Optimal Monetary Policy and the Asset Market:

A Non-cooperative Game

Christos Ioannidis,
Oreste Napolitano

Department of Economics and Finance,
Brunel University, UK

December, 2003

ABSTRACT

In this paper we construct a model of a policy game in order to analyse the optimal reaction function of the Central Bank to a shock in the asset market. In doing so, we consider three different non-cooperative games: Nash equilibrium, Stackelberg equilibrium with “FED” as leader and “ECB” Stackelberg as leader. Three major conclusions can be drawn from our work in the presence of asset market shocks. First, in the Nash equilibrium the ECB will adopt a less restrictive monetary policy compared to the FED’s behaviour. Second, comparing the Nash and Stackelberg non-cooperative equilibria, the Stackelberg solution is certainly superior when the FED is the leader, but the Nash solution is superior for the follower. Finally, irrespective of where the shocks originate, if the FED would choose the Stackelberg leader equilibrium the ECB would minimize its social loss along with a lower level of interest rates.

Keywords: Monetary Policy, Non Cooperative Game, Asset Market.
JEL Classification: E58, E61,C72

1 We are indebted to Virginie Boinet, Floro Ernesto Caroleo and Norma Sephuma for their most valuable comments and suggestions. Any remaining errors are sole responsibility of the authors.
Corresponding author: Oreste Napolitano, Brunel University Uxbridge, Middlesex, UB8 3PH.
oreste.napolitano@brunel.ac.uk.
1. Introduction

The purpose of this paper is to analyse interactions among monetary policymakers in the presence of shocks in asset markets. This analysis will be undertaken in the context of a simple theoretical game with no uncertainty. In this framework the concepts of co-ordination, co-operation and commitment between two countries are fundamental in the evaluation of the resulting policy rules that will emerge under different behavioural assumptions regarding the relationship of the monetary authorities with each other.

In particular, we address the following issues: the impact on the Central Bank’s policy response to a shock in the asset market and how the resulting policy change in one country will affect both the Central Bank response and the asset market in the other country.

The first step in providing answers to the question considered above is to describe how asset markets respond to a monetary policy change initiated either by the home or foreign Central Bank. It is evident that financial markets’ responses to monetary policy actions undertaken by either the home or foreign Central Bank depend on a combination of domestic and foreign influences. These influences manifest themselves through two channels. The first and most immediate relates to movements in the quoted prices such as exchange rates and interest rates in the international money, capital and foreign exchange markets. The second channel is due to changes in domestic real activity and prices. These channels have both direct effects and indirect effects on the economy, and the latter can partially or totally offset the initial effects of the former. For example, changes in equilibrium prices will affect both private incomes and wealth. The existence of a wealth effects associated with asset market fluctuations has been analysed by Morck, Shleifer and Vishny (1990), Goodhart and Hoffman (2000) and Mishkin (2001) and is beyond dispute. A fall in asset market prices due to restrictive monetary policy will erode personal wealth. In addition, lower asset prices are associated with lower private sector investment resulting in greater employment uncertainty and lower confidence, particularly because layoffs typically increase during such periods, so that individuals will stop spending. Since consumption represents a great percentage of GDP, even small changes in consumer spending could affect economic growth.

Higher inflation due to lax monetary policy can have a negative impact on the asset market, because increasing inflation results in moderating long-term interest rates, thus reducing the present value of future profits. In addition, as higher inflation is normally associated with variable inflation, this has a further negative effect on the firms because typically it incites investors to demand higher risk premiums. This takes the form of increased spreads of corporate bond and commercial paper interest rates relative to Treasury yields.

The present paper uses a formal model within a policy game in order to analyse an optimal reaction of the Central Bank to a shock in the asset market. We consider two countries,
“USA” and “Europe”, and different games in which we assume that both central banks react to a shock in their asset markets.

The structure of the paper is as follows: section 2 reviews the literature. The model is developed and the basic results are derived in section 3. Section 4 analyses the impact of shocks in the asset markets and their effects on monetary policy and discusses the welfare implications of different forms of non-cooperative behaviour. Section 5 contains the conclusions.

2. Review of the literature

Beginning with early work by Hamada (1976), a large number of studies have analyzed strategic interactions between monetary authorities. It has been argued that cooperation is Pareto efficient (Carzoneri and Gray, 1985) but it is also well known that enforcing the cooperative outcome is unlikely (Persson and Tabellini, 1995).

Hamada (1976 and 1985) analysed the interactions between monetary policy and exchange regimes. He conducted the analysis within a static monetary approach to the balance of payments with fixed exchange rates. In these models, it was shown that the non-cooperative solutions were inferior compared to the coordinated equilibrium, as the latter was located on the Pareto contract curve. This provides the classical argument for benefits from coordination in Hamada's (1976) seminal article: all countries could do better by agreeing not to try to export inflation.

Canzoneri & Gray (1985) reached the conclusion that in regimes with externalities coordination is desirable because of improved welfare for both countries than the Nash or Stackelberg equilibria. This result is derived by analysing the impact of the same exogenous shock in two groups of counties, the US and the rest of the world. However, they also showed that there is a special case in which non-cooperation is Pareto optimal. This is the case when, following a common supply shock in a symmetric model, one of the policymakers is acting as a fixed-exchange-rate leader. The paper of Walsh (1998) reaches similar conclusions.

One of the most prominent critiques to this conclusion is that of Rogoff (1985), who showed that policy cooperation between countries may be counterproductive if there are domestic credibility problems. He augments a two-country model with a Barro-Gordon (1983) dynamic inconsistency problem, and shows that the cooperative outcome reduces welfare compared to the non-cooperative one when the policy commitment is considered infeasible by the private sector. Kehoe (1991), and Carraro and Giavazzi (1991) rejected Rogoff's (1985) point of view, presenting a counter example. However in these models, there are questions about credibility and inter-temporal inconsistency, as the assumption of the existence of a common strategy between the private agents and the government cannot be justified.
In the last decade it has been recognised that asset prices play an important role in determining business cycles conditions. A significant impact can be found in the role that capital markets play in the modern economic environment. Their impact has gone beyond indirect intermediation; it has a direct effect on activity due to both the deepening and widening of the capital markets. The existence of a wealth effect associated with asset market fluctuations has been analysed by Morck, Shleifer and Vishny (1990), Goodhart and Hoffman (1999) and Mishkin (2001) in the recent literature. In the Dynan and Maki (2001) study that analyses the response of individual households to changes in stock market wealth, it was found that, over the period 1983-1999, there was a positive relationship between spending of U.S. households that own stocks and movements in the stock market. A second study by Maki and Palumbo (2001) has estimated that, in the second half of the 1990s, US households with high levels of income showed the largest consumption increases, consistent with the fact that these households owned the most stocks and experienced the largest gains in wealth.

Although the statistical link between asset prices and output is not well established (Poterba, 2000, Poterba and Samwick, 1996), it is impossible to negate that an increasing part of households’ wealth is locked into the stock market and that at the same time the amount of firms’ external financing has increased as never before. With such a central role for asset prices, it is essential for the monetary authorities that pursue an inflation target to take them into consideration, as they will affect aggregate demand. This does not assert that the Central Bank should target asset prices, but it implies that they should be considered for their effect on inflation indirectly via their impact on private sector spending.

In the subsequent section we will develop a model of strategic interaction between two monetary authorities and will allow for an explicit role of the asset markets in the structure of the economy in the light of the above discussion.

3. The model

Following the pioneering contributions of Hamada (1976, 1985), Canzoneri and Gray (1985), Cooper (1985), Canzoneri and Henderson (1991) and more recently Lambertini (1998), Frowen and Karakitsos (2000) and Lambertini and Rovelli (2001), we develop a formal model within a policy game in order to analyse an optimal reaction of the Central Bank to a shock in the asset market. In doing this, we consider different games in which we assume that both central banks, Federal Reserve and European Central Bank (FED and ECB henceforth) react to a shock in their asset markets.
We assume that monetary policy is the result of equilibria of a policy game. Some games are co-operative and others non co-operative\(^2\). Central banks react to each other on the basis of some knowledge of the interdependence of their various policies. We consider three different games with various equilibria: the first two are based on Stackelberg equilibria, and the third is based on Nash equilibrium. In the first we consider that the Federal Reserve is the leader and the ECB is the follower. In the second we reverse the role. The third is based on a Nash equilibrium that implies also a non-co-operative solution\(^3\).

The subsequent analysis is based on the following assumptions:

1) The world exists for a single period or one-shot game and consists of two countries: “United States” and “Europe”;
2) each central bank optimises an objective function that penalises deviations of inflation from target, and output gap. Of crucial importance is the weight that the central bank attaches to each component of the objective function. Hence, in the present paper the FED is assumed, according to its mandate, to be balanced in its pursuit of monetary policy and this implies that the weights attached to inflation and growth are the same; while, the ECB is tied by its narrow mandate to maintain price stability and this implies that the weight attached to inflation is higher than that of the FED;
3) the impact of an increase in the price of imported raw materials (e.g. oil price) has a stronger impact on the European economy compared to the US economy;
4) the spillover effect of the FED’s monetary policy on Europe is more pronounced than that of the ECB on the US;
5) asset market fluctuations impact future consumption choices and consequently future rates of inflation;
6) in accordance with Bernanke and Gertler (1999) we assume that central banks do not respond to asset prices movements unless they affect their inflation forecast;

\(^2\) We adopt the terminology of Canzoneri and Henderson where coordination refers to the way policymakers settle on one solution out of several in a non-cooperative game. (1991, page 4)
\(^3\) In general, the problem with any cooperative game is that policymakers have an incentive to cheat. Implicit in any cooperative game structure is the ability of policymakers to commit to binding agreements.
A comparison of the outcome of the non-cooperative Nash equilibrium with the equilibrium in which the policymakers cooperate, and the public expect cooperation, confirms the following proposition:
Rogoff proposition: under complete information, policy cooperation lowers welfare. Hence, international policy cooperation is counterproductive.
However, the above proposition has been criticized by Carraro and Giavazzi (1991). In fact, they affirm that is the policymakers have the choice to cooperate or not, the not-cooperative equilibrium is not sub-game perfect. The Carraro-Giavazzi proposition is: assume complete information. If the central bank can sign binding agreements to cooperate, the non-cooperative Nash equilibrium is not a sub-game perfect equilibrium of the sequential game.
The proof of the above proposition is that once the public has formed its expectation, there are only two players left. In this context, co-operation between them is unambiguously superior.
We present a simple symmetric model\(^4\) which is the static equivalent of a conventional aggregate demand–aggregate supply model augmented with the asset market. Hamada (1979) called this approach “strategic” because it “is based on the joint reactions and counteractions of each participating country”\(^5\).

The model describing the US economy that the FED uses consists of the following\(^6\):

\[
\begin{align*}
\pi_t &= \pi_t^\ast + \alpha(\gamma_1 - \pi_t^\ast) + \alpha_{ia} X \\
\gamma_t &= \gamma_1 - \beta(\gamma_t - \pi_t^\ast) + \beta_t S_t \\
S_t &= -\gamma(\gamma_t - \pi_t^\ast) - \gamma_{ia}(\gamma_t - \pi_t^\ast) + \gamma_2 (\gamma_t - \pi_t^\ast) + \epsilon_t
\end{align*}
\]

Equation (1) is the Phillips curve where inflation “\(\pi\)” will increase or decrease relative to the target level \(\pi^\ast\) in response to positive/negative values of output and imported inflation. In the absence of shocks \(\pi\) is also its expected value.

Equation (2) links the output gap to the domestic interest rates and the asset market \((S_t)\).

\(r_t - \pi_t\) (\(\Delta r_t\) henceforth) is the deviation of the interest rate from its equilibrium value, that is, the value that ensures the Bank’s loss function is at the bliss point.

Equation (3) describes the behaviour of the asset market. It allows for both the domestic and foreign interest rates to influence the value of the asset market\(^7\). Both interest rates (domestic and foreign) have a negative impact on the asset markets and this can be rationalized as follows. A rise in the domestic interest rate has a negative effect because higher interest rates decrease investment and subsequently aggregate demand. Meanwhile, a rise in the foreign interest rate will have a contractionary effect on the foreign economy thus reducing exports to that economy. The reduction in profits of the domestic firms will lead investors to expect a decrease in domestic asset prices.

Moreover, in eq. (3) an increase in output will boost the profits of firms, which in turn causes an increase in the asset values. Finally, we consider an unexpected shock in the asset market denoted by \((\epsilon_t)\).

Under these assumptions, if a country is experiencing an unexpected increase in the value of the asset market, the central bank could use the interest rate (e.g. increase) in order to “cool down” the market.

---

\(^4\) The size of the Europe economy is almost the same of the US economy. Hence, Europe and the United States are symmetric to each another. The structural parameters for the two countries are equal.

\(^5\) Hamada (1979), pag. 299.

\(^6\) Subscript 1 denotes FED and subscript 2 denotes ECB.

\(^7\) The fundamental question about the relation between interest rates and asset prices hinges on the relation between money tomorrow and money today. A stock share (or some other asset) represents a claim to receive some amount of money tomorrow.
The equivalent model for the EU used by the ECB is:

\[
\pi_2 = \pi + \alpha(y_2 - \bar{y}_2) + \alpha_{le}X + \epsilon_2
\]  

(4)

\[
y_2 = \bar{y}_2 - \beta_2(r_2 - \bar{r}_2) + \beta_2S_2
\]  

(5)

\[
S_2 = -\gamma(r_2 - \bar{r}_2) - \gamma_{le}(\rho - \bar{r}_2) + \gamma_2(y_2 - \bar{y}_2) + \epsilon_2
\]  

(6)

Since we are assuming a symmetric model, equations (4)-(6) follow the same descriptions we made above.

We assume that both shocks \(e_1\) and \(e_2\) are independent and identically distributed (iid) with zero mean and constant variance. An unexpected shock in the asset market implies a persistent deviation from its long-run equilibrium. For the sake of simplicity, we assume that only policy makers can observe the shocks in real time. This allows us to avoid additional terms describing surprises for the economic agents.

In each model the following restrictions apply. All the coefficients are positive but less than one. The following inequality, \(\gamma > \gamma_{1e}\), implies that domestic monetary policy has a larger impact on the domestic asset market than on the foreign one. The second inequality, \(\beta > \beta_1\), implies that the effect on output of domestic monetary policy exceeds the wealth. The third inequality \(\alpha_{le} > \alpha_{le}\) implies that an increase in the price of raw materials has a greater impact on the European economy compared to the US. The fourth assumption regarding the comparative spillover effect is expressed by the following inequality, \(\gamma_{1e} > \gamma_{le}\).

In both models the transmission of monetary impulses operates through one main channel: the interest rate. More precisely, the effects of a monetary contraction have a negative impact on the domestic asset market. This causes a further decrease in output due to the wealth effect in addition to the contraction of aggregate demand. As output contracts domestic inflation decreases.

The general form of the loss function of the central banks is given by:

\[
L_i = \frac{1}{2} \left[ w_i (\pi_i - \pi)^2 + (y_i - \bar{y})^2 \right]
\]  

(7)

where the subscript ‘i’ refers to the country. \(w_i\) is the degree of inflation aversion that policymakers attach to inflation. When \(w_i\) is one, the central bank is balanced in its pursuit of monetary policy with respect to the two conflicting targets of inflation and unemployment.
The higher the value of the weight \((w_i)\) associated with the inflation deviation, the greater the bank’s inflation aversion. \(\bar{y}\) denotes the potential output and \(\bar{p}\) the inflation rate that corresponds to the potential output.

The two bliss points are consistent with the level of potential output and inflation target such that \(L_i = f[\bar{\pi}_i, \bar{y}_i, \bar{r}_i] = 0\).

Subject to equation (7), each central bank optimises its own objective function with respect to the economic models, eq. (1)-(6), which allows for the interdependence of these economies.

The optimal combination of policy instruments is achieved, for each country, when the loss function is maximized subject to the economic model. In this game, the choice of a single central bank is conditioned (taking as given) on the choices of the other.

The following equation describes the optimal monetary policy for each country:

\[
\frac{\partial L_i}{\partial \Delta r_i} = \frac{\partial L_i}{\partial \Delta \pi_i} \left[ \frac{\partial \Delta \pi_i}{\partial \Delta y_i} \frac{\partial \Delta y_i}{\partial \Delta r_i} + \frac{\partial \Delta \pi_i}{\partial \Delta r_i} \right] + \frac{\partial L_i}{\partial \Delta y_i} \left[ \frac{\partial \Delta y_i}{\partial \Delta r_i} + \frac{\partial \Delta y_i}{\partial \Delta r_i} \frac{\partial S_i}{\partial \Delta r_i} \right] = 0 \tag{8}
\]

Equation (8) describes the solution and can be applied to all cases in the subsequent discussion. Substituting eq. (1)-(3) into eq.(7) and setting the partial derivative equal to zero as in eq.(8), yields the first-order condition and the following FED reaction function:

\[
\Delta r_1 = A_1 \Delta r_2 + A_2 \tag{9}
\]

where \(A_1 = -\frac{\beta_1 \gamma_{1w}}{\beta + \beta_1 \gamma} < 0\), \(A_2 = \frac{\beta_1}{\beta + \beta_1 \gamma} \epsilon_1 + \frac{(1 - \beta_1 \gamma_2) \alpha \alpha_{lw}}{\alpha^2 + 1} \beta_1 \gamma_{1w} X > 0\).

Under the assumption that the model is defined as deviations from full employment, the loss function of the central banks should be equal to zero if there are no shocks in the system. That is, both economies are at their bliss points.

The equivalent reaction function for the ECB is:

\[
\Delta r_1 = B_1 \Delta r_2 + B_2 \tag{10}
\]

where \(B_1 = -\frac{\beta + \beta_1 \gamma}{\beta_1 \gamma_{1w}} < 0\), \(B_2 = \frac{1}{\gamma_{1w}} \epsilon_2 + \frac{(1 - \beta_1 \gamma_2) \alpha \alpha_{lw}}{\alpha^2 + 1} \beta_1 \gamma_{1w} X > 0\).

From the above solution we derive the following conclusion:

\footnote{The model has been simplified somewhat here in order to focus on asset market.}
Proposition 1

Given that \( \gamma > \gamma_{lu}, \gamma_{le} ; \beta > \beta_1 ; \alpha_{le} > \alpha_{lu} \) and \( \gamma_{lu} > \gamma_{le} \), the reaction function of the ECB is steeper than the reaction function of the FED, and the ECB intercept lies above that of the FED.

Proof:

From equations (9) and (10) we compare the slopes and the intercepts of both reaction functions; given our assumptions ( \( \gamma > \gamma_{lu}, \gamma_{le} ; \beta > \beta_1 ; \alpha_{le} > \alpha_{lu} \) and \( \gamma_{lu} > \gamma_{le} \) ) we conclude that for the intercepts the following inequality holds:

\[
\frac{\beta + \beta_1 \gamma}{\beta_1 \gamma_{lu}} > \frac{\beta_1 \gamma_{le}}{\beta + \beta_1 \gamma}
\]

Furthermore, based on the same assumption the relationship between slopes is given by

\[
\frac{1}{\gamma_{lu}} > \frac{\beta_1}{\beta + \beta_1 \gamma} \quad \text{and} \quad \frac{(1 - \beta_1 \gamma_2)w \alpha \alpha_{le}}{(w \alpha^2 + 1) \beta_1 \gamma_{1u}} > \frac{(1 - \beta_1 \gamma_2)w \alpha \alpha_{1u}}{(\alpha^2 + 1)(\beta + \beta_1 \gamma)}
\]

Once we have established the reaction functions of both countries and defined their relative forms, next step is to derive three equilibria as results of non-cooperative games. In doing so, we consider a one-shot game, but with the so-called “pre-play stage” (Lambertini and Rovelli, 2003). This stage implies that both monetary authorities, before choosing the optimal level of their respective interest rates, have to make a preliminary decision regarding a non-cooperative or cooperative game. “Assume that there are two instants, \( t_1 \) and \( t_2 \), at which the two authorities can move. These instants are purely logical entities, and do not belong to calendar time; they represent the pure strategies available to players at the first (pre-play) stage”\(^9\).

The most interesting thing of the pre-play stage is that, if both authorities want to choose at the same instant (\( t_1 \) or \( t_2 \)), the consequent equilibrium of the second stage is the Nash equilibrium. If, on the other hand, one authority chooses \( t_1 \) while the other chooses \( t_2 \), the final

---

solution in stage two will be a Stackelberg equilibrium with the country that has chosen $t_1$ to act as leader\textsuperscript{10}.

3.1 Game 1: Nash equilibrium

In this section, each Central Bank is supposed to behave in a non-cooperative way, and to set, at the same instant, the policy on the basis of the objective function, without considering the consequences on the other players’ welfare.

The Nash equilibrium point $(N)$ is achieved through the intersection of eq. (9-10), solving we obtain:

\begin{align*}
\Delta r_1^N &= \frac{\beta_1^3 \gamma_{1u} \gamma_{1u}}{(\beta + \beta_1 \gamma) \Omega} e_1 + \frac{\Omega - \beta_1^2 \gamma_{1u} \gamma_{1u}}{\gamma_{1u} \Omega} e_2 + \left(\frac{(\beta + \beta_1 \gamma)^2}{\Omega} - \frac{\beta_1^2 \gamma_{1e} \gamma_{1u}}{\Omega} Z_1 \right) X \\
\Delta r_2^N &= -\frac{\beta_2^2 \gamma_{1u}}{\Omega} e_1 + \beta_2 (\beta + \beta_1 \gamma) e_2 + \frac{(\beta + \beta_1 \gamma) \beta \gamma_{1u}}{\Omega} (z_1 - Z) X
\end{align*}

(13)

(14)

where $\Omega = (\beta + \beta_1 \gamma)^2 - \beta_1^2 \gamma_{1e} \gamma_{1u}$, $Z = \left(\frac{1 - \beta_1 \gamma_2}{\alpha^2 + 1} \right) (\beta + \beta_1 \gamma)$ and $Z_1 = \left(\frac{1 - \beta_1 \gamma_2}{w^2 \alpha^2 + 1}\right) \beta_i \gamma_{1u}$

and we deduce

**Proposition 2** Assume that the change in the price of raw materials is zero ($X = 0$). Then the following holds:

\[ e_1 > \frac{2 \beta_1^2 \gamma_{1u} \gamma_{1e} + (\beta + \beta_1 \gamma) \left[\beta_1 \gamma_{1u} (\beta + \beta_1 \gamma)\right]}{\beta_1^2 \gamma_{1e} \left[\beta_1 \gamma_{1u} (\beta + \beta_1 \gamma)\right]} e_2 \]

**Proof:**

given that $\gamma > \gamma_{1u}, \gamma_{1e}, \beta > \beta_1$ it follows that $(\beta + \beta_1 \gamma)^2 - \beta_1^2 \gamma_{1e} \gamma_{1u} > 0$. This implies that, in a Nash game, it is possible to identify a wide range of shocks in the US asset market such that

\[ \frac{2 \beta_1^2 \gamma_{1u} \gamma_{1e} + (\beta + \beta_1 \gamma) \left[\beta_1 \gamma_{1u} (\beta + \beta_1 \gamma)\right]}{\beta_1^2 \gamma_{1e} \left[\beta_1 \gamma_{1u} (\beta + \beta_1 \gamma)\right]} e_2 \propto e_1 \propto \infty \]

for which the FED is forced to use a more restrictive monetary policy, compared to the ECB ($\Delta r_1^N > \Delta r_2^N$).

\textsuperscript{10} “The extended game is a two-stage game where the first stage concerns the choice of timing, while the second stage is the proper policy game where policy instruments are to be set according to the sequence selected at the previous stage”, Lambertini
3.3 Game2: Stackelberg equilibria

Under the Stackelberg regime, monetary authorities no longer act simultaneously. It is now assumed that one player (the Stackelberg leader) has a first-mover advantage when selecting policy, and takes into account the response of the other player (the follower) to the policy measures. Thus, the leader chooses the optimal strategy subject to the follower’s reaction function, and the follower’s committed response is to simply take the leader’s policy as given and minimise its loss. We think that the leader-follower policy regime is interesting in as much as it allows to highlight of the strategic aspects of the decision-making process in the context of differential inflation-aversion coefficients.

In the Stackelberg equilibrium, monetary leadership is the usual way to capture the notion of central bank independence [Petit (1989), Hughes Hallett and Petit (1990), Debelle (1996)].

3.3.1 Stackelberg equilibrium with FED as leader

The leader’s problem is:

\[
\min_{\Delta r_1} L_1 = \frac{1}{2} \left[ w_1 (\pi_1 - \bar{\pi}_1)^2 + (\gamma_1 - \bar{\gamma}_1)^2 \right] \tag{15}
\]

s.t.: \( \Delta r_1 = B_1 \Delta r_2 + B_2 \)

where \( w_1 \), the weight that the FED attaches to inflation is unity because the US central bank is assumed, according to its mandate, to be “balanced” in its pursuit of monetary policy.

Proceeding by substitution and setting the partial derivative with respect to \( \Delta r_1 \) of the leader’s objective function equal to zero, we obtain the first-order condition:

\[
\frac{\partial L_1}{\partial \Delta r_1} = 0 \tag{16}
\]

yielding:

\[
\Delta r_{1us}^S = \frac{\beta_1}{\Omega} \epsilon_1 - \frac{\beta_1^2 \gamma_{1u} \epsilon_2}{(\beta + \beta_1 \gamma) \Omega} - \frac{\beta_1^2 \gamma_{1u} \gamma_{1u} Z_1}{(\beta + \beta_1 \gamma) \Omega} - \frac{(1 - \beta_1 \gamma_2) \alpha \alpha_{1u}}{(\alpha^2 + 1) \Omega} X \tag{17}
\]

and

\[
\Delta r_{2us}^S = -\frac{\beta_1^2 \gamma_{1u} \epsilon_1}{(\beta + \beta_1 \gamma) \Omega} + \frac{\beta_1 \Psi}{(\beta + \beta_1 \gamma)^2 \Omega} \epsilon_2 + \frac{\beta_1 \gamma_{1u} \Psi Z_1}{(\beta + \beta_1 \gamma)^2 \Omega} - \frac{(1 - \beta_1 \gamma_2) \beta_1 \gamma_{1u} \alpha \alpha_{1u}}{(\alpha^2 + 1) \Omega (\beta + \beta_1 \gamma)} X \tag{18}
\]

where $\Psi = \beta_1^2 \gamma_{ue} \gamma_{le} + (\beta + \beta_I) \Omega$.

From the above it follows:

**Proposition 3** Assume that the change in the price of raw materials is zero ($X = 0$). Then the following inequality holds:

$$
\epsilon_1 > \frac{\beta_1^2 \gamma_{ue} \gamma_{le} (\beta + \beta_I \gamma - 1) - (\beta + \beta_I \gamma)^2}{\beta_I \gamma_{ue} \beta_1^2 \gamma_{le} \gamma_{ue} (\beta + \beta_I \gamma)} \epsilon_2 \text{ when the FED acts as leader.}
$$

**Proof:**

Provided that $\beta_I \gamma_{ue} \beta_1^2 \gamma_{le} \gamma_{ue} (\beta + \beta_I \gamma)$ is positive for all $\beta_I > \frac{\beta + \beta_I \gamma}{\gamma_{ue} \gamma_{ue}}$, in the Stackelberg equilibrium with FED as leader, it is possible to identify a narrow range of shocks in the US asset market $0 < \epsilon_1 < \frac{\beta_1^2 \gamma_{ue} \gamma_{le} (\beta + \beta_I \gamma) - 1 - (\beta + \beta_I \gamma)^2}{\beta_I \gamma_{ue} \beta_1^2 \gamma_{le} \gamma_{ue} (\beta + \beta_I \gamma)} \epsilon_2$ such that FED’s monetary policy is less restrictive compare to its Nash equilibrium.

A corollary of the above is that for this range of asset shocks the ECB’s policy when acts as follower will be more restrictive compare to its Nash equilibrium.

### 3.3.2 Stackelberg equilibrium with ECB as leader

In this case, the leader’s problem is given by:

$$
\min_{\Delta r_2} L_2 = \frac{1}{2} \left[ w_2 (\pi_2 - \bar{\pi}_2)^2 + (y_2 - \bar{y}_2)^2 \right]
$$

s.t. $\Delta r_1 = A_1 \Delta r_2 + A_2$

where $w_2$, the weight that the ECB attaches to inflation, is assumed to be greater than unity because the European Central Bank is tied by its narrow mandate to “maintain price stability”. Solving by substitution and setting the partial derivative with respect to $\Delta r_2$ of the leader’s objective function equal to zero, we obtain the first- order condition:

$$
\frac{\partial L_2}{\partial \Delta r_2} = 0
$$

(20)
\[
\Delta r_{1ou}^S = \frac{\beta_1}{1 - \beta_1^2} e_1 + \frac{\beta_1^2}{1 - \beta_1^2} e_2 + \left( \frac{\Omega - \beta_1^2}{1 - \beta_1^2} \right) X - \left( \frac{w^2 a a_{1u} (1 - \beta_1) \beta_1}{\Omega (w^2 \alpha^2 + 1)} \right) X
\]

(21)

\[
\Delta r_{2ou}^S = -\frac{\beta_1^2}{\Omega} e_1 + \frac{\beta_1}{\Omega} e_2 + \left( \frac{\Omega - \beta_1^2}{\Omega} \right) X + \left( \frac{w^2 a a_{1u} (1 - \beta_1) \beta_1}{\Omega (w^2 \alpha^2 + 1)} \right) X
\]

(22)

An interesting aspect of this solution is that both central banks’ reactions to a US asset market shock are identical independently of whether they act as leader or follower. The banks’ reactions differ significantly in the face of a European asset market shock.

3.4 International non-coordination policy: an evaluation

In the final part of this section we provide an analysis of the outcomes of the Nash and Stackelberg equilibria. In figure 1, we depict the change of the US interest rate (\(\Delta r_1\)) on the vertical axis and the equivalent change for Europe (\(\Delta r_2\)) on the horizontal axis, their positions and slopes follow the solutions of equations (9) and (10).

The intersection of the two reaction functions denotes the Nash equilibrium (\(N\)). The locus of points between \(\Phi_{US}\) and \(\Phi_{EU}\) (respective bliss points), derived when the isoloss curves of the two countries are tangential to each other, represents the contract curve.

We start evaluating these outcomes commencing with the Nash non-cooperative equilibrium which is reached when there is no incentive for either economy to change its policy position, taking the other’s policy as given. Consistent with proposition 1, ECB’s reaction function is steeper than that of the FED. This implies that in the presence of a domestic asset market shock, the FED will follow a tighter monetary policy compared to the one followed by the ECB.

In the Stackelberg game, one of the two players (the leader) realises that there is a better position to be achieved than the Nash equilibrium. This occurs when the leader chooses policy assuming that it will influence the policy choice of the follower, and ignoring the latter’s choice. Assuming that the FED acts as leader, the Stackelberg equilibrium is denoted at point \(S_{US}\) (Fig. 1). At this point the isoloss curve \(G_{US}\) is tangential to ECB’s reaction function. This is the closest isoloss curve to point \(\Phi_{us}\) that the US can reach given the whole range of possible reactions of Europe. Alternatively when the ECB acts as leader, the equivalent Stackelberg equilibrium is point \(S_{EU}\).
Comparing the Nash and Stackelberg non-cooperative equilibria, the Stackelberg solution is certainly superior when the US acts as leader. However, when the US acts as follower, whether the Nash solution dominates the Stackelberg solution will depend crucially upon the slope of its reaction function. The greater the slope of the FED’s reaction function, the more likely that the Nash solution will be the more desirable of the two. When the ECB acts as leader, it achieves a lower isoloss curve compared with the Nash equilibrium. The Stackelberg equilibrium is preferable to the Nash even in the case where the ECB acts as follower to the FED’s leadership. Both the Nash and Stackelberg equilibria are inefficient, as they do not lay on the contract curve $\Phi_{ue}$, $\Phi_{ue}$ that is derived from the joint minimization of the loss functions.

4 Non-cooperative equilibria in the presence of a shock in the US asset market.

In this section we analyse the impact of a shock in the US asset market on domestic and foreign monetary policy.

Consider a positive shock in the US asset market that shifts the FED’s reaction function to the right as shown in equation (9). The new Nash equilibrium (fig.2) implies a tighter monetary policy for the FED, while for the ECB a less tight monetary policy is required. Table 1 shows the impact of this shock on $\Delta r_1$ and $\Delta r_2$ through the coefficients $\beta_1$, $\gamma$, $\gamma_{1u}$ and $\gamma_{1e}$.

For the FED, the higher the response of asset prices to domestic interest rates (coefficient $\gamma$), the smaller the required change in the interest rates to re-establish the Nash equilibrium. Moreover, the required increase in the US rate would vary positively with the size of the wealth effect (coefficient $\beta$) and the sensitivity of the asset prices (in both countries) to domestic and foreign rates ($\gamma_{1u}$ and $\gamma_{1e}$). The increase in the US rate will be ameliorated the higher the response of asset prices to domestic interest rate (coefficient $\gamma$). Considering the problem from the ECB’s point of view, whether a positive shock in the US asset market requires an easing of monetary stance ($\Delta r_2$) will depend upon the coefficients $\beta_1$, $\gamma$, $\gamma_{1u}$ and $\gamma_{1e}$.

What is interesting in this case is that the sign of the partial derivative of $\Delta r_2$ with respect to $\beta_1$ that is negative. The greater the impact of wealth in aggregate demand the less able will the ECB be to reduce rates.

The direction of the impact of a shock in the US asset market on $\Delta r_1$ when the FED acts as leader is, in terms of signs of the coefficients $\beta_1$, $\gamma$, $\gamma_{1u}$ and $\gamma_{1e}$, the same as the Nash.

---

11 The summary of the results are presented in table 1.
The Stackelberg equilibrium is not affected by the shock (fig.2). However, in terms of social welfare, the equilibrium now is on a higher isoloss curve compared with the previous FED’s reaction function.

Finally, we consider the Stackelberg EU leader equilibrium and a shock in the US asset market. The new equilibrium is worse for Europe compared to the Stackelberg US leader equilibrium and, in terms of social welfare, the equilibrium is on a higher isoloss curve compared with the previous Stackelberg EU leader equilibrium.

Moreover, comparing the Nash and Stackelberg non-cooperative equilibria, the Stackelberg solution is certainly superior for the FED leader implying a less tight monetary policy. However, the Nash solution dominates the Stackelberg solution for the ECB when acts as follower.

5 Non-cooperative equilibria in the presence of a shock in the EU asset market.

In this section we analyse the impact of a shocks in the EU asset market on domestic and foreign monetary policy.

Assuming a positive shock in the EU asset market that shifts the ECB’s reaction function to the right as shown in equation (10). The new Nash equilibrium (fig.3) implies tighter monetary policy for the ECB while for the FED, a less tight monetary policy is required. Table 2 shows the impact of this shock on $\Delta r_1$ and $\Delta r_2$ through the coefficients $\gamma_{1u}$, $\gamma_{1e}$, $\beta_1$, and $\gamma$.

For the ECB, the higher the response of asset prices to domestic interest rates (coefficient $\gamma$), the smaller the required change in interest rates to re-establish the Nash equilibrium. The domestic interest rate will increase and this will have a negative effect on the EU asset market. On the other hand, the decrease of the US interest rate will reinforce the negative effect for Europe because of the increase in the interest rate differential. Hence, inflation pressure may arise. Moreover, the required increase in the EU interest rate would vary positively with the size of the wealth effect (coefficient $\beta$) and the sensitivity of asset prices in both countries to domestic and foreign rates. However, the sign of the coefficient $\gamma_{1u}$ is ambiguous for $\Delta r_1$. Whether the ECB can re-establish the Nash equilibrium depends crucially upon value of the coefficient $\gamma$. In fact, the increase in the EU rate will be ameliorated the higher the response of asset prices to domestic interest rate (coefficient $\gamma$).

Considering the problem from the FED’s point of view, whether a positive shock in the EU asset market requires an easing of monetary stance ($\Delta r_1$) will depend upon the coefficients $\beta_1$, $\gamma$, $\gamma_{1u}$ and $\gamma_{1e}$. 
Finally, we analyse the impact of the shock in the EU asset market on $\Delta r_1$ and $\Delta r_2$ through the coefficients $\beta_1$, $\gamma$, $\gamma_{1u}$ and $\gamma_{1e}$ considering the Stackelberg equilibrium when the FED acts as leader.

The Stackelberg equilibrium is affected by the shock (fig.3). The new equilibrium will be at a lower level of interest rates for the FED and at a higher level for the ECB. Moreover, in terms of social welfare, the equilibrium now is on a lower isoloss curve compared with the previous one. In this scenario, the leader has, undoubtedly, an advantage. As matter of fact, the FED would set its interest rate at low level in order to aid output to converge to $\bar{y}$, whilst the ECB is faced with a positive shock in the asset market that requires a rising of the rate with the subsequent real economy slowing. Therefore, the follower has to fix the interest rate at a higher level compared with the previous Stackelberg equilibrium. Table 2 shows that the partial derivatives of $\Delta r_2$ with respect to $\beta_1$ and $\gamma$ are ambiguous. This cannot allow us to interpret the sign of the wealth effect on inflation via the output gap. If, however, the link between monetary policy and the EU asset market is weak, then the final benefit for the ECB of an increase in the domestic interest rate will depend essentially on the value of the coefficient $\beta_1$.

If we consider the Stackelberg EU leader equilibrium assuming a shock in the EU asset market, the equilibrium is not affected by the shock. However, in terms of social welfare, the equilibrium for the leader is on a lower isoloss curve compared with the previous EU reaction function.

Table 3 summarizes and compares all the possible scenarios. In order to simplify the analysis, we have defined the choice of the Central Banks as “Most preferred”, “Less preferred” and “Least preferred”.

“Most preferred” is referred to a scenario where both optimal monetary policy ($\Delta r_1$ or $\Delta r_2$) and social loss are minimized; “Less preferred” is referred to a scenario where only one of two is minimised and “Least preferred” when neither measure achieve their minimum;

When we look at the situation in which the FED is facing a shock in the domestic asset market, both the players (FED and ECB) would prefer to be the followers. In fact, this is the situation in which they minimize their loss functions. However, since the FED is assumed, according to its mandate, to be balanced in its pursuit of monetary policy, the Stackelberg leader equilibrium could be chosen especially if the shock is not of such a big magnitude that could have strong effect on the inflation. This is clear in the situation in which the ECB is facing a shock in its domestic asset market. In this case both Central Banks would prefer the scenario where the FED is the leader and the ECB acts as follower. In fact, the optimal monetary policy for the leader would imply a lower level of interest rate that could help the
economy while for the follower (ECB) the higher level of interest rate could cool down the “asset market bubble” reducing the pressure that the wealth effect has on inflation.

6. Conclusions

This paper has re-examined the issue of international macroeconomic policy coordination, taking advantage of recent developments in theoretical methods used in the literature to study monetary policy optimization.

All the recent attention on the asset market and on monetary policy rules has inspired a natural question: should a central bank also react to asset price movements when it sets its monetary policy? The movements in the asset markets have stimulated a great discussion among economists about the role the asset market should play in influencing monetary policy decisions.

The review of the literature, however, does not offer a conclusive answer to whether, and how, a central bank should respond to asset “shocks”. This paper examines, theoretically, in a non-cooperative game framework, the optimal monetary policy assuming that the central bank considers the information from the asset market. In particular, we examined the impact of shocks in the asset markets on domestic and foreign monetary policy.

Comparing the Nash and Stackelberg non-cooperative equilibria, the Stackelberg solution is dominant for the FED leader compared to the Nash solution. In the all scenarios analyzed, the Nash equilibrium for the ECB is the one that allows the central banks to manage the economy with the lowest level of interest rates.

The Stackelberg EU leader equilibrium is worse for Europe compared to the Stackelberg US leader and, in terms of social welfare, the equilibrium is on a higher isoloss curve.

When we analysed the impact of a shock in the EU asset market, the subsequent Nash equilibrium implied a tighter monetary policy for the ECB and, a less tight monetary policy for the FED. The analysis presents different partial optimal equilibria related to different cases where FED is the leader and ECB the follower and vice versa. However, the following general conclusion can be achieved. In terms of social welfare, comparing the Nash and Stackelberg equilibrium when the ECB is the leader, the Stackelberg US solution is certainly superior for the leader because this implies a less tight monetary policy for the US. The same conclusion is found for the ECB when it acts as the follower.

In conclusion, irrespective of where the shocks originate, if the FED would choose the Stackelberg leader equilibrium, the ECB would minimaze its social loss and, in the case of a shock in the US, at a lower level of interest rate.
## Appendix

### Table 1
Partial derivatives of $\Delta r_1$ and $\Delta r_2$ with respect to $\beta_1$, $\gamma_{1u}$, $\gamma_{1e}$, and $\gamma$ assuming a shock in the US Market

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\gamma_{1u}$</th>
<th>$\gamma_{1e}$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta r_1^a$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>$\Delta r_2^a$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&gt;0$</td>
</tr>
<tr>
<td>$\Delta r_{lus}$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>$\Delta r_{2us}$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&gt;0$</td>
</tr>
<tr>
<td>$\Delta r_{lue}$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>$\Delta r_{2ue}$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&gt;0$</td>
</tr>
</tbody>
</table>

The result for the coefficient $\gamma$ is ambiguous for $\Delta r_{lus}$ and $\Delta r_{2us}$.

The result for the coefficient $\beta_1$ is ambiguous for $\Delta r_{2us}$.

The result for the coefficient $\gamma_{lu}$ is ambiguous for $\Delta r_1^a$.

### Table 2
Partial derivatives of $\Delta r_1$ and $\Delta r_2$ with respect to $\beta_1$, $\gamma_{1u}$, $\gamma_{1e}$, and $\gamma$ assuming a shock in the EU Market

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\gamma_{1u}$</th>
<th>$\gamma_{1e}$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta r_1^a$</td>
<td>$&lt;0$</td>
<td>--</td>
<td>$&lt;0$</td>
<td>$&gt;0$</td>
</tr>
<tr>
<td>$\Delta r_2^a$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>$\Delta r_{lus}$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>--</td>
</tr>
<tr>
<td>$\Delta r_{2us}$</td>
<td>--</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>--</td>
</tr>
<tr>
<td>$\Delta r_{lue}$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>--</td>
</tr>
<tr>
<td>$\Delta r_{2ue}$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
</tbody>
</table>

The result for the coefficient $\gamma$ is ambiguous for $\Delta r_{lus}$, $\Delta r_{2us}$, and $\Delta r_{lue}$.
### Table 3

<table>
<thead>
<tr>
<th>Stackelberg equilibria</th>
<th>Shock in the US Asset Market</th>
<th>Shock in the EU Asset Market</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FED Leader</strong></td>
<td>$\Delta r_1$ Social Welfare</td>
<td>$\Delta r_1$ Social Welfare</td>
</tr>
<tr>
<td>FED Leader</td>
<td>Unchanged Higher Isoloss curve</td>
<td>Decrease the interest rate differential</td>
</tr>
<tr>
<td>ECB Follower</td>
<td>$\Delta r_2$ Social Welfare</td>
<td>$\Delta r_2$ Social Welfare</td>
</tr>
<tr>
<td>ECB Follower</td>
<td>Decrease the interest rate differential Lower Isoloss curve</td>
<td>Increase the interest rate differential</td>
</tr>
<tr>
<td>ECB Leader</td>
<td>$\Delta r_2$ Social Welfare</td>
<td>$\Delta r_2$ Social Welfare</td>
</tr>
<tr>
<td>ECB Leader</td>
<td>Increase the interest rate differential Higher Isoloss curve</td>
<td>Unchanged</td>
</tr>
<tr>
<td>FED Follower</td>
<td>$\Delta r_1$ Social Welfare</td>
<td>$\Delta r_1$ Social Welfare</td>
</tr>
<tr>
<td>FED Follower</td>
<td>Increase the interest rate differential Lower Isoloss curve</td>
<td>Increase the interest rate differential</td>
</tr>
</tbody>
</table>

**Legend**

We define the choice of the Central Banks as Most preferred, Less preferred, and Least preferred.

- **Most preferred** is referred to a scenario where both optimal monetary policy and social loss are minimized;
- **Less preferred** is referred to a scenario where only one of the two is minimized;
- **Least preferred** is when neither measure achieve their minimum;

**Fig. 1** Nash and Stackelberg equilibria
Fig. 2 Shock in the US asset market

Fig. 3 Shock in the EU asset market
Bibliography


Blinder A.S., 1999, Central Bank Credibility: Why Do We Care? How Do We Build It? NBER Working Papers, n.7161


Cooper, R., 1985, Economic Interdependence and Coordination of Economic Policies, in Jones and Kenen , pp.1195-1239.


