FRACTIONAL COINTEGRATION AND AGGREGATE MONEY DEMAND FUNCTIONS

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Abstract

This paper examines aggregate money demand relationships in five industrial countries by employing a two-step strategy for testing the null hypothesis of no cointegration against alternatives which are fractionally cointegrated. Fractional cointegration would imply that, although there exists a long-run relationship, the equilibrium errors exhibit slow reversion to zero, i.e. that the error correction term possesses long memory, and hence deviations from equilibrium are highly persistent. It is found that the null hypothesis of no cointegration cannot be rejected for Japan. By contrast, there is some evidence of fractional cointegration for the remaining countries, i.e., Germany, Canada, the US, and the UK (where, however, the negative income elasticity which is found is not theory-consistent). Consequently, it appears that money targeting might be the appropriate policy framework for monetary authorities in the first three countries, but not in Japan or in the UK.

Keywords: Money Demand, Velocity, Fractional Integration, Fractional Cointegration

JEL Classification: C32, C22, E41

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1.   Introduction

The existence of a stable long-run relationship linking real money balances to real income and interest rates has been extensively investigated, as it would provide support for money targeting as a policy strategy for monetary authorities. Whilst early empirical studies seemed to suggest that a log-linear equation of this kind exhibited stability (see, e.g., Goldfeld, 1973), subsequently it became apparent that the relationship had broken down, both in the US and in the UK, possibly as a result of policy changes. Goldfeld and Sichel (1990) concluded that the existing empirical models suggested instability in the money demand function (see Goodhart, 1989).

The claim that a stable money demand exists, more specifically that real M1, real income and short-term interest rates are cointegrated, or, alternatively, that velocity is a stationary variable (the two statements being equivalent, as long as interest rates are I(0)), has resurfaced in some recent studies. A typical example is the work of Hoffman, Rasche and Tieslau (1995) and Hoffman and Rasche (1996), who, using the Johansen (1988, 1991) procedure, find cointegrated long-run demand functions for narrowly defined money (M1) in five industrial countries including the UK.

This result is in contrast to some of the stylised facts about money demand. Consider, for instance, the case of the UK (see Goodhart, 1989). Existing empirical studies generally reach the conclusion that monetary aggregates are not cointegrated with nominal income[1]; however, cointegration can be achieved by adding other variables, such as wealth, financial innovation variables or cumulated interest rates, to a standard money demand function (see, e.g., Hall *et al*. 1990, Hendry and Ericsson, 1991, 1993). This is because velocity is highly trended, and only by including a similar effect is it possible to build a well-balanced equation. There is a very clear downward trend until 1981, after which year an increasingly important component of M1 began to earn interest and so M1 began to behave more like broad money. Then this is followed by a clear

reversal and a strong upward trend. Even if one ignores this break in definition it is not clear that velocity is a stationary process.

Caporale et al (2001) argue that the test procedure in Hoffman et al (1995) and Hoffman and Rasche (1996) is crucially affected by the specification of the VAR, and show that when an adequate VAR can be achieved the assumption of cointegration is no longer supported. Furthermore, where there are important structural breaks which are not adequately dealt with it may be impossible to achieve correct inference from this procedure.

In this paper we provide further evidence on whether or not there exist stable money demand functions by using fractional integration and cointegration techniques. The motivation is the following. Earlier studies rely on standard cointegration analysis, and either conclude that all variables are I(1) and the equilibrium errors are an I(0) process, which is not persistent (e.g., Hoffman et al, 1995), or, alternatively, that they are I(1) (e.g., Friedman and Kuttner, 1992). However, the discrete options I(1) and I(0) offered by this type of analysis are rather restrictive, as adjustment to equilibrium might take a longer time than suggested by standard cointegration tests if, in fact, the equilibrium errors respond more slowly to shocks, which results in highly persistent deviations from equilibrium. In other words, real money balances, real income and nominal interest rates might be tied together through a fractionally integrated I(d)-type process such that the equilibrium errors are I(d), with $d < 1$, and exhibit slow reversion to zero. Consequently, it is possible that a failure to identify a long-run equilibrium consistent with money demand theory simply reflects the adoption of a classical cointegration framework. Therefore, we adopt a testing procedure which allows for the possibility of a long-memory cointegrating relationship, and which enables us to gain a better understanding of money demand.

Specifically, we test the null hypothesis of no cointegration against alternatives which are fractionally cointegrated using the two-step strategy presented in Gil-Alana (2003) and Caporale

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[1] For a survey of the evidence, see Goodhart (1989) and also Temperton (1991).

and Gil-Alana (2004). By applying this methodology and using data for five major industrial countries (namely, Canada, US, Japan, Germany, and UK), we find mixed results. In particular, the null hypothesis of no cointegration cannot be rejected for Japan, but there is some evidence of fractional cointegration for Germany, Canada, the US, and the UK (where, however, the income elasticity is found to be negative). In such cases, although there exists a long-run relationship, the error correction term may possess long memory, which means that deviations from equilibrium are highly persistent. Hence, our findings are to some extent, but not wholly, consistent with the conclusions reached by other studies of the behaviour of monetary aggregates in the industrial countries (see, e.g., Caporale et al, 2001), and suggest that money targeting might be the appropriate policy framework for monetary authorities in some countries (where a stable relationship appears to exist), though not in some others (where instability prevails, or elasticities are not theory-consistent).[2]

The layout of the paper is the following. Section 2 briefly describes the concepts of fractional integration and cointegration, and the procedure adopted for testing the null hypothesis of no cointegration against fractionally cointegrated alternatives. The empirical results are presented in Section 3. Finally, Section 4 offers some concluding remarks.

2. Testing for fractional integration and cointegration

As already mentioned, in studies relying on standard cointegration analysis the equilibrium errors are restricted to be an I(0) process, which is not persistent. However, it might be the case that the equilibrium errors respond more slowly to shocks, which results in highly persistent deviations from equilibrium. The testing procedure described below allows for the possibility of such long-memory cointegrating relationships.

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[2] Some recent papers investigate possible instabilities as well as nonlinearities using the smooth transition regression techniques developed by, e.g., Granger and Terasvirta (1993) and Lin and Terasvirta (1994), and tend to find stable linear relationships (see, e.g., Wolters and Lütkepohl, 1997, and Wolters, Teräsvirta and Lütkepohl, 1998).

For the purpose of the present paper, we define an I(0) process $\{u_t, t = 0, \pm1, \ldots\}$, as a covariance stationary process with spectral density that is positive and finite at zero frequency. In this context, an I(d) process, $\{x_t, t = 0, \pm1, \ldots\}$, is defined by:

$$(1 - L)^d \, x_t \; = \; u_t, \quad t = 1, 2, \ldots , \tag{1}$$

$$x_t = 0 \qquad t \leq 0,^3 \tag{2}$$

where L is the lag operator and u_t is I(0). The macroeconomic literature focuses on the cases $d = 0$ and $d = 1$ (see, e.g., Nelson and Plosser, 1982), whereas we define $(1 - L)^d$ for all real d by:

$$(1 - L)^d \; = \; \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} L^k \; = \; 1 \; - \; d\,L \; + \; \frac{d\,(d-1)}{2} L^2 \; - \; \frac{d\,(d-1)\,(d-2)}{6} L^3 \; + \; \ldots .$$

The process u_t in (1) could be a stationary and invertible ARMA sequence, with an exponentially decaying autocovariance function. This property can be said to characterise a "weakly autocorrelated" process. When $d = 0$, $x_t = u_t$, so a "weakly autocorrelated" x_t is allowed for. When $d = 1$, x_t has a unit root, while for a general integer d, x_t has d unit roots. For $0 < d < 0.5$, x_t is still covariance stationary, but its lag-j autocovariance γ_j decreases very slowly, like the power law j^{2d-1} as $j \to \infty$, and so the γ_j are non-summable. The distinction between I(d) processes with different values of d is also important from an economic point of view: if a variable is an I(d) process with $d \in [0.5, 1)$, it will be covariance nonstationary but mean-reverting, since an innovation will have no permanent effect on its value. This is in contrast to an I(1) process, which will be both covariance nonstationary and non-mean-reverting, in which case the effect of an innovation will persist forever.

Robinson (1994a) proposes Lagrange Multiplier (LM) tests for testing unit roots and other nonstationary hypotheses, embedded in fractional alternatives. A very simple version of his tests consists in testing the null hypothesis:

$$H_o: \; \theta \; = \; 0 \tag{3}$$

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[3] For an alternative definition of fractional integration (the type I class), see Marinucci and Robinson (1999).

in the model

$$(1 - L)^{d + \theta} x_t = u_t, \qquad t = 1, 2, \dots, \qquad (4)$$

where x_t is the time series we observe; u_t is I(0), and d is a given value that may be 1 but also any other real number. The functional form of the test statistic ($\hat{R}$) is given in Appendix A.

Robinson (1994a) showed that under certain regularity conditions:

$$\hat{R} \rightarrow_d \chi_1^2, \qquad as \quad T \rightarrow \infty. \qquad (5)$$

Consequently, unlike in other procedures, we are in a classical large-sample testing situation: an approximate one-sided $100\alpha\%$ level test of H_o (3) against the alternative: H_a: $d > d_o$ ($d < d_o$) will be given by the rule: "Reject H_o if $\hat{r} = \sqrt{\hat{R}} > z_\alpha$ ($\hat{r} < - z_\alpha$)", where the probability that a standard normal variate exceeds z_α is α.

Having defined fractional integration and described a way of testing I(d) statistical models, next we introduce the concept of fractional cointegration. By adopting the simplest possible definition, it can be said that a given vector X_t is fractionally cointegrated if:

a)   all its components (X_{it}) are integrated of the same order (say d), i.e.,

$$(1 - L)^d x_{it} = u_{it}, \quad t = 1, 2, \dots, \text{ for all } i,$$

with I(0) u_i's, and

b)   there is at least one linear combination of these components which is fractionally integrated of order b, with $b < d$.[4]

We use here a two-step procedure suggested in Gil-Alana (2003) and Caporale and Gil-Alana (2004) for testing the null hypothesis of no cointegration against the alternative of fractional cointegration. In the first step, Robinson's (1994a) tests are used to test the order of integration of each of the individual series and, if all are integrated of the same order (say d), as a second step, the degree of integration of the residuals from the cointegrating regression is tested. In standard cointegration analysis (where cointegration of order 1,0 is considered), Stock (1987),

Phillips (1991) and Johansen (1991) showed that the least squares estimate of the cointegrating parameter was consistent and converged in probability at the rate $T^{1-\delta}$ for any $\delta > 0$, rather than the usual $T^{1/2}$. In the context of fractional integration, the estimation of the parameter of the cointegrating relationship is more complex, and crucially depends on the values of d and b. Thus, if d < 0.5, the least squares estimate (LSE) produces an inconsistent estimate. To solve this problem, Robinson (1994b) showed that a narrow-band frequency-domain least squares estimate (NBLSE) is consistent, and the same method was subsequently studied by Robinson and Marinucci (2001b) in the case of d > 0.5. Here, the LSE is consistent, with the convergence rate depending on the specific locations of d and b, though the NBLSE still sometimes converges faster.[5] The present paper, however, focuses on cases where the individuals series are I(1), and, given the consistency of the LS estimators in our case, we can use Robinson's (1994a) univariate tests for testing the integration order of the equilibrium errors. A difficulty in this context is that the residuals are not actually observed but obtained from the cointegrating regression, and thus there might be a bias in favour of stationary residuals.[6] In order to solve this problem, finite-sample critical values of the tests will be computed in the next section. We can consider the model,

$$(1 - L)^{d+\theta} e_t = v_t, \qquad t = 1, 2, \ldots \qquad (6)$$

where e_t are the OLS residuals from the cointegrating regression and I(0) v_t, and test H_o (3) against the one-sided alternative:

$$H_a: \quad \theta < 0. \qquad (7)$$

Note that if we cannot reject H_o (3) on the estimated residuals, we will find evidence of no cointegration, since the residuals will be integrated of the same order as the univariate series. On

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[4] A more general definition of fractional cointegration, allowing different integration orders for each series, can be found, for example, in Robinson and Marinucci (2001a).

[5] For related results on fractional cointegration, see Dolado and Marmol (1997), Jeganathan (1999, 2001), Kim and Phillips (2000), Robinson and Hualde (2002, 2003), Robinson and Yajima (2002), etc.

[6] Note that this problem is similar to the one encountered by Engle and Granger (1987) when testing cointegration with the tests of Fuller (1976) and Dickey and Fuller (1979).

the other hand, rejections of H_o (3) against (8) will provide evidence of cointegration of a certain degree, since the residuals will be integrated of a smaller order than that of the individual series.

3. An application to aggregate money demand relationships

The existence of long-run stable money demand relationships is examined in this section applying fractional integration and cointegration techniques. In particular, we use the methodology described in Section 2, testing initially for the order of integration of the individual series, and then for the possibility of their being fractionally cointegrated.

Money is measured by M1 for all countries except for the UK where M0 is used instead (since in UK M1 there is a break in 1981, as already discussed, and another one in 1992, owing to a redefinition of the series to include deposits at Building Societies). The CPI is used to obtain real money balances and also to construct real income by deflating the nominal GDP series. Finally, the 3-month Treasury bill rate is used as the nominal (short-term) interest rate. All series are quarterly and seasonally adjusted, and were obtained from Datastream. The sample period is 60q1-04q2 for Canada; 70q1-03q4 for Germany; 70q1-04q1 for the UK, and 70q1-04q2 for the US and Japan.

We start the analysis by carrying out unit roots tests on the individual series. In Table 1, the results from the tests of Robinson (1994a), specifically designed against fractional alternatives, are reported. In addition, in Table 2, the Dickey and Fuller's (1979, 1981) test statistics (designed against autoregressive alternatives) are also presented.

Specifically, Table 1 reports values of the one-sided statistic $\hat{r}$ in Appendix A, in a model given by (4) with d = 1, for different types of disturbances. In particular, we model u_t in (4) as white noise and AR processes of form:

$$u_t = \sum_{j=1}^{q} \phi_j u_{t-js} + \varepsilon_t, \qquad t = 1, 2, \ldots$$

with s = 1 (non-seasonal) and s = 4 (seasonal), and q = 1 and 2. Higher AR orders were also considered obtaining similar results. A first glance at this table indicates that the unit root hypothesis is almost never rejected for any series and any country. The main exception is the interest rate for Japan, where the unit root null is rejected with autocorrelated disturbances but it is not if u_t is white noise. The null is also rejected in some cases for money and income in Japan, but in general it cannot be rejected for most types of disturbances.

(Tables 1 and 2 about here)

The results from the Dickey-Fuller tests on the same univariate series are similar, with the unit root hypothesis possibly being rejected only in the case of Canadian income. On the whole, the results in these two tables provide strong evidence in favour of the presence of unit roots in the three series for the five countries examined.[7]

As a final step to verify that all series are truly nonstationary I(1), we also applied a semiparametric estimation procedure (Robinson, 1995) in order to determine the order of integration of the series. This method is basically a 'Whittle estimator' in the frequency domain, considering a band of frequencies that degenerates to zero. The explicit form of the estimator is described in Appendix B.

(Insert Figure 1 about here)

Figure 1 displays the estimates of the order of integration for each of the series and each country for a range of values of m (a bandwidth parameter, see Appendix B) from 2 to T/3. It also shows the 95% confidence interval corresponding to the I(1) case. We observe that even though some of the estimates are above the I(1) interval, most of them are within it, implying the existence of a unit root in all cases.

Before investigating the possibility of the three series being fractionally cointegrated we compute, in Table 3, finite-sample critical values of the tests of Robinson (1994a) for testing the null hypothesis of no cointegration against fractionally cointegrated alternatives. In doing so, we

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follow the pioneering work by Cheung and Lai (1993). Alternatively, we could have used other approaches, based, for example, on bootstrap (Davidson, 2002); asymptotic theory (Hassler and Breitung, 2002) or multivariate analysis (Breitung and Hassler, 2002). However, given the empirical focus of the present study, for our purposes it is sufficient to rely on finite-sample critical values obtained by simulation.

We consider samples of size equal to 50, 100, and to those used in the empirical application, that is, 136 (for Germany), 138 (for Japan, UK and US), and 178 (for Canada), and carry out Monte Carlo simulations based on 50,000 replications. The true system is assumed to consist of three I(1) series which are not cointegrated, and Robinson's (1994a) tests are then performed for white noise, seasonal and non-seasonal AR disturbances.

(Table 3 about here)

Table 3 reports the critical values for testing H_o (3) against (8) on the residuals from the cointegrating regression at the 5% significance level. It appears that the finite-sample critical values are smaller than those for the Normal distribution, which is consistent with the earlier discussion according to which, when testing H_o (3) against (7), the use of standard critical values will result in the cointegrating tests rejecting the null hypothesis of no cointegration too often.

Table 4 contains the OLS estimates of the coefficients of the cointegrating regressions for each country. We have regressed real money balances against nominal (short-term) interest rates and real income and, consistently with theory, the former has negative coefficients while the latter positive ones, the UK being the only exception.[8]

(Tables 4 – 5 and Figure 2 about here)

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[7] Other procedures such as KPSS (Kwiatkowski et al., 1992) were also performed and evidence was again found in favour of unit roots.

[8] We have adopted the same specification for the money demand equation as in Hoffman et al (1995), and Hoffman and Rasche (1996), which are representative studies claiming to have found evidence of classical cointegration. One should note, though, that such a specification is often criticised. For instance, some studies emphasise the importance of including money's own rate as the opportunity cost (see, e.g. Baba et al, 1992) or inflation, which represents the costs of not holding goods, and can also be important if short- and long-run elasticities with respect to prices are not the same (see Rose, 1985). Further, in some cases the long-term rather than the short-term interest rates is employed (see, e.g., Lucas, 1988), where the former could be seen as a proxy for expected average future short rates which drive demand in transaction cost models.

The results from Robinson's (1994a) tests on the residuals from the cointegrating regression discussed above are displayed in Table 5. They correspond to the one-sided statistic given by $\hat{r}$ in Appendix A, in a model given by (6) with d = 1, testing H_o (3) against (7) with different types of I(0) disturbances. We find that the null hypothesis of I(1) residuals cannot be rejected for Japan for any type of disturbances. However, for the remaining countries, there is at least one case where H_o is rejected in favour of fractional cointegrated alternatives. For instance, for the UK, fractional cointegration is found if the disturbances are white noise but also if they follow a non-seasonal AR(1) process, while this hypothesis is rejected for the remaining types of disturbances. The same happens for Canada and the US, but now fractional cointegration is also found if the disturbances are seasonal AR(1). Finally, for Germany, the null hypothesis of I(1) residuals is rejected in favour of I(d) with d < 1 if the disturbances are white noise. Moreover, we also performed the tests for the null of d = 0 on these estimated residuals, and the null hypothesis was rejected for all types of disturbances in all countries in favour of alternatives with d > 0, implying lack of cointegration at least in its classical sense (these results are not reported for brevity). As a final step, we also estimate the order of integration of the estimated residuals from the long run relationships, using the Whittle estimation method of Robinson (1995). Note that this should be seen as a simple illustration, since formal statistical inference would require calculating appropriate critical values. Figure 2 suggests that virtually all the estimates are below unity, the only exception being the series for Japan. Thus, we can conclude that there is some degree of fractional cointegration at least for the UK, the US, Canada and Germany.

As the existence of a stable and theory-consistent demand for money is a prerequisite for effective money targeting, it appears that this monetary policy framework would be suitable for Canada, the US and Germany, but not for the UK or Japan. In fact, following the example set by the Bundesbank and the Federal Reserve, monetary targets were popular in the 1980s, when they were often used to combat inflation. However, financial deregulation, which made monetary aggregates less stable, and a growing body of evidence that the links between money growth and

inflation are imprecise, led many central banks to switch to inflation targets in the 1990s (e.g., Canada, Japan and the UK of the countries being analysed here), whilst the Fed now takes into account various indicators when setting interest rates, and the newly created ECB had adopted a two-pillar strategy, one being a money growth reference target, the other being defined with reference to asset prices etc.[9] One advantage of simply and clearly defined targets is the resulting transparency and accountability of the central bank (see Leiderman and Svensson, 1995), but their effectiveness requires the transmission mechanism of monetary policy to be stable, a condition which, as our findings indicate, is not always satisfied. A more pragmatic policy strategy might be more effective in such cases.

4.   Conclusions

This paper has investigated whether aggregate money demand relationships can be characterised as fractionally cointegrated I(d)-type processes. The analysis has been carried out for five industrial countries using the two-step testing strategy for fractional cointegration described in detail in Gil-Alana (2003) and Caporale and Gil-Alana (2004). This procedure is based on Robinson's (1994a) univariate tests, and it involves initially testing the order of integration of the individual series, and then testing the degree of integration of the estimated residuals from the cointegrating regression. These tests have higher power than other alternative tests for the null of no cointegration against fractional alternatives.

The results for Germany, Canada, the US and the UK indicate the possibility of a fractionally cointegrated relationship, consistently with the claim made in Hoffman et al (1995) and Hoffman et al (1996) that a stable money demand exists. These findings have important policy implications, as the crucial condition for the adoption of a money targeting regime is the existence of a stable demand for money as well as theory-consistent elasticities. Therefore, money targeting would appear to be a suitable monetary policy framework for the first three

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[9] For more details on the successful performance of inflation targeting regimes, see Arestis, Caporale and Cipollini

countries, but not in either the UK, where the income elasticity is estimated to be negative, or Japan, where the relationship is not stable, implying that the standard transmission mechanisms of monetary policy cannot be relied upon. The Japanese case could be interpreted in terms of a "liquidity trap" (see Krugman, 1999), i.e. a situation where money and bonds become perfect substitutes as the lower zero bound on interest rates has already been reached; this causes a fundamental shift in the equilibrium money demand relationship, with money demand becoming indeterminate and conventional monetary policies being ineffective to raise output or prices. In cases such as the Japanese or UK ones alternative policy settings, for instance inflation or exchange rate targeting (see, e.g., Goodfriend, 1991), or pragmatic strategies based on a variety of indicators, might be more appropriate, although policy frameworks which reduce the transparency and accountability of central banks can unsettle financial markets and increase the cost of borrowing (see Svensson, 1999).

Further issues to be analysed are possible structural breaks and other types of instabilities and nonlinearities in the context of fractional cointegration. Future research will focus upon them.

(2002).

Appendix A

The score test statistic proposed by Robinson (1994a) is:

$$\hat{R} = \hat{r}^2; \quad \hat{r} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2},$$

where T is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j)$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 - \sum_{j=1}^{T-1}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1} \sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau}\log g(\lambda_j; \hat{\tau}),$$

where $I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1 - L)^d x_t$, evaluated at $\lambda_j = 2\pi j/T$, and g is a known

function related to the spectral density function of $\hat{u}_t$: $f(\lambda_j; \tau) = \dfrac{\sigma^2}{2\pi} g(\lambda_j; \tau)$, with $\hat{\tau}$ obtained

by minimising $\sigma^2(\tau)$.

Appendix B

Robinson (1995) proposed a 'Whittle estimate' in the frequency domain, considering a band of

frequencies that degenerates to zero. The estimate is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2d \frac{1}{m}\sum_{j=1}^{m}\log \lambda_j \right),$$

$$\overline{C(d)} = \frac{1}{m}\sum_{j=1}^{m} I(\lambda_j) \lambda_j^{2d}, \qquad \lambda_j = \frac{2\pi j}{T}, \qquad \frac{m}{T} \to 0.$$

Under finiteness of the fourth moment and other mild conditions, Robinson (1995) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \to_d N(0, 1/4) \quad as\ T \to \infty,$$

where d_o is the true value of d and with the only additional requirement that m $\rightarrow \infty$ slower than

T. Robinson (1995) shows that m must be smaller than T/2 in order to avoid aliasing effects.

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TABLE 1

Testing for unit roots with the tests of Robinson (1994a)

CANADA

Series / u_t	White noise	AR(1)	AR(2)	Seasonal AR(1)	Seasonal AR(2)
Income	**-0.46'**	**-0.13'**	**-0.76'**	**-0.45'**	**-0.60'**
Interest rate	**0.92'**	-2.18	**-0.61'**	**1.07'**	**1.12'**
Money	**-1.17'**	**0.10'**	**-1.07'**	**-1.28'**	**-0.81'**

UNITED STATES

Series / u_t	White noise	AR(1)	AR(2)	Seasonal AR(1)	Seasonal AR(2)
Income	**-1.02'**	**-0.48'**	**-0.44'**	**-0.84'**	**-0.80'**
Interest rate	**0.11'**	-2.08	**-0.27'**	**0.12'**	**0.04'**
Money	**-0.38'**	**-0.25'**	**-0.93'**	**-0.60'**	**-0.63'**

UNITED KINGDOM

Series / u_t	White noise	AR(1)	AR(2)	Seasonal AR(1)	Seasonal AR(2)
Income	**-0.76'**	**-0.68'**	**0.65'**	**-0.91'**	**-0.83'**
Interest rate	**0.94'**	**-1.45'**	**-1.25'**	**0.98'**	**1.00'**
Money	**-0.30'**	**-0.37'**	**-0.41'**	**-0.44'**	**-0.44'**

JAPAN

Series / u_t	White noise	AR(1)	AR(2)	Seasonal AR(1)	Seasonal AR(2)
Income	**-0.34'**	**-0.49'**	-2.07	**-0.32'**	**-0.45'**
Interest rate	**1.47'**	-4.97	-2.64	2.58	2.22
Money	-2.03	**-1.15'**	**-1.19'**	**-0.55'**	**0.11'**

GERMANY

Series / u_t	White noise	AR(1)	AR(2)	Seasonal AR(1)	Seasonal AR(2)
Income	**-0.13'**	**0.86'**	**0.46'**	**-0.20'**	**-0.21'**
Interest rate	2.98	**-1.32'**	**-0.63'**	2.74	2.68
Money	**-0.34'**	**-0.35'**	**-0.48'**	**-0.26'**	**-0.25'**

' and in bold: Non-rejection values of the null hypothesis of a unit root at the 95% significance level. We test H_o (3) in (4) with d = 1.

TABLE 2

Testing for unit roots with the tests of Fuller (1976) and Dickey & Fuller (1979, 1981)

CANADA

Series / u_t	k = 0	k = 1	k= 2	k = 3	k = 4
Income	-3.71'	-3.13'	**-2.62'**	-2.99'	-2.96'
Interest rate	**-1.65'**	**-2.27'**	**-1.97'**	**-2.09'**	**-1.94'**
Money	**-0.40'**	**- 0.09'**	**-0.18'**	**0.001'**	**-0.40'**

UNITED STATES

Series / u_t	k = 0	k = 1	K = 2	k = 3	k = 4
Income	**-0.37'**	**-0.58'**	**-0.60**	**-0.90**	**-0.28**
Interest rate	**-1.37'**	**-1.87'**	**-1.21'**	**-1.94'**	**-1.73'**
Money	**-0.97'**	**0.50'**	**0.19**	**1.09**	**-0.43**

UNITED KINGDOM

Series / u_t	k = 0	k = 1	k = 2	k = 3	k = 4
Income	**-0.28'**	**-0.13**	**0.22**	**0.31**	**0.07**
Interest rate	**-1.67**	**-2.34**	**-2.28**	**-2.17**	**-2.15**
Money	**-1.79**	**-1.23**	**-0.92**	**-0.37**	**-1.69**

JAPAN

Series / u_t	k = 0	k = 1	k = 2	k = 3	k = 4
Income	-2.91	-3.09	**-2.27**	**-2.11**	**-1.93**
Interest rate	**-1.08**	**-2.14**	**-2.42**	**-1.99**	**-1.96**
Money	**0.34**	**0.93**	**1.03**	**-2.14**	**-0.44**

GERMANY

Series / u_t	k = 0	k = 1	k = 2	k = 3	k = 4
Income	**-1.20**	**-0.90**	**-0.88'**	**-0.81'**	**-0.82'**
Interest rate	**-1.06'**	**-2.12'**	**-2.19'**	**-2.06'**	**-2.64**
Money	**-0.31'**	**0.51'**	**0.49'**	**-1.55'**	**-0.09'**

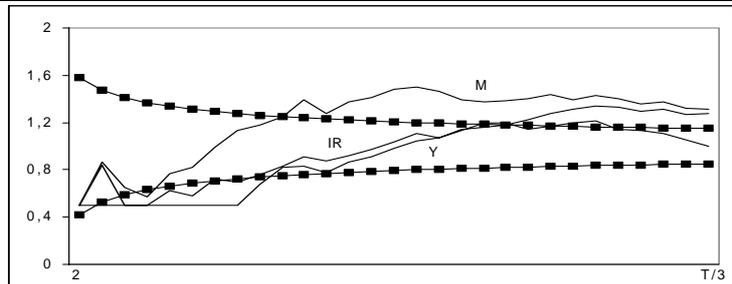' and in bold: Non-rejection values of the null hypothesis of a unit root at the 95% significance level.

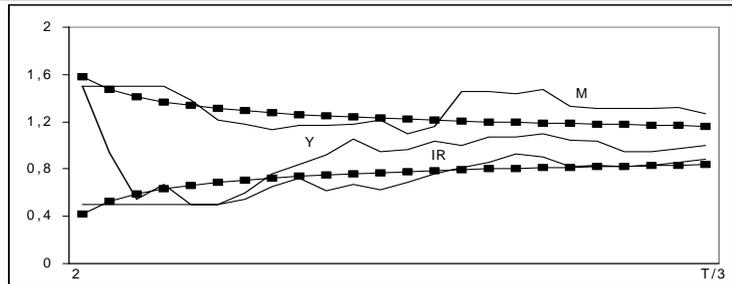**FIGURE 1**
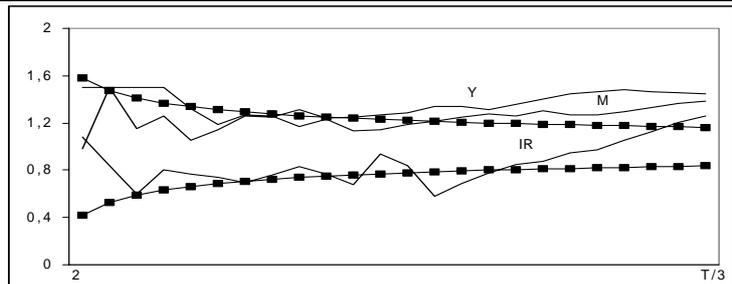
Whittle estimates of d based on Robinson (1995a)

CANADA
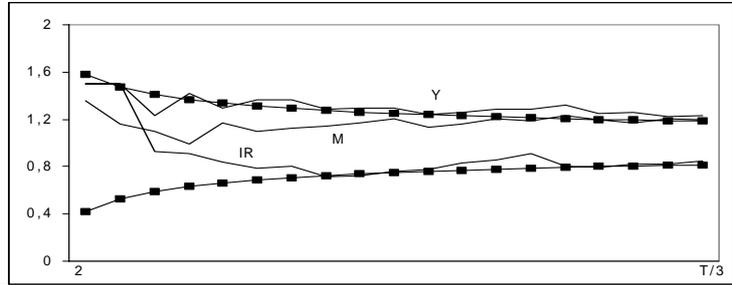
UNITED STATES
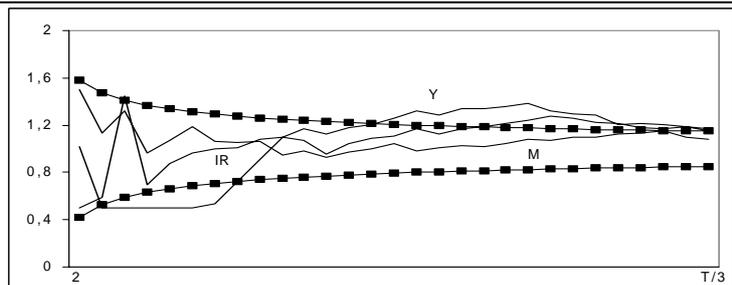
UNITED KINGDOM

JAPAN

GERMANY

The horizontal axe refers to the bandwidth parameter number m, while the vertical one corresponds to the estimated values of d

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TABLE 3

Critical values of the tests of Robinson (1994a) for testing the null
hypothesis of no cointegration against fractional cointegration

Residuals / T	50	100	136	138	178
White noise	-2.00	-1.91	-1.90	-1.90	-1.90
AR(1)	-2.85	-2.32	-2.21	-2.21	-2.12
AR(2)	-4.43	-2.86	-2.66	-2.65	-2.44
Seasonal AR(1)	-1.97	-1.91	-1.92	-1.92	-1.93
Seasonal AR(2)	-1.99	-1.93	-1.94	-1.94	-1.93

The null hypothesis consists of three independent I(1) processes which are non-cointegrated.
50,000 replications were used in each case. The nominal size is 5%.

TABLE 4

Estimated values of the coefficients in the cointegrating regression

MONEY $= \alpha + \beta_1$ INT. RATES $+ \beta_2$ INCOME

Country	α	β_1	β_2
CANADA	-1.835 (-43.23)	-0.053 (-19.12)	1.167 (51.38)
UNITED STATES	0.475 (4.66)	-0.024 (-13.22)	0.499 (22.19)
UNITED KINGDOM	6.047 (112.42)	-0.024 (-6.35)	-0.289 (-5.42)
JAPAN	-6.956 (-8.31)	-0.055 (-7.24)	0.902 (9.197)
GERMANY	4.690 (64.98)	-0.034 (-7.88)	1.746 (54.64)

The estimation was carried out using Ordinary Least Squares. t-values in parenthesis.

TABLE 5

Testing the null of no cointegration against fractional cointegration on the
estimated residuals from the cointegrating regression.

Country/ u_t	White noise	AR(1)	AR(2)	Seasonal AR(1)	Seasonal AR(2)
CANADA	**-2.02'**	**-2.38'**	-0.77	**-1.93'**	-1.43
UNITED STATES	**-4.74'**	**-2.80'**	-1.29	**-2.53'**	-1.61
UNITED KINGDOM	**-2.18'**	**-2.27'**	-0.02	0.65	1.88
JAPAN	0.04	0.42	-0.31	1.50	2.30
GERMANY	**-4.59'**	-1.24	-1.64	-1.73	-1.79

' and in bold: Non-rejection values of the null hypothesis of no cointegration against fractional
cointegration at the 5% significance level. We test H_o (3) against (8) in (7) with d =1.

FIGURE 2

Estimates of d based on Whittle (Robinson, 1995) on the estimated
residuals from the cointegrating regression.