Price Cap Regulation, Revenue Sharing and Information Acquisition*

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July 2003

Abstract

We study the incentives of regulated firms to acquire costly information under price cap regulation. We show that revenue sharing plans, in the spirit proposed by Sappington and Weisman (1996), can provide greater incentives for information acquisition than pure price capping and increase social welfare.

JEL Classification: D82, D83, L51.

Keywords: information acquisition, price cap regulation, revenue sharing.

*Financial support from C.N.R. and M.U.R.S.T. in Rome is gratefully acknowledged.

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1 Introduction

Economists and policy makers have widely discussed how to design regulatory mechanisms for public utilities so as to promote efficiency, competition and help the transition to an unregulated environment. This is not an easy task, for regulators have to deal with their lack of information on the characteristics of the industries and of the firms that operate therein. The access to this information is undoubtedly easier for regulated firms and this explains why the economic literature on regulation has devoted so much attention to the design of regulatory mechanisms when the firm has private information about cost and demand conditions.

However, information acquisition is often costly also for regulated firms. Expensive marketing research may be needed to learn the realization of a demand that fluctuates randomly, as is often the case in the service sector. Engineering and economic research may be required to asses the operating costs of complex firms as utilities. In unregulated markets, firms gain from expensive information gathering about cost and demand conditions for information helps to make better decisions and thus generate higher profits. In regulated markets, the firm's discretion and its action set are by definition constrained and this may severely limit the profit opportunities that better information generates. Thus, it may be misleading to presume that regulated firms always possess precise information about cost and demand conditions. Indeed, when discussing RPI-X regulation, Beesley and Littlechild (1989, p. 467) state:

> *"RPI-X does not assume that costs and demands are given or known: indeed, the problem is to provide adequate incentives for the company to discover them. The aim is to stimulate alertness to lower cost techniques and hitherto unmet demands".*

In this paper we analyze how the information acquisition problem affects the desirability of introducing revenue sharing arrangements within price cap regulation. Our motivation is twofold. First there is now a wide literature that shows how the information acquisition problem can have an impact on the design and performance of the regulatory mechanism. For example, Lewis and Sappington (1993) extend the standard agency model of adverse selection to incorporate the possibility of ignorance and show that this calls for less discretion to the agent. Lewis and Sappington (1997) examine how to motivate a private firm to acquire valuable information about its cost conditions in a standard procurement model. They find that concerns about information management result in super-high powered contracts with pronounced cost sharing. Cremer *et al.* (1998) also endogenize the information structure in a regulated environment and analyze when to induce or deter information acquisition about costs. While this strand of literature has produced important insights, it limits itself to optimal mechanisms that, due to their complexity, are rarely implemented in practice. Instead, price cap regulation is widely used worldwide.

Second, in a recent paper, Iossa and Stroffolini (2002) show that price cap regulation with downward price flexibility provides weak incentives to acquire information about costs since prices cannot fully adjust upwards. When an increase in exogenous costs cannot be translated into higher prices for prices are capped, the firm does not gain anything by learning about such increase. But this implies that the firm has lower incentives to acquire information about its own costs than when it is unregulated; and if incentives are too low, profitable (and socially optimal) price reduction will be foregone when technological conditions improve. While this problem can be at least partially resolved by raising the price cap, this is not very appealing for profits would rise and higher prices would result in the presence of high costs (i.e. when the price cap binds). Giving up price capping as a regulatory mechanism is not particularly appealing either, because of its being a simple and easy to implement mechanism that only requires price monitoring to ensure that it is properly enforced. It is therefore important to ask whether some simple modifications to the pure form of price cap regulation can help the information acquisition problem.

We show that the introduction of revenue sharing under price cap regulation, where a fraction of revenues is rebated to consumers via lump sum transfers, can increase the regulated firm's incentives to acquire information. The condition required is that the fraction of revenues retained by the firm decreases in the price it chooses. Since higher costs are associated with higher prices and lower profits, this form of revenue sharing increases the sensitivity of profits to the (exogenous) technological conditions, which raises the value of information. This has important welfare effects. Revenue sharing increases the level of the price cap that ensures the firm's participation but reduces the profit-maximizing price that the firm can charge under good technological conditions (where the price cap does not bind). This implies that revenue sharing does not always lead to overall lower prices than pure price cap. However, when under pure price cap, the level of price cap needs to be increased in order to provide incentives to acquire information, this ambiguity is resolved. Under revenue sharing, prices will be lower for any given technological condition, and welfare will be higher. Furthermore, revenue sharing does not compromise productive efficiency: the firm remains residual claimant of any cost reduction.

Like profit sharing, revenue sharing regulation has usually been advocated for it allows consumers to share with the firm some of the gain of production. Furthermore, as pointed out by Sappington and Weisman (1996) and Sappington (2002), revenue sharing plans avoid some of the drawbacks of profit sharing.[1] For example, they do not reduce the regulated

[1]See for example Schmalensee (1989), Gasmi *et al.* (1994), Lyon (1996), Mayer and Vickers (1996) for a

firm's incentives to minimize its operating costs and do not create incentives for the regulator to expropriate the firm by disallowing costs incurred.[2] However, revenue sharing plans may reduce regulated firms' incentives to increase quality, although in an empirical study Ai and Sappington (1998) have shown that such effect may not constitute a real concern.[3] By showing that revenue sharing can increase efficiency by raising the firm's incentives to acquire information, our paper emphasizes another advantage of revenue sharing which goes beyond redistributive concerns.

The paper is organized as follows. Section 2 outlines the model. Section 3.1 discusses the value of information while Section 3.2 compares pure price cap regulation to price cap regulation with revenue sharing in terms of incentives to acquire information and social welfare. Section 4 concludes. All the proofs are in the Appendix.

2 The model

We build a simple model of regulation of a single-product monopolist who aims to maximize profit; we assume away any redistributive concern on the part of the regulator, who maximizes the sum of the net consumer surplus and profit. It is immediate to show that all our results would continue to hold in the presence of a rent-extraction concern on the part of the regulator.

The firm's realized costs are $C = (\beta - e)q$ where q is the quantity produced, e the managerial effort and β a random variable representing exogenous cost conditions. β is

discussion on profit sharing.

[2]In the case where the fraction of revenues rebated to consumers is constant, Sappington and Weisman (1996) also show that revenue sharing limits price discrimination and lobbying by regulated firm.

[3]Ai and Sappington (1998) provide a comprehensive study of the impact of incentive regulation on the quality of service in the telecommunication industry in US during the period (1990-1996). They consider four regimes: rate of return regulation, rate moratorium, revenue sharing and price cap, and shows no systematic link between incentives regulation and service quality.

distributed on the interval $[\underline{\beta}, \overline{\beta}]$ according to the distribution function $F(\beta)$, with density function $f(\beta)$. The disutility of effort (in monetary units) is $\psi(e)$ with $\psi' > 0$, $\psi'' > 0$, $\psi''' \geq 0$, for $e > 0$. The demand function is $q(p)$ with $q^{'} < 0$ and $q^{''} = 0$ (for simplicity), where p denotes the price. $S(p)$ with $S'(p) = -q(p)$ and $S''(p) = -q'(p)$ denotes the net consumer surplus. The regulator observes the price level but not the total costs and the firm's effort.

We focus on the firm's incentives to acquire costly information about its exogenous cost conditions and assume that the realization of β is initially unknown to both the regulator and the firm. However, upon expenditure of K, the firm can observe β while information acquisition is prohibitively costly for the regulator and third parties. The regulator knows the value of K but cannot observe the information acquisition process, which prevents the regulator from simply instructing the firm to acquire information.

In practice, regulated firms are required to report data about the costs of their operations and it is likely that they will have some but not perfect information about their cost conditions. Our assumption of a binary (all or nothing) information structure is made for simplicity; it is meant to capture the possibility that the firm's data do not accurately reflect its real cost conditions and it takes time and effort for the regulatory agency to assess this.

The assumption that the regulator can observe K can be justified when K is interpreted as the cost of engineering studies, for there is no asymmetry of information about this cost. K may also depend on a variety of observable factors, such as the size and age of the firm, that can help the regulator to derive a reliable estimate of these information-acquisition costs.

Within this setting, we consider a price cap mechanism with downward flexibility: given a price cap $\overline{P}$ set by the regulator, the firm is free to choose any price $p \leq \overline{P}$. Furthermore, the regulatory mechanism may provide for a fraction of the firm's revenue to be transferred to consumers via lump sum transfers, where this fraction can be a function of the price the firm charges. Formally, let $\alpha(p)$ be the fraction of revenues retained by the firm, with $0 < \alpha(\cdot) \leq 1$ for each p. For simplicity, we restrict attention to a linear function: $\alpha(p) = a - bp$, where $a \in (bp, 1 + bp]$. In this way we can distinguish four cases:

1) $b = 0$ and $a = 1$, there is no revenue sharing.

2) $b = 0$ and $a < 1$, there is revenue sharing where the fraction of revenues retained by the firm is constant.

3) $b > 0$ and $a = 1$, there is revenue sharing where the fraction of revenues retained by the firm decreases with the price it chooses.[4]

4) $b < 0$, and $a \in (bp, 1 + bp]$, there is revenue sharing where the fraction of revenues retained by the firm increases with the price it chooses.

Case (1) represents pure price cap regulation (PC), while cases (2) to (4) are different forms of price cap regulation with revenue sharing (RS). Note that, given a, higher values of b correspond to greater levels of revenue sharing (if b is negative, this implies that we are considering smaller absolute values): $\alpha(\cdot)$ decreases in b for any p. Given b, higher values of a represent lower levels of revenue sharing: $\alpha(\cdot)$ increases in a for any p.

Denote by $\Pi(\beta, a, b, p) = (a - bp)pq(p) - (\beta - e(p))q(p) - \psi(e(p))$ the firm's profit, where $e(p)$ is the firm's choice of effort that solves $\psi'(e) = q(p)$, and let $p^M(\beta, a, b)$ be the level of p that maximizes $\Pi(\beta, a, b, p)$.

When the firm is informed about the realization of β it chooses $p^I(\beta, a, b) = min \left\{\overline{P}, p^M(\beta, a, b)\right\}$. Since $p^M(\beta, a, b)$ is non-decreasing in β for any a, b, then for each price cap $\overline{P}$ and revenue sharing plan (a, b) there exists a level of β, denoted by $\beta^M \in [\underline{\beta}, \overline{\beta}]$, where $\overline{P} = p^M(\beta^M, a, b)$.

[4] In fact, $a = 1$ is not necessary when $b > 0$; any $a \in (bp, 1 + bp]$ is feasible. We focus on the case where $a = 1$ when $b > 0$ for it simplifies the comparison with pure price cap regulation.

Thus, for $\beta < \beta^M(\cdot)$ an informed firm charges the profit-maximizing price while for $\beta \geq \beta^M(\cdot)$ the price cap is binding and the firm charges $\overline{P}$. Note that β^M increases with the price cap and decreases with the profit-maximizing price. Instead, when the firm is not informed about the realization of β it chooses $p^N \equiv \min\{\overline{P}, p^M(E(\beta), a, b)\}$, where $p^M(E(\beta), a, b)$ maximizes the expected profit of the monopolist, with $E(\beta) = \int_{\underline{\beta}}^{\overline{\beta}} \beta f(\beta) d\beta$.

Let $\overline{P}(a, b, K)$ be the optimal level of the price cap, for a given revenue sharing plan (i.e. for given a and b, including the case of pure price cap regulation) when the regulator wishes to induce information acquisition, thus $\overline{P}(a, b, K)$ solves

$$\max_{\overline{P}} \int_{\underline{\beta}}^{\overline{\beta}} \left\{V(p^I(\beta, a, b)) + \Pi(\beta, a, b, p^I(\beta, a, b))\right\} f(\beta) d\beta - K \tag{P1}$$

$$s.t. \qquad E_\beta \Pi(\beta, a, b, p^I(\beta, a, b)) - K \geq 0, \tag{1}$$

$$\Pi(\beta, a, b, p^I(\beta, a, b)) \geq 0 \qquad \text{for all } \beta \in [\underline{\beta}, \overline{\beta}], \tag{2}$$

$$E_\beta \Pi(\beta, a, b, p^I(\beta, a, b)) - K \geq E_\beta \Pi(\beta, a, b, p^N), \tag{3}$$

$$p^I(\beta, a, b) = min\left\{\overline{P}, p^M(\beta, a, b)\right\},$$

where $V(p^I(\beta, a, b)) = S(p^I(\cdot)) + [1 - (a - bp^I(\cdot))]p^I(\cdot)q(p^I(\cdot))$ denotes the total consumer surplus. Constraint (1) ensures that the firm anticipates non-negative expected profits when it acquires information. Constraint (2) guarantees that, after discovering the state of the world, the firm agrees to produce. Under (3) the firm prefers to incur K to become informed about the realization of β rather than remaining uninformed.

In the next sections we shall consider the desirability of revenue sharing in the presence of an information acquisition problem. We shall derive the conditions under which at the solution of the above program welfare is higher when revenue sharing is allowed. Thus, under such conditions the optimal regulatory mechanism allows for revenue sharing.

3 The regulatory mechanism

3.1 The value of information

Let us consider the incentives to acquire information under the standard mechanism designed for the case where the firm privately observes β at no cost. Denote by $\overline{p}(a,b)$ the level of the price cap that solves program (P1) when we disregard the information acquisition constraints (1) and (3). At this level of price cap $\overline{p}\,(a,b)$, the value of information is measured by the difference between the expected profits of an informed and uninformed firm when the former chooses $p^I(\beta,a,b) = min\left\{\overline{p}\,(a,b)\,, p^M(\beta,a,b)\right\}$ and the latter chooses $p^N \equiv \min\{\overline{p}\,(a,b)\,, p^M(E(\beta),a,b)\}$. For expositional simplicity, we focus on the case where $p^M(E(\beta),a,b) \geq \overline{p}(a,b)$ and therefore $p^N = \overline{p}(a,b)$.[5] The value of information is (this expression is derived in the appendix)

$$I(a,b) = \int_{\underline{\beta}}^{\beta^M(a,b)} \int_{\beta}^{\beta^M(a,b)} [q^M(p^M\,(s,a,b)) - q(\overline{p}(a,b))] ds dF(\beta), \tag{4}$$

where $\beta^M(a,b)$ solves: $\overline{p}(a,b) = p^M(\beta^M,a,b)$. First, $I(a,b)$ increases in the level of β^M. When $\beta \geq \beta^M$ information about β is valueless ex post for it does not affect the firm's choice (since the price cap is binding, both an informed and an ignorant firm choose the price cap). This is because of the very nature of price cap regulation that breaks the link between prices and costs. Second, within the range $\beta < \beta^M$, the value of information is proportional to the difference in output levels, for each β, when the firm acquires information and when it

[5] See the appendix for the remaining case.

remains ignorant. Indeed the greater this difference, the greater the sensitivity of the firm's profits with respect to the technological parameter and the higher the gain from acquiring information. It follows that a higher price cap and a lower profit maximizing price increase the value of information.

¿From the above, it also follows that if $I(a,b) \geq K$, the regulatory mechanism induces the firm to acquire information at the level of price cap $\overline{p}(a,b)$. Thus, $\overline{p}(a,b)$ is the solution to (P1).[6] However, for $K > I(a,b)$, the firm prefers to remain ignorant and never decrease the price below the price cap. This goes at the expense of consumers whose welfare the regulated firm does not take into account. Consumers suffer from the firm's lack of incentives to acquire information for they cannot benefit from the lower prices that good cost conditions and the downward flexibility of the price cap mechanism could generate. Therefore, when $K > I(a,b)$ it will be optimal to modify the regulatory mechanism so as to increase the firm's incentives to acquire information (unless K is so high that inducing information acquisition becomes suboptimal). In the absence of revenue sharing, such incentives can be provided only by raising the level of price cap and increasing the firm's expected profits at least proportionally to K, as shown by Iossa and Stroffolini (2002). We now analyze whether the introduction of revenue sharing, may result in a better response to the information acquisition problem.

3.2 Pure price cap regulation versus revenue sharing

First, consider the effect of revenue sharing on the level of price cap, when we disregard the information acquisition problem.

Lemma 1 *Revenue sharing increases the level of price cap.*

[6] Note that since $E_\beta \Pi(\beta, a, b, p^N) > 0$, constraint (1) is automatically satisfied when (3) holds.

In the absence of an information acquisition problem, the optimal price cap corresponds to the minimum level of $\overline{P}$ that ensures the firm's participation. Since the profit function is decreasing in β, $\Pi_{\beta}(\beta, a, b, p^{I}(\beta, a, b)) = -q(p^{I}(\beta, a, b)) < 0$, the price cap $\overline{p}(a, b)$ solves: $\Pi(\overline{\beta}, a, b, \overline{p}(a, b)) = 0$. As some of the firm's revenues are transferred to consumers, the firm's profit decreases for any given price; therefore the level of price cap that ensures the firm's participation increases.

The above Lemma allows us to provide a sufficient condition for the value of information to increase when pure price cap regulation is modified so as to introduce some form of revenue sharing.

Lemma 2 *A sufficient condition for the value of information to increase with revenue sharing is that* $p^{M}(\beta, a, b) \leq p^{M}(\beta, 1, 0)$*, with strict inequality for some* $\beta \leq \beta^{M}(a, b)$*: under RS the profit maximizing price is at most the same as under PC.*

If RS leads not only to a higher level of price cap (Lemma 1), but also to a lower profit-maximizing price, $\beta^{M}(\cdot)$ unambiguously rises, and there is a wider range of βs where acquiring information is valuable ex post. Further, within this range, the difference between $q^{M}(p^{M}(\beta, a, b))$ and $q(\overline{p}(a, b))$ is greater, which in light of (4) increases the value of information.

Proposition 1 *The necessary condition for* $p^{M}(\beta, a, b) \leq p^{M}(\beta, 1, 0)$ *is* $b > 0$*. For* $b \in (0, -\frac{q'}{q(p=0)}) > 0$*, the sufficient condition is* $\left|\eta(p^{M}(\beta^{M}, 1, 0))\right| \leq 2$*, where* $\eta(p)$ *denotes the elasticity of demand.*

In order to induce the firm to charge a lower monopoly price under RS than under PC the fraction of revenues retained by the firm must be lower the higher the price the firm chooses. Since higher costs are associated with higher prices and lower profits, this form of revenue sharing increases the sensitivity of profits to the technological conditions so as to raise the value of information. Thus, we need to focus on case (3), where $b > 0$ and $a = 1$. For $b \in (0, -\frac{q'}{q(p=0)})$ and $\left|\eta(p^M(\beta^M, 1, 0))\right| \leq 2$, the reduction in firm's retained revenues due to an increase in b, is greater the higher the price charged by the firm.[7] This makes the profit maximizing price non-increasing in b, which, in light of Lemma 2, implies that the value of information is higher under RS than under PC.

Having established that RS can be designed so as to increase the value of information, we now discuss the welfare effects of revenue sharing. Let $I(1,0)$ denote the value of information under pure price cap regulation. For $K \leq I(1,0)$, where information acquisition occurs under both PC and RS, RS is welfare superior to PC only if the increase in the expected welfare due to the reduction of the profit maximizing price for low βs (Proposition 1) overcompensates the reduction in expected welfare due to the increase in price cap for high βs (Lemma 1). This clearly depends on the shape of the function $F(\beta)$. However, for some $K > I(1,0)$, this ambiguity is resolved. To see this, let us consider the case where at $K = I(1,0) + \varepsilon$, where $\varepsilon > 0$ and small, under pure price cap regulation it is optimal to induce information acquisition. Formally, we assume

Assumption 1. $\int_{\underline{\beta}}^{\overline{\beta}} \left\{V(p^I(\beta,1,0)) + \Pi(\beta,1,0,p^I(\beta,1,0))\right\} dF(\beta) - K$ *is greater than* $V(p^N(1,0)) + \int_{\underline{\beta}}^{\overline{\beta}} \Pi(\beta,1,0,p^N(1,0)) dF(\beta)$.

Proposition 2 *When* $a = 1$, $b \in (0, -\frac{q'}{q(p=0)})$ *and* $\left|\eta(p^M(\overline{\beta}, 1, 0))\right| \leq 2$, *there exists a level*

[7] Due to the linear form of revenue sharing assumed here, the marginal revenue can become increasing in price for high levels of b and $|\eta|$. The upper bound imposed on the value of the revenue sharing parameter b ensures that the second order conditions of the maximization problem of the firm are satisfied in the relevant range (i.e. $|\eta(\cdot)| \leq 2$).

of K, denoted by $\overline{K}$ with $\overline{K} > I(1,0)$, such that for all $K \in (I(1,0), \overline{K}]$ price cap regulation with revenue sharing welfare dominates pure price cap regulation.

The intuition behind Proposition 2 can be understood as follows. If $K > I(1,0)$, under PC the price cap needs to increase in order to provide incentives to acquire information (since the higher the price cap the greater the range of β where information is valuable ex post).[8] Then, when the conditions in Proposition 2 are satisfied, the value of information is higher under RS than under PC *for the same price cap*. Therefore, RS can induce information acquisition at a lower price cap (and lower $p^M(\cdot)$) than PC. This suggests that *if* it is optimal to induce information acquisition and if the conditions in Proposition 2 are met, then RS is welfare superior to PC. However, whether information acquisition is optimal depends on the value of K and on the shape of the distribution function $F(\beta)$ that affects the probability that an informed firm will choose a price lower than the price cap (this explains the upper bound $\overline{K}$). In industries where it is perceived that there are good technological conditions or great scope for cost reductions, it will be likely that information acquisition is optimal since it allows to capture the gain from the downward flexibility of the price cap mechanism. In these cases, RS appears a better choice than PC.

4 Conclusions

In general, profit or revenue sharing arrangements are introduced in order to let consumers share with the firm some of the gains from production. In this paper we have provided a rationale for revenue sharing that goes beyond redistributive concerns. In particular, we have shown that price cap regulation with revenue sharing, where the fraction of revenues

[8]For a formal proof see Iossa and Stroffolini (2002).

retained by the firm decreases in the price it chooses, can be more efficient than pure price cap regulation in the presence of an information-acquisition problem.

It should be apparent at this stage that the form of sliding scale arrangement proposed here is by no means the only one that can increase incentives to acquire information under PC. Take for example a profit sharing arrangement, where the fraction of profits retained by the firm decreases with the price it chooses, as suggested for example by Burns *et al.* (1998). Like revenue sharing, this regulatory mechanism could result in the firm choosing lower prices for low values of the technological parameter (that is when the price cap is not binding) and for any given level of effort in cost reduction, so as to increase the range of β where information is valuable. However, the productive inefficiency associated with profit sharing would generate an opposite effect on prices due to the reduction in effort for any given level of the technological parameter (see e.g. Mayer and Vickers, 1996). This point, made by Iossa and Stroffolini (2002), suggests that from the point of view of incentives to acquire information, revenue sharing is preferable to profit sharing. However, profit sharing might be better in terms of incentives to increase quality, as shown by Sappington and Weisman (1996) for the case where the fraction of revenues retained by the firm is constant. A better understanding of the trade off between incentives for quality and for information acquisition could constitute the scope of future research.

5 Appendix

Derivation of expression (4). Integrating $\Pi_\beta(\beta, \overline{p}(a,b), a, b) = -q(\overline{p}(a,b))$ we obtain $\Pi(\beta^M(a,b), \overline{p}(a,b), a, b) = \int_{\beta^M(a,b)}^{\overline{\beta}} q(\overline{p}(a,b))d\beta$, since $\Pi(\overline{\beta}, \cdot) = 0$. Then, by integrating $\Pi_\beta(\beta, p^M(\beta,a,b), a, b) = -q(p^M(\beta,a,b))$ and substituting for $\Pi(\beta^M(a,b), \overline{p}(a,b), a, b)$, the profits for $\beta < \beta^M(a,b)$ are

$$\Pi(\beta, p^M(\beta,a,b), a, b) = \int_{\beta^M(a,b)}^{\overline{\beta}} q(\overline{p}(a,b))d\beta + \int_{\beta}^{\beta^M(a,b)} q^M(s,a,b)ds,$$

where $q^M(s,a,b) \equiv q\left(p^M(s,a,b)\right)$. Applying the same procedure yields $\Pi(\beta, \overline{p}(a,b), a, b) = \int_{\beta}^{\overline{\beta}} q(\overline{p}(a,b))ds$ when $\beta \geq \beta^M(a,b)$. Then, expression (4) follows from the difference between the expected profit of an informed firm and that of an ignorant firm that chooses $p^N = \overline{p}(a,b)$. There are respectively given by

$$\int_{\underline{\beta}}^{\beta^M(\cdot)} \left[\int_{\beta^M(\cdot)}^{\overline{\beta}} q(\overline{p}(a,b))d\beta + \int_{\beta}^{\beta^M(\cdot)} q^M(s,a,b)ds\right] dF(\beta) + \int_{\beta^M(\cdot)}^{\overline{\beta}} \int_{\beta}^{\overline{\beta}} q(\overline{p}(a,b))dsdF(\beta),$$
$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\beta}^{\overline{\beta}} q(\overline{p}(a,b))dsdF(\beta)$$

where $\beta^M(\cdot) \equiv \beta^M(a,b)$.

Proof of Lemma 1. Let $\Pi(\overline{\beta}, \overline{P}, \alpha(\overline{P})) = \alpha(\overline{P})\overline{P}q(\overline{P}) - (\overline{\beta} - e(\overline{P}))q(\overline{P}) - \psi(e(\overline{P}))$ be the firm's profit at $\overline{\beta}$, where $e(\overline{P})$ solves $\psi'(e) = q(\overline{P})$. Then, simply notice that $\frac{\partial \overline{P}}{\partial \alpha(\overline{P})} = -\frac{\overline{P}q(\overline{P})}{\Pi_p(\overline{\beta}, \overline{P}, \alpha(\overline{P}))} < 0$ since $\Pi_p\left(\overline{\beta}, \overline{P}, \alpha(\overline{P})\right)$ is positive for $\overline{P} < p^M\left(\overline{\beta}, \alpha(\overline{P})\right)$, as is always the case for a monopolist under price cap regulation.

Proof of Lemma 2. Compared to pure price cap regulation, revenue sharing entails either a higher level of b or a lower level of a. It follows that it suffices to prove that

$\frac{\partial I(a,b)}{\partial a} < 0, \frac{\partial I(a,b)}{\partial b} > 0$. By differentiating (4) with respect to a and b it is easy to show that the above conditions are equivalent to prove that $p^M(\beta, a, b) \leq p^M(\beta, 1, 0)$.

Proof of Proposition 1. Consider the first order condition of profit maximization

$$-bp^M(\beta,a,b)q^M(\beta,a,b) + \alpha(p^M(\beta,a,b))\left[p^M(\beta,a,b))q' + q^M(\beta,a,b)\right] - (\beta - e^M(\beta,a,b))q' = 0 \tag{A1}$$

where $q^M(\beta,a,b) \equiv q^M(p^M(\beta,a,b))$ and $e^M(\beta,a,b)$ solves $\psi'(e) = q^M(\beta,a,b)$. Expression (A1) under PC ($a = 1, b = 0$) becomes

$$p^M(\beta,1,0)q' + q^M(\beta,1,0) - (\beta - e^M(\beta,1,0))q' = 0$$

Provided that the profit function is concave, a necessary condition for $p^M(\beta,1,0) \geq p^M(\beta,a,b)$ is

$$p^M(\beta,a,b)q' + q^M(\beta,a,b) - (\beta - e(\beta,a,b))q' \geq 0 \tag{A2}$$

Adding and subtracting $\alpha(p^M(\beta,a,b))(\beta - e^M(\beta,a,b))q'$ in (A1)

$$-bp^M(\beta,a,b)q^M(\beta,a,b) + \alpha(p^M(\beta,a,b))[p^M(\beta,a,b)q' + q^M(\beta,a,b) +$$
$$-(\beta - e^M(\beta,a,b))q'] - (1 - \alpha(p^M(\beta,a,b)))(\beta - e^M(\beta,a,b))q' = 0$$

thus, $b > 0$ is required for (A2) to hold.

Now, let $a = 1, b > 0$ so that $\alpha(p) = 1 - bp$. Then, total differentiation of first order conditions yields

$$\frac{\partial p^M(\beta,1,b)}{\partial b} = -\frac{\psi'' p^M(\beta,1,b)(p^M(\beta,1,b)q' + 2q^M(\beta,1,b))}{-\psi''(2q' - 4bp^M(\beta,1,b)q' - 2bq^M(\beta,1,b)) - (q')^2}$$

that is no-greater than zero (provided that the denominator is positive by the second order conditions of profit maximization) if

$$\frac{\partial MR(\beta,1,b)}{\partial b} = -p^M(\beta,1,b)(p^M(\beta,1,b)q' + 2q^M(\beta,1,b)) \leq 0 \tag{A3}$$

where $MR(\beta,1,b)$ is the marginal revenue (w.r.t. the price) of a β-firm, evaluated at $p^M(\beta,1,b)$ and given by

$$MR(\beta,1,b) = p^M(\beta,1,b)q' + q^M(\beta,1,b) - b\left(p^M(\beta,1,b)\right)^2 q' - 2bq^M(\beta,1,b)p^M(\beta,1,b)$$

Under pure price cap regulation, $p^I(\beta,1,0) = min\left\{\overline{p}(1,0), p^M(\beta,1,0)\right\}$ and $\overline{p}(1,0) = p^M(\beta^M(1,0),1,0)$, with $\frac{\partial p^M(\beta,1,0)}{\partial \beta} > 0$. Thus if

$$\frac{\partial^2 MR(\beta,1,b,p)}{\partial b \partial p^M} = -4q' - 2q^M(\beta,1,b,p) \geq 0, \tag{A4}$$

that is if $\left|\eta(p^M(\beta,1,b))\right| \geq \frac{1}{2}$ for all $p^M(\beta,1,b)$, then a sufficient condition for $(A3)$ to hold for all β is that $\left|\eta(p^M(\beta^M(1,0),1,0))\right| \leq 2$. The Lemma below, discussing second order conditions, completes the proof.

Lemma 3 *In the case of a linear demand function, the range of admissible values of* b *is* $b \in (0, -\frac{q'}{q(p=0)})$; *further, over this range* $|\eta(p)| \geq \frac{1}{2}$.

Proof of Lemma 3. When $\alpha(p) = 1 - bp$, with $b > 0$, the second order conditions of the maximization problem of the firm require[9]

$$\Pi_{pp}(\beta, 1, b) = MR_p(\beta, 1, b) = 2q' - 4bp^M(\beta, 1, b)q' - 2bq^M(\beta, 1, b) < 0$$

Rewriting the above expression in elasticity terms and simplifying, we have

$$-(2bp^M(\beta, 1, b) - 1)\eta(p^M(\beta, 1, b)) - bp^M(\beta, 1, b) < 0 \tag{A5}$$

which is always satisfied for $|\eta(\cdot)| \leq 1/2$. For $|\eta(\cdot)| \geq 1/2$ $(A5)$ increasing in b. Let $p_{\max}$ denote the level of p such that $\eta(p_{\max}) = -2$. Then $(A5)$ is satisfied for $b \leq \frac{2}{3p_{\max}}$. Let the demand function be given by $q(p) = c - dp$, then $b \leq \frac{2}{3p_{\max}}$ is equivalent to $b \leq \frac{d}{c} \equiv -\frac{q'}{q(p=0)}$. Finally note that $|\eta(\cdot)| = \frac{1}{2}$ at $p_{\min} = \frac{1}{3}\frac{c}{d}$, and $MR(p_{\min}, 1, b) = 0$ at $b = \frac{d}{c}$. Since MR is decreasing in b for $|\eta(\cdot)| \leq 2$ and $|\eta(\cdot)|$ is increasing in p, it follows that at the profit-maximizing price $\left|\eta(p^M(\cdot))\right| > \frac{1}{2}$, which implies $(A4)$.

Proof of Proposition 2. Let $\widehat{K} = I(1, 0)$. For $K > \widehat{K}$, by Assumption 1, under PC information acquisition calls for a price cap $\overline{P}(1, 0, K) > \overline{p}(1, 0)$, where $\overline{P}(1, 0, K)$ solves (3), with $\frac{\partial \overline{P}(1,0,K)}{\partial K} \geq 0$. Now, consider the following three cases, that differ in terms of the choice of price made by the ignorant firm.

i) $p^N = \overline{P}$ under both PC and RS.

ii) $p^N = \overline{P}$ under PC and $p^N = p^M(E(\beta), \cdot)$ under RS.

iii) $p^N = p^M(E(\beta), \cdot)$ under both RS and PC.

Clearly, the higher the price cap, the more likely that $\overline{P} > p^M(E(\beta), \cdot)$. Thus, case (i)

[9]Second order conditions are given by $\Pi_{pp} < 0$, $\Pi_{ee} = -\psi'' < 0$, which always hold and $-\psi''\Pi_{pp}(\cdot) - (q')^2 > 0$, which holds for ψ'' high or q' small only if $\Pi_{pp} < 0$.

is likely to occur for low values of K, case (ii) for intermediate values and case (iii) for high values.

Consider case (i). By Lemma 1 and Proposition 1, if $\left|\eta(p^M(\beta^M,1,0))\right| \leq 2$, information acquisition under RS can be induced at $\overline{p}\,(1,b) < \overline{P}(1,0,K)$ and $p^I(\beta,1,b) \leq p^I(\beta,1,0,K)$ for all $\beta \in [\underline{\beta},\overline{\beta}]$.

Now consider case (iii). Following the same procedure as for the derivation of expression (4), the value of information for a given level of b is given by

$$\int_{\underline{\beta}}^{\beta^M(1,b)}\int_{\beta}^{E(\beta)}(q^M(s,1,b)) - q^M(E(\beta),1,b))dsdF(\beta) +$$
$$+\int_{\beta^M(1,b)}^{\overline{\beta}}\int_{\beta^M(1,b)}^{E(\beta)}(q^M(\beta,1,b)) - q^M(E(\beta),1,b))d\beta dF(\beta) +$$
$$+\int_{\beta^M(1,b)}^{\overline{\beta}}\int_{\beta^M(1,b)}^{\beta}(q^M(E(\beta),1,b) - q(\overline{p}(1,b)))dsdF(\beta)$$

which, after integration by parts, yields

$$\int_{\underline{\beta}}^{\beta^M(1,b)}(q^M(\beta,1,b)) - q^M(E(\beta),1,b))F(\beta)d\beta +$$
$$+\int_{\beta^M(1,b)}^{E(\beta)}(q^M(\beta,1,b)) - q^M(E(\beta),1,b))d\beta +$$
$$+\int_{\beta^M(1,b)}^{\overline{\beta}}(q^M(E(\beta),1,b) - q(\overline{p}(1,b)))(1-F(\beta))d\beta$$

Differentiating the above expression with respect to b, it is easy to show that this derivative is positive if $\frac{\partial}{\partial b}(\frac{\partial p^M(\beta,1,b)}{\partial \beta}) \geq 0$. Thus, the value of information positively depends on the sensitivity of the profit-maximizing price function with respect to the technological parameter β over the range $\left[\underline{\beta},\overline{\beta}\right]$. This is equal to prove that $\frac{\partial \Pi_{pp}(\beta,1,b)}{\partial b} = \frac{\partial MR_p(\beta,1,b)}{\partial b} \geq 0$ in the relevant range (since the lower, in the absolute value, the sensitivity of the marginal revenue with respect to the price, the greater the increase in the profit-maximizing price for any increase in marginal cost), which was shown to hold in the proof of Lemma 3.

Sufficient condition for case (ii) is that the result holds, when under PC we calculate the value of information at $p^N = p^M(E(\beta), \cdot)$ rather than at $p^N = \overline{p}$ (i.e. at the optimal level of p^N). Then, by the proof of case (iii) the result follows.

Finally, note that as K rises, the expected welfare when the firm acquires information decreases, while the expected welfare when the firm does not acquire information remains constant. This suggests that there will be a value of $K > \widehat{K}$, denoted by K_1, above which inducing information acquisition is no longer optimal. Further, there exists a value of $K > \widehat{K}$ denoted by K_2,where the constraint $b \leq -\frac{q'}{q(p=0)}$ binds. Thus, $\overline{K} = \min\left[K_1, K_2\right].$

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