NELSON AND PLOSSER REVISITED:
EVIDENCE FROM FRACTIONAL ARIMA MODELS

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Abstract

In this paper fractionally integrated ARIMA (ARFIMA) models are estimated using an extended version of Nelson and Plosser's (1982) dataset. The analysis employs Sowell's (1992) maximum likelihood procedure. Such a parametric approach requires the model to be correctly specified in order for the estimates to be consistent. A model-selection procedure based on diagnostic tests on the residuals, together with several likelihood criteria, is adopted to determine the correct specification for each series. The results suggest that all series, except unemployment and bond yields, are integrated of order greater than one. Thus, the standard approach of taking first differences may result in stationary series with long memory behaviour.

Keywords: *Nonstationarity; Long memory; ARFIMA models*

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1. Introduction

Economists widely agree that numerous macroeconomic time series evolve smoothly over time. Such smooth movements were initially modelled by assuming that the series fluctuate around a deterministic trend. Subsequently, Nelson and Plosser (1982), following the work of Box and Jenkins (1970), suggested that allowing for unit roots would result in a better understanding of their stochastic behaviour. Mandelbrot (1969) and his co-authors advocated a third modelling approach. Specifically, they argued that the persistent trend-cyclical behaviour of many macroeconomic time series tends to disappear when they are examined over long time periods. In particular, the autocorrelations take far longer to decay to zero than the exponential rate associated with the autoregressive moving average (ARMA) class of models. Therefore, "long memory" models, which are characterised by significant dependence between distant observations, are more appropriate. One such model is the so-called fractionally integrated one. This involves using the fractional differencing operator ∇^d, where

$$\nabla^{d} = (1 - L)^{d} = \sum_{j=0}^{\infty} (-1)^{j} \binom{d}{j} L^{j},$$

and L is the lag operator. To illustrate this in the case of a scalar time series, x_t, t = 1, 2, …, suppose that u_t is an unobservable covariance stationary sequence with spectral density that is bounded and bounded away from zero at any frequency, and

$$(1 - L)^{d} x_{t} = u_{t}, \qquad t = 1, 2, ... \tag{1}$$

and $x_t = 0$ for $t \leq 0$.[1] The process u_t could itself be a stationary and invertible ARMA sequence, with its autocovariances decaying exponentially; however, these could also decay much slower than exponentially. When d = 0 in (1), $x_t = u_t$, and x_t is 'weakly autocorrelated', or 'weakly dependent'. If $0 < d < 0.50$, x_t is still stationary, but its lag-j autocovariance γ_j decreases very slowly, as the power law j^{2d-1} as $j \to \infty$, and so the γ_j are non-summable. Finally, as d in (1) increases beyond 0.5 and towards 1 (the unit root case), x_t can be viewed as becoming 'more nonstationary', in the sense, for example, that the variance of the partial sums increases in magnitude. Processes like (1) with a positive non-integer d are called fractionally integrated, and when u_t is ARMA(p,q), x_t is known as a fractional ARIMA (ARFIMA(p,d,q)) process. Thus, the model becomes

$$\phi(L)(1 - L)^d x_t = \theta(L)\varepsilon_t, \quad t = 1, 2, ... \qquad (2)$$

where ϕ and θ are polynomials of order p and q respectively, with all zeroes of $\phi(L)$ outside the unit circle, and those of $\theta(L)$ outside or on the unit circle, and ε_t is a white noise. This kind of models were introduced by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981), and were theoretically justified by Robinson (1978), Granger (1980) and, more recently, by Parke (1999) and Diebold and Inoue (2001).

In view of the preceding remarks, it is obviously interesting to estimate the fractional differencing parameter d, along with the other parameters of the ARMA representation. Specifically, we claim in this paper that many macroeconomic time series may be described as fractionally ARIMA models, and show that the classical trend-stationary I(0) and unit roots (I(1)) representations may be too restrictive with respect to the low-frequency dynamics of the series. The layout of the paper is the following. Section 2 briefly summarises the earlier literature analysing the Nelson and Plosser's (1982) dataset. In Section 3 ARFIMA models are estimated for each of the series in an extended version of this dataset. The procedure employed is due to Sowell (1992); it allows a quick evaluation of the likelihood function in the time domain, and yields exact maximum likelihood estimates of the parameters of fractional ARIMA models such as (2). Section 4 offers some concluding remarks.

2. Earlier studies analysing the Nelson and Plosser's dataset

[1] For an alternative definition of fractional integration (the type I class), see Marinucci and Robinson (1999).

Nelson and Plosser (1982) analysed fourteen annual macroeconomic time series for the US to establish whether they could be better characterised as trend-stationary or difference-stationary processes. The time period covered by the data ranged from 1860 to 1909 until 1970, and in all cases but one logged series were examined. Applying the tests of Fuller (1976), and Dickey and Fuller (1979), strong evidence of unit roots was found. More specifically, let x_t, t = 1,2,…, be the series under investigation. The unit-root model tested by Nelson and Plosser (1982) was essentially

$$(1 - L)\, x_t = \alpha + u_t, \qquad t = 1, 2, ... \tag{3}$$

where

$$\phi(L) u_t = \varepsilon_t, \quad t = 1, 2, ... \tag{4}$$

ϕ is a pth degree polynomial, with all zeroes lying outside the unit circle, and ε_t is a white noise sequence. In the terminology of Box and Jenkins (1970), (3) and (4) constitute an ARIMA(p,1,0) model. Nelson and Plosser (1982) tested (3) in

$$(1 - \rho L) x_t = \alpha + \beta t + u_t, \qquad t = 1, 2, ... \tag{5}$$

where the null hypothesis is

$$H_o: \ \rho = 1 \quad and \quad \beta = 0, \tag{6}$$

with $|\rho| < 1$ corresponding to a trend-stationary model. The tests failed to reject the unit root null (3) in all cases, with the exception of the unemployment rate, for various values of p in (4).

The influential study of Nelson and Plosser (1982) spawned much subsequent research. Stock (1991) provided asymptotic confidence intervals for the largest autoregressive root of a time series when this root was close to one. He reported that the confidence intervals for the Nelson and Plosser’s (1982) data were typically wide, containing the unit root for all series except unemployment and bond yields, but also including values significantly different from one. Perron (1988) analysed the same dataset, using the tests of Phillips (1987) and Phillips and Perron (1988), and questioned the findings of Nelson and Plosser (1982) in favour of unit roots.

Kwiatkowski et al. (1992) pointed out that formulating the null hypothesis of a I(1) rather than I(0) series might led to a bias in favour of the unit root hypothesis; they proposed an I(0) test in which the null is a zero variance in a random walk, and applied it to the Nelson and Plosser (1982) data. They concluded that the unemployment rate is I(0); consumer prices, real wages, velocity and stock prices have unit roots; real GNP, nominal GNP and bond yields might also have a unit root; for the remaining seven series neither the unit root nor the trend-stationary representations can be rejected.

The possibility of structural breaks was then analysed by several authors. Perron (1989) showed that the Dickey and Fuller (1979) tests are invalid if the true alternative is that of trend-stationarity with a structural break. He proposed new tests and found that the unit root null could be rejected in ten out of fourteen cases for the Nelson and Plosser's (1982) data. He treated the break as exogenous. Zivot and Andrews (1992) proposed a variation of his tests, allowing the structural break to be endogenous, finding less evidence against the presence of unit roots. Stock (1994) applied a Bayesian procedure that consistently classifies the stochastic component of a series as I(1) or I(0), with both linear detrending and piecewise linear detrending, and found support for the Nelson and Plosser's (1982) conclusions in the former, but not in the latter case.

As for fractional models, Crato and Rothman (1994) applied an ARFIMA approach to an extended version of the Nelson and Plosser's (1982) dataset to distinguish between trend- and difference-stationary models, and found stronger evidence for the latter. Gil-Alana and Robinson (1997) analysed the same series using Robinson's (1994) procedure for testing unit roots and other hypotheses. These tests allow one to consider the unit root (I(1)) and the trend-stationary I(0) hypotheses as special cases of I(d) processes. Their results varied substantially across the series and the various models for the I(0) disturbances, but virtually all series were found to be nonstationary with d greater than 0.5.

3. An empirical application

In this section we analyse an extended version of the Nelson and Plosser's (1982) dataset by estimating fractionally integrated ARMA models. As with their original data, the starting date is 1860 for consumer prices and industrial production; 1869 for velocity; 1871 for stock prices; 1889 for GNP deflator and money stock; 1890 for employment and unemployment rate; 1900 for bond yield, real wages and wages; and 1909 for nominal and real GNP and GNP per capita. All series except bond yields are transformed to natural logarithms and end in 1988. We estimate for each of them different ARFIMA(p,d,q) models with p and q smaller than or equal to three, using Sowell's (1992) maximum likelihood estimation procedure.[2] Other parametric methods for estimating d based on the frequency domain were proposed, amongst others, by Fox and Taqqu (1986) and Dahlhaus (1989). Small sample properties of these and other estimates were examined in Smith et al. (1997) and Hauser (1999). The former authors compare several semi-parametric procedures with the maximum likelihood estimation method of Sowell (1992), finding that Sowell's (1992) procedure outperforms the semi-parametric ones in terms of the bias and the mean square errors. Hauser (1999) also compares it with others based on the exact and the Whittle likelihood function in both the time and in the frequency domain, and shows that it dominates the others in case of fractionally integrated models. Having carried out the estimation, we then perform several diagnostic tests for each model to ensure white noise residuals - in particular, normality, heteroscedasticity, autoregressive conditional heteroscedasticity (ARCH) and Ljung & Box tests.

Table 1 summarises the estimated d's for the different ARMA representations of each series. It can be seen that for all of them the estimated values of d are greater than 1, except in the case of the unemployment rate (for which they are smaller than 1), and in some cases for velocity and bond yields. For all but four series the difference between the minimum and the maximum value of d is smaller than 0.4, the main exception being again unemployment, with d ranging

[2] See Sowell (1992) for more details. To ensure stationarity and following standard practice, the models were estimated in first differences and then converted back to levels.

from –1.45 to 0.87. Also, money stock, wages, GNP deflator, nominal GNP and consumer prices appear to be the most nonstationary series, with d ranging from 1.21 to 1.49. On the other hand, the unemployment rate, and bond yields, appear to be the least nonstationary ones, with d ranging from 0.87 to 1.17 in the case of the latter series. Similar conclusions were reached by Gil-Alana and Robinson (1997) when applying Robinson's (1994) tests to the same series. Finally, it should be mentioned that only in six out of the fourteen series we found at least one model where the residuals passed all the diagnostic tests at the 1% level. Most of the models failed to pass normality tests, largely because of the presence of outliers corresponding to World War II.

(Tables 1 and 2 about here)

Table 2 reports the results based on post-war data. They are similar to those in Table 1, with all values of d greater than 1 except for the unemployment rate, and in some cases for velocity and bond yields. The latter is the only series for which we are unable to fit a model passing all the diagnostic tests on the residuals, evidence of heteroscedasticity being found. As in Table 1, money stock, nominal GNP, consumer prices, wages and GNP deflator appear to be the most nonstationary series, while unemployment, followed by velocity and bond yields, are the closest to stationarity.

As mentioned above, all these models were estimated by maximum likelihood. It is well known that parametric approaches such as this one yield inconsistent estimates of d if the model is not correctly specified. In particular, misspecification of the short-run components of the series invalidates the estimation of its long-run behaviour. To choose the best specification in each case we focused on the models passing all the diagnostic tests, and used model-selection criteria based on LR tests along with the Akaike (AIC) and Schwarz (SIC) information criteria This improves upon the earlier study by Crato and Rothman (1994), who only used AIC and SIC for model selection (these criteria not necessarily being the best ones for applications involving fractional differences - see e.g. Hosking, 1981), and accounts for differences in the results. For the series which failed to pass the tests in Table 1 we analysed the post-war data instead, adopting the same type of procedure. The results were the following.

a) Real GNP

Eight models pass the diagnostic tests at the 1% significance level. The results are presented in Table 3a. The values of d range between 1.17 and 1.49, rejecting the null of d = 0 in all cases and the unit root null in five. The most general specifications are the ARMA(2,2) and the ARMA(3,1) ones. In the former, the second MA coefficient is not significantly different from zero, and in the latter the last two AR coefficients are also insignificant. Going backwards from the ARMA(3,1) to an ARMA(1,1), a LR test rejects the former model, but the two coefficients of the latter one are now close to zero. If the MA coefficient is dropped, the AR(1) model has an insignificant coefficient, whereas if the AR parameter is deleted the MA(1) specification seems adequate. On the other hand, if we move from an ARMA(3,1) to an AR(3) model, a LR test indicates that the latter model should be preferred, but the last two AR coefficients are insignificantly different from zero. When comparing the ARMA(2,2) with the AR(2) specification, the former seems to be preferable but the second AR and MA coefficients are again close to zero. In view of these results, it can be concluded that the best two models describing the short-run dynamics of this series are the white noise and the MA(1) ones. The AIC suggests the MA(1) but the SIC indicates that the white noise specification might be more appropriate. If a LR test is performed, the white noise model appears to be preferable. In addition, the standard error of d is smaller under this final parameterisation. Thus, it appears that real GNP can be well described by an ARFIMA(0, 1.30, 0) model.

b) Real per capita GNP

In this case there are nine models with possibly white noise residuals. The most general specification is an ARMA(3,2), but all the AR and the second MA coefficients are insignificantly different from zero. In the ARMA(3,1), the values are similar to the previous case, with smaller standard errors, though we still find non-significant coefficients. A LR test suggests that the

ARMA(1,1) is to be preferred, but both coefficients in this model are now insignificant. When suppressing either of these two coefficients, both the AR(1) and the MA(1) seem appropriate. The AR(3) and the AR(2) models are both inappropriate in view of the t-values. On the other hand, when comparing the ARMA(3,2) with the ARMA(2,2) the latter seems to be preferable, all the coefficients being highly significant. Thus, there are three models that might be appropriate for this series: the ARMA(2,2), the AR(1) and the MA(1). The AIC indicates that the ARMA(2,2) is the best specification, but the SIC, leading to a less heavily parameterised model, suggests the AR(1). We have chosen the ARMA(2,2) since it has smaller standard errors and the highest AIC of all models. Visual inspection of the residuals also corroborates this choice, the final parameterisation yielding the closest residuals to a white noise process. Therefore, the best model for this series appears to be an ARFIMA(2, 1.11, 2).

(Tables 3a – 3d about here)

c) Employment

Six models were selected for this series, with a narrower range of values of d, from 1.14 to 1.22. In all these models the coefficients are insignificantly different from zero in all cases except for the ARMA(1,1) and the MA(1) models. In the former, the AR parameter is not significant, and thus the MA(1) seems to be more appropriate. In addition, both the AIC and the SIC have the highest values in this case. Therefore, the best model specification for employment seems to be an ARFIMA(0, 1.14, 1).

d) Unemployment rate

Again six models were selected for this series. The values of d oscillate now between –0.58 and 0.25.[3] In the most general specification, i.e. the ARMA(3,3), all except the first MA coefficient appear significant. The ARMA(2,2) is clearly rejected, since all coefficients are insignificant, and when eliminating the second MA component the two AR coefficients are close to zero. We see

[3] Models with d ranging between –0.5 and 0 are short memory and have been described as anti-persistent by Mandelbrot (1977) because the spectral density function is dominated by high frequency components.

that the AR(3) and the AR(2) models have insignificant coefficients, suggesting that the AR(1) might be more appropriate. The AIC suggests that the ARMA(3,3) is preferable to the AR(1) model; however, in view of the smaller number of parameters used in the AR(1) case, (leading to a higher value at the SIC), and the easier interpretation of the fractional differencing parameter (which also has a smaller standard error), we select this model, choosing as a final specification the ARFIMA(1, 0.25, 0).

e) Real wages

Ten models were considered here, with the values of d ranging between 1.16 and 1.41. Starting from the ARMA(3,1), we see that the last two AR coefficients are insignificantly different from zero. Going backwards to an ARMA(2,1), the second AR coefficient still remains insignificant, and in the ARMA(1,1) both parameters are close to zero. In the AR(3) model, which is preferred to the ARMA(3,1) on the basis of a LR test, all parameters are insignificant, as in the AR(2) and AR(1) models. The ARMA(1,2) has an insignificant second MA coefficient, and the MA(1) also appears inappropriate. A white noise specification appears to be the best to describe the short-run dynamics of this series, as it yields the smallest standard error for d and the highest values of the AIC and SIC. Thus, real wages can be characterised as an ARFIMA(0, 1.22, 0).

f) Velocity

Five models were selected in this case, with the values of d oscillating between 1.01 and 1.35. The ARMA(3,3) has various insignificant parameters. When dropping the first, the third and then the second MA coefficients, we still find that the AR parameters are not significantly different from zero. In the ARMA(2,2) specification all the parameters are significant, unlike in the ARMA(2,1), and the standard error of d is smaller. Therefore, velocity appears to be well described by an ARFIMA(2, 1.01, 2).

(Tables 3e – 3h about here)

g) Bond yields

This series requires special attention since none of the models passed the diagnostics at the 1% level, as a result of lack of normality in the residuals. Visual inspection suggests that this might be due to the presence of outliers corresponding to World War II. One possibility would be to focus on the post-war data, though even then heteroscedastic residuals are obtained. We decided to consider only those models which pass all the diagnostics at the 0.1% level. Three potential models were then found. The values of d are 0.87 and 0.96 and the unit root hypothesis cannot be rejected in any of them. The ARMA(3,3) has insignificant parameters in both the AR and the MA components. On the basis of LR tests, the ARMA(3,2) and the ARMA(2,3) seem to be more appropriate. In the ARMA(3,2), the first AR and MA coefficients are insignificant, and in the ARMA(2,3) only the first AR coefficient is close to zero. The standard errors are much smaller in the latter model and the AIC and SIC also suggest that this might be the correct specification. Therefore, we model this series as an ARFIMA(2, 0.96, 3).

As mentioned before, for the remaining seven series, all the estimated ARFIMA models failed to pass the diagnostic tests on the residuals, presumably owing to the presence of outliers corresponding to World War II. Thus, for these series we only modelled the post-war data.

h) Nominal GNP

Ten models were selected, with the values of d ranging between 1.43 and 1.49, and both the I(0) and the I(1) hypotheses being rejected in all cases, clearly indicating the nonstationary nature of this series. The most general specification is the ARMA(3,3), but all its parameters are not significantly different from zero. There is an improvement when adopting an ARMA(3,1) specification, as the last AR and MA coefficients both appear to be significant. Going one step further, we move to an AR(3), and the parameters change substantially with respect to the previous parameterisation, with the first two coefficients being significantly different from zero. A LR test indicates that this model is to be preferred to the ARMA(3,1), but the AR(2) seems an even better specification. LR tests suggest that the AR(2) model is preferable to the AR(1) and the white noise specification. Also, from the ARMA(3,1), we can move to an ARMA(2,1) and since the MA coefficient is not significantly different from zero, and again the AR(2) appears preferable. A MA(2) has a second parameter close to zero, suggesting that a MA(1) is a better fit, and similarly, the MA(1) seems more appropriate than the ARMA(1,1). Therefore, we have to decide between the AR(2) and the MA(1) models. The AIC indicates that the AR(2) is more adequate, but the SIC suggests the MA(1) instead. Visual inspection of the residuals indicates that the AR(2) produces residuals which are closer to being white noise. Thus, we can conclude that nominal GNP can be well described as an ARFIMA(2, 1.49, 0).

i) Industrial production

Eight models were selected for this series, with the values of d ranging between 1.15 and 1.49. The ARMA(3,1) appears to be outperformed by the ARMA(2,1), which in turn is outperformed by the MA(1) model, indicating the importance of the MA coefficient. Evidence in favour of the MA(1) is also found from the MA(2), where the second coefficient appears insignificant. On the other hand, an AR(3) is clearly rejected in favour of an AR(2), which is to be preferred to an AR(1) on the basis of a LR test. Therefore, we have to choose between the AR(2) and the MA(1) specifications, and, since both the AIC and the SIC have higher values in the case of the MA representation, the specification finally selected is an ARFIMA(0, 1.48, 1).

j) GNP deflator

Only four models were found to be adequate, with the values of d oscillating between 1.43 and 1.47, and the unit root hypothesis being rejected in all cases. The ARMA(2,1) is rejected since the AR coefficients are both insignificantly different from zero. Similarly, the ARMA(1,1) is rejected because of the insignificance of the AR coefficient. A LR test favours the AR(2) to the AR(1) specification. The former also has smaller standard errors and higher values of AIC and SIC. Thus, the GNP deflator can be characterised as an ARFIMA(2,1.47, 0).

(Tables 3i – 3l about here)

k) Consumer prices

Nine models were selected here. When modelling the series as an ARMA(3,2), the two MA coefficients are close to zero, and similarly, if we estimate an ARMA(3,1), the MA coefficient is insignificantly different from zero. The AR(3) seems to be appropriate, but when comparing it with the AR(2) and AR(1) models, the test statistics suggest that the AR(2) might be more adequate. In addition, this model has smaller standard errors. A LR test was again performed to choose between the ARMA(3,1) and the ARMA(2,1), and evidence was found in favour of the latter model, but the AR(2) appears to produce a better fit. The MA(3) has the last two coefficients close to zero, and the MA(1) seems preferable, having a highly significant coefficient. Therefore, potential models are the AR(2) and the MA(1). We choose the MA(1) as the correct specification in view of the higher values of both the AIC and the SIC. Thus, the series may be described as an ARFIMA(0, 1.39, 1).

l) Nominal wages

Four models were selected for this series. The most general specification is an ARMA(3,1), but the first two AR coefficients are not significantly different from zero. Moving to an ARMA(2,1), the first AR coefficient is still insignificant along with the MA one. The corresponding AR(2) without the MA parameter has again a first coefficient close to zero. It seems difficult to determine the best specification in this case. Visual inspection of the residuals suggests that the AR(3) and the ARMA(3,1) exhibit the closest residuals to a white noise process, and performing a LR test indicated that the latter model is to be preferred. Thus, we adopt an ARFIMA(3, 1.36, 1) specification for this series.

m) Money stock

Five models were selected on the basis of the diagnostic tests, with values of d between 1.47 and 1.48. The most general specification, which is an ARMA(3,1), has the second AR coefficient close to zero. A LR tests suggests that an AR(3) model is preferable, though all its coefficients appear to be insignificant, and this model is also rejected in favour of the white noise specification. The MA(2) model has the second coefficient close to zero, and when moving backwards to the MA(1) the coefficient becomes highly significant. Thus, we need to choose between the MA(1) and the white noise model, a LR test favouring the former. This model also has the highest values at the AIC and the SIC of all possible specifications. Therefore, the money stock is modelled as an ARFIMA(0, 1.47, 1).

(Tables 3m and 3n about here)

n) Common stock prices

Fourteen out of the sixteen ARMA representations passed all the diagnostics. Starting with an ARMA(3,3), the first two MA coefficients are insignificantly different from zero. Going backwards, either to an ARMA(3,2) or to an ARMA(2,3), we find that in the former model both MA parameters are close to zero, and in the latter only the last MA coefficient appears insignificant. Deleting this parameter and thus moving to an ARMA(2,2), all parameters become significant. This model also appears more appropriate than the ARMA(2,1) and the ARMA(1,2). Similarly, the AR(3) model seems to provide a better fit than the ARMA(3,1) and the ARMA(3,2), given the non-significance of the MA coefficients in the latter models. LR tests indicate that amongst the AR(3) and the AR(2), AR(1) and white noise specifications, the AR(3) is the best one. The ARMA(1,1) has an insignificant AR coefficient and the MA(1) appears more appropriate. Therefore, we have to choose between the ARMA(2,2), the AR(3) and the MA(1) models. In view of the fact that it has the lowest standard errors and the highest value at the AIC, the AR(3) appears to be the best specification to characterise the short-run dynamics in this series. Thus, the final selected model is an ARFIMA(3, 1.46, 0).

Table 4 reports the best model specification for each series. One can see that the unemployment rate is the only series for which we cannot reject the null of I(0) stationary residuals (i.e., d = 0). For five series (real per capita GNP, employment, wages, velocity and bond yield), the unit root (i.e., d = 1) cannot be rejected. For the remaining eight series, both hypotheses are rejected, with all the orders of integration being greater than one. In other words, even when taking first differences, there is still significant dependence between observations far apart in time.

In brief, nominal GNP, industrial production, GNP deflator, money stock, common stock prices and consumer prices are the most nonstationary series, with the unit root null rejected in all cases; the unit root hypothesis is also rejected for real GNP and real wages; it cannot be rejected instead for real per capita GNP, employment, wages and velocity, though higher orders of integration are also estimated; bond yields may be modelled as a unit root process, though the order of integration seems slightly smaller than one. Finally, the unemployment rate seems to be stationary, and although the I(0) hypothesis cannot be rejected, it can be better modelled with an order of integration greater than zero, thus indicating the presence of mean reversion. These results are consistent with those of Gil-Alana and Robinson (1997), where it was shown that these series could be better characterised using fractional integration rather than the classical I(1) or I(0) models. The only exception is industrial production, for which d is estimated to be equal to 1.48, while in Gil-Alana and Robinson (1997) this series was found to be close to stationarity.

(Tables 4 and 5 about here)

The estimates of the remaining parameters are also of interest. Consider, for instance, the unemployment rate for which the best model appears to be a short-memory one (i.e., the estimated d is insignificantly different from zero). In this case, the short-run dynamics are described by the AR(1) model, with an estimated parameter of 0.62, which implies that more than 95% of the effects of a shock die away in approximately six years. However, if we allow d to be fractional rather than zero (specifically, $d = 0.25$), they disappear after a much longer time.

Table 5 reports the impulse responses for the first 17 periods for the growth rates of the all series except the unemployment rate and bond yields (which are in levels), based on the previously selected models. We see that shocks to the unemployment rate, though disappearing in the long run, still have 10% of their initial impact after 15 years. This illustrates the importance of distinguishing between short memory ($d = 0$) and long memory ($d > 0$) behaviour.

In the case of bond yields, shocks seem to persist over time, though, as the estimated value of d is smaller than one, they disappear in the long run. For the remaining twelve series, shocks to the growth rates also tend to disappear, though at different rates. Thus, for example, in the case of wages, 26.5% of the initial impact is still present after 17 years; the corresponding percentages for industrial production, money stock, GNP deflator and consumer prices are 23%, 15.8%, 15.3%, and 13.9% respectively. These results corroborate the finding of Gil-Alana and Robinson (1997) that these series are the most nonstationary ones, the only exception being again industrial production. On the other hand, almost 90% of the initial shock to the growth rates of real wages, real per capita GNP, velocity and employment disappear after three years.

4. Conclusions

Different ARFIMA models have been estimated in this paper using an extended version of the Nelson and Plosser's (1982) series. They provide a greater degree of flexibility in modelling the low-frequency dynamics compared with the standard ARMA and ARIMA specifications, which can be seen as special cases.

We have employed Sowell's (1992) maximum likelihood estimation procedure, and first selected various models for each series on the basis of several diagnostic tests on the residuals. Then, a model selection procedure, based on LR tests and other likelihood criteria, was adopted to choose the best specification in each case. This is crucial when adopting parametric estimation approaches, since misspecification of the short-run components invalidates the estimation of the fractional differencing parameter. Our approach represents an improvement relative to the earlier study of Crato and Rothman (1994), where model selection was based only on AIC and SIC, which might not be the best criteria in the case of fractional differencing; our using a larger set of model selection criteria also accounts for differences in the chosen specification in each case.

The empirical findings indicate that the unemployment rate is the only stationary series, with an order of integration of approximately 0.25. The t-value on this parameter implies that the

I(0) hypothesis cannot be rejected. The remaining thirteen series all appear to be non-stationary, with orders of integration ranging from 0.96 (bond yields) and 1.01 (velocity) to 1.49 (nominal GNP). The I(1) hypothesis cannot be rejected for bond yields, velocity, real per capita GNP, wages and employment. For all the other series, d appears to be much higher than one, and thus the standard approach of taking first differences does not guarantee I(0) stationary residuals. In fact, the impulse response functions based on the growth rates of the series show that, even ten years after a shock has occurred, almost 20% of its impact still remains in the case of industrial production, GNP deflator, wages, consumer prices and money stock, clearly indicating the presence of long memory.

Possible extensions to our analysis, aimed at shedding further light on the stochastic behaviour of macroeconomic time series, include testing the Bloomfield (1973) exponential model for the description of the short-run components of the series, and adopting semi-parametric and non-parametric methods of estimating d. These issues will be addressed in future work.

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TABLE 1

Maximum likelihood estimates of d in ARFIMA(p,d,q) models for the extended Nelson and Plosser (1982) dataset.

	ARMA(p, q)															
Series	(0,0)	(1,0)	(0,1)	(1,1)	(2,0)	(0,2)	(2,1)	(1,2)	(2,2)	(3,0)	(0,3)	(3,1)	(3,2)	(1,3)	(2,3)	(3,3)
Real GNP	1.30’	1.17’	1.20’	1.17’	1.18’	1.15	1.49	--	1.49’	1.24’	1.19	1.49’	--	--	--	1.18
Nominal GNP	1.39	1.26	1.28	1.27	1.29	1.28	1.48	1.48	1.31	1.32	--	1.42	1.47	1.48	--	1.29
Real cap. GNP	1.24’	1.02’	1.09’	1.02’	1.03’	1.06	--	1.49	1.11’	1.10’	1.07	1.49’	1.49’	1.49’	1.04	1.06
Industrial prod.	1.15	1.17	1.18	1.49	1.23	1.49	1.49	1.14	1.49	1.23	1.49	1.49	1.49	1.49	1.49	1.49
Employment	1.28	1.16	1.14’	1.17’	1.22’	1.21	1.21’	--	--	1.22’	1.48	1.21’	--	--	--	1.49
Unemployment	0.87	0.25	0.43	-0.42	-.58’	0.40	-.26’	-0.30	-.28’	0.11’	0.31	-1.43	-1.38	-0.36	-1.45	-.41’
GNP deflator	1.40	1.29	1.32	1.28	1.28	--	1.28	1.32	1.35	1.29	--	1.32	1.27	1.28	1.36	1.32
Cons. prices	1.46	1.21	1.24	1.27	1.31	--	1.23	1.26	1.23	1.24	--	1.24	1.26	--	1.21	--
Wages	1.40	1.27	1.28	1.29	1.31	--	1.38	1.28	1.42	1.33	1.35	1.38	1.49	1.49	--	1.40
Real wages	1.22’	1.16’	1.16’	1.17’	1.21’	1.24’	1.41’	1.41’	1.38	1.24’	--	1.41’	--	1.41	1.41	1.42
Money stock	1.47	1.39	1.39	1.38	1.39	1.38	1.38	--	--	1.38	--	1.38	--	--	1.42	1.42
Velocity	1.07	0.99	0.99	1.00	1.00	0.97	1.34’	1.35	1.01’	1.06	1.38	1.34’	1.35’	1.34	1.36	1.35’
Bond Yield	1.09	0.94	0.88	0.96	1.17	1.01	1.11	1.01	1.04	0.92	0.98	0.88	0.87	0.87	0.96	0.87
C. Stock prices	1.12	1.00	1.12	1.01	1.08	1.09	1.07	--	1.42	1.08	1.42	1.07	1.42	1.42	1.42	1.45

--: The model failed to achieve convergence after 240 iterations. ‘: The corresponding model passed the diagnostic tests of normality, heteroscedasticity, ARCH and Ljung & Box at the 1% level.

TABLE 2

Maximum likelihood estimates of d in ARFIMA(p,d,q) models for the extended Nelson and Plosser dataset, starting in 1947.

	ARMA(p, q)															
Series	(0,0)	(1,0)	(0,1)	(1,1)	(2,0)	(0,2)	(2,1)	(1,2)	(2,2)	(3,0)	(0,3)	(3,1)	(3,2)	(1,3)	(2,3)	(3,3)
Real GNP	1.33	1.31’	1.31	1.31	1.34’	--	1.34’	1.33	--	1.30’	1.23	1.31’	1.32	--	1.27	1.30
Nominal GNP	1.43’	1.47’	1.49’	1.49’	1.49’	1.48’	1.49’	1.47	1.47	1.48’	1.48	1.48’	--	--	1.47	1.47’
Real cap. GNP	1.24	1.17’	1.7	1.20	1.19’	1.20	1.20’	--	1.40	1.14’	--	1.14’	1.15	--	--	1.34
Industrial prod.	1.15’	1.24’	1.48’	1.49	1.36’	1.49’	1.48’	1.49	--	1.38’	1.48	1.49’	1.49	1.48	--	1.49
Employment	1.31	1.26’	1.21	1.25’	1.38’	1.40	1.36’	--	1.36	1.34’	--	1.34’	1.26	--	1.39	--
Unemployment	0.79	0.62’	0.59	0.65	0.80’	0.77	0.78’	1.41	0.76	0.75	1.42	0.75	0.64’	--	1.49	1.42’
GNP deflator	1.48	1.43’	1.45	1.44’	1.47’	1.43	1.46’	1.44	1.46	1.39	1.43	1.38	1.38	1.45	1.44	1.40
Cons. prices	1.47’	1.41’	1.39’	1.40	1.47’	1.41	1.46’	1.44	--	1.41’	1.44’	1.38’	1.37	1.44	1.44	--
Wages	1.46	1.47	1.48	1.48	1.48’	--	1.48’	1.48	--	0.93’	--	1.36’	1.36	1.45	--	1.39
Real wages	1.30’	1.06’	1.12’	1.17’	1.17’	--	1.08’	1.15	0.62’	1.17’	--	0.65’	1.16	1.08	--	1.18
Money stock	1.48’	1.46	1.47’	1.47	1.48	1.47’	1.48	1.48	1.47	1.48’	1.48	1.48’	--	--	1.48	1.48
Velocity	1.00	0.97	0.81	0.93	1.26	1.30	1.18	1.31	1.06’	1.09	1.25	1.10’	--	1.31	1.13’	--
Bond Yield	1.09	0.94	0.88	0.95	1.17	1.02	1.12	1.02	1.06	0.96	1.01	0.92	0.90	0.92	1.00	0.90
C. Stock prices	1.19’	1.24’	1.42’	1.42’	1.36’	1.42	1.43’	1.42’	1.39’	1.46’	1.40’	1.45’	1.43’	1.26	1.40’	1.42’

--: The model failed to achieve convergence after 240 iterations. ‘: The corresponding model passed the diagnostic tests of normality, heteroscedasticity, ARCH and Ljung & Box at the 1% level.

TABLE 3a

Parameter estimates of ARFIMA models for real GNP

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	**1.30 (0.08)**	**16.25**	**3.75**	**--**	**--**	**--**	**--**	**--**	**--**	**225.50**	**223.13**
(1, 0)	1.17 (0.11)	10.63	1.54’	0.25 (0.16)	--	--	--	--	--	226.06	221.32
(0, 1)	1.20 (0.09)	13.33	2.22	--	--	--	0.21 (0.10)	--	--	226.01	221.28
(1, 1)	1.17 (0.11)	10.63	1.54’	0.15 (0.37)	--	--	0.09 (0.32)	--	--	224.16	217.05
(2, 0)	1.18 (0.11)	10.72	1.63’	0.24 (0.16)	-0.05 (0.11)	--	--	--	--	224.28	217.17
(2, 2)	1.49 (0.01)	149.0	49.0	1.16 (0.39)	-0.43 (0.28)	--	-1.34 (0.41)	0.36 (0.41)	--	227.08	215.23
(3, 0)	1.24 (0.11)	11.27	2.18	0.24 (0.11)	0.18 (0.15)	-0.03 (0.11)	--	--	--	224.26	214.79
(3, 1)	1.49 (0.01)	149.0	49.0	0.77 (0.11)	-0.09 (0.14)	-0.11 (0.11)	-0.97 (0.12)	--	--	227.37	215.51

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses. ‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3b

Parameter estimates of ARFIMA models for real GNP per capita

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.24 (0.10)	12.40	2.40	--	--	--	--	--	--	224.72	222.35
(1, 0)	1.02 (0.14)	7.28	0.14’	0.34 (0.18)	--	--	--	--	--	226.92	222.18
(0, 1)	1.09 (0.10)	10.90	0.90’	--	--	--	0.27 (0.12)	--	--	226.54	221.80
(1, 1)	1.02 (0.14)	7.28	0.14’	0.25 (0.31)	--	--	0.09 (0.25)	--	--	225.07	217.96
(2, 0)	1.03 (0.14)	7.35	0.21’	0.35 (0.18)	-0.05 (0.11)	--	--	--	--	225.19	218.07
(2, 2)	**1.11 (0.09)**	**12.33**	**1.22’**	**0.81 (0.10)**	**-0.89 (0.08)**	**--**	**-0.65 (0.11)**	**0.98 (0.18)**	**--**	**229.45**	**217.60**
(3, 0)	1.10 (0.13)	8.46	0.76	0.28 (0.17)	-0.02 (0.11)	-0.15 (0.11)	--	--	--	224.86	215.38
(3, 1)	1.49 (0.05)	29.80	9.80	0.77 (0.11)	-0.09 (0.14)	-0.10 (0.11)	-0.97 (0.15)	--	--	225.94	214.09
(3, 2)	1.49 (0.05)	29.80	9.80	0.69 (1.12)	-0.02 (0.90)	-0.11 (0.21)	-0.89 (1.12)	-0.08 (1.11)	--	223.95	209.72

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses. ‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3c

Parameter estimates of ARFIMA models for employment

ARMA	d	T $_{d=0}$	t $_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 1)	**1.14 (0.08)**	**14.25**	**1.75**	**--**	**--**	**--**	**0.33 (0.12)**	**--**	**--**	**379.61**	**374.43**
(1, 1)	1.17 (0.09)	13.00	1.88’	-0.20 (0.29)	--	--	0.48 (0.22)	--	--	378.05	370.28
(2, 0)	1.22 (0.10)	12.20	2.20	0.21 (0.14)	-0.18 (0.10)	--	--	--	--	378.43	370.67
(2, 1)	1.21 (0.10)	12.10	2.10	-0.01 (0.51)	0.12 (0.18)	--	0.24 (0.54)	--	--	376.51	366.16
(3, 0)	1.22 (0.11)	11.09	2.00	0.21 (0.15)	-0.18 (0.10)	0.008 (0.11)	--	--	--	376.43	366.09
(3, 1)	1.21 (0.11)	11.00	1.90'	-0.10 (0.83)	-0.10 (0.21)	-0.02 (0.19)	0.33 (0.81)	--	--	374.53	361.61

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3d

Parameter estimates of ARFIMA models for unemployment rate

ARMA	d	t $_{d=0}$	t $_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(1, 0)	**0.25 (0.24)**	**1.04'**	**-3.12**	**0.62 (0.18)**	**--**	**--**	**--**	**--**	**--**	**226.92**	**222.18**
(2, 0)	-0.58 (0.75)	-0.77'	-2.10	1.43 (0.62)	-0.52 (0.49)	--	--	--	--	225.19	218.07
(2, 1)	-0.26 (0.28)	-0.92'	-4.50	0.59 (0.36)	0.17 (0.22)	--	0.71 (0.14)	--	--	229.45	217.60
(2, 2)	-0.28 (0.32)	-0.87'	-4.00	0.53 (0.64)	0.22 (0.51)	--	0.78 (0.72)	0.05 (0.47)	--	229.45	217.60
(3, 0)	0.11 (0.43)	0.25'	-2.06	0.90 (0.38)	-0.37 (0.17)	0.18 (0.10)	--	--	--	224.86	215.38
(3, 3	-0.41 (0.27)	-1.51'	-5.22	1.15 (0.43)	-1.00 (0.39)	0.59 (0.18)	0.27 (0.26)	0.57 (1.17)	0.60 (1.11)	223.95	209.72

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3e

Parameter estimates of ARFIMA models for real wages

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	**1.22** **(0.07)**	**17.42**	**3.14**	**--**	**--**	**--**	**--**	**--**	**--**	**333.49**	**331.00**
(1, 0)	1.16 (0.10)	11.60	1.60’	0.11 (0.14)	--	--	--	--	--	332.14	327.18
(0, 1)	1.16 (0.09)	12.88	1.77’	--	--	--	0.14 (0.14)	--	--	332.36	327.40
(1, 1)	1.17 (0.09)	13.00	1.88’	-0.26 (0.68)	--	--	0.39 (0.62)	--	--	330.54	323.10
(2, 0)	1.21 (0.10)	12.10	2.10	0.08 (0.15)	-0.10 (0.11)	--	--	--	--	331.00	323.56
(0, 2)	1.24 (0.16)	7.75	1.50’	--	--	--	0.03 (0.23)	-0.12 (0.18)	--	330.92	323.48
(2, 1)	1.41 (0.11)	12.81	3.72	0.66 (0.15)	-0.12 (0.11)	--	-0.82 (0.13)	--	--	331.32	321.41
(1, 2)	1.41 (0.11)	12.81	3.72	0.48 (0.21)	--	--	-0.65 (0.24)	0.15 (0.14)	--	331.22	321.31
(3, 0)	1.24 (0.11)	11.27	2.18	0.04 (0.15)	-0.11 (0.11)	-0.07 (0.11)	--	--	--	329.44	319.53
(3, 1)	1.41 (0.11)	12.81	3.72	0.65 (0.16)	-0.11 (0.12)	-0.02 (0.11)	-0.81 (0.15)	--	--	329.35	316.95

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3f

Parameter estimates of ARFIMA models for velocity

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(2, 1)	1.34 (0.18)	7.44	1.88’	0.60 (0.16)	-0.10 (0.10)	--	-0.83 (0.10)	--	--	313.63	302.50
(2, 2)	**1.01** **(0.07)**	**14.42**	**0.14’**	**0.89** **(0.20)**	**-0.83** **(0.16)**	**--**	**-0.76** **(0.21)**	**0.76** **(0.26)**	**--**	**312.21**	**298.30**
(3, 1)	1.34 (0.17)	7.88	2.00	0.53 (0.14)	-0.03 (0.10)	-0.14 (0.10)	-0.78 (0.15)	--	--	313.72	299.82
(3, 2)	1.35 (0.18)	7.50	1.94’	0.79 (0.45)	-0.20 (0.30)	-0.13 (0.11)	-1.05 (0.48)	0.23 (0.38)	--	312.03	295.34
(3, 3)	1.35 (0.17)	7.94	2.05	0.40 (0.48)	0.35 (0.45)	-0.35 (0.23)	-0.65 (0.55)	-0.43 (0.53)	0.28 (0.36)	309.98	290.52

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3g

Parameter estimates of ARFIMA models for bond yield

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(3, 2)	0.87 (0.23)	3.78	-0.56'	0.21 (0.24)	-0.75 (0.11)	0.54 (0.23)	0.16 (0.13)	0.75 (0.11)	--	-153.91	-168.76
(2, 3)	**0.96 (0.10)**	**9.60**	**-0.40'**	**-0.06 (0.07)**	**-0.86 (0.07)**	**--**	**0.33 (0.10)**	**0.92 (0.08)**	**0.49 (0.11)**	**-153.19**	**-168.04**
(3, 3)	0.87 (0.23)	3.78	-0.56'	0.11 (0.33)	-0.78 (0.13)	0.41 (0.33)	0.29 (0.19)	0.79 (0.10)	0.18 (0.21)	-155.13	-172.46

All these models pass the diagnostic tests on the residuals at the 0.1% significance level. Standard errors in parentheses.
' stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3h

Parameter estimates of ARFIMA models for nominal GNP starting from 1947.

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.43 (0.05)	28.60	8.60	--	--	--	--	--	--	155.50	153.73
(1, 0)	1.47 (0.03)	49.00	15.66	0.31 (0.16)	--	--	--	--	--	156.62	153.09
(0, 1)	1.49 (0.01)	149.0	49.00	--	--	--	-0.46 (0.13)	--	--	159.54	156.01
(1, 1)	1.49 (0.01)	149.0	49.00	-0.003 (0.26)	--	--	-0.46 (0.20)	--	--	157.54	152.25
(2, 0)	**1.49 (0.01)**	**149.0**	**49.00**	**-0.46 (0.15)**	**-0.41 (0.15)**	**--**	**--**	**--**	**--**	**161.03**	**155.73**
(0, 2)	1.48 (0.01)	148.0	48.00	--	--	--	-0.47 (0.26)	-0.008 (0.27)	--	157.54	152.25
(2, 1)	1.49 (0.01)	149.0	49.00	-0.54 (0.34)	-0.43 (0.17)	--	0.09 (0.36)	--	--	159.10	152.05
(3, 0)	1.48 (0.01)	148.0	48.00	-0.44 (0.16)	-0.38 (0.17)	0.05 (0.17)	--	--	--	159.13	152.07
(3, 1)	1.48 (0.03)	49.33	16.00	0.27 (0.38)	-0.04 (0.21)	0.38 (0.18)	-0.69 (0.35)	--	--	157.64	148.83
(3, 3)	1.47 (0.03)	49.00	15.66	0.24 (0.62)	-0.004 (0.60)	0.22 (0.50)	-0.68 (0.59)	-0.04 (0.77)	0.17 (0.63)	153.89	141.55

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
' stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3i

Parameter estimates of ARFIMA models for industrial production starting from 1947.

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.15 (0.08)	14.37	1.87’	--	--	--	--	--	--	106.42	104.65
(1, 0)	1.24 (0.10)	12.40	2.40	-0.25 (0.19)	--	--	--	--	--	105.90	102.37
(0, 1)	**1.48 (0.02)**	**74.00**	**24.00**	**--**	**--**	**--**	**0.84 (0.12)**	**--**	**--**	**111.34**	**107.81**
(2, 0)	1.36 (0.09)	15.11	4.00	-0.49 (0.18)	-0.39 (0.16)	--	--	--	--	108.39	103.09
(0, 2)	1.49 (0.01)	149.0	49.00	--	--	--	-0.72 (0.21)	-0.14 (0.22)	--	109.82	104.53
(2, 1)	1.48 (0.02)	74.00	24.00	0.08 (0.21)	-0.13 (0.20)	--	0.82 (0.18)	--	--	108.94	101.07
(3, 0)	1.38 (0.10)	13.80	3.80	-0.47 (0.20)	-0.42 (0.19)	-0.05 (0.18)	--	--	--	106.46	99.41
(3, 1)	1.49 (0.02)	74.50	24.50	0.19 (0.25)	-0.10 (0.21)	0.18 (0.22)	-0.92 (0.25)	--	--	106.83	98.01

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses. ‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3j

Parameter estimates of ARFIMA models for GNP deflator starting from 1947.

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(1, 0)	1.43 (0.08)	17.87	5.37	0.51 (0.17)	--	--	--	--	--	209.72	206.16
(1, 1)	1.44 (0.06)	24.00	7.33	0.12 (0.22)	--	--	0.60 (0.13)	--	--	214.37	209.07
(2, 0)	**1.47 (0.03)**	**49.00**	**15.66**	**0.60 (0.15)**	**-0.36 (0.16)**	**--**	**--**	**--**	**--**	**212.43**	**207.13**
(2, 1)	1.46 (0.05)	29.20	9.20	0.21 (0.25)	-0.16 (0.22)	--	0.50 (0.22)	--	--	212.89	205.83

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses. ‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3k

Parameter estimates of ARFIMA models for consumer prices starting from 1947.

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.47 (0.03)	49.00	15.66	--	--	--	--	--	--	182.10	180.33
(1, 0)	1.41 (0.09)	15.66	4.55	0.39 (0.17)	--	--	--	--	--	185.72	182.19
(0, 1)	**1.39 (0.07)**	**19.85**	**5.57**	**--**	**--**	**--**	**0.77 (0.13)**	**--**	**--**	**196.29**	**192.75**
(2, 0)	1.47 (0.03)	49.00	15.66	0.51 (0.13)	-0.53 (0.14)	--	--	--	--	193.99	188.69
(2, 1)	1.46 (0.04)	36.50	11.50	0.17 (0.21)	-0.40 (0.18)	--	0.51 (0.20)	--	--	195.81	188.75
(3, 0)	1.41 (0.13)	10.84	3.15	0.73 (0.22)	-0.69 (0.16)	0.41 (0.21)	--	--	--	196.66	189.61
(0, 3)	1.44 (0.06)	24.00	7.33	--	--	--	0.75 (0.17)	-0.12 (0.20)	-0.27 (0.15)	195.06	188.01
(3, 1)	1.38 (0.18)	7.66	2.11	1.02 (0.42)	-0.85 (0.29)	0.54 (0.22)	-0.29 (0.46)	--	--	194.81	185.99
(3, 2)	1.37 (0.22)	6.22	1.68	0.98 (0.33)	-0.69 (0.31)	0.48 (0.20)	-0.19 (0.31)	-0.19 (0.24)	--	193.40	182.83

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3l

Parameter estimates of ARFIMA models for wages starting from 1947.

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(2, 0)	1.48 (0.01)	148.0	48.00	-0.21 (0.17)	-0.30 (0.17)	--	--	--	--	194.24	188.95
(2, 1)	1.48 (0.01)	148.0	48.00	-0.48 (0.34)	-0.37 (0.16)	--	0.28 (0.32)	--	--	192.96	185.91
(3, 0)	0.93 (0.18)	5.16	-0.38’	0.38 (0.20)	0.006 (0.18)	0.58 (0.16)	--	--	--	196.46	189.41
(3, 1)	**1.36 (0.26)**	**5.23**	**1.38’**	**0.37 (0.24)**	**-0.11 (0.18)**	**0.55 (0.16)**	**-0.47 (0.19)**	**--**	**--**	**196.61**	**187.79**

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
‘ stands for non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3m

Parameter estimates of ARFIMA models for money stock starting from 1947.

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.48 (0.01)	148.0	48.00	--	--	--	--	--	--	198.53	196.75
(0, 1)	**1.47 (0.03)**	**49.00**	**15.66**	**--**	**--**	**--**	**0.34 (0.13)**	**--**	**--**	**200.60**	**197.07**
(0, 2)	1.47 (0.04)	36.75	11.75	--	--	--	0.32 (0.24)	-0.02 (0.21)	--	198.61	193.31
(3, 0)	1.48 (0.01)	148.0	48.00	0.29 (0.16)	-0.28 (0.16)	-0.03 (0.16)	--	--	--	198.51	191.45
(3, 1)	1.48 (0.01)	148.0	48.00	-0.54 (0.18)	-0.02 (0.18)	-0.39 (0.15)	0.86 (0.14)	--	--	199.37	190.55

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
‘ stands non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 3n

Parameter estimates of ARFIMA models for common stock prices starting from 1947.

ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.19 (0.09)	13.22	2.11	--	--	--	--	--	--	52.42	50.65
(1, 0)	1.24 (0.11)	11.27	2.18	-0.12 (0.19)	--	--	--	--	--	50.79	47.25
(0, 1)	1.42 (0.10)	14.20	4.20	--	--	--	-0.50 (0.19)	--	--	52.64	49.11
(1, 1)	1.42 (0.10)	14.20	4.20	0.19 (0.26)	--	--	-0.62 (0.22)	--	--	51.25	45.95
(2, 0)	1.36 (0.09)	15.11	4.00	-0.27 (0.17)	-0.34 (0.15)	--	--	--	--	52.76	47.47
(2, 1)	1.43 (0.07)	20.42	6.17	-0.05 (0.23)	-0.34 (0.16)	--	-0.35 (0.23)	--	--	52.45	45.39
(1, 2)	1.42 (0.10)	14.20	4.20	-0.13 (0.43)	--	--	-0.24 (0.42)	-0.22 (0.20)	--	50.02	42.97
(2, 2)	1.39 (0.10)	13.90	3.90	0.40 (0.20)	-0.74 (0.15)	--	-0.84 (0.17)	0.73 (0.23)	--	55.67	46.85
(3, 0)	**1.46 (0.04)**	**36.50**	**11.50**	**-0.49 (0.15)**	**-0.51 (0.14)**	**-0.38 (0.14**	**--**	**--**	**--**	**55.98**	**48.93**
(0, 3)	1.40 (0.10)	14.00	4.00	--	--	--	-0.60 (0.17)	-0.07 (1.14)	0.37 (0.15)	52.32	45.27
(3, 1)	1.45 (0.05)	29.00	9.00	-0.73 (0.29)	-0.59 (0.17)	-0.47 (0.15)	0.29 (0.32)	--	--	54.82	46.01
(3, 2)	1.43 (0.08)	17.87	5.37	-0.67 (0.32)	-0.74 (0.21)	-0.48 (0.16)	0.27 (0.39)	0.26 (0.27)	--	53.60	43.03
(2, 3)	1.40 (0.11)	12.72	3.63	0.40 (0.21)	-0.75 (0.17)	--	-0.85 (0.26)	0.74 (0.33)	-0.01 (0.20)	53.67	43.09
(3, 3)	1.42 (0.08)	17.75	5.25	-0.57 (0.23)	-0.41 (0.22)	-0.73 (0.15)	0.09 (0.25)	-0.02 (1.21)	0.59 (0.28)	53.03	40.69

All these models pass the diagnostic tests on the residuals at the 1% significance level. Standard errors in parentheses.
‘ stands non-rejection values of the hypotheses d = 0 and d = 1.

TABLE 4

Best model specification for the extended version of Nelson and Plosser's dataset.

Series	ARFIMA	$t_{d=0}$	$t_{d=1}$	AR estimates			MA estimates		
				ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
Real GNP	(0, 1.30, 0)	16.25	3.75	--	--	--	--	--	--
Nominal GNP[+]	(2, 1.49, 0)	149.0	49.00	-0.46	-0.41	--	--	--	--
Real cap. GNP	(2, 1.11, 2)	12.33	1.22'	0.81	-0.89	--	-0.65	0.89	--
Industrial prod.[+]	(0, 1.48, 1)	74.00	24.00	--	--	--	0.84	--	--
Employment	(0, 1.14, 1)	14.25	1.75'	--	--	--	0.33	--	--
Unemployment	(1, 0.25, 0)	1.04'	-3.12	0.62	--	--	--	--	--
GNP deflator[+]	(2, 1.47, 0)	49.00	15.66	0.60	-0.36	--	--	--	--
Cons. prices[+]	(0, 1.39, 1)	19.85	5.57	--	--	--	0.77	--	--
Wages[+]	(3, 1.36, 1)	5.23	1.38'	0.37	-0.11	0.55	-0.47	--	--
Real wages	(0, 1.22, 0)	17.42	3.14	--	--	--	--	--	--
Money stock[+]	(0, 1.47, 1)	49.00	15.66	--	--	--	0.34	--	--
Velocity	(2, 1.01, 2)	14.42	0.14'	0.89	-0.83	--	-0.76	0.76	--
Bond Yield	(2, 0.96, 3)	9.60	-0.40'	-0.06	-0.86	--	0.33	0.92	0.49
C. Stock prices[+]	(3, 1.46, 0)	36.50	11.50	-0.49	-0.51	-0.38	--	--	--

[+] indicates that the series were analysed only for the post-war data. ' stands for non-rejection values of the null hypotheses: d = 0 and d = 1.

TABLE 5

Impulse response functions for the growth rate in the extended version of Nelson and Plosser's dataset.

Series	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Real GNP	.300	.195	.149	.123	.106	.093	.084	.076	.070	.065	.061	.061	.058	.052	.049	.047	.045
Nom. GNP	.030	-.058	.317	.142	.041	.139	.120	.070	.093	.094	.079	.079	.078	.073	.071	.070	.067
Cap. GNP	.270	.208	.029	-.101	-.064	.073	.146	.079	-.043	-.085	-.011	-.011	.083	.016	-.056	-.047	.024
Ind. prod.	1.319	.758	.591	.502	.443	.401	.369	.343	.322	.304	.289	.289	.276	.254	.245	.237	.230
Employm.	.470	.126	.083	.063	.051	.043	.038	.033	.030	.028	.025	.025	.023	.020	.019	.018	.017
Unemp.	.870	.695	.548	.435	.350	.288	.241	.207	.181	.160	.145	.145	.132	.113	.106	.100	.094
GNP def.	1.070	.627	.275	.186	.233	.273	.266	.234	.208	.195	.189	.189	.184	.169	.163	.158	.153
Cons. Pric.	1.159	.571	.424	.349	.301	.268	.242	.223	.207	.193	.182	.182	.173	.157	.150	.144	.139
Wages[+]	.260	.061	.621	.437	.192	.424	.431	.270	.334	.377	.294	.294	.292	.286	.267	.279	.265
Real wages	.219	.134	.099	.079	.067	.058	.052	.047	.042	.039	.036	.036	.034	.030	.028	.027	.026
Money st.	.810	.505	.401	.343	.304	.276	.254	.236	.222	.210	.199	.190	.190	.175	.169	.163	.158
Velocity	.139	.052	-.062	-.095	-.030	.054	.075	.023	-.040	-.054	-.013	.033	.033	.010	-.025	-.030	-.005
Bond Yield	1.229	1.243	1.479	1.402	1.173	1.229	1.405	1.330	1.711	1.233	1.356	1.286	1.286	1.176	1.319	1.257	1.182
C. Stock P.	-.030	-.159	-.011	.336	.113	-.029	.007	.134	.098	.028	.025	.070	.070	.070	.035	.048	.053

All the series have been first-differenced except the unemployment rate.