On Design of Quantized Fault Detection Filters with Randomly Occurring Nonlinearities and Mixed Time-Delays

Hongli Dong*, Zidong Wang and Huijun Gao

Abstract

This paper is concerned with the fault detection problem for a class of discrete-time systems with randomly occurring nonlinearities, mixed stochastic time-delays as well as measurement quantizations. The nonlinearities are assumed to occur in a random way. The mixed time-delays comprise both the multiple discrete time-delays and the infinite distributed delays that occur in a random way as well. A sequence of stochastic variables is introduced to govern the random occurrences of the nonlinearities, discrete time-delays and distributed time-delays, where all the stochastic variables are mutually independent but obey the Bernoulli distribution. The main purpose of this paper is to design a fault detection filter such that, in the presence of measurement quantization, the overall fault detection dynamics is exponentially stable in the mean square and, at the same time, the error between the residual signal and the fault signal is made as small as possible. Sufficient conditions are first established via intensive stochastic analysis for the existence of the desired fault detection filters, and then the explicit expression of the desired filter gains is derived by means of the feasibility of certain matrix inequalities. Also, the optimal performance index for the addressed fault detection problem can be obtained by solving an auxiliary convex optimization problem. A practical example is provided to show the usefulness and effectiveness of the proposed design method.

Keywords

Fault detection; Networked control systems; Randomly occurring nonlinearities; Randomly occurring mixed time-delays; Signal quantization.

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H. Dong is with the College of Electrical and Information Engineering, Northeast Petroleum University, Daqing 163318, China, and also with the Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, China. (Email: `shiningdhl@gmail.com`)

Z. Wang is with the Department of Information Systems and Computing, Brunel University, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: `Zidong.Wang@brunel.ac.uk`)

H. Gao is with the Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, China. (Email: `hjgao@hit.edu.cn`)

* Corresponding author.

1. Introduction

In the past decades, the problem of fault detection in dynamic systems has been attracting extensive research attention owing to the ever increasing demand for higher performance, higher safety and reliability standards [4, 6, 7, 11, 22, 23, 31, 32, 35–37]. Roughly speaking, the aim of fault detection and isolation (FDI) is to construct a residual signal and compute a residual evaluation function which can then be compared with a pre-defined threshold. When the residual has a value larger than the threshold, the fault is detected and an alarm of fault is generated. Among different methods for fault detection, the model-based approach has been widely used in recent years as it makes explicit use of the mathematical model for designing a fault detection filter/observer to detect the fault signal. So far, the problem of fault detection has been thoroughly investigated for a variety of systems including linear uncertain systems [15, 20], fuzzy systems [33], time-delay systems [21, 38], Markovian jump linear systems [2, 40] and networked control systems [24], to name just a few.

Networked control systems (NCSs) have recently received a great deal of research attention because of the rapid development of network technologies and their successful industrial applications [1, 5, 14, 17–19, 39]. Nevertheless, compared with the rich literature on filtering and control problems for NCSs [1, 8, 9, 25], only a limited number of results have been available on the general topic of fault detection for NCSs [13, 30]. In the context of network-induced communication delays, there is a need to discuss the *distributed delays* that occur very often in practical systems. The engineering significance of distributed delays has been widely recognized and a number of corresponding results have been published, see e.g. [16, 28, 29, 34]. Note that almost all relevant literature has been concerned with the continuous-time systems involving continuously distributed delays that are described in the form of either a finite or infinite integral. The distributed delays in the discrete-time setting, on the other hand, have received little attention despite their clear engineering insight due to the spatially distributed nature of NCSs. Up to now, little attention has been paid to the FDI problem for networked control systems with infinite distributed communication delays, not to mention the case where the measurement quantization (another typical network-induced phenomenon) is also involved.

It is well known that nonlinearities exist universally in practice and it is quite common to describe them as additive nonlinear disturbances. In a networked system such as the internet-based three-tank system for leakage fault diagnosis, such nonlinear disturbances may occur in a probabilistic way due to the random occurrence of networked-induced phenomenon. For example, in a particular moment, the transmission channel for a large amount of packets may encounters severe network-induced congestions due to the bandwidth limitations, and the resulting phenomenon could be reflected by certain randomly occurring nonlinearities

(RONs) where the occurrence probability can be estimated via statistical tests. Note that some initial work has been carried out for systems with RONs, see [27] and the references therein. Also, the network-induced time-delays are typically time-varying and random, and therefore should be modeled in a probabilistic way as well. In other words, randomly occurring time-delays can better reflect the signal transmission lags for networked systems. However, up to date, the fault detection problem for discrete networked systems involving *randomly occurring multiple delays under quantized measurement outputs with or without RONs* is still an open yet challenging issue. This situation has motivated our current investigation with hope to shorten such a gap by addressed the quantized fault detection problem with randomly occurring mixed delays as well as nonlinearities.

Summarizing the above discussion, in this paper, we are motivated to study the fault detection problem for a class of discrete-time systems involving stochastic mixed time delays, randomly occurring nonlinearities and measurement quantization of the logarithmic type. By augmenting the states of the original system and the fault detection filter, the addressed fault detection problem is converted into an auxiliary $\mathcal{H}_\infty$ filtering problem. Sufficient conditions are established for the existence of the desired fault detection filter, and then the explicit expression of the desired filter gains is derived. A practical example is provided to show the usefulness and effectiveness of the proposed design method. *The main contributions of this paper can be listed as follows. 1) A combination of important factors contributing to the complexity of the systems are investigated within an unified framework that comprises randomly occurring nonlinearities, randomly occurring multiple time-varying communication delays, randomly occurring infinite distributed delays and measurement quantization. 2) The model-based fault detection problem is put forward in the presence of network-induced phenomena occurring with given probabilities. 3) Intensive stochastic analysis is carried out to enforce the $\mathcal{H}_\infty$ performance for the addressed 'complex' systems in addition to the usual stability requirement.*

Notation. The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ and $\mathbb{Z}$ ($\mathbb{Z}^+$, $\mathbb{Z}^-$) denote, respectively, the n-dimensional Euclidean space, the set of all $n\times m$ real matrices and the set of integers (nonnegative integers, negative integers). $l_2[0,\infty)$ is the space of square summable vectors. $\|A\|$ refers to the norm of a matrix A defined by $\|A\| = \sqrt{\mathrm{trace}(A^TA)}$. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of the matrix M. I and 0 represent the identity matrix and zero matrix of compatible dimension, respectively. $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|\,y\}$ will, respectively, mean expectation of the stochastic variable x and expectation of x conditional on y. $\mathrm{Prob}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following discrete-time systems with randomly occurring nonlinearities and mixed stochastic time-delays:

$$\begin{cases} x(k+1) = Ax(k) + A_{d1}\sum_{i=1}^{q}\alpha_i(k)x(k-\tau_i(k)) + \beta(k)A_{d2}\sum_{d=1}^{\infty}\mu_d x(k-d) \\ \qquad + \gamma(k)g(k,x(k)) + D_1 w(k) + Gf(k) \\ y(k) = Cx(k) + D_2 w(k) + Hf(k) \\ x(k) = \psi(k), \forall k \in \mathbb{Z}^- \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ represents the state vector; $y(k) \in \mathbb{R}^m$ is the process output; $w(k) \in \mathbb{R}^p$ is the unknown input belonging to $l_2[0,\infty)$; and $f(k) \in \mathbb{R}^l$ is the fault to be detected. $\tau_i(k)$ $(i = 1, 2, ..., q)$ denote the discrete time-delays while d $(d = 1, 2, ..., \infty)$ describe the distributed time-delays, $\psi(k)$ is a given initial sequence, and $A, A_{d1}, A_{d2}, D_1, G, C, D_2, H$ are all constant matrices with appropriate dimensions.

The nonlinear function $g(k, x(k))$ satisfies the following condition:

$$\|g(k,x(k))\|^2 \le \varepsilon(k)\|E(k)x(k)\|^2 \tag{2}$$

where $\varepsilon(k) > 0$ is a known positive scalar and $E(k)$ is a known constant matrix.

The constants $\mu_d \ge 0$ $(d = 1, 2, \ldots, \infty)$ satisfy the following convergence condition:

$$\bar{\mu} := \sum_{d=1}^{\infty}\mu_d \le \sum_{d=1}^{\infty} d\mu_d < +\infty \tag{3}$$

The stochastic variables $\alpha_i(k)$ $(i = 1, 2, \cdots q)$, $\beta(k)$ and $\gamma(k)$ are mutually uncorrelated Bernoulli distributed white sequences that account for, respectively, the phenomena of randomly occurring discrete time-delays, distributed time-delays and nonlinearities. A natural assumption on the sequences $\alpha_i(k)$ $(i = 1, 2, \cdots q)$, $\beta(k)$ and $\gamma(k)$ are made as follows:

$$\begin{aligned} &\mathrm{Prob}\{\alpha_i(k)=1\} = \mathbb{E}\{\alpha_i(k)\} = \bar{\alpha}_i,\ \mathrm{Prob}\{\alpha_i(k)=0\} = 1-\bar{\alpha}_i, \\ &\mathrm{Prob}\{\beta(k)=1\} = \mathbb{E}\{\beta(k)\} = \bar{\beta},\ \mathrm{Prob}\{\beta(k)=0\} = 1-\bar{\beta}, \\ &\mathrm{Prob}\{\gamma(k)=1\} = \mathbb{E}\{\gamma(k)\} = \bar{\gamma},\ \mathrm{Prob}\{\gamma(k)=0\} = 1-\bar{\gamma}, \end{aligned} \tag{4}$$

where $\bar{\alpha}_i \in [0,\ 1]$, $\bar{\beta} \in [0,\ 1]$ and $\bar{\gamma} \in [0,\ 1]$ are known constants.

Remark 1: As discussed in the introduction, the nonlinearities described by $g(k, x(k))$ could occur in a probabilistic way based on an individual probability distribution specified *a prior* through statistical tests.

The concept of such randomly occurring nonlinearities (RONs) has been put forward in [27] to reflect the stochastic nonlinearities for complex networks. In this paper, the RONs are addressed for the fault detection problems which render more practical significance in a networked environment. On the other hand, the term $\sum_{d=1}^{\infty} \mu_d x(k-d)$ in (1) represents the so-called infinitely distributed delay in the discrete-time setting, which can be regarded as the discretization of the infinite integral form $\int_{-\infty}^{t} k(t-s)x(s)ds$ for the continuous-time system. The importance of distributed delays has been widely recognized, but the corresponding results for discrete-time systems have been very few especially when the fault detection problem becomes a research focus.

Assumption 1: The communication delays $\tau_i(k)$ $(i = 1, 2, \cdots, q)$ are time-varying and satisfy $d_m \leq \tau_i(k) \leq d_M$, where d_m and d_M are constant positive scalars representing the lower and upper bounds on the communication delays, respectively.

Remark 2: The description of the communication delays in (1) exhibits the following two features: 1) the communication delays are allowed to occur in three fashions, i.e., discrete, successive, or even distribute ways; and 2) each possible delay could occur independently according to an individual probability distribution that can be specified *a prior* through statistical test.

In a networked environment, it is quite common that the measurements $y(k)$ of the system are quantized during the signal transmission. Let us denote the quantizer as $h(\cdot) = \begin{bmatrix} h_1(\cdot) & h_2(\cdot) & \cdots & h_m(\cdot) \end{bmatrix}^T$ which is symmetric, i.e., $h_j(-v) = -h_j(v)$, $j = 1, \ldots, m$. The map of the quantization process is

$$\tilde{y}(k) = h(y(k)) = \begin{bmatrix} h_1(y^{(1)}(k)) & h_2(y^{(2)}(k)) & \cdots & h_m(y^{(m)}(k)) \end{bmatrix}^T.$$

In this paper, we are interested in the logarithmic static and time-invariant quantizer. For each $h_j(\cdot)$ $(1 \leq j \leq m)$, the set of quantization levels is described by

$$\mathscr{U}_j = \{\pm\hat{\mu}_i^{(j)}, \hat{\mu}_i^{(j)} = \chi_j^i \hat{\mu}_0^{(j)}, i = 0, \pm 1, \pm 2, \cdots\} \cup \{0\}, \;\; 0 < \chi_j < 1, \; \hat{\mu}_0^{(j)} > 0,$$

and each of the quantization level corresponds to a segment such that the quantizer maps the whole segment to this quantization level.

According to [12], the logarithmic quantizer is given by

$$h_j(y^{(j)}(k)) = \begin{cases} \hat{\mu}_i^{(j)}, & \frac{1}{1+\delta_j}\hat{\mu}_i^{(j)} \leq y^{(j)}(k) \leq \frac{1}{1-\delta_j}\hat{\mu}_i^{(j)} \\ 0, & y^{(j)}(k) = 0 \\ -h_j(-y^{(j)}(k)), & y^{(j)}(k) < 0 \end{cases}$$

where $\delta_j = (1-\chi_j)/(1+\chi_j)$. It can be easily seen from the above definition that $h_j(y^{(j)}(k)) = (1+\Delta_k^{(j)})y^{(j)}(k)$ with $|\Delta_k^{(j)}| \leq \delta_j$. According to the transformation discussed above, the quantizing effect can be transformed into the sector-bounded uncertainties.

Defining $\Delta_k = \text{diag}\{\Delta_k^{(1)}, \cdots, \Delta_k^{(m)}\}$, the measurements with quantization effect can be expressed as

$$\tilde{y}(k) \;=\; (I+\Delta_k)y(k) = (I+\Delta_k)Cx(k) + (I+\Delta_k)D_2 w(k) + (I+\Delta_k)Hf(k) \tag{5}$$

Consider a full-order fault detection filter of the following structure:

$$\begin{cases} \hat{x}(k+1) = A_F\hat{x}(k) + B_F\tilde{y}(k) \\ r(k) = C_F\hat{x}(k) + D_F\tilde{y}(k) \end{cases} \tag{6}$$

where $\hat{x}(k) \in \mathbb{R}^n$ represents the filter state vector, $r(k) \in \mathbb{R}^l$ is the so-called residual that is compatible with the fault vector $f(k)$, and A_F, B_F, C_F, D_F are appropriately dimensioned filter matrices to be determined.

By defining

$$\bar{\Delta} = \text{diag}\{\delta_1, \cdots, \delta_m\}, \quad F_k = \Delta_k\bar{\Delta}^{-1},$$

we can obtain an unknown real-valued time-varying matrix satisfying $F_kF_k^T \le I$. From (1), (5) and (6), we have the overall fault detection dynamics governed by the following system

$$\begin{cases} \bar{x}(k+1) = (\bar{A}+\Delta\bar{A})\bar{x}(k) + \sum_{i=1}^{q}(\bar{A}_{di}+\tilde{A}_{di})\bar{x}(k-\tau_i(k)) + (\bar{A}_d+\tilde{A}_d)\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d) \\ \qquad + (\bar{\gamma}+\tilde{\gamma}(k))Zg(k,x(k)) + (\bar{D}+\Delta\bar{D})v(k) \\ \bar{r}(k) = (\bar{C}+\Delta\bar{C})\bar{x}(k) + (\bar{D}_F+\Delta\bar{D}_F)v(k) \end{cases} \tag{7}$$

where

$$\begin{aligned}
\bar{x}(k) &= \begin{bmatrix} x^T(k) & \hat{x}^T(k) \end{bmatrix}^T, \ \bar{r}(k) = r(k) - f(k), \ v(k) = \begin{bmatrix} w^T(k) & f^T(k) \end{bmatrix}^T, \\
\bar{A} &= \begin{bmatrix} A & 0 \\ B_FC & A_F \end{bmatrix}, \ \bar{A}_{di} = \begin{bmatrix} \bar{\alpha}_iA_{d1} & 0 \\ 0 & 0 \end{bmatrix}, \ \tilde{A}_{di} = \begin{bmatrix} \tilde{\alpha}_i(k)A_{d1} & 0 \\ 0 & 0 \end{bmatrix}, \\
\bar{A}_d &= \begin{bmatrix} \bar{\beta}A_{d2} & 0 \\ 0 & 0 \end{bmatrix}, \ \tilde{A}_d = \begin{bmatrix} \tilde{\beta}(k)A_{d2} & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{D} = \begin{bmatrix} D_1 & G \\ B_FD_2 & B_FH \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
Z &= \begin{bmatrix} I & 0 \end{bmatrix}^T, \ \bar{C} = \begin{bmatrix} D_FC & C_F \end{bmatrix}, \ \bar{D}_F = \begin{bmatrix} D_FD_2 & D_FH - I \end{bmatrix}, \\
\Delta\bar{A} &= H_FF_kE_C, \ \Delta\bar{D} = H_FF_kE_D, \ \Delta\bar{C} = D_FF_kE_C, \ \Delta\bar{D}_F = D_FF_kE_D, \\
H_F &= \begin{bmatrix} 0 & B_F^T \end{bmatrix}^T, \ E_C = \begin{bmatrix} \bar{\Delta}C & 0 \end{bmatrix}, \ E_D = \begin{bmatrix} \bar{\Delta}D_2 & \bar{\Delta}H \end{bmatrix},
\end{aligned}$$

with $\tilde{\alpha}_i(k) = \alpha_i(k) - \bar{\alpha}_i$, $\tilde{\beta}(k) = \beta(k) - \bar{\beta}$ and $\tilde{\gamma}(k) = \gamma(k) - \bar{\gamma}$. It is clear that $\mathbb{E}\{\tilde{\alpha}_i(k)\} = 0, \mathbb{E}\{\tilde{\alpha}_i^2(k)\} = \bar{\alpha}_i(1-\bar{\alpha}_i), \mathbb{E}\{\tilde{\beta}(k)\} = 0, \mathbb{E}\{\tilde{\beta}^2(k)\} = \bar{\beta}(1-\bar{\beta})$ and $\mathbb{E}\{\tilde{\gamma}(k)\} = 0, \mathbb{E}\{\tilde{\gamma}^2(k)\} = \bar{\gamma}(1-\bar{\gamma})$.

Definition 1: [26] The fault detection dynamics in (7) is said to be exponentially stable in the mean square if, in case of $v(k)=0$ and for any initial conditions, there exist constants $\delta>0$ and $0<\kappa<1$ such that

$$\mathbb{E}\{\|\bar{x}(k)\|^2\}\leq\delta\kappa^k\sup_{i\in\mathbb{Z}^-}\mathbb{E}\{\|\psi(i)\|^2\},\quad\forall k\geq 0.$$

Our aim in this paper is to design a filter of the form (6) that makes the error between residual and fault signal as small as possible. By means of definition 1, the aim of this paper can be restated as finding the filter parameters A_F, B_F, C_F and D_F such that the following two requirements are satisfied simultaneously:

$(R1)$ The overall fault detection dynamics (7) is exponentially stable in the mean square.

$(R2)$ Under zero initial condition, the residual error $\bar{r}(k)$ satisfies

$$\sum_{k=0}^{\infty}\mathbb{E}\{\|\bar{r}(k)\|^2\}\leq\gamma^2\sum_{k=0}^{\infty}\mathbb{E}\{\|v(k)\|^2\} \tag{8}$$

for all non-zero $v(k)$, where $\gamma>0$ is made as small as possible in the feasibility of (8).

We further adopt a residual evaluation stage including an evaluation function $J(k)$ and a threshold J_{th} of the following form:

$$J(k)=\left\{\sum_{h=0}^{k}r^T(h)r(h)\right\}^{\frac{1}{2}},\quad J_{th}=\sup_{w_k\in l_2,f_k=0}\mathbb{E}\{J(L)\}. \tag{9}$$

where L denotes the maximum time step of the evaluation function. Based on (9), the occurrence of faults can be detected by comparing $J(k)$ with J_{th} according to the following rule:

$$J(k)>J_{th}\Longrightarrow\text{with faults}\Longrightarrow\text{alarm},$$

$$J(k)\leq J_{th}\Longrightarrow\text{no faults}.$$

3. Main results

In this section, let us investigate the both the analysis and synthesis problems for the fault detection filter design of system (1) in the presence of measurement quantization (5). The following lemmas will be used in deriving our main results.

Lemma 1: [3] Let $x\in\mathbb{R}^n$, $y\in\mathbb{R}^n$ and matrix $Q>0$. Then, we have $x^TQy+y^TQx\leq x^TQx+y^TQy$.

Lemma 2: [16] Let $\mathcal{M}\in\mathbb{R}^{n\times n}$ be a positive semidefinite matrix, $x_i\in\mathbb{R}^n$, and constant $a_i>0$ $(i=1,2,\cdots)$. If the series concerned is convergent, then we have

$$\left(\sum_{i=1}^{\infty}a_ix_i\right)^T\mathcal{M}\left(\sum_{i=1}^{\infty}a_ix_i\right)\leq\left(\sum_{i=1}^{\infty}a_i\right)\sum_{i=1}^{\infty}a_ix_i\mathcal{M}x_i \tag{10}$$

For presentation convenience, we first discuss the *nominal* system of (7) (i.e., without the parameter uncertainties $\Delta\bar{A}$, $\Delta\bar{D}$, $\Delta\bar{C}$ and $\Delta\bar{D}_F$) and will eventually extend our main results to more general case. In the following theorem, a sufficient condition is presented for the residual dynamics (7) to be exponentially stable with (8) satisfied under zero initial conditions.

Theorem 1: Consider the nominal system of (7) with given filter parameters and a prescribed $\mathcal{H}_\infty$ index $\gamma > 0$. The fault detection dynamics is exponentially stable in the mean square and satisfies (8) if there exist matrices $P > 0$, $Q_j > 0$ $(j = 1, 2, \ldots, q)$, $Q > 0$ and positive constant scalar ρ satisfying

$$\Phi = \begin{bmatrix} \Omega_{11} + \bar{C}^T\bar{C} & * & * & * \\ \hat{Z}^T P\bar{A} & \Omega_{22} & * & * \\ \bar{A}_d^T P\bar{A} & \bar{A}_d^T P\hat{Z} & \Omega_{33} & * \\ \bar{D}^T P\bar{A} + \bar{D}_F^T\bar{C} & \bar{D}^T P\hat{Z} & \bar{D}^T P\bar{A}_d & \Omega_{44} \end{bmatrix} < 0 \tag{11}$$

$$Z^T PZ \leq \rho I \tag{12}$$

where

$$\Omega_{11} = 2\bar{A}^T P\bar{A} + \rho\bar{E}(k) + \bar{\mu}Q + \textstyle\sum_{j=1}^{q}(d_M - d_m + 1)Q_j - P,$$

$$\Omega_{22} = 2\hat{Z}^T P\hat{Z} + \mathrm{diag}\{-Q_1 + \tilde{A}_1, -Q_2 + \tilde{A}_2, \cdots, -Q_q + \tilde{A}_q\},$$

$$\Omega_{33} = 2\bar{A}_d^T P\bar{A}_d + \bar{\beta}(1-\bar{\beta})\hat{A}_{d2}^T P\hat{A}_{d2} - \tfrac{1}{\bar{\mu}}Q, \;\; \Omega_{44} = 2\bar{D}^T P\bar{D} + \bar{D}_F^T\bar{D}_F - \gamma^2 I,$$

$$\tilde{A}_i = \bar{\alpha}_i(1-\bar{\alpha}_i)\hat{A}_{d1}^T P\hat{A}_{d1}, \;\; i = 1, 2, \ldots, q. \;\; \bar{E}(k) = \mathrm{diag}\{(4\bar{\gamma}^2 + \bar{\gamma})\varepsilon(k)E^T(k)E(k), 0\},$$

$$\hat{Z} = \begin{bmatrix} \bar{A}_{d1} & \bar{A}_{d2} & \cdots & \bar{A}_{dq} \end{bmatrix}, \;\; \hat{A}_{d1} = \mathrm{diag}\{A_{d1}, 0\}, \;\; \hat{A}_{d2} = \mathrm{diag}\{A_{d2}, 0\}.$$

Proof: Choose the following Lyapunov functional for system (7):

$$V(k) = \sum_{i=1}^{4} V_i(k) \tag{13}$$

where

$$V_1(k) = \bar{x}^T(k)P\bar{x}(k), \;\; V_2(k) = \sum_{j=1}^{q}\sum_{i=k-\tau_j(k)}^{k-1}\bar{x}^T(i)Q_j\bar{x}(i),$$

$$V_3(k) = \sum_{j=1}^{q}\sum_{m=-d_M+1}^{-d_m}\sum_{i=k+m}^{k-1}\bar{x}^T(i)Q_j\bar{x}(i), \;\; V_4(k) = \sum_{d=1}^{\infty}\mu_d\sum_{\tau=k-d}^{k-1}\bar{x}^T(\tau)Q\bar{x}(\tau)$$

with $P > 0, \; Q > 0, \; Q_j > 0$ $(j = 1, 2, \ldots, q)$ being matrices to be determined.

Notice that

$$\mathbb{E}\left\{\tilde{A}_{di}^T P\tilde{A}_{di}\right\} = \bar{\alpha}_i(1-\bar{\alpha}_i)\hat{A}_{d1}^T P\hat{A}_{d1} \tag{14}$$

$$\mathbb{E}\{\tilde{A}_d^T P\tilde{A}_d\} = \bar{\beta}(1-\bar{\beta})\hat{A}_{d2}^T P\hat{A}_{d2}. \tag{15}$$

According to Lemma 1, we have

$$\begin{aligned}
2\bar{\gamma}\bar{x}^T(k)\bar{A}^T PZg(k,x(k)) &\leq \bar{x}^T(k)\bar{A}^T P\bar{A}\bar{x}(k) + \bar{\gamma}^2 g^T(k,x(k))Z^T PZg(k,x(k)), \qquad (16)\\
2\bar{\gamma}g^T(k,x(k))Z^T P\bar{D}v(k) &\leq \bar{\gamma}^2 g^T(k,x(k))Z^T PZg(k,x(k)) + v^T(k)\bar{D}^T P\bar{D}v(k), \qquad (17)\\
2\bar{\gamma}\bigg(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\bigg)^T PZg(k,x(k)) &\leq \bigg(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\bigg)^T P\bigg(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\bigg)\\
&\quad +\bar{\gamma}^2 g^T(k,x(k))Z^T PZg(k,x(k)), \qquad (18)\\
2\bar{\gamma}\bigg(\bar{A}_d\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg)^T PZg(k,x(k)) &\leq \bigg(\bar{A}_d\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg)^T P\bigg(\bar{A}_d\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg)\\
&\quad +\bar{\gamma}^2 g^T(k,x(k))Z^T PZg(k,x(k)). \qquad (19)
\end{aligned}$$

Also, it follows from (2) that

$$g^T(k,x(k))(4\bar{\gamma}^2+\bar{\gamma})Z^T PZg(k,x(k)) \leq x^T(k)(4\bar{\gamma}^2+\bar{\gamma})\rho\varepsilon(k)E^T(k)E(k)x(k) = \bar{x}^T(k)\rho\bar{E}(k)\bar{x}(k) \qquad (20)$$

Then, along the trajectory of system (7), we have from (14)-(20) that

$$\begin{aligned}
\mathbb{E}\{\Delta V_1(k)\} &= \mathbb{E}\{\bar{x}^T(k+1)P\bar{x}(k+1) - \bar{x}^T(k)P\bar{x}(k)\}\\
&\leq \mathbb{E}\bigg\{\bar{x}^T(k)(2\bar{A}^T P\bar{A} - P + \rho\bar{E}(k))\bar{x}(k) + 2\bar{x}^T(k)\bar{A}^T P\bigg(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\bigg)\\
&\quad +2\bar{x}^T(k)\bar{A}^T P\bar{A}_d\bigg(\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg) + 2\bar{x}^T(k)\bar{A}^T P\bar{D}v(k)\\
&\quad +2\bigg(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\bigg)^T P\bigg(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\bigg)\\
&\quad +\sum_{i=1}^{q}\bar{x}^T(k-\tau_i(k))\tilde{A}_{di}^T P\tilde{A}_{di}\bar{x}(k-\tau_i(k)) + 2\bigg(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\bigg)^T P\bar{D}v(k)\\
&\quad +2\bigg(\sum_{i=1}^{q}\bar{A}_{di}\bar{x}(k-\tau_i(k))\bigg)^T P\bar{A}_d\bigg(\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg) + 2v^T(k)\bar{D}^T P\bar{D}v(k)\\
&\quad +2\bigg(\bar{A}_d\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg)^T P\bigg(\bar{A}_d\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg) + 2\bigg(\bar{A}_d\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg)^T P\bar{D}v(k)\\
&\quad +\bigg(\tilde{A}_d\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg)^T P\bigg(\tilde{A}_d\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\bigg)\bigg\}. \qquad (21)
\end{aligned}$$

Next, it can be derived that

$$\begin{aligned}
\mathbb{E}\{\Delta V_2(k)\} &\leq \mathbb{E}\bigg\{\sum_{j=1}^{q}\bigg(\bar{x}^T(k)Q_j\bar{x}(k) - \bar{x}^T(k-\tau_j(k))Q_j\bar{x}(k-\tau_j(k)) + \sum_{i=k-d_M+1}^{k-d_m}\bar{x}^T(i)Q_j\bar{x}(i)\bigg)\bigg\}\\
\mathbb{E}\{\Delta V_3(k)\} &= \mathbb{E}\bigg\{\sum_{j=1}^{q}\bigg((d_M-d_m)\bar{x}^T(k)Q_j\bar{x}(k) - \sum_{i=k-d_M+1}^{k-d_m}\bar{x}^T(i)Q_j\bar{x}(i)\bigg)\bigg\}\\
\mathbb{E}\{\Delta V_4(k)\} &= \mathbb{E}\bigg\{\bar{\mu}\bar{x}^T(k)Q\bar{x}(k) - \sum_{d=1}^{\infty}\mu_d\bar{x}^T(k-d)Q\bar{x}(k-d)\bigg\}. \qquad (22)
\end{aligned}$$

From Lemma 2, it can be easily seen that

$$-\sum_{d=1}^{\infty}\mu_d\bar{x}^T(k-d)Q\bar{x}(k-d) \le -\frac{1}{\bar{\mu}}\Big(\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\Big)^T Q\Big(\sum_{d=1}^{\infty}\mu_d\bar{x}(k-d)\Big) \tag{23}$$

where $\bar{\mu}$ is defined in (3). For notational convenience, we denote the following matrix variables

$$\xi(k) := \left[\begin{array}{cccccc} \bar{x}^T(k) & \bar{x}^T(k-\tau_1(k)) & \cdots & \bar{x}^T(k-\tau_q(k)) & \sum_{d=1}^{\infty}\mu_d\bar{x}^T(k-d) & v^T(k) \end{array}\right]^T,$$
$$\zeta(k) := \left[\begin{array}{ccccc} \bar{x}^T(k) & \bar{x}^T(k-\tau_1(k)) & \cdots & \bar{x}^T(k-\tau_q(k)) & \sum_{d=1}^{\infty}\mu_d\bar{x}^T(k-d) \end{array}\right]^T.$$

We are now ready to prove the exponential stability of the system (7) with $v(k) = 0$. Obviously, the combination of (21)-(23) results in

$$\mathbb{E}\{\Delta V(k)\} \le \mathbb{E}\{\zeta^T(k)\Omega\zeta(k)\} \tag{24}$$

where

$$\Omega = \left[\begin{array}{ccc} \Omega_{11} & * & * \\ \hat{Z}^T P\bar{A} & \Omega_{22} & * \\ \bar{A}_d^T P\bar{A} & \bar{A}_d^T P\hat{Z} & \Omega_{33} \end{array}\right].$$

It follows immediately from Theorem 1 that $\Omega < 0$. Furthermore, along the same line of the proof for Theorem 1 in [26], the exponential stability of system (7) can be confirmed in the mean square sense.

Let us now move to the proof of the $\mathcal{H}_\infty$ performance for the system (7). To do so, we assume zero initial condition and consider the following index:

$$\begin{aligned} J_N &= \mathbb{E}\sum_{k=0}^{\infty}[\bar{r}^T(k)\bar{r}(k) - \gamma^2 v^T(k)v(k)] \\ &= \mathbb{E}\sum_{k=0}^{\infty}[\bar{r}^T(k)\bar{r}(k) - \gamma^2 v^T(k)v(k) + \Delta V(k)] - \mathbb{E}V(k+1) \\ &\le \mathbb{E}\sum_{k=0}^{\infty}[\bar{r}^T(k)\bar{r}(k) - \gamma^2 v^T(k)v(k) + \Delta V(k)] = \xi^T(k)\Phi\xi(k) \end{aligned}$$

According to Theorem 1, we have $J_N \le 0$ and therefore (8), which completes the proof of Theorem 1. ■

Remark 3: The conditions derived in this paper are based on the quadratic Lyapunov function approach. This method has some advantages, for example, the definition of Lyapunov function is simple and the computation cost in the design procedure is low. However, the common quadratic Lyapunov functions tend to be conservative and might not exist for some highly nonlinear systems. It can be shown that with the use of basis-dependent Lyapunov function and delay partitioning approach, less conservative results can be obtained than those with the use of single quadratic Lyapunov function at the cost of higher computational burden.

Having established the analysis results, we are in a position to deal with the filter design problem. In the following theorem, sufficient conditions are provided for the existence of the desired fault detection filters.

Theorem 2: Consider the nominal system of (7) and let $\gamma > 0$ be a given scalar. A desired full-order fault detection filter of the form (6) exists if there exist positive definite matrices P, Q, Q_j $(j = 1, 2, \ldots, q)$, positive constant scalar ρ and matrices X, K satisfying

$$\Lambda = \begin{bmatrix} \hat{\Lambda}_{11} & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ \hat{\Lambda}_{31} & P\hat{D}_0 + X\hat{R}_2 & -P & * & * \\ \hat{\Lambda}_{41} & 0 & 0 & -\bar{P} & * \\ \hat{\Lambda}_{51} & \hat{\Lambda}_{52} & 0 & 0 & -\hat{P} \end{bmatrix} < 0 \tag{25}$$

$$Z^T P Z \leq \rho I \tag{26}$$

where

$$\hat{\Lambda}_{11} = \mathrm{diag}\{\Lambda_{11}, \Lambda_{22}, \Lambda_{33}\}, \quad \hat{\Lambda}_{31} = \begin{bmatrix} P\hat{A}_0 + X\hat{R}_1 & P\hat{Z} & P\bar{A}_d \end{bmatrix},$$

$$\hat{\Lambda}_{41} = \mathrm{diag}\{P\hat{A}_0 + X\hat{R}_1, P\hat{Z}, P\bar{A}_d\}, \quad \Lambda_{11} = \rho\bar{E}(k) + \bar{\mu}Q + \sum_{j=1}^{q}(d_M - d_m + 1)Q_j - P,$$

$$\Lambda_{22} = \mathrm{diag}\{-Q_1 + \tilde{A}_1, \cdots, -Q_q + \tilde{A}_q\}, \quad \Lambda_{33} = \bar{\beta}(1-\bar{\beta})\hat{A}_{d2}^T P\hat{A}_{d2} - \frac{1}{\bar{\mu}}Q,$$

$$\hat{\Lambda}_{51} = \begin{bmatrix} 0 & 0 & 0 \\ K\hat{R}_1 & 0 & 0 \end{bmatrix}, \quad \hat{\Lambda}_{52} = \begin{bmatrix} P\hat{D}_0 + X\hat{R}_2 \\ K\hat{R}_2 - \hat{E}_1^T \end{bmatrix}, \quad \hat{E} = \begin{bmatrix} 0_{n\times n} \\ I_{n\times n} \end{bmatrix},$$

$$\bar{P} = \mathrm{diag}\{P, P, P\}, \quad \hat{P} = \mathrm{diag}\{P, I\}, \quad \hat{E}_1 = \begin{bmatrix} 0_{l\times p} & I_{l\times l} \end{bmatrix}^T, \quad \hat{E}_2 = \begin{bmatrix} 0_{m\times n} & I_{m\times m} \end{bmatrix}^T,$$

$$\hat{A}_0 = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{D}_0 = \begin{bmatrix} D_1 & G \\ 0 & 0 \end{bmatrix}, \quad \hat{R}_1 = \begin{bmatrix} 0 & I \\ C & 0 \end{bmatrix}, \quad \hat{R}_2 = \begin{bmatrix} 0 & 0 \\ D_2 & H \end{bmatrix}.$$

Furthermore, if (P, Q, Q_j, X, K, ρ) is a feasible solution of (25)-(26), then the fault detection filter parameters in the form of (6) are given as follows:

$$\begin{bmatrix} A_F & B_F \end{bmatrix} = [\hat{E}^T P \hat{E}]^{-1}\hat{E}^T X,$$

$$\begin{bmatrix} C_F & D_F \end{bmatrix} = K.$$

Proof: In order to avoid partitioning the positive definite matrices P, Q and Q_j, we rewrite the parameters in Theorem 1 in the following form

$$\bar{A} = \hat{A}_0 + \hat{E}K_1\hat{R}_1, \quad \bar{D} = \hat{D}_0 + \hat{E}K_1\hat{R}_2,$$

$$\bar{C} = K\hat{R}_1, \quad \bar{D}_F = K\hat{R}_2 - \hat{E}_1^T, \quad H_F = \hat{E}K_1\hat{E}_2, \tag{27}$$

where $K_1 = \begin{bmatrix} A_F & B_F \end{bmatrix}$. Noticing (27) and using the Schur Complement Lemma, (11) can be rewritten as

$$\begin{bmatrix} \hat{\Lambda}_{11} & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ \check{\Lambda}_{31} & \hat{D}_0 + \hat{E}K_1\hat{R}_2 & -P^{-1} & * & * \\ \check{\Lambda}_{41} & 0 & 0 & -\bar{P}^{-1} & * \\ \hat{\Lambda}_{51} & \check{\Lambda}_{52} & 0 & 0 & -\hat{P}^{-1} \end{bmatrix} < 0 \tag{28}$$

where

$$\check{\Lambda}_{31} = \begin{bmatrix} \hat{A}_0 + \hat{E}K_1\hat{R}_1 & \hat{Z} & \bar{A}_d \end{bmatrix}, \quad \check{\Lambda}_{41} = \mathrm{diag}\{\hat{A}_0 + \hat{E}K_1\hat{R}_1, \hat{Z}, \bar{A}_d\}, \quad \check{\Lambda}_{52} = \begin{bmatrix} \hat{D}_0 + \hat{E}K_1\hat{R}_2 \\ K\hat{R}_2 - \hat{E}_1^T \end{bmatrix}.$$

Pre- and post-multiplying the inequality (28) by $\mathrm{diag}\{I, I, P, \bar{P}, \hat{P}\}$ and letting $X = P\hat{E}K_1$, we can obtain (25) readily, and the proof is then complete. ■

So far, we have obtained the main results for nominal systems, and let us show how the results can be extended to the general case where the parameter uncertainties are included.

Theorem 3: Consider the uncertain system (7) and let $\gamma > 0$ be a given scalar. A desirable full-order fault detection filter of the form (6) exists if there exist positive definite matrices P, Q, Q_j $(j = 1, 2, \ldots, q)$, positive constant scalars ρ, φ and matrices X, K satisfying

$$\Psi = \begin{bmatrix} \hat{\Lambda}_{11} & * & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * & * \\ \hat{\Lambda}_{31} & P\hat{D}_0 + X\hat{R}_2 & -P & * & * & * & * \\ \hat{\Lambda}_{41} & 0 & 0 & -\bar{P} & * & * & * \\ \hat{\Lambda}_{51} & \hat{\Lambda}_{52} & 0 & 0 & -\hat{P} & * & * \\ 0 & 0 & \bar{X} & \hat{X} & \bar{K} & -\varphi I & * \\ \bar{E}_C & \bar{E}_D & 0 & 0 & 0 & 0 & -\varphi I \end{bmatrix} < 0 \tag{29}$$

$$Z^T P Z \leq \rho I \tag{30}$$

where

$$\bar{E}_C = \begin{bmatrix} \varphi E_C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{X} = \begin{bmatrix} \hat{E}_2^T X^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{K} = \begin{bmatrix} 0 & \hat{E}_2^T K^T \\ \hat{E}_2^T X^T & \hat{E}_2^T K^T \end{bmatrix},$$

$$\bar{E}_D = \begin{bmatrix} 0 & \varphi E_D^T \end{bmatrix}^T, \quad \bar{X} = \begin{bmatrix} X\hat{E}_2 & X\hat{E}_2 \end{bmatrix}^T,$$

with $\hat{\Lambda}_{11}$, $\hat{\Lambda}_{31}$, $\hat{\Lambda}_{41}$, $\hat{\Lambda}_{51}$, $\hat{\Lambda}_{52}$, $\bar{P}$, $\hat{P}$ defined in Theorem 2. Furthermore, if $(P, Q, Q_j, X, K, \rho, \varphi)$ is a feasible solution of (29)-(30), then the fault detection filter parameters in the form of (6) are given as follows:

$$\begin{bmatrix} A_F & B_F \end{bmatrix} = [\hat{E}^T P \hat{E}]^{-1} \hat{E}^T X, \tag{31}$$

$$\begin{bmatrix} C_F & D_F \end{bmatrix} = K. \tag{32}$$

Proof: In (25), let us replace $\bar{A}, \bar{C}, \bar{D}, \bar{D}_F$ with $\bar{A} + \Delta\bar{A}, \bar{C} + \Delta\bar{C}, \bar{D} + \Delta\bar{D}, \bar{D}_F + \Delta\bar{D}_F$, respectively, where $\Delta\bar{A} = \hat{E}K_1\hat{E}_2F_kE_C$, $\Delta\bar{D} = \hat{E}K_1\hat{E}_2F_kE_D$, $\Delta\bar{C} = K\hat{E}_2F_kE_C$ and $\Delta\bar{D}_F = K\hat{E}_2F_kE_D$. Then, rewrite (25) in terms of S-procedure as $\Lambda + MF_kN + N^TF_k^TM^T < 0$ with

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & \hat{E}_2^TX^T & \hat{E}_2^TX^T & 0 & 0 & 0 & \hat{E}_2^TK^T \\ 0 & 0 & 0 & 0 & \hat{E}_2^TX^T & 0 & 0 & 0 & \hat{E}_2^TX^T & \hat{E}_2^TK^T \end{bmatrix}^T,$$

$$N = \begin{bmatrix} E_C & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_D & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

From Schur Complement and the S-procedure [3], (29) can be easily obtained which ends the proof. ■

Remark 4: In Theorem 3, sufficient conditions are presented that ensure the residual dynamics to be exponential stable in the mean square with a guaranteed performance index γ. It is shown that the feasibility of the fault detection filter design problem can be readily checked by the solvability of inequalities (29) and (30). Among these feasible solutions, the optimal performance index γ^* can be found by solving the following convex optimization problem: minimize γ subject to (29) and (30) over matrix variables P, Q, Q_j ($j = 1, 2, \ldots, q$), X, K and scalars ρ, φ.

4. An illustrative example

In this section, we aim to demonstrate the effectiveness and applicability of the proposed method. Following [10], we consider the networked fault detection problem for an industrial continuous-stirred tank reactor system, where chemical species A reacts to form species B. Fig. 1 illustrates the physical structure of the system. Assuming that the network-induced delays and randomly occurring nonlinearities exist in this system, a discrete-space model is obtained as

$$x(k+1) = Ax(k) + Bu(k) + A_{d1}\sum_{i=1}^{2}\alpha_i(k)x(k-\tau_i(k)) + \gamma(k)g(k, x(k)) + D_1w(k)$$

$$y(k) = Cx(k) + D_2w(k)$$

where the state variables are chosen as $x_1 = C_A$ and $x_2 = T_C$, the input variables are chosen as $u_1 = T$ and $u_2 = C_{Ai}$, C_A, T_C, T, C_{Ai} are, respectively, the output concentration of chemical species A, the cooling medium temperature, the reaction temperature and the input concentration of a key reactant A. Our purpose

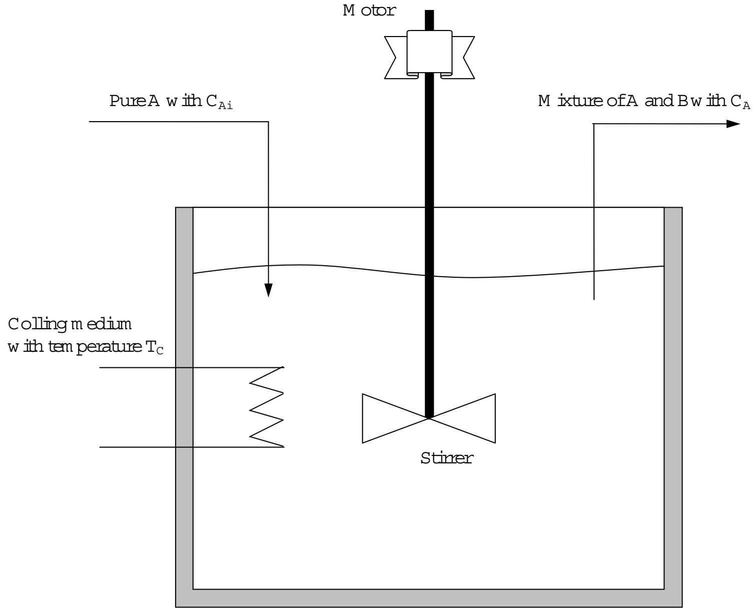

Fig. 1. A continuous-stirred tank reactor model

is to detect the fault appearing on the cooling medium temperature T_C. Therefore, the above system can be represented in the form of (1) with matrices given by

$$A = \begin{bmatrix} 0.9719 & -0.0013 \\ -0.0340 & 0.8628 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.14 & 0.2 \\ 0 & 0.2 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}, \quad G = \begin{bmatrix} -0.0839 & 0.0761 \end{bmatrix}^T, \quad B = 0, \quad H = 0.$$

Let the time-varying communication delays satisfy $1 \leq \tau_i(k) \leq 3$ $(i = 1, 2)$ and assume that $\bar{\alpha}_1 = \mathbb{E}\{\alpha_1(k)\} = 0.9$, $\bar{\alpha}_2 = \mathbb{E}\{\alpha_1(k)\} = 0.7$, $\bar{\gamma} = \mathbb{E}\{\gamma(k)\} = 0.8$. The nonlinear function $g(k, x(k))$ is selected as $g(k, x(k)) = 0.5x_1(k)\sin(x_2(k))$. It is easy to see that the constraint (2) is met with $\varepsilon(k) = 1$ and $E(k) = \mathrm{diag}\{0.2, 0.15\}$. For the measurement quantization, the parameters of the logarithmic quantizer are set as $\hat{\mu}_0 = 2$ and $\chi = 0.8$. Then, the fault detection filter parameters can be obtained from Theorem 3 as follows:

$$A_F = \begin{bmatrix} -0.3276 & 0.2003 \\ -0.2621 & -0.1353 \end{bmatrix}, \quad B_F = \begin{bmatrix} -0.0057 \\ -0.0027 \end{bmatrix}, \quad C_F = \begin{bmatrix} -0.2984 & -0.0015 \end{bmatrix}, \quad D_F = 0.0063,$$

and the optimal performance index given in (8) is $\gamma^* = 1.0007$.

It is worth noting that the obtained optimal performance index γ^* will change as the values of $\bar{\alpha}_1$, $\bar{\alpha}_2$ and $\bar{\gamma}$ change. Letting $\bar{\alpha}_1 = 0.9$, for different combinations of $\bar{\alpha}_2$ and $\bar{\gamma}$, the corresponding optimal performance indices γ^* are shown in Table I. It can be concluded from Table I that the optimal trade-off between the

TABLE I

THE FAULT DETECTION OPTIMAL PERFORMANCE INDEX FOR DIFFERENT $\bar{\alpha}_2$ AND $\bar{\gamma}$

γ^*	$\bar{\alpha}_2 = 0.9$	$\bar{\alpha}_2 = 0.7$	$\bar{\alpha}_2 = 0.5$
$\bar{\gamma} = 0.8$	1.0004	1.0007	1.0011
$\bar{\gamma} = 0.6$	1.0010	1.0012	1.0106

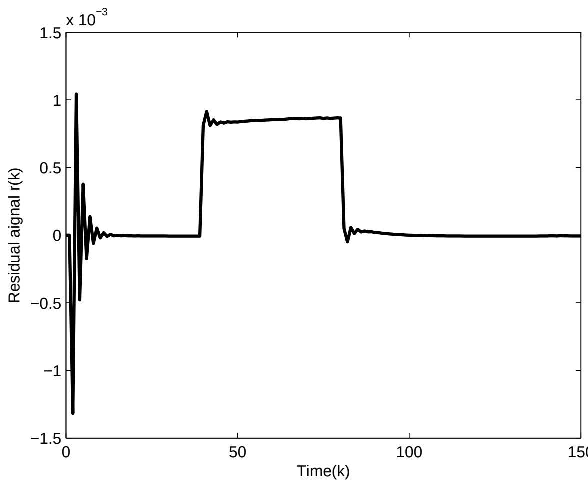

Fig. 2. Residual signal without $w(k)$

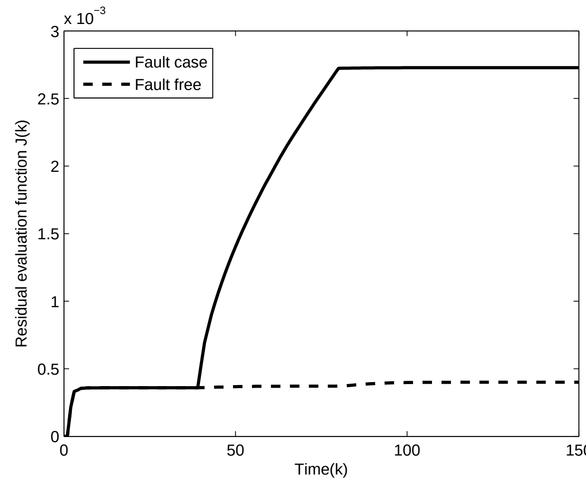

Fig. 3. Evolution of residual evaluation function $J(k)$ without $w(k)$

robustness and sensitivity is affected by not only the randomly occurring communication time-delays but also the randomly occurring nonlinearities.

To further illustrate the effectiveness of the designed fault detection filter, for $k = 0, 1, \ldots, 150$, let the fault signal $f(k)$ be given as:

$$f(k) = \begin{cases} 1, & 40 \leq k \leq 80 \\ 0, & \text{else.} \end{cases} \tag{33}$$

First, in the case that the external disturbance is $w(k) = 0$, the residual response $r(k)$ and evolution of residual evaluation function $J(k)$ are shown in Fig. 2 and Fig. 3, respectively, which indicate that the designed filter can detect the fault effectively when it occurs.

Next, assume that the disturbance is given by

$$w(k) = \begin{cases} \begin{bmatrix} \text{rand}[0,\ 1] & 1.2\,\text{rand}[0,\ 1] \end{bmatrix}^T, & 0 \leq k \leq 50 \\ 0, & \text{else} \end{cases} \tag{34}$$

where the `rand` function generates arrays of random numbers whose elements are uniformly distributed in the interval $[0 \quad 1]$. The residual response $r(k)$ and evolution of residual evaluation function $J(k)$ are shown in

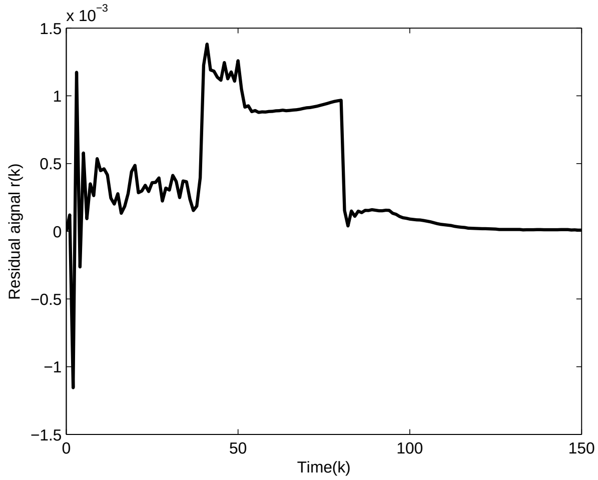

Fig. 4. Residual signal with $w(k)$

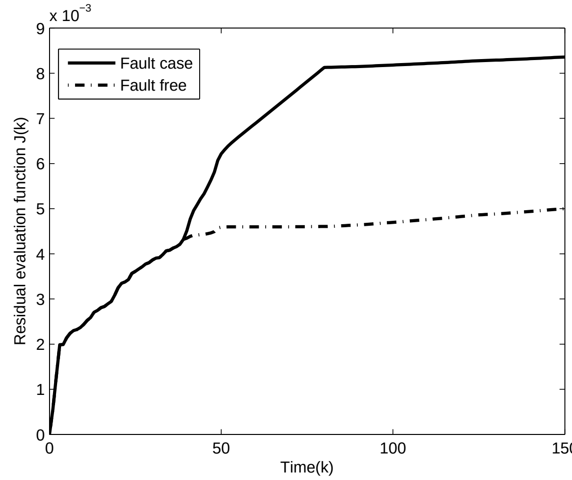

Fig. 5. Evolution of residual evaluation function $J(k)$ with $w(k)$

Fig. 4 and Fig. 5, respectively. Selecting a threshold as $J_{th} = \sup_{f=0}\mathbb{E}\left\{ \sum_{s=0}^{150} r'(s)r(s) \right\}^{1/2}$, after 200 runs of the simulations, we get an average value of $J_{th} = 0.0031$. From Fig. 5, we can see that $0.0026 = J(45) < J_{th} < J(46) = 0.0034$, which means that the fault can be detected in 6 time steps after its occurrence. Therefore, it can be seen that the residual can not only reflect the fault in time, but also detect the fault without confusing it with the disturbance $w(k)$.

In summary, all the simulation results have further confirmed our theoretical analysis for the problem of quantized fault detection for networked systems with randomly occurring nonlinearities and mixed time-delays.

5. Conclusions

In this paper, the fault detection problem has been dealt with for a class of discrete-time systems with randomly occurring nonlinearities, mixed stochastic time-delays as well as measurement quantizations. A fault detection filter has been designed such that, in the presence of measurement quantization, the overall fault detection dynamics is exponentially stable in the mean square and, at the same time, the error between the residual signal and the fault signal is made as small as possible. Sufficient conditions have been established via intensive stochastic analysis for the existence of the desired fault detection filters, and then the explicit expression of the desired filter gains has been derived by means of the feasibility of certain matrix inequalities. Also, the optimal performance index for the addressed fault detection problem has been obtained by solving an auxiliary convex optimization problem. A practical example has been provided to show the usefulness and effectiveness of the proposed design method. Other possible future research directions include real-time applications of the proposed fault detection theory in telecommunications, and further extensions of the present results to more complex systems with unreliable communication links, such as sampled-data systems, bilinear systems, and time varying systems, etc.

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