Measurement of the ratio of the inclusive 3-jet cross section to the inclusive 2-jet cross section in pp collisions at $\sqrt{s} = 7$ TeV and first determination of the strong coupling constant in the TeV range

The CMS Collaboration

Abstract

A measurement is presented of the ratio of the inclusive 3-jet cross section to the inclusive 2-jet cross section as a function of the average transverse momentum, $\langle p_{T1,2} \rangle$, of the two leading jets in the event. The data sample was collected during 2011 at a proton-proton centre-of-mass energy of 7 TeV with the CMS detector at the LHC, corresponding to an integrated luminosity of 5.0 fb$^{-1}$. The strong coupling constant at the scale of the Z boson mass is determined to be $\alpha_s(M_Z) = 0.1148 \pm 0.0014$ (exp.) $\pm 0.0018$ (PDF) $\pm 0.0050$ (theory), by comparing the ratio in the range $0.42 < \langle p_{T1,2} \rangle < 1.39$ TeV to the predictions of perturbative QCD at next-to-leading order. This is the first determination of $\alpha_s(M_Z)$ from measurements at momentum scales beyond 0.6 TeV. The predicted ratio depends only indirectly on the evolution of the parton distribution functions of the proton such that this measurement also serves as a test of the evolution of the strong coupling constant. No deviation from the expected behaviour is observed.

1 Introduction

As a consequence of the non-Abelian nature of quantum chromodynamics (QCD), the renormalization group equation (RGE) [1–3] predicts that the strong force becomes weaker at short distances corresponding to large momentum transfers, a property of QCD referred to as asymptotic freedom. The strength of the strong force, $\alpha_S(Q)$, at a given distance or momentum scale $Q$ is not predicted and has to be extracted from experiment. Measurements at different $Q$ can then be compared for consistency with QCD via the RGE, which precisely describes the evolution of $\alpha_S(\mu_r)$, where the renormalization scale $\mu_r$ is identified with $Q$. By convention, the consistency is tested by evolving all values of $\alpha_S(Q)$ to the common scale $\mu_r = Q = M_Z$, i.e. the precisely known mass of the Z boson. The current world average value is $\alpha_S(M_Z) = 0.1184 \pm 0.0007$ [4].

Measurements of the *running* of $\alpha_S(Q)$ provide a stringent test of QCD. Previous collider experiments at LEP and HERA have established the validity of the RGE up to momentum transfers $Q$ of 208 GeV [4]. A recent publication by the D0 Collaboration extends this range up to 400 GeV [5]. The determination of $\alpha_S(Q)$ from jet cross sections as in [6] or [7] depends directly on parton distribution functions (PDFs) that have been evolved from small to very high momentum scales via the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [8–10], which assume the validity of the RGE. This dependence on the evolution of the PDFs can be reduced by investigating cross section ratios. The ratio $R_{32}$ of the inclusive 3-jet cross section to the inclusive 2-jet cross section is proportional to $\alpha_S(Q)$ where $Q$ is defined as the average transverse momentum of the two jets leading in $p_T$,

$$Q = \langle p_{T1,2} \rangle = \frac{p_{T1} + p_{T2}}{2}.$$  

Many theoretical systematic uncertainties related to the choice of the renormalization and factorization scales, $\mu_r$ and $\mu_f$, or to nonperturbative effects are reduced in the cross section ratio. In addition, experimental uncertainties such as those due to the jet energy scale largely cancel in the measurement of $R_{32}$. The uncertainty on the integrated luminosity measurement cancels completely. The Compact Muon Solenoid (CMS) Collaboration has previously measured $R_{32}$ [11], and the predictions of various Monte Carlo (MC) event generators were found to be in general agreement with the measurement.

This measurement is performed using a sample of multijet events, collected during 2011 by the CMS experiment at the Large Hadron Collider (LHC), corresponding to an integrated luminosity of 5.0 fb$^{-1}$ of pp collisions at a centre-of-mass energy $\sqrt{s} = 7$ TeV. The transverse momentum $p_T$ and the rapidity $y$ of a jet with energy $E$ and momentum $\vec{p} = (p_x, p_y, p_z)$ (where $p_z$ is the momentum component along the direction of the anticlockwise proton beam) are defined as $p_T = \sqrt{p_x^2 + p_y^2}$ and $y = \frac{1}{2} \ln[(E + p_z)/(E - p_z)]$, respectively. Jets are reconstructed using the infrared- and collinear-safe anti-$k_T$ clustering algorithm [12, 13] with a size parameter of 0.7. This measurement uses jets with $p_{T > 150}$ GeV and $|y| < 2.5$.

The large number of multijet events collected over a wide range of $\langle p_{T1,2} \rangle$, 420 $< \langle p_{T1,2} \rangle < 1390$ GeV, allows $\alpha_S(Q)$ to be determined with only a small dependence on the evolution of the PDFs, thus testing the validity of the RGE in an extended range of transverse momenta.

2 The CMS detector

The central feature of the CMS apparatus is a superconducting solenoid, 13 m in length and 6 m in diameter, providing an axial magnetic field of 3.8 T. The field volume of the solenoid
Event selection and reconstruction

The CMS detector records events using a two-level trigger system consisting of a hardware-based level-1 (L1) trigger and a software-based high level trigger (HLT). In this study, single-jet triggers that reconstruct jets from calorimeter energy deposits at L1 and HLT are used to select events based on three HLT $p_T$ thresholds, 190, 240, and 370 GeV. All except the highest-threshold trigger were prescaled during the 2011 run. The corresponding integrated luminosity $\mathcal{L}$ for each of the three samples is shown in Table 1. The efficiency of each of the triggers is estimated using lower-$p_T$-threshold triggers. These three jet trigger thresholds ensure 100% trigger efficiency in the three jet samples for $\langle p_{T1,2} \rangle > 215, 269, \text{and } 409 \text{ GeV}$.

<table>
<thead>
<tr>
<th>HLT $p_T$ threshold (GeV)</th>
<th>190</th>
<th>240</th>
<th>370</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{L}$ (fb$^{-1}$)</td>
<td>0.15</td>
<td>0.51</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Each event is required to have at least one offline-reconstructed vertex [15] along the beam line that is within 24 cm of the nominal interaction point. The four-vectors of particle candidates reconstructed by the CMS global event reconstruction algorithm (also called particle-flow event reconstruction [16]) are used as input to the jet-clustering algorithm. The clustering is performed by the FASTJET package [13] using four-momentum summation. The global event reconstruction algorithm reconstructs and identifies each particle with an optimized combination of subdetector information. The energy of photons is obtained directly from the ECAL measurements after being corrected for zero-suppression effects. The energy of electrons is determined from a combination of the track momentum at the main interaction vertex, the corresponding ECAL cluster energy, and the energy sum of all bremsstrahlung photons attached to the track. The energy of muons is derived from the corresponding track momentum. The energy of charged hadrons is determined from a combination of the track momentum and the corresponding ECAL and HCAL energies, corrected for zero-suppression effects, and calibrated for the nonlinear response of the calorimeters. Finally, the energy of neutral hadrons is obtained from the corresponding calibrated ECAL and HCAL energies.

Jet energy corrections [17] are derived using simulated events, generated by PYTHIA 6.4.22 [18] and processed through the CMS detector simulation based on GEANT4 [19], and in situ measurements with dijet, photon+jet, and Z+jet events. An offset correction is applied to take into account the extra energy clustered into jets from additional proton-proton interactions within the same or neighbouring bunch crossings (in-time and out-of-time pileup) [17]. Pileup effects
are important only for low-$p_T$ jets and become negligible for jets with $p_T > 200$ GeV. The current measurement is therefore largely insensitive to pileup effects. The jet energy corrections, which depend on the $\eta$ and $p_T$ of the jet, are applied to the jet four-momentum vector as a multiplicative factor. The multiplicative factor is in general smaller than 1.2, approximately uniform in $\eta$, with typical values of 1.1 for jets having $p_T = 100$ GeV and decreasing to 1.0 for higher values of $p_T$.

To suppress nonphysical jets, i.e. jets resulting from noise in the ECAL and/or HCAL calorimeters, tight identification criteria are applied: each jet should contain at least two particles, one of which is a charged hadron, and the jet energy fraction carried by neutral hadrons, photons, muons, and electrons should be less than 90%. These criteria have an efficiency greater than 99% for genuine jets.

The selection of multijet events requires two or more jets with transverse momentum greater than 150 GeV and $|y| < 2.5$. The final sample is extracted by rejecting events if either or both of the leading jets in $p_T$ have $|y| > 2.5$.

4 Measurement of $R_{32}$ and comparison with theoretical predictions

The measured ratio $R_{32}$ as a function of $\langle p_{T1,2} \rangle$ is the ratio of the number of selected inclusive 3-jet events to the number of selected inclusive 2-jet events in each $\langle p_{T1,2} \rangle$ bin. The ratio $R_{32}$ is corrected for detector smearing effects and unfolded to stable-particle level. The unfolding method is the iterative Bayesian method [20], as implemented in the ROOUnfold software package [21]. Unfolding uses a response matrix that maps the true distribution onto the measured one. The response matrix is derived from a simulation, which uses as input the true $R_{32}$ distribution from PYTHIA6 tune Z2 and introduces the smearing effects by taking into account the $\langle p_{T1,2} \rangle$ resolution [17]. After unfolding $R_{32}$ to stable-particle level, the final statistical uncertainties include the correlation among the various $\langle p_{T1,2} \rangle$ bins.

Two main sources of systematic uncertainties on $R_{32}$ are considered. The first is due to the jet energy scale (JES) and the second due to the unfolding.

The JES uncertainty has been estimated to be 2.0–2.5% for particle-flow jets [22], depending on the jet $p_T$ and $\eta$. All mutually uncorrelated JES uncertainty sources are considered following the procedure described in Ref. [23]. The total systematic uncertainty on $R_{32}$ due to the JES uncertainty is 1.2%.

The unfolding method takes into account three different mutually uncorrelated uncertainty sources. The first arises from insufficient knowledge of the simulated inclusive 3-jet and 2-jet $\langle p_{T1,2} \rangle$ spectra, which are used to construct the simulated ratio $R_{32}$ used in the unfolding. The uncertainty is estimated by varying the 3-jet and 2-jet spectra slopes by ±10%. This is a conservative estimate and is motivated by the observed difference in the 3-jet and 2-jet spectra slopes between simulations using the event generators PYTHIA6 tune Z2 and HERWIG++ [24] with its default tune in version 2.3. Simulations of 3-jet and 2-jet spectra using the MadGRAPH [25, 26] event generator are in agreement with those of PYTHIA6. The second uncertainty arises from the insufficient knowledge of the $\langle p_{T1,2} \rangle$ resolution and is estimated by varying it by ±10%. This variation is motivated by the observed difference between data and simulation in the jet energy resolution [17]. Finally, the third uncertainty arises from non-Gaussian components in the $\langle p_{T1,2} \rangle$ resolution and is estimated by adding non-Gaussian tails to the simulation. The overall systematic uncertainty on $R_{32}$ due to unfolding is less than 1%. A potential bias originat-
ing from the unfolding technique is studied by comparing the unfolding result of the Bayesian method with that of the singular-value decomposition (SVD) method [27]. The bias is found to be negligible.

The theoretical calculation of the ratio $R_{32}$ is based on next-to-leading-order (NLO) perturbative QCD (pQCD) calculations multiplied by a nonperturbative factor, which corrects for multiparton interactions (MPI) and hadronization effects. The NLO calculations assume $N_f = 5$ massless quark flavours and are based on the parton-level generator NLOJET++ [28, 29]. The computations with NLOJET++ are performed within the FastNLO framework [30, 31] using the following four PDF sets: NNPDF2.1 [32, 33], ABM11 [34], MSTW2008 [35, 36], and CT10 [37, 38]. In each case the PDF version employing next-to-next-to-leading order (NNLO) evolution code is chosen, and for comparisons the respective default values of $\alpha_s(M_Z)$, which are 0.119, 0.1134, 0.1171, and 0.118, are used. NNPDF2.1, MSTW2008, and CT10 utilize a variable-flavour-number scheme with the maximal number of flavours $N_{f,\text{max}}$ equal to 6, 5, and 5 respectively, while the ABM11 PDF set was developed in a fixed-flavour-number scheme with $N_f = 5$. The renormalization and factorization scales are set to the average transverse momentum $\langle p_T \rangle$.

The nonperturbative effects are estimated using the PYTHIA6 tune Z2 and HERWIG++ tune 2.3 event generators. The chosen MC models feature different descriptions of the phenomena and are representative of the possible values of the nonperturbative corrections. The nonperturbative correction (NPC) factor is defined as the ratio of $R_{32}$ predicted with the nominal generator settings to that obtained with the MPI and hadronization switched off. This factor is calculated considering the average of the two MC generators and has typical values of $\approx 1.02$ for $\langle p_T \rangle = 250$ GeV, decreasing to 1.0 for higher $\langle p_T \rangle$. The uncertainty is considered to be half of the difference between the NPC values obtained using the two MC generators and amounts to $\approx 0.1\%$, leading to a negligible influence on the final result.

Finally, uncertainties due to the renormalization and factorization scales are evaluated by varying from the default choice of $\mu_r = \mu_f = \langle p_{T1,2} \rangle$ between $\langle p_{T1,2} \rangle/2$ and $2\langle p_{T1,2} \rangle$, simultaneously in the numerator and denominator of the ratio $R_{32}$, in the following six combinations: $(\mu_r/\langle p_{T1,2} \rangle, \mu_f/\langle p_{T1,2} \rangle) = (1/2, 1/2), (1/2, 1), (1, 1/2), (1, 2), (2, 1),$ and $(2, 2)$.

Figure 1 presents the measured ratio $R_{32}$ together with NLO predictions using the NNPDF2.1 (top left), the ABM11 (top right), the MSTW2008 (bottom left), and the CT10 (bottom right) NNLO PDF sets. The upper panel of each plot shows the ratio $R_{32}$ (solid circles) together with the NLO prediction (solid line) corrected for nonperturbative effects, the scale uncertainty, and the PDF uncertainty. At the bottom of each plot, the ratio of data over theory is shown together with bands representing the scale (dotted lines) and PDF uncertainties (solid lines). The error bars in the figure correspond to the total uncertainty, for which the statistical and systematic uncertainties are added in quadrature. For each PDF set the respective default value of $\alpha_s(M_Z)$ is used in this comparison as indicated.

The measured ratio rises with increasing $\langle p_{T1,2} \rangle$ as the phase space opens up for the production of a third jet, reaching a plateau value for $600 < \langle p_{T1,2} \rangle < 1000$ GeV. At higher $\langle p_{T1,2} \rangle$ $R_{32}$ decreases again because of the running of $\alpha_s$, smaller gluon fractions in the total parton luminosity, and because 3-jet configurations reach kinematic limits earlier than dijet events.

Scale uncertainties have a very similar behaviour for all PDF sets and dominate the region up to $\langle p_{T1,2} \rangle \approx 400$ GeV. A comparison to jets with a size parameter of $R = 0.5$ reveals consistent results but with larger scale uncertainties.

The PDF uncertainties are different for each individual PDF set. The CT10 set exhibits the
Figure 1: Measurement of $R_{32}$ and NLO predictions using the NNPDF2.1 (top left), the ABM11 (top right), the MSTW2008 (bottom left), and the CT10 (bottom right) NNLO PDF sets. In the upper panel of each plot, the ratio $R_{32}$ (solid circles) together with the NLO prediction (solid line) corrected for nonperturbative effects (NPC), the scale uncertainty, and the PDF uncertainty are shown. The bottom panels show the ratio of data to the theoretical predictions, together with bands representing the scale (dotted lines) and PDF (solid lines) uncertainties. The error bars correspond to the total uncertainty. For each PDF set the respective default value of $\alpha_S(M_Z)$ is used as indicated.
largest PDF uncertainties, which are of the order of 2% at $\langle p_{T1,2} \rangle = 400$ GeV, increasing to 2.5% in the 1 TeV region. For the NNPDF2.1 set PDF uncertainties are of the order of 1.5% at 400 GeV, increasing to 2.3% at 1 TeV. Finally, for the MSTW2008 and ABM11 sets PDF uncertainties are of the order of 1% throughout the range of this measurement.

The comparison of data with the predictions of pQCD in Fig. 1 demonstrates that the NLO calculations using the NNPDF2.1, MSTW2008, and CT10 PDF sets are in agreement with the measured ratio $R_{32}$ throughout the range of this measurement. The NLO result employing the ABM11 PDF set underestimates $R_{32}$, especially for $\langle p_{T1,2} \rangle < 600$ GeV.

5 Determination of $\alpha_S(M_Z)$

The measurement of the ratio $R_{32}$ is used for the determination of the strong coupling constant $\alpha_S(M_Z)$. Figure 2 shows the predictions using the NNPDF2.1 (top left), the ABM11 (top right), the MSTW2008 (bottom left), and the CT10 (bottom right) NNLO PDF sets for a series of values of $\alpha_S(M_Z)$, together with the measured $R_{32}$. The $\alpha_S(M_Z)$ value is varied in the range 0.106–0.124, 0.104–0.120, 0.107–0.127, and 0.110–0.130 in steps of 0.001 for the NNPDF2.1, ABM11, MSTW2008, and CT10 PDF sets, respectively.

From Fig. 2 one observes that the sensitivity of the ratio $R_{32}$ to variations of the strong coupling by $\Delta \alpha_S(M_Z) = \pm 0.001$ is different for each of the four PDF sets. This translates into differences in the experimental uncertainty in the value of $\alpha_S(M_Z)$ obtained for each PDF set.

The value of $\alpha_S(M_Z)$ is determined by minimizing the $\chi^2$ between the experimental measurement and the theoretical predictions. The $\chi^2$ is defined as

$$\chi^2 = M^T C^{-1} M,$$  \hspace{1cm} (2)

where $M$ is the vector of the differences between the data ($R_{32}^i$) and the theoretical values ($T_{32}^i$) in each bin $i$,

$$M^i = R_{32}^i - T_{32}^i,$$  \hspace{1cm} (3)

and $C$ is the covariance matrix including all experimental (statistical, JES, and unfolding) uncertainties. $C$ is defined as

$$C = \text{Cov}^{\text{Stat}} + \sum \text{Cov}^{\text{JES Sources}} + \sum \text{Cov}^{\text{Unfolding Sources}},$$  \hspace{1cm} (4)

where $\text{Cov}^{\text{Stat}}$ is the statistical covariance matrix that accounts for the correlations due to unfolding, and $\text{Cov}^{\text{JES Sources}}$, $\text{Cov}^{\text{Unfolding Sources}}$ are the covariance matrices that account for the JES and unfolding systematic uncertainty sources, respectively. Each systematic uncertainty source for the JES and unfolding is treated as 100% correlated across the $\langle p_{T1,2} \rangle$ bins.

To avoid the region with large scale uncertainties close to the minimal jet $p_T$ requirements, visible in Fig. 1, $\alpha_S(M_Z)$ is extracted only for $\langle p_{T1,2} \rangle > 420$ GeV. The central result is obtained by minimizing the $\chi^2$ (Eq. 2) with respect to $\alpha_S(M_Z)$ for the NNPDF2.1 PDF set, which is the only one that permits the propagation of PDF uncertainties to the fits for each value of $\alpha_S(M_Z)$. The experimental uncertainties are obtained from the $\alpha_S(M_Z)$ values for which $\chi^2$ is increased by one with respect to the minimum value. The result of a fit to the region of 420–1390 GeV is

$$\alpha_S(M_Z) = 0.1148 \pm 0.0014 \text{ (exp.)},$$  \hspace{1cm} (5)
Figure 2: The NLO predictions using the NNPDF2.1 (top left), the ABM11 (top right), the MSTW2008 (bottom left), and the CT10 (bottom right) NNLO PDF sets for a series of values of $\alpha_S(M_Z)$, together with the measured $R_{32}$. The $\alpha_S(M_Z)$ value is varied in the range 0.106–0.124, 0.104–120, 0.107–0.127, and 0.110–0.130 in steps of 0.001 for the NNPDF2.1, ABM11, MSTW2008, and CT10 PDF sets, respectively.
with $\chi^2/N_{\text{dof}} = 22.0/20$ at minimum. The experimental uncertainty contains the statistical, JES, and unfolding sources (Eq. (4)), with the JES uncertainty being the dominant one.

The contribution of PDFs to the uncertainty of the measurement is evaluated by repeating the fit for each of the 100 PDF replicas of the NNPDF2.1 set at the relevant value for $\alpha_S(M_Z)$. In this way 100 determinations of $\alpha_S(M_Z)$ are obtained, whose distribution corresponds to the propagation of the underlying probability density from the PDFs to the fitted strong coupling. The PDF uncertainty of the measurement is then computed as the standard deviation of this distribution. A more detailed description of the method can be found in Ref. [39].

The uncertainties due to the renormalization and factorization scales are treated separately by varying the default choice of $\mu_r = \mu_f = \langle p_{T1,2} \rangle$ between $\langle p_{T1,2} \rangle/2$ and $2\langle p_{T1,2} \rangle$ in six combinations as explained in Section 4. The $\chi^2$ minimization with respect to $\alpha_S(M_Z)$ is repeated for these six combinations. The contribution from the $\mu_r, \mu_f$ scale variations to the uncertainty in the measurement is evaluated by considering the differences between the NNPDF2.1 $\alpha_S(M_Z)$ central value and the highest and lowest values found in these six scale combinations. Out of all scale combinations the lowest $\alpha_S(M_Z)$ value corresponds to the default scale choice of $\mu_r = \mu_f = \langle p_{T1,2} \rangle$ and the highest to the scale choice of $\mu_r = \mu_f = \langle p_{T1,2} \rangle/2$. The frequent observation of asymmetric scale uncertainties with larger downward uncertainties in the case of NLO cross sections is transformed into a purely upward uncertainty for the ratio, as can be seen in Table 2.

Table 2: The values of $\alpha_S(M_Z)$ at the central scale and for the six scale factor combinations.

<table>
<thead>
<tr>
<th>$\mu_r/\langle p_{T1,2} \rangle$</th>
<th>$\mu_f/\langle p_{T1,2} \rangle$</th>
<th>$\alpha_S(M_Z) \pm \text{(exp.)}$</th>
<th>$\chi^2/N_{\text{dof}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$1$</td>
<td>$0.1148 \pm 0.0014$</td>
<td>$22.0/20$</td>
</tr>
<tr>
<td>$1/2$</td>
<td>$1/2$</td>
<td>$0.1198 \pm 0.0021$</td>
<td>$30.6/20$</td>
</tr>
<tr>
<td>$1/2$</td>
<td>$1$</td>
<td>$0.1149 \pm 0.0014$</td>
<td>$22.2/20$</td>
</tr>
<tr>
<td>$1$</td>
<td>$1/2$</td>
<td>$0.1149 \pm 0.0014$</td>
<td>$22.2/20$</td>
</tr>
<tr>
<td>$1$</td>
<td>$2$</td>
<td>$0.1150 \pm 0.0015$</td>
<td>$21.9/20$</td>
</tr>
<tr>
<td>$2$</td>
<td>$1$</td>
<td>$0.1159 \pm 0.0014$</td>
<td>$20.7/20$</td>
</tr>
<tr>
<td>$2$</td>
<td>$2$</td>
<td>$0.1172 \pm 0.0018$</td>
<td>$21.3/20$</td>
</tr>
</tbody>
</table>

A cross check on the impact of the top quark by imposing $N_f = 6$ massless flavours in the NLO matrix elements revealed an increase by $+0.0009$ in the fitted value of $\alpha_S(M_Z)$. Further effects, for example from the evolution of $\alpha_S$ and the PDFs with five or six flavours, multijet production via fully hadronic decays in the reaction $pp \rightarrow t\bar{t} + X$, or an incomplete cancellation of electroweak corrections between numerator and denominator, are estimated to contribute each at a $\pm 1\%$ level to the theoretical uncertainty. These residual effects are taken into account by symmetrizing the scale uncertainty such that the largest deviation is adopted as the total symmetric theory uncertainty.

The final result is

$$\alpha_S(M_Z) = 0.1148 \pm 0.0014 \text{(exp.)} \pm 0.0018 \text{(PDF)} \pm 0.0050 \text{(theory)},$$

in agreement with the world average value of $\alpha_S(M_Z) = 0.1184 \pm 0.0007$ [4], with the Tevatron results [5, 6, 40], and a recent result obtained with LHC data [7].
The determination of $\alpha_S(M_Z)$, which is based on the NNPDF2.1 PDF set, is also in agreement with the results obtained using the MSTW2008 or CT10 PDF sets.

\begin{align*}
\text{MSTW2008: } \alpha_S(M_Z) &= 0.1141 \pm 0.0022 \text{ (exp.)}, \\
\text{CT10: } \alpha_S(M_Z) &= 0.1135 \pm 0.0019 \text{ (exp.)},
\end{align*}

with $\chi^2/N_{\text{dof}} = 20.6/20$ and $21.1/20$, respectively. If PDF sets with NLO evolution are used instead the impact on the results of the fits to the ratio observable $R_{32}$ is negligible. This is in contrast to fits to cross sections, where NNLO PDF sets usually lead to smaller values of $\alpha_S(M_Z)$ than NLO ones, and confirms the reduced dependence of $R_{32}$ on details of the PDF evolution.

In the case of the ABM11 PDFs the series in values of $\alpha_S(M_Z)$ ends at 0.120, which is insufficient for a derivation of the complete shape of the $\chi^2$ curve at minimum such that a fit value for $\alpha_S(M_Z)$ including uncertainties can only be extrapolated to give around $\alpha_S(M_Z) = 0.1214 \pm 0.0020$ (exp.) with $\chi^2/N_{\text{dof}} = 20.6/20$. For the ABM11 PDF set at NLO with a default value of $\alpha_S(M_Z) = 0.118$, the series in $\alpha_S(M_Z)$ values ends at 0.130 such that a fit can be performed which yields $\alpha_S(M_Z) = 0.1214 \pm 0.0018$ (exp.), consistent with the extrapolation above. The fit exhibits, however, a somewhat larger value of $\chi^2/N_{\text{dof}} = 28.5/20$ compared to the other results.

It is observed that with ABM11 PDFs a higher value of $\alpha_S(M_Z)$ is preferred. This is in accord with the fact that the ABM11 gluon density in the phase space relevant for this analysis is significantly smaller than that of all other PDF sets. Thus, the fit favors a larger $\alpha_S(M_Z)$ value to compensate for this effect. In summary, the ABM11 PDF set does not describe the data as well as the alternative PDF sets, as shown in Fig. [1] which leads to an inferior fit quality and a less consistent result for the strong coupling.

To investigate the running of the strong coupling constant in more detail, the fitted region of $420–1390 \text{ GeV}$ is split into three bins of $\langle p_{T1,2} \rangle$ and the fitting procedure is repeated in each of these bins. The three separate extractions of $\alpha_S(M_Z)$ are reported in Table 3. The experimental uncertainties in the three obtained values are correlated. These $\alpha_S(M_Z)$ determinations are then evolved back to the corresponding values $\alpha_S(Q)$ using the 3-loop solution to the RGE from the NNPDF2.1 set. For each fit region the cross-section-weighted average of $\langle p_{T1,2} \rangle$ from the inclusive dijet calculation at NLO with NLOJET++ is chosen as the momentum scale $Q$ and is computed to be $Q = 474, 664, \text{ and } 896 \text{ GeV}$, respectively. These values, derived again with the FASTNLO framework, are identical within about 1 GeV for different PDFs and vary at most by a few GeV when using inclusive 3-jet events.

To emphasize that theoretical uncertainties limit the achievable precision, Table 4 presents the decomposition of the total uncertainty for the three bins in $\langle p_{T1,2} \rangle$ into the experimental, PDF, and theory components.

Figure 3 presents the strong coupling $\alpha_S(Q)$ (solid line) and its total uncertainty (band) as evolved from the CMS determination, $\alpha_S(M_Z) = 0.1148 \pm 0.0055$, using the 3-loop solution to the RGE from NNPDF2.1, as before. The extractions of $\alpha_S(Q)$ in three separate ranges of $Q$ as presented in Table 3 are also shown. In the same figure the values of $\alpha_S$ at lower scales determined by the H1 [41, 42], ZEUS [43], and D0 [5, 40] Collaborations are shown for comparison. The results on $\alpha_S$ reported here are consistent with the energy dependence predicted by the RGE and extend the range, in which the RGE is tested, to the region of several hundred GeV.
Figure 3: The strong coupling $\alpha_S(Q)$ (solid line) and its total uncertainty (band) evolved from the CMS determination $\alpha_S(M_Z) = 0.1148 \pm 0.0055$ using a 3-loop solution to the RGE as a function of the momentum transfer $Q = \langle p_{T1,2} \rangle$. The extractions of $\alpha_S(Q)$ in three separate ranges of $Q$ as presented in Table 3 are shown together with results from the H1 [41, 42], ZEUS [43], and D0 [5, 40] experiments at the HERA and Tevatron colliders.
6 Summary

The ratio $R_{32}$ of the inclusive 3-jet cross section to the inclusive 2-jet cross section, for jets with $p_T > 150$ GeV and $|y| < 2.5$, has been measured in the range $250 < \langle p_{T1,2} \rangle < 1390$ GeV for proton-proton collisions at a centre-of-mass energy of 7 TeV. The results have been compared with predictions of QCD at NLO obtained with various PDF sets. The NLO calculations using the NNPDF2.1, MSTW2008, and CT10 NNLO PDF sets are in agreement with the measured ratio $R_{32}$ throughout the range of $\langle p_{T1,2} \rangle$ studied. However, calculations using the ABM11 PDF sets underestimate $R_{32}$ for $\langle p_{T1,2} \rangle < 600$ GeV.

Measurements of $R_{32}$ over the range $420 < \langle p_{T1,2} \rangle < 1390$ GeV have been used to determine the strong coupling constant $\alpha_S$ at the scale of the Z boson mass. The final result is

$$\alpha_S(M_Z) = 0.1148 \pm 0.0014 \text{ (exp.)} \pm 0.0018 \text{ (PDF)} \pm 0.0050 \text{ (theory)} = 0.1148 \pm 0.0055,$$

where experimental, PDF, and theory uncertainties have been added quadratically to give the total uncertainty. The result is in agreement with the world average value of $\alpha_S(M_Z) = 0.1184 \pm 0.0007$ and represents the first determination of the strong coupling constant from jet measurements with momenta of the order of 1 TeV. The dominant uncertainties are of theoretical origin and limit the currently achievable precision. The predicted ratio depends only indirectly on the evolution of the parton distribution functions of the proton and consequently this measurement also serves as a test of the evolution of $\alpha_S(Q)$. No deviation from the expected behaviour is observed.

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