The specification of cross exchange rate equations used to test Purchasing Power Parity.

John Hunter
Department of Economics and Finance, Brunel University, Uxbridge, Middlesex, UB8 3PH, England

Mark Simpson
National Audit Office, 157-197 Buckingham Palace Road Victoria, London SW1W 9SP

November 18, 2004

Abstract

The Article considers the specification of models used to test Purchasing Power Parity when applied to cross exchange rates. Specifically, conventional dynamic models used to test stationarity of the real exchange rate are likely to be misspecified, except when the parameters of each exchange rate equation are the same.

Keywords: Arbitrage; misspecification; non-stationarity; Purchasing Power Parity; real exchange rate

JEL Classifications: C32, F31

1 Introduction

The Article considers the problem of testing non-stationarity on cross exchange rates. Smith and Hunter (1985) addressed a theoretical proposition that only specific exchange rate models are coherent under cross arbitrage constraints. Here it is shown that cross arbitrage has similar implications for the specification of dynamic models used to test Purchasing Power Parity and the associated proposition that the real exchange rate is stationary. To test the proposition that the real exchange rate is stationary, either requires independent cross rate data; that the coefficients of every dynamic model on which a cross rate is based are the same or that the cross rate equations are correctly specified, when in addition to the cross rate variables they include a set of dollar variables for one of the two exchange rates used to derive the cross rate. Furthermore, arbitrage is also likely to bind, when the data collected is generated by transactions that are calculated via an intermediary $ rate conversion; a common practice in many offices dealing in foreign exchange. The final proposition implies that cross rate equations ought to have up to double the number of parameters of $ equations. By implication, the power and size of tests based on correctly specified cross rate equations is likely to be affected by the inclusion of the extra variables.

2 Cross Equation Dynamics and tests of Stationarity.

Consider a small dynamic extension to the model of the exchange rate presented in Smith and Hunter (1985):

$$\Delta e_{ijt} = \delta_{oij} + \gamma_{ij} e_{ijt-1} + \beta_{ij}(x_i - x_j)_{t-1} + \beta_{0ij}\Delta(x_i - x_j)_t + \varepsilon_{ijt} \quad (1)$$

where e_{ijt} values the home currency in terms of country j, lowercase variables are in logarithms, $x_i - x_j$ is a $j \times 1$ vector of parity conditions associated with, prices, interest rates and money, β_{ij} a $1 \times j$ vector of parameters and ε_{ijt} is the disturbance term for the ij^{th} exchange rate equation. When international transactions occur through the intermediary of an international means of exchange and country j defines that denomination then application of the associated arbitrage condition means that $e_{ikt} = e_{ijt} - e_{kjt}$ and the cross rate equation is:

$$\begin{aligned}\Delta e_{ikt} \quad = \quad & \delta_{oik} + \gamma_{ij} e_{ijt-1} - \gamma_{ik} e_{kjt-1} + \beta_{ij}(x_i - x_j)_{t-1} - \beta_{kj}(x_k - x_j)_{t-1} \\ & + \beta_{0ij}\Delta(x_i - x_j)_t - \beta_{0kj}\Delta(x_k - x_j)_t + \varepsilon_{ijt} - \varepsilon_{kjt} \quad (2)\end{aligned}$$

To simplify the exposition it will be assumed that $\varepsilon_{jkt} = \varepsilon_{ijt} - \varepsilon_{kjt}$ are white noise innovations, $x_m - x_l$ are weakly exogenous for all m and l.[1] and that the

[1] If the errors are not innovations and the regressors not weakly exogenous (see Ericsson and Irons(1994)), then the degree of inconsistency and the order of misspecification is likely to be magnified.

the following reparameterisation can be applied:

$$\begin{aligned}\Delta e_{ikt} &= \delta_{oik} + \gamma_{ij} e_{ikt-1} - (\gamma_{kj} - \gamma_{ij}) e_{ikt-1} + \beta_{ij}(x_i - x_k)_{t-1} - \\ &\quad (\beta_{kj} - \beta_{ij})(x_k - x_j)_{t-1} + \beta_{0ij}\Delta(x_i - x_k)_t - \\ &\quad (\beta_{0kj} - \beta_{0ij})\Delta(x_k - x_j)_{t-1} + \varepsilon_{ikt}.\end{aligned}$$

Hence:

$$\Delta e_{ikt} = \delta_{oik} + \gamma_{ij} e_{ikt-1} + \beta_{ij}(x_i - x_k)_{t-1} + \beta_{0ij}\Delta(x_i - x_k)_{t-1} + \varepsilon_{ikt}, \quad (3)$$

with ε_{ikt} an innovation process, if and only if $(\gamma_{kj} - \gamma_{ij}) = 0$, $(\beta_{kj} - \beta_{ij}) = 0$ and $(\beta_{0kj} + \beta_{0ij}) = 0$. Otherwise, (3) is misspecified and estimates of γ_{ij}, β_{ij} and β_{0ij} are biased and inconsistent. To emphasize this point, models used to tests long-run Purchasing Power Parity (PPP) and stationarity are considered next.[2]

Let $x_l = p_l$ a price index for country $l = i, j$. In the context of (1), PPP is said to hold in the long-run when $\beta_{ij} = -\gamma_{ij}$ and:

$$\Delta e_{ijt} = \delta_{oij} + \gamma_{ij}(e_{ij} - (p_i - p_j))_{t-1} + \beta_{0ij}\Delta(p_i - p_j)_t + \varepsilon_{ijt} \quad (4)$$

Tests of the proposition that $\rho_{ijt} = e_{ijt} - (p_i - p_j)_t$ is integrated of order 1 or I(0) are based on the coefficient on the error correction term $\gamma_{ij} < 0$.The distribution of this test statistic under the null for the case considered here is asymptotically normal (see Kremers et al (1992)), but the power and performance of the test relies on the proposition that the model is correctly specified. In general, tests of PPP when the cross rate is considered need to be applied to:

$$\begin{aligned}\Delta e_{ikt} &= \delta_{oik} + \gamma_{ij}(e_{ij} - (p_i - p_j))_{t-1} - (\gamma_{kj} - \gamma_{ij})(e_{ik} - (p_k - p_j))_{t-1} \\ &\quad + \beta_{0ij}\Delta(p_i - p_k)_t - (\beta_{0kj} - \beta_{0ij})\Delta(p_k - p_j)_{t-1} + \varepsilon_{ikt},\end{aligned}$$

which implies that the joint proposition $\gamma_{ij} < 0$ and $\gamma_{kj} < 0$ be satisfied, except for the case where $\gamma_{kj} - \gamma_{ij} = 0$ or $\gamma_{kj} = \gamma_{ij}$. It follows that PPP can only be tested coherently across all \$ exchange rates and their cross rates using equations of the form of (4), when:

$$\gamma_{kj} = \gamma_{ij},\ \beta_{kj} = \beta_{ij} \text{ and } \beta_{0kj} = \beta_{0ij}.$$

If PPP is tested using the test due to Dickey and Fuller (1979) via the proposition that the real exchange rate is I(0), then in addition to $\beta_{ij} = -\gamma_{ij}$, it is required that $\beta_{0ij} = 1$ and for the Dickey-Fuller test:

$$\begin{aligned}\Delta e_{ijt} - \Delta(p_i - p_j)_t &= \delta_{oij} + \gamma_{ij}(e_{ij} - p_i + p_j)_{t-1} + \varepsilon_{ijt} \\ &\quad \textit{or} \\ \Delta \rho_{ijt} &= \delta_{oij} + \gamma_{ij}\rho_{ijt-1} + \varepsilon_{ijt}. \end{aligned} \quad (5)$$

[2] It should be noted that these results readily extend to the case where the variables used are generated by a potentially cointegrating, vector moving average model, see the discussion of the Granger representation theorem in Engle and Granger(1987).

If j is the \$, then the value of the currency for country k relative to the \$ is given by the same type of dynamic equation:

$$\Delta\rho_{kjt} = \delta_{okj} + \gamma_{kj}\rho_{jkt-1} + \varepsilon_{kjt} \tag{6}$$

It follows from (5) and (6) that the cross rate equation is:

$$\begin{aligned} \Delta\rho_{ijt} - \Delta\rho_{jkt} &= \Delta\rho_{ikt} = \delta_{oij} + \delta_{okj} + \gamma_{ij}\rho_{ijt-1} + \gamma_{kj}\rho_{jkt-1} + \varepsilon_{ijt} + \varepsilon_{kjt} \\ & \quad or \\ \Delta\rho_{ikt} &= \delta_{oik} + \gamma_{ik}\rho_{ikt-1} - (\gamma_{ik} - \gamma_{kj})\rho_{jkt-1} + \varepsilon_{ikt}. \end{aligned}$$

Hence the usual Dickey-Fuller test applied to the cross rate equation suffers from omitted variable bias as it assumes that $(\gamma_{ik} - \gamma_{kj}) = 0$ or $(\gamma_{ik} = \gamma_{kj})$. The following results associated with univariate tests of stationarity for real \$ exchange rates show γ_{ik} is not usually equal to γ_{kj}.[3]

Table 1 Summary of Augmented Dickey Fuller Tests and their Respective Coefficients

Country	Parameter γ_{ik}	Test Statistic	Result
Italy	-0.0695	-1.96	n-s
Spain	-0.0504	-1.39	n-s
Belgium	-0.1188	-2.36	n-s
Denmark	-0.0612	-1.83	n-s
Finland	-0.0707	-1.36	n-s
France	-0.2207	-2.96	s
Germany	-0.0781	-1.76	n-s
Ireland	-0.2208	-3.69	s
Luxembourg	-0.2023	-3.61	s
Holland	-0.1955	-2.70	n-s
Portugal	-0.0204	-0.84	n-s
UK	-0.1543	-2.83	n-s

(n-s non-stationary t>-2.89, s stationary, t<-2.89)

Based on table 1, Dickey-Fuller tests associated with cross rates will be similar when Italy, Spain and Denmark are analysed, but very different when applied to Portugal or France. The question of specification is further complicated when one considers that the results in table 1 based on models with 5^{th} order lags are also likely to be affected by the significance of different lags and a key assumption that prices are weakly exogenous.

3 Conclusion

Any differences in inference based on cross exchange rate data and their original \$ equations is likely to be due to misspecification of the cross rate equations used

[3] Quarterly observations on dollar real exchange rates were drawn from the Datastream Database for the period (1980q1 – 1998q1) for twelve countries: Italy, Spain, Belgium, Denmark, Finland, France, Germany, Ireland, Luxembourg, Holland, Portugal and UK.

to test either for PPP or stationarity of the real exchange rate. It is argued, on the basis of the results in Smith and Hunter (1985), that the use of conventional dynamic models (4) and (5) is inappropriate for cross rate equations except when the estimated parameters of the dollar rate equations are the same. Otherwise, the cross rate equations will be misspecified and the parameter estimates of γ_{ik} used to test stationarity of the real exchange rate will be inconsistent and biased. Where more appropriate equations with dynamic cross rate and dollar rate terms are used, then the power of tests based on cross rate equations would be reduced in small and moderate samples due to the inclusion of an extra set of regressors. An appropriately specifoed augmented Dickey-Fuller test with k lags on the dollar equation would require $2k$ lagged terms in the equivalent cross rate equation.

The problem is not ameliorated by using an effective exchange rate as any rate generated from n dollar rates would require $n-1$ dynamic terms for comparable inference. Nor can it be fully resolved by considering a system or non-linear models. Estimating a system only obviates the requirement that the price, interest rate or money supply differentials are weakly exogenous. While nonlinearity may eliminate exact cross restrictions, but not the inconsistency caused by the exclusion of linear or non-linear terms from the $ equations.

4 References

Dickey, D.A. and Fuller, W.A., 1979. Distribution of the Estimators for Autoregressive Time Series with a Unit Root. Journal of the American Statistical Association, 74, 427-431.

Engle, R.F. and Granger, C.W.J., 1987. Co-Integration and Error-Correction: Representation, Estimation and Testing. Econometrica, 55, 251-276.

Ericsson, N.R. and Irons, J.S., 1994. Testing Exogeneity. Oxford University Press, Oxford.

Hunter, J and Simpson, M, 2001. A Panel Test of Purchasing Power Parity under the Null of Stationarity. Paper presented at the Money Macro Finance Conference, Belfast, September 2001 and Brunel Discussion Paper, 01- 01.

Kremers, J.J.M., Ericsson, N.R. and Dolado, J. 1992. The power of cointegration tests. Oxford Bulletin of Economics and Statistics. 54, 325-348.

Smith, R.P and Hunter, J, Cross, 1985..Arbitrage and Specification in Exchange Rate Models. Economics Letters, 18, 375-376.