Self Employment, Wage Employment and Informality in a Developing Economy

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Abstract

We construct a simple model incorporating various urban labour market phenomena obtaining in developing economies and we give a diagrammatic formulation of the market equilibrium. Our initial formulation assumes an integrated labour market and allows for entrepreneurship, self-employment and wage employment. We then introduce labour market segmentation. In equilibrium voluntary and involuntary self-employment, formal and informal wage employment, and formal and informal entrepreneurship may all coexist. We illustrate the model by an example calibrated on Latin American data, examining individual labour market transitions and implications of education/training and labour market policies.

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1 Introduction

Urban labour markets in developing economies exhibit considerable diversity, typically including substantial segments of both voluntary and involuntary self-employment, and of formal and informal wage employment. In Latin America and the Caribbean, for example, Perry *et al.* (2007) find that, on average, 24% of urban employment is informal self-employment and 30% is informal wage employment, and that each of these segments have significant voluntary and involuntary elements.[1] The aim of the present paper is to formulate a simple model that incorporates all of these labour market states, as well as different types of entrepreneurship, and to develop a diagrammatic analysis of market equilibrium. Some recent literature on informality based on search-and-matching theory also incorporates the interaction of several of these labour market states (Albrecht *et al.*, 2009; Margolis *et al.*, 2012), but is rather complex and relies extensively on simulations to generate results.

In contrast to most of the theoretical literature on informality, we model in detail the supply as well as the demand side of the labour market.[2] We assume that each agent can allocate his or her labour to one of three activities: self-employment, wage employment, or entrepreneurship (running a firm and providing wage employment to others).[3] An agent is characterized in terms of two skills, Y and Z, where, loosely speaking, Y is the ability

[1] See Fields (2009) for a general discussion of the complexity of labour markets in developing economies and of approaches to modeling them.

[2] Galiani and Weinschelbaum (2012) analyse both sides of the market, but their focus, in particular their concern with the effects of social programs, is different from ours.

[3] In the absence of functioning systems of unemployment insurance and protection from job loss, open unemployment is 'rarely an option' for workers in developing economies (Ghosh, 2012).

to produce and sell an output, and Z is managerial ability. Success as a self-employed worker would depend on the amount y of skill Y possessed; but, following the Lazear (2005) 'jack-of-all-trades' formulation, success as an entrepreneur would depend on applying both skills together, specifically, on the value of $\min(y, z)$, where z is the amount of Z the agent possesses. In wage employment, however, everyone is assumed to be equally able.[4]

We develop two versions of the model. First, in our benchmark case, we assume that the market for wage employment clears, so that all labour states are 'voluntary.' This is useful for expositional purposes, providing the groundwork for the second version of the model, in which we assume labour market segmentation. However, it is also of interest in its own right because empirical evidence suggests that in some developing economies labour markets may be largely integrated (see, e.g., El Badaoui *et al.* (2008) on South Africa and for general discussion of the empirical literature). In the second version we assume that segmentation is the result of a minimum wage, with which there is full compliance in formal employment. The simplicity of our diagrammatic formulation is thus bought at the cost of this limitation of the model. In practice, however, the wage floor in formal work might instead be the result of union bargaining or of efficiency wage considerations, and there may be limited compliance with a minimum wage. In the concluding section we discuss briefly the potential for generalizing the model to allow for these factors.

For the benchmark model, we begin by characterizing the supply function

[4]This simplifying assumption is common in the informality literature. Its justification is that across the population we may expect differences in the ability to perform wage work to be relatively small, compared to ability in self-employment and entrepreneurship.

of an agent to the three activities. Two cases are developed (depending on parameter values) which may be interpreted as corresponding to different underlying macroeconomic conditions, and the implications for labour market transitions that may occur as agents acquire greater skills are discussed. The model also generates a demand for wage labour by those agents who choose entrepreneurship. Given the joint distribution of y and z across individuals, and a flexible wage rate, we characterize the labour market equilibrium and examine its comparative statics. We depict this equilibrium in a diagram which we later adapt to the second version of the model.

In practice, informality is generally associated with smaller size (Perry *et al.*, 2007). This is because formal regulations may only apply to larger firms, and, insofar as they apply to all firms, informal firms may eschew larger size to avoid detection.[5] In the second version of the model we therefore assume that the minimum wage rate applies only for firms above a certain size, with such firms being regarded as formal, and smaller firms as informal. For this case we show that informal and formal wage employment can coexist with voluntary and involuntary self-employment. If an agent who is rationed out of a formal wage job chooses self-employment, this is involuntary in the sense that it is not the agent's first choice – though it is voluntary in the sense that it is chosen freely from the remaining options. Involuntary entrepreneurship may also obtain, i.e., agents who are rationed out of formal wage employment may choose, according to their second preference, to run a firm and employ others.[6] Our analysis is not restricted to any particular rationing scheme

[5] See Ahsan and Pages (2007) on India, and Almeida and Carneiro (2009) on Brazil.

[6] A related phenomonen in developed economies is the worker who is made redundant and then uses his or her redundancy payment or savings to set up a business.

to allocate formal jobs between the subset of agents who would like to take them. We note, however, that a potential inefficiency exists (in addition to the distortion caused by the minimum wage rate): agents who are lucky enough to gain formal jobs may actually have a comparative advantage in self-employment. In equilibrium this has an adverse effect on output by both informal and formal firms.

We end this section by considering briefly the transitions of workers between different labour market states. The empirical literature on Latin America indicates that young people tend to get informal jobs when they leave school, and that these jobs are often used as a stepping-stone to acquire skills. Formal employment may later be obtained, but for many the ultimate destination is voluntary self-employment (Perry *et al.*, 2007; Bosch and Maloney, 2010; Cunningham and Salvagno, 2011). Although our model is not dynamic, it is found to be consistent with these observations.

To illustrate the model we explore an example calibrated so that it generates values that correspond broadly to Latin American data. This generates some implications for policy, albeit tentative given the stylized nature of the model. First, to diminish informality, a cut in the cost of formality may be more effective than an increase in the cost of informality. Second, it is more effective to provide education and training that improves the ability to produce and sell, rather than managerial skills. Third, policy changes that might have been expected to favour entrepreneurship may reduce the total number of entrepreneurs, while increasing (formal) employment and output by the relatively able ones.

Following the lead of Rauch (1991), many contributions to the informality

literature, including Fortin *et al.* (1997), Amaral and Quintin (2006), Fiess *et al.* (2010) and de Paula and Scheinkman (2011), assume agents differ with respect to a single ability parameter.[7] Antunes and Cavalcanti (2007) also include 'bequests' as a second dimension by which agents are characterized, while Galiani and Weinschelbaum (2012) incorporate into their analysis secondary workers (from the same household as the head), showing that these workers are likely to choose informal work. Yet, although these analyses lay bare various important issues underlying informality in practice, they do not allow for the simultaneous existence of informal wage labour and informal self-employment. Gollin (2008) is an exception, developing a dynamic equilibrium model of capital accumulation in which an agent's time is split between self employment and working as a wage employee. However, none of these contributions allow for the simultaneous existence of voluntary and involuntary informality.

Recently, a separate branch of the literature has grown, which develops search-and-matching models of informality. In this framework additional labour market states can be added at the cost of some complexity. In particular, in the formulation by Albrecht *et al.* (2009), all choices are voluntary, and a worker can be in one of four states: unemployment, informal self-employment, or formal wage employment either as a new hire (an outsider)

[7] Rauch's formulation incorporates informality into the framework developed by Lucas (1978). Jovanovic (1994) generalizes Lucas's approach in a different direction, including heterogeneity of both labour and management skills, but he is not concerned with informality (see also Poschke, 2013). A variation of the approach in the present paper is formulated by Bennett *et al.* (2012), with individuals characterized in terms of both ability and risk aversion. It is found, for example, that the allocation of workers between informal and formal employment depends on the source of the risk involved in informal work (opportunistic non-payment by the employer or detection and penalties imposed by the authorities).

or as an insider with a higher wage. Ability is one-dimensional, and it is assumed only to affect an agent's productivity in a formal sector job. A simulation gives insight into how workers respond to informal or formal job offers according to their ability, and into the effects of different tax policies. A variation on this approach, with four labour market states (unemployment, self-employment, formal wage employment and informal wage employment) is formulated by Margolis *et al.* (2012) as the basis for empirical analysis of the Malaysian labour market.

Section 2 formulates the benchmark version of the model, with a market-clearing wage rate. Section 3 introduces labour market segmentation, and Section 4 applies the model in the Latin American context, examining the policy conclusions. Section 5 concludes. Proofs are given in the Appendix.

2 The Benchmark Model

Consider a large population P of agents, each of which is characterized in terms of two skills, Y and Z. Y may be thought of as the ability to produce and sell, and Z as managerial skill. An agent's levels of Y and Z are distributed on the non-negative intervals $y \in \left[\underline{y}, \bar{y}\right]$ and $z \in [\underline{z}, \bar{z}]$, respectively. Skills are distributed across P according to $f(y, z)$. Throughout, for simplicity, we assume that $f(.)$ is continuous and positive for all y and z.

Any agent may have one of three occupations: wage employment, self-employment or entrepreneurship. We assume a self-employed person does not employ others – rather, any employment of others qualifies the person to be categorized as an entrepreneur.[8] Regardless of an agent's (y, z)-

[8] Our assumption is consistent with the definition that Lazear (2005) gives of an entre-

characteristics, he or she has the same ability to do wage work as any other person. However, for self-employment and entrepreneurship, ability matters. If an agent with characteristics (y, z) is self-employed, he or she produces the quantity y; that is, for self-employment 'the ability to produce and sell' matters, but 'managerial skills' do not.[9] If, alternatively, he or she is an entrepreneur, the relevant measure of skill is $\min(y, z) \equiv A$; that is, a balance of both types of skill matters.[10] Such a person runs a firm for which the production function is

$$x = Al^{\alpha}, \quad \alpha \in (0, 1), \tag{1}$$

where x is output and l is the number of people the firm employs. Our specifications of production are chosen to keep the analysis simple. The same qualitative results would be obtained without the Leontief specification of A, provided the two types of skill are sufficiently weak substitutes; and with some concavity of the production function for the self-employed.

Let q and p be the prices for the output of the self-employed and entrepreneurial firms, respectively, and let w be the money wage rate. An entrepreneur's profit is therefore $px - wl$, which, given (1), is maximized at $l = \hat{l}(A)$, where

$$\hat{l}(A) = \left(\frac{Ap\alpha}{w}\right)^{\frac{1}{1-\alpha}}. \tag{2}$$

preneur as being conceptually distinct from a self-employed person.

[9] In practice self-employment covers a wide range of activity. Self-employed production with low y may be, e.g., construction work or street vending, while that with high y may be, e.g., professional work. A similar comment applies to entrepreneurial output.

[10] Since we are concerned with relatively small firms, the productive and sales skills of the entrepreneur will generally matter for a firm's success. De Mel *et al.* (2008) suggest that the case for a jack-of-all-trades characterization is stronger if the market for business services is thin, as typically obtains in developing economies. Bloom *et al.* (2013) report on recent field experiments in developing countries that show that some forms of basic business training and advice can have significant effects on performance in small enterprises.

We assume that both self-employment and entrepreneurship give an agent a non-pecuniary benefit v, which may be thought of as the desire for independence.[11] Thus, letting UW, US and UE denote the utility from working, self-employment and entrepreneurship, respectively, we have

$$UW = w; \qquad US = qy + v; \qquad UE = pA\hat{l}^{\alpha} - w\hat{l} + v. \tag{3}$$

We shall only consider cases in which $w > v$, which is necessary for wage employment to exist in equilibrium.

We partition P into three sets, W, S and E, according to whether an agent's first preference is for wage employment, self employment or entrepreneurship, respectively.[12] Thus, the sets are defined by

$$\begin{aligned} W &: UW > \max(UE, US); \\ S &: US > \max(UW, UE); \\ E &: UE > \max(UW, US). \end{aligned} \tag{4}$$

Using (2) and (3), we can determine the borderline values of parameters underlying (4):

$$UW \gtreqless UE \text{ as } A \lesseqgtr B(w); \tag{5}$$

$$UW \gtreqless US \text{ as } y \lesseqgtr C(w); \tag{6}$$

$$UE \gtreqless US \text{ as } A \gtreqless [D(w)]^{\alpha} y^{1-\alpha} \equiv \tilde{z}(y). \tag{7}$$

[11] Perry *et al.* (2007) stress the significance of this non-pecuniary benefit in Latin America and the Caribbean. An implication in our model is that in equilibrium, depending on ability, some agents could earn more in informal wage employment than from self-employment, while for others the reverse is true. This is consistent with the mixed empirical evidence on which of these types of earnings is the higher (see, e.g., Agénor, 2007). With minor amendments, our analysis would still apply if $v = 0$ or even if $v < 0$, in which case v might be interpreted as the disutility of extra effort required from independent/entrepreneurial work.

[12] Throughout, we simplify the exposition by only specifying strong preference.

where $B(w) \equiv (w/\alpha)^{\alpha}\left((w-v)/(1-\alpha)\right)^{1-\alpha}/p$; $C(w) \equiv (w-v)/q$; and $D(w) \equiv (w/p\alpha)\left(q/\left(p\left(1-\alpha\right)\right)\right)^{(1-\alpha)/\alpha}$. Note that $B(w)-C(w) \gtreqless 0$ as $q/p \gtreqless Q(w)$, where

$$Q(w) = \alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\left(\frac{w-v}{w}\right)^{\alpha}. \tag{8}$$

A relatively low q/p might be interpreted as reflecting strong aggregate demand, being tilted towards the largely higher-quality output of entrepreneurial firms. Since $\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha} \in (1/2, 1)$, $Q(w) \in (0,1)$. Thus, if $q < p$ then either $q/p > Q(w)$ or $q/p < Q(w)$; but if $q \geq p$, $q/p > Q(w)$.

Using (5)-(8), Proposition 1 characterizes the allocation of agents to the three sets, W, S and E and Figures 1-2 give an intuitive illustration.

Proposition 1 *Consider agent i_{yz} with characteristics (y,z). (i) for $q/p > Q(w)$, $i_{yz} \in W$ if $y < C(w)$; $i_{yz} \in S$ if either $y \in (C(w), D(w))$ or both $y > D(w)$ and $z < \tilde{z}(y)$; and $i_{yz} \in E$ otherwise. (ii) for $q/p < Q(w)$, $i_{yz} \in W$ if either $y < B$, or both $y \in (B(w), C(w))$ and $z < B(w)$; $i_{yz} \in S$ if $y > C(w)$ and $z < \tilde{z}(y)$; and $i_{yz} \in E$ otherwise.*

In Figure 1(a) $q/p > Q(w)$ and in Figure 1(b) $q/p < Q(w)$. Thus, for given w, in 1(a) self-employment is relatively more attractive, compared to entrepreneurship, than in 1(b). For simplicity, it is assumed in these figures that $\bar{y} = \bar{z}$ and $\underline{y} = \underline{z} = 0$. Consider Figure 1(a), in which $q/p > Q(w)$. For individuals with $y < C(w)$ self-employment and entrepreneurship both offer relatively low rewards and so wage employment is preferred. If $y > C(w)$ either self-employment or entrepreneurship is preferred. For $C\left(w\right) < y < D\left(w\right)$ self-employment is preferred, but when $y > D(w)$, entrepreneurship is

preferred if z is large enough, with the critical value of z increasing in y.[13]

[Figure 1 about here]

The figures may be interpreted in terms of an agent's transition between labour market states as skills are acquired. Consider, for example, an agent with skill $z = z_1$ in Figure 1(a). Starting from a low level, the acquisition of greater skill y enables a transition from W to S, and then from S to E; but the acquisition of sufficiently high skill y enables a transition back to S. Thus, for some agents, even for changes in y alone, the transition between labour market states may be non-monotonic.[14] Figure 1(b) is significantly different to 1(a) in that, as y rises, an agent may switch directly from W to E, with no intermediate stage S. An implication is that, for increases in y (or z) alone, monotonicity obtains, though, as in 1(a), some agents belong to S at the highest values of y even though at lower y they would belong to E.

Remark 1 *The mobility implications of education and training that affect individuals' ability Y can depend on macroeconomic factors (p and q) as well as individual-specific ones (Z here).*

In Figure 1(a), where q/p is low (strong aggregate demand) transitions may occur straight from W to E, however small the increase in Y. But in Figure 1(b), where q/p is high (weak aggregate demand) only a relatively large addition to skill Y would enable direct transition from W to E; in

[13] If there were some limited substitutability of the two skills in the entrpreneurial production function, there would be no straight boundaries between E and the other sets, but the properties of the figure would be unaffected.

[14] If there is incremental acquisition of both skills y and z, and this occurs sequentially over time, then repeated switches between E and S are also possible.

the absence of such large additions to skill, self-employment may play an important transitional role.

Aggregating over P, we obtain the supplies of labour to the three activities. We denote the total supplies to wage employment, self-employment and entrepreneurship by L^s, SE^s and E^s, respectively. For each entrepreneur the demand for labour is given by $\hat{l}(A)$ in (2) and thus we obtain the total demand for labour, L^d.

Lemma 1 *The comparative statics of the supply and demand for wage labour are as follows:*

$$L_p^d > 0;\ L_q^d < 0;\ L_w^d < 0;\ L_q^s < 0;\ L_w^s > 0;\ L_v^s < 0;$$
$$L_v^d \begin{cases} = 0 & \text{for } q/p > Q(w) \\ > 0 & \text{for } q/p < Q(w) \end{cases};\ L_p^s \begin{cases} = 0 & \text{for } q/p > Q(w) \\ < 0 & \text{for } q/p < Q(w) \end{cases}.$$

The demand for wage labour is increasing in the price of the firms' output and decreasing in the money wage. It is decreasing in the price paid for the output of the self-employed because a higher price for this output makes entrepreneurship relatively less attractive. A greater benefit, v, from independence makes entrepreneurship (and self employment) more attractive relative to wage employment. However, this is only associated with more agents choosing entrepreneurship if there are agents on the margin of choice between entrepreneurship and wage employment (Figure 1(b)).

The supply of wage labour is increasing in the money wage rate, and decreasing in the price of self-employed output and benefit from independence. If the output price p is higher then, again, provided there are agents on the margin of choice between entrepreneurship and wage employment (Figure

1(b)), wage employment becomes less attractive relative to entrepreneurship for these agents, and so the supply of wage labour is lower.

We can now specify sufficient conditions for equilibrium in the labour market, including the coexistence of wage employment and self employment. We denote the lowest and highest levels of A in P by $\underline{A}$ and $\overline{A}$, respectively, and we define $\underline{w}$ and $\overline{w}$ as

$$UE\left(\underline{A},\underline{w}\right)=UW\left(\underline{w}\right);\qquad UE\left(\overline{A},\overline{w}\right)=UW\left(\overline{w}\right). \tag{9}$$

Thus, $\underline{w}$ is the level of the wage w at which an agent with $A=\underline{A}$ would be indifferent between being an entrepreneur and a worker, and $\overline{w}$ is defined similarly for $A=\overline{A}$.

Proposition 2 *If* $UE\left(\underline{A},\underline{w}\right)<US\left(\underline{y}\right)<UW\left(\overline{w}\right)<US\left(\bar{y}\right)<UE\left(\overline{A},\underline{w}\right)$ *then there exists a wage* $w^{*}\in\left(\underline{w},\overline{w}\right)$ *such that* $L^{d}\left(w^{*}\right)=L^{s}\left(w^{*}\right)$ *and the sets* E,S,W *are non-empty.*

Depending on whether the market-clearing wage rate w^{*} is such that $q/p>Q(w^{*})$ or $q/p<Q(w^{*})$, Figure 1(a) or 1(b), respectively, can be interpreted as representing this equilibrium.

Lemma 2 *In equilibrium* ($w=w^{*}$), $dw/dp>0$, $dw/dq\gtrless 0$ *and* $dw/dv>0$; *and total wage employment* L *satisfies*

$$\begin{aligned}\frac{dL}{dp} &> 0\begin{cases}\text{if } q/p>Q(w^{*}),\\ \text{if } q/p<Q(w^{*}) \text{ and } L_{p}^{d}L_{w}^{s}-L_{w}^{d}L_{p}^{s}>0;\end{cases}\\ dL/dq &< 0;\\ \frac{dL}{dv} &= \begin{cases}<0 & \text{if } q/p>Q(w^{*}),\\ \gtrless 0 & \text{if } q/p<Q(w^{*}).\end{cases}\end{aligned}$$

If the price p of the entrepreneurial output is higher then L^d is greater, as is w^*. Set W is therefore larger, subject, when $q/p < Q(w^*)$, to a stability condition. If the output price q for the self-employed is higher, the greater attractiveness of self-employment is associated with lower supply of and lower demand for wage labour, the latter effect arising because the supply of entrepreneurship is smaller. Thus, W is smaller, but the net effect on w^* may be of either sign. A greater desire for independence v stimulates both self-employment (reducing the supply of wage labour) and entrepreneurship (increasing the demand for wage labour). The latter effect implies a greater demand for wage labour, but as the supply of wage labour is smaller we can only sign the effect on W when $q/p > Q(w^*)$.

3 Labour Market Segmentation

We now examine the equilibrium that obtains when the wage rate w is fixed by law, at w_f, above the market-clearing level w^*. As first specified by Rauch (1991), we assume that only firms above a certain threshold employment level $l = l_0$ pay the minimum wage w_f, whereas firms with $l \leq l_0$ pay the market-clearing wage $w = w_i$. The former firms are denoted 'formal' and the latter 'informal'. In Rauch's model (in which skill is one-dimensional) there is a critical entrepreneurial skill level above which formality is chosen, with informality being chosen otherwise. In our model there is a critical level of A, $A = \tilde{A}$, that plays a similar role. This is the level of A at which the entrepreneur achieves the same utility from operating informally at the maximum employment level l_0 as from operating formally at the higher,

profit-maximizing employment level $\hat{l}(A)$; i.e.,

$$UE(\tilde{A}, w_i, l_0) = UE(\tilde{A}, w_f, \hat{l}(\tilde{A})). \tag{10}$$

Of agents choosing entrepreneurship, those with $A > \tilde{A}$ choose formality. As in Rauch's model there is a gap in the size-distribution of firms at $A = \tilde{A}$.

With this revised model, the utility from self-employment is the same as in (3), but we now distinguish the respective utilities, U_f and U_i from formal and informal wage work:

$$US = qy + v; \qquad UW_f = w_f; \qquad UW_i = w_i. \tag{11}$$

The utilities from formal and informal entrepreneurship are denoted by UE_f and UE_i, where

$$UE_j = pAl_j^{\alpha} - w_j\hat{l}_j + v, \quad j = f, i. \tag{12}$$

For an informal entrepreneur ($A \leq \tilde{A}$), if there were no constraint on informal employment we would have $l_i = \hat{l}_i(A) = (Ap\alpha/w_i)^{1/(1-\alpha)}$. So the constraint $l \leq l_0$ binds exactly if $(Ap\alpha/w_i)^{1/(1-\alpha)} = l_0$; i.e., if $A = w_i l_0^{1-\alpha}/p\alpha \equiv A_0$. Thus, for firms operating informally,

$$l_i = \begin{cases} \hat{l}_i(A) & \text{if } A_0 > A; \\ l_0 & \text{if } \tilde{A} \geq A \geq A_0; \end{cases}$$

and for firms operating formally

$$l_f = \hat{l}_f = \left(\frac{Ap\alpha}{w_f}\right)^{\frac{1}{1-\alpha}}.$$

The population P can be partitioned into four sets according to their first preferences in the labour market.[15] In the appendix we specify the equalities parallel to (5)-(8) that underlie these first preferences (as well as those

[15] For each agent, the first preference is 'voluntary', but we only use this term in naming a set if the distinction will be necessary below where we specify 'involuntary' sets (for which a similar comment applies).

underlying second preferences). Our notation will be to write in parentheses f for formal and i for informal, and then to add a subscript V for voluntary and I for involuntary if a further distinction is necessary. Thus, all agents belong to one of the following sets.

1. Formal entrepreneurship, denoted $E(f)$; defined by $UE_f > \max(UE_i, US, UW_f)$.
2. Voluntary informal entrepreneurship, denoted $E_V(i)$; defined by $UE_i > \max(UE_f, US, UW_f)$.
3. Voluntary self-employment, denoted S_V; defined by $US > \max(UE_f, UE_i, UW_f)$.
4. Formal employment, denoted $W(f)$; defined by $UW_f > \max(UE_f, UE_i, US)$.

Set $W(f)$ can be partitioned into those agents who obtain a formal job (set $W(f)^+$) and those who do not (set $W(f)^-$). Members of set $W(f)^-$ attain their second preferences; i.e., they allocate their labour 'involuntarily'. Each belongs to one of the following sets.[16]

1. Involuntary informal entrepreneurs, denoted $E_I(i)$; defined by $UE_i > \max(US, UW_i)$.
2. Involuntary self-employed, denoted S_I; defined by $US > \max(UE_i, UW_i)$.
3. Informal employees, denoted $W(i)$; defined by $UW_i > \max(UE_i, US)$.

Proposition 3 *Suppose firms may be formal, with $l > l_0$ and paying wage w_f, where $w_f > w^*$, or informal, with $l \leq l_0$ and paying the market clearing*

[16] No (y, z)-combinations exist for which both (i) formal employment is first preference and (ii) formal entrepreneurship second preference; i.e., involuntary formal entrepreneurship is not feasible.

wage w_i. Then the sets $E(f)$, $E_V(i)$, S_V, $W(f)^+$, $E_I(i)$, S_I and $W(i)$ may, simultaneously, all be non-empty in equilibrium.

We prove the proposition by giving an example in which, indeed, the sets $E(f)$, $E_V(i)$, S_V, $W(f)^+$, $E_I(i)$, S_I and $W(i)$ are, simultaneously non-empty in equilibrium. We delay giving this example until Section 5, however, where we relate it to Latin American data, because we wish to consider this example in its own right. The sets listed in the proposition are not necessarily non-empty, and degenerate equilibria may easily be formulated (e.g., if q/p were sufficiently high all agents would belong to set S_V). But we shall focus on cases in which all the sets (except possibly $E_I(i)$) are non-empty because these correspond to the labour markets observed in practice.

Proposition 3 is illustrated in Figure 2, which is a development of Figure 1, and can be interpreted as representing the equilibrium with endogenous adjustment of w_i. As previously, the cases shown correspond to different ranges of q/p relative to Q, but whereas in Figure 1 Q was a function of the single wage rate w^*, now there are two wage rates, w_f and w_i in the model. The relevant formulation is $Q(w_i, w_j)$ with $j = f, i$, where the first argument is the unit cost of labour to the entrepreneur and the second is the wage earned in activity j.[17] This is derived in the appendix, along with the borderline parameter values $B(\cdot,\cdot)$, $C(\cdot)$ and $D(\cdot)$ shown in Figure 2. Panel (i) illustrates the case in which $q/p > Q(w_i, w_f) > Q(w_i, w_i)$, which corresponds to the case shown in Figure 1(a); in panel (ii) $Q(w_i, w_f) > q/p >$

[17] In Figures 1(a) and 1(b) $Q(w^*)$ is the critical value of q/p determining whether the borderline value of y (and of z for entrepreneurship) at which $UW = UE$ is greater or less than that at which $UW = US$. Now two different values of Q come into play, depending on whether an employed agent earns w_f or w_i.

$Q(w_i, w_i)$, which is essentially a hybrid of the Figure 1(a)- and 1(b)-cases; and in panel (iii) $Q(w_i, w_f) > Q(w_i, w_i) > q/p$, which corresponds to Figure 1(b). Each of the panels can be explained in three steps.[18]

[Figure 2 about here]

First, using equations (1)-(7) with $w = w_f$ and (10), we determine the (y, z)-characteristics of the members of the 'first-preference' sets $E(f)$, $E_V(i)$, S_V and $W(f)$. The first three of these sets are shown unshaded, while set $W(f)$ is shown by the entire shaded area in each panel.

Second, because the rationing scheme has not been specified, note that membership of set $W(f)^+$ may come from anywhere in the shaded area (set $W(f)$) in each panel.

Third, disregarding temporarily the allocation of agents to set $W(f)^+$, we treat the shaded area in the same way as we did the whole of (y, z)-space in Figures 1(a) and 1(b). Thus, for the agents concerned, we show the preference among the three options of entrepreneurship, self-employment and informal wage employment, given that all three options are involuntary in the sense that these agents would prefer formal wage employment. Thus we determine the 'second-preference' sets $E_I(i)$, S_I and $W(i)$, with the proviso that a selection of agents with (y, z)-characteristics consonant with these sets, belong instead to set $W(f)^+$.

For a given (y, z)-distribution, we assume that w_i adjusts endogenously such that informal wage labour supply (from set $W(i)$) equals informal wage labour demand (from set $E_V(i) \cup E_I(i)$). The other allocations are determined

[18] If w_f is not significantly above w^* the horizontal boundary of the set $E(f)$ will meet the upward-sloping boundary of set $E_V(i)$ and terminate there.

simultaneously. It can be seen that relatively highly-skilled agents with a balanced skill set become formal entrepreneurs, while those not quite so highly skilled and/or with not quite so balanced skill sets become voluntary informal entrepreneurs. Agents with a high y, but sufficiently low z, become voluntarily self-employed.

In panel (i), the return to self-employment is relatively high ($q/p > Q(w_i, w_f) > Q(w_i, w_i)$). Consequently, there is no involuntary informal entrepreneurship, involuntary self-employment being preferred instead. However, the return to self-employment is not so high in panels (ii) and (iii) and so some involuntary entrepreneurship obtains, with the agents concerned having lower values of $A = \min(y, z)$ than voluntary entrepreneurs. Roughly speaking, involuntary informal entrepreneurs have high values of z, but intermediate values of y, although a member of set S_I may have more of both skills than a member of set $E_I(i)$.

Remark 2 *The rationing scheme for formal wage employment may create an (additional) inefficiency, with output being forgone from self-employment, and both informal and formal entrepreneurial firms.*

Unless the formal wage employees are those with the smallest y endowments in the shaded area in each panel of Figure 2, some output by the involuntarily self-employed is forgone. Also, in panels (ii) and (iii), insofar as some agents from the shaded area associated with $E_I(i)$ gain formal employment, there is a negative effect on informal wage labour demand and the supply of informal output. This impacts negatively on the informal wage rate w_i, causing substitution out of formal wage employment and output.

In each panel of Figure 2 an agent with low skills will – unless they manage to obtain formal wage employment – begin in set $W(i)$. As they acquire skills, they will move north-east in the figure, perhaps shifting into set S_I. Nonetheless, there is a possible non-monotonic transition in the sense that, if an agent moves from (involuntary) self-employment into a formal wage job, they may then go back to (voluntary) self-employment if their skills develop sufficiently.[19] Our model is thus broadly consistent with the empirical evidence on Latin American transitions. However, there are significant differences between the panels in Figure 2 in terms of the possible transitions from informal wage work to entrepreneurship. If skills are acquired incrementally, in Figure 2(i) an agent will move through intermediate stages of involuntary and voluntary self-employment. In Figure 2(ii), where q/p not as great as in Figure 2(i), the only intermediate stage is involuntary self-employment, while in Figure 2(iii), where q/p is lower still, there is no intervening stage of self-employment.

4 An Application To Latin American Data

To illustrate the model we calibrate it using Latin American data. We assume a joint log-normal distribution of skills Y and Z:

$$f(y,z) = \frac{e^{-k/2\sigma^2}}{2\pi\sigma^2 yz},$$

where $k \equiv (\log y)^2 + (\log z)^2$ and σ is a constant. We fix $\alpha = 0.5$, for there appears no compelling justification for any particular calibration of

[19] As in the benchmark model, if q/p is sufficiently large, there can be non-monotonic transitions from self-employment to entrepreneurship and back again.

this parameter. We set $l_0 = 5$ on the basis of the survey of informality by Oviedo *et al.* (2009), who note that informal firms 'mostly' have five or fewer employees.

The remaining parameters – p,q,v,w_f,σ – are calibrated such that our model matches some recent statistics for various countries in Latin America and the Caribbean reported by Perry *et al.* (2007). These are that the proportions of paid private nondomestic employment in urban areas (excluding entrepreneurs) are 37% formal waged, 28% informal waged, and 34% and self-employed, while the informal wage is around 59% of the formal wage.[20] As we have one additional parameter to calibrate relative to the number of statistics, we impose one further restriction to identify a unique calibration, that the benefit from independence v is 10% of the minimum wage w_f.[21]

We search numerically for the parameter values that obtain in our model under the assumption that the rationing scheme for allocating agents in set $W(f)$ to set $W(f)^+$ is random. As we use a dense lattice to approximate the infinite population of agents assumed in the model, a small degree of coarseness prevents us from matching these statistics with absolute precision. However, after adjusting the first three statistics above to account for entrepreneurs, the reported estimates (shown under Table 1 below) match to within one percentage point. For these calibrated parameter values, the

[20] The proportions of employment are cross-country averages derived from Table 2.1 of Perry *et al.*, which uses the social protection/legal definition of informality. There is wide variation across countries and types of worker in the formal-informal wage gap. The figure of 59% in the text relates to an average-earnings job in Argentina.

[21] This value is consistent with recent research reviewed by Carter (2010) – albeit for developed economies – which argues that the compensating differential from self-employment is substantially smaller than the widely-cited estimate of Hamilton (2000) of 35% of equivalent employment income.

Table 1: An example for Latin America

	Parameter Change						
Baseline (% of P)	q	p	v	w_f	l_0	Y	Z
$\lvert E(f)\rvert = 0.63$	0	+	+	−	−	+	+
$\lvert E_V(i)\rvert = 6.86$	−	−	−	+	−	−	+
$\lvert E_I(i)\rvert = 0.09$	−	−	−	+	−	−	−
$\lvert E(f) \cup E_V(i) \cup E_I(i)\rvert$	−	+	−	+	−	−	+
$\lvert S_V\rvert = 23.35$	+	−	+	−	+	+	−
$\lvert S_I\rvert = 7.77$	+	−	+	+	−	−	−
$\lvert S_V \cup S_I\rvert$	+	−	+	+	−	+	−
$\lvert W(f)^+\rvert = 35.40$	0	+	+	−	−	+	+
$\lvert W(i)\rvert = 25.99$	−	−	−	+	+	−	+
$\lvert W(f)^+ \cup W(i)\rvert$	−	+	−	−	+	−	+
$w_i/w_f = 0.5807$	−	+	+	−	+	+	+

$q = 0.55, p = 0.9, \alpha = 0.5, v = 0.1, w_f = 1.1, l_0 = 5, \sigma = 1.63.$

price of entrepreneurial output p is around two-thirds higher than the price of self-employed output q. The table shows the comparative statics of various parameter changes. The first five columns show the signs for small increases in the value of each of the parameters listed and last two columns give the effects of positive incremental shifts in the distributions shown.

The comparative statics signs shown in the table can be understood intuitively for any parameter by first considering the effect on first preferences and then on second preferences. Consider, for example, an increase in the self-employed price q. Because there are no agents on the borderline of choice between formal entrepreneurship and self-employment, this has no effect on set $E(f)$, and therefore none on $W(f)^+$. However, it causes a switch in first preferences away from formal employment and informal entrepreneurship towards self-employment, and so set $E_V(i)$ becomes smaller and S_V larger. The higher value of q also causes a shift towards self-employment as a sec-

ond preference, and so set S_I becomes larger, but $W(i)$ and $E_I(i)$ smaller. Although the supply of informal labour falls, the decrease in the demand dominates and so w_i falls. Overall, there are more self-employed and fewer entrepreneurs and wage workers. Similar explanations can be given for other parameter changes, but, for brevity, we focus on some potential policy tools.

First, consider changes in parameters w_f and l_0. l_0 can be regarded as a policy tool even if it is not fixed by government regulation; changes in the probability of detection of informality or in the penalties when caught would affect the informal employment level that entrepreneurs are willing to set. A lower w_f is a reduced cost of formality, while a lower l_0, limiting informal firm size further, can be interpreted as an increased cost of informality. We might expect each of these changes to result in less informality. Indeed, reducing w_f does cause $E(f)$ and $W(f)^+$ to become larger, while $E_V(i)$ and $W(i)$ become smaller. However, while a reduction in l_0 also causes $E(f)$ and $W(f)^+$ to expand and $W(i)$ to contract, the effect on informal activity is not clear-cut, for it expands $E_V(i)$ and $E_I(i)$. Thus, in this example, if the aim is to reduce informality, a reduction in w_f might be preferred. Nonetheless, although a reduction in the minimum wage rate is a shift towards the (static) first best, the reduction it causes in the number of informal entrepreneurs may also have an adverse longer-term effect if informal entrepreneurship provides a learning experience for some potential future formal entrepreneurs.

Second, the results are suggestive of the effects of different types of education/training. In Table 1 a general increase in skill Z expands both $E(f)$ and $E_V(i)$, as well as $W(f)^+$ and $W(i)$, while both S_V and S_I are diminished. However, a general increase in skill Y, with or without an associated increase

in Z, diminishes $E_V(i)$ and $W(i)$, while a switch occurs into S_V from S_I. Suppose that both general education and on-the-job training would increase the stock of Y, whereas specialist management training is required to increase the stock of Z. This suggests that, if the aim is to reduce informality then, because of the role played by voluntary self-employment, general education and on-the-job training is more effective.

5 Concluding Comments

We construct a parsimonious model that captures some of the complexity that obtains in urban labour markets in developing economies. In the benchmark version the labour market is unsegmented and agents may be in one of three states – self employment, wage employment and entrepreneurship. The second version of the model adds labour market segmentation. In equilibrium, voluntary and involuntary self-employment, formal and informal wage employment, and formal and informal entrepreneurship (the latter possibly dividing into voluntary and involuntary components) may all coexist. We also develop a diagrammatic interpretation of both the segmented and the unsegmented cases.

Our analysis suggests the importance of underlying macroeconomic conditions in determining the effects of education and training on transitions of individuals between labour market states, and that these transitions may be non-monotonic. The role of the rationing scheme by which workers are selected for formal jobs is also highlighted. As an illustration, a numerical example is developed that generates results that correspond closely to Latin American experience. In this example, if the government wishes to reduce

informality, reduction of the costs of formality is generally more effective than increasing the costs of informality, while education and training that improves the ability of individuals to produce and sell is more effective than increasing managerial skills.

These results are obtained from a highly stylized model, in particular from the assumption of a competitive labour market, the only imperfection being a minimum wage for formal employers with which there is full compliance. However, we conjecture that some alternative assumptions could be accommodated into our framework relatively easily. For example, partial compliance by formal firms with the minimum wage law would shift the boundary between formal and informal entrepreneurship to the south-west in Figure 2 (assuming that expected penalties for detection are not too large). The potential effects would include a substitution into formal from informal entrepreneurship and, as the informal wage would be driven up, there would also be a substitution out of informal entrepreneurship into voluntary and involuntary self-employment.

Similar effects would be obtained if trade unions were included in the model, with insiders and outsiders among formal-firm employees. However, a more radical overhaul of the analysis would be required to allow for decentralized bargaining between unions and employers, or for efficiency wages. For example, in a moral hazard or nutritional model it would have to be taken into account how worker productivity depends on wage rates. A more straightforward generalization that might be made would be to allow for heterogenous innate ability in wage work. Among other factors that might be included are free labour provided by the family, and wealth and liquidity

constraints that might hold back both self-employment and entrepreneurship.

Appendix

Proposition 1 Consider first the conditions under which wage employment is preferred. If $q/p > Q(w)$ then $C < B$. Since $A \leq y$, we have that $y < C(w) \Rightarrow A \leq y < C(w) < B(w)$; i.e., (6) is sufficient for (5) to be satisfied. If $q/p < Q(w)$ then $B(w) < C(w)$. To satisfy (5) and (6), we need either $y < B(w)$ or $y \in (B(w), C(w))$ and $z < B(w)$ (since, $A \leq z$, so that $z < B(w)$ is sufficient for $A < B(w)$).

Now consider the conditions under which self-employment is preferred. We have seen that $y > C(w) \Rightarrow US > UW$, so now consider what is required for $US > UE$. First, suppose $A = z$; then, from (7), $UE > US$ if $A > [D(w)]^{\alpha} y^{1-\alpha} \equiv \tilde{z}(y)$. For this to be consistent with $A = z$ we require $y \geq \tilde{z}(y)$. Note that, for $y > 0$, $\tilde{z}(y)$ has a unique fixed point, $\tilde{z}(D(w)) = D(w)$, and that $\tilde{z}'(y) = (1-\alpha)[D(w)]^{\alpha} y^{-\alpha} > 0$, so that $\tilde{z}'(D(w)) = 1-\alpha < 1$. Since also $\tilde{z}''(y) = -\alpha(1-\alpha)[D(w)]^{\alpha} y^{-\alpha-1} < 0$, this implies that $y \gtrless D(w) \Leftrightarrow y \gtrless \tilde{z}(y)$. Hence, if $y > D(w)$, we have $US > UE \Leftrightarrow z < \tilde{z}(y)$. Alternatively, suppose $A = y$. Then, from (7) $y < D(w) \Rightarrow US > UE$.

Hence $US > UE$ if either (i) $y < D(w)$ or (ii) $y > D(w)$ and $z < \tilde{z}(y)$. Therefore $US > \max(UW, UE)$ when either (i) $y \in (\min(C(w), D(w)), D(w))$ or (ii) $y > \max(C(w), D(w))$ and $z < \tilde{z}(y)$. But also, from (5), (7) and (8) we have that $B(w) - D(w) \gtrless 0 \Leftrightarrow q/p \lessgtr Q(w)$. Therefore, since $B(w) - C(w) \gtrless 0$ as $q/p \gtrless Q(w)$, we have $q/p \gtrless Q(w) \Leftrightarrow C(w) \gtrless B(w) \gtrless D(w)$, and the conditions stated in the proposition under which self-employment is preferred follow. The conditions under which entrepreneurship is preferred then follow.

Lemma 1 First we find from (2) and (6)-(7) that $\hat{l}_p > 0$; $\hat{l}_q = 0$; $\hat{l}_w < 0$; $\hat{l}_v = 0$; $B_p(w) < 0$; $B_q(w) = 0$; $B_w(w) > 0$; $B_v(w) < 0$; $C_p(w) = 0$; $C_q(w) < 0$; $C_w(w) > 0$; $C_v(w) < 0$; $D_p(w) < 0$; $D_q(w) > 0$; $D_w(w) > 0$; $D_v(w) = 0$; $\tilde{z}_p < 0$; $\tilde{z}_q > 0$; $\tilde{z}_w > 0$; $\tilde{z}_v = 0$. Using Proposition 1, first we specify the supply of individuals to wage employment and entrepreneurship:

$$L^s = \begin{cases} \int_{\underline{y}}^{C} \int_{\underline{z}}^{\bar{z}} f(y,z)dzdy & \text{for } q/p > Q(w); \\ \int_{\underline{y}}^{B} \int_{\underline{z}}^{\bar{z}} f(y,z)dzdy + \int_{B}^{C} \int_{\underline{z}}^{B} f(y,z)dzdy & \text{for } q/p < Q(w). \end{cases}$$

$$E^s = \begin{cases} \int_{D}^{\bar{y}} \int_{\tilde{z}(y)}^{\bar{z}} f(y,z)dzdy & \text{for } q/p > Q(w); \\ \int_{B}^{C} \int_{B}^{\bar{z}} f(y,z)dzdy + \int_{C}^{\bar{y}} \int_{\tilde{z}(y)}^{\bar{z}} f(y,z)dzdy & \text{for } q/p < Q(w). \end{cases}$$

Inserting $\hat{l}(A)$ into each double integral in E^s we obtain labour demand, L^d. Using $A \equiv \min(y, z)$, this can be written

$$L^d = \begin{cases} \int_{D}^{\bar{y}} \int_{\tilde{z}(y)}^{y} \hat{l}(z) f(y,z)dzdy + \int_{D}^{\bar{y}} \int_{y}^{\bar{z}} \hat{l}(y) f(y,z)dzdy & q/p > Q(w), \\ \int_{B}^{C} \int_{B}^{y} \hat{l}(z) f(y,z)dzdy + \int_{B}^{C} \int_{y}^{\bar{z}} \hat{l}(y) f(y,z)dzdy & \\ \quad + \int_{C}^{\bar{y}} \int_{\tilde{z}(y)}^{y} \hat{l}(z) f(y,z)dzdy + \int_{C}^{\bar{y}} \int_{y}^{\bar{z}} \hat{l}(y) f(y,z)dzdy & q/p < Q(w). \end{cases}$$

Differentiating L^s and L^d by (p, q, w, v) and using these inequalities, the lemma is obtained.

Proposition 2 From (9), $w = \underline{w} \Rightarrow W = \varnothing$; $w = \overline{w} \Rightarrow E = \varnothing$. Therefore, $L^s(\underline{w}) = 0$; $L^d(\overline{w}) = 0$. If $w = \underline{w}$, $UE\left(\overline{A}, \underline{w}\right) > US\left(\overline{A}\right) \Rightarrow E \neq \varnothing \Rightarrow L^d > 0$; and if $w = \overline{w}$, $UW(\overline{w}) > US\left(\underline{y}\right) \Rightarrow W \neq \varnothing \Rightarrow L^s > 0$. It follows that, if both $UE\left(\overline{A}, \underline{w}\right) > US\left(\overline{A}\right)$ and $UW(\overline{w}) > US\left(\underline{y}\right)$ the excess demand functions satisfy $L^d(\underline{w}) - L^s(\underline{w}) > 0$ and $L^d(\overline{w}) - L^s(\overline{w}) < 0$. Then, by the continuity of $L^d - L^s$, there must exist a $w^* \in (\underline{w}, \overline{w})$ such that $L^d(w^*) - L^s(w^*) = 0$. Additionally, if $w = \underline{w}$ then $UE(\underline{A}, \underline{w}) < US(\underline{A}) \Rightarrow S \neq \varnothing$, and if $w = \overline{w}$ then $UW(\overline{w}) < US(\bar{y}) \Rightarrow S \neq \varnothing$. The proposition follows.

Lemma 2 Writing labour supply and supply as $L^s(w, i)$ and $L^d(w, i)$, respectively, where $i = (p, q, v)$, when $w = w^*$, $dw/di = (L_i^d - L_i^s)/(L_w^s - L_w^d)$. Using Lemma 1 with this equation yields $dw/dp > 0$, $dw/dq \gtreqless 0$ and $dw/dv > 0$. Thus, (i) $dL/dp = L_w^d(dw/dp) + L_p^d = (L_p^d L_w^s - L_w^d L_p^s)/(L_w^s - L_w^d)$; from Lemma 1, $L_w^d - L_w^s < 0$ and if $q/p > Q(w)$, $L_p^s = 0$ and the result for dL/dp follows; (ii) $dL/dq = (L_q^d L_w^s - L_w^d L_q^s)/(L_w^s - L_w^d) < 0$; (iii) $dL/dv = (L_v^d L_w^s - L_w^d L_v^s)/(L_w^s - L_w^d)$ and the result in the lemma follows.

Borderline Preferences with Labour Market Segmentation To compare the utilities from the different activities we use (11) and (12).

Self employment versus wage employment. Since $w_f > w_i$, $UW_f > UW_i$. Thus, to consider first preferences, we compare US with UW_f. If the agent is rationed out of a formal job, second preferences matter, so we compare US with UW_i. Thus we obtain

$$US \gtreqless UW_j \text{ as } y \gtreqless \frac{1}{q}(w_j - v) \equiv C(w_j), \quad j = f, i.$$

Since $w_f > w_i$, $C(w_f) > C(w_i)$.

Entrepreneurship versus self employment. As an entrepreneur, an individual chooses formality if $A > \tilde{A}$, but informality otherwise. This gives two comparisons with self employment:

$$UE_j \gtreqless US \text{ as } A \gtreqless \frac{1}{p\hat{l}_j^{\alpha}}(qy + w_j \hat{l}_j) \equiv \tilde{z}_j(y), \quad j = f, i.$$

As in Section 2, denote the fixed points of $\tilde{z}_j(y)$ as $D(w_j)$, $j = f, i$; i.e., $\tilde{z}_j(D(w_j)) = D(w_j)$.

Entrepreneurship versus wage employment. With respect to the agent's first preference, we compare UE_i with UW_f, and if the agent is rationed out of a formal job, we compare UE_i with UW_i:

$$UE_i \gtreqless UW_j \text{ as } A \gtreqless \frac{1}{pl_i^{\alpha}}(w_j + w_i l_i - v) \equiv B(w_i, w_j), \quad j = f, i.$$

where the first argument of $B(.,.)$ is the wage paid as an entrepreneur and the second argument is the wage received as an employee.

We can now define corresponding values of $Q(.,.)$. $B(w_i, w_j) - C(w_j) \gtrless 0$ as $q/p \gtrless Q(w_i, w_j)$,where

$$Q(w_i, w_j) \equiv \frac{w_j - v}{l_i(w_j + w_i l_i^{\alpha} - v)}, \quad j = f, i.$$

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Figures

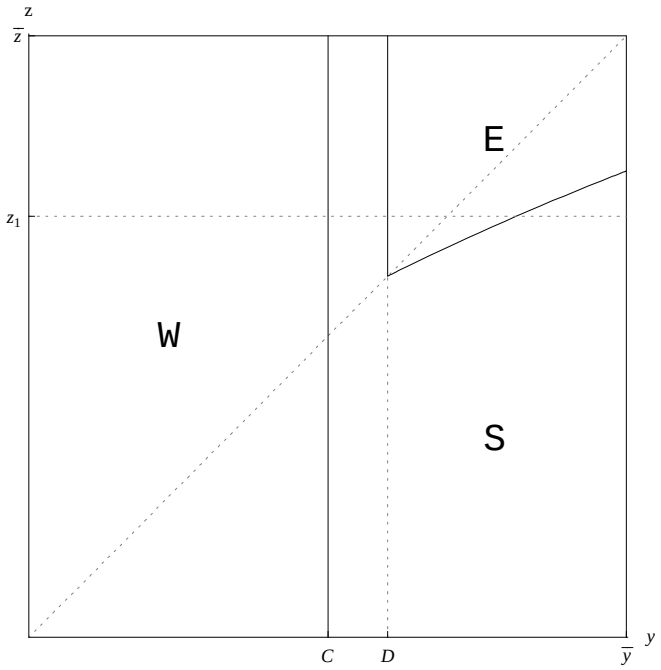

Figure 1(a): Labour allocation for $q/p > Q(w)$

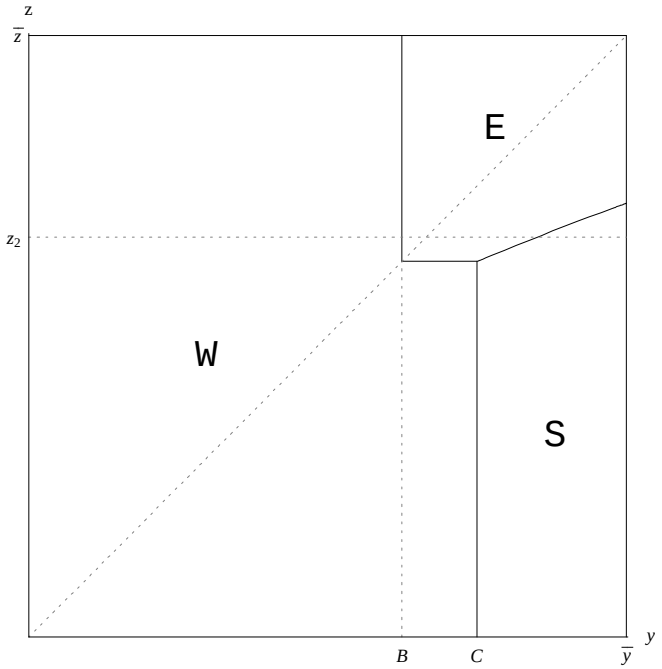

Figure 1(b): Labour allocation for $q/p < Q(w)$

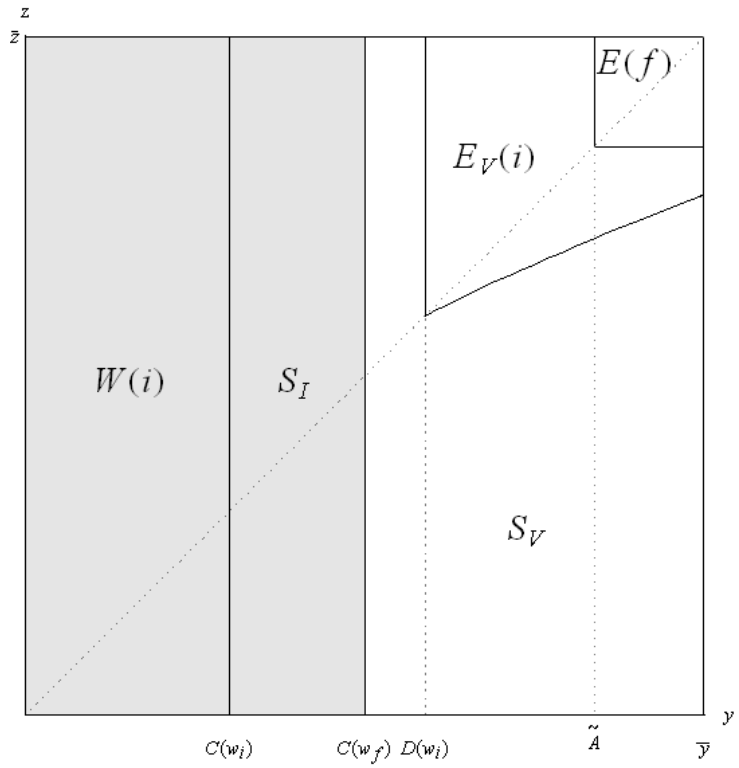

Figure 2(i): Labour allocation for $q/p > Q(w_i, w_f) > Q(w_i, w_i)$. Shaded area denotes the set $W(f)$.

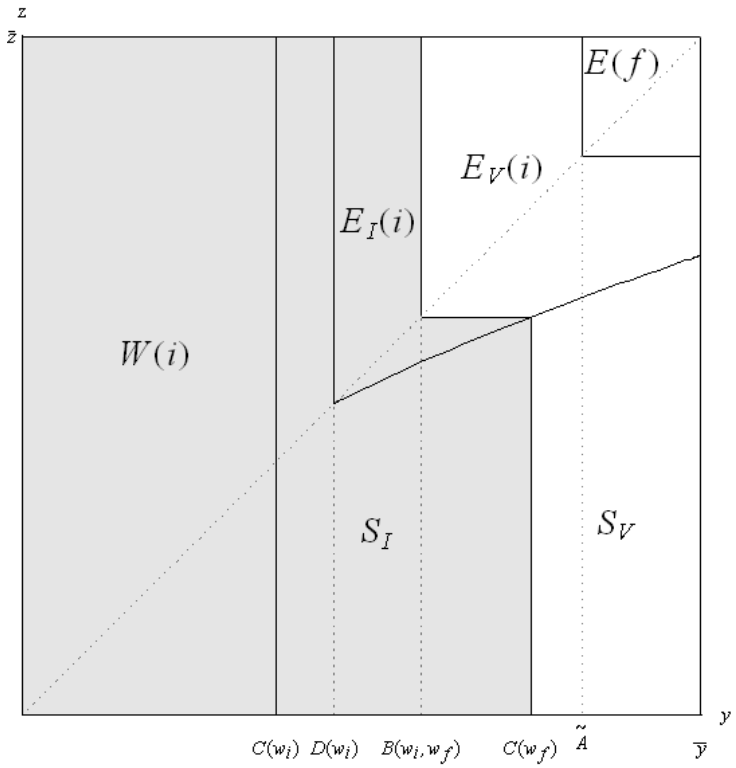

Figure 2(ii): Labour allocation for $Q(w_i, w_f) > q/p > Q(w_i, w_i)$. Shaded area denotes the set $W(f)$.

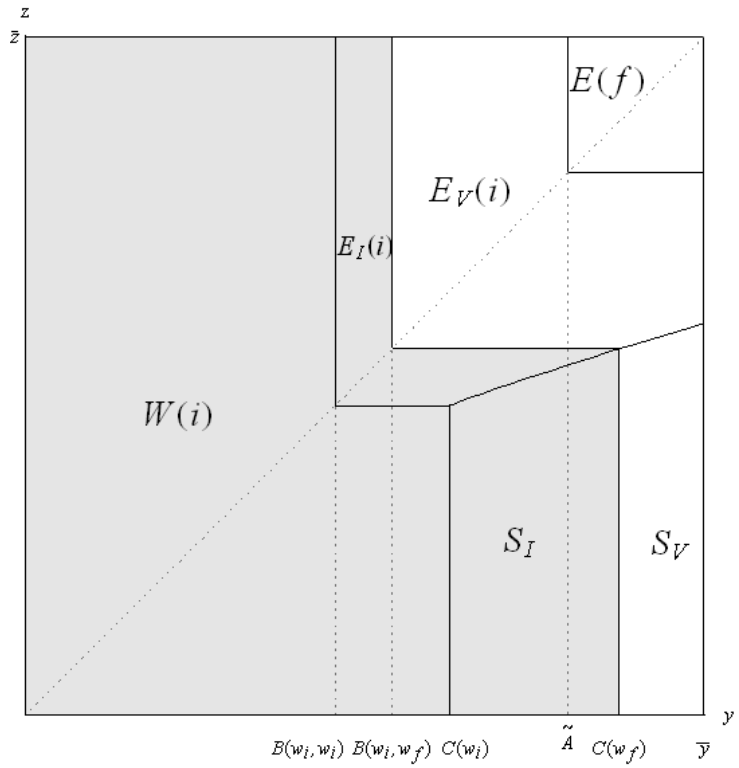

Figure 2 (iii): Labour allocation for $Q(w_i, w_f) > Q(w_i, w_i) > q/p$. Shaded area denotes the set $W(f)$.