Tax Evasion and Exchange Equity: A Reference-Dependent Approach

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Abstract

The standard portfolio model of tax evasion with a public good produces the perverse conclusion that when taxpayers perceive the public good to be under-(over-) provided, an increase in the tax rate increases (decreases) evasion. I treat taxpayers as thinking in terms of gains and losses relative to an endogenous reference level which reflects perceived exchange equity between the value of taxes paid and the value of public goods supplied. With these alternative behavioral assumptions I overturn the aforementioned result in a direction consistent with the empirical evidence. I also find a role for relative income in determining individual responses to a change in the marginal rate of tax.

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1 Introduction

Since the seminal contribution to the economic theory of tax compliance of Allingham and Sandmo (1972) a number of apparent failings of the standard portfolio model have been identified. First, assuming decreasing absolute risk aversion, the standard portfolio model predicts that the level of tax evasion is a decreasing function of the tax rate (Yitzhaki 1974). As well as being counterintuitive, this prediction is contradicted by the balance of the empirical evidence: while support is offered by Feinstein (1991) and Alm, Sanchez, and De Juan (1995), the opposite finding is reported by the majority of studies (e.g. Friedland, Maital, and Rutenberg 1978; Clotfelter 1983; Slemrod 1985; Crane and Nouruzad 1986). Thus, while it seems there may be circumstances in which the prediction of the standard model holds true, the evidence suggests that the relationship between evasion and the marginal tax rate is more normally observed to be positive than negative.

Second, a wide range of research - including experiments (Spicer and Becker 1980; Becker, Buchner, and Sleeking 1987; Kim 2002), attitudinal surveys (Spicer and Lundstedt 1976; Citrin 1979; Wallschutzky 1984; Scholz and Lubell 1998), and empirical studies (e.g. Alm, Bahl, and Murray 1990) - argues that public expenditure affects tax compliance. Specifically, adverse discrepancies between tax payments and the perceived value of public expenditures are found to be positively related to tax evasion. However, when Cowell and Gordon (1988) extend the standard portfolio model by adding a government financed public good, their principal comparative static result implies exactly the opposite of the empirical evidence: individual evasion is decreasing (increasing) in the tax rate when the public good is over-provided (under-provided). Falkinger (1988) finds the same perverse result in a similar model. Subsequent authors have felt the need to distinguish between the ‘exchange-equity’ and ‘economic’ explanations of the role of public expenditures on tax compliance, the two being irreconcilable.

Third, when extended to allow for obligatory advanced tax payments, the portfolio model predicts that such payments have no effect on compliance. However, both empirical and experimental studies show that evasion is decreasing in the amount of refund that taxpayers expect to receive upon the filing of a tax return, and increasing in the amount of taxes they still have to pay (Chang and Schultz 1990; Robben et al.
These three shortcomings appear to question the reasonableness of representing the taxpayer as a risk-averse agent gambling over levels of net consumption. Alternatively, a substantial literature spanning economics and psychology - reviewed in Kahneman and Tversky (2000) - proposes that individuals judge outcomes relative to a reference level.\(^2\)

The dominant reference-dependent model of choice under risk is (cumulative) prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992). As applied to tax compliance, prospect theory supposes, first, that taxpayers are risk averse over outcomes that fall above the reference level of net consumption, but risk seeking over outcomes that fall below the reference level (diminishing sensitivity). Second, utility is steeper over consumption below the reference level than it is above, so outcomes below the reference level are more painful than the corresponding outcomes above the reference level are satisfying (loss aversion). Third, taxpayers psychologically overweight low probabilities but underweight high probabilities (probability weighting).

For an arbitrary exogenous reference level, Bernasconi and Zanardi (2004) show that replacing expected utility theory with prospect theory in the standard portfolio model can reverse Yitzhaki’s negative relationship between the tax rate and evasion. Furthermore, prospect theory can offer an account of the advanced payment effect (see Schepanski and Shearer 1995; Yaniv 1999).\(^3\)

Given the evidence that public expenditure can influence tax compliance, it seems unlikely that the reference level employed by the taxpayer is simply exogenous as in Bernasconi and Zanardi (2004). Dhami and al-Nowaihi (2007) allow the reference level to be endogenously determined as the taxpayer’s legal level of net consumption. However, this specification remains independent of public expenditure and the compliance behavior of other taxpayers.

In this paper I propose a model in which the reference level reflects a concern for exchange equity between taxpayers and government, thereby capturing both the role of public good provision and the behavior of other taxpayers. My notion of exchange equity is specific to each taxpayer, measuring the balance between a taxpayer’s valuation of the public goods provided by government, and the value of taxes paid.
With this approach I am able to overturn Cowell and Gordon’s result on the relationship between tax rates and evasion in the presence of public expenditures in a direction consistent with the empirical evidence. Also, I am able to analyze the role of income distribution on tax evasion. I find that when taxpayers are heterogeneous in income, the response of individual evasion to a change in the marginal tax rate depends upon relative income (income relative to the mean income). This implies that the aggregate relationship between evasion and the marginal tax rate depends on properties of the income distribution. I also analyze the impact of the structure of the tax system on compliance, and show that raising the marginal tax rate of one taxpayer type lowers the evasion of the remaining taxpayer types, the aggregate impact being undetermined. I argue that this indeterminacy at the aggregate level may help to explain the mixed empirical findings on the relationship between tax rates and compliance.

Earlier contributions by Cowell (1992) and Falkinger (1995) also examine the role of exchange equity on tax compliance, but not in a reference-dependent framework. Falkinger (1995) extends Cowell and Gordon’s (1988) model to allow concern for a common equity index that is exogenous to the individual taxpayer, and reflects the overall equity of the economic order. By contrast, I develop a model with a personalized notion of equity. Cowell (1992) also introduces a concern for equity, but is unable to decisively overturn the perverse prediction of Cowell and Gordon (1988).

My approach also relates to a wider ‘behavioral’ literature on tax compliance. For instance, research has considered the possible role in compliance behavior of stigma or reputation costs (Gordon 1989; Kim 2003); and social norms (Myles and Naylor 1996; Traxler 1999). This literature, however, does not allow for the role of public expenditure on compliance outcomes.

The plan of the paper is as follows: Section 2 describes a reference-dependent model of tax compliance, and Section 3 explores its key comparative static properties. Section 4 explores two extensions of the model: allowing for heterogeneous taxpayers and alternative ethical rules. Section 5 concludes.
2 A Model of Tax Compliance

2.1 Government

My implementation of government is identical to that of Cowell and Gordon (1988). There are \( n \) identical taxpayers, each with an exogenous taxable income of \( y \), on which the government levies a proportional income tax at marginal rate \( \theta \). Taxpayers underdeclare their income by an amount denoted \( e \), so the evaded tax is given by \( \theta e \). Each year, a proportion \( p \) of all taxpayers are chosen at random for an income tax audit at a cost of \( \phi [p] \) per person, where \( \phi' [p] > 0 \). If audited, all current year tax evasion is detected, and the taxpayer must pay an amount \( s \theta e \), where \( s > 1 \). The population is sufficiently large that government revenue is certain, and given by:

\[
R = n (\theta y - \phi [p] - q \theta e) > 0,
\]

where \( q \equiv 1 - ps \) is the expected rate of return to evading tax. The government uses \( R \) to finance a publicly provided good (\( G \)). To allow for possible congestion effects I suppose:

\[
G \equiv \frac{R}{\tau [n]},
\]

where \( \tau \) is a function such that \( 1 \leq \tau [n] \leq n \), and it is assumed that:

\[
\lim_{n \to \infty} \frac{1}{\tau [n]} = 0; \quad \lim_{n \to \infty} \frac{n}{\tau [n]} = \frac{1}{\psi} > 0.
\]

Assuming a large economy (\( n \to \infty \)) it follows that

\[
G = \frac{1}{\psi} (\theta y - q \theta e - \phi [p]).
\]

2.2 Taxpayers

Taxpayers are modelled as viewing outcomes in terms of gains and losses relative to a reference level of net income (consumption). I therefore represent taxpayers’ preferences with a ‘value’ function, \( V [\cdot] \), defined on changes in consumption about an endogenous reference level of consumption \( c_r \). Consumption outcomes \( c \geq c_r \) are considered as ‘gains’, while outcomes \( c < c_r \) are considered as ‘losses’. Individual consumption is composed of the consumption of private goods and the public good.
Following Bordignon (1993) I take a taxpayer’s per unit valuation of the public good to be their marginal willingness to pay \( (m) \). An individual’s valuation of the supply of the public good is then \( mG \). I can therefore write:

\[
V [c_p, G; c_r] = V [c_p + mG - c_r],
\]

where \( c_p \) is a random variable representing the private component of consumption (disposable income plus proceeds from evasion). It follows that, holding \( c_r \) constant,

\[
\frac{V_G[c_p, G; c_r]}{V_{cp}[c_p, G; c_r]} = m \text{ is the marginal willingness to pay for the public good.}
\]

The existing literature models \( m \) as a constant, which implies a zero income effect. My results also obtain if I assume \( m \) constant. It is perhaps more realistic, however, to allow the marginal willingness to pay to be a function of the legal private consumption level (the level of consumption if all legally due tax is paid), so that \( m \equiv m [y (1 - \theta)] \). Higher incomes appear to be associated with more public services: Bergstrom and Goodman (1973) find that US general municipal expenditures show an income elasticity of 0.64, while Borcherding and Deacon (1972) present evidence, again from the US, that the income elasticities for various public services range between 0.2 and 1.0. Accordingly, I suppose \( m' \geq 0 \).

The specification of preferences in (1) assumes that taxpayers use a single reference level against which to compare their combined consumption of public and private goods. A possible alternative is that taxpayers make separate comparisons of their consumption of public and private goods against two different reference levels, yielding a specification of the form:

\[
V [c_p, G; c_{rp}, c_{rG}] = V [c_p - c_{rp}, mG - c_{rG}],
\]

It remains debated in the psychology literature as to whether decision-makers refer simultaneously to multiple referents or, in order to simplify decision tasks, combine aspects of several potential referents into a single composite referent (Copeland and Cuccia 2002). For the purposes of this paper, however, the distinction appears of relatively little importance for - as I comment further in Section 3 - the two specifications yield qualitatively similar results. As, however, the specification in (2) introduces additional degrees of complexity into the analysis, I focus here on the simpler specification provided by (1).
2.2.1 Reference Level

Dhami and al-Nowaihi (2007) suppose the reference level to be the legal consumption level, by which I refer to the level of post-tax consumption if all legally due tax is paid. Here legal consumption is given by the sum of legal disposable income and the money value of the public good:

\[ c_l = y(1 - \theta) + mG. \]

I propose a generalization that accommodates the concept of exchange equity. Having paid their taxes, taxpayers look for good value from the government services provided in return, and are sensitive to perceived adverse discrepancies between the value of taxes paid and the value of the government services provided. In essence, taxpayers think of taxes as being the ‘price’ paid in return for the provision of public services.

I define exchange equity for each taxpayer by comparing the value of taxes owed \((\theta y)\) with the value of the public good provided \((mG)\). Exchange inequity occurs if \(\theta y \neq mG\), while exchange equity corresponds to the special case in which \(\theta y = mG\).

I assume the reference level of consumption corresponds to the level that is achieved under exchange equity:

\[ c_r = c_l + \theta y - mG = y \]

The intuition for (3) is that under exchange equity, the value of tax paid is exactly offset by the value of the public goods provided in return, so the reference level of consumption exactly matches private income. Note from (3) that under exchange equity it holds that \(c_r = c_l\), so in this instance any outcome involving some undetected evasion is considered a gain. However, when \(\theta y > mG\) it holds that \(c_r > c_l\), so any outcome that involves undetected evasion of less than \((\theta y - mG)\) will be perceived as a loss, since the amount of evasion is still insufficient to equalize the value of taxes paid and public good provided. Undetected evasion must exceed \((\theta y - mG)\) for the outcome to be perceived as a gain.

In principle, two forms of exchange inequity are possible. If \(\theta y > mG\) the taxpayer is said to experience unfavorable exchange inequity, but if \(\theta y < mG\) the taxpayer experiences favorable inequity. Evidence from attitudinal surveys suggests that the case of perceived unfavorable exchange inequity is by far the more pervasive: while
one can find epochs in which governments have enjoyed widespread satisfaction with their spending decisions\(^5\), most studies reveal pervasive beliefs across countries and time that government is a wasteful bureaucracy which overtaxes the majority of its citizens because of opportunities for tax avoidance for those with higher incomes and wealth (Citrin 1979; Wallschutzky 1984). From a theoretical perspective this may not be surprising: difficulties due to x-inefficiency can thwart productive efficiency (Liebenstein 1966), while the problem of achieving allocative efficiency for public good provision is formidable. More generally, when taxpayers differ in their valuations of each of the various types of public good, the allocation chosen by government will, in general, be sub-optimal from the perspective of any individual taxpayer.\(^6\)

In light of these arguments, I make the focus of the paper the case in which a degree of perceived exchange inequity exists: \(\theta y > mG\). I assume an asymmetric notion of exchange inequity, whereby taxpayers experience disutility from unfavorable inequity, but are ambivalent towards favorable inequity. This assumption reflects the idea that people are less worked up by favorable inequity than by unfavorable inequity, which is consistent with the emerging economic literature on inequity aversion (see e.g. Fehr and Schmidt 1999) and the evidence in the Introduction.\(^7\) However, I discuss the consequences of a symmetric notion of exchange equity in Section 4.2. Incorporating this asymmetric notion of exchange inequity into the reference level of consumption gives:

\[
c_r \equiv c_l + \max[\theta y - mG, 0] = \begin{cases} 
c_l & \text{if } \theta y \leq mG \\
y & \text{if } \theta y > mG \end{cases}.
\]

For the case of favorable exchange inequity \((\theta y \leq mG)\) I then have that \(c_r = c_l\), in which case the model collapses to a special case of Dhami and al-Nowaihi (2007). In what remains, I devote my attention to the more interesting case in which \(\theta y > mG\).

### 2.3 Individual Maximization

Drawing on the insights of prospect theory I make the following assumptions on \(V[\cdot]\):

\[
\begin{array}{l}
A0. V[c] \text{ is continuous for all } c, \text{ twice differentiable for } c \neq 0, \text{ and } V[0] = 0. \\
A1. V[c] \text{ is strictly increasing.} \\
A2. V'[c] < V'[-c] \text{ for } c > 0. \\
A3. V''[c] < 0 \text{ for } c > 0 \text{ and } V''[c] > 0 \text{ for } c < 0.
\end{array}
\]
Assumptions A0 and A1 are standard technical assumptions, needed for tractability and to ensure the existence of an equilibrium. Assumption A2 is loss aversion (the disutility of a loss exceeds the utility of an equivalent gain), and assumption A3 is diminishing sensitivity (marginal utility is a decreasing function in distance from the reference level). Diminishing sensitivity implies risk seeking preferences over outcomes in the loss domain and risk averse preferences over outcomes in the gain domain. Together, loss aversion and diminishing sensitivity imply that $V[\cdot]$ has a kink-point at the reference level of consumption.

The overweighting of low probabilities and underweighting of high probabilities is captured by the rank-dependent theory of Quiggin (1982), which applies a transformation to the cumulative probability distribution. For the pertinent case of a binary probability distribution, the probabilities $(p, 1 - p)$ are subjectively transformed to $(w[p], 1 - w[p])$, where $w[p]$ is a continuous and strictly increasing probability weighting function, with $w[0] = 0$ and $w[1] = 1$. It is worth noting, however, that the paper does not require probability weighting: the results still hold if $w[p]$ is taken to be the identity function. The reason is that, although probability weighting is potentially important in explaining the level of tax evasion, since $w'[p] > 0$, it has no qualitative implications for signing changes in tax evasion. In the special case when taxpayers’ subjective decision weights correspond to the objective probabilities, the model differs from expected utility theory only in respect of reference-dependence, and therefore satisfies the axioms of Sugden’s (2003) formulation of reference-dependent subjective expected utility theory.

The taxpayer’s objective function can now be written as follows:

$$
\Psi \equiv (1 - w[p])V [c_l + \theta e - c_r] + w[p] V [c_l - (s - 1) \theta e - c_r].
$$

When $\theta y > mG$ the reference level (4) is simply $c_r = y$, so I can rewrite $\Psi$ as:

$$
\Psi = (1 - w[p])V [\theta e - (\theta y - mG)] + w[p] V [-(s - 1) \theta e - (\theta y - mG)].
$$

(5)

In Cournot fashion, the taxpayer chooses $e$, and hence $\theta e$, to maximize $\Psi$ taking the
average evasion of others \((\theta \bar{e})\) as given. This implies that \(G\) is replaced in (5) with:

\[
\bar{G} = \frac{1}{\psi} (\theta y - q \theta \bar{e} - \phi [p]).
\]

The first-order condition with respect to \(\theta e\) is then:

\[
\frac{V' [Y]}{V' [Z]} = \frac{w [p] (s - 1)}{1 - w [p]},
\]

where, as throughout, the derivatives of \(V [:]\) are defined for \(Y, Z \neq 0\), and:

\[
Y \equiv \theta e - (\theta y - m \bar{G}) ; \quad Z \equiv Y - s \theta e.
\]

The second-order condition for an interior maximum requires that:

\[
-(1 - w [p]) V' [Y] \{ A [Y] + (s - 1) A [Z] \} < 0,
\]

where \(A [\cdot] \equiv -\frac{V'' [\cdot]}{V' [\cdot]}\) is the Arrow-Pratt coefficient of absolute risk aversion. The assumption that the value function is convex for losses implies that there is no meaningful restriction which ensures that (7) is satisfied. If (7) is not satisfied, then the taxpayer chooses a corner solution at which they either declare all their income or none of it. Although from a normative perspective these corner solutions cannot be dismissed, from a positive standpoint it is clear that almost everyone pays at least some fraction of their taxes, but often do not pay all their taxes. Therefore, I argue that interior solutions in which taxpayers evade some fraction of their taxes are of greater interest from a positive perspective than either corner solution. In what follows, I therefore focus my attention where the model is strongest from a positive perspective: interior solutions that satisfy \(e \in (0, y)\), which is the case if (7) is satisfied and:

\[
\frac{V' [m \bar{G}]}{V' [m \bar{G} - s \theta y]} < \frac{w [p] (s - 1)}{1 - w [p]} < 1.
\]

The second inequality implies \(q > 0\), which is the standard restriction that the tax gamble must have a positive expected return.
2.4 Nash Equilibrium

The individual taxpayer chooses $\theta e$ treating $\theta \bar{e}$ as exogenous. I now set $\bar{e} = e$ to solve for the symmetric Nash equilibrium between taxpayers. The equilibrium expressions for $Y$ and $Z$ are:

$$Y = \theta e \delta - \frac{\theta y}{\psi} (\psi - m) - \frac{m}{\psi} \phi [p] > 0; \quad Z = Y - s \theta e < 0; \quad (9)$$

where $\delta \equiv 1 - \frac{mq}{\psi}$. The assumptions of an interior optimum, and of unfavorable exchange inequity ($\theta y > mG$), are sufficient to guarantee that these equilibrium values of $Y$ and $Z$ satisfy $Y > 0$ and $Z < 0$. Therefore, by diminishing sensitivity, I have that $A[Y] > 0$ and $A[Z] < 0$. From (6) I have that:

$$\frac{\partial (\theta e)}{\partial (\theta \bar{e})} = \frac{(1 - \delta) (A[Y] - A[Z])}{A[Y] + (s - 1) A[Z]} > 0; \quad \frac{\partial^2 (\theta e)}{(\partial (\theta \bar{e}))^2} = 0; \quad (10)$$

so the reaction function of each taxpayer is a linear and increasing function of average evasion $\theta \bar{e}$, which is sufficient to guarantee uniqueness of the equilibrium. Unlike much of the behavioral literature on tax compliance cited in the Introduction, the model therefore does not suffer from the predictive difficulties associated with multiple equilibria.\(^9\) I assume local stability of the Nash equilibrium, which requires that $\left| \frac{\partial (\theta e)}{\partial (\theta \bar{e})} \right| < 1$ (Cornes and Sandler 1986). Using the expression for $\frac{\partial (\theta e)}{\partial (\theta \bar{e})}$ in (10), local stability can be shown to imply that $\delta A[Y] + (s - \delta) A[Z] > 0$.

3 Analysis

I begin by investigating the taxpayer’s optimal rate of taxation, which, for a given level of evasion, balances the marginal benefit from receipt of public goods with the marginal cost of additional taxation. Using the equilibrium expressions for $Y$ and $Z$ in (9) I have that:

$$\left. \frac{\partial \Psi}{\partial \theta} \right|_{\theta e = \text{constant}} = \frac{y}{\psi} (m - \psi - m' \psi G) \{ (1 - w [p]) V' [Y] + w [p] V' [Z] \}. \quad (11)$$

Therefore, if $m - \psi - m' \psi G > 0$ (high $m$) individual utility is increasing in the tax rate and the public good is said to be under-provided. If $m - \psi - m' \psi G < 0$ (low $m$) individual utility is decreasing in the tax rate, and the public good is said to
be over-provided. The individually optimal provision of the public good is therefore when \( m - \psi - m'\psi G = 0 \).^{10}

At the Nash equilibrium the response of evasion to a change in the tax rate is given by:

\[
\frac{\partial (\theta e)}{\partial \theta} = -\frac{\psi (m - \psi - m'\psi G) \left( A[Y] - A[Z] \right)}{\delta A[Y] + (s - \delta) A[Z]}.
\] (12)

Local stability guarantees that the denominator is positive, and as \((A[Y] - A[Z])\) is positive, it follows that (12) turns on the sign of \(- (m - \psi - m'\psi G)\). I therefore have the following Proposition:

**Proposition 1** When the public good is over- (under-) provided, the effect on tax evasion of an increase in the tax rate is positive (negative):

\[
\frac{\partial (\theta e)}{\partial \theta} \geq 0 \iff -(m - \psi - m'\psi G) \geq 0.
\]

Proposition 1 is the opposite result to that of Cowell and Gordon (1988, 312). The intuition behind the result is that when the public good is under-provided, increasing the tax rate increases consumption, so the evasion required to at least achieve exchange equity (beyond which taxpayers become risk averse over further evasion) falls. Conversely, if the public good is over-provided, further increases in the tax rate decrease consumption, so increasing the level of evasion required to achieve exchange equity.

Similar to Proposition 1, Traxler (2009) also finds that the response of evasion to the tax rate switches sign about a threshold. However, in Traxler’s model the threshold is in terms of a critical degree of internalization of the social norm for compliance, and therefore does not correspond to the optimal provision of the public good as in the present model.

In respect of the alternative specification of preferences in (2): if, for instance, additive separability is assumed such that \( V [c_p, G; c_{rp}, c_{rG}] = V [c_p - c_{rp}] + V [mG - c_{rG}] \), with \( c_{rp} = y (1 - \theta) \) (the legal level of private consumption) and \( c_{rG} = \theta (y - e) \) (the value of taxes paid), then an equivalent statement to that in Proposition 1 can be derived.

Turning to the role of income, I have that:
\[
\frac{\partial (\theta e)}{\partial e} = -\frac{1}{\psi} \left\{ \theta (m - \psi) + (1 - \theta) m'G \right\} \frac{A [Y] - A [Z]}{\delta A [Y] + (s - \delta) A [Z]},
\]
which has the sign of \(- \{ \theta (m - \psi) + (1 - \theta) m'G \}\). Consequently, when the public good is under-provided (13) is negative, and remains so at the individual optimum. However, for a sufficient degree of over-provision, i.e. \(\theta (m - \psi) < -(1 - \theta) m'G\), the effect on evasion of an increase in income becomes positive. In this latter case, the prediction of (13) coincides with that of the standard portfolio model and the empirical evidence (Clotfelter 1983; Baldry 1987).

I summarize the comparative static properties of the remaining variables in Proposition 2:

**Proposition 2** For the first-order condition given by (6) it holds that:

\[
\frac{\partial (\theta e)}{\partial \psi} = \frac{mG}{\psi} \frac{A [Y] - A [Z]}{\delta A [Y] + (s - \delta) A [Z]} > 0;
\]
\[
\frac{\partial (\theta e)}{\partial \phi [p]} = \frac{m}{\psi} \frac{A [Y] - A [Z]}{\delta A [Y] + (s - \delta) A [Z]} > 0;
\]
\[
\frac{\partial (\theta e)}{\partial m} = -\frac{G (A [Y] - A [Z])}{\delta A [Y] + (s - \delta) A [Z]} < 0;
\]
\[
\frac{\partial (\theta e)}{\partial p} = -\frac{mw' [p]}{\psi} \frac{(s\theta e - \phi' [p]) (A [Y] - A [Z]) + \frac{w' [p]}{w [p] (1 - w [p])}}{\delta A [Y] + (s - \delta) A [Z]} \geq 0 \iff s\theta e \leq \phi' [p];
\]
\[
\frac{\partial (\theta e)}{\partial s} = -\frac{mw' [p] \theta e (A [Y] - A [Z]) + \theta e A [Z] + \frac{1}{s - 1}}{(1 - \delta) A [Y] + (s - 1 + \delta) A [Z]} \geq 0.
\]

The first three results of Proposition 2 are intuitive. The second result is obtained (as \(\phi [p]\) is a function) by writing \(\phi [p] + \varepsilon\) in (6) and computing \(\lim_{\varepsilon \to 0} \frac{\partial (\theta e)}{\partial e}\). The third derivative (with respect to \(m [\cdot] \)) is computed analogously. However, as noted by Bernasconi and Zanardi (2004), under prospect theory there is ambiguity over the response of evasion to both a change in the probability of detection (\(p\)) and the fine rate (\(s\)). In the case of \(p\), the derivative \(\frac{\partial (\theta e)}{\partial p}\) takes its expected negative sign if the marginal increase in government revenue raised by increasing \(p (s\theta e)\) outweighs the extra cost of raising \(p (\phi' [p])\).
The difficulty in signing the effect of the fine rate arises from the assumption of diminishing sensitivity (A3), as it implies that an increase in $s$ leads the taxpayer to wish to evade more in the loss state. Although there is no easily interpreted condition under which $\frac{\partial (\theta_e)}{\partial s}$ takes its expected negative sign, the necessary condition for this to occur does not appear unduly restrictive. For instance, if the value function takes the constant-relative-risk-aversion form proposed by Tversky and Kahneman (1992), which the authors show to provide a good fit to experimental data, it can be shown that $\frac{\partial (\theta_e)}{\partial s} < 0$.

The one issue so far ignored in the above analysis is the impact of loss aversion (A2), which describes the phenomenon whereby the disutility of a loss exceeds the utility of an equivalent gain. Kahneman and Tversky (1979) model loss aversion by writing the value function in the form:

$$V[c, \lambda] = \begin{cases} v[c] & c \geq 0 \\ -\lambda v[-c] & c < 0 \end{cases},$$

where $\lambda > 1$ is an index of loss aversion. In the loss domain I therefore have that $\frac{\partial^2 V}{\partial c \partial \lambda} = v'[c] > 0$. Substituting $V[Y] = V[Y, \lambda]$ and $V[Z] = V[Z, \lambda]$ in (6), I have that:

$$\frac{\partial (\theta_e)}{\partial \lambda} = \frac{w[p] (s - 1) \frac{\partial^2 V}{\partial c \partial \lambda}}{-(1 - w[p]) V'[Y, \lambda] \{A[Y, \lambda] + (s - 1) A[Z, \lambda]\}} < 0,$$

where the result follows from the denominator being negative (by the assumption of an interior maximum). The predicted negative effect of loss aversion on evasion agrees with the finding of Dhami and al-Nowaihi (2007) in their model.

4 Extensions

4.1 Heterogeneous Taxpayers

I now relax the assumption of homogeneous taxpayers along the lines suggested in Bordignon (1993). This enables, amongst other things, an investigation into the role of the income distribution, and the structure of the tax system, in determining tax compliance. In particular, I investigate whether taxpayers of different incomes may respond differently to a change in the tax rate, and how shifting the burden of taxation between taxpayers affects their compliance. Although an individual taxpayer
is unlikely to know how much each other taxpayer in a large economy has evaded, it might be reasonable to suppose that the individual knows the average evasion of taxpayers within different classes of society. On this basis, let there be two types of taxpayer, indexed $i = 1, 2$, where the number of each type is assumed to be $n/2$ for simplicity.

### 4.1.1 Income Distribution and Compliance

I first suppose that taxpayer types are distinguished by their income, $y_i$, $i = 1, 2$, but remain otherwise identical. In this case I have that:

$$
\bar{G} = \frac{1}{\psi} (\theta \bar{y} - q \theta \bar{e} - \phi [p]),
$$

where:

$$
\bar{e} = \frac{1}{2} (\bar{e}_1 + \bar{e}_2); \quad \bar{y} = \frac{1}{2} (y_1 + y_2).
$$

The first-order condition for an $i$-type taxpayer who takes $\bar{e}$ as given is then:

$$
\frac{V' [Y_i]}{V' [Z_i]} = \frac{w [p] (s - 1)}{1 - w [p]}, \quad (14)
$$

where:

$$
Y_i = \theta e_i - (\theta y_i - m_i \bar{G}); \quad Z_i = Y_i - s \theta e_i.
$$

The second-order condition is analogous to (7). Local stability of the (unique) Nash equilibrium here requires that $\frac{\partial (\theta e_1)}{\partial (\theta e_2)} \frac{\partial (\theta e_2)}{\partial (\theta e_1)} < 1$, which can be shown to imply that:

$$
\omega_i A [Y_i] + (s - \omega_i) A [Z_i] > 0,
$$

where $\omega_i \equiv 1 - \frac{m_i q}{2 \psi}$. Proceeding as in (11) I have that the public good is under-(over-) provided from the perspective of an $i$-type taxpayer as $m_i \bar{y} - \psi y_i (1 + m'_i G)$ is greater than (less than) zero.

I now examine the effect of a change in the tax rate on the equilibrium level of evasion. Differentiating the first-order condition (14) yields:

$$
\frac{\partial (\theta e_i)}{\partial \theta} = -\frac{1}{\psi} \left\{ m_i \bar{y} - \psi y_i (1 + m'_i G) \right\} \frac{(A [Y_i] - A [Z_i])}{\omega_i A [Y_i] + (s - \omega_i) A [Z_i]}. \quad (15)
$$
As the denominator of (15) is positive by local stability, it can therefore be seen that - as was the case for homogenous taxpayers - individual evasion turns on the over- or under-provision of the public good. Further insight is gained by noting from (15) that:

\[
\frac{\partial (\theta e_i)}{\partial \theta} \geq 0 \Leftrightarrow \frac{y_i}{\psi} \geq \frac{m_i}{\psi (1 + m'_G)}.
\] (16)

Eqn. (16) demonstrates that the response of evasion to an increase in the tax rate depends on a measure of relative income: the ratio of income to mean income. Although \(\frac{\partial (\theta e_i)}{\partial \theta}\) may be of the same sign for both taxpayer types, it is possible that the two types alter their evasion in opposite directions. This occurs when one type perceives the public good to be under-provided while the other type perceives the public good to be over-provided. In this case, the aggregate effect on compliance depends upon how income is distributed above and below the mean income level. Suppose that there is a ‘high’ income type \((i = 1)\) and a ‘low’ income type, \((i = 2)\), with \(y_1 > y_2\). At this level of generality it is not possible to determine which taxpayer type might increase their evasion and which might lower it. However, if I follow the earlier literature in making the simplifying assumption that \(m\) is constant (implying zero income effects) then (16) yields the following Proposition:

**Proposition 3** If \(m\) is constant (zero income effects) then in response to an increase in the tax rate either:

i) Taxpayers of both types increase their tax evasion (if the public good is over-provided) or decrease their tax evasion (if the public good is under-provided);

ii) Type-1 (high income) taxpayers increase their tax evasion and type-2 (low income) taxpayers decrease their tax evasion.

Proposition 3 shows that if \(m'\) is small enough, when the two taxpayer types respond differently, it is the high income taxpayers who increase their tax evasion, and the low income taxpayers who decrease their evasion. The intuition for the result is that high income taxpayers pay a greater amount of tax than low income taxpayers, yet all taxpayers consume the same level of the public good. As such, if the valuation of the public good is common across both taxpayer types, high income taxpayers are the more likely to perceive the public good to be over-provided.
The result paints a relatively complex picture for the aggregate level of evasion. If both taxpayer types perceive the public good to be either under- or over-provided then the aggregate impact is clear. However, if falling compliance by high income taxpayers is offset by increased compliance by low income taxpayers then the aggregate outcome cannot easily be determined, and depends critically on properties of the income distribution such as the difference in income between the two types \((y_1 - y_2)\) and the abundance of high income taxpayers relative to low income. This complexity over the aggregate impact may help to account for the mixed evidence on the relationship between aggregate evasion and tax rates found in the empirical literature.

The next case of interest is that of a pure increase in inequality, i.e. a mean preserving spread of the income distribution. To implement the mean preserving spread I raise \(y_1\) and lower \(y_2\) such that \(\bar{y}\) is unchanged. The comparative static result is given by:

\[
\left. \frac{\partial (\theta e_1)}{\partial y_1} \right|_{\bar{y}=\text{constant}} = \left( \theta - (1 - \theta) m' G \right) \frac{(A[Y_1] - A[Z_1])}{\omega_1 A[Y_1] + (s - \omega_1) A[Z_1]}.
\] (17)

The first term in the numerator reflects a tendency to increase tax evasion due to an increase in the tax liability for no change in public good supply. The second term, however, reflects the increased valuation of the public good from an increase in income. In general, as these two effects contradict, the net effect in (17) can be of either sign. However, if \(m\) is assumed to be constant, I have the following Proposition:

**Proposition 4** If \(m\) is constant (zero income effects) then a mean preserving spread of the income distribution that increases \(y_1\) and decreases \(y_2\) increases evasion by the high income type:

\[
\left. \frac{\partial (\theta e_1)}{\partial y_1} \right|_{\bar{y}=\text{constant}} > 0.
\]

Proposition 4 shows that when \(m'\) is small, a mean preserving shift of the income distribution raises the evasion of the high income type. It is also straightforward to show, as a corollary, that the mean preserving spread lowers the evasion of the low income type. The high income taxpayers increase their evasion as a response to the worsened exchange inequity entailed by the requirement to pay greater tax for no increase in the supply of public goods. Conversely, the low income taxpayers experience a lessening of exchange inequity, which induces a fall in evasion. While
the standard portfolio model is consistent with this finding - it predicts that evasion is an increasing function of income (Christiansen 1980) - it only does so under the assumption of decreasing absolute risk aversion, which is not a feature of the current model.

The opposing behavior of the two taxpayer types prevents the drawing of a clear conclusion as to the aggregate compliance effect. However, the Proposition suggests that income inequality can generate disparity between the compliance of the rich and the poor - a result consistent with a common finding from attitudinal surveys that the rich are perceived as being the bigger evaders (Citrin 1979; Wallschutzky 1984).

4.1.2 Tax Rate Structure and Compliance

I now suppose that, in addition to the above analysis, each taxpayer type also faces a different marginal rate of tax: $i$-type taxpayers pay tax at the marginal rate $\theta_i$. Kim (2003) also allows for agents that are heterogenous in income and face an income-dependent marginal rate of taxation. However, the author does not present any results regarding parameter variations, instead focusing on the existence of multiple equilibria. I begin by defining:

$$G = \frac{1}{\psi} \left( \theta \tilde{y} - q \tilde{\theta} \tilde{e} - \phi [p] \right),$$

where:

$$\tilde{e} = \frac{1}{2} (\theta_1 \tilde{e}_1 + \theta_2 \tilde{e}_2); \quad \tilde{\theta} = \frac{1}{2} (\theta_1 + \theta_2).$$

The first-order condition for an $i$-type taxpayer, taking $\tilde{e}$ as given, is symbolically identical to (14) with:

$$Y_i = \theta_i e_i - (\theta_i \tilde{y} - m_i \tilde{G}); \quad Z_i = Y_i - s \theta_i e_i.$$

An interesting question to explore in this framework is the effect of shifts in the burden of taxation between taxpayer types on evasion. To do this I increase the tax burden on $j$-type taxpayers relative to $i$-type taxpayers by allowing $\theta_j$ to increase, holding $\theta_i$ constant. My result is summarized in the following Proposition (again assuming local stability):
Proposition 5 An increase in the marginal tax rate of one taxpayer type lowers the level of evasion by the other taxpayer type:

$$\frac{\partial (\theta_i e_i)}{\partial \theta_j} = -\frac{m_i g_i (A[Y_i] - A[Z_i])}{\xi_i A[Y_i] + (s - \xi_i) A[Z_i]} < 0 \quad i \neq j,$$

where $\xi_i \equiv 1 - \frac{m_i g_i}{2\psi} \left(1 + \sum_{j \neq i} \frac{\partial e_j}{\partial e_i}\right)$. The intuition for Proposition 5 is that when the tax payments of the one type of taxpayer increase, the associated increase in public good supply moves the other taxpayer group closer to a position of exchange equity. It follows that if I also allow $\theta_i$ to fall to offset the increase in $\theta_j$ (the case of a revenue neutral redistribution of the tax burden) I obtain an even more significant reduction in evasion. The effect of such a redistribution is simply to lower the tax owed by $i$-type taxpayers without any offsetting fall in public good provision.

Under progressive tax systems Proposition 5 implies that we should expect to observe greater evasion by high income taxpayers relative to low income, as high income taxpayers face higher marginal rates of tax. By highlighting the externalities between the compliance behavior of the two taxpayer types, the Proposition also makes clear that a concern for exchange equity engenders a similar interdependency in compliance decisions to that of the social norm for compliance considered in Traxler (2009). As such, the idea in Traxler (2009) that it is important to enforce high tax compliance among taxpayers who are influential to the compliance behavior of others also applies in the current model. For instance, the model would predict that a campaign that reduced evasion and avoidance of high income taxpayers (whose behavior seems emotive to taxpayers more generally) would additionally lead to the improved compliance of lower income taxpayers.

4.2 Ethical Rules

Until now I have considered an asymmetric notion of exchange equity in which taxpayers are concerned about unfavorable exchange inequity but are ambivalent towards favorable inequity. Bordignon (1993) proposes a stronger stance whereby taxpayers never choose to evade beyond the point of exchange equity. Although he did not do so, Bordignon’s approach can be motivated from the standpoint of a large sociopsychological literature on equity theory (e.g. Homans 1958; Adams 1965). The present model could be generalized in this direction by appropriate modification of
the reference level. For instance, the reference level could be taken to be the legal consumption level ($c_l$), and it could be assumed that the proceeds of evasion are only perceived as a gain if they reduce unfavorable exchange inequity, but not if they lead to favorable inequity. Given the tendency of the standard portfolio model to overpredict levels of tax evasion for plausible levels of risk aversion (Alm, McClelland, and Schulze 1992), the most likely outcome of such a model would be for the taxpayer to choose the constrained solution $\max[y - mG, 0]$. That is, under favorable exchange inequity the taxpayer would be fully compliant, but would evade up to the point of exchange equity under unfavorable exchange inequity.

While this approach can explain the unwillingness of some experimental subjects to evade any tax, even when the tax evasion gamble is clearly better than fair (Baldry 1986), equilibrium evasion is entirely mechanical, in the sense that it depends only on $(y - mG)$, so does not directly reflect the preferences over risk and return embodied in the utility function. An alternative approach that can explain this phenomenon within my framework without such a consequence is to include stigma or reputation costs in the payoffs of the respective states (see e.g. Gordon 1989; Kim 2003).

5 Conclusion

This paper presents a model of tax evasion predicated on two important modifications of the standard portfolio model. First, I use a reference-dependent model of choice under risk, the key feature of which is that taxpayers are risk seeking over outcomes that fall below a reference level. Second, I model in the direction of the empirical evidence by replacing the coercive relationship between taxpayer and government with one of exchange whereby taxpayers care about inequity between the amount they pay in taxes and the amount they perceive themselves to receive in public services.

I find that evasion is increasing (decreasing) in the tax rate when the public good is over- (under-) provided. This result, which accords well with intuition and existing empirical evidence, overturns the opposing result of Cowell and Gordon (1988). Moreover, it is achieved without introducing perverse comparative static properties for the remaining variables. In a more general setting, allowing for heterogeneous agents, I find that differential responses of evasion to changes in the tax rate are possible when agents differ in income. Moreover, whether individuals increase or de-
crease their evasion in response to a change in the tax rate depends upon a measure of their relative income. The aggregate effect therefore depends on the properties of the income distribution.

The findings for heterogeneous agents, as well as the mixed empirical results on the relationship between tax rates and compliance, suggest that more attention should be placed, in both empirical and theoretical work, on the fact that trends in aggregate evasion may well mask significant heterogeneity between different types of taxpayers. While this issue must await further exploration, I think the present exposition manages to retain much of the tractability of the standard portfolio model while improving its performance against the empirical evidence.

Notes

1See Feld and Frey (2007) for a review of the literature in this area.

2Specific evidence of reference-dependence in the context of taxation is provided by Carroll (1992), who documents diaries of taxpayers’ tax-related thoughts and behavior.

3In addition to tax compliance, prospect theory has also been applied to a wide range of economic phenomena: examples include the endowment effect (Knetsch 1989), the equity premium puzzle (Benartzi and Thaler 1995), and consumption smoothing (Bowman, Minehart, and Rabin 1999).

4Allowing for congestion effects implies that the publicly provided good is not a public good in the technical sense of being perfectly non-rival and non-excludable. I shall nevertheless refer to the publicly provided good as simply the public good throughout.

5An example might be from the USA in the early 1960’s, when less than fifty percent of Americans reported that they felt the tax burden was too high (Citrin 1979, 114).

6In extreme cases government can even spend a taxpayer’s contributions on goods that may cause them disutility (e.g. nuclear weapons).

7Specifically, Fehr and Schmidt (1999) propose a utility function $U_{i} [x; \alpha_{i}, \beta_{i}]$, where $\alpha_{i}$ weighs the disutility of unfavorable inequity and $\beta_{i}$ weighs the disutility of favorable inequity. The authors assume that $\alpha_{i} \geq \beta_{i}$.

8Tversky and Kahneman (1992) also allow for different probability weighting functions in the gain and loss domains. I apply a common weighting function as, empirically, the authors find the same weighting function to hold in both domains.

9Equally, however, the model does not explain features of compliance that so far appear to rely on the existence of multiple equilibria. Principally, this is the
phenomenon of two countries experiencing very different levels of compliance despite possessing similar compliance enforcement regimes (see e.g. Kim 2003; Traxler 2009).

The Samuelson condition for Pareto optimality requires $m = \psi$, so the individual optimum is not Pareto optimal except in the zero-income effect case ($m' = 0$). Gottlieb (1985) provides a more detailed analysis of Pareto optimality.
References


