. Engrs., 1970, Vol. 184,
Pt. 3L.
24. O'DONOGHUE, J.P.
25. ROWE, W.B., and STOUT, K.J.
26. STOUT, K.J. and ROWE, W.B.
'Viscosity variation in hydrostatic bearings. Triboloty, December 1972.
'Externally pressurized bearings design for manufacture'. Triboloty Int. June 1974.
27. STOUT, K.J. and ROWE, W.B.
'Externally Pressurized journal bearings for manufacture'. Tribology Int. October 1974
28. DIXON, R.S., and LENARD, R.
'Formulae defining the performance of hydrostatic journal bearings incorporating 3,4,5 and 6 recesses'. Int.Mach.Tool Des. Res., Vol. 14, pp 85-94, 1974.
29. ETTLES, C.
'The vane bearing: A self-pressurized journal bearing'. Proc. Inst.Mech.Engrs. Vo1. 189, 8/75.
30. ROHDE, S.M., and EZZAT, H.A.
'On the dynamic behaviour of hybrid journal bearings'. Journal of Lubrication Technology, June 1976.
31. GHOSH, M.K., and MAJUMDAR, B.C.
'Stiffness and damping characteristics
of hydrostatic multi-recessed oil
journal bearings'. Advanced Mach.Tool
Des. Res., Vol. 18, pp 139-151.

## APPENDIX <br> AI

A1. 1 A COMPUTER PROGRAMME TO EVALUATE THE PRESSURE, LOAD, FLOW AND THE STIFFNESS PARAMETERS

WFITE (3,80)


[^8]A.l.la. SAMPLE OF THE OUTPUT.
100 FORMAT ( $7 \mathrm{X}, \mathrm{F} 15.8, F 15,8, F 15,8, F 15,8, F 15,8, F 15,8, F 15,8$ )

al. 2 A COMPUTER PROGRamME TO CALCULATE the oil flow, SUPPLY PRESSURE, STIFFNESS AND PUMPING POWER
00006:
COMMON E1,E2,E3,S1,S2,S3,AK1, AK2,AK3,STIF1,STIF2,STIF3,ALFHA,OFFSE $2 \mathrm{~T}, \mathrm{CMIN}, \mathrm{EF}, \mathrm{E}, \mathrm{EL} 1, \mathrm{EL} 2, \mathrm{EL} 3, \mathrm{P} 1, \mathrm{P} 2, \mathrm{P} 3, \mathrm{FS}, \mathrm{FT}, \mathrm{QCT}, \mathrm{QAT}, \mathrm{QT}, \mathrm{QA}, \mathrm{QA} 2, \mathrm{QA} 3, \mathrm{STIF}$ $3 F N, H F, Q C 1, R C 2, Q C 3, C 11, C 22$, DUMY
$\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ WFITE(3,10)
THE CALCULATION OF THE OIL
2 FLOW RATES, SUFFLY FRESSURE, FAN PRESSURE/,94H
10 FORIMAT (96H
3 GEARING STIFFNESS,AND THE FUMFING FOWER FOR A THREE LORE
4JOURNAL/99H GESUR GEARINGS WITH EXTERNAL F SRESSURIZATION ANI SFECIFIEN LOAD,ECCENTFICITY/,45H
$\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ WRITE (3,20)
20 FORMAT (69H
$\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$
$\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ~$ $\mathrm{C}=8.0 / 39300.0$
10 $1 \quad 1=1,6$
C IYY IS THE JOURNAL IISFLACEMENT UNDER THE EXTERNAL LOAI
$\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ LIO $2 \mathrm{~J}=1,4$







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## ヨวงษyษヨาコ <br> FAIIIAL <br> 3HL

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 WFITE $(3,25)$ 25 FORMAT（ 119 H
C＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊
FT
B1 $\mathbf{I} \quad \mathbf{C} \quad$ Z

$$
\text { WRITE }(3,30) \mathrm{B}, \mathrm{B1}, \mathrm{I}, \mathrm{C}, \mathrm{Z}, \mathrm{~T}, \mathrm{E}, \mathrm{IIY}, \mathrm{FT}
$$

 C＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊ WRITE（3，40）
40 FORMAT（100H＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊

## ()0079: 2*********************************************)





| 00150： | CALL SAMTA |
| :---: | :---: |
| 00151：22220 | MUMY $=0.0$ |
| （0）152： | $\mathrm{FT}=\mathrm{FFT}+7000.0$ |
| $00153: 7$ | CONTINUE |
| 00154 | IIY $=$ IIYt（1．0／39300．0） |
| O） 055 | CONTTNUE |
| 00156 | $\mathrm{C}=\mathrm{C}+(1.0 / 39300.0)$ |
| 00157： 1 | CONTINUE |
| 00158： | STOF＇ |
| 00159 ： | ENa |
| 00160 ： | SUFFOUT INE HESHAM |
| 00161： | COMMON E1，E2，E3，S1，S2， 3 ，AK゙1，AK゙2，AK゙3，STIF1，STIF2，STIF3，ALFHA，OFFSE |
| $00162:$ |  |
| （0163： |  |
| 00164 ： | THETA1 $=60.0 * 3.14 / 180.0$ |
| $00165:$ | $A 1=-\mathrm{EL} 1+\mathrm{COS}$（THETA1） |
| 00166： | EE1 $=1.0-(E L 1 * C O S(T H E T A 1))$ |
| 00167： | IF（ELI．GT．1．0）GO TO 12121 |
| 00168： | GAMA1 $=A C O S(A 1 / E H 1)$ ． |
| 00169： | $\times 1=(1,0 \cdots$（ELI＊＊2）＊＊ 2 ， 5 |
| 00170 ： |  |
| 00171 ： | $X 3=2.0 * E L 1 * S I N(G A M A 1) ~$ |
| 00172 | $\times 4=(E L 1 * E L 1 / 2.0) * S I N(G A M A 1) * C O S(G A M A 1)$ |
| 00173： | $51=(1,0 / \times 1) *(\times 2+\times 3+\times 4)$ |
| 00174 ： | GAMA $=0.0$ |
| 00175： | $X$ S $=$ COS（GAMA $)+((E L 1 / 4.0) * \operatorname{COS}(2.0 * G A M A))$ |
| 00176 | $\times 6=\operatorname{COS}(G A M A 1)+((E L 1 / 4.0) * \operatorname{COS}(2.0 * G A M A 1))$ |
| 00177 | $x 7=(1.0-(E L 1 * * 2)) * * 0.5$ |
| 00178： | $E 1=(\times 5-\times 6) /(\times 2+\times 3+\times 4)) * \times 7$ |
| 00179 | THETA2 $=60.0$ \％3．14／180．0 |
| 00180： | X $9=$ THETA2＊（ $1.0+(1.5 *(E L 1 * * 2))$ ） |
| ）（181： |  |
| 00182 ； | $\times 99=$（EL． $1 * * 3) * S I N(T H E T A 2) ~$ |
| 00183： | $\times 10=0.75 *(E L 1 * * 2) * S I N(2.0 * T H E T A 2) ~$ |
| 00184 | $\mathrm{X} 11=0.3333 *$（ELI＊＊3）＊SIN（3．0＊THETA2） |
| 00185： | AK゙ $1=(\times 8-\times 9+\times 10+\times 11-\times 99)$ |


$\mathrm{x} 27=(1.0-(E L 2 * * 2)) * * 0.5$




the total oil flow fate for the bear



## A.1.2a. SAMPLE OF THE OUTPUT



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## APPENDIX A2

A2.1 A COMPUTER PROGRAMME FOR THE NUMERICAL SOLUTION OF REYNOLDS EQUATION


## $00001: J 0 \mathrm{~B}(\mathrm{GREM} 114, J 12, T 600, \mathrm{M} 7600)$

00002: MNF (T)
$00003 ; L \operatorname{DSET}$ ( $M A F=B / Z Z Z Z M F$, FFRESET=NGINF) GO.
00005
00006
FROGFAM S44(INFUT, OUTFUT, TAFE4=INFUT, TAFE $3=0 U T F U T)$
IIMENSION F1 $(30,52), F 11(30,52), F 2(30,52), F 22(30,52), F 3(30,52), F 33($ $230,52), \mathrm{H} 1(30,52), \mathrm{H} 2(30,52), \mathrm{H} 3(30,52)$

$00035: \mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ II IS THE BEAFIING LAIMETEF
C******************************************************************************
MuUu und
 $\begin{array}{lll}10 & 10 & J=1, N+1 \\ 10 & 10 & I=1, N+1\end{array}$ Fi $1(I, J)=0.0$
F11(I,J)=0.0
$F \cdot 2(I, J)=0.0$
$F 22(I, J)=0.0$
P $3(I, J)=0.0$
$F \cdot 33(I, J)=0,0$
$H 1(I, J)=0.0$
H2 (I.J)
JHNI.NOD
C******************************************************************************



WRITE 3,680$)$ DMEGA
,F8.4/, 45H**********
米

## MEGA

 $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$
ALLFHA IC THE ATTITULIE ANGLE
ALPHA $=35.0 * 3.14 / 180.0$ ALFHA $=35.0 * 3.14 / 180.0$
ALFHA $=A L F H A * 180.0 / 3.14$ WFITE ( 3,730 ) ALPHA
730 FORMAT(91H


$$
A L F H A=A L P H A * 3.14 / 180.0
$$

E IS THE EEAFING ECCENTFICITY FATIO (《BEAFING ECCENTRICITTY/LOBE FAIIIAL CLEARANCE))

## EL. 1, EL2, ANI EL 3 ARE THE IIFFERENT LOEES ECCENTRICITY RATIO ALFHA1, ALFHA?, ANI ALFHA3 ARE THE LOBESATTITUIIE ANGLES

 THETA1, THETA2, ANII THETA3 ARE THE ANGLES MEASUREII FFOM THE OIL INLETFOINT TO THE FOINT OF MINIMUM OIL FILM THICKNESS IN THE CONUEFGINCE
C************************************************************************** ELI $1=((E * * 2)+(E L I F I T * * 2)+(2.0 * E * E L I F I T * C O S(A L F H A))) * * 0.5$ ALFHA1 $=($ ASIN $((E * S I N(A L F H A) / E L 1))) * 180.0 / 3.14$ THETA1 $=50.0$ +ALFHA1
 414 AL.FHA2 $=-$ AL.FHAO THETA $2=50.0+120$ EL $3=((E * * 2+(E L I F I T * * 2)+(2.0 * E * E L I F I T * C O S((3.14 * 2.0 / 3.0)-A L F H A)$ 2))) **0.5 $20.0 / 3.14$
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 J) $* \operatorname{IIX} * \operatorname{II} * \operatorname{COS}(G 11)$


 응ㅇㅇㅇㅇㅇㅇ OO181:C***************************************************
$00182: \mathrm{C}$ SUMC11,SUMC 00183:C HIFECTION THE ATTITUTIE ANE
120 CONTINUF


$00215:$
$00216:$
$00217:$
$00218:$
00219
$00220:$
$00221:$
00222
00223
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$00225:$
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00229
00230
00231
00232
00233
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00238
00239
00240
00241
00242
00243
0
IF (DA.GT, EFS ) GO TO 92
SUMC2 $=0.0$
SUMC22 $=0.0$
SUMS2 0.0



(TE
$33=($ THETA3 $) * 3.14 / 180.0$
$33 \quad J=1, M+1$
G22 $=(180.0-$ THETA2 $) * 3.14 / 180.0$


[^9]
$73 \mathrm{FB}(1 ; J)=0.0$
10 0
 $\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ $\mathrm{F} 33(I, J)=F 3(I, J)$
FESII =ABS (PNEW3-F33(I,J))
IF ( $\mathrm{FESIII}, \mathrm{GT}, \mathrm{IA}$ ) DA=RESII
883 F3 $3(I, J)=$ FNEW3
103 CONTINUE **********



693 FBRAT

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00314: $\quad$ IF (J.GT. (THETA3/2.0).AND.F.3(I,J).LT. O. O) F3 (I, J) $=0.0$

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$00323:$
$00324:$
$00325:$
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SUMC $33=$ SUMC3 3 SUMC 33
SUMS33＝SUMS3＋SUMS33
SUMC3 $=0.0$
SUMS $=0.0$
＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊
FCos3＝SUMC33
$\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ $\qquad$ $\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ a 3 HL OL $7 甘 \cap 03$ SI 1 na ロ
c＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊ ALFHA $1=$ ALFHA $1 * 3,14 / 180.0$
AL＿FHA2 $=$ AL＿PHA2 $* 3+14 / 180.0$
AL＿PHA3＝AL＿FHA3＊3．14／180．
IF（FSIN1，EQ．O．O．ANI，FCOS1．EQ．O．O）GO TO 7171
FHI1＝ATAN（－FSIN1／FCOS1）

9898 IF (ALFHA3.GT.1.57) FHI 33=(ALFHA3-1.57-FHI3) $\mathrm{FH} 3=-\mathrm{FT} 3 * \operatorname{COS}(\mathrm{FHI} 33)$
 WFITE $(3,810)$ IYY, IIX
FDRMAT $25 H$

$$
\mathrm{IVY}=, F 15.8,45 \mathrm{H}
$$

$$
\begin{aligned}
& .8,45 \mathrm{H} \\
& * * * * * * * * * * * * * * * * * * * * * * * * * * \\
& * * * * * * * * * * * * * * * * * * * * * * * * * * *
\end{aligned}
$$

$$
0\lrcorner \quad 7 \forall 1 \mathrm{NOZI} \triangle O H
$$

$$
\begin{aligned}
& \text { THE ATTITUNE } \\
& * * * * * * * * * *
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$$
\begin{aligned}
& \text { 4**************************************************) } \\
& \text { WFIITE(3,5555)FHT }
\end{aligned}
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WFITE 3,6666 )FUT
FORMAT 59 H



FLOCO2 $=\mathrm{FLOCO}+((2.0 /(9,42)) *((\mathrm{~L} / \mathrm{FLOAT}(\mathrm{L})) * * 2) *(\mathrm{~F} 2(\mathrm{~N}, \mathrm{~J})) *((\mathrm{H} 2(\mathrm{~N}, \mathrm{~J})$
FLOCO3 $=\mathrm{FLOCO} 3+((2, \mathrm{O} /(9,42)) *((\mathrm{I} / \mathrm{FLOAT}(\mathrm{L})) * * 2) *(\mathrm{~F} 3(\mathrm{~N}, \mathrm{~J})) *((\mathrm{H} 3(\mathrm{~N}, \mathrm{~J})$ 2)**3))

## WFITE (3,7777)FLOCO1 7777 FOFMAT(75H

 an****)
***



 $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$
TFLOCO IS THE TOTAL 0IL FLOW COEFFICITNT
TFLOCO=FLOCO1 FLOCO2 FLLOCO 3
WRITE 3,7770 TFLOCO
WRITE $(3,7770)$ TFLOCO
7770 FORMAT 73 H
THE TOTAL FLOW COEF
THE SECONI LOBE FLOW COEFF
$* * * * * * * * * * * * * * * * * * * * * * * * * *$
THE THIFII LOBE FLOW COEFF

## 为

******~** (*****

$$
\begin{aligned}
& \begin{array}{l}
\text { 2FICIENT TFLOCO = F15. 8/, } 100 \text { H************************************* } \\
3 * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *) ~
\end{array} \\
& \text { WFiITE }(3,713)((\mathrm{Fi}(I, J) y, J=3, M+1,2) y I=2,14) \\
& \text { FORMAT (2X, 12(FO 5) } \\
& \begin{array}{l}
\text { C************* } \\
67676 \text { CONTINUE }
\end{array}
\end{aligned}
$$





```
The following subroutine can be used with the main computer
programme to solve for non-zero boundary conditions at the low
pressure inlets,. The solution is required to calculate the total
oil flow coefficient.
```


## $\mathrm{PF}=3.0$

no $50 \quad \mathrm{I}=3,27$
no $50 \mathrm{~J}=1,5,2$
no $50 \mathrm{~J}=1,5,2$
P1 $(1, J)=P F$
3nNIINOS
no $60 \mathrm{I}=3,27$
no $60 \mathrm{I}=3,27$
$\mathrm{no} 60 \mathrm{~J}=57,61,2$
$\mathrm{Fi}(\mathrm{I}, \mathrm{J})=\mathrm{FF}$
in 60 CONTINUE $\mathrm{nO} 100 \mathrm{~J}=3, \mathrm{M}, 2$
$\operatorname{IF}(\mathrm{I}, \mathrm{GT}, 1, \mathrm{ANH}, \mathrm{I} \cdot \mathrm{LT}, 28)$ GO TO 1515 GO TO 2525
IF (J.GT. O.ANI.J.LT . 6 ) GO TO 100
IF (J.GT. $56, A N D . J . L T . G 2) ~ G O ~ T O ~$
100 $0052 \quad \mathrm{I}=3 \mathrm{n} 27$
$1052 \mathrm{~J}=1,5,2$
F2 (I, J) $=\mathrm{FF}$
52 CONTINUE
1515

5 IF (J.GT.O.ANH.J.LT.6) GO TO 103

$$
5 \begin{aligned}
& \mathrm{FB}(\mathrm{I}, \mathrm{~J})=F \\
& \mathrm{CONTINUE}
\end{aligned}
$$

10 $63 \mathrm{I}=3,27$
100 63 J=53,61,2
F3 $3(1, J)=F F$
IF (J.GT.O.ANI.J.LT 6) GO TO 102
IF (J.GT. $56 . A N I+J . L T .62) ~ G O ~ T O ~$
102
[10 $53 \mathrm{I}=3,27$
$1062 \mathrm{I}=3,27$
$1062 \mathrm{~J}=57.61,2$
$F 2(I, J)=F \cdot F$
$\mathrm{nO} 53 \mathrm{~J}=1,5,2$
$\mathrm{~F} 3(\mathrm{I}, \mathrm{J})=\mathrm{FF}$
43
$M$
$N 7$

IF (J.GT.56.ANII.J.LT.62) GO TO 103
IF (J.GT.56.AND.J.LT.62) GO TO 103




 $00299:$
$00300:$
$00301:$
$00302:$
$00303:$
$00304:$
$00305:$
$00320:$
$00321:$

## A.2.la. SAMPLE OF THE OUTPUT.





 $=11.11101 y 51$



 Locol＝－11170ヶ111



＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊ －S114くりくら1）


ALPHAS IHETAI IHEIAC
1HETAC





 －\ll1／5б13
く－くつういノかりU







．101316

fitt IHIKG LUbt FLUW CUEFHICLENT FLOCUS＝


A2. 2 THE DESIGN PROGRAMME

- NULLL
00001: JOB (GREM114A, J9, T30.M7600) E
$\sum_{2}^{2}$
0
8
8
8
00003:LLSET (MAF $=$ E/ZZZZMF', FRESET $=$ ZERO $)$ LGO.
REAL LI,L,M,N,MEU1,MEU2, MEU11, MEU22
INTEGER $V$


… .t.0......+*+*
FROGRAM EEA (INFIT, OUTFUT, TAFE4:=INFUT, TAFE3=OUTFUT)
COMMON L, LI,F, U,EO,C,OFFSET,HF,FO,T1,FLOCO,MEU1,N,TO,MEU2,OD,OH, OT, 2CMIN, CMAX,CM, A11, A12, A21, A22,FHI, IYY,N1,F,HS,T2,FX1T,FY1T,FTOTAL,AL 3FHA, S, MEU11, MEUZ2
MECH. ENG. IEET.
W
$+$
$\llbracket$
6
M.
T.

$00004:$
00006
00007
00009
o
13 :
$\stackrel{-}{8}$
00017
00019
20
88
88

8
**...
$\frac{\text { TEMF }}{30.0}$

| F41 | TELLUS OIL |
| :---: | :---: |
| 1.4 | $-\frac{\mathrm{F} 33}{0.877}$ |

0
1
10
0
0

| $M$ | 0 |
| :---: | :---: |
| 8 | $M$ |
| 8 | 0 |



| ```CMIN=C-(2.0*OFFSET) CMAX=CMIN*(1.0-(EO/2.0))/(1.0-EO)``` |
| :---: |
|  |  |
|  |
| $E B=0.0$ |
| $\mathrm{QF}=0.0$ |
| $\mathrm{QH}=0.0$ |
| no $5 k=1,5$ |
| CALL TAREK3 |
| T $1=30.0$ |
| 10 $20 \mathrm{I}=1,3$ |
| IF (T1, EQ 30,0) MEU1=0.53 |

MEU1 IS THE OIL UISCOSITY AT THE OIL INLETS
IF (T1.EQ.40.0) MEU1 $=0.33$
IF (T1.EQ.50.0) MEU1 $=0.22$
$N=1000.0$
$N \quad$ IS THE SHAFT ROTATON SFEEEI
NO $30 \quad \mathrm{~J}=1,10$
MEU2=MEU11*( 2.73 )**(-ALFHA*(TO-35.0))

$\mathrm{FLOCO}=0.3+(0.5 * \mathrm{FO})$
$\mathrm{FLOCO}=0.115+(0.152 * \mathrm{FO})$
$\mathrm{FLOCO}=0.17+(0.1906 * \mathrm{FO})$



MEU2 IS THE OIL UISCOSITY AT THE OUTLET
TO IS THE OIL TEMFERATURE AT THE OUTLET
$M=(1.0 /((\mathrm{C} / 2.0) * * 3)) *((\mathrm{H} 1 * * 3)+(\mathrm{H} 2 * * 3)+(\mathrm{H} 3 * * 3))$ $\xrightarrow{M}$
$\mathrm{B} 2=(3.14 * \mathrm{~B}) / 22.5$

UU0


## $A=T 2-T O$

## IF (A.LE.2.O.ANH.A.GT.0) GO TO 9

 IF (A.LT, O.O.ANI.A.GE,-2.0) GO TO 9N
$2 I$

00 $\therefore$ 54)) DUTY $=2$
54)) DUTY $=2$

8 | 1 |
| :--- |
| 0 |
| 2 |
| 2 |
| 2 | N 124

 .54)) DUTY=0.6
aUTY $=0.73$
 nuTY=1.0 LUTY $=1.2$
 EO.EQ.0.45.ANI.CMIN.EQ.0.004*2.54 5 ( $0.004 * 2.54)$ - Z*100 0 - $\dot{0}$
 $\begin{aligned} & \text { IF (A.LT. O.O.ANH.A.GT. } \\ & \text { IF (T2.LT.TO) GO TO } 7 \\ & \text { IF (T2.GT.TO) GO TO } 11 \\ 9 & \text { IF (EO.EQ.O.O.ANL.CMIN }\end{aligned}$

[^10]

[^11]
30 CONTINUE $\qquad$
0
$\dot{0}$
+
$\vdots$
$\vdots$
$H$

## $\mathrm{EE}=\mathrm{EB}+0.2$

5 CONTINUE
 $23 \mathrm{X}, \mathrm{F} 6.2,3 \mathrm{X}, \mathrm{F} 6.2,3 \mathrm{X}, \mathrm{F} 6.2,2 \mathrm{X}, \mathrm{F} 15,2, \mathrm{~F} 9.1)$ RETURN
SUEFOUTINE TAFEKB CONTINUE
STOF
ENII
SUBROUTI
COMMON L
2CMIN,CMA
3FHA,SyST
R

COMMON L， $\mathrm{II}, \mathrm{F}, \mathrm{V}, \mathrm{EO}, \mathrm{C}, \mathrm{OFFSET}, \mathrm{HF}, \mathrm{FO}, \mathrm{T} 1, \mathrm{FLOCO}, \mathrm{MEU}, \mathrm{N}, \mathrm{TO}, \mathrm{MELZ}, \mathrm{QF}, \mathrm{QH}, \mathrm{QT}$, 2CMIN，CMAX，CM，A11，A12，A21，A22，FHI，IYY，N1，F，HS，T2，FX1T，FY1T，FTOTAL，AL 3FHA，SPSTIFN，LOAII
FEAL L，M，N，K，MEU1，MEUZ，LOAI WFITE 3,40 ）L
40 FDFMAT（ 100 H

WFITE 3,60 ）F
60 FORMAT $(100 \mathrm{H}$ WFITE $(3,80)$ EO
THE OFFSET RATIO
$E 0=, F 20.8)$
the diameter cleafiance
$\mathrm{C}=, \mathrm{F} 20.8$ ）
THE BEAFING DFFSET DISTANCE
OFFET $=$, F20．8）
THE HYIROIYNAMIC SIDE－LEAKAGE FLOCO $=$ ，F20．8）
$\begin{aligned} & \text { THE UISCOSITY CONSTANT } \\ & \text { ALFHA }=, F 20.8)\end{aligned}$ THE MINIMUM GIAMETERAL CLEAR
THE MAXIMUM DIAMETERAL CLEAFA
CMAX $=, F 20.8)$ CMAX $=$ ，F20．8）

$$
\begin{aligned}
& \text { THE BEARING WIDTH } \\
& \text { L }=, F 20.8)
\end{aligned}
$$

$\mathrm{I}=, \mathrm{F} 20.8$ ）
THE SUFFLY FRESSURE
$F=, F 20.8$ ）
THE BEAFING DFFSET IISTANCE
OFFET $=$, F20．8）
－EMax＝or20．a）
THE BEAKING WIITH
$L=, F 20.8)$

| $\left(8^{\circ} 0 \mathrm{c} \exists^{4}=5\right.$ <br>  |
| :---: |
| $\begin{aligned} & \text { (8.0ZJ }=03 \\ & \text { OII甘y } 1 \exists 5 \pm \pm 0 \text { \#H1. } \end{aligned}$ |
|  |
| $\text { <8.0z.a4 }=\\|$ <br> タヨ1ヨW甘II ONTY甘ヨG 3HI |


THE MEAN IIAMETRAL CLEARANCE
QH
LOAD $)$
$C M=, F 20.8)$
안
MEU2
STIFN
$\stackrel{\text { ¹ }}{\underline{\Sigma}}$
$z$
8
$\stackrel{\Gamma}{\overrightarrow{3}}$
$\overbrace{}^{\text {n }}$


A2.2a. SAMPLE OF THE OUTPUT FOR THE SELF-GENERATING OPERATION.

0
.04130147
.04150197
.01416000
.01289558 .01193600 LUAD
4004.6
 $n$
$\hat{n} \hat{n}$
0
$n$
$n$ $n$
$n$
$n$
0

0 | $n$ |
| :---: |
|  |
|  | $\begin{array}{ll}1 & \infty \\ n & n \\ n & n \\ 0 & 0\end{array}$ 2 2

0
0

0 | 0 |
| :---: | :---: |
| 0 |
| 0 |
| 4 |
| 4 | $n$

$n$
2
0
0 0
0
0
$n$
$n$




$$
\begin{array}{r}
5000.00 \\
6000.00 \\
7000.00 \\
8000.00 \\
4000.00 \\
10000.00 \\
11000.000 \\
1000.000 \\
2000.000 \\
5000.00 \\
4000.00 \\
5000.00 \\
8000.00 \\
7000.00 \\
0000.00 \\
9000.010 \\
110010.00 \\
11000.00
\end{array}
$$



$$
\begin{aligned}
& \begin{array}{l}
51=14 \\
45=05 \\
511.06 \\
50=20 \\
62=08 \\
68=93 \\
68=43 \\
10=01 \\
10.06 \\
25=11 \\
52=54 \\
54=15 \\
45=05 \\
52=55 \\
54=05 \\
65=114
\end{array}
\end{aligned}
$$

OUTLT ItMPERATUKE. IHE URERAIING VISCOSITY AND IHE SIIfFivesS FOR THE THREE LOBE BEARING
$\star \star$
12

| $\mathrm{L} .=$ | 12. (u0usubu |
| :---: | :---: |
| $\nu=$ |  |
| $P=$ | 1.buduouva |
| EO = | -45uviluou |
| C = | -1184\% |
| OFFET= | -004 15050 |
| FLOCO = | $u$ |
| ALPHA = | .U415U191 |
| CHIN = | - Ulutoulle |
| ChiAx $=$ | - U1451050 |
| CM = | . 11200104 |






5000.00
6000.00 2
0
0
0
0 8000.00 33
03
03
03 ？ 3.
0
30
0
20 M
 30
0
0
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0 5
2
0
0 5
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ヘッツッヘッn～～～～～～～～～～～


STIFトNESS

| L. $=$ | 12.10000000 |
| :---: | :---: |
| D $=$ | 15. לuounuou |
| $\mathrm{P}=$ | 1.5000000u |
| t0 = | - 70000000 |
| $\mathrm{C}=$ | . US506001 |
| OFFEI= | . 01185335 |
| FLOCO = |  |
| ALPHA $=$ | .04150197 |
| CHIN $=$ | - 01016000 |
| $C M A X=$ | -102015s5 |
| $C M=$ | - U1786467 |
| SIIFN | LOAD |




 $N$
$\underset{Z}{3}$
$-$

 $\sum_{\Perp} \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega \Omega n \mathfrak{n}$ $\sum$








A2.2b SAMPLE OF THE OUTPUT FOR THE

EXTERNALLY PRESSURIZED OPERATION.







## APPENDIX A. 3.

A.3.1. A COMPUTER PROGRAMME FOR THE NUMERICAL SOLUTION FOR THE THREE-LOBE EXTERNALLY PRESSURIZED JOURNAL BEARING .
"THE HIGH PRESSURE OIL IS ADMITTED AT THE CENTRE OF EACH LOBE".
18．15．36
PROGRAIA HYS（INPUI，OUIPUT，TAPEG＝INPUT，IAPES＝OUTPUI）
 COQLT，QST，FHTOFVI

INTEGER V
$E B=1,0$
00 $71717 \mathrm{~K}=1.10$
EPS＝U．JUS
$H A X I T=80$
$L=2$
$L=2$
$\mathrm{H}=18 \mathrm{U}$
ELIPIT＝U．4
UX＝3．14／FLUAI（M／く）
$N=20$

| ouj fo | 103 H |  |  |  |  |  | IHE | SULUIIOf： | OF | kernu |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CLDS | guvation | FUR | 1HE | EXte | ckivaluy |  | SSURI | ED／．90H |  |  |
| 3 | ThKEE |  | JE J | Ukival | DEARINGS | ， | ＋0k | eró Speti | CASt |  |
| 4 |  |  |  |  |  |  |  |  |  |  |


OOO WKIIE（3．606）
THE HIGH rRESSURE UIL IS sUrPLI
＊＊

> | $=$ |
| :--- |
| $\frac{1}{2}$ |
| $\vdots$ |

WRITE $(S O O S J) N$
FURMAI $(24 H$
SA）
WRITE $(3.01 U)$
O10 FORFAT（ 54 H

s1／0b／77 UNIVERSITY

0246076
0246150
0246156
0246328

33U FORMAT(91H
2 LISPLACEMENT ANGLE ALPHA = F1U.O/.10UH

2**************************************************)
EMEU=1.U-(( (S.14**2)*(4.U+(S.14*D/FLUA1(L))**2))/(C.U* ((FLOAT(M/2)

OMEGA=2*((1.U-(1.1)-EMEU**Z)**U.5)/(EMEU**2.U))

*********
ALPHA $=$ U.U
WRITE(S) 3U)ALPHA
4***************)
$4 \star \star \star * H A=A L \nmid H A * S .14 / 1811.0$
E=EB*(1.11-ELIPIT)

ALPHAT=(ASIN((E*SIN(ALPHA))/tLI))*181.0/S. 14
ELL=( (E**く+ELIPIT**2-2.U*E*ELIPIT*COS( (S.14/S.0)-ALHHA))**(U.b)
ALPHA

ALPHAS = ((2.U*S.14/S.U)-(ASIN(E*SIN((S.14*2.(J/S.U)-ALPHA)/ELS)))*16 ALPHA1
ELS
tLL
ELI
20.U/3.14
IHETA1=01). 11+ALPHA1
THETA $C=1 \varnothing 0.11$-ALPHAC
THETAS =ALPHAS-万0. 15
FORMAT(1101 $t$
040 FORMAT(11011


1rtTAS//.1U SUH**************************************************************** 4************************************)

 $G 1=(18 U . U-T H E T A 1) * S .14 / 18 U . U$
$003 U J=1 乌 1-1 \measuredangle 1$
$0020 \quad I=1, N+1$
$H 1(I, J)=(1 . U+E L 1 * \operatorname{CUS}(G 1)) / L . U$
$G 1=G 1+(2.0 * 3.14 / 18 U . U)$
$G 4=(18(J . J-I H E T A 1+6 U . U) * S .14 / 10 U . U$
$D 04 U J=1 . S 1$
$004 b I=1-N+1$
$H 1(I, J)=(1 . U+E L 1 \star \operatorname{COS}(G 4)) / 2.1 J$
CONTINUE
$G 4=64+(\angle$

$\begin{array}{ll}\circ & 0 \\ 0 & \text { - }\end{array}$



 (1)25355B $n$
$n$
$n$
$n$
$n$
$n$ 0255508 $n$
$n$
$n$
$n$
$n$

$n$ | $n$ |
| :--- |
| 0 |
| $n$ |
| $n$ |
| $n$ |
|  | 0255628 $n$

$n$
$n$
$n$
0 $n$
$n$
$n$
$n$
$n$
$n$ 0254046 025404 B 0254226 $n$
$\sim$
$\sim$
$\sim$
$\sim$
$\beth$
$\beth$ 0254528 025450 B 02546118 02546 ut 025476 s 025306́s 02553un 112554118 U25 46UB uくら57us U25 5711n 02560615 $n 1$
0
0
0
0
0
0
0


 0261138 11261108 026127 B 026135 H 026144 t 02617 (1) 02617 nts 0261778 0262018 0202038 $026<14 B$ 026214 H 026217 B | $n$ |
| :---: |
|  |
|  |
|  | $020262 B$ $026262 B$ 026263B 1026265b (12053uts 026532 H 0203326 $n$

0
$n$
$n$
$n$
$\sim$

$\underset{c}{n}$ | $n$ |
| :--- |
| 0 |
| $n$ |
| $n$ |
| 0 |
| $\vdots$ |
|  | 2

$\vdots$
0
0
0

$\vdots$ 026401 t 020404 H | 1 |
| :--- |
|  |
|  |
| 0 |
| 0 |
| 0 |
| 0 | | $n$ |
| :--- |
|  |
| 0 |
| 0 |
| $\vdots$ | $n$

0
0
0
0
0

CALL HESHAM



|  |
| :---: |
| 0261006 |
| 02670 St |
| 026/006 |
| 126\%15B |
| 0<612UB |
| 0267208 |
| 0261203 |
| 026122 B |
| 026/24 |
| 026125B |
| 0267258 |
| 026726 B |
| 026731 B |
| 026135B |
| 0)261466 |
| 026764 E |
| 026773日 |
| 027001 |
| 02700113 |
| 0210051 |
| 02700) |
| 027U0/8 |
| U27ucirb |
| $02700 \%$ - |
| 027L1SH |
| 127014 B |
|  |
|  |


 $0 \star * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)$ WRITE $S$, （116）
sumsi3）
 $\stackrel{0}{2}$
v






COMTINIE
$F \cos t=\sin \cos$
FSLHA＝SUFStst
$+\cos 1=F \cos A++\operatorname{costs}$
FSIM1＝FSIWH＋FSI：A
WHITE（S．（2いて）FSINIot LOS 1
FURMAT（F12．b）
HHIIE（S．（US）

> -

シ

253．
1270156
112701） 3 0270163 Li／7U1／$H$ 02702．10 fく／u2ひは いて7いてbo 027い3ub 0． 2711416 027ば16 127u7uts
121174 11271745 0くないての （1271111s $\underset{y}{v}$
$\underset{\sim}{v}$
$\underline{y}$ い2716iくn いく，10んn ij 2710 OH ＂く71010 $112711 \mathrm{us}^{2}$ li27112 10． 1114 s


027212 b
259
261
261
202
263
204
265
266
208
269
210
27
27

$$
\begin{aligned}
& \text { SUMSて二SUmSく+P1(IfJ)*DX*DY*SIN(G22) } \\
& \text { CONTINUE } \\
& \text { WRITE } 3,912) \mathrm{J} \text {, SUIFCL, SUMS } 2 \\
& \text { FORHAT ( } 2 x \text { X, } 14,25 X, F 1 U .1 \text {. } \\
& \text { F } 2=\mathrm{G} 22+(4,0 * 5.14 / 18(1.0) \\
& \text { SUIAC } 22=\text { SUMC } 2+\text { SUMC } 22 \\
& \text { SUMS } 22=\text { SUMS } 2+\text { SUMS } 22 \\
& \text { SUMC } 2=11.11 \\
& \text { SUMS }
\end{aligned}
$$

$\begin{aligned} & \text { +COS } 2=\text { SUMIC } 22 \\ & \text { FSIN }=\text { SUMS } 22\end{aligned}$
WKIIE (3.12L) +COSCOFSINL






PHI1＝ATAN（－FSIN1／＋COS1）



|  | GOTO 4141 |
| :---: | :---: |
| 7171 | PHI $1=0 . \mathrm{U}$ |
| 4141 | IF（PHI1．GT．ALPHAI）G0 109191 |
|  | GO TO 9292 |
| 9191 | PHI11＝（ $\mathrm{PH} 11-\mathrm{ALPHA1}$ ） |
|  | FH1 $=\mathrm{F}$ T1＊SIN（ H H111） |
|  |  |
|  | GO TO 9393 |
| 9292 | IF（PHI1．LT．ALPHA1）PHI11＝（ALPHA1－HHI1） |
|  | FH1＝－FT1＊SIN（HH111） |
|  | FV1＝FT1＊COS（PH111） |
| 9393 | FT2＝SURT（（FCOSく＊＊2）＋（FSIN2＊＊2）） |
|  | IF（FSINZ．EQ．U．U．AND．FCOS2．EW．U．U）GO TO 1212 |
|  | PHI2＝ATAN（－FSIN2／FCOS |
|  | GO TO 4242 |
| 7272 | PH12＝0．0 |
| 4242 | IF（ALPHAC．GT．1．ST）GO TO 4494 |
|  | G0 10 YSY5 |
| 9494 | PHI22＝（AL．PHA＜－1． $37+$ HHIC） |
|  | FH2＝ F T $2 \star \operatorname{COS}(1+H I 2<)$ |
|  | FVC＝－FT2＊S1H（PHIく2） |
|  | GO TO 9096 |
| 9393 | IF（ALPHAく．LT．1．5才）PHI2C＝（1．S7－ALPHAく－PHIC） |
|  | FH2 F F 2 2＊COS（PHI22） |
|  | FVL＝＋T $2 *$ S IN（HH12く） |
| 9090 |  |
|  |  |
|  | PHI3＝ATAN（ - FSIN3／FCOSS） |
|  | GO 104543 |
| 1575 | PHI $3=0.0$ |
| 4545 | IF（ALPHAS．LT．1．51）G0 10 9／91 |
|  | GU 10 Y®y8 |
| Y／91 | PHISS $=(1 . S(-A L P H A S+P H I S)$ |




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## WRITE（S．25O）


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APPENDIX A.3.1a

SAMPLE OF THE OUTPUT

HIGHPKESSUKE GRUOVES

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$S_{P}=0.03$



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$S_{P}=0.03$

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# A.3.2. A COMPUTER PROGRAMME FOR THE NUMERICAL SOLUTION FOR THE THREE-LOBE JOURNAL BEARING . 

"THE hIGH PRESSURE OIL IS ADMITTED AT THE CONJUNCTION OF the lobes (i.e. At the axial grooves)."
等



[^13]

[^14]



GO TO 2


[^15][10 $1005 \mathrm{I}=1, \mathrm{~N}+1$
QC201 $=\mathrm{QC} 201+\langle\mathrm{FL}$

| $2 \mathrm{I}, 33$ ) |  |
| :---: | :---: |
| 1005 CONTINUE |  |
| QC.203 $=0.0$ |  |
| [10 1006 $I=1, N+1$ |  |
| $21(I ; 149))$ |  |
|  |  |
| 1006 CONTINUE |  |
| $\mathrm{QC2}=\mathrm{QC2O1+0C203}$ |  |
| QC102=0.0 |  |
| [10 $1007 \mathrm{I}=1, \mathrm{~N}+1$ |  |
| QC102 $=$ QC102+(FLOAT (L)/H)* |  |


$\mathrm{HO} 1008 \mathrm{I}=1, \mathrm{~N}+1$
$\mathrm{QC} 103=\mathrm{aC} 103+(\mathrm{FLOAT}(\mathrm{L}) / \mathrm{II}) *(\mathrm{IYY} /(2.0 * \mathrm{IIX})) *(\mathrm{H} 1(\mathrm{I}, 91) * * 3) *(\mathrm{~F} 1(\mathrm{I}, 89)-\mathrm{F} 1($
21:93)
GONIINOJ 8001

21.8NTINUE
$\mathrm{aC} 302=0.0$
$\operatorname{LO} 1010 \mathrm{I}=1, \mathrm{~N}+1$
$\mathrm{QC} 302=\mathrm{QC} 302+(\mathrm{FLOAT}(\mathrm{L}) / \mathrm{II}) *(\mathrm{IY} /(2.0 * \mathrm{IIX})) *(\mathrm{H} 1(\mathrm{I}, 151) * * 3) *(\mathrm{~F} 1(\mathrm{I}, 149)-\mathrm{F}$. 21(I, 153))
CONTINUE
QC $3=Q C 302+Q C 301$
QA2O1=0.0
10 $1000 \quad \mathrm{~J}=3,31,2$






SUMS1=SUMS1+F1 (I, J) *IIX*LIY*SIN(G11) CONTINUE
WFITE $(3,910)$ J, SUMC 1, SUMS 1
910 FOFMAT $28 X, 12,25 X, F 10.7,23 X$
910 FOKMAT $(28 X, 12,25 X, F 10.7,23 X, F 10.7)$ $\mathrm{G} 11=\mathrm{G} 11+(4.0 * 3.14 / 180.0)$
SUMC11=SUMC1+SUMC11
SUMS $11=$ SUMS $1+$ STUMS11
SUMC1 $=0.0$
110 SUMS1=0.0
FSIN $=$ SUMS1
WRITE 3,720$)$ FCOS1,FSIN1
720 FORMAT (77HTHE TOTAL HYDIR FORCE IN TH ************ HYLROSTATIC E LINE ************************
ITUL A*******
Jums2) 6********************************)
WRITE (3,702)
702 FORMAT (95H











## $\mathrm{FHI} 33=(1,57$-ALFHA3 + FHI3) <br> 9797

$\mathrm{FH} 3=-\mathrm{FT} 3 * \operatorname{COS}(\mathrm{PHI} 33)$ FU3 $=+\mathrm{FT} 3 * \operatorname{SIN}(\mathrm{FHI} 33$ )
GO TO 9899

## 9898

## $\mathrm{FH} 3=-\mathrm{FT} 3 * \mathrm{COS}(\mathrm{PHI} 33)$ FU3=-FT3*SIN(PHI33) $F H T=F H_{1}+F H_{2}+F H_{2}+F V 3$ <br> IF (ALLFHA3, GT , 1.57) FHI 33 = (ALPHA3-1,57-FHI3) <br> FTO=SART (FHT**2+FUT**2)

 IF (FHT .GT + O.0) ALFH11=ACOS (FUT/FTO)
2,FU1,FU2,FU3
WRITE (3,88885)FT1,FT2,FT3
FORMAT ( $3(F 10.5,3 X)$ )
WRITE (3,99993)FHT, FUT
ALFH11=ALFH11*180.0/3. 14
WRITE (3,99994)FTO,ALFH11
FORMAT (12(F8.5,2X))
FORMAT (2(F10.5))
WRITE(3,99996)PHI1, PHI11, FHI2, $\mathrm{FHI} 22, \mathrm{FHI} 3, \mathrm{PHI} 33$ FORMAT ( 6 (F10.5,2X))
CALL HALLAH
WRITE (3,77777)FHT
FORMAT(2(F10.5))
FHI $1=$ FHI 1 * $180.0 / 3.14$
PHI11=PHI $11 * 180.0 / 3.14$
PHI2=FHI2*180.0/3.14.14
PHI $3=$ PHI $3 * 180.0 / 3.14$

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15 FLRMAT (10SH





The following subroutine can be used with the computer programmes
which are given in Appendices A. 2 and A. 3 when a solution is
required for a vertical force which is applied upwards and also this force can be swung $\pm 60^{\circ}$.


| 9595 | ```IF(ALFHA2.GT.1.57) PHI22=(ALFHA2-1.57-FHI2) FH2=-FT2*COS(PHI22) FV2=-FU2*SIN(FHI22)``` |
| :---: | :---: |
| 9696 | FT3 $=$ SQRT ( $($ COS $3 * * 2)+($ SSIN3**2) ) |
|  | IF (FSIN3.EQ.O.O.AND.FCOS3.EQ.0.0) G0 TO 7373 |
|  | PHI3=ATAN(-FSIN3/FCOS3) |
|  | GO TO 4343 |
| 7373 | $\mathrm{PHI} 3=0.0$ |
| 4343 | IF(ALPHA3.LT.1.57) G0 TO 9797 |
|  | GO TO 9898 |
| 9797 | PHI33 $=(1.57-$ ALPHA3-PHI3) |
|  | FH3 $=$ FT3*COS (FHI33) |
|  | FV3=FT3*SIN(PHI33) |
|  | G0 T0 9899 |
| 9898 | IF (ALPHA3.GT.1.57) PHI33= (ALPHA3-1.57+FHI3) |
|  | $\mathrm{FH} 3=\mathrm{FT} 3 * \operatorname{COS}(\mathrm{PHI} 33)$ |
|  |  |



APPENDIX A. 4.

# Appendix A.4. The Design of the Bearing Shell by the Machining Method. 


#### Abstract

A.4.1. Introduction

The two bearing shells which have been tested were manufactured by cutting a hollow cylinder into three segments. The segments were then assembled together with shims between them and the assembly was bored circular. The required shape of the bearing shell was then obtained by removing the shims.


Theoretically this method should produce the exact shape proposed for the bearing. However, it was found that the segments have to be butted very accurately to obtain an accurate form.

## A.4.2. The Bearing Shell Material.

The first bearing shell was machined from a cylinder of $60 / 40$ brass which was cast in the Brunel University foundary. Problems with porosity and inclusions were recognised and attempts were made to alleviate them.

The second bearing shell was machined from a cylinder of gun bronze known commercially as Jml. The metal was stress relieved and tested iur porosity by the manufacture.
A.4.3. The Manufacture of the First Bearing Shells.

The bearing shell was machined in six stages, as follows:-

Stage 1.

The brass cylinder was roughly machined. The screw and dowel holes were drilled into the brass cylinder on a vertical milling machine and in conjunction with a universal dividing head. The recesses for the screw and dowel holes were then milled using slot drills and a ball nosed slot drill.

Stage 2.

The brass cylinder was milled into three equal segments using a slitting saw, a horizontal miller and a universal dividing head.

Stage 3.

The three segments were de-burred and assembled together with the $0.102 \mathrm{~mm}\left(0.004^{\prime \prime}\right)$ thick shims and the screws. Standard silver steel dowels were then fitted. After boring and removing the shims, the resultant shape was found to have large steps at the joints. To remedy the situation stage 3 was recommenced. The faces of the segments were scraped and lapped to obtain good mating surfaces at each joint. The bearing shell was assembled again by means of the screws, and oversize dowel holes were reamed, the shims were tested for constant thickness, inserted between the segments, and the bearing was then assembled with oversize dowels. Chucking of the assembly would have brought distortion so the bearing was mounted on a lathe face plate and bolted in place (figure A.4.1.). Finally the shell was bored to a surface finish of 22 micro inches and by means of "Roundness Measuring Machine" it was found to be circular to within $\pm 20$ micro inches. For the good surface finish required (in the CLA


```
from 6 to }12\mathrm{ micro inches) it was decided to use a diamond tool
to bore the shell. However, the best CLA valUe obtained was }1
micro inches.
```

Stage 4.

The shims were then removed and the bearing shell re-assembled After extensive inspection, the shell was partially disassembled to mill the low pressure oil supply grooves with an end mill.

Stage 5.

The outside diameter of the shell was machined to obtain 0.051 mm interference for the shrink fit. The shell was again mounted on a face plate, with locating spigot, to prevent distortion from a chuck. Finally, pressure relief grooves were machined on the outside of the bearing shell. The aim of these grooves is to prevent oil infiltrating the joints and causing them to separate.

## Stage 6.

Due to the high pressure required to operate the bearing at the extreme design conditions, a substantial bearing body was envisaged. A mild steel cylinder of 356 mm . diameter was used. The shell was shrunk into the steel body by heating up the steel body to approximateIy $200^{\circ} \mathrm{C}$. The bearing shell was suspended above the bearing housing and lowered into it. To keep the brass cool until it was in position, the bearing was packed with solid $\mathrm{CO}_{2}$. The high pressure inlets were drilled into the housing and bearing.


#### Abstract

A.4.4. Inspection of the Bearing Shell.

At the end of stages 4 and 5, the bearing shell was tested again on the "Roundness Measuring machine" and a typical result is shown by figure A.4.2. From this figure it is found that the average lobe depth is $0.029 \mathrm{~mm} .\left(0.00115^{\prime \prime}\right)$.


The full bearing dimensions are listed in table 4.1 (Chapter 4), and the bearing was tested on the simple test rig (Chapter 4).


#### Abstract

At this stage the bearing was not provided with the low pressure oil inlet, and it was judged that the axial grooves for the low pressure oil were too small. The bearing shell was then removed from its housing by cooling and by utilizing the differential expansion between brass and steel. The axial grooves were machined to dimensions given in table 6.1. Some difficulties with steps at the joints arose and the bearing was treated from stage (3) onwards. The full final dimensions are given in table 6.1. The bearing was then reassembled in the housing, the journal of the shaft was ground to suit the bearing and the bearing was tested in the main test rig, (Chapter 6).


## A.4.5. The Manufacture of the Second Bearing Shell

The first bearing shell suffered from an inadequate appreciation by everyone of the great care required at each stage of manufacture, if the intended result is to be obtained clearly, and without remedial action which can never be fully satisfactory. This lesson had been well learnt when the second bearing came to be manufactured and the careful attention given to the fit at the butts, and to the fit of the dowels, produced a sequence of manufacture ending in the intended

result, and without any back-tracking (Figures A.4.3-A.4.6). This second experience demonstrates that there is nothing impracticable about the method of manufacture, all that is required is an appreciation where extreme care has to be taken. The form achieved is shown in figure A.4.7. The bearing shell was shrunk into the steel housing by freezing the bearing shell to around - $50^{\circ} \mathrm{C}$ for about 5 hours.

The second bearing shell was tested on the main test rig (Chapter 6).






FIG 4 4.7 Profile of finished bearing.

## APPENDIX A. 5.

THE DEFORMING METHOD FOR THE PRODUCTION OF THE THREE-LOBE JOURNAL BEARING.

Appendix A.5. The Deforming Method for the Production of a three-lobe Journal Bearing.


#### Abstract

A method of manufacture described in Appendix A. 4 has produced a very satisfactory form but a simpler method is clearly desirable. A method which has been tried only experimentally is illustrated in Figure A.5.1. It comprises the elastic distortion of the bearing ring within three jaws of a chuck. To give the closest approximation to circular lobes the bearing ring is made less rigid in bending at three places by axial grooves. The distorted ring is machined circular, and then released, and its outer surface is then turned circular. A roundness trace from an experimental ring of 4 mm wall thickness cut from a piece of steel tubing is shown in figure A.4.2. The experiment shows promise.


This method of elastic distortion has also been tried with a circular ring with no axial groove. The theoretical analysis of this situation had suggested that a form of three-lobe could be achieved. However, the experimental test was not a success. The point can be made that the lobes themselves do not necessarily have to be circular and that forms including terms beyond the second degree, such as produced by elastic distortion, might also give satisfactory hydrostatic and hydrodynamic performance. This matter has not been investigated, but the finite difference method does provide a means by which other forms could be examined numerically.


FIG.A5.1
Jig for manufacturing method.


FIG.A5. 2 CMT Chart for the deformed bearing shell.


FIG.A.5.3 JIG FOR MAUFACTURING MET HOD.

APPENDIX A. 6.

THE DESIGN AND THE SPECIFICATION
OF THE MIST LUBRICATOR.

## Appendix A. 6.

## A.6.1. SPECIFICATION of the Mist Lubricator

$$
\begin{aligned}
& \text { The venting area for each bearing is given by: } \\
& A_{\nabla}=\pi / 16\left(D^{2}-d^{2}\right)
\end{aligned}
$$

where
D is diameter of the cover plate bore

```
d " " " " shaft at the cover plate.
```

And the venting diameter is given by

$$
\begin{aligned}
& D_{v}=\sqrt{4 A v / \pi} \\
& D_{v} \geq \text { The recommended venting diameter for the selected fittings. }
\end{aligned}
$$

A.6.1. Recommended mist fittings.

1. Tail Bearings and Slave Bearings.

MIST fitting No. 381290-4
Venting diameter $=1.575 \mathrm{~mm}$.
AIR FLOW FOR EACH BEARING AT LINE PRESSURE OF 20" HZO

$$
=.00452 \mathrm{~m}^{3} \mathrm{~min}^{-1}
$$

TOTAL AIR FLOW $=0.00805 \mathrm{~m}^{3} \mathrm{~min}^{-1}$
2. Thrust Bearing

```
MIST fitting No. 381290-6
Venting diameter = 1.981 mm.
AIR FLOW AT PRESSURE LINE OF 2O" HO2 =0.0068 m}\mp@subsup{\textrm{m}}{}{3}\mp@subsup{\textrm{min}}{}{-1
```

A.6.3. The Design Factors on which the Fittings were selected.

Mist line pressure $=20^{\prime \prime} \mathrm{H} 2 \mathrm{O}$
Mean bearing shafts diameter $=80 \mathrm{~mm}$.
Light/medium operating conditions.
A.6.4. Setting the lubricator for operation.

The air pressure gauge (4) should be set at $15 \mathrm{psi}(=1 \mathrm{~atm})$, initially, by using the reducing valve (3), with the oil regulating screw half turned open.

- Unless a manometer is installed in the mist line, for the 20 " to be set by the air input valve, the indication of correct system is
a) No excessive emission of mist from the vents, (bearing in mind that revolving bearing balls will knock the majority of oil from the mist and so reduce the vented material).
b) Overheating of bearing housing.
A.6.5. Lubricating oils
i - High speed oil VIF. 21 (VExC.s. at $40^{\circ} \mathrm{C}$ )
ii - Low speed oil SHELL Vit. 25 ( $v=25 \mathrm{c} . \mathrm{s}$. at $40^{\circ} \mathrm{C}$ ).


FIG.A.6.1 Diagram of oil mist lubricator.

APPENDIX A.7.

SPECIFICATION OF THE CONSTANT VOLUME

FLOW VALVES.

Appendix A.7. Specifications of the Constant Volume Flow Valves.

Three constant flow valves were needed to couple the high pressure pump to the bearing.


#### Abstract

These control the flow rate by varying the area of an orifice within the valve (see table 1.1). To nullify the effects of fluctuating back pressure the valves contain a spring controlled by-pass loop.


Three valves of this type, "Keelavite" hydraulics VFSl/1000 valves were provided. The Manufacturer would not provide performance details for these valves but a data sheet was obtained from them.

The valves were subjected to varying back pressures whilst the pump line pressure was held constant. The flow rates were measured by the "bucket and stop watch" technique, and the results showed that the valves were working correctly. Tables of the results obtained are given, and these show that the valves are controlling the flow despite the variation in the back pressure or the oil temperature.

TABLE 7.1 Specification of Keelavite constant volume valve (VFS $1 / 100$ )

| Maximum pressure | $2.04 \times 10^{7} \mathrm{Nm}^{-2}$ |
| :---: | :---: |
| Flow Range | $0 \quad 3.6 \mathrm{gpm}\left(0.273 \mathrm{Ls}^{-1}\right)$ |
| Pressure drop across valve (assuming normal temperatures) | $1.02 \times 10^{6} \mathrm{Nm}^{-2}$ |
| Error in flow setting | $\pm 1 \%$ |
| Filtration required for valves | 15~20 microns |
| Frequency response | $60 \times 10^{-6} \mathrm{sec}$. |

TABLE AT. 2 Results of tests on the flow control valves.

Valve No. 3
Valve Setting: 2
Volume: 7.5 litres

| $\begin{aligned} & \text { Pump Press. } \\ & \times 10^{-7} \mathrm{Nm}^{-2} \end{aligned}$ | $\begin{aligned} & \text { Back Press. } \\ & \times 10^{-7} \mathrm{Nm}^{-1} \end{aligned}$ | $\begin{gathered} 0 i 1 \text { Temp. } \\ { }^{\circ} \mathrm{C} \end{gathered}$ | Time <br> Second | Flow L $\mathrm{S}^{-1}$ | Flow GPM |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0667 | $\begin{aligned} & 0.132 \\ & 0.264 \\ & 0.396 \\ & 0.528 \end{aligned}$ | $\begin{aligned} & 26 \\ & 28 \\ & 30 \\ & 31 \end{aligned}$ | $\begin{aligned} & 462 \\ & 461 \\ & 461 \\ & 461 \end{aligned}$ | $\begin{aligned} & 0.0163 \\ & 0.0167 \\ & 0.0167 \\ & 0.0166 \end{aligned}$ | $\begin{aligned} & 0.214 \\ & 0.220 \\ & 0.220 \\ & 0.218 \end{aligned}$ |
| 1.334 | 0.132 0.264 0.396 0.528 0.667 0.792 0.924 1.056 1.188 | $\begin{aligned} & 27 \\ & 30 \\ & 31 \\ & 34 \\ & 36 \\ & 39 \\ & 41 \\ & 43 \\ & 47 \end{aligned}$ | 462 <br> 462 <br> 462 <br> 462 <br> 462 <br> 459.4 <br> 456.3 <br> 456.3 <br> 453 | $\begin{aligned} & 0.0163 \\ & 0.0163 \\ & 0.0163 \\ & 0.0163 \\ & 0.0163 \\ & 0.01633 \\ & 0.01644 \\ & 0.01644 \\ & 0.01654 \end{aligned}$ | $\begin{aligned} & 0.214 \\ & 0.214 \\ & 0.214 \\ & 0.214 \\ & 0.214 \\ & 0.215 \\ & 0.217 \\ & 0.217 \\ & 0.218 \end{aligned}$ |
| $\therefore 2.000$ | $\begin{aligned} & 0.132 \\ & 0.264 \\ & 0.396 \\ & 0.528 \\ & 0.667 \\ & 0.792 \\ & 0.924 \\ & 1.056 \\ & 1.188 \\ & 1.320 \\ & 1.452 \end{aligned}$ | $\begin{aligned} & 50 \\ & 55 \\ & 55 \\ & 63 \\ & 78 \\ & 80 \\ & 88 \\ & 88 \\ & 90 \\ & 92 \\ & 93 \end{aligned}$ | 461 461 461 452 452 449 449 449 438 438 438 | $\begin{aligned} & 0.0163 \\ & 0.0163 \\ & 0.0163 \\ & 0.0166 \\ & 0.0166 \\ & 0.0167 \\ & 0.0167 \\ & 0.0167 \\ & 0.0171 \\ & 0.0171 \\ & 0.0171 \end{aligned}$ | $\begin{aligned} & 0.215 \\ & 0.215 \\ & 0.215 \\ & 0.219 \\ & 0.219 \\ & 0.22 \\ & 0.22 \\ & 0.22 \\ & 0.225 \\ & 0.225 \\ & 0.225 \end{aligned}$ |

TABLE A.7.3 Results of tests on the flow control valves.

Valve No. 3
Valve Setting: 3.5 Volume: 7.5 litres

| Pump Press. $\times 10^{-7} \mathrm{Nm}^{-2}$ | Back Press. $\times 10^{-7} \mathrm{Nm}^{-7}$ | $\begin{gathered} 0 i 1 \text { Temp. } \\ { }^{\circ} \mathrm{C} \end{gathered}$ | Time <br> Second | $\begin{aligned} & \text { Flow } \\ & \text { L } S^{-1} \end{aligned}$ | Flow GPM |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.667 | $\begin{aligned} & 0.132 \\ & 0.264 \\ & 0.396 \\ & 0.528 \end{aligned}$ | $\begin{aligned} & 24 \\ & 35 \\ & 27 \\ & 28 \end{aligned}$ | $\begin{aligned} & 119 \\ & 118 \\ & 118 \\ & 118 \end{aligned}$ | $\begin{aligned} & 0.063 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \end{aligned}$ | $\begin{aligned} & 0.831 \\ & 0.839 \\ & 0.839 \\ & 0.839 \end{aligned}$ |
| 1.334 | $\begin{aligned} & 0.132 \\ & 0.264 \\ & 0.396 \\ & 0.528 \\ & 0.667 \\ & 0.792 \\ & 0.924 \\ & 1.056 \\ & 1.188 \end{aligned}$ | $\begin{aligned} & 27 \\ & 30 \\ & 35 \\ & 37 \\ & 39 \\ & 41 \\ & 43 \\ & 45 \\ & 47 \end{aligned}$ | 119 <br> 118 <br> 118 <br> 118 <br> 118 <br> 118 <br> 118 <br> 118 <br> 118 | $\begin{aligned} & 0.063 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \end{aligned}$ | $\begin{aligned} & 0.831 \\ & 0.839 \\ & 0.839^{\prime} \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \end{aligned}$ |
| 1.7 | $\begin{aligned} & 0.132 \\ & 0.264 \\ & 0.396 \\ & 0.529 \\ & 0.667 \\ & 0.792 \\ & 0.924 \\ & 1.056 \\ & 1.188 \\ & 1.320 \\ & 1.452 \end{aligned}$ | $\begin{aligned} & 32 \\ & 37 \\ & 45 \\ & 46 \\ & 48 \\ & 52 \\ & 54 \\ & 57 \\ & 57 \\ & 57 \\ & 57 \end{aligned}$ | $\begin{aligned} & 118 \\ & 118 \\ & 118 \\ & 118 \\ & 118 \\ & 118 \\ & 118 \\ & 118 \\ & 118 \\ & 118 \\ & 118 \end{aligned}$ | $\begin{aligned} & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \\ & 0.0636 \end{aligned}$ | $\begin{aligned} & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \\ & 0.839 \end{aligned}$ |

TABLE A. 7.4 Results of tests on the flow control valves.

Valve No. 3
Valve Setting: 5.5
Volume: 7.5 litres

| Pump Press. | Back Press | Oil Temp. | Time | $\begin{gathered} \text { Flow } \\ \mathrm{L} \mathrm{~S}^{-1} \end{gathered}$ | Flow GPM |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.667 | $\begin{aligned} & 0.132 \\ & 0.264 \\ & 0.396 \\ & 0.528 \end{aligned}$ | $\begin{aligned} & 34 \\ & 37 \\ & 39 \\ & 41 \end{aligned}$ | $\begin{aligned} & 56.23 \\ & 55.83 \\ & 55.83 \\ & 56.0 \end{aligned}$ | $\begin{aligned} & 0.133 \\ & 0.134 \\ & 0.134 \\ & 0.134 \end{aligned}$ | $\begin{aligned} & 1.76 \\ & 1.77 \\ & 1.77 \\ & 1.77 \end{aligned}$ |
| 1.334 | $\begin{aligned} & 0.132 \\ & 0.264 \\ & 0.396 \\ & 0.528 \\ & 0.667 \\ & 0.792 \\ & 0.924 \\ & 1.056 \\ & 1.188 \end{aligned}$ | $\begin{aligned} & 38 \\ & 39 \\ & 40 \\ & 41 \\ & 42 \\ & 44 \\ & 50 \\ & 54 \\ & 59 \end{aligned}$ | 57.4 <br> 57 <br> 56.42 <br> 56.42 <br> 56.42 <br> 56.42 <br> 56.0 <br> 56.0 <br> 56.0 | $\begin{aligned} & 0.131 \\ & 0.1316 \\ & 0.133 \\ & 0.133 \\ & 0.133 \\ & 0.133 \\ & 0.134 \\ & 0.134 \\ & 0.134 \end{aligned}$ | $\begin{aligned} & 1.726 \\ & 1.734 \\ & 1.753 \\ & 1.753 \\ & 1.753 \\ & 1.753 \\ & 1.765 \\ & 1.765 \\ & 1.765 \end{aligned}$ |
| 1.88 | $\begin{aligned} & 0.132 \\ & 0.264 \\ & 0.396 \\ & 0.528 \\ & 0.667 \\ & 0.792 \\ & 0.924 \\ & 1.056 \\ & 1.188 \\ & 1.320 \\ & 1.452 \\ & 1.584 \end{aligned}$ | 30 40 45 50 53 55 55 57 58 59 61 63 | 57.0 <br> 56.91 <br> 56.81 <br> 56.61 <br> 56.61 <br> 56 <br> 55.83 <br> 55.83 <br> 55.63 <br> 55.63 <br> 55.63 <br> 55.63 | 0.1316 0.132 0.132 0.132 0.1325 0.134 0.1342 0.1342 0.135 0.135 0.135 0.135 | $\begin{aligned} & 1.734 \\ & 1.738 \\ & 1.738 \\ & 1.746 \\ & 1.746 \\ & 1.765 \\ & 1.77 \\ & 1.77 \\ & 1.78 \\ & 1.78 \\ & 1.78 \\ & 1.78 \end{aligned}$ |

TABLE A.7.5 Results of tests on the flow control valves.

Valve No. 3
Valve Setting: 7
Volume: 7.5 litres


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TABLE A.7.6 Results of tests on the flow control valves.

Valve No. 1
Valve Setting: 5.5 Measured volume: 7.5 litres

| Pump Press | Back Press | Oil Temp. | Time | $\begin{aligned} & \text { Flow } \\ & \text { L } S^{-1} \end{aligned}$ | Flow GPM |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.334 | 0.132 | 41 | 56.1 | 0.134 | 1.77 |
|  | 0.264 | 44 | 56.1 | 0.134 | 1.77 |
|  | 0.396 | 47 | 56.1 | 0.134 | 1.77 |
|  | 0.528 | 51 | 56.1 | 0.134 | 1.77 |
|  | 0.667 | 53 | 56 | 0.134 | 1.77 |
|  | 0.792 | 56 | 55.6 | 0.135 | 1.78 |
|  | 0.924 | 58 | 54.6 | 0.137 | 1.8 |
|  | 1.056 | 61 | 54.6 | 0.137 | 1.8 |
|  | 1.188 | 64 | 54.6 | 0.137 | 1.8 |

Valve No. 1
Valve Setting: 5.5 Measured volume: 7.5 litres


Results in Table A8.1 were obtained by using Shell Vit (150) ofl.

TABLE A.7.7 Results of tests on the flow control valves.

Valve No. 2

| Pump Press. | Back Press. | $0 i 1$ Temp. | Time | Flow | Flow |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.334 | 0.132 | 43 | 118 | 0.0636 | 0.839 |
|  | 0.264 | 45 | 118 | 0.0636 | 0.839 |
|  | 0.396 | 50 | 118 | 0.0636 | 0.839 |
|  | 0.528 | 54 | 118 | 0.0636 | 0.839 |
|  | 0.667 | 56 | 118 | 0.0636 | 0.839 |
|  | 0.792 | 57 | 118 | 0.0636 | 0.839 |
|  | 0.924 | 60 | 118 | 0.0636 | 0.839 |
|  | 1.056 | 62 | 118 | 0.0636 | 0.839 |
|  | 1.188 | 62 | 118 | 0.0636 | 0.839 |

Valve Setting: 7
Volume: 7.5 litres

| $\mathrm{P}_{s}$ | $\mathrm{P}_{\mathrm{i}}$ | T ${ }^{\circ} \mathrm{C}$ | Time | $\begin{gathered} \text { Flow } \\ \mathrm{L} \mathrm{~S}^{-1} \\ \hline \end{gathered}$ | Flow GPM |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.334 | 0.132 | 38 | 43.0 | 0.174 | 2.3 |
|  | 0.264 | 40 | 42.8 | 0.175 | 2.3 |
|  | 0.396 | 42 | 42.7 | 0.175 | 2.3 |
|  | 0.528 | 45 | 42.7 | 0.175 | 2.3 |
|  | 0.667 | 47 | 42.1 | 0.178 | 2.35 |
|  | 0.792 | 47 | 42.1 | 0.178 | 2.35 |
|  | 0.924 | 49 | 42 | 0.178 | 2.35 |
|  | 1.056 | 51 | 42 | 0.178 | 2.35 |
|  | 1.188 | 53 | 41.70 | 0.18 | 2.37 |

TABLE A7. 8 Results of tests on the flow meters.
Valve Setting: 3.5
Measured volume: 7 litres
Oil Viscosity: $37 \mathrm{C} . S$ at $40^{\circ} \mathrm{C}$


Valve Setting: 4.5


## APPENDIX A.8.

A COMPUTER PROGRAMME FOR

THE CALCULATION OF SPINDLE DEFLECTION.



$\operatorname{MAT}_{Q=Q+1}^{Y=X}$
IF $Q=N+1$ THEN 730
GOTO 660
$C(1,1)=Y(3,1)$


 8888888888888888888888888888888880
00103:1020 $E(3,1)=0$

$A(1,3)=L \cdots 2 /(2 * E * I)$
$A(1,4)=\left(L^{\sim}{ }^{\sim} 3 /(6 * E * I)\right)-\left((2,6 * L) /\left(3,14 *\left(\mathrm{~F}^{m m 2}\right) * E\right)\right)$

$A(2,3)=L /(E * I)$
$A(2,4)=\left(L^{m m} 2 /(2 * E * I)\right)-\left((2.6) /\left(3.14 *\left(\mathrm{R}^{m} 2\right) * E\right)\right)$ $A(3,1)=L *\left(\left(M * W^{m m 2}\right)-K 3\right)$
$A(3,2)=0$
(M*Wの-2)-K3
 RETUFN
IIATA $27,2,1 E+11$
IIATA $016,0275,0,1485,0,0,0,0$
IIATA $04,03,0,259,0,0,0,25000$
IIATA .02,.045,0, $007,0,0,0,0$
IIATA .0375,.0675,0,2,556,0,0,0,0
IIATA .0375,.0675,0,4,12,0,0,0,0
IIATA .0435, 06735, 0675,3,E+08,4,49,0,0,0,0
IIATA . $0435,0675,6, E+08,4,86,0,0,0,0$ LIATA $27,2,1 E+11$
IIATA $016,0275,0,1485,0,0,0,0$
IIATA $04,03,0,259,0,0,0,25000$
IIATA $02,045,0,0,607,0,0,0,0$
IIATA .0375, 0675,0,2,556,0,0,0,0
IIATA .0375,.0675,0,4,12,0,0,0,0
IIATA $0435,0675,3, E+08,4,49,0,0,0,0$
IIATA . $0435,+0675,6, E+08,4,86,0,0,0,0$ LIATA $27,2,1 E+11$
IIATA $016,0275,0,1485,0,0,0,0$
IIATA $04,03,0,259,0,0,0,25000$
IIATA $02,045,0,0,607,0,0,0,0$
IIATA .0375, 0675,0,2,556,0,0,0,0
IIATA .0375,.0675,0,4,12,0,0,0,0
IIATA $0435,0675,3, E+08,4,49,0,0,0,0$
IIATA . $0435,+0675,6, E+08,4,86,0,0,0,0$ LIATA $27,2,1 E+11$
IIATA $016,0275,0,1485,0,0,0,0$
IIATA $04,03,0,259,0,0,0,25000$
IIATA $02,045,0,0,607,0,0,0,0$
IIATA .0375, 0675,0,2,556,0,0,0,0
IIATA .0375,.0675,0,4,12,0,0,0,0
IIATA $0435,0675,3, E+08,4,49,0,0,0,0$
IIATA . $0435,+0675,6, E+08,4,86,0,0,0,0$ LIATA $27,2,1 E+11$
IIATA $016,0275,0,1485,0,0,0,0$
IIATA $04,03,0,259,0,0,0,25000$
IIATA $02,045,0,0,607,0,0,0,0$
IIATA .0375, 0675,0,2,556,0,0,0,0
IIATA .0375,.0675,0,4,12,0,0,0,0
IIATA $0435,0675,3, E+08,4,49,0,0,0,0$
IIATA . $0435,+0675,6, E+08,4,86,0,0,0,0$ LIATA $27,2,1 E+11$
IIATA $016,0275,0,1485,0,0,0,0$
IIATA $04,03,0,259,0,0,0,25000$
IIATA $02,045,0,0,607,0,0,0,0$
IIATA .0375, 0675,0,2,556,0,0,0,0
IIATA .0375,.0675,0,4,12,0,0,0,0
IIATA $0435,0675,3, E+08,4,49,0,0,0,0$
IIATA . $0435,+0675,6, E+08,4,86,0,0,0,0$ $3=K$
$4=0$





LIATA .0375,.0675,0,4,49,0,0,0,0
$.0375, .0675,0,4 \cdot 12,0,0,0,0$
02,.0475,0,2.61,0,0,0,0
$.007, .04,0,687,0,0,0,0$
$.007,04,0, .274,0,0,0,0$
$007,04,0, .274,0,0,0,0$ $032,04,0,1+262,0,0,0,0$

 007, 03,1.2E+08, 154,0,0,0,0 $.007, .03,6, E+07, .154,0,0,0,0$
$.03, .03,0,408,0,0,0,0$ $.03, .03,0,+408,0,0,0,0$
$.011, .0275,0, .433,0,0$, .011,.0275,0, 433,0,0,0,0 $.033, .025,0, .372,0,0,0,0$
$.033,025,0, .54,0,0,0,0$ $033, .025,0, .54,0,0,0,0$ $0,+025,0,27,0,0,0,0$




 8

A.8.1. SAMPLE OF THE OUTPUT FOR THE TEST

SPINDLE .

RESFONSE TO UNBALANCE FORCES ANI IMFRESSEI MOMENTS
S.I.UNITS.
to avoin re-entry of mata run 510.
INSERT SFEEII Z IN R.F.M. IN LINE 510.
ENTEF IIATA AS N,E THEN L,Fi,K,M,M1,F1,F'2,DELTA(M*R),FOF EACH STATION DATA STARTS ON LINE 1540
SUB ROUTINE FOR REARING SUFFORT ON LINES 1510.1520.
II A T A
NUMEER OF STATION 27
yOUNGS MONULUS 210000000000

| STATION NO | SEGMENT L. | Fi A II |
| :---: | :---: | :---: |
| EFG STIFFNESS | MASS |  |
| 1 | . 016 | . 0275 |
| 0 | . 1485 |  |
| 2 | . 04 | . 03 |
| 0 | . 259 |  |
| 3 | . 02 | . 045 |
| 0 | . 607 |  |
| 4 | . 0375 | . 0675 |
| 0 | 2.556 |  |
| 5 | . 0375 | . 0675 |
| 0 | 4.12 |  |
| 6 | . 0435 | . 0675 |
| 300000000 | 4.49 |  |
| 7 | . 0435 | . 0675 |
| 600000000 | 4.86 |  |
| 8 | . 0435 | .0675 |
| 300000000 | 4.86 |  |
| 9 | . 0375 | . 0675 |
| 0 | 4.49 |  |
| 10 | . 0375 | . 0675 |
| 0 | 4.12 |  |
| 11 | . 02 | .0475 |
| 0 | 2.61 |  |
| 12 | . 007 | . 04 |
| 0 | . 687 |  |
| 13 | . 007 | . 04 |
| 0 | . 274 |  |
| 14 | . 007 | . 04 |
| $\bigcirc$ | . 274 |  |
| 15 | . 032 | . 04 |
| 0 | 1.262 |  |
| 16 | . 076 | . 0375 |
| 0 | 2.435 |  |
| 17 | . 076 | . 0375 |
| 0 | 2.62 |  |
| 18 | . 076 | . 0375 |
| 0 | 2.62 |  |
| 19 | . 007 | . 03 |
| 60000000 | 1.387 |  |
| 20 | . 007 | . 03 |
| 120000000 | . 154 |  |
| 21 | . 007 | . 03 |
| $60000000 \ldots$ | . 154 |  |
| 22 | . 03 | . 03 |


| 0 | . 408 |  |
| :---: | :---: | :---: |
| 23 | . 011 | . 0275 |
| 0 | . 433 |  |
| 24 | . 033 | . 025 |
| $\bigcirc$ | . 372 |  |
| 25 | . 033 | . 035 |
| 0 | . 54 |  |
| 26 | . 033 | . 025 |
| 0 | +54 |  |
| 27 | $\bigcirc$ | . 025 |
| 0 | +27 |  |
| STATION NO | II S T | ACCMASS |
| BASE MASS |  |  |
| 1 | 0 | . 1485 |
| 0 |  |  |
| 2 | . 016 | . 4075 |
| 0 |  |  |
| 3 | . 056 | 1.0145 |
| 0 |  |  |
| 4 | . 076 | 3.5705 |
| 0 |  |  |
| 5 | . 1135 | 7.6905 |
| 0 |  |  |
| 6 | . 151 | 12.1805 |
| 0 |  |  |
| 7 | . 1945 | 17.0405 |
| 0 |  |  |
| 8 | . 238 | 21.9005 |
| 0 |  |  |
| 9 | . 2815 | 26.3905 |
| $\bigcirc$ |  |  |
| 10 | . 319 | 30.5105 |
| $\bigcirc$ |  |  |
| 1.1 | . 3565 | 33.1205 |
| 0 |  |  |
| 12 | . 3765 | 33.8075 |
| $\bigcirc$ |  |  |
| 13 | + 3835 | 34.0815 |
| $\bigcirc$ |  |  |
| 14 | . 3905 | 34.3555 |
| 0 |  |  |
| 15 | . 3975 | 35.6175 |
| 0 |  |  |
| 16 | . 4295 | 38.0525 |
| 0 |  |  |
| 17 | . 5055 | 40.6725 |
| 0 |  |  |
| 18 | . 5815 | 43.2925 |
| 0 |  |  |
| 19 | . 6575 | 44.6755 |
| 0 |  |  |
| 20 | . 6645 | 44.8335 |
| $\bigcirc$ |  |  |
| 21 | . 6715 | 44.9875 |
| $\bigcirc$ |  |  |
| 22 | . 6785 | 45.3955 |
| $\bigcirc$ |  |  |
| 23 | . 7085 | 45.8285 |
| 0 |  |  |


| 24 | . 7194999999999 | 46.2005 |
| :---: | :---: | :---: |
| 0 |  |  |
| 25 | . 7524999999999 | 46.7405 |
| $\bigcirc$ |  |  |
| 26 | . 7854999999999 | 47.2805 |
| $\bigcirc$ |  |  |
| 27 | . 8184999999999 | 47.5505 |
| 0 |  |  |
| StATION NO | MOMENT P1 M | MOMENT F2 |
| DELTA (M*F) |  |  |
| 1 | 0 | 0 |
| 0 |  |  |
| 2 | 0 | 0 |
| 25000 |  |  |
| 3 | 0 | 0 |
| 0 |  |  |
| 4 | 0 | 0 |
| OUTFUT FOF O | F.F.M. |  |
| STATION NO | IISFLACEMENT | SLOFE |
| M M | SHEAK |  |
| 0 I ST | ERGGF | $U N B A L F$ |
| EASE/FOTOR EFF STIFFNESS |  |  |
| $1275789797.457$ |  |  |
| 1 | . 0001838713537495 | -. 000816471742853.1 |
| 0 | 0 |  |
| 0 | 0 | 0 |
| 0 0 |  |  |
| 2 | . 0001708078058638 | $-.0008164717428531$ |
| 0 | $\bigcirc$ |  |
| . 016 | 0 | 95000 |
| 0 O |  |  |
| 3 | . 0001357649257449 | -.0007762181640291 |
| 1000 | 25000 |  |
| . 056 | $\bigcirc$ | 0 |
| 0 0 |  |  |
| 4 | . 0001196121612989 | $-.0007879140072017$ |
| 1500 | 25000 |  |
| . 076 | $\bigcirc$ | 0 |
| $\bigcirc$ | 0 |  |
| 5 | 8.96264708777E-05 | $-.0007879756453739$ |
| 2437.5 | 25000 |  |
| .1135 | 0 | 0 |
| $\bigcirc$ | 0 |  |
| 6 | 5.983108831315 ECOE | - - . 00007777642548482 |
| 3375 | 25000 |  |
| . 151 | 17949.32649394 | $\bigcirc$ |
| $\bigcirc$ | 300000000 |  |

7
3681.704297513 .1945
0

8
3291.688288616 .238
0
9
2955.614469863 .2815
0
10
2665.895660593 .319
0
11
2376.176851323 .3565
0

## 12

2221.660153046 .3765
0
13
2167.579308649 .3835
0

## 14

2113.498464252
.3905
0

15
2059.417619855 .3975
0
16
1812.190902611
.4295
0

17
1225.027449158 .5055
0
18
637.8639957046 .5815
0

| $\begin{aligned} & 2.669426461343 \mathrm{E}-05 \\ & 7050.673506055 \end{aligned}$ | -. 0007390164555891 |
| :---: | :---: |
| 16016.55876806 | 0 |
| 600000000 |  |
| -4.133501160469E-06 | -. 0006869374216548 |
| -8965.885262001 |  |
| -1240.050348.141 | 0 |
| 300000000 |  |
| -3.284536628204E-05 | -. 0006405462495068 |
| -7725.83491386 |  |
| 0 | 0 |
| 0 |  |
| -5.602770768468E-05 | $-.0006030603471776$ |
| -7725.83491386 |  |
| 0 | 0 |
| 0 |  |
| -7.786385354282E-05 | -.0005687491537998 |
| -7725.83491386 |  |
| 0 | 0 |
| 0 |  |
| -8.84147827905E-05 | $-.0005004590382484$ |
| -7725.83491386 |  |
| 0 | 0 |
| 0 |  |
| -9.165679010469E-05 | -. 0004450175231723 |
| -7725.83491386 |  |
| 0 | () |
| 0 |  |
| -9.451384645897E-05 | $-.0003904730497032$ |
| -7725.83491386 |  |
| 0 | 0 |
| 0 |  |
| -9.69922311446E-05 | $-.0003368256178409$ |
| -7725.83491386 |  |
| 0 | 0 |
| 0 |  |
| -. 0001047628401985 | -.0001710010882736 |
| -7725.83491386 |  |
| 0 | 0 |
| 0 |  |
| -. 0001017923228599 | . 0002046958512815 |
| $-7725.83491386$ |  |
| 0 | 0 |
| 0 |  |
| -7.547050076203E-05 | . 0004435069317081 |
| -7725.83491386 |  |
| 0 | ) |
| 0 |  |

$-4.133501160469 E-06-.0006869374216548$
-
240.050348141

```
-7725.83491386
O
```

$-5.602770768468 \mathrm{E}-05 \quad-.0006030603471776$
$-7725.83491386$
0 0
0
$-7.786385354282 \mathrm{E}-05 \quad-.0005687491537998$
-7725.83491386
0
$-8.84147827905 \mathrm{E}-05 \quad-.0005004590382484$
$-7725.83491386$
0 0
$-9.165679010469 \mathrm{E}-05-.0004450175231723$
$-7725.83491386$
0
$-9.451384645897 E-05-.0003904730497032$
$-7725.83491386$
0
$-9.69922311446 \mathrm{E}-05 \quad-.0003368256178409$
$-7725.83491386$
0
0
$-.0001047628401985-.0001710010882736$
$-7725.83491386$
0
$-.0001017923228599 .0002046958512815$
$-7725.83491386$
0
$-7.547050076203 \mathrm{E}-05 \quad .0004435069317081$
0
0

| 19 | $-3+62006991986 E-05$ | . 0005454321530063 |
| :---: | :---: | :---: |
| 50.70054225117 | -7725.83491386 |  |
| .6575 | -2172.041951916 | 0 |
| 0 | 60000000 |  |
| 20 | $-3+220542766981 E-08$ | . 0005714026742878 |
| 11.82399151756 | $-5553.792961944$ |  |
| . 6645 | $-3864.651320378$ | 0 |
| 0 | 120000000 |  |
| 21 | -2.815236069031E-05 | .0005791128772086 |
| $2.659635356395 E-08$ | $-1689+141641567$ |  |
| . 6715 | $-1689.141641419$ | 0 |
| 0 | 60000000 |  |
| 22 | $-2.409857054985 E-05$ | .0005791128772106 |
| $2.556203071435 E-08$ | $-1.477310433984 \mathrm{E}-07$ |  |
| .6785 | 0 | 0 |
| 0 | 0 |  |
| 23 | $-6.725184233426 E \cdots 06$ | . 0005791128772165 |
| $2.11300994124 E-08$ | $-1.477310433984 E-07$ |  |
| . 7085 | 0 | 0 |
| 0 | 0 |  |
| 24 | $-3.549425840334 E-07$ | . $000579112877219 \%$ |
| 1.950505793502E-08 | $-1.477310433984 E-07$ |  |
| . 7194999999999 | $0$ | 0 |
| 0 | 0 |  |
| 25 | 1.875578236441E-05 | . 0005791128772293 |
| 1.462993350287E-08 | $-1.477310433984 E \cdots 0 \%$ |  |
| . 7524999999999 | 0 | 0 |
| 0 | 0 |  |
| 26 | $3.786650731311 \mathrm{E}-05$ | .000679112977236 |
| 9.754809070727E-09 | $-1.477310433984 E-$ - 07 |  |
| $0^{.7854999999999}$ | $\begin{aligned} & 0 \\ & 0 \end{aligned}$ | ) |
| 27 | $5.697723296202-05$ | .0005791.28772411 |
| 4.87968463858E-09 | $-1.477310433984 \mathrm{E}-07$ |  |
| . 8184999999999 | 0 | 0 |
| 0 | 0 |  |
| 28 | E.697723226202E-05 | -0005791128792421 |
| 4.87968463858E-09 | $-1.477310433984 E-07$ |  |

A.8.2. SAMPLE OF THE OUTPUT FOR A CONSTANT DIAMETER SPINDLE.

## RESFONSE TO UNBALANCE FORCES ANI IMFRESSEI MOMENTS

> S.I.UNITS.

TO AUOII RE-ENTRY OF IIATA RUN 510.
INSERT SFEED Z IN R.F.M. IN LINE 510.
ENTER IIATA AS N,E THEN L,R,K゙,M,M1,F1,F2,DELTA(M*R),FOR EACH STATIOR
IIATA STAFTS ON LINE 1540
SUB ROUTINE FOR REARING SUPFORT ON LINES 1510.1520.
II A T A
NUMBEF OF STATION 27
YOUNGS MODULUS 210000000000

| STATION NO BRG STIFFNESS | $\begin{aligned} & \text { SEGMENT L } \\ & M A S S \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | . 016 | . 2 |  |
| 1 | 7.84 |  |  |
| 2 | . 04 | . 2 |  |
| 0 | 27.44 |  |  |
| 3 | . 02 | . 2 |  |
| 0 | 29.4 |  |  |
| 4 | . 0375 | . 2 |  |
| 0 | 28.175 |  |  |
| 5 | . 0375 | . 2 |  |
| 0 | 36.75 |  |  |
| 6 | . 0435 | . 2 |  |
| 300000000 | 39.695 |  |  |
| 7 | . 0435 | . 2 |  |
| 600000000 | 42.64 |  |  |
| 8 | . 0435 | . 2 |  |
| 300000000 | 42.64 |  |  |
| 9 | . 0375 | . 2 |  |
| 0 | 39.695 |  |  |
| 10 | +0375 | . 2 |  |
| 0 | 36.75 |  |  |
| 11 | . 02 | . 2 |  |
| 0 | 28.175 |  |  |
| 12 | . 007 | . 2 |  |
| 0 | 13.23 |  |  |
| 13 | . 007 | . 2 |  |
| 0 | 6.86 |  |  |
| 14 | . 007 | . 2 |  |
| 0 | 6.86 |  |  |
| 15 | . 032 | . 2 |  |
| 0 | 19.11 |  |  |
| 16 | . 076 | . 2 |  |
| 0 | 52.92 |  |  |
| 17 | . 076 | . 2 |  |
| 0 | 74.48 |  |  |
| 18 | +046 | . 2 |  |
| 0 | 59.74 |  |  |
| 19 | . 0417 | . 2 |  |
| 300000000 | 42.93 |  |  |
| 20 | . 0417 | . 2 |  |
| 600000000 | 40.86 |  |  |



| 23 | . 7658 | 756.8899999999 |
| :---: | :---: | :---: |
| 0 |  |  |
| 24 | . 779 | 769.8299999999 |
| 0 |  |  |
| 25. | . 7922 | 782.7699999999 |
| 0 |  |  |
| 26 | . 8054 | 795.7099999999 |
| 0 |  |  |
| 27 | . 8186 | 802.1799999999 |
| 0 |  |  |
| STATION NO | MOMENT P1 | MOMENT FI2 |
| IELTA (M*F) |  |  |
| 1 | 0 | 0 |
| 0 |  |  |
| 2 | 0 | 0 |
| 30000 |  |  |
| 3 | 0 | 0 |
| 0 |  |  |
| 4 | 0 | 0 |
| 0 |  |  |
| 5 | 0 | 0 |
| 0 |  |  |
| 6 | 0 | 0 |
| 0 |  |  |
| 7 | 0 | 0 |
| 0 |  |  |
| 8 | 0 | 0 |
| 0 |  |  |
| 9 | 0 | 0 |
| 0 |  |  |
| 10 | 0 | 0 |
| 0 |  |  |
| 11 | 0 | 0 |
| 0 |  |  |
| 12 | $\bigcirc$ | 0 |
| 0 |  |  |
| 13 | 0 | 0 |
| 0 |  |  |
| 14 | 0 | 0 |
| 0 |  |  |
| 15 | 0 | 0 |
| $\bigcirc$ |  |  |
| 16 | 0 | 0 |
| 0 |  |  |
| 17 | 0 | 0 |
| 0 |  |  |
| 18 | 0 | 0 |
| 0 |  |  |
| 19 | 0 | 0 |
| 0 |  |  |
| 20 | 0 | 0 |
| 0 |  |  |
| 21 | 0 | 0 |
| 0 |  |  |
| 22 | 0 | 0 |
| $\bigcirc$ |  |  |
| 23 | 0 | 0 |
| 0 |  |  |
| 24 | 0 | 0 |
| 0 |  |  |
| 25 | 0 | 0 |


| OUTFUT FOR O | R.F.M. |  |
| :---: | :---: | :---: |
| STATION NO | IIISFLLACEMENT | 5 L 0 FE |
| M O M | SHEAR |  |
| D I S T | ERGGF | UNEALF |
| BASE/ROTOR | EFF STIFFNESS |  |
| -913807607.0668 | $-2300215632.271$ | -260745191.6254 |
| -1017597909.298 |  |  |
| 1 | $5.254708103785 E-05$ | -9.569427551826E-05 |
| 0 | 0 |  |
| 0 | 0 | 0 |
| 0 | 0 |  |
| 2 | 5.101597262956E-O5 | -9.569427551826E-05 |
| $\bigcirc$ | 0 |  |
| . 016 | 0 | 30000 |
| 0 | 0 |  |
| 3 | 4.718941483297E-05 | -9.560328370753E-05 |
| 1200 | 30000 |  |
| . 056 | 0 | 0 |
| $\bigcirc$ | 0 |  |
| 4 | 4.527841072994E-05 | -9.548954394411E-05 |
| 1800 | 30000 |  |
| . 076 | 0 | 0 |
| 0 | $\bigcirc$ |  |
| 5 | 4.170335089419E-05 | $-9.515365620525 E-05$ |
| 2925 | 30000 |  |
| .1135 | 0 | 0 |
| $\bigcirc$ | 0 |  |
| 6 | 3.814388584632E-05 | -9.465782192409E-05 |
| 4050 | 30000 |  |
| . 151 | 11443.1657539 | 0 |
| 0 | 300000000 |  |
| 7 | 3.404176340419E-05 | -9.392332054101E-05 |
| 4857.222289705 | 18556.8342461 |  |
| . 1945 | 20425.05804252 | 0 |
| 0 | 600000000 |  |
| 8 | 2.997342497532E-05 | -9.312895598095E-05 |
| 4775.954554561 | -1868.223796414 |  |
| . 238 | 8992.027492596 | 0 |
| $\bigcirc$ | 300000000 |  |
| 9 | 2.593888219426E-05 | -9.238024932386E-05 |
| 4303.533623489 | $-10860.25128901$ |  |
| .2815 | $\bigcirc$ | 0 |
| $\bigcirc$ | 0 |  |
| 10 | 2.248573321257E-05 | -9.179734669764E-05 |
| 3896.274200151 | $-10860.25128901$ |  |
| $.319$ | 0 | 0 |
|  | 0 |  |


| 16 | 1.242305252268E-05 | -9.041641286589E- |
| :---: | :---: | :---: |
| 2696.216432715 | -10860.25128901 |  |
| . 4295 | 0 | 0 |
| 0 | 0 |  |
| 17 | 5.577914502295E-06 | -8.97584357535E-0 |
| 1870.83733475 | -10860.25128901 |  |
| . 5055 | 0 | 0 |
| $\bigcirc$ | 0 |  |
| 18 | -1.22625462044E-06 | -8.933828398023E- |
| 1045.458236785 | $-10860.25128901$ |  |
| . 5815 | 0 | 0 |
| 0 | 0 |  |
| 19 | -5.33229008231E-06 | -8.919951793144E- |
| 545.8866774905 | -10860.25128901 |  |
| . 6275 | -1599.687024693 | 0 |
| 0 | 300000000 |  |
| 20 | -9.050534859186E-06 | -8.914374024281E- |
| 159.7211476685 | -9260.564264318 |  |
| . 6692 | $-5430.3209155 .2$ | 0 |
| $\bigcirc$ | 600000000 |  |
| 21 | -1.276747782955E-05 | -8.913111442415E- |
| 2.324668457732E-08 | $-3830 \cdot 243348806$ |  |
| . 7109 | -3830.243348866 | 0 |
| 0 | 300000000 |  |
| 22 | -1.648424530104E-05 | -8.913111442415E- |
| 2,573506208137E-08 | 5.972106009722E-08 |  |
| . 7526 | 0 | 0 |
| 0 | $\bigcirc$ |  |
| 23 | -1.766077601144E-05 | -8.913111449415E- |
| $2.652338007465 \mathrm{E}-08$ | $5.972106009722 \mathrm{E}-08$ |  |
| . 7658 | 0 | 0 |
| 0 | $\bigcirc$ |  |
| 24 | -1.883730672184E-05 | -8.913111442415E- |
| 2,731169806793E-08 | 5.972106009722E-08 |  |
| . 779 | 0 | 0 |
| $\bigcirc$ | 0 |  |
| 25 | -2.001383743224E-05 | -8.913111442415E- |
| 2.810001606121E-08 | $5.972106009722 \mathrm{E}-08$ |  |
| . 7922 | 0 | 0 |
| 0 | $\bigcirc$ |  |
| 26 | -2.119036814264E-05 | -8.913111442415E- |
| 2.88883340545E-08 | 5.972106009722E-08 |  |
| . 8054 | 0 | 0 |
| 0 | 0 |  |
| 27 | $-2.236689885304 \mathrm{E}-05$ | -8.913111442415E- |
| 2.967665204778E-08 | 5.972106009722E-08 |  |
| . 8186 | 0 | 0 |
| 0 | 0 |  |
| 28 | -2.236689885304E-05 | -8.913111442415E- |
| 2.967665204778E-08 | $5.972106009722 \mathrm{E}-08$ |  |

APPENDIX A. 9

METALLIC CONTACT
MONITORING.


#### Abstract

In the hydrostatic tests in the simple test rig (chapter 4), the need arose for a device which would indicate whether the shaft was supported by an oil film or was supported by metal to metal contact. With no load on the journal a metal to metal contact may arise due to a blockage in a high pressure line or in a constant volume flow valve. It was suggested that a measurement of the electric resistance due to the oil film should be monitored. That idea was used on the simple test rig and was also used on the main test rig (chapter 6).


In chapter 6, it has been described how the shaft was insulated from the bearings and from the frame, to make possible measurements of the resistance of the oil film. Duruing the assemply process the bearing housing and the brass shell were cleaned and every care had been taken to ensure that no swarf remained on the warp. After some tests, it was found that the resistance reading was always zero. Several attemps were made to find the reason and finally the shaft was taken out of the bearing. It was found that the journal and bearing surfaces were slightly damaged due to the presence of some metal swarf which had hidden in the high pressure passages. When the high pressure system was operated, that metal swarf came into contact with both the journal bearing surfaces and caused some scoring. The journal was lapped and the bearing was dressed.

[^16]surface of the journal were as they were after dressing. Figure A.9.1 shows a photograph for the first bearing shell after long operation.

That experience showed the usefulness of the metallic contact monitoring. It can be seen that without this safeguard the bearing and the journal might be rendered useless. It also emphasized that one cannot rely upon shop statements that components have been thoroughly cleaned.


FIG.A. 91 SCORED BEARING SHELL

APPENDIX A. 10

A COMPUTER PROGRAMME FOR THE NUMERICAL SOLUTION OF REYNOLDS EQUATION FOR THE FULL CIRCULARE BORE JOURNAL BEARING ( $360^{\circ}$ JOURNAL BEARING).
-NULL
00001: J0B(GREM114, J12,M7600,T900) 00002: MNF (T)
00003: LISET (MAF: $=$ B/ZZZZMF $;$ FRESET $=Z E F O$ )
官
FROGFAM A 35 (INFUT, OUTFUT, TAFE $4=$ INFUT, TAFE $3=O U T F U T$ )
IIMENSION F1 $(40,122), \mathrm{Fi} 1(40,122), \mathrm{H} 1(40,122)$
INTEGER U.V
$L=1$
IO 3
$U=1,3$
$E=0.1$
no 1
$001 K=1,9$
110 $54545 \quad U=1,4$
$E F S=0.0001$
MAXIT $=200$
$1=2.0$

C**************************************************************************** IN THE SOLUTION
OF ITERTATIONS

C***************************************************************************** LIAIMETER:
C**************************************************************************** M IS THE NUMEEF OF MESH FOINTS AROUND THE EEARING
C***************************************************************************** [10 $10 \mathrm{~J}=1, \mathrm{MH} 1$

$00035:$
00036
00037
00038
00039
00040
00041
00042
00043
00044
00045
00046
00047
00048
00049
00050
00051
00052
00053
00054
00055
00056
00057
00058
00059
00060
00061
00062
00063
00064
00065
00066
00067
00068
00069
00070

| 110 50 I=2,N |  |
| :---: | :---: |
| $60 \mathrm{~F} 1(\mathrm{I}, 1)=0.0$ |  |
|  | [10 $60 \mathrm{I}=2, \mathrm{~N}$ |
| 60 | $F 1(T, M+1)=0.0$ |
|  | $1070 \mathrm{~J}=2 \mathrm{ym}$ |
| 70 | F1 $(1, J)=0.0$ |
|  | no $80 \mathrm{~J}=2 \mathrm{~m}$ |
|  | $\mathrm{F} 1(\mathrm{~N}+1, \mathrm{~J})=0.0$ |
| : $\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ |  |
| : | ITN=1 |
| : | WFITE (3,810) ITY, IIX |
| 810 | FOFMAT $25 H \quad$ UY $=$, F15.8,45H |
| 2 | $2 \mathrm{IX}=\mathrm{F}$ F15.8/,100H****************************** |
| 4 | 4****************************************************************** |
| 三 | こ****) |
| : | $\mathrm{F}=(\mathrm{I} / \mathrm{FLOAT}(\mathrm{L})$ ) $* * 2$ |
| : | S=0x**2 |
| + | T=[Y**2 |
| : | B=1.0-OMEGA |
| : 90 | [1A $=0.0$ |
| : $\mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ |  |
| : | $\square 100 \mathrm{I}=2$, N |
| : | $110100 \mathrm{~J}=3, \mathrm{M}, 2$ |
| \% C | THE NEXT STATEMEET IS THE FEYNOLIS EQUATION FOR STEAIIY STATE COITION, |
| : C | FEFFSENTEII IN A FINITE IIFFEFEENCE FOFM |
| : C | F(I;J) IS THE UIMENIONSLESS FRESSURE AT THE MESH FOINTS |
| : | $A 1=((18,8 /[1 \times) *(H 1(I, J-1)-H 1(I, J+1))$ ) |
| : | B1 $=((\mathrm{F} / \mathrm{T}) *((H 1(I+1, J)) * * 3) *(F \cdot 1(1+1, J))$ ) |
| : | $\mathrm{Ci}=((1,0 / 5) *((H 1(I, J+1)) * * 3) *(F 1(I, J+2))$ ) |
| : |  |
| : |  |
| : | F1=( $\mathrm{F} / \mathrm{T}) *((\mathrm{H} 1$ ( $\mathrm{I}+1, \mathrm{~J})$ ) **3) $)$ |
| : |  |
| : | Y1=( $(\mathrm{F} / \mathrm{T}) *((H 1(I-1, J)) * * 3))$ |
| : |  |
| : |  |


$00107 \ddagger \mathrm{C} * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ 00108: IF(J.GE. (180.0/2.0)) 60 T0 888


N FERFENDICULAF ON THE ATTITUNE LINE


```
This paper has been presented at the 6th Conference
'The Optimization Problems in Design and Production of
Machines', held at INSTYTUT KONSTUKCJI IEKSPLOR TACJI
MASZYN i INSTYTUT TECHNOLOGII BUDOWY MASZYN POLITECHNIKI
WROCLAW SKIGJ POLAND, 21-25 September, 1978.
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# DEVELOPMENT OF A BEARING FOR A WIDE SPEED RANGE WITH REFERENCE TO MACHINE TOOLS APPLICATION 

A.W. CROOK* and T.M. SALEM**

The paper describes work carried out in the development of a bearing for a wide speed range, and of high stiffness. The intended speed range is from zero to 9000 rpm , and the intended load capacity is approximately $6.6 \times 10^{4} \mathrm{~N}$ in the low speed range and $4.0 \times 10^{3} \mathrm{~N}$ in the high speed range.

The bearing is conceived as a hydrostatic bearing at low speeds, and as a self generating bearing at high speeds. The bearing is of three lobe form with the intention of enhancing stiffness in the self generating regime. High pressure inlets with no pockets are provided in the mid lobe positions, and low pressure inlets are provided at the apices of the lobes.

A development bearing of 135 mm bore and 127 mm width has been made and is being tested. Hydrostatic tests without rotation of the journal have shown the stiffness to be satisfactory and substantially as predicted from the analysis of the bearing. Tests with rotation of the journal are currently in progress and tests have been carried out up to a speed of 3000 rpm . The measured power loss and temperature rise are substantially those which had been expected.

[^17]
## 1. Introduction

The development work described in this paper was originated by an enquiry for a machine tool bearing of 152 mm diameter and with the specification given in Table 1.

It seemed possible, from a feasibility study, that the specification could be met by a bearing which operates hydrostatically at low speed, and which becomes self generating as the speed is increased. In a self-generating bearing of circular bore the stiffness of the bearing arises in the locality of the position of minimum film thickness, and the remainder of the surface contributes little to stiffness but a great deal to the loss within the bearing. However, if the bearing is a multi-lobe bearing then there is a position of film convergence at each lobe, and each lobe contributes to the overall stiffness. By adopting a multi-lobe bearing it was considered that the required stiffness in the self-generating regime could be obtained at a lower power loss than would be possible with a bearing of circular bore. This concept reacted upon the design of the bearing in its hydrostatic aspect.

In a bearing which is entirely hydrostatic the advantages of pockets are well known, but as reported by SHELDON et al ${ }^{(1) *}$, and ROWE, KOSHAL, and STOUT ${ }^{(2)}$ the consequent removal of surface greatly diminishes the load carrying capacity of the bearing as a self generating bearing. Furthermore, turbulence within the pockets increases the loss. Consequently it was decided that the bearing would be designed without pockets, and that the required characteristics in the hydrostatic regime would be obtained by employing a sufficiently high inlet pressure.

[^18]| Regime | Speed rpm | Maximum <br> bearing <br> pressure Nm | Deflection of <br> spindle m | Mean stiff- <br> ness <br> $\mathrm{Nm}^{-1}$ |
| :--- | :---: | :---: | :---: | :---: |
| Low speed | $8-40$ | $3.86 \times 10^{6}$ | Unimportant | - |
| Roughing | $40-80$ | $3.86 \times 10^{6}$ | Unimportant |  |
| High speed | 40 to 1800 | $1.18 \times 10^{6}$ | $2.5 \times 10^{-5}$ | $8 \times 10^{8}$ |
| 1500 to 9000 | $2.34 \times 10^{5}$ | $1.25 \times 10^{-5}$ | $3.2 \times 10^{8}$ |  |

For the above bearing pressures maximum loads for a bearing of 135 mm diameter and 127 mm long become $6.6 \times 10^{4} \mathrm{~N}, 6.6 \times 10^{4} \mathrm{~N}$, $2.0 \times 10^{4} \mathrm{~N}$ and $4.0 \times 10^{3} \mathrm{~N}$ respectively.

Table 1. Specification

| C | Diametral clearance | m |
| :---: | :---: | :---: |
| $0^{=}$ | Minimum diametral clearance | m |
| C | Maximum diametral clearance | m |
| $\square$ | Lobe diameter | m |
| d | Shaft diameter | m |
| $F_{t}$ | External load | N |
| $\mathrm{H}_{s}$ | Power loss | KW |
| L | Bearing axial width | m |
| N | Rotational speed | rpm |
| $P_{5}$ | Hydrostatic supply pressure | $\mathrm{Nm}^{-2}$ |
| $Q_{t}$ | Total rate of flow | Iitres s ${ }^{-1}$ |
| Z | Absolute viscosity | CP |
| $\Delta r$ | Offset | m |

3. The Bearing

The concept of the bearing is illustrated by Figure 1. The bearing comprises three lobes of circular bore with the centre of each lobe displaced from the axis of the bearing. With no external load, the axis of the journal and bearing will be co-linear, but the journal will be forced to run eccentrically with respect to each of the three lobes. Bearings with three lobes have been analysed by PINKUS ${ }^{(3)}$. They have been proposed as having good anti-whirl properties, and as having high stiffness.

A high pressure oil inlet for the hydrostatic regime is provided at the centre of each lobe, and a low pressure inlet is provided for the self-generating regime at each conjunction of the lobes.
3.1 The development bearing was made by splitting a brass cylinder into three segments, by reassembling the segments but separated by

| 1 | Steel housing |  |  |
| :--- | :--- | :--- | :--- |
| 2 | Brass shell |  |  |
| 3 | High pressure inlet |  |  |
| 4 | Low pressure inlet |  |  |

FIG.I Three lobe-bearing for hydrodynamic and hydrostatic operation
shims, by boring the assembly circular, by removing shims and finally by turning the outer surface circular. The thickness of the shims was calculated to provide the required offset of the centres of the lobes (Figure 1). Particular care had to be taken with the abutments of the three segments so that no bending was introduced on tightening. The dimensions of the bearing are given in Table 2, and the form achieved by the process is shown by the roundness chart of Figure 2. The departures from the intended form are negligible.

## 4. Hydrostatic Tests

### 4.1 Test Rig

The bearing as arranged for hydrostatic test is shown in Figure 3. The rig does not permit continuous rotation, but the effect of applying the force in different directions with respect to the lobes was tested by rotating the bearing in its frame. The force was applied by a hydraulic ram, the movement of the journal with respect to the bearing was measured with dial gauges, and the absence of metal to metal contact between journal and bearing was monitored by measuring the electrical resistance between journal and bearing. High pressure oil was supplied to the high pressure inlets via individual volume control valves. Volume control valves were chosen in contrast with orifice plates or capillaries to minimise the loss and therefore the pump power, to minimise the pressure of the pump, and also as reported by Malonski and Loeb ${ }^{(4)}$ to provide a greater stiffness. The supply pressure from the pump was measured and oil flow, oil temperature and pressure were measured downstream of each of the three volume control valves.


TABLE 2 DIMENSIONS OF TEST BEARINGS


FIG. 2 PROFILE OF FINISHED bEARING


FIG. 3 TEST RIG FOR EXTERNAL PRESSURIZATION

### 4.2 Results of hydrostatic tests

The tests were carried out with an oil of viscosity of 20 cp at $75^{\circ} \mathrm{C}$ (SHELL VITREA 69 - Industrial Oil). Typical conditions of test were a supply pressure of $1.7 \times 10^{7} \mathrm{Nm}^{-2}$, a total volume flow of 0.122 litres $s^{-1}$, a pump power of 3 KW and an oil temperature at bearing inlets of $50^{\circ} \mathrm{C}$ at which the oil has an absolute viscosity of 65 cp . A typical result is a stiffness of $1 \times 10^{9} \mathrm{Nm}^{-1}$, and absence of metal to metal contact up to the maximum load which was applied of $5.6 \times 10^{4} \mathrm{~N}$.

Specific test results are shown in Figures 4 and 5. In Figure 4 displacement is shown as a function of load for three rates of oil flow. In all tests the supply pressure was $1.7 \times 10^{7} \mathrm{Nm}^{-2}$ ( 170 bar). The results show that the displacement load relationship is not exactly linear and that an increase in volume flow produces an increase in stiffness provided that an adequate pressure drop is maintained across the constant volume valves. Figure 5 shows some test results at three different diametral clearances (clearance based upon lobe and journal diameters). The stiffness falls rapidly as the diametral clearance is increased despite the increase in volume flow. The interrupted line shows calculated stiffnesses for the conditions of the tests, and is based upon the infinitely wide bearing theory. However, as may be derived from Figure 4, a stiffness of $1.2 \times 10^{9} \mathrm{Nm}^{-1}$ was achieved at a load of $6.4 \times 10^{4} \mathrm{~N}$ with a supply pressure of $1.7 \times 10^{7} \mathrm{Nm}^{-2}$, a total volume flow of 0.185 litres $\mathrm{s}^{-1}$ and with a pumping power of 4.5 KW . The diametral clearance was $3 \times 10^{-4} \mathrm{~m}$. There was no metal to metal contact.

The experimentally determined performance exceeds the specification

for stiffness in finishing ( $1.2 \times 10^{9} \mathrm{Nm}^{-1}$ from test of $8 \times 10^{8} \mathrm{Nm}^{-1}$ ), and also closely approaches the specification for pressure on projected area in roughing ( $3.38 \times 10^{6} \mathrm{Nm}^{-2}$ in test cf $3.86 \times 10^{6} \mathrm{Nm}^{-2}$ ).

## 5. Tests with Rotation

### 5.1 Test rig

A layout of the test rig is shown in Figure 6. The shaft is supported by the wide speed range bearing, and by rolling element thrust and tail bearings in the housing on the right hand side. The hydraulic ram on the left is employed to apply side load, via a slave bearing, to the overhang of the shaft. The conception of the rig is that it should correspond as closely as is practicable with an actual spindle; for instance the tail and thrust races have been chosen with machine tool duty in mind, and are mist lubricated. The displacement of the journal with respect to the bearing is measured with inductive displacement transducers at each end of the bearing. Thermocouples are provided to measure oil and bearing metal temperature. Flow meters and pressure gauges are provided at each high pressure oil inlet. A photograph of the test rig with its instrumentation is shown in Figure 7.

### 5.2 Tests and test results

In the first instance the rig was used for tests without rotation of the shaft. A load was applied to the shaft overhang by the hydraulic jack and the displacement of the shaft was measured at the available axial positions. The measured displacements are shown by the circular points in Figure 8. The crosses show displacement as calculated by means of a computer programme in which the wide speed range bearing and the tail bearing were represented by springs of appropriate stiffness. There is substantial agreement between the


FIG. 6 MAIN TEST RIG


FIG. 7 TEST RIG AND INSTRUMENTATIONS

observed and calculated displacement. An implication of Figure 8 is that the extension of the shaft to the tail bearing should be made considerably stiffer because much of the stiffness of the wide speed range bearing is being lost in the flexibility of the shaft itself. The reason why the shaft extension was made with a reduced diameter was to allow a rolling element thrust bearing to be used.

The measured displacement-load relationship, again without rotation, is shown in Figure 9. The magnitudes of the displacements are plotted but the displacements of line $A$ were in fact of oppositesign to those of line $C$; the change in sign reflects the bending of the shaft itself. However, line $B$ may be used to derive a mean stiffness which is $1.2 \times 10^{9} \mathrm{Nm}^{-1}$, and which equals the stiffness as found in the static rig.

The power loss in the bearing when the shaft was rotated has been assessed from the oil flow through the bearing, and from the rise in temperature of the oil. An assessment of the heat loss to the surroundings was made by taking cooling curves. The results are shown in Figure 10 in which the triangular points show the measured power loss as calculated from oil flow and temperature rise, and the circular points have been adjusted to take into consideration the heat loss from the rig. The loss is plotted against speed on a $\log \log$ basis and the experimental points lie on a straight line. The dotted extrapolation indicates the power loss to be expected at high speeds. At the time of writing very limited tests of the stiffness of the bearing when the shaft is rotating have been made. The results indicate a somewhat greater stiffness when rotating than when the journal

$A$ and $C$ measured. $B$ deduced displacement at centre.

FIG. 9 Load displacement relationship


FIG. 10 Variation of the power loss with speed
is stationary and supported only hydrostatically. For instance at a speed of 1000 rpm a mean stiffness of $1.25 \times 10^{9} \mathrm{Nm}^{-1}$ has been found.

## 6. Manufacture

The method of manufacture outlined in paragraph 3 has produced a very satisfactory form but a simpler method is clearly desireable. A method which has so far been tried only experimentally is illustrated by Figure ll. It comprises the elastic distortion of the bearing ring within the three jaws of a chuck. To give the closest approximation to circular lobes the bearing ring is made less rigid in bending at three places by axial grooves machined in its outer surface. The bore of the distorted ring is machined circular, the ring is then released, and its outer surface is turned circular. A roundness trace from an experimental ring cut from a piece of steel tubing is shown in Figure 12. The experiment shows promise.
7. Power Loss

It is inevitable that the power loss in the bearing should be very considerable at high speed. The measured power loss over a limited range of speed has been presented already in Figure 10. Predicted power losses as calculated by the Petroff relationship are given in Figure 13. The calculations were carried out iteratively so that the power loss and the viscosity of the oil at its outlet temperature are self consistent. The measured power losses are shown by the crossed points in Figure 13, and are substantially in agreement with the predicted losses.


| 1 | CHUCK |
| :--- | :--- |
| 2 | JAW |
| 3 | ELASTICALLY DISTORTED RING |
| 4 | CIRCULAR BORE |
|  |  |

FIG.ll Jig for manufacturing method


FIG. 12 OMT chart for a deformed bearing shell


FIG. 13 Variation of the Predicted Power Loss with Speed


#### Abstract

A principal point of the three lobe bearing in contrast with a bearing of circular bore is to achieve the required stiffnesses with the least possible power loss. A comparison of three lobe and circular bore designs of the same mean stiffness has been carried out on the basis that if a particular oil outlet temperature is permissible with a bearing of circular bore then the same oil outlet temperature is also permitted with a three lobe bearing. The comparison indicates that the power loss in the bearing of circular bore will be approximately 1.5 times greater than the loss in a three lobe bearing of the same mean stiffness, that the oil inlet temperature of the circular bore bearing would have to be impracticably low, and that small clearances difficult to provide would be necessary.


## 8. Conclusion

From the tests which have been carried out with the three lobe bearing in its hydrostatic regime and without rotation of the journal, and from the limited tests of the bearing with journal rotation which have been carried out upto the time of writing, it seems probable that the three lobe bearing will substantially satisfy the onerous functional specification of Table l. A point which has become emphasized by the work is that with a bearing of such high stiffness the flexibility of the shaft may become the predominant factor, and shafts of very high stiffness have to be provided if the capability of the bearing is to be realized.

## REFFERENCES

1. SHELDON, D.F. et al. 'A hydrodynamic pocket journal bearing', Proc.Inst.Mech.Eng. 1970, 184 (Part 32), 70-74.
2. ROWE, W.B., KOSEAL, D., STOUT, K.J., "Slot entry bearing for hydrodynamic and hydrostatic operation', Journal of Mechanical Engineering Science, Vol.18, No.2. 1976.
3. PINKUS, O., 'Analysis and characteristics of three lobe bearing', Journal of Basic Engineering, March 1959.
4. MALANOSKI, Stanley B., LOEB, Alfred M. 'The effect of compensation on hydrostatic bearing stiffness'. Journal of Basic Engineering, June 1961.

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#### Abstract

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Mr. H. Lister, Metallurgy.
Mr. E. Cave, Production Technology.
Miss P. Robinson, Metallurgy.


[^0]:    * See 'Terminology'

[^1]:    ${ }^{*} P_{0}$ is the pressure downstream of the control valves when the journal is centred within the bearing.

[^2]:    *The writer designed, drew, assembled and commissioned the rig himself.

[^3]:    * The computer programme has been written by Professor A.W. Crook.

[^4]:    * Reynolds' number corresponds to the critical Taylor's number.

[^5]:    In figures 8.12 and 8.13 the measured and the calculated power loss, oil temperature rise and oil flow are plotted versus the journal speed. The method which has been used in the calculation has previously been described in paragraph 3.8. It may be seen from figures 8.12 and 8.13 that there is close agreement between the measured and calculated power loss, oil flow and oil temperature rise. The agreement implies that laminar conditions persisted throughout, and indeed calculations of the Reynolds' number according to the

[^6]:    'Investigation on the dynamic behaviour of hydrostatic spindle systems', Proc. loth Int. MTDR Conf., 1969, Pergamon Press, Oxford.

[^7]:    'An Experimental Investigation into the temperature effects in hydrostatic journal
    bearings'. Tribology, August 1970.

[^8]:    

[^9]:    *******************************************************************************J I no $53 \mathrm{I}=2$, N
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[^16]:    The bearing and journal were tested for a long period. High oil film resistance was always observed and no evidence that operation was affected by the scoring. The journal and bearing were removed to fit the second bearing, the surface of the first bearing and the

[^17]:    * Professor of Mechanical Engineering at Brunel University.
    ** Research Student, Department of Mechanical Engineering, Brunel University. (Formerly with Helwan University, Egypt).

[^18]:    * Numbers in parentheses refer to the References at the end of the paper.

